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The Asian Option

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Outline

1. Introduction
2. Foundation of Asian option
3. Methods to pricing Asian option
4. Results and Comparison

Introduction

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Background

- Importance:
  - Risk Management
  - Investment Strategies
  - Investment decisions
- Path Dependency Issue
  - Traditional model like Black-Scholes can’t directly applied due to the average feature.
- **Two methods:**
  - **Basic Monte Carlo simulation**
  - **Enhanced version using the Control Variates techniques**

Foundation of Asian Option

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- ***An Asian option is a type of financial option whose payoff depends on average price of the underlying asset over a specific period.***
- Benefit of Path dependency: Reduced Volatility Impact
- Examine the better model:
    - Comparing the accuracy, efficiency and adaptability of two methods
    - Evaluating the convergence of estimates of option price, computation time, and se
    - Test whether correlation coefficient affects the effectiveness of variance reduction

Method 1-

Basic Monte Carlo Simulation Approach

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1. **Model setup**
    a. **asset price S(t); r is the risk-free rate; σ is the volatility; W(t) is a standard Brownian motion under risk-neutral measure Q,;**
2. **Discretization for Simulation**
    a. **Time increment** $\Delta t = \frac{T}{t}$
    b. **discretized version of the asset price path** $S(t_{i+1}) = S(t_i)e^{((r-\frac{1}{2}\sigma^2)\Delta t+\sigma\sqrt{\Delta t}Z_i)}$
3. **Simulation of Average Prices**
    a. **arithmetic average of the asset price** $A = \frac{1}{n}\sum_{i=1}^{n} S(t_i)$.
4. **Monte Carlo Estimation**

Method 2-

Enhanced Monte Carlo with Control Variates

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1. Simulate paths for the underlying asset price using geometric Brownian motion.

2. Calculate the arithmetic average for each path

3. Calculate the payoffs for both the arithmetic average (actual payoff) and the geometric average (control variate). Calculate the payoff using the geometric average of the asset prices as a control variate.

4. Compute beta using the covariance between the arithmetic and geometric payoffs and the variance of the geometric payoffs.

5. Average the adjusted payoffs and discount back

Analysis:

- Assessing the Convergence and Accuracy of two methods by varying M and n

- Evaluating the Impact of Parameters such as Volatility σ, Risk-Free Rate r, and Maturity T on the Option Price

- Comparison between basic Monte Carlo simulation and an enhanced version under different replications

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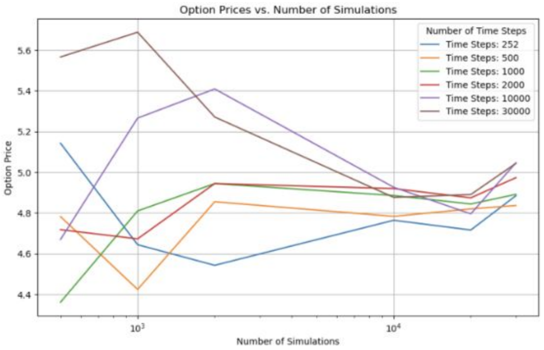
Results

#1- Assessing the Convergence and Accuracy of the Simulations by Varying M and n:

```
results_p = pd.DataFrame(np.array(price_simulation).reshape((6,
results_p
```

	500	1000	2000	10000	20000	30000
252	5.142281	4.644185	4.542999	4.764185	4.716312	4.884772
500	4.781955	4.424472	4.855075	4.783325	4.820062	4.836738
1000	4.361778	4.810042	4.944360	4.885688	4.844012	4.891433
2000	4.718035	4.672857	4.944437	4.919785	4.874339	4.973256
10000	4.670074	5.265812	5.408878	4.926131	4.795679	5.046402
30000	5.565791	5.687507	5.271123	4.876523	4.890530	5.043582



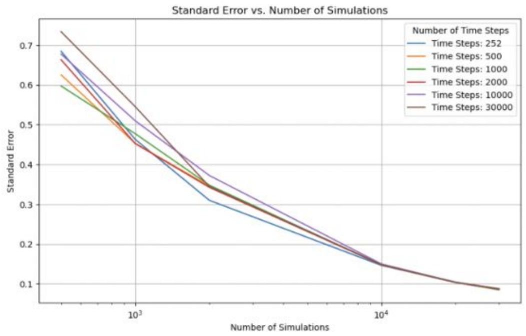
As M increases: Price converges
As N increases: Price increases

Prices Table and Plot

As M increases: Error converges to 0
As N increases,: little change on Error

```
results_s=pd.DataFrame(np.array(Stand_Error).reshape((6,6)), cc
results_s
```

	500	1000	2000	10000	20000	30000
252	0.684492	0.463050	0.309850	0.146277	0.103245	0.085640
500	0.625229	0.453060	0.345277	0.147645	0.103639	0.084748
1000	0.597187	0.477372	0.347859	0.147660	0.103436	0.085421
2000	0.663155	0.452524	0.342479	0.147720	0.103727	0.086875
10000	0.677352	0.509830	0.372245	0.149759	0.103578	0.087794
30000	0.733741	0.545271	0.344751	0.148182	0.104067	0.086944



Standard Errors Table and Plot

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Results

#2：Impact of Parameters Such as Volatility σ, Risk-Free Rate r, and Maturity T on the Option Price：

```
Volatility: 0.1, Estimated Call Price: 0.8816
Volatility: 0.2, Estimated Call Price: 7.5863
Volatility: 0.3, Estimated Call Price: 16.9625
Volatility: 0.4, Estimated Call Price: 27.1696
Risk-Free Rate: 0.01, Estimated Call Price: 3.6670
Risk-Free Rate: 0.05, Estimated Call Price: 5.3201
Risk-Free Rate: 0.1, Estimated Call Price: 8.0913
Maturity: 0.5, Estimated Call Price: 1.0764
Maturity: 1.0, Estimated Call Price: 4.4365
Maturity: 2.0, Estimated Call Price: 12.4371
```


```
Volatility: 0.1, Estimated Call Price: 0.8816
Volatility: 0.2, Estimated Call Price: 7.5863
Volatility: 0.3, Estimated Call Price: 16.9625
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Maturity: 0.5, Estimated Call Price: 1.0764
Maturity: 1.0, Estimated Call Price: 4.4365
Maturity: 2.0, Estimated Call Price: 12.4371
```


- **Higher volatility levels result in greater returns and risks associated with options.**
- **As r increases, the discount factor decreases, which raises the present value of expected payoffs.**
- **Longer maturity makes price more fluctuate over time.**

Results

#3: Comparison between basic Monte Carlo simulation and an enhanced version under different replications:

Replications	Basic Price	Basic Std Error	Basic Time	Enhanced Price	Enhanced Std Error	Enhanced Time	Correlation
1000	4.4245	0.4531	0.0303s	4.8726	0.0187	0.0340s	0.9993
10000	4.7833	0.1476	0.2535s	5.045	0.0062	0.6538s	0.9992
100000	4.8987	0.0471	3.1667s	4.8741	0.002	5.7862s	0.9991
500000	4.9121	0.0211	13.4660s	4.8502	0.0009	29.3288s	0.9991
1000000	4.8767	0.0149	26.9068s	4.8684	0.0006	56.5744s	0.9991

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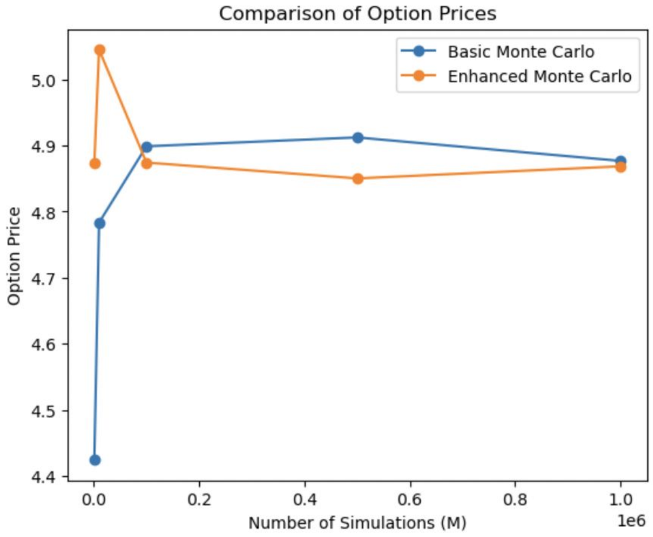
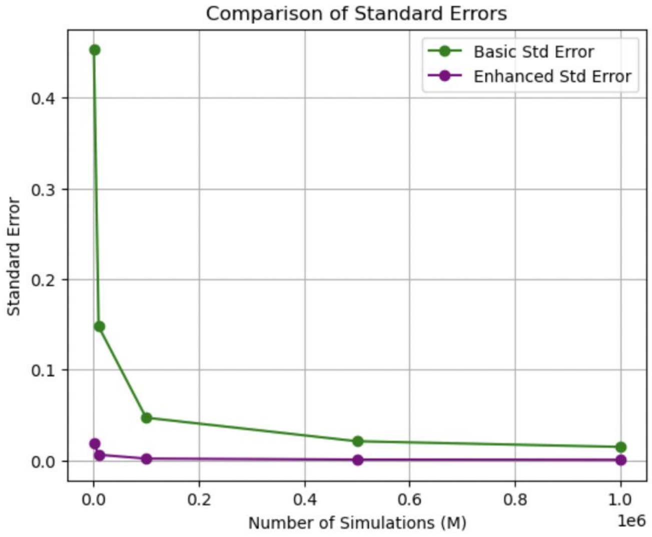
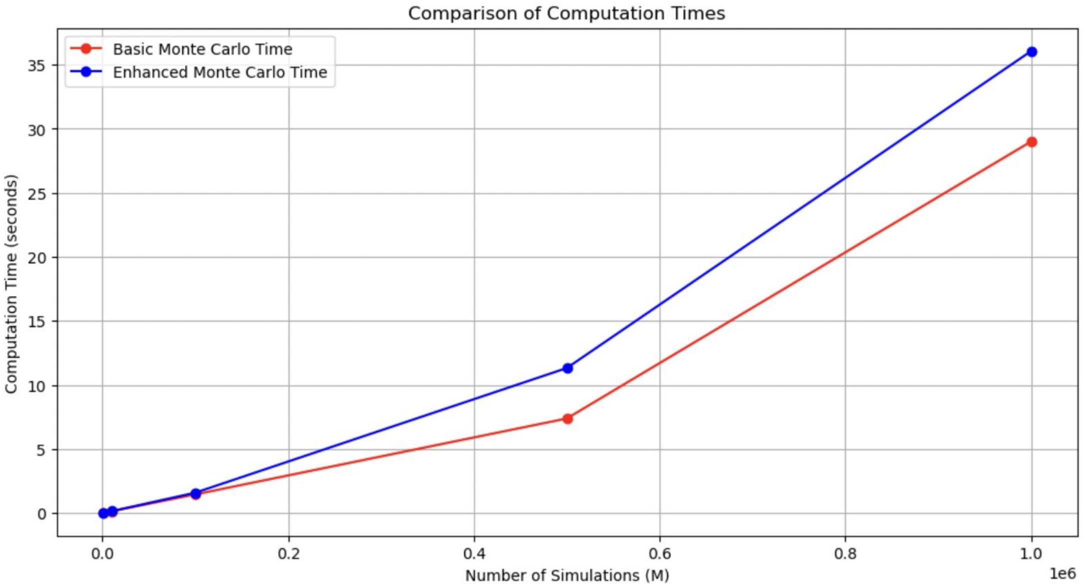
Comparison of Prices and Standard Error





- The estimated prices converge toward a more consistent value for both methods
- The enhanced method is more effective because it consistently reduces variance with control variates.

Comparison of Computation times



- Basic Monte Carlo is faster for scenarios with a high demand for speed, regardless of simulation size.
- Enhanced method may be worth the additional computation time

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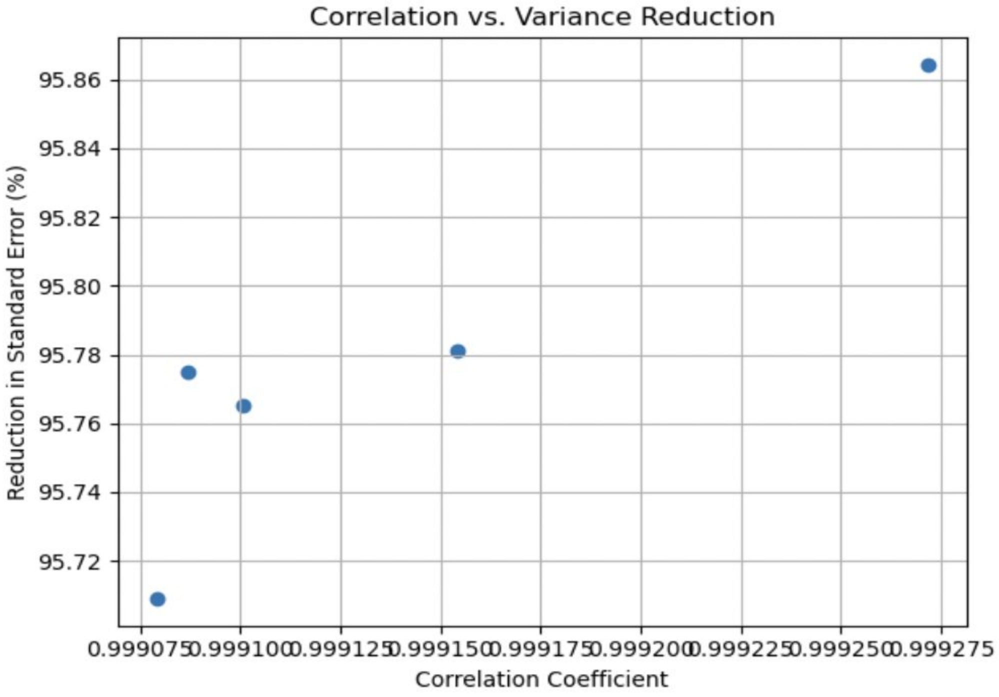
Impact of Correlation Coefficients



- High correlation coefficients substantially reduce the variance in the pricing estimates.
- Given such a huge efficiency, fewer simulations might be needed to achieve a certain accuracy.
- Control variates is a powerful tool for improving financial model precision because of this strong correlation.

Conclusion

- **Both methods show price convergence and less variability with increasing simulations.**
- **The enhanced method, which utilizes control variates, consistently provides more accurate and stable estimates.**
- **Due to the relation between correlation coefficient and significant reduction in variance, it is suitable for scenarios requiring high precision.**
- **The basic method, however, is preferred when calculations need to be performed quickly with moderate accuracy.**

Reference

1. Glasserman, Paul. “Monte Carlo Methods in Financial Engineering.” Https://Www.Bauer.Uh.Edu/Spirrong/Monte_Carlo_Methods_In_Financial_Enginee.Pdf.

2. Avellaneda , M, et al. STEVE_SHREVESTOCHASTIC_CALCUL...., cms.dm.uba.ar/academico/materias/2docuat2016/analisis_cuantitativo_en_finanzas/Steve_ShreveStochastic_Calculus_for_Finance_II.pdf.