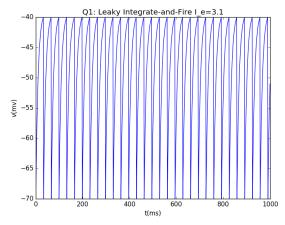
1. Simulate an integrate and fire model

The purpose in this question is to simulate and implement the leaky integrate and fire model. This model's formula is below:

$$\tau_m \frac{dV}{dt} = E_L - V + R_m I_E \tag{1}$$



We simulate this model for 1s, and as you can see in the figure above, the simulated neuron produces spikes or action potentials repeatedly when $I_E = 3.1$.

Related to question 1

1. Compute analytically the minimum current I_E From the equation (1) we can see that the value where V stops changing, is

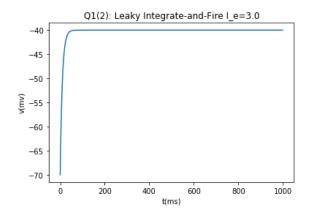
$$\bar{V} = E_L + R_m I_E \tag{2}$$

Hence, if $ar{V} > V_{threshold}$ the neuron will generate spikes. Therefore, the $I_{threshold}$ should be

$$I_{threshold} = \frac{V_{threshold} - E_L}{R_m} \tag{3}$$

Substituting the parameters in Question 1 to the equation (3), and then the result is $I_{threshold}$ = 3.0. So if $I_E > I_{threshold}$ (the minimum current $I_{minimum}$ = 3.1), the neuron will produce spikes.

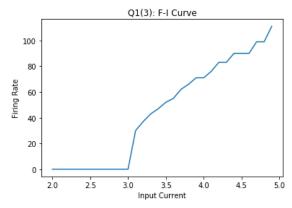
2. Simulate the neuron when $I_E = 3.0$



The figure shows the changing voltage of the neuron when I_E equals 3.0. Other parameters are as same as those in question 1. As shown in the figure above, there is no spikes when I_E = 3.0. In other words, the voltage have never reached V_T .

3. F-I curve

Obviously we just try to change the value of I_E from 2 [nA] to 5 [nA] in steps of 0.1 [nA], and count the number of spikes. As shown in the following figure, there is no firing until the input current is larger than 3.0.



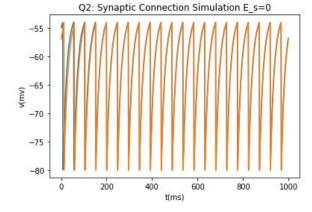
2. Simulate synaptic connections (simple synapse model)

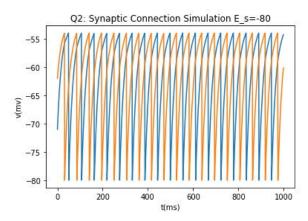
The neurons are coupled through synaptic currents, so these two neurons should satisfy the following equations

$$\tau_m \frac{dV}{dt} = E_L - V_1 + R_m I_E + R_m \bar{g}_S S_2 (E_S - V_1)$$

$$\tau_m \frac{dS_2}{dt} = -S_2$$

$$\begin{cases} V_1 = V_r, S_1 -> S_1 + P & V_1 > V_t \\ V_2 = V_r, S_2 -> S_2 + P & V_2 > V_t \end{cases}$$





The excitatory synaptic connections are simulated in the left figure. As we can see from this graph, the changing voltage of those two neurons are similar through excitatory synaptic connections. However, in the right figure, the variation of the neurons' voltage are inhibitory to each other. In other words, the inhibitory synapse makes a postsynaptic neuron less likely to generate an action potential.