1. $E(W_n) = 94\% \cdot 21746 + 5\% \cdot 19046 + 1\% \cdot 16546 = 21559$

$E(W_p) = 99\% \cdot 21584 + 1\% \cdot 19084 = 21559$

$E(W_f) = 1 \cdot 21559 = 21559$

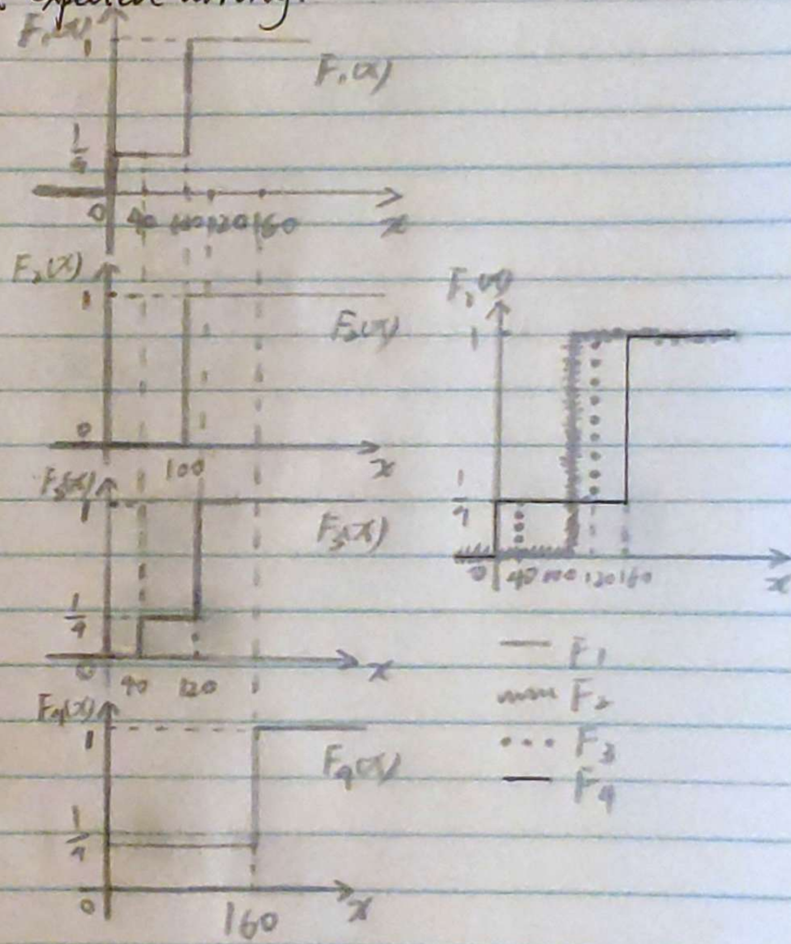
All 3 contracts give the same expected wealth.

$E(u(W_n)) = 94\% \cdot \sqrt{21746} + 5\% \cdot \sqrt{19046} + 1\% \cdot \sqrt{16546} \approx 146.80401$

$E(u(W_p)) = 99\% \cdot \sqrt{21584} + 1\% \cdot \sqrt{19084} \approx 146.82724$

$E(u(W_f)) = \sqrt{21559} \approx 146.82983$

Full coverage gives the highest expected utility.



2. (a)

$$F_1(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{4} & \text{if } 0 \le x < 100 \\ 1 & \text{if } x \ge 100 \end{cases}$$

$$F_2(x) = \begin{cases} 0 & \text{if } x < 100 \\ 1 & \text{if } x \ge 100 \end{cases}$$

$$F_3(x) = \begin{cases} 0 & \text{if } x < 40 \\ \frac{1}{4} & \text{if } 40 \le x < 120 \\ 1 & \text{if } x \ge 120 \end{cases}$$

$$F_4(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{4} & \text{if } 0 \le x < 160 \\ 1 & \text{if } x \ge 160 \end{cases}$$

(b) $\because F_2 \le F_1, \forall x \in \mathbb{R}$

$\therefore F_2$ FOSD F_1 by definition

$F_3 \le F_1, \forall x \in \mathbb{R}$

$\therefore F_3$ FOSD F_1 by definition

$F_4 \le F_1, \forall x \in \mathbb{R}$

$\therefore F_4$ FOSD F_1 by definition.

Thus, F_1 is dominated by the other 3 c.d.f's in the sense of FOSD

(c). $E_1[X] = \frac{1}{4} \cdot 0 + \frac{3}{4} \cdot 100 = 75$

$E_2[X] = 100$

$E_3[X] = \frac{1}{4} \cdot 40 + \frac{3}{4} \cdot 120 = 10 + 90 = 100$

$E_4[X] = \frac{1}{4} \cdot 0 + \frac{3}{4} \cdot 160 = 120$

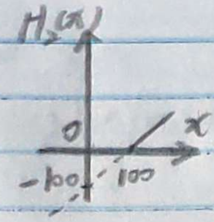
$E_4[X] > E_3[X] = E_2[X]$.

For $i=2$,

if $x<100$, $H_2(x)=\int_{-\infty}^{x} 0\,dy=0$.

if $x\ge 100$, $H_2(x)=\int_{100}^{x} 1\cdot dy = x-100$

$\therefore H_2(x)=\begin{cases} 0 & \text{if } x<100 \\ x-100 & \text{if } x\ge 100 \end{cases}$



For $i=3$,

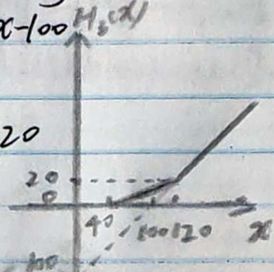
if $x<40$, $H_2(x)=0$

if $40\le x<120$, $H_3(x)=\int_{40}^{x}\frac{1}{4}dy=\frac{1}{4}(x-40)$

if $x\ge 120$, $H_3(x)=\int_{40}^{120}\frac{1}{4}dy+\int_{120}^{x}1\cdot dy$

$=\frac{80}{4}+x-120=x-100$

$\therefore H_3(x)=\begin{cases} 0 & \text{if } x<40 \\ \frac{1}{4}(x-40) & \text{if } 40\le x<120 \\ x-100 & \text{if } x\ge 120 \end{cases}$



$\forall x\in\mathbb{R}$, $H_3(x)\ge H_2(x)$.

Thus, F4 cannot be compared, and F_2 SOSD F_3.

3. Let a_1 be the quantity of bond, a_2 be the quantity of stock.

$w_0=a_1+qa_2 \Rightarrow a_1=w_0-qa_2$

In the state of bad economy,

$u(a_1+0)=u(a_1)=u(w_0-qa_2)=(w_0-qa_2)-0.05(w_0-qa_2)^2$

$=w_0-qa_2-0.05(w_0^2+q^2a_2^2-2w_0qa_2)$

$=w_0-qa_2-0.05w_0^2-0.05q^2a_2^2+0.1w_0qa_2$

In the state of good economy

$u(a_1+2a_2)=u(w_0-qa_2+2a_2)=u(w_0+(2-q)a_2)$

$=w_0+(2-q)a_2-0.05(w_0^2+(2-q)^2a_2^2-2w_0(2-q)a_2)$

$=w_0+(2-q)a_2-0.05w_0^2-0.05(2-q)^2a_2^2-0.1w_0(2-q)a_2$

$u(a_1)+u(a_1+2a_2)=2w_0+(2-2q)a_2-2\times 0.05w_0^2-0.05a_2^2[q^2+(2-q)^2]+0.1w_0a_2[q-(2-q)]$

The expected utility:

$E[U]=w_0+(1-q)a_2-0.05w_0^2-\frac{0.05}{2}a_2^2(q^2+q^2-4q+4)+0.1w_0a_2(q-2+q)\times\frac{1}{2}$

$=w_0+(1-q)a_2-0.05w_0^2-0.05a_2^2\cancel{\frac{2q^2-4q+4}{2}}+0.1w_0a_2(\frac{2q-2}{2})$

$=w_0+(1-q)a_2-0.05w_0^2-0.05a_2^2(q^2-2q+2)+0.1w_0a_2(q-1)$

$\frac{\partial E[U]}{\partial a_2}=(1-q)-0.1w_0(q-1)=(1-q))$

$$\frac{\partial E[U]}{\partial a_2} = (1-q) - 0.1 a_2 (q^2 - 2q + 2) + 0.1 W_0 (q-1) = 0 \quad \text{FOC.}$$

$$\Rightarrow 0.1 a_2 (q^2 - 2q + 2) = (1-q) + 0.1 W_0 (q-1) = (1-q) - 0.1 W_0 (1-q) = (1-q)[1 - 0.1 W_0]$$

$$\Rightarrow a_2 (q^2 - 2q + 2) = (1-q)(10 - W_0)$$

$$\Rightarrow a_2 = \frac{(1-q)(10-W_0)}{(1-q)^2 + 1}.$$

(b) $a_2 = \frac{(1-q)(10-W_0)}{(1-q)^2+1}.$

Assume $q \in (0,1)$, $W_0 < 10$.

$(1-q)^2 + 1 \geq 1 > 0.$

$\therefore a_2 > 0.$

$$\frac{\partial a_2}{\partial q} = \frac{1}{[(1-q)^2+1]^2} \{(10-W_0)\cdot(-1)[(1-q)^2+1] - (1-q)(10-W_0)\cdot 2(q-1)\}$$

$$= \frac{1}{[(1-q)^2+1]^2} \{-(10-W_0)[(1-q)^2+1] + (1-q)(10-W_0)(1-q)\cdot 2\}$$

$$= \frac{(10-W_0)}{[(1-q)^2+1]^2} \{-[q^2-2q+2] + 2(1-q)^2]$$

$$= \frac{10-W_0}{[(1-q)^2+1]^2} \{-[(1-q)^2+1] + 2(1-q)^2]$$

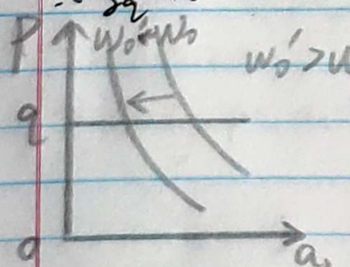
$$= \frac{10-W_0}{[(1-q)^2+1]^2} [-(1-q)^2 - 1 + 2(1-q)^2]$$

$$= \frac{10-W_0}{[(1-q)^2+1]^2} [(1-q)^2 - 1]$$

$\because 0 < q < 1 \quad \therefore 1-q < 1 \Rightarrow (1-q)^2 < 1 \Rightarrow (1-q)^2 - 1 < 0.$

$\because 10 - W_0 > 0, \quad [(1-q)^2+1]^2 \geq 1^2 = 1 > 0$

$\therefore \frac{\partial a_2}{\partial q} < 0.$



In this diagram, price of stock is a variable, (P)
q is a constant given by the question,
W_0 & W_0' are wealth.

Obviously, from the expression of a_1 we can see $W_0 \uparrow$, $10 - W_0 \downarrow$, $a_2 \downarrow$.
In other words, increase in income leads to decrease in a_1.
Thus stock is NOT a normal good for this investor.

3. (c). $a_1 = w_0 - q a_2$

$$= w_0 - \frac{q(1-q)(10-w_0)}{(1-q)^2+1}$$

By $a_2 = \frac{(1-q)(10-w_0)}{(1-q)^2+1}$, we know that if $q \geq 1$, $a_2 \leq 0$.

If $q \geq 1$ and short sales are not allowed, all of w_0 will be ~~spend~~ spent on bond.

4. (a).

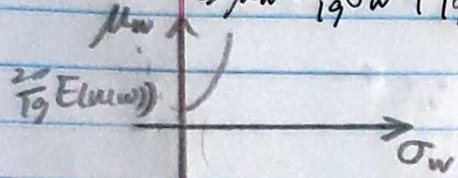
$u(w) = w - 0.05w^2$

$E(u(w)) = E(w) - 0.05E(w^2)$

$= \mu_w - 0.05(\sigma_w^2 + \mu_w^2)$

$= 0.95\mu_w - 0.05\sigma_w^2.$

$\Rightarrow \mu_w = \frac{1}{19}\sigma_w^2 + \frac{20}{19}E(u(w))$



(b).

$\mu = \frac{1}{2}\cdot 0 + \frac{1}{2}\cdot\frac{2}{q} = \frac{1}{q}$

$\sigma^2 = \frac{1}{2}(0-\frac{1}{q})^2 + \frac{1}{2}(\frac{2}{q}-\frac{1}{q})^2$

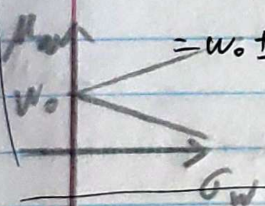
$= \frac{1}{q^2}.$

By $\mu_w = w_0 R \pm \frac{\mu - R}{\sigma}\sigma_w$,

we have:

$\mu_w = w_0\cdot 1 \pm \frac{\frac{1}{q}-1}{\frac{1}{q^2}}\sigma_w$

$= w_0 \pm q(1-q)\sigma_w$



X.

(b). ~~$w = w_0 + \frac{a(r-q)}{2}$~~

(b). $w = a_1\cdot 1 + a_2\cdot r$

$= a_1\cdot 1 + a_2\cdot q + a_2\cdot r - a_2\cdot q$

$= (a_1\cdot 1 + a_2\cdot q) + a_2(r-q)$

$= w_0 + a_2(r-q).$

$E[r] = \frac{1}{2}\cdot 0 + \frac{1}{2}\cdot 2 = 1$

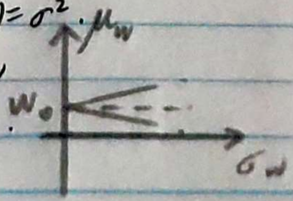
$var(r) = \frac{1}{2}\cdot(1-0)^2 + \frac{1}{2}(1-2)^2 = 1$

$\sigma_w = a_2^2\sigma^2 = a_2^2.$ plug in μ_w.

Let $\mu = E[r]$, $var(r) = \sigma^2$.

$\mu_w = w_0\cdot 1 \pm \frac{1-q}{1}\sigma_w$

$= w_0 \pm (1-q)\sigma_w.$



5. (a)

$$F(x)=\begin{cases}0 & \text{if } x<20\\ p_1 & \text{if } 20\le x<40\\ p_1+p_2 & \text{if } 40\le x<60\\ 1 & \text{if } x>60\end{cases} \qquad G(x)=\begin{cases}0 & \text{if } x<20\\ \frac{1}{4} & \text{if } 20\le x<40\\ \frac{3}{4} & \text{if } 40\le x<60\\ 1 & \text{if } x>60\end{cases}$$

(b). If G FOSD F, then $p_1 \geq \frac{1}{4}$, $p_1+p_2 \geq \frac{3}{4}$.

(c). $E^F[X] = 20p_1+40p_2+60p_3$ ①

$E^G[X] = 20\times\frac{1}{4}+40\times\frac{1}{2}+60\times\frac{1}{4} = 40$ ②

$\therefore 20p_1+40p_2+60p_3 = 40 \Leftrightarrow \cancel{p_1+p_2+p_3=4} \; 2p_1+4p_2+6p_3=4$

$p_1+p_2+p_3=1$ ③

By ②③, we have $p_1+3p_2+5p_3=3$ ④ and $2p_2+4p_3=2 \Rightarrow p_2+2p_3=1$ ⑤

$\begin{cases} p_1+2p_3=1 \\ p_1+p_2+p_3=1 \end{cases} \Rightarrow p_1=p_3$

If $x<20$, $H^F(x)=H^G(x)=0$

If $20\le x<40$, $H^F(x)=\int_{20}^{x} p_1 dy = p_1(x-20)$ $\quad H^G(x)=\frac{x-20}{4}$

If $40\le x<60$, $H^F(x)=\int_{20}^{40} p_1 dy+\int_{40}^{x}(p_1+p_2)dy = 20p_1+(p_1+p_2)(x-40)$

$H^G(x) = 5+\frac{3}{4}x-\frac{3}{4}\cdot 40 = \frac{3}{4}x-25$

If $x>60$, $H^F(x)=\int_{20}^{40} p_1 dy+\int_{40}^{60}(p_1+p_2)dy+\int_{60}^{x}1dy = 20p_1+20(p_1+p_2)+(x-60) = 20(2p_1+p_2)+(x-60)$

$= x-40$

$H^G(x) = 20\cdot\frac{1}{4}+20\cdot\frac{3}{4}+(x-60) = x-40 = H^F(x)$.

If $H^F(x)\geq H^G(x)$, then

C_1: $p_1(x-20)\geq\frac{1}{4}(x-20)$ if $20\le x<40 \Rightarrow p_1\geq\frac{1}{4}$

C_2: $20p_1+(p_1+p_2)(x-40)\geq\frac{3}{4}x-25$ if $40\le x<60 \Leftrightarrow 20p_1+(p_1+p_2)x-40p_1-40p_2\geq\frac{3}{4}x-25$

$\Rightarrow (p_1+p_2)x-20p_1-40p_2\geq\frac{3}{4}-25$.