1. (a)

The probability of minor damage:

$0.1 \times 0.25 = 0.025$

The value after minor damage:

$1{,}000K \times (1-10\%) = 1{,}000K \times 0.9 = 900K.$

The probability of moderate damage:

$0.1 \times 0.5 = 0.05$

The value after moderate damage:

$1{,}000K \times (1-25\%) = 1{,}000K \times 75\% = 750K$

The probability of major damage:

$0.1 \times 0.25 = 0.025$

The value after major damage:

$1{,}000K \times (1-50\%) = 1{,}000K \times 0.5 = 500K$

Full coverage:

$1{,}000K - 27.5K = 972.5K$, regardless of states

$w_f = 972{,}500$ is deterministic.

Partial coverage:

If major damage happens:

$w_p = 500{,}000 - 21{,}250 + 250{,}000 = 728{,}750$ with probability 0.025

If moderate damage happens:

$w_p = 750{,}000 - 21{,}250 + 250{,}000 = 978{,}750$ with probability 0.05

If minor damage happens:

$w_p = 900{,}000 - 21{,}250 + 100{,}000 = 978{,}750$ with probability 0.025.

If no accident happens:

$w_p = 1{,}000{,}000 - 21{,}250 + 0 = 978{,}750$ with probability $1 - 0.1 = 0.9$

$\therefore w_p = \{728{,}750;\ 978{,}750\}$ with probability distribution $P_p = (0.025, 0.975)$

Mandatory insurance:

If major damage happens:

$w_m = 500{,}000 - 10{,}000 + 100{,}000 = 590{,}000$ with probability 0.025.

If moderate damage happens:

$w_m = 750{,}000 - 10{,}000 + 100{,}000 = 840{,}000$ with probability 0.05

If minor damage happens:

$w_m = 900,000 - 10,000 + 100,000 = 990,000$ with probability 0.025

If no damage happens:

$w_m = 1,000000 - 10,000 + 0 = 990,000$ with probability 0.9

$\therefore w_m = \{590,000; 840,000; 990,000\}$ with probability distribution $P_m = (0.025, 0.05, 0.925)$

(b)

$$F_1(w) = \begin{cases} 0 & \text{if } w < 972,500 \\ 1 & \text{if } w \geq 972,500 \end{cases}$$

$$F_2(w) = \begin{cases} 0 & \text{if } w < 728,750 \\ 0.025 & \text{if } 728,750 \leq w < 978,750 \\ 1 & \text{if } w \geq 978,750 \end{cases}$$

$$F_3(w) = \begin{cases} 0 & \text{if } w < 590,000 \\ 0.025 & \text{if } 590,000 \leq w < 840,000 \\ 0.075 & \text{if } 840,000 \leq w < 990,000 \\ 1 & \text{if } w \geq 990,000 \end{cases}$$

(c).

Cannot.

$E[w_f] = 972,500$

$E[w_p] = 728,750 \times 0.025 + 978,750 \times 0.975$

$= 18,218.75 + 954,281.25$

$= 972,500$

$E[w_m] = 590,000 \times 0.025 + 840,000 \times 0.05 + 990,000 \times 0.925$

$= 14,750 + 42,000 + 915,750$

$= 972,500$

$\therefore E[w_f] = E[w_p] = E[w_m]$

According to the FOSD, cannot identify preferences over insurance plans.

(d)

Full coverage:

if $w < 972,500$, $H_1(w) = 0$

if $w \geq 972,500$, $H_1(w) = \int_{-\infty}^{w} F_1(y)dy = \int_{-\infty}^{972,500} 0\,dy + \int_{972,500}^{w} 1\,dy$

$= w - 972,500$

Partial Coverage:

if $w<728{,}750$, $H_2(w)=0$.

if $728{,}750\le w<978{,}750$

$$H_2(w)=\int_{-\infty}^{w}F_2(y)dy=\int_{-\infty}^{728{,}750}0dy+\int_{728{,}750}^{w}0.025dy=0.025(w-728{,}750)$$

if $w\ge 978{,}750$

$$H_2(w)=\int_{-\infty}^{w}F_2(y)dy=\int_{-\infty}^{728{,}750}0dy+\int_{728{,}750}^{978{,}750}0.025dy+\int_{978{,}750}^{w}1dy$$

$$=0+0.025(978{,}750-728{,}750)+(w-978{,}750)$$

$$=0.025\times 250{,}000+w-978{,}750=6{,}250+w-978{,}750$$

$$=w-972{,}500$$

Mandatory insurance:

if $w<590{,}000$

$H_3(w)=0$

if $590{,}000\le w<840{,}000$

$$H_3(w)=\int_{-\infty}^{w}F_3(y)dy=\int_{-\infty}^{590{,}000}0dy+\int_{590{,}000}^{w}0.025dy=0.025(w-590{,}000)$$

if $840{,}000\le w<990{,}000$

$$H_3(w)=\int_{-\infty}^{w}F_3(y)dy=\int_{-\infty}^{590{,}000}0dy+\int_{590{,}000}^{840{,}000}0.025dy+\int_{840{,}000}^{w}0.075dy$$

$$=0.025(840{,}000-590{,}000)+0.075(w-840{,}000)$$

$$=0.025\times 250{,}000+0.075w-0.075\times 840{,}000$$

$$=6250+0.075w-63{,}000$$

$$=0.075w-56{,}750$$

if $w\ge 990{,}000$

$$H_3(w)=\int_{-\infty}^{w}F_3(y)dy=\int_{-\infty}^{590{,}000}0dy+\int_{590{,}000}^{840{,}000}0.025dy+\int_{840{,}000}^{990{,}000}0.075dy+\int_{990{,}000}^{w}1dy$$

$$=0.025\times(840{,}000-590{,}000)+0.075\times(990{,}000-840{,}000)+(w-990{,}000)$$

$$=6250+0.075\times 150{,}000+w-990{,}000$$

$$=11{,}250+w-983{,}750$$

$$=w-972{,}500.$$

In all,

$$H_1(w)=\begin{cases}0 & ,\text{if } w<972{,}500\\ w-972{,}500 & \text{if } w\ge 972{,}500\end{cases}$$

$$H_2(w)=\begin{cases}0 & \text{if } w<728{,}750\\ 0.025(w-728{,}750) & \text{if } 728{,}750\le w<978{,}750\\ w-972{,}500 & \text{if } w\ge 978{,}750\end{cases}$$

$$H_3(w)=\begin{cases} 0 & \text{if } w<590{,}000 \\ 0.025(w-590{,}000) & \text{if } 590{,}000\le w<840{,}000 \\ 0.075w-56{,}750 & \text{if } 840{,}000\le w<990{,}000 \\ w-972{,}500 & \text{if } w\ge 990{,}000 \end{cases}$$

If $w<590{,}000$

$H_1(w)=H_2(w)=H_3(w)=0$

If $590{,}000\le w<728{,}750$

$H_1(w)=H_2(w)=0\le 0.025(w-590{,}000)=H_3(w)$

If $728{,}750\le w<840{,}000$

$H_1(w)=0\le 0.025(w-728{,}750)=H_2(w)<0.025(w-590{,}000)=H_3(w)$

If $840{,}000\le w<972{,}750$

$H_1(w)=0$

$H_2(w)=0.025(w-728{,}750)>0$

$H_3(w)=0.075w-56{,}750$

$H_3(w)-H_2(w)$

$=0.05w-56{,}750+0.025\times 728{,}750=0.05w-56750+18218.75=0.05w-38{,}531.25$

$\ge 42{,}000-38{,}531.25>0.$

If $972{,}500\le w<978{,}750$

$H_1(w)=w-972{,}500$

$H_2(w)=0.025(w-728{,}750)$

$H_3(w)=0.075w-56{,}750$

$H_2(w)-H_1(w)$

$=-0.975w-0.025\times 728{,}750+972{,}500=-0.975w-18{,}218.75+972{,}500$

$=-0.975w+954{,}281.25$

$>-0.975\times 978{,}750+954{,}281.25=954{,}281.25-954{,}281.25$

$=0.$

$H_3(w)-H_2(w)$

$=0.075w-56{,}750-0.025w+0.025\times 728{,}750=0.05w-56{,}750+18{,}218.75=0.05w-38{,}531.25$

$\ge 0.05\times 972{,}500-38{,}531.25=48625-38{,}531.25>0.$

If $978{,}750\le w<990{,}000$

$H_1(w)=w-972{,}500$

$H_2(w)=w-972{,}500=H_1(w)$

$H_3(w) = 0.075w - 56{,}750$

$H_3(w) - H_2(w) = H_3(w) - H_1(w)$

$= -0.925w + \cancel{915}, -56{,}750 + 972{,}500 = -0.925w + 915{,}750$

$> -0.925 \times 990{,}000 + 915{,}750 = -915{,}750 + 915{,}750$

$= 0.$

If $w \geq 990{,}000$

$H_1(w) = H_2(w) = H_3(w) = w - 972{,}500.$

$\therefore \forall w \in \mathbb{R},\ H_1(w) \leq H_2(w) \leq H_3(w)$

F_1 SOSD F_2, F_2 SOSD F_3, F_1 SOSD F_3.

Risk averse decision maker would prefer full coverage most.

2. (a)

Denote the wealth in "good" state as w_1.

Denote the wealth in "bad" state as w_2.

$w_1 = 1.37\alpha w_0 + 1.02(1-\alpha)w_0 = 1.37\alpha w_0 + 1.02w_0 - 1.02\alpha w_0$

$= 0.35\alpha w_0 + 1.02w_0$

$= (0.35\alpha + 1.02)w_0$

$w_2 = 1.01\alpha w_0 + 1.02(1-\alpha)w_0 = 1.01\alpha w_0 + 1.02w_0 - 1.02\alpha w_0$

$= -0.01\alpha w_0 + 1.02w_0$

$= (-0.01\alpha + 1.02)w_0.$

(b).

$E[w] = \frac{2}{3} \times w_1 + \frac{1}{3} \times w_2 = \frac{2}{3} \times (0.35\alpha + 1.02)w_0 + \frac{1}{3} \times (-0.01\alpha + 1.02)w_0$

$= w_0[\frac{2}{3} \times (0.35\alpha + 1.02) + \frac{1}{3} \times (-0.01\alpha + 1.02)] = w_0[\frac{2}{3} \times 0.35\alpha + \frac{2}{3} \times 1.02 + \frac{1}{3} \times (-0.01\alpha) + \frac{1}{3} \times 1.02]$

$= w_0[\frac{1}{3} \times 0.7\alpha + \frac{1}{3} \times (-0.01\alpha) + 1.02] = w_0[\frac{\alpha}{3}(0.7 - 0.01) + 1.02] = w_0[\frac{\alpha}{3} \times 0.69 + 1.02)$

$= (0.23\alpha + 1.02)w_0$

(c)

$E[U] = \frac{2}{3}u(w_1) + \frac{1}{3}u(w_2)$

$= \frac{2}{3} \cdot (1 - e^{-0.01w_1}) + \frac{1}{3}(1 - e^{-0.01w_2})$

$= \frac{2}{3} \cdot 1 - \frac{2}{3}e^{-0.01w_1} + \frac{1}{3} - \frac{1}{3}e^{-0.01w_2}$

$= 1 - \frac{2}{3}e^{-0.01w_1} - \frac{1}{3}e^{-0.01w_2}$

$= 1 - \frac{2}{3}e^{-0.01(0.35\alpha + 1.02)w_0} - \frac{1}{3}e^{-0.01(-0.01\alpha + 1.02)w_0}$

(d) $\max_{\alpha} E[U] = \max_{\alpha}\left(1-\frac{2}{3}e^{-0.01(0.35\alpha+1.02)w_0}-\frac{1}{3}e^{-0.01(-0.01\alpha+1.02)w_0}\right)$

FOC:

$$-\frac{2}{3}e^{-0.01(0.35\alpha+1.02)w_0}\times(-0.01w_0)\times 0.35-\frac{1}{3}e^{-0.01(-0.01\alpha+1.02)w_0}\times(-0.01w_0)\times(-0.01)=0$$

$$\Rightarrow \frac{2}{3}e^{-0.01(0.35\alpha+1.02)w_0}\times(0.01w_0)\times 0.35-\frac{1}{3}e^{-0.01(-0.01\alpha+1.02)w_0}\times(0.01w_0)\times(-0.01)=0$$

$$\Rightarrow 2e^{-0.01(0.35\alpha+1.02)w_0}\times 0.35+e^{-0.01(-0.01\alpha+1.02)w_0}\times(-0.01)=0$$

$$\Rightarrow 2\times 35\times e^{-0.01(0.35\alpha+1.02)w_0}+e^{-0.01(-0.01\alpha+1.02)w_0}\times(-1)=0$$

$$\Rightarrow 70e^{-0.01(0.35\alpha+1.02)w_0}=e^{-0.01(-0.01\alpha+1.02)w_0}$$

$$\Rightarrow 70e^{-0.01(0.35\alpha+1.02)w_0}\cdot e^{0.01(-0.01\alpha+1.02)w_0}=1$$

$$\Rightarrow 70e^{-0.01(0.35\alpha+1.02)w_0+0.01(-0.01\alpha+1.02)w_0}=1$$

$$\Rightarrow e^{-0.01\times 0.35\alpha w_0-0.01\times 1.02w_0+(0.01\times(-0.01)\times\alpha w_0)+0.01\times 1.02w_0}=\frac{1}{70}$$

$$\Rightarrow e^{-0.01\times 0.35\alpha w_0-0.01\times 0.01\alpha w_0}=\frac{1}{70}$$

$$\Rightarrow -0.01\times 0.36\alpha w_0=\ln\frac{1}{70}=-\ln 70$$

$$\Rightarrow 0.01\times 0.36\alpha w_0=\ln 70$$

$$\Rightarrow \alpha=\frac{\ln 70}{0.01\times 0.36w_0}=\frac{2500\ln 70}{9w_0}\approx\frac{1180.138\times\ln 70}{w_0}$$

Check SOC:

$$\frac{\partial^2 E[U]}{\partial\alpha^2}=-\frac{2}{3}e^{-0.01(0.35\alpha+1.02)w_0}\cdot(-0.01w_0)\cdot 0.35\cdot(-0.01w_0)\cdot(0.35)-\frac{1}{3}e^{-0.01(-0.01\alpha+1.02)w_0}\times(-0.01w_0)\times(-0.01)\times(-0.01w_0)\times(-0.01)$$

$$=-\frac{2}{3}e^{-0.01(0.35\alpha+1.02)w_0}\times[(-0.01w_0)\times 0.35]^2-\frac{1}{3}e^{-0.01(-0.01\alpha+1.02)w_0}[(-0.01w_0)\times(-0.01)]^2$$

$\because [(-0.01w_0)\times 0.35]^2\geq 0$, $[(-0.01w_0)\times(-0.01)]^2\geq 0$, $e^{-0.01(0.35\alpha+1.02)w_0}>0$, $e^{-0.01(-0.01\alpha+1.02)w_0}>0$,

$\therefore \frac{\partial^2 E[U]}{\partial\alpha^2}<0.$

In conclusion, $\alpha=\frac{\ln 70}{0.01\times 0.36w_0}=\frac{2500\ln 70}{9}\approx\frac{1180.138\times\ln 70}{w_0}$ maximize expected utility.

(e) As w_0 increases, α decreases. The more wealthy investor is, the less fraction of wealth will be spent on ~~sa~~ risky asset.

(f) $\alpha w_0=\frac{\ln 70}{0.01\times 0.36}=\frac{2500\ln 70}{9}\approx 1180.138$, is a constant.

αw_0 is independent of w_0.

However wealthy the investor is, the amount of money invested on risky asset doesn't change.