1. Denote X as a R.V. indicates where the hurricane lands at the east FL coastline.

X is a R.V. uni. dist. on the interval $[0, 447]$.

The p.d.f. of X is: $f(x) = \begin{cases} \frac{1}{447}, & \text{if } x \in \text{the coastline of east FL.} \\ 0, & \text{otherwise} \end{cases}$

Let the coastline of Miami starts at $a \in [0, 447]$ belonging to east FL. coastline.

$P(X \le a) = \int_{-\infty}^{a} f(x)dx = \int_{0}^{a} \frac{1}{447} dx = \frac{a}{447}$

$P(X \le a+9) = \int_{-\infty}^{a+9} f(x)dx = \int_{0}^{a+9} \frac{1}{447} dx = \frac{a+9}{447}$.

$\therefore$ P(The eye of the next hurricane that hits east Florida crosses Miami Beach)

$= P(a \le X \le a+9) = P(X \le a+9) - P(X \le a) = \frac{a+9}{447} - \frac{a}{447} = \frac{9}{447}$.

2. There are 3 possible actions m, p, f for me.

Denote minor accident as w_1, moderate accident as w_2, major accident as w_3, no accident as w_4.

The state space $\Omega = \{w_1, w_2, w_3, w_4\}$.

The following table represents consequences of each action:

	w_1	w_2	w_3	w_4
m	$22500-\pi_m$	$19800-\pi_m$	$17300-\pi_m$	$22500-\pi_m$
p	$22500-\pi_p$	$22500-\pi_p$	$20000-\pi_p$	$22500-\pi_p$
f	$22500-\pi_f$	$22500-\pi_f$	$22500-\pi_f$	$22500-\pi_f$

The set of all possible consequences is

$C = \{22500-\pi_m, 19800-\pi_m, 17300-\pi_m, 22500-\pi_p, 20000-\pi_p, 22500-\pi_f\}$.

The consequences:

$c(m) = (22500-\pi_m, 22500-\pi_m-5600+2900, 22500-\pi_m-8100+2900, 22500-\pi_m)$

$= (22500-\pi_m, 19800-\pi_m, 17300-\pi_m, 22500-\pi_m)$

$c(p) = (22500-\pi_p, 22500-\pi_p, \cancel{}\,22500-\pi_p-8100+5600, 22500-\pi_p)$

$= (22500-\pi_p, 22500-\pi_p, 20000-\pi_p, 22500-\pi_p)$

$c(f) = (22500-\pi_f, 22500-\pi_f, 22500-\pi_f, 22500-\pi_f)$.

3. (a) The state space is $\Omega = \{w_1, w_2, w_3, w_4\}$.
As defined above, w_1 is minor accident w/ prob. 20%,
w_2 is moderate accident w/ prob. 5%,
w_3 is major accident w/ prob. 1%,
w_4 is no accident w/ prob. 74%,
where $74\% = 1 - 20\% - 5\% - 1\%$.

(b) $E[\pi_m] = (20\% + 5\% + 1\%)(\pi_m - 2900) + 74\% \cdot \pi_m$
$= 26\%(\pi_m - 2900) + 74\% \pi_m$
$= \pi_m - 754$

$E[\pi_p] = 20\%(\pi_p - 2900) + (5\% + 1\%)(\pi_p - 5600) + 74\% \pi_p$
$= \pi_p - 20\% \cdot 2900 - 6\% \cdot 5600$
$= \pi_p - 916$

$E[\pi_f] = 20\%(\pi_f - 2900) + 5\%(\pi_f - 5600) + 1\%(\pi_f - 8100) + 74\% \cdot \pi_f$.
$= \pi_f - 20\% \cdot 2900 - 5\% \cdot 5600 - 1\% \cdot 8100$
$= \pi_f - 941$

$\because E[\pi_m] = E[\pi_p] = E[\pi_f] = 0$

$\therefore \pi_m = 754, \pi_p = 916, \pi_f = 941$.

(c) $22500 - \pi_m = 22500 - 754 = 21746$, $19800 - \pi_m = 19800 - 754 = 19046$
$17300 - \pi_m = 16546$ $22500 - \pi_p = 21584$ $20000 - \pi_p = 19084$
$22500 - \pi_f = 21559$.
Thus, the set of all possible consequences is
$C = \{21746, 19046, 16546, 21584, 19084, 21559\}$
The set of lotteries $\mathcal{L} = \{L_1, L_2, L_3\}$, where

~~$L_1 =$~~

~~94%~~	~~5%~~	~~0~~	~~0~~	~~0~~
~~21746~~	~~19046~~	~~16546~~	~~19084~~	~~21559~~

~~for the lottery m induces,~~

		~~99%~~	~~1%~~	~~0~~
~~21746~~	~~19046~~	~~16546~~	~~19084~~	~~21559~~

$L_1 =$

94%	5%	1%	0	0	0
21746	19046	16546	21584	19084	21559

for the lottery m induces,

$L_2 =$

0	0	0	99%	1%	0
21746	19046	16546	21584	19084	21559

for the lottery p induces,

$\dfrac{0}{21746}\Big|\dfrac{0}{19046}\Big|\dfrac{0}{16546}\Big|\dfrac{0}{21584}\Big|\dfrac{0}{19084}\Big|\dfrac{1}{21559}$ for the lottery f induces.

The c.d.f of L_1 is
$$F(w_m)=\begin{cases}0 & \text{if } w_m<16546\\ 1\% & \text{if } 16546\le w_m<19046\\ 6\% & \text{if } 19046\le w_m<21746\\ 1 & \text{if } w_m\geq 21746\end{cases}$$

The c.d.f. of L_2 is
$$F(w_p)=\begin{cases}0 & \text{if } w_p<19084\\ 1\% & \text{if } 19084\le w_p<21584\\ 1 & \text{if } w_p\geq 21584\end{cases}$$

The c.d.f. of L_3 is $F(w_f)=\begin{cases}0 & \text{if } w_f<21559\\ 1 & \text{if } w_f\geq 21559\end{cases}$

4. The set of consequences is $C=\{-300, 0, 120, 240, 300\}$
The $\mathcal{L}=\{L_1, L_2, L_3, L_4\}$, where

$L_1=\dfrac{\frac{1}{3}}{-300}\Big|\dfrac{0}{0}\Big|\dfrac{\frac{1}{3}}{120}\Big|\dfrac{0}{240}\Big|\dfrac{\frac{1}{3}}{300}$ for the lottery a_1 induces

$L_2=\dfrac{\frac{1}{3}}{-300}\Big|\dfrac{0}{0}\Big|\dfrac{\frac{1}{3}}{120}\Big|\dfrac{0}{240}\Big|\dfrac{\frac{1}{3}}{300}$ for the lottery a_2 induces

$L_3=\dfrac{0}{-300}\Big|\dfrac{\frac{2}{3}}{0}\Big|\dfrac{0}{120}\Big|\dfrac{\frac{1}{3}}{240}\Big|\dfrac{0}{300}$ for the lottery a_3 induces

$L_4=\dfrac{0}{-300}\Big|\dfrac{1}{0}\Big|\dfrac{0}{120}\Big|\dfrac{0}{240}\Big|\dfrac{0}{300}$ for the lottery a_4 induces

5. (a) Denote X as R.V. indicates the amount of time between 2 successive hurricane struck TX. coast.
Assume $X\sim Exp(\lambda)$
$\therefore E(X)=\frac{1}{\lambda}$.
On average, the amount of time between 2 successive hurricane struck TX. coast is
is: $\frac{64}{2022-1851+1}\approx 0.372\approx\frac{1}{3}$ year. In other words, $E(X)=3$. per year
$\cancel{\frac{1}{\lambda}=\frac{64}{172}\approx\frac{1}{3}}\Rightarrow\lambda=2.6875\approx 3$.

(b)
(i) $2017-2008-1=8$.

$\frac{2022-1851+1}{64} = 2.6875 \approx 3$.

In other words, $E(X) \approx 3$ years

$\therefore \lambda = \frac{1}{2.6875} \approx \frac{1}{3}$.

(b) (i) $2017 - 2008 - 1 = 8$.

$P(X>8) = \int_8^\infty \lambda e^{-\lambda x} dx = -e^{-\lambda x}\Big|_8^\infty = e^{-8\lambda}$

Plug in $\lambda = \frac{1}{2.6875} \approx \frac{1}{3}$, we get $P(X>8) \approx 0.1$

But since Ike and Harvey did hit TX and X is a continuous Random variable, the probability of $P(X=8) = 0$, since Ike hit in 2008 and Harvey hit in 2017.

(or, $P(X=9)$).

(ii) $P(X>2) = \int_2^\infty \lambda e^{-\lambda x} dx = e^{-\lambda x}\Big|_2^\infty = e^{-2\lambda}$

Plug in $\lambda = \frac{1}{2.6875} \approx \frac{1}{3}$, we get $P(X>2) \approx 0.5$.