Yu Xia's Answer for Problem Set 2

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I use a electronic version for this time, because of so many repeated notations, and also because it is so likely to make mistake in calculation.

1

(a)

$w_0 = q\bar{a} = 2 \times 5 + 1 \times 10 = \boxed{20}$

(b)

$R = \begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$

$r_{22} = 1$ $r_{21} = 3$

$q_2 = 1$ $q_1 = 2$

$\frac{r_{22}}{q_2} = 1$ $\frac{r_{21}}{q_1} = \frac{3}{2}$ $\frac{r_{22}}{q_2} - \frac{r_{21}}{q_1} = -\frac{1}{2}$

$r_{11} = 1$ $r_{12} = 2$

$\frac{r_{11}}{q_1} = \frac{1}{2}$ $\frac{r_{12}}{q_2} = 2$ $\frac{r_{11}}{q_1} - \frac{r_{12}}{q_2} = -\frac{3}{2}$

$\therefore \frac{\frac{r_{22}}{q_2} - \frac{r_{21}}{q_1}}{\frac{r_{11}}{q_1} - \frac{r_{12}}{q_2}} > 0$

(c)

$w_1 = 1 \cdot a_1 + 2 \cdot a_2 = a_1 + 2a_2$ for space 1,

$w_2 = 3 \cdot a_1 + 1 \cdot a_2 = 3a_1 + a_2$ for space 2.

$\therefore w = (w_1, w_2) = \boxed{(a_1 + 2a_2, 3a_1 + a_2)}$

(d)

It's possible.

$\det R = \det\begin{vmatrix}1 & 2\\3 & 1\end{vmatrix} = 1 - 2\times 3 \neq 0$

(e)

The investor's problem, as a portfolio problem, is:

$$\begin{aligned}&\max_{(a_1,a_2)} \frac{2}{5}u\left(w_1\left(a\right)\right)+\frac{3}{5}u\left(w_2\left(a\right)\right)\\ &\text{subject to}\\ &2a_1+a_2=20.\end{aligned} \implies \begin{aligned}&\max_{(a_1,a_2)} \frac{2}{5}\sqrt{a_1+2a_2}+\frac{3}{5}\sqrt{3a_1+a_2}\\ &\text{subject to}\\ &2a_1+a_2=20.\end{aligned}$$

Alternaltively, by (c) we haven

$a_1 = -\frac{1}{5}w_1+\frac{2}{5}w_2,\ a_2 = \frac{3}{5}w_1-\frac{1}{5}w_2$

The constraint becomes:

$2\times\left(-\frac{1}{5}w_1+\frac{2}{5}w_2\right)+\left(\frac{3}{5}w_1-\frac{1}{5}w_2\right)=20$

$\implies -\frac{2}{5}w_1+\frac{4}{5}w_2+\frac{3}{5}w_1-\frac{1}{5}w_2=20$

$\implies \frac{w_1}{5}+\frac{3}{5}w_2=20$

$\implies w_1+3w_2=100$

The investor's problem, as a state contingent wealth problem, is:

$$\begin{aligned}&\max_{(a_1,a_2)} \frac{2}{5}\sqrt{w_1}+\frac{3}{5}\sqrt{w_2}\\ &\text{subject to}\\ &w_1+3w_2=100.\end{aligned}$$

(f)

Solve the state contingent wealth problem:

$MU_1 = \frac{2}{5}\cdot\frac{1}{2}w_1^{-\frac{1}{2}} = \frac{1}{5}\cdot\frac{1}{\sqrt{w_1}}$

$MU_2 = \frac{3}{5}\cdot\frac{1}{2}w_2^{-\frac{1}{2}} = \frac{3}{10}\cdot\frac{1}{\sqrt{w_2}}$

$$\frac{MU_1}{MU_2} = \frac{\dfrac{1}{5\sqrt{w_1}}}{\dfrac{3}{10\sqrt{w_2}}} = \frac{\dfrac{2}{\sqrt{w_1}}}{\dfrac{3}{\sqrt{w_2}}} = \frac{2\sqrt{w_2}}{3\sqrt{w_1}}$$

On the other hand,

$$\frac{MU_1}{MU_2} = \frac{1}{3} \implies \frac{2\sqrt{w_2}}{3\sqrt{w_1}} = \frac{1}{3} \implies \frac{2\sqrt{w_2}}{\sqrt{w_1}} = 1 \implies 2\sqrt{w_2} = \sqrt{w_1} \implies 4w_2 = w_1$$

$\because w_1 + 3w_2 = 100$

$\therefore 7w_2 = 100$

$$\boxed{\begin{cases} w_1 = \dfrac{400}{7} \\ w_2 = \dfrac{100}{7} \end{cases}}$$

$$a_1 = -\frac{1}{5}w_1 + \frac{2}{5}w_2 = -\frac{1}{5} \cdot \frac{400}{7} + \frac{2}{5} \cdot \frac{100}{7} = -\frac{-400+200}{5 \cdot 7} = -\frac{200}{5 \cdot 7} = \boxed{-\frac{40}{7}}$$

$$a_2 = 20 - 2a_1 = 20 - \left(-\frac{80}{7}\right) = 20 + \frac{80}{7} = \frac{140+80}{7} = \boxed{\frac{220}{7}}$$

(g)

She doesn't hedge all risks completely. As we see, $w_1 \neq w_2$

2

(a)

$w_0 = q\bar{a} = 2 \times 10 + 1 \times 10 = \boxed{30}$

(b)

$w_1 = 5 \cdot a_1 + 1 \cdot a_2 = 5a_1 + a_2$ for space 1,

$w_2 = (-1) \cdot a_1 + 1 \cdot a_2 = -a_1 + a_2$ for space 2,

$w_3 = 2 \cdot a_1 + 1 \cdot a_2 = 2a_1 + a_2$ for space 3.

$\therefore w = (w_1, w_2, w_3) = \boxed{(5a_1 + a_2, -a_1 + a_2, 2a_1 + a_2)}$

(c)

Possible.

Because $w_3 = 2a_1 + a_2 = 30$, as the budget constraint tells us.

Now that there are 2 equations, 2 unknowns (a_1, a_2) left.

$\det R = \det\begin{vmatrix} 5 & 1 \\ -1 & 1 \end{vmatrix} = 5 - (-1) \neq 0$

(d)

$$\max_{(a_1,a_2)} \frac{2}{3}u\left(w_1\left(a\right)\right) + \frac{1}{6}u\left(w_2\left(a\right)\right) + \frac{1}{6}u\left(w_3\left(a\right)\right)$$

subject to

$$2a_1 + a_2 = 30.$$

$\Longrightarrow$

$$\max_{(a_1,a_2)} \frac{2}{3}\ln\left(5a_1 + a_2\right) + \frac{1}{6}\ln\left(-a_1 + a_2\right) + \frac{1}{6}\ln\left(2a_1 + a_2\right)$$

subject to

$$2a_1 + a_2 = 30.$$

$MU_1 = \frac{2}{3}\cdot\frac{1}{5a_1+a_2}\cdot 5 + \frac{1}{6}\cdot\frac{1}{-a_1+a_2}\cdot(-1) + \frac{1}{6}\cdot\frac{1}{2a_1+a_2}\cdot 2 = \frac{10}{3}\cdot\frac{1}{5a_1+a_2} + \frac{1}{6}\cdot\frac{1}{a_1-a_2} + \frac{1}{3}\cdot\frac{1}{2a_1+a_2}$

$MU_2 = \frac{2}{3}\cdot\frac{1}{5a_1+a_2}\cdot 1 + \frac{1}{6}\cdot\frac{1}{-a_1+a_2}\cdot 1 + \frac{1}{6}\cdot\frac{1}{2a_1+a_2}\cdot 1 = \frac{2}{3}\cdot\frac{1}{5a_1+a_2} + \frac{1}{6}\cdot\frac{1}{-a_1+a_2} + \frac{1}{6}\cdot\frac{1}{2a_1+a_2}$

$\frac{MU_1}{MU_2} = \dfrac{\frac{10}{3}\cdot\frac{1}{5a_1+a_2} + \frac{1}{6}\cdot\frac{1}{a_1-a_2} + \frac{1}{3}\cdot\frac{1}{2a_1+a_2}}{\frac{2}{3}\cdot\frac{1}{5a_1+a_2} + \frac{1}{6}\cdot\frac{1}{-a_1+a_2} + \frac{1}{6}\cdot\frac{1}{2a_1+a_2}}$

On the other hand,

$\frac{MU_1}{MU_2} = 2 \implies \dfrac{\frac{10}{3}\cdot\frac{1}{5a_1+a_2} + \frac{1}{6}\cdot\frac{1}{a_1-a_2} + \frac{1}{3}\cdot\frac{1}{2a_1+a_2}}{\frac{2}{3}\cdot\frac{1}{5a_1+a_2} + \frac{1}{6}\cdot\frac{1}{-a_1+a_2} + \frac{1}{6}\cdot\frac{1}{2a_1+a_2}} = 2$

$\implies \frac{10}{3}\cdot\frac{1}{5a_1+a_2} - \frac{1}{6}\cdot\frac{1}{-a_1+a_2} + \frac{1}{3}\cdot\frac{1}{2a_1+a_2} = \frac{4}{3}\cdot\frac{1}{5a_1+a_2} + \frac{1}{3}\cdot\frac{1}{-a_1+a_2} + \frac{1}{3}\cdot\frac{1}{2a_1+a_2}$

$\implies \frac{6}{3}\cdot\frac{1}{5a_1+a_2} + \left(-\frac{1}{6} - \frac{1}{3}\right)\cdot\frac{1}{-a_1+a_2} + 0 = 0 \implies 2\cdot\frac{1}{5a_1+a_2} + \left(-\frac{1}{6} - \frac{2}{6}\right)\cdot\frac{1}{-a_1+a_2} = 0$

$\implies 2\cdot\frac{1}{5a_1+a_2} = \frac{3}{6}\cdot\frac{1}{-a_1+a_2} \implies 2\cdot\frac{1}{5a_1+a_2} = \frac{1}{2}\cdot\frac{1}{-a_1+a_2} \implies 4\cdot\frac{1}{5a_1+a_2} = \frac{1}{-a_1+a_2}$

$\implies \frac{4}{5a_1+a_2} = \frac{1}{-a_1+a_2} \implies 4\left(-a_1+a_2\right) = 5a_1 + a_2 \implies -4a_1 + 4a_2 = 5a_1 + a_2$

$\implies 4a_2 = 9a_1 + a_2 \implies 3a_2 = 9a_1$

$\implies a_2 = 3a_1$

$\because 2a_1 + a_2 = 30$

$\therefore 2a_1 + 3a_1 = 30 \implies 5a_1 = 30$

$\boxed{\begin{cases} a_1 = 6 \\ a_2 = 18 \end{cases}}$

(e)

$w_1 = 5a_1 + a_2 = 5 \times 6 + 18 = 30 + 18 = \boxed{48}$ for space 1,

$w_2 = -a_1 + a_2 = -6 + 18 = \boxed{12}$ for space 2,

$w_3 = 2a_1 + a_2 = \boxed{30}$ for space 3.

She doesn't hedge all risks completely. As we see, w_1, w_2 and w_3 are not equal.

3

$w_0 = q \cdot \bar{a} = \sum_{k=1}^{K} q_k \cdot \bar{a}_k$

The original portfolio problem is:

$$\max_{(a_1, a_2, \ldots, a_K)} \sum_{s=1}^{S} p_s u\left(w_s\left(a\right)\right)$$

$$\text{subject to}$$

$$q \cdot \bar{a} = w_0.$$

$$w_s\left(a\right) = \sum_{k=1}^{K} r_{sk} a_k \implies 1 = \frac{\sum_{k=1}^{K} r_{sk} a_k}{w_s}$$

The question now becomes:

$$\max_{(a_1, a_2, \ldots, a_K)} \sum_{s=1}^{S} p_s u\left(\sum_{k=1}^{K} r_{sk} a_k\right)$$

$$\text{subject to}$$

$$\sum_{k=1}^{K} q_k \cdot \bar{a}_k = w_0.$$

Denote

$$\bar{\alpha}_k = \frac{q_k a_k}{w_0}, \sum_{k=1}^{K} \bar{\alpha}_k = \frac{\sum_{k=1}^{K} q_k a_k}{w_0} = \frac{w_0}{w_0} = 1$$

$$\alpha_{sk} = \frac{r_{sk} a_k}{w_s}, \text{ note that } \frac{w_s \alpha_{sk}}{r_{sk}} = a_k = \frac{w_0 \bar{\alpha}_k}{q_k}$$

$$\sum_{k=1}^{K} \alpha_{sk} = \frac{\sum_{k=1}^{K} r_{sk} a_k}{w_s} = \frac{w_s}{w_s} = 1,$$

where $k = 1, 2, 3, ..., K$, and $s = 1, 2, 3, ..., S$ above.

Now the problem is:

$$\max_{(\alpha_1, \alpha_2, \ldots, \alpha_K)} \sum_{s=1}^{S} p_s u \left(w_s \sum_{k=1}^{K} \alpha_{sk} \right)$$

subject to

$$\sum_{k=1}^{K} \bar{\alpha}_k = 1.$$

$$w_s \alpha_{sk} q_k = w_0 \bar{\alpha}_k r_{sk} \implies \alpha_{sk} = \frac{w_0 \bar{\alpha}_k r_{sk}}{w_s q_k}$$

$$w_s \sum_{k=1}^{K} \alpha_{sk} = w_0 \cdot 1 = w_0 = \sum_{k=1}^{K} r_{sk} a_k = \sum_{k=1}^{K} r_{sk} a_k = \sum_{k=1}^{K} r_{sk} \frac{w_0 \bar{\alpha}_k}{q_k} = w_0 \sum_{k=1}^{K} r_{sk} \frac{\bar{\alpha}_k}{q_k}$$

The problem becomes:

$$\max_{(\alpha_1, \alpha_2, \ldots, \alpha_K)} \sum_{s=1}^{S} p_s u \left(w_0 \sum_{k=1}^{K} \frac{r_{sk} \bar{\alpha}_k}{q_k} \right)$$

subject to

$$\sum_{k=1}^{K} \bar{\alpha}_k = 1.$$

We konw the expression of $u(x)$, we know p_s, r_{sk}, q_k and w_0 are given.

So the problem is solvable, and it is equivalent to the former one.

4

(a)

$$R = \begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix} = \begin{pmatrix} 8 & 5 \\ 4 & 1 \end{pmatrix}$$

$$q = (q_1, q_2) = (2, 1)$$

$$\rho_1 = (\rho_{11}, \rho_{21}) = \left(\frac{r_{11}}{q_1}, \frac{r_{21}}{q_1}\right) = \left(\frac{8}{2}, \frac{4}{2}\right) = \boxed{(4, 2)}$$

$$\rho_2 = (\rho_{12}, \rho_{22}) = \left(\frac{r_{12}}{q_2}, \frac{r_{22}}{q_2}\right) = \left(\frac{5}{1}, \frac{1}{1}\right) = \boxed{(5, 1)}$$

(b)

$$\begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ 2 & 1 \end{pmatrix}$$

$$\mu_1 = \rho_{11} p_1 + \rho_{21} p_2 = 4 \times \frac{1}{3} + 2 \times \frac{2}{3} = \frac{4}{3} + \frac{4}{3} = \boxed{\frac{8}{3}}$$

$$\mu_2 = \rho_{12} p_1 + \rho_{22} p_2 = 5 \times \frac{1}{3} + 1 \times \frac{2}{3} = \frac{5}{3} + \frac{2}{3} = \boxed{\frac{7}{3}}$$

$$\begin{aligned}
\sigma_{11} &= (\rho_{11} - \mu_1)^2 p_1 + (\rho_{21} - \mu_1)^2 p_2 \\
&= \left(4 - \frac{8}{3}\right)^2 \times \frac{1}{3} + \left(2 - \frac{8}{3}\right)^2 \times \frac{2}{3} = \left(\frac{12-8}{3}\right)^2 \times \frac{1}{3} + \left(\frac{6-8}{3}\right)^2 \times \frac{2}{3} = \left(\frac{4}{3}\right)^2 \times \frac{1}{3} + \left(\frac{2}{3}\right)^2 \times \frac{2}{3} \\
&= \frac{16}{9} \times \frac{1}{3} + \frac{4}{9} \times \frac{2}{3} = \frac{16+8}{27} = \frac{24}{27} = \frac{3 \times 8}{3 \times 9} \\
&= \frac{8}{9}
\end{aligned}$$

$$\begin{aligned}
\sigma_{12} &= (\rho_{11} - \mu_1)(\rho_{12} - \mu_2) p_1 + (\rho_{21} - \mu_1)(\rho_{22} - \mu_2) p_2 \\
&= \left(4 - \frac{8}{3}\right)\left(5 - \frac{7}{3}\right) \times \frac{1}{3} + \left(2 - \frac{8}{3}\right)\left(1 - \frac{7}{3}\right) \times \frac{2}{3} \\
&= \left(\frac{12-8}{3}\right)\left(\frac{15-7}{3}\right) \times \frac{1}{3} + \left(\frac{6-8}{3}\right)\left(\frac{3-7}{3}\right) \times \frac{2}{3} \\
&= \left(\frac{4}{3}\right)\left(\frac{8}{3}\right) \times \frac{1}{3} + \left(-\frac{2}{3}\right)\left(-\frac{4}{3}\right) \times \frac{2}{3} = \frac{32}{9} \times \frac{1}{3} + \frac{8}{9} \times \frac{2}{3} = \frac{32+16}{27} = \frac{48}{27} = \frac{3 \times 16}{3 \times 9} \\
&= \frac{16}{9}
\end{aligned}$$

$$\begin{aligned}
\sigma_{22} &= (\rho_{12} - \mu_2)^2 p_1 + (\rho_{22} - \mu_2)^2 p_2 \\
&= \left(5 - \frac{7}{3}\right)^2 \times \frac{1}{3} + \left(1 - \frac{7}{3}\right)^2 \times \frac{2}{3} = \left(\frac{15-7}{3}\right)^2 \times \frac{1}{3} + \left(\frac{3-7}{3}\right)^2 \times \frac{2}{3} = \left(\frac{8}{3}\right)^2 \times \frac{1}{3} + \left(\frac{4}{3}\right)^2 \times \frac{2}{3} \\
&= \frac{64}{9} \times \frac{1}{3} + \frac{16}{9} \times \frac{2}{3} = \frac{64+32}{27} = \frac{96}{27} = \frac{3 \times 32}{3 \times 9}
\end{aligned}$$

$= \frac{32}{9}$

$$\boxed{\sum = \begin{pmatrix} \frac{8}{9} & \frac{16}{9} \\ \frac{16}{9} & \frac{32}{9} \end{pmatrix}}$$

(c)

$\rho_w = (\rho_{1w}, \rho_{2w}) = (\alpha_1\rho_{11} + \alpha_2\rho_{12}, \alpha_1\rho_{21} + \alpha_2\rho_{22})$
$= (\alpha_1 \times 4 + (1-\alpha_1) \times 5, \alpha_1 \times 2 + (1-\alpha_1) \times 1) = (4\alpha_1 + 5 - 5\alpha_1, 2\alpha_1 + 1 - \alpha_1)$
$= \boxed{(-\alpha_1 + 5, \alpha_1 + 1)}$

$\mu_w = \rho_{1w}p_1 + \rho_{2w}p_2 = (-\alpha_1 + 5) \times \frac{1}{3} + (\alpha_1 + 1)\,\frac{2}{3} = \frac{-\alpha_1 + 5}{3} + \frac{2\alpha_1 + 2}{3} = \boxed{\frac{\alpha_1 + 7}{3}}$

$\sigma_w^2 = (\rho_{1w} - \mu_w)^2 p_1 + (\rho_{2w} - \mu_w)^2 p_2$
$= \left(-\alpha_1 + 5 - \frac{\alpha_1 + 7}{3}\right)^2 \times \frac{1}{3} + \left(\alpha_1 + 1 - \frac{\alpha_1 + 7}{3}\right)^2 \times \frac{2}{3}$
$= \left(\frac{-3\alpha_1 + 15}{3} - \frac{\alpha_1 + 7}{3}\right)^2 \times \frac{1}{3} + \left(\frac{3\alpha_1 + 3}{3} - \frac{\alpha_1 + 7}{3}\right)^2 \times \frac{2}{3}$
$= \left(\frac{-3\alpha_1 + 15 - \alpha_1 - 7}{3}\right)^2 \times \frac{1}{3} + \left(\frac{3\alpha_1 + 3 - \alpha_1 - 7}{3}\right)^2 \times \frac{2}{3}$
$= \left(\frac{-4\alpha_1 + 8}{3}\right)^2 \times \frac{1}{3} + \left(\frac{2\alpha_1 - 4}{3}\right)^2 \times \frac{2}{3} = \left(\frac{4}{3} \cdot (2 - \alpha_1)\right)^2 \times \frac{1}{3} + \left(\frac{2}{3} \cdot (\alpha_1 - 2)\right)^2 \times \frac{2}{3}$
$= (\alpha_1 - 2)^2 \cdot \frac{16}{9} \cdot \frac{1}{3} + (\alpha_1 - 2)^2 \cdot \frac{4}{9} \cdot \frac{2}{3} = \left(\frac{16}{27} + \frac{8}{27}\right)(\alpha_1 - 2)^2 = \frac{24}{27}(\alpha_1 - 2)^2$
$= \boxed{\frac{8}{9}(\alpha_1 - 2)^2}$

(d)

Clearly, by the expression of σ_w^2 in part (c), $\alpha_1 = 2$ minimize the variance.

It can also be proved in another way. Using the notation in the Lecture Note 2:

$\bar{\mu} = \alpha_1\mu_1 + \alpha_2\mu_2 = \frac{8}{3}\alpha_1 + \frac{7}{3}(1 - \alpha_1) = \frac{8}{3}\alpha_1 + \frac{7}{3} - \frac{7}{3}\alpha_1 = \frac{1}{3}\alpha_1 + \frac{7}{3}$

$A = \sigma_{11} - 2\sigma_{12} + \sigma_{22} = \frac{8}{9} - 2 \times \frac{16}{9} + \frac{32}{9} = \frac{8}{9}$

$B = (\sigma_{11} - \sigma_{12})\,\mu_2 + (\sigma_{22} - \sigma_{12})\,\mu_1 = \left(\frac{8}{9} - \frac{16}{9}\right) \times \frac{7}{3} + \left(\frac{32}{9} - \frac{16}{9}\right) \times \frac{8}{3} = \left(-\frac{8}{9}\right) \times \frac{7}{3} + \frac{16}{9} \times \frac{8}{3} = \frac{-56+128}{27} = \frac{72}{27} = \frac{8 \times 9}{3 \times 9} = \frac{8}{3}$

$\mu_2 - \mu_1 = \frac{7}{3} - \frac{8}{3} = -\frac{1}{3}$

$\sigma_{11}\sigma_{22} - \sigma_{12}^2 = \frac{8}{9} \times \frac{32}{9} - \left(\frac{16}{9}\right)^2 = \frac{32 \times 8 - 16 \times 16}{9} = \frac{16 \times 2 \times 8 - 16 \times 16}{9} = 0$

To minimize

$$\sigma_w^2 = \frac{A}{(\mu_2 - \mu_1)^2}\left(\bar{\mu} - \frac{B}{A}\right)^2 + \frac{\sigma_{11}\sigma_{22} - \sigma_{12}^2}{A}$$

$\bar{\mu} = \frac{B}{A} \iff \frac{1}{3}\alpha_1 + \frac{7}{3} = \frac{\frac{8}{3}}{\frac{8}{9}} = 3 = \frac{9}{3} \iff \frac{\alpha_1}{3} = \frac{2}{3} \iff \boxed{\alpha_1 = 2}$

$\boxed{\alpha_2 = 1 - \alpha_1 = -1}$

The minimum of σ_w^2 is $\boxed{0}$, since $\bar{\mu} - \frac{B}{A} = 0$ and $\sigma_{11}\sigma_{22} - \sigma_{12}^2 = 0$

In other words, the investor hedge away all risks, and get a certain amount of wealth.

$\mu_w = \frac{\alpha_1 + 7}{3} = \frac{2+7}{3} = \boxed{3}$

The share of 1st asset is $\frac{\alpha_1 w_0}{q_1} = \frac{\alpha_1 q\bar{a}}{q_1} = 2 \times (2 \times 10 + 1 \times 10) \div 2 = \boxed{30}$

The share of 2nd asset is $\frac{\alpha_2 w_0}{q_2} = \frac{\alpha_2 q\bar{a}}{q_2} = -1 \times (2 \times 10 + 1 \times 10) \div 1 = \boxed{-30}$, that is, to short 30 shares of asset 2.