1. (a)

Option 1: Full coverage.

$w_f = 22500 - 941 = 21559$, regardless of space.

~~$\therefore w_f = 21559$~~

$\therefore w_f = 21559$ with probability 100%.

Option 2: Partial coverage.

If major accident happens,

$w_p = 22500 - 916 - 8100 + 5600 = 19084$, with probability 1%.

If in all other states:

$w_p = 22500 - 916 = 21584$ with probability $1 - 1\% = 99\%$.

$\therefore w_p = \{19084, 21584\}$ with the probability distribution $P_p = (0.01, 0.99)$.

Option 3: madatory insuarance

If major accident happens

$w_m = 22500 - 754 - 8100 + 2900 = 16546$ with probability 1%.

If moderate accident happens.

$w_m = 22500 - 754 - 5600 + 2900 = 19046$ with probability 5%.

In other cases:

$w_m = 22500 - 754 = 21746$ with probability $1 - 5\% - 1\% = 94\%$

$\therefore w_m = \{16546, 19046, 21746\}$ with probability distribution $P_m = (0.01, 0.05, 0.94)$.

(b).

$$F_1(w) = \begin{cases} 0 & \text{if } w < 21559 \\ 1 & \text{if } w \geq 21559 \end{cases}$$

$$F_2(w) = \begin{cases} 0 & \text{if } w < 19084 \\ 0.01 & \text{if } 19084 \leq w < 21584 \\ 1 & \text{if } w \geq 21584 \end{cases}$$

$$F_3(w) = \begin{cases} 0 & \text{if } w < 16546 \\ 0.01 & \text{if } 16546 \leq w < 19046 \\ 0.06 & \text{if } 19046 \leq w < 21746 \\ 1 & \text{if } w \geq 21746. \end{cases}$$

(c).

Cannot.

Option 1: $E[w_f]=21559$

Option 2:

$E[w_p]=0.01\times19084+0.99\times21584=21559$

Option 3:

$E[w_m]=0.01\times16546+0.05\times19046+0.94\times21746=21559.$

$\therefore E[w_f]=E[w_p]=E[w_m]=21559.$

Additionally, we cannot find $F_i(x)\le F_j(x)$, where $i=1,2,3$, $j=1,2,3$, $i\neq j$, holds for all $x\in\mathbb{R}$.

Thus, using FOSD, cannot completely identify preferences over insurance plans.

(d).

Option 1:

if $w<21559$, $H_1(w)=0.$

if $w\ge21559$, $H_1(w)=\int_{-\infty}^{w}F_1(y)dy=\int_{0}^{21559}0\,dy+\int_{21559}^{w}1\,dy=w-21559$

Option 2:

if $w<19084$, $H_2(w)=0.$

if $19084\le w<21584$: $H_2(w)=\int_{-\infty}^{w}F_2(y)dy=\int_{-\infty}^{19084}0\,dy+\int_{19084}^{w}0.01\,dy=0.01(w-19084)$

if $w\ge21584$: $H_2(w)=\int_{-\infty}^{w}F_2(x)dy=\int_{-\infty}^{19084}0\,dy+\int_{19084}^{21584}0.01\,dy+\int_{21584}^{w}1\,dy$

$=0.01\cdot(21584-19084)+(w-21584)=0.01\times2500+(w-21584)$

$=w-21584+25$

$=w-21559.$

Option 3:

if $w<16546$ $\quad H_3(w)=0$

if $16546\le w<19046$: $H_3(w)=\int_{-\infty}^{16546}0\,dy+\int_{16546}^{w}0.01\,dy=0.01(w-16546)$

if $19046\le w<21746$: $H_3(w)=\int_{-\infty}^{16546}0\,dy+\int_{16546}^{19046}0.01\,dy+\int_{19046}^{w}0.06\,dy$

$=0+0.01(19046-16546)+0.06\cdot(w-19046)$

$=0.01\times2500+0.06w-0.06\times19046$

$=25+0.06w-1142.76$

$=0.06w-1117.76$

if $w \geq 21746$: $H_3(w) = \int_{-\infty}^{16546} 0\,dy + \int_{16546}^{19046} 0.01dy + \int_{19046}^{21746} 0.06dy + \int_{21746}^{w} 1dy$

$= 25 + 0.06(21746 - 19046) + (w - 21746)$

$= w - 21746 + 0.06 \times 2700 + 25 = w - 21721 + 162$

$= w - 21559$

Thus:

$$\underline{H_1(w)} = \begin{cases} 0 & \text{if } w < 21559 \\ w - 21559 & \text{if } w \geq 21559 \end{cases}$$

$$\underline{H_2(w)} = \begin{cases} 0 & \text{if } w < 19084 \\ 0.01(w - 19084) & \text{if } 19084 \leq w < 21584 \\ w - 21559 & \text{if } w \geq 21584 \end{cases}$$

$$\underline{H_3(w)} = \begin{cases} 0 & \text{if } w < 16546 \\ 0.01(w - 16546) & \text{if } 16546 \leq w < 19046 \\ 0.06w - 1117.76 & \text{if } 19046 \leq w < 21746 \\ w - 21559 & \text{if } w \geq 21746 \end{cases}$$

if $w < 16546$, $H_1(w) = H_2(w) = H_3(w) = 0$.

if $16546 \leq w < 19046$, $H_1(w) = H_2(w) = 0 \leq H_3(w)$

if $19046 \leq w < 19084$ $H_1(w) = H_2(w) = 0 < H_3(w)$.

if $19084 \leq w < 21559$ $H_2(w) = 0.01w - 190.84$

$H_3(w) = 0.06w - 1117.76$

$H_3(w) - H_2(w) = 0.05w - 1117.76 + 190.84 = 0.05w - 926.92 \geq 0.05 \times 19084 - 926.92$

$= 954.2 - 926.62 > 0$

$H_1(w) = 0 \leq H_2(w) < H_3(w)$.

if $21559 \leq w < 21584$

$H_1(w) = w - 21559$

$H_2(w) = 0.01w - 190.84$

$H_3(w) = 0.06w - 1117.76$

$H_3(w) - H_2(w) = 0.05w - 926.92 \geq 0.05 \times 21559 - 926.92 > 0.05 \times 19084 - 926.92 > 0$.

$H_2(w) - H_1(w) = (0.01w - 190.84) - (w - 21559) = 0.01w - 190.84 - w + 21559 = (21559 - 190.84) - 0.99w$

$= 21368.16 - 0.99w > 21368.16 - 0.99 \times 21584 = 0$.

$\therefore H_1(w) < H_2(w) < H_3(w)$.

if $21584 \le w < 21746$,

$H_1(w) = w - 21559$

$H_2(w) = w - 21559 = H_1(w)$

$H_3(w) = 0.06w - 1117.76$.

$H_3(w) - H_2(w) = \cancel{-0.94w + 2044}$

$-0.94w - 1117.76 + 21559 = -0.94w + 20441.24 > -0.94 \times 21746 + 20441.24 = 0$

$\therefore H_1(w) = H_2(w) < H_3(w)$.

if $w \geq 21746$: $H_1(w) = H_2(w) = H_3(w) = w - 21559$

$\therefore$ ~~$\forall w \in \mathbb{R}$,~~

$\therefore$ ~~$\forall w \in \mathbb{R}$, 1 SOSD 2, 2 SOSD 3,~~ 1 SOSD 3.

$\therefore \forall w \in \mathbb{R}$, Plan 1 SOSD Plan 2, Plan 2 SOSD Plan 3, Plan 1 SOSD Plan 3.

2. (a).

In "good" state: $[1.32\alpha w_0 + 1.02(1-\alpha)w_0] = [1.32\alpha + 1.02 - 1.02\alpha]w_0 = (0.3\alpha + 1.02)w_0$

In "bad" state: $(0.9\alpha w_0 + 1.02(1-\alpha)w_0) = (0.9\alpha + 1.02 - 1.02\alpha)w_0 = (-0.12\alpha + 1.02)w_0$

(b).

$E[w] = \frac{1}{3} \times (0.3\alpha + 1.02)w_0 + \frac{2}{3} \times (-0.12\alpha + 1.02)w_0 = (0.1\alpha + \frac{1}{3} \times 1.02 - \frac{2}{3} \times 0.12\alpha + \frac{2}{3} \times 1.02)w_0$

$= (0.1\alpha + 1.02 - 2 \times 0.04\alpha)w_0 = (0.1\alpha - 0.08\alpha + 1.02)w_0 = (0.02\alpha + 1.02)w_0$.

(c).

In "good" state: $\ln[(0.3\alpha + 1.02)w_0]$

In "bad" state: $\ln[(-0.12\alpha + 1.02)w_0]$

$E[U] = \frac{1}{3}\ln[(0.3\alpha + 1.02)w_0] + \frac{2}{3} \times \ln[(-0.12\alpha + 1.02)w_0]$

$= \ln[(0.3\alpha + 1.02)w_0]^{\frac{1}{3}} + \ln[(-0.12\alpha + 1.02)w_0]^{\frac{2}{3}} = \ln\{[(0.3\alpha + 1.02)w_0]^{\frac{1}{3}}[(-0.12\alpha + 1.02)w_0]^{\frac{2}{3}}\}$

$= \ln[(0.3\alpha + 1.02)^{\frac{1}{3}}(-0.12\alpha + 1.02)^{\frac{2}{3}} w_0]$.

(d)

FOC: $\frac{1}{3} \cdot \frac{1}{(0.3\alpha + 1.02)w_0} \cdot 0.3w_0 + \frac{2}{3} \cdot \frac{1}{(-0.12\alpha + 1.02)w_0} \cdot (-0.12w_0) = 0$

$\Rightarrow \frac{1}{3} \cdot \frac{0.3}{0.3\alpha + 1.02} + \frac{2}{3} \cdot \frac{-0.12}{-0.12\alpha + 1.02} = 0 \Rightarrow \frac{0.1}{0.3\alpha + 1.02} + 2 \cdot \frac{-0.04}{-0.12\alpha + 1.02} = 0$

$\Rightarrow \frac{1}{3\alpha + 10.2} + \frac{-0.08}{-0.12\alpha + 1.02} = 0 \Rightarrow \frac{1}{3\alpha + 10.2} + \frac{-0.04}{-0.06\alpha + 0.51} = 0 \Rightarrow \frac{1}{3\alpha + 10.2} + \frac{-4}{-6\alpha + 51} = 0$

$\Rightarrow \frac{1}{3\alpha + 10.2} = \frac{4}{-6\alpha + 51} \Rightarrow -6\alpha + 51 = 12\alpha + 40.8 \Rightarrow 51 = 18\alpha + 40.8$

$\Rightarrow 18\alpha = 51 - 40.8 = 10.2$

$\Rightarrow \alpha = \frac{10.2}{18} = \frac{5.1}{9} = \frac{1.7}{3} = \frac{17}{30} \approx 0.5667$

check SOC:

$$\frac{d\left((3\alpha+1/0.2)^{-1}-4(-6\alpha+51)^{-1}\right)}{d\alpha}=-(3\alpha+1/0.2)^{-2}\cdot 3-4\times(-1)(-6\alpha+51)^{-2}\cdot(-6)$$

$$=\frac{-3}{(3\alpha+1/0.2)^2}+4\times(-6)\times\frac{1}{(-6\alpha+51)^2}=\frac{-3}{(3\alpha+1/0.2)^2}-\frac{24}{(-6\alpha+51)^2}<0$$

Reasonable. Since $\ln w$ is strictly concave.

In conclusion, utility is maximized at $\alpha=\frac{17}{30}\approx 0.5667$.

(e)

α is independent of (current) wealth.

In fact, we can calculate

$$r_R(w)=\frac{-wu''(w)}{u'(w)}=\frac{-w\cdot(-\frac{1}{w^2})}{\frac{1}{w}}=\frac{\frac{1}{w}}{\frac{1}{w}}=1,\text{ a constant.}$$

(f).

α is positively related to (current) wealth.

If current wealth goes up, the optimal amount of money αw_0 increases.

In fact, we can calculate

$$r_A(w)=-\frac{u''(w)}{u'(w)}=-\frac{-\frac{1}{w^2}}{\frac{1}{w}}=\frac{w}{w^2}=\frac{1}{w}$$

r_A is an decreasing function.

Investor invests more amount of money in the risky asset, and treat risky asset as normal good.