1. By no-arbitrage price formula:

$$0=\frac{1}{1+r}(\lambda(\Phi_0-\Phi_{1u})+(1-\lambda)(\Phi_0-\Phi_{1d}))$$
$$+\frac{\lambda}{1+r}(\lambda(\Phi_{1u}-1.14)+(1-\lambda)(\Phi_{1u}-1.10))$$
$$+\frac{1-\lambda}{1+r}(\lambda(\Phi_{1d}-1.10)+(1-\lambda)(\Phi_{1d}-1.06)) \quad ①$$

$$0=\frac{1}{1+r}(\lambda(\Phi_{1u}-1.14)+(1-\lambda)(\Phi_{1u}-1.10)) \quad ②$$
$$0=\frac{1}{1+r}(\lambda(\Phi_{1d}-1.10)+(1-\lambda)(\Phi_{1d}-1.06)) \quad ③.$$

By ②, we have:

$$0=-1.14\lambda+\Phi_{1u}\lambda-1.10(1-\lambda)+(1-\lambda)\Phi_{1u}$$
$$\Rightarrow \Phi_{1u}=1.14\lambda+1.10-1.10\lambda=0.04\lambda+1.10$$
$$=0.04\times 0.6+1.10$$
$$\boxed{=1.124}$$

By ③, we have:

$$0=-1.10\lambda+\Phi_{1d}\cdot\lambda-1.06(1-\lambda)+(1-\lambda)\Phi_{1d}$$
$$\Rightarrow \Phi_{1d}=1.10\lambda+1.06-1.06\lambda$$
$$=1.06+0.04\lambda=1.06+0.04\times 0.6$$
$$=1.06+0.024$$
$$\boxed{=1.084}$$

Plug into ①, we have:

$$\lambda(\Phi_0-\Phi_{1u})+(1-\lambda)(\Phi_0-\Phi_{1d})=0$$
$$\Rightarrow \Phi_0-\lambda\Phi_{1u}-(1-\lambda)\Phi_{1d}=0$$
$$\Rightarrow \Phi_0=0.6\times 1.124+0.4\times 1.084$$
$$\boxed{=1.108}$$

The notional amount of each future contract is (in USD)

$\Phi_0\times 1000=1108$

Total amount of ~~each~~ future contract: $\frac{200,000}{1,000}=200$.

$$200\times(-\Phi_{1u}+\Phi_0)\times 1000$$
$$=200\times(-1.124+1.108)\times 1000$$
$$=200\times(-0.016)\times 1000$$
$$=200\times 16$$
$$=-3200$$

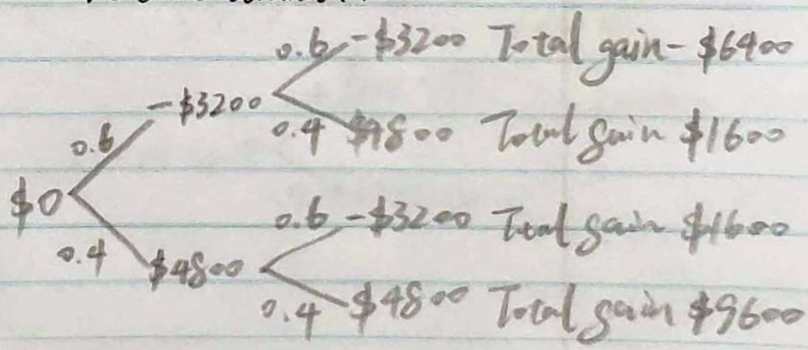
$$200\times(\Phi_0-\Phi_{1d})\times 1000$$
$$=200\times(1.108-1.084)\times 1000$$
$$=200\times 0.024\times 1000$$
$$=200\times 24$$
$$=4800$$

$$200\times(\Phi_{1u}-1.14)\times 1000$$
$$=200\times(1.124-1.14)\times 1000$$
$$=200\times(-0.016)\times 1000$$
$$=200\times(-16)$$
$$=-3200.$$

$$200\times(-1.10+\Phi_{1u})\times 1000$$
$$=200\times(1.124-1.10)\times 1000$$
$$=200\times 0.024\times 1000$$
$$=200\times 24$$
$$=4800$$

$$200\times(-1.06+\Phi_{1d})\times 1000=-3200$$
$$200\times(\Phi_{1d}-1.06)\times 1000=4800.$$

Evolution of futures' cash stream, in each contract:



$200,000\times 1.14=228,000$

$200,000\times 1.10=220,000$

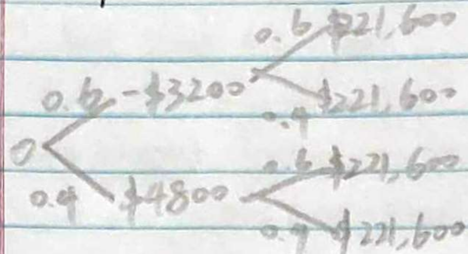
$200,000\times 1.06=212,000$

$228,000-6400=221,600$

~~228,000~~ 220,000 $+1600=221,600$

~~228,000~~ 212,000 $+9600=221,600$

The producer's cash flow:



(b) By no-arbitrage pricing formula, we have:

$0=\frac{1}{1+r}(\pi(X-1.12)+(1-\pi)(X-1.08))$
$\quad+\frac{\pi}{(1+r)^2}(\pi(X-1.14)+(1-\pi)(X-1.10))$
$\quad+\frac{1-\pi}{(1+r)^2}(\pi(X-1.10)+(1-\pi)(X-1.06))$

$\Rightarrow 0=(0.6(X-1.12)+0.4(X-1.08))$
$\quad+\frac{0.6}{1+r}(0.6(X-1.14)+0.4(X-1.10))$
$\quad+\frac{0.4}{1+r}(0.6(X-1.10)+0.4(X-1.06))$

$\Rightarrow 0=(X-0.6\times1.12-0.4\times1.08)$
$\quad+\frac{0.6}{1+r}(X-0.6\times1.14-0.4\times1.1)$
$\quad+\frac{0.4}{1+r}(X-0.6\times1.10-0.4\times1.06)$

$\Rightarrow 0=(1+r)(X-0.672-0.432)$
$\quad+(0.6X-0.36\times1.14-0.24\times1.1)$
$\quad+(0.4X-0.24\times1.1-0.16\times1.06)$

$\Rightarrow 0=1.005(X-1.104)+X$
$\quad-(0.36\times1.14+0.24\times1.1)$
$\quad-(0.24\times1.1+0.16\times1.06)$

$\Rightarrow 0=2.005X-1.005\times1.004$
$\quad-0.4104-0.264\times2-0.1696$

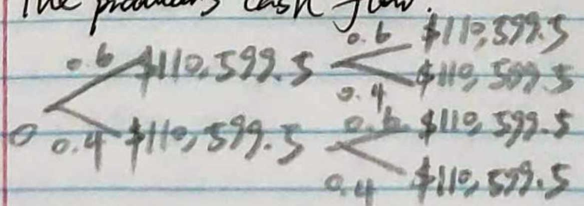
$\Rightarrow 0=2.005X-1.10952-1.108$

$\Rightarrow 2.005X=2.21752$

$\Rightarrow \boxed{X=\frac{2.21752}{2.005}\approx1.105995}$

$100{,}000X\approx110{,}599.5$

The producer's cash flow:

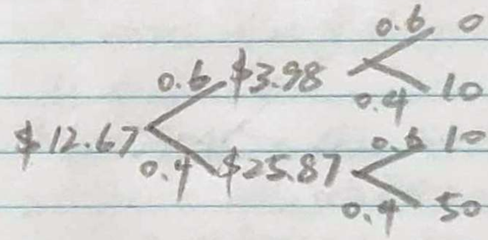


(c).

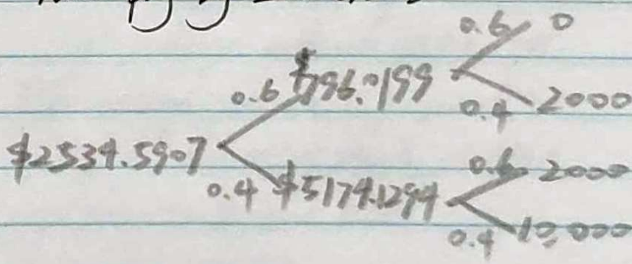
$q_{p_1}(q_{1u})=\frac{1}{1+r}(\pi\cdot0+(1-\pi)(K-1.10\times1000))$
$\quad=\frac{1-\pi}{1+r}(K-1100)$
$\quad=\frac{0.4}{1.005}\times10$
$\quad\approx3.9800995$

$q_{p_1}(q_{1d})=\frac{1}{1+r}(\pi(K-1.10\times1000)+(1-\pi)(K-1.06\times1000))$
$\quad=\frac{1}{1+r}(K-(1.10\pi+1.06(1-\pi))\times1000)$
$\quad=\frac{1}{1+r}(K-1.084\times1000)$
$\quad=\frac{1}{1+r}(K-1084)$
$\quad=\frac{1}{1.005}(1110-1084)$
$\quad=\frac{26}{1.005}$
$\quad\approx25.870647$

$q_0(q_0)=\frac{1}{1.005}(0.6\times\frac{4}{1.005}+0.4\times\frac{26}{1.005})$
$\quad\approx12.672954$



Multiply by 200 shares:



3. (a) I don't think it's right.
History data may not reflect what happens right now perfectly.
Some new factors may affect current price:
news, profit, inflation, new risk...
If \$98.18 is actually at its value, there also exists a risk-neutural probability measure $\{\pi, 1-\pi\}$:
By no-arbitrage price formula:

$q_0 = \frac{1}{1+r}(\pi q_u + (1-\pi) q_d)$

$\Rightarrow 98.18 = \frac{1}{1.1}(120\pi + 90(1-\pi))$

$\Rightarrow 98.18 \times 1.1 = 120\pi + 90 - 90\pi$

$\Rightarrow 107.998 = 30\pi + 90$

$\Rightarrow 17.998 = 30\pi$

$\Rightarrow \pi = \frac{17.998}{30} \approx 0.599933 \approx 0.6$

$1-\pi \approx 0.40067 \approx 0.4$

(b)

If $q_1 = \$120 > K = \$100$,
exercise the option,
payoff is \$120 − \$100 = \$20
If $q_1 = \$90 < K$, the investor won't exercise,
the payoff is 0.
Denoting the replicating portofolio cosisting of α_c shares of the stock, and β_c shares of the bond.

$20 = 120\alpha_c + 1.1\beta_c$

$0 = 90\alpha_c + 1.1\beta_c$

$\Rightarrow 30\alpha_c = 20 \Rightarrow \alpha_c = \frac{2}{3}$.

$1.1\beta_c = -90\alpha_c = -90 \times \frac{2}{3} = -30 \times 2 = -60$.

$\Rightarrow \beta_c = -\frac{60}{1.1} \approx -54.545$

Hence the European call can be replicated by buying $\frac{2}{3}$ shares of the stock and shorting 54.545 shares of bond.
The option price at $t=0$ is:

$q_c = \frac{2}{3} \times 98.18 - 54.545$
≈ 10.907879

(c)

Suppose that $q_c < 10.907879$.
Consider a portofolio consisting of 1 share of option, $\alpha_c = \frac{2}{3}$ shares of stock, and buys 54.545 shares of bond.
The market value of the portfolio is:

$q_c - 98.18\alpha_c - \beta_c < 0$.

Suppose $q_1 = \$120$, then investor exercise the call, gets \$120, and gets back $\$1.1 \times 54.545 = \$60$ on the bond holding.
∴ The investor ends up \$80 to get.
The investor uses these payment to buy $\alpha_c = \frac{2}{3}$ shares of stock at \$120 and pays back the debt.
Since $\frac{2}{3} \times 120 = 80$,
the net payoff on the portfolio is 0.
If $q_1 = \$90$, the investor won't exercise the call, using the payment of $1.1 \times 54.545 = 60$ to buy $\frac{2}{3}$ shares of stock at \$90, paying back the debt.
Since $\frac{2}{3} \times \$90 = \$60$, the net payoff on the portfolio is 0.

Suppose now $q_c > 10.907879$.
Then at $t=0$, the investor shorts 1 call option, buys $\frac{2}{3}$ shares of the stock and shorts 54.545 shares of the bond.
The market value of portfolio is $-q_c + 98.18\alpha_c + \beta_c < 0$.
If $q_1 = \$120$, the value of the call is \$20, the investor sells $\frac{2}{3}$ shares of the stock, gets $\frac{2}{3} \times \$120 = \$80$.
The investor returns \$20 for the call option and pays back debt of bond: $1.1 \times \$54.545 = \$60$.
Since $\$20 + \$60 = \$80$, the net payoff is 0.
If $q_1 = \$90$, the value of the call is 0, the investor sells $\frac{2}{3}$ shares of the stock, gets $\frac{2}{3} \times \$90 = \$60$
The ~~k~~debt on the bond this investor needs to payback is $1.1 \times \$54.545 = \$60$.
The net payoff is 0.
Hence there is arbitrage opportunity.

(d)
The American put is exercised at $t=0$ iff:

$$K - q_0 \geq \frac{1-\pi}{1+r}(K - q_L)$$

$$\Rightarrow K - q_0 \geq \frac{1-\pi}{1+r}K - \frac{1-\pi}{1+r}q_L$$

$$\left(\frac{1+r}{1+r} - \frac{1-\pi}{1+r}\right)K \geq q_0 - \frac{1-\pi}{1+r}q_L$$

$$(r+\pi)K \geq (1+r)q_0 - (1-\pi)q_L$$

$$K \geq \frac{(1+r)q_0 - (1-\pi)q_L}{r+\pi}$$

$$K \geq \frac{1.1 \times 98.18 - 0.40067 \times 90}{0.1 + 0.599933}$$

$$K \geq 102.85551.$$