# Yu Xia's A6

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Fall 2022

## Multiple Choice

1-5:
cdeac

6-10:
badac

11-15:
bbcea

16-20:
adacb

## Long

### 1. Labor Search Model A

a.

Consumer's job-finding constraint:

$$\boxed{(1-\rho)\, n_{t-1}^{s} + p_t^{f} s_t = n_t^{s}} \tag{1.1}$$

Consumer's labor is determined by the proportion of people keeping their job, the probability of finding a job, and the search effort $s_t$. It is different from the classical framework which focuses on the choice of labor and consumption, without probability.

Consumer's budget constraint:

$$\boxed{c_t = (1-t_t)\, w_t n_t^{s} + \left(1 - p_t^{f}\right) s_t b_t^{ue}} \tag{1.2}$$

When consumers find a job (in other words, those people who have a job), they get the salary according to their labor, otherwise (unemployment pool) they spend the time to find a job and get the unemployment benefit. It can also be expressed as the expectation: the case of having a job and getting paid, and the case of finding a job and getting unemployment benefits.

The difference between the dynamic model and the static model is that when we introduce $\rho$, there are workers who stay in their job for the next period. Variables could be different in each time period, so we have the subscript.

b.

The Lagrangian:

$$
\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Bigg( & \left( u\left(c_t\right) - h\left(\left(1 - p_t^f\right) s_t + n_t^s\right)\right) \\
& - \lambda_t^h \left(c_t - \left(1 - t_t\right) w_t n_t^s - \left(1 - p_t^f\right) s_t b_t^{ue}\right) \\
& - \mu_t^h \left(\left(1 - \rho\right) n_{t-1}^s + p_t^f s_t - n_t^s\right)\Bigg) \\
= \dots & \\
& + u\left(c_t\right) - h\left(\left(1 - p_t^f\right) s_t + n_t^s\right) \\
& - \lambda_t^h \left(c_t - \left(1 - t_t\right) w_t n_t^s - \left(1 - p_t^f\right) s_t b_t^{ue}\right) \\
& - \mu_t^h \left(\left(1 - \rho\right) n_{t-1}^s + p_t^f s_t - n_t^s\right) \\
& + \beta u\left(c_{t+1}\right) - h\left(\left(1 - p_{t+1}^f\right) s_{t+1} + n_{t+1}^s\right) \\
& - \beta \lambda_{t+1}^h \left(c_{t+1} - \left(1 - t_{t+1}\right) w_{t+1} n_{t+1}^s - \left(1 - p_{t+1}^f\right) s_{t+1} b_{t+1}^{ue}\right) \\
& - \beta \mu_{t+1}^h \left(\left(1 - \rho\right) n_t^s + p_{t+1}^f s_{t+1} - n_{t+1}^s\right) \\
& + \dots
\end{aligned}
$$

The choice variables: $\boxed{c_t,\ n_t^s,\ s_t}$ (in each period).

The given variables: $\boxed{p_t^f,\ w_t,\ b_t^{ue},\ t_t,\ \rho,\ \beta}$.

c.

FOCs:
w.r.t $c_t$:

$$u' - \lambda_t^h = 0 \tag{1.3}$$

w.r.t $n_t^s$:

$$-h'\left(\left(1-p_t^f\right)s_t + n_t^s\right) + \lambda_t^h\left(1-t_t\right)w_t + \mu_t^h - \beta\mu_{t+1}^h\left(1-\rho\right) = 0 \tag{1.4}$$

w.r.t $s_t$:

$$-h'\left(\left(1-p_t^f\right)s_t + n_t^s\right)\left(1-p_t^f\right) + \lambda_t^h\left(1-p_t^f\right)b_t^{ue} - \mu_t^h p_t^f = 0 \tag{1.5}$$

(1.4) $\iff$

$$h'\left(\left(1-p_t^f\right)s_t + n_t^s\right) = u'\left(c_t\right)\left(1-t_t\right)w_t + \mu_t^h - \beta\mu_{t+1}^h\left(1-\rho\right)$$

(1.5) $\iff$

$$h'\left(\left(1-p_t^f\right)s_t + n_t^s\right)\left(1-p_t^f\right) = u'\left(c_t\right)\left(1-p_t^f\right)b_t^{ue} - \mu_t^h p_t^f$$

$\iff$

$$h'\left(\left(1-p_t^f\right)s_t + n_t^s\right)\left(1-p_t^f\right) - u'\left(c_t\right)\left(1-p_t^f\right)b_t^{ue} = -\mu_t^h p_t^f$$

$\iff$

$$u'\left(c_t\right)\left(1-p_t^f\right)b_t^{ue} - h'\left(\left(1-p_t^f\right)s_t + n_t^s\right)\left(1-p_t^f\right) = \mu_t^h p_t^f$$

$\iff$

$$\frac{u'\left(c_t\right)\left(1-p_t^f\right)b_t^{ue} - h'\left(\left(1-p_t^f\right)s_t + n_t^s\right)\left(1-p_t^f\right)}{p_t^f} = \mu_t^h$$

$\iff$

$$\left(u'\left(c_t\right)b_t^{ue} - h'\left(\left(1-p_t^f\right)s_t + n_t^s\right)\right)\frac{1-p_t^f}{p_t^f} = \mu_t^h$$

And

$$\left(u'\left(c_{t+1}\right)b_{t+1}^{ue} - h'\left(\left(1-p_{t+1}^f\right)s_{t+1} + n_{t+1}^s\right)\right)\frac{1-p_{t+1}^f}{p_{t+1}^f} = \mu_{t+1}^h$$

$$\therefore h'(lfp_t) = u'(c_t)(1-t_t)w_t + \left(u'(c_t)b_t^{ue} - h'(lfp_t)\right)\frac{1-p_t^f}{p_t^f}$$
$$-\beta\left(u'(c_{t+1})b_{t+1}^{ue} - h'(lfp_{t+1})\right)\frac{1-p_{t+1}^f}{p_{t+1}^f}(1-\rho)$$

$$\iff$$

$$h'(lfp_t) = u'(c_t)(1-t_t)w_t + u'(c_t)b_t^{ue}\frac{1-p_t^f}{p_t^f} - h'(lfp_t)\frac{1-p_t^f}{p_t^f}$$
$$-\beta\frac{1-p_{t+1}^f}{p_{t+1}^f}(1-\rho)u'(c_{t+1})b_{t+1}^{ue} + \beta\frac{1-p_{t+1}^f}{p_{t+1}^f}(1-\rho)h'(lfp_{t+1})$$

$$\iff$$

$$h'(lfp_t) + h'(lfp_t)\frac{1-p_t^f}{p_t^f} = u'(c_t)\left((1-t_t)w_t + b_t^{ue}\frac{1-p_t^f}{p_t^f}\right)$$
$$-\beta\frac{1-p_{t+1}^f}{p_{t+1}^f}(1-\rho)u'(c_{t+1})b_{t+1}^{ue} + \beta\frac{1-p_{t+1}^f}{p_{t+1}^f}(1-\rho)h'(lfp_{t+1})$$

$$\iff$$

$$h'(lfp_t)\left(1 + \frac{1-p_t^f}{p_t^f}\right) = u'(c_t)\left((1-t_t)w_t + b_t^{ue}\frac{1-p_t^f}{p_t^f}\right)$$
$$-\beta\frac{1-p_{t+1}^f}{p_{t+1}^f}(1-\rho)u'(c_{t+1})b_{t+1}^{ue} + \beta\frac{1-p_{t+1}^f}{p_{t+1}^f}(1-\rho)h'(lfp_{t+1})$$

$$\iff$$

$$h'(lfp_t)\left(1 + \frac{1}{p_t^f} - 1\right) = u'(c_t)\left((1-t_t)w_t + b_t^{ue}\frac{1-p_t^f}{p_t^f}\right)$$
$$-\beta\frac{1-p_{t+1}^f}{p_{t+1}^f}(1-\rho)u'(c_{t+1})b_{t+1}^{ue} + \beta\frac{1-p_{t+1}^f}{p_{t+1}^f}(1-\rho)h'(lfp_{t+1})$$

$$\iff$$

$$h'(lfp_t)\frac{1}{p_t^f} = u'(c_t)\left((1-t_t)w_t + b_t^{ue}\frac{1-p_t^f}{p_t^f}\right)$$
$$-\beta\frac{1-p_{t+1}^f}{p_{t+1}^f}(1-\rho)u'(c_{t+1})b_{t+1}^{ue} + \beta\frac{1-p_{t+1}^f}{p_{t+1}^f}(1-\rho)h'(lfp_{t+1})$$

$\iff$

$$h'(lfp_t) = u'(c_t)\left(p_t^f(1-t_t)w_t + b_t^{ue}\left(1-p_t^f\right)\right) - \beta p_t^f \frac{1-p_{t+1}^f}{p_{t+1}^f}(1-\rho)u'(c_{t+1})b_{t+1}^{ue} + \beta p_t^f \frac{1-p_{t+1}^f}{p_{t+1}^f}(1-\rho)h'(lfp_{t+1})$$

$\iff$

$$\frac{h'(lfp_t)}{u'(c_t)} = \left(p_t^f(1-t_t)w_t + b_t^{ue}\left(1-p_t^f\right)\right) - \frac{\beta p_t^f \frac{1-p_{t+1}^f}{p_{t+1}^f}(1-\rho)u'(c_{t+1})b_{t+1}^{ue}}{u'(c_t)} + \frac{\beta p_t^f \frac{1-p_{t+1}^f}{p_{t+1}^f}(1-\rho)h'(lfp_{t+1})}{u'(c_t)}$$

Assume $b_t^{ue} = b_{t+1}^{ue} = b$

$\iff$

$$\frac{h'(lfp_t)}{u'(c_t)} = \left(p_t^f(1-t_t)w_t + b\left(1-p_t^f\right)\right) + p_t^f \frac{1-p_{t+1}^f}{p_{t+1}^f}(1-\rho)\left(\frac{\beta h'(lfp_{t+1})}{u'(c_t)} - \frac{\beta u'(c_{t+1})b}{u'(c_t)}\right) \tag{1.6}$$

$\iff$

$$\frac{h'(lfp_t)}{u'(c_t)} = \left(p_t^f(1-t_t)w_t + b\left(1-p_t^f\right)\right) + p_t^f \frac{1-p_{t+1}^f}{p_{t+1}^f}(1-\rho)\left(\frac{\beta u'(c_{t+1})}{u'(c_t)}\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \frac{\beta u'(c_{t+1})b}{u'(c_t)}\right)$$

d.

A job is an asset for consumers.

All these relative to the value of unemployment benefit $b$.

The pricing kernel: $\frac{\beta u'(c_{t+1})}{u'(c_t)}$, working as a type of asset long lasting relationship. The opportunity cost across periods. If workers found a job in one period, they will not need to search for jobs in the next period if they keep their jobs.

The asset value (of a job) is the avoided disutility from searching a job in period $t+1$: $\frac{h'(lfp_{t+1})}{u'(c_{t+1})}$.

The adjustment for relative probability of not finding a job next period: $\frac{1-p_{t+1}^f}{p_{t+1}^f}$.

If $p_{t+1}^{f}$ increases, $-\dfrac{h'\left(lfp_t\right)}{u'\left(c_t\right)}$ increases (the absolute value decreases.) Higher probability of successfully finding work in the future encourages workers to wait for a period.

If $b$ increases, $-\dfrac{h'\left(lfp_t\right)}{u'\left(c_t\right)}$ decreases (usually it is the case). Consumers will be better off and consume more.

If $\beta$ increases, $-\dfrac{h'\left(lfp_t\right)}{u'\left(c_t\right)}$ decreases. Workers will be less impatient, and attach importance to the longer term. It would be easier for workers to find jobs successfully as time goes by.

e.

$$\boxed{n_{t+1} = \left(1-\rho\right) n_t + v_{t+1} q_{t+1} \qquad (1.7)}$$

$n_t$ represents labor demand $n_t^d$.

The labor in the $t+1$ period is determined by two terms: for the fraction of keeping their job, they stay in the labor market in the next period $(t+1)$. For the vacancy, the firm will find the worker with probability $q$.

In the classical model we assume that the firm can make the best of their labor demand, and we didn't take the matching into consideration (in other words, firm only consider $n^d$ and we suppose the equilibrium $n^s = n^d$).

Similar to the consumer's problem, the difference between the dynamic model and the static model is that when we introduce $\rho$, there are workers who stay in their job for the next period. Variables could be different in each time period, so we have the subscript.

f.

The Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \left( \frac{1}{\prod_{i=0}^{t} (1+r_i)} \Big( (A_t f(n_t) - w_t n_t - \omega_t v_t) - \mu_t^f (n_t - (1-\rho) n_{t-1} - v_t q_t) \Big) \right)$$

$$\begin{aligned}
&= \dots \\
&\quad + A_t f(n_t) - w_t n_t - \omega_t v_t \\
&\qquad - \mu_t^f (n_t - (1-\rho) n_{t-1} - v_t q_t) \\
&\quad + \frac{1}{1+r_t} A_{t+1} f(n_{t+1}) - w_{t+1} n_{t+1} - \omega_{t+1} v_{t+1} \\
&\qquad - \frac{1}{1+r_t} \mu_{t+1}^f (n_{t+1} - (1-\rho) n_t - v_{t+1} q_{t+1}) \\
&\quad + \dots
\end{aligned}$$

The choice variables: $\boxed{n_{t+1},\, v_{t+1}}$ (in the period $t$).

The given variables: $\boxed{r_t,\, A_t,\, q_t,\, w_t,\, \rho,\, \omega}$.

g.

FOCs:
w.r.t $n_t$:

$$A_t f'(n_t) - w_t - \mu_t^f + \frac{1}{1+r_t} \mu_{t+1}^f (1-\rho) = 0 \tag{1.8}$$

w.r.t $v_t$:

$$-\omega_t + \mu_t^f q_t = 0 \tag{1.9}$$

(1.9) $\iff$

$\mu_t^f q_t = \omega_t$

$\iff$

$\mu_t^f = \dfrac{\omega_t}{q_t}$

And

$\mu_{t+1}^f = \dfrac{\omega_{t+1}}{q_{t+1}}$

Plug into (1.8), we get:

$$A_t f'(n_t) - w_t - \frac{\omega_t}{q_t} + \frac{1}{1+r_t}\frac{\omega_{t+1}}{q_{t+1}}(1-\rho) = 0$$

$$\frac{\omega_t}{q_t} = A_t f'(n_t) - w_t + \frac{1}{1+r_t}\frac{\omega_{t+1}}{q_{t+1}}(1-\rho) \tag{1.10}$$

h.

Vacancy Posting Condition is determined by:

For the workers already hired by the firm, maximize the profit in the factory.

Make the decision of keeping workers and posting new vacancies, across periods.

If $q^f_{t+1}$ increases, the marginal benefit of posting a vacancy decreases. Lower probability of successfully hiring a suitable job candidate discourages the firm from posting.

If $\rho$ increases, the marginal benefit of posting a vacancy decreases. The firm has to post less jobs due to the job retention "constraint".

If $r_t$ increases, the marginal benefit of posting a vacancy decreases. Firms will be more impatient, attach importance to the workers' performance in the short run, thus increasing the vacancy posted once the firm is unsatisfied with workers.

i.

The matching function:

$$m(s_t, v_t) = s_t^{\gamma} v_t^{1-\gamma} \tag{1.11}$$

Market tightness:

$$\theta_t = \frac{v_t}{s_t} \tag{1.12}$$

By definition:

$$m(s_t, v_t) = s_t p_t^f \tag{1.13}$$

$$m(s_t, v_t) = v_t q_t \tag{1.14}$$

In equilibrium:

$$n_t = n_t^s \tag{1.15}$$

Good market clearing condition:

$$c_t = A_t f(n_t) \tag{1.16}$$

We also have:

$$\frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{1}{1+r_t} \tag{1.17}$$

By (1.1), (1.7) & (1.15) we get:

$$(1-\rho) n_{t-1} + p_t^f s_t = (1-\rho) n_{t-1} + v_t q_t$$

$$p_t^f s_t = v_t q_t \equiv m(s_t, v_t)$$

$$\therefore p_t^f = \frac{s_t^\gamma v_t^{1-\gamma}}{s_t} = \left(\frac{v_t}{s_t}\right)^{1-\gamma} = \theta_t^{1-\gamma}$$

$$q_t = \frac{s_t^\gamma v_t^{1-\gamma}}{v_t} = \left(\frac{s_t}{v_t}\right)^{\gamma} = \left(\frac{v_t}{s_t}\right)^{-\gamma} = \theta_t^{-\gamma}$$

$$p_t^f = \theta_t^{1-\gamma} \tag{1.18}$$

$$q_t = \theta_t^{-\gamma} \tag{1.19}$$

We assume

$$b = 0 \tag{1.20}$$

$$t_t = 0 \tag{1.21}$$

Plug (1.20) & (1.21) into (1.2) we have:

$$c_t = w_t n_t^s \tag{1.22}$$

Plug (1.20) & (1.21) into (1.6) we have:

$$\frac{h'(lfp_t)}{u'(c_t)} = p_t^f w_t + \frac{\beta p_t^f \dfrac{1-p_{t+1}^f}{p_{t+1}^f}(1-\rho) h'(lfp_{t+1})}{u'(c_t)} \tag{1.23}$$

By (1.18) & (1.23):

$$\frac{h'(lfp_t)}{u'(c_t)} = \theta_t^{1-\gamma} w_t + \frac{\beta \theta_t^{1-\gamma} \dfrac{1-\theta_{t+1}^{1-\gamma}}{\theta_{t+1}^{1-\gamma}}(1-\rho) h'(lfp_{t+1})}{u'(c_t)} \tag{1.24}$$

By (1.10) & (1.19) & (1.17):

$$\frac{\omega_t}{\theta_t^{-\gamma}} = A_t f'(n_t) - w_t + \frac{\beta u'(c_{t+1})}{u'(c_t)} \frac{\omega_{t+1}}{\theta_{t+1}^{-\gamma}} (1-\rho) \tag{1.25}$$

By (1.16) & (1.22):

$$A_t f(n_t) = w_t n_t^s$$

$$\therefore A_t f'(n_t) = \frac{\partial w_t}{\partial n_t} n_t + w_t$$

$$\frac{\omega_t}{\theta_t^{-\gamma}} = A_t f'(n_t) - w_t + \frac{\beta u'(c_{t+1})}{u'(c_t)} \frac{\omega_{t+1}}{\theta_{t+1}^{-\gamma}} (1-\rho) \tag{1.26}$$

With (1.15), (1.22), (1.24) & (1.26):

$$\boxed{\begin{aligned} &\frac{h'(lfp_t)}{u'(w_t n_t)} = \theta_t^{1-\gamma} w_t + \frac{\beta \theta_t^{1-\gamma} \frac{1-\theta_{t+1}^{1-\gamma}}{\theta_{t+1}^{1-\gamma}} (1-\rho) h'(lfp_{t+1})}{u'(w_t n_t)} \\ &\frac{\omega_t}{\theta_t^{-\gamma}} = A_t f'(n_t) - w_t + \frac{\beta u'(w_{t+1} n_{t+1})}{u'(w_t n_t)} \frac{\omega_{t+1}}{\theta_{t+1}^{-\gamma}} (1-\rho) \end{aligned}}$$

## 2. Optimal Policy in the NK model

a.

Solving consumer's problem $\Longrightarrow$ Solving retail firm's problem $\Longrightarrow$ Solving wholesale firm's problem and get NKPC $\Longrightarrow$ Introducing government's problem $\Longrightarrow$ Summarize equilibrium conditions and substitute, solving system of equation $\Longrightarrow$ Imposing steady state $\Longrightarrow$ Maximizing government's goal $\Longrightarrow$ Modifying conditions to get optimal policy.

b.

Consumer's problem:

$$\boxed{\begin{aligned} &\max \sum_{s=0}^{\infty} \beta^s u(c_t, 1-n_t) \\ &\text{subject to} \\ &P_t c_t + P_t^b B_t + S_t a_t = P_t w_t n_t + B_{t-1} + (S_t + D_t) a_{t-1}. \end{aligned}}$$

The Lagrangian:

$$
\begin{aligned}
\mathcal{L} = \sum_{s=0}^{\infty} \beta^s u\left(c_t, 1-n_t\right) \\
& - \lambda_t \left(P_t c_t + P_t^b B_t + S_t a_t - P_t w_t n_t - B_{t-1} - \left(S_t + D_t\right) a_{t-1}\right) \\
& - \beta \lambda_{t+1} \left(P_{t+1} c_{t+1} + P_{t+1}^b B_{t+1} + S_{t+1} a_{t+1} - P_{t+1} w_{t+1} n_{t+1} - B_t - \left(S_{t+1} + D_{t+1}\right) a_t\right) \\
& + \ldots
\end{aligned}
$$

FOCs

w.r.t $c_t$:

$$u_1\left(c_t, 1-n_t\right) - \lambda_t P_t = 0 \tag{2.1}$$

w.r.t $n_t$:

$$u_2\left(c_t, 1-n_t\right) + \lambda_t P_t w_t = 0 \tag{2.2}$$

where $u_2 = u_l = -u_n$

w.r.t $B_t$:

$$-\lambda_t P_t^b + \beta \lambda_{t+1} = 0 \tag{2.3}$$

w.r.t $a_t$:

$$-\lambda_t S_t + \beta \lambda_{t+1}\left(S_{t+1} + D_{t+1}\right) = 0 \tag{2.4}$$

w.r.t $\lambda_t$:

$$P_t c_t + P_t^b B_t + S_t a_t = P_t w_t n_t + B_{t-1} + \left(S_t + D_t\right) a_{t-1} \tag{2.5}$$

c.

Resource constraint:

$$c_t + \frac{\psi}{2}\left(\pi_t\right)^2 = n_t \tag{2.6}$$

Since we assume CRS production function

$$y_t = n_t,$$

resource constraint implies that the market clears. Otherwise, resources haven't been used or overused, and both consumer and firm can make the best of this situation and become better.

d.

Divide (2.2) with (2.1) we have:

$$\boxed{\frac{u_2(c_t, 1-n_t)}{u_1(c_t, 1-n_t)} = w_t \qquad (2.7)}$$

By (2.1) we have:

$$u_1(c_t, 1-n_t) = \lambda_t P_t \tag{2.8}$$

$$u_1(c_{t+1}, 1-n_{t+1}) = \lambda_{t+1} P_{t+1} \tag{2.9}$$

Divide (2.8) with (2.9) we have:

$$\frac{u_1(c_t, 1-n_t)}{u_1(c_{t+1}, 1-n_{t+1})} = \frac{\lambda_t P_t}{\lambda_{t+1} P_{t+1}} \tag{2.10}$$

By the definition of the inflation, we have:

$$1 + \pi_{t+1} = \frac{P_{t+1}}{P_t} \tag{2.11}$$

By (2.10) and (2.11) we get:

$$\frac{u_1(c_t, 1-n_t)}{u_1(c_{t+1}, 1-n_{t+1})} = \frac{\lambda_t}{\lambda_{t+1}} \cdot \frac{1}{(1+\pi_{t+1})} \tag{2.12}$$

By (2.3) we have:

$$\lambda_t P_t^b = \beta \lambda_{t+1}$$

$$\iff$$

$$\lambda_t = \frac{\beta \lambda_{t+1}}{P_t^b}$$

$$\frac{1}{P_t^b} = \frac{\lambda_t}{\beta \lambda_{t+1}} \tag{2.13}$$

By the definition of the interest rate, we have:

$$\frac{1}{P_t^b} = 1 + i_t \tag{2.14}$$

Plug in (2.13) we get:

$$\frac{\lambda_t}{\beta \lambda_{t+1}} = 1 + i_t$$

Plug this equation back to the (2.12) we get:

$$\boxed{\frac{u_1\left(c_t, 1-n_t\right)}{\beta u_1\left(c_{t+1}, 1-n_{t+1}\right)}=\frac{1+i_t}{1+\pi_{t+1}} \qquad (2.15)}$$

We also have:

$$mc_t = w_t \tag{2.16}$$

Because $mp_t = f'\left(n_t\right) = \dfrac{\partial n_t}{\partial n_t} = 1 > mc_t$

Labor demand condition:

$$\boxed{w_t < 1 \qquad (2.17)}$$

By the previous chapter and A5, we have the NKPC:

$$\boxed{\frac{1}{1-\varepsilon}\left(1-\varepsilon mc_t\right) n_t - \psi \pi_t\left(1+\pi_t\right) + \beta \psi \pi_{t+1}\left(1+\pi_{t+1}\right) = 0 \qquad (2.18)}$$

And the resource constraint (2.6):

$$\boxed{c_t + \frac{\psi}{2}\left(\pi_t\right)^2 = n_t}$$

e.

From (2.6) & (2.7) & (2.16) we get:

$$\frac{u_2\left(c_t, 1-c_t-\dfrac{\psi}{2}\left(\pi_t\right)^2\right)}{u_1\left(c_t, 1-c_t-\dfrac{\psi}{2}\left(\pi_t\right)^2\right)} = mc_t \tag{2.19}$$

Substitute (2.19) into (2.15) we have:

$$\frac{u_1\left(c_t, 1-c_t-\dfrac{\psi}{2}\left(\pi_t\right)^2\right)}{\beta u_1\left(c_{t+1}, 1-c_{t+1}-\dfrac{\psi}{2}\left(\pi_{t+1}\right)^2\right)} = \frac{1+i_t}{1+\pi_{t+1}} \tag{2.20}$$

Substitute (2.19) into (2.18) we have:

$$\frac{1}{1-\varepsilon}\left(1-\frac{\varepsilon u_2\left(c_t, 1-c_t-\frac{\psi}{2}\pi_t^2\right)}{u_1\left(c_t, 1-c_t-\frac{\psi}{2}\pi_t^2\right)}\right)\left(c_t+\frac{\psi}{2}\pi_t^2\right)-\psi\pi_t\left(1+\pi_t\right)+\beta\psi\pi_{t+1}\left(1+\pi_{t+1}\right)=0 \tag{2.21}$$

In the steady state, rewriting (2.19) & (2.20) & (2.21):

$$\boxed{\frac{u_2\left(c, 1-c-\frac{\psi}{2}\pi^2\right)}{u_1\left(c, 1-c-\frac{\psi}{2}\pi^2\right)}=w} \tag{2.22}$$

$$\boxed{\frac{u_1\left(c, 1-c-\frac{\psi}{2}\pi^2\right)}{\beta u_1\left(c, 1-c-\frac{\psi}{2}\pi^2\right)}=\frac{1+i}{1+\pi}} \tag{2.23}$$

$$\frac{1}{1-\varepsilon}\left(1-\frac{\varepsilon u_2\left(c, 1-c-\frac{\psi}{2}\pi^2\right)}{u_1\left(c, 1-c-\frac{\psi}{2}\pi^2\right)}\right)\left(c+\frac{\psi}{2}\pi^2\right)-\psi\pi\left(1+\pi\right)+\beta\psi\pi\left(1+\pi\right)=0 \tag{2.24}$$

Also,

$$g=\pi \tag{2.25}$$

in a steady state. Plug into (2.24):

$$\boxed{\frac{1}{1-\varepsilon}\left(1-\frac{\varepsilon u_2\left(c, 1-c-\frac{\psi}{2}g^2\right)}{u_1\left(c, 1-c-\frac{\psi}{2}g^2\right)}\right)\left(c+\frac{\psi}{2}g^2\right)-\psi g\left(1+\pi\right)+\beta\psi g\left(1+g\right)=0} \tag{2.26}$$

f.

Optimal monetary policy problem:

$$\boxed{\max_g \sum_{s=0}^{\infty} \beta^s u\left(\bar{c}(g), 1-\bar{c}(g)-\frac{\psi}{2}g^2\right)}$$

$\iff$

$$\max_g \frac{u\left(\bar{c}(g), 1-\bar{c}(g)-\frac{\psi}{2}g^2\right)}{1-\beta}$$

The policy tries to maximize the utility of the society. In the representative case, it is the same with maximizing consumer's utility.

The choice variable is $g$.

g.

FOC w.r.t $g$:

$$u_1 \frac{\partial \bar{c}(g)}{\partial g} + u_2 \cdot \frac{\partial\left(1-\bar{c}(g)-\frac{\psi}{2}g^2\right)}{\partial g} = 0$$

$\iff$

$$u_1 c'(g) + u_2\left(-c'(g) - \psi g\right) = 0$$

$\iff$

$$u_1 c'(g) = u_2\left(c'(g) + \psi g\right)$$

$\iff$

$$\frac{u_2\left(c(g), 1-c(g)-\frac{\psi}{2}g^2\right)}{u_1\left(c(g), 1-c(g)-\frac{\psi}{2}g^2\right)} = \frac{c'(g)}{c'(g)+\psi g}$$

h.

Introduce $\varepsilon$ in the steady state NKPC (2.26):

$$\frac{1}{1-\varepsilon}\left(1-\frac{\varepsilon u_2\left(c, 1-c-\frac{\psi}{2}g^2\right)}{\boxed{\varepsilon} u_1\left(c, 1-c-\frac{\psi}{2}g^2\right)}\right)\left(c+\frac{\psi}{2}g^2\right) - \psi g(1+\pi) + \beta\psi g(1+g) = 0$$

And we assume $\psi = 0$.

By inspection, $\frac{u_2(c(g), 1-c(g))}{u_1(c(g), 1-c(g))} = 1$ solves this problem. That is, $g^* = 0$.

Zero inflation eliminates the menu cost, which has no benefit. Thus there is no need to change prices in this model, and the private market achieves efficiency.