A Truthful Online Mechanism for Virtual Cluster Provisioning in Geo-Distributed Clouds

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Motivation

- VM auction → VC auction

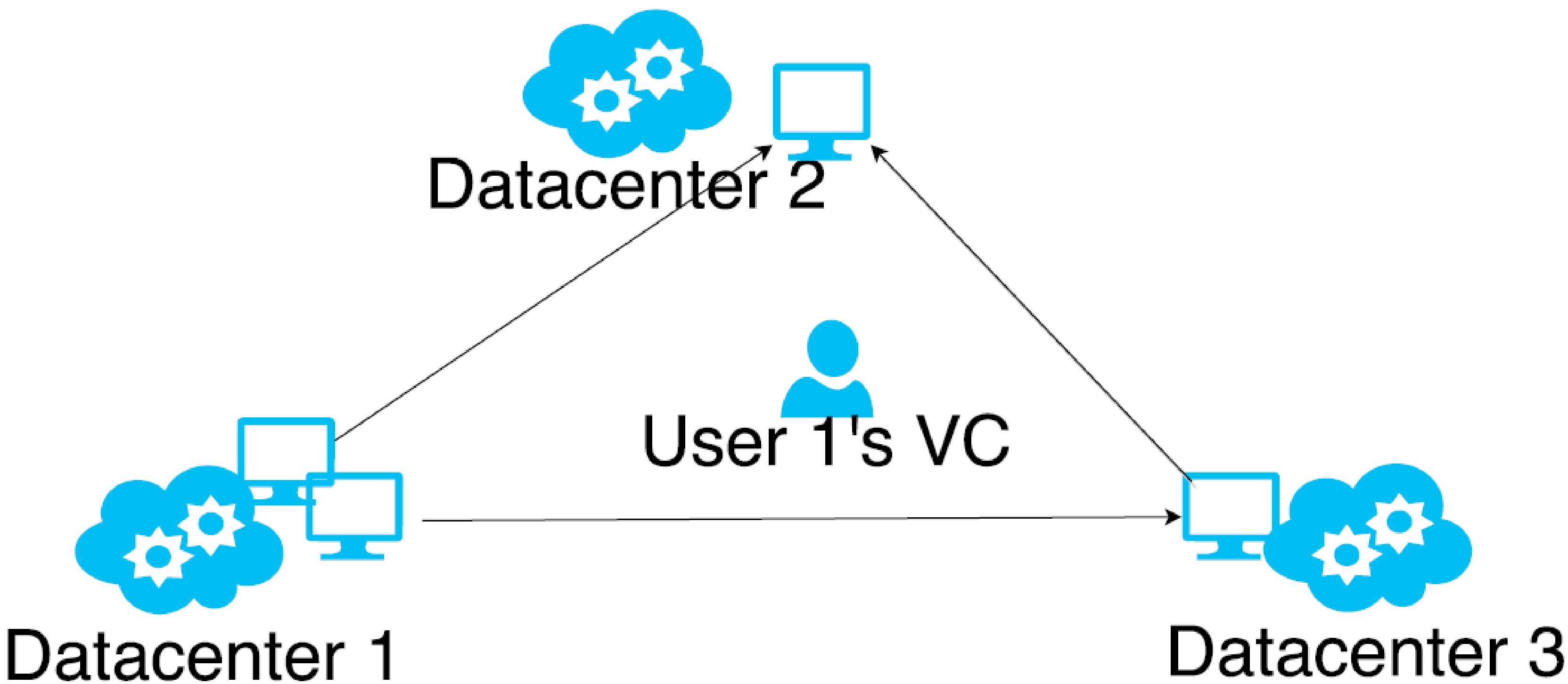


Fig. 1. An example of virtual cluster provisioning.

Model

N users
V_n VMs
P DCs
R resources

b_n: valuation (bid)

x_n: 0/1, accept the user or not

$$\text{maximize} \quad \sum_{n\in[N]} b_n x_n \qquad (1)$$

Z: 0/1, put user n's VM v to DC p or not

$$\text{s.t.} \quad \sum_{p\in[P]} z^n_{v,p} = x_n \quad \forall n \in [N], v \in [V_n] \quad (1a)$$

Resource constraint

$$\sum_{n\in N_t} \sum_{v\in[V_n]} z^n_{v,p} a^n_{v,r} \le \hat{A}_{p,r} \quad \forall p \in [P], r \in [R], t \quad (1b)$$

$$\sum_{n\in N_t} \sum_{v\in[V_n]} \sum_{v'\in[V_n]} z^n_{v,p}(1 - z^n_{v',p})\Gamma^n_{v,v'} \le \hat{B}_{p,out} \quad \forall p \in [P], t \quad (1c)$$

$$\sum_{n\in N_t} \sum_{v\in[V_n]} \sum_{v'\in[V_n]} z^n_{v,p}(1 - z^n_{v',p})\Gamma^n_{v',v} \le \hat{B}_{p,in} \quad \forall p \in [P], t \quad (1d)$$

$$x_n, z^n_{v,p} \in \{0,1\} \quad \forall p \in [P], n \in [N], v \in [V_n] \quad (1e)$$

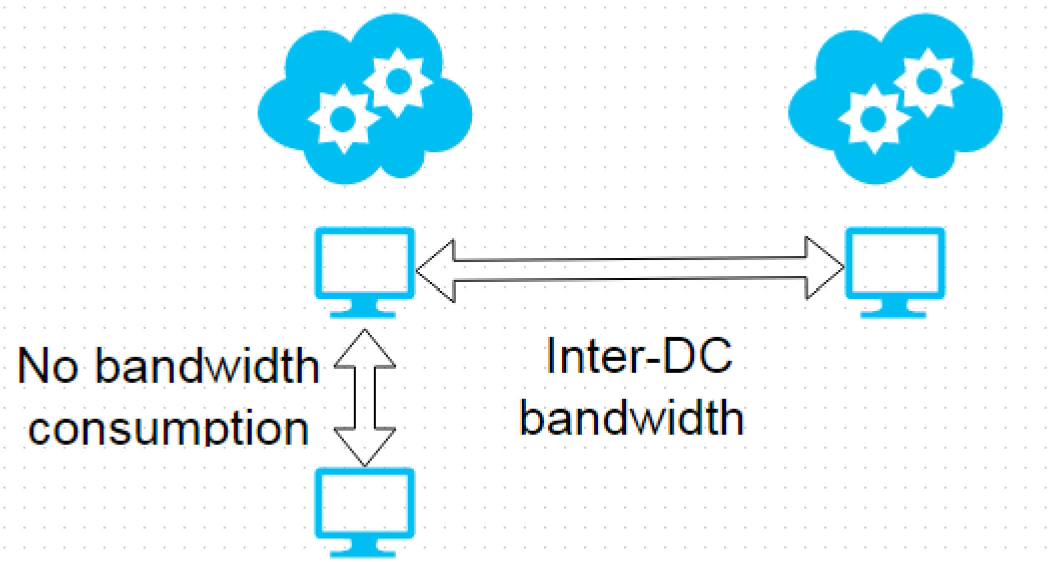
Model

Traffic from v to v'

Out-bound bandwidth at DC p

$$\sum_{n\in N_t}\sum_{v\in[V_n]}\sum_{v'\in[V_n]} z^n_{v,p}(1-z^n_{v',p})\Gamma^n_{v,v'} \leq \hat{B}_{p,out} \quad \forall p\in[P],t \quad (1c)$$

$$\sum_{n\in N_t}\sum_{v\in[V_n]}\sum_{v'\in[V_n]} \boxed{z^n_{v,p}(1-z^n_{v',p})}\Gamma^n_{v',v} \leq \hat{B}_{p,in} \quad \forall p\in[P],t \quad (1d)$$



Difficulties

- Online
- Auction
- NP-hard
- Quadratic

Online auction framework

- Re-formulate the problem:

- VM mapping scheme $\leftrightarrow$ Bundle β
- M=R+2 types of resources
- $r(n,\beta,m,p,t)$
- $y_{n,\beta}$: 0/1 accept user n's bundle β or not

Online auction framework

$$\text{maximize} \quad \sum_{n \in [N]} \sum_{\beta \in \mathbb{B}_n} b_n y_{n,\beta} \tag{2}$$

$$\text{s.t.} \quad \sum_{\beta \in \mathbb{B}_n} y_{n,\beta} \leq 1 \qquad \forall n \in [N] \tag{2a}$$

$$\sum_{n \in [N]} \sum_{\beta \in \mathbb{B}_n} r(n, \beta, m, p, t) y_{n,\beta} \leq 1 \qquad \forall m, t, p \tag{2b}$$

$$y_{n,\beta} \geq 0 \qquad \forall n \in [N], \beta \in \mathbb{B}_n \tag{2c}$$

Online auction framework

- Virtual unit cost:

- $c_{m,p}(t,n) = \mu^{Consumed\_Amount} - 1$
- Cost for resource m at DC p at time t for user n
- Increase exponentially with the consumed amount
- Reflect the shortage of the resource

Online auction framework

- Cost of a bundle $C(\beta,n)$
- Suppose an oracle can choose a "cheap" bundle for us
- Compare the bundle cost $C(\beta,n)$ with bundle valuation b_n
- User n's payment = $C(\beta,n)$ if accepted

Online auction framework

- Conclusion of the online algorithm:

- Truthful and individual rational
- No constraint violation
- $(1+2\alpha \log \mu)$-competitive in social welfare
- α is the approximation ratio of the oracle

One-round oracle

- Find a cheap bundle with (1+ε)-optimal cost

- Combine some unit cost constants
- Let $w_{v,v',p} = z_{v,p}(1 - z_{v',p})$

$$\text{minimize} \quad \sum_{p\in[P],v\in[V]} c_{v,p} z_{v,p} + \sum_{p\in[P],v,v'\in[V]} c_{v,v',p} w_{v,v',p} \quad (3)$$

One-round oracle

$$\mathbb{U}: \quad \sum_{p \in [P]} z_{v,p} = 1 \quad \forall v \in [V] \quad (3a)$$

$$\sum_{v \in [V]} a_{v,r} z_{v,p} \leq A_{p,r} \quad \forall p \in [P], r \in [R] \quad (3b)$$

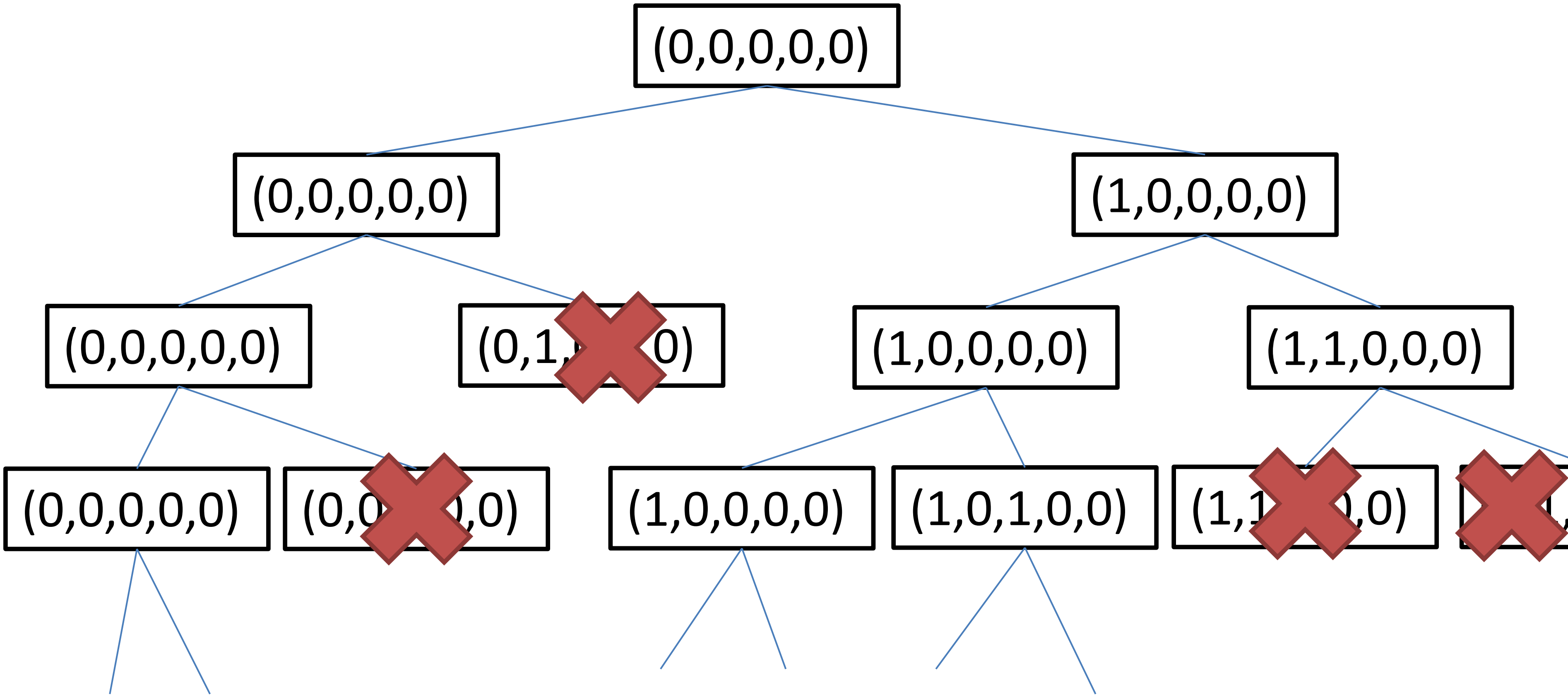
$$\sum_{v,v' \in [V]} \Gamma_{v,v'} w^n_{v,v',p} \leq B_{p,out} \quad \forall p \in [P] \quad (3c)$$

$$\sum_{v,v' \in [V]} \Gamma_{v',v} w^n_{v,v',p} \leq B_{p,in} \quad \forall p \in [P] \quad (3d)$$

$$z_{v,p} - z_{v',p} \leq w_{v,v',p} \quad \forall p \in [P], v, v' \in [V] \quad (3e)$$

$$z_{v,p}, w_{v,v',p} \in \{0, 1\} \quad \forall p \in [P], v \in [V] \quad (3f)$$

Exact Algorithm



Pareto optimal set

Exact Algorithm

- “Dominate”: use less resource to achieve better objective function (cost)
- Pareto optimal solution: not dominated by other solutions
- Property of Pareto set: “inheritable”
- Find a Pareto set in U and optimal solution in U
- Time complexity $O(|P_i|^2 P^2V^4)$

Exact Algorithm

- Problem: starting vector infeasible
- Problem: intermediate vector infeasible
- Expand the solution space
- Find Pareto set in U', and the optimum in U

$$\mathbb{U}': \quad \text{Constraint } (3b)(3c)(3d)(3f) \text{ and}$$

$$\sum_{p \in [P]} z_{v,p} \leq 1 \quad \forall v \in [V] \quad (3g)$$

$$\sum_{p \in [P]} -z_{v,p} \leq 0 \quad \forall v \in [V] \quad (3h)$$

$$z_{v,p} - z_{v',p} - w_{v,v',p} \leq 1 \quad \forall p \in [P], v, v' \in [V] \quad (3i)$$

Random perturbation

- $|P_i|$ can be very large
- Random perturbation, use a slightly different objective function $\text{Minimize } \boldsymbol{c}' \cdot \boldsymbol{u} \text{ for } \boldsymbol{u} \in \mathbb{U}'.$
- Get solution u by running the exact algorithm
- Near-optimal solution: u^f (fractional)
- Smooth analysis guarantees poly-time in expectation

Random decomposition

- Find a distribution/decomposition equal/smaller to u^f

$$Pr = \begin{cases} 1 - \boldsymbol{\theta} \cdot \boldsymbol{u}' & \text{For } \boldsymbol{u}' \\ \boldsymbol{\theta} \cdot \boldsymbol{u}'/I & \text{For } \boldsymbol{v}_i, \forall i \in [I] \end{cases}$$