The Performance of Deferred-Acceptance Auctions

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Outline

- Briefly define the deferred-acceptance auction.
- How good are the DA auctions?
- Formally define the DA auctions.
- Locally highest bid (LHB) mechanism
- DA auction implementation of LHB mechanism
- Conclusion

Brief Description of Deferred-acceptance Auction (DA Auction)

- Traditional greedy algorithms [Lehmann et al. 2002] accept the most promising candidate.
- By analogy, the deferred-acceptance auctions use the "backward greedy algorithms", which reject the least promising candidate.
- It is iterated, rejecting the candidates one by one.
- Use the "scoring functions" to decide which candidate should be rejected.

How Good Are the DA Auctions?

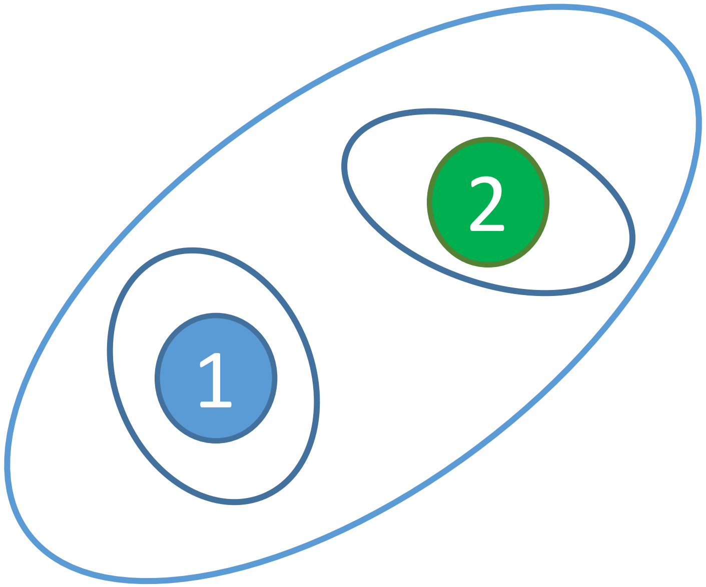
- Milgrom and Segal [2014] prove these properties:
  - Incentive compatible
  - Weakly group-strategyproof
  - The dominant-strategy outcome of a DA auction is the same as the unique Nash equilibrium.

(Example) Group-strategy (in VCG mechanism)

- A = { item1, item2 }
- B = { item1 }
- C = { item2 }

- $b_A = 1$
- $b_B, b_C = \epsilon$
- (ε: a very small value)

- $b'_A = 1$
- $b'_B, b'_C = 1 + \epsilon$



Formal Definition of DA Auctions (1/2)

- A DA auction operates in stage $t \geq 1$. In each stage t, a set of bidders $A_t \subseteq N$.
- Initially, the $A_1 = N$.
- Scoring rules $\sigma_i^{A_t}(b_i, b_{N \setminus A_t})$

  that are non-decreasing in the first argument.

Formal Definition of DA Auctions (2/2)

- Stage t proceeds as follows
- If A_t is feasible, accept the bidders in A_t, and charge each bidder in A_t its threshold price $p_i(b_i) = \inf\{b'_i | i \in A(b'_i, b_{-i})\}$ ($A(b'_i, b_{-i})$ denotes the set of bidders that have been accepted when bid b'.
- Otherwise, set $A_{t+1} = A_t \setminus \{i\}$ where bidder $i \in \arg\min_{i \in A_t}\{\sigma_i^{A_t}(b_i, b_{N \setminus A_t})\}$

Problem Model (Single-minded Combinatorial Auctions)

- M denotes the set of m heterogeneous items.
- N denotes the set of n bidders.
- Each bidder bids for its required bundle (e.g. multiple items).
- Bidders are single-minded.

Two useful concepts

- Bundle graph G_b
  - an edge-weighted hypergraph whose vertices correspond to the set of items and whose hyperedges correspond to the \$n\$ bundles of the single-minded bidders.
- Conflict graph G_c
  - a vertex-weighted graph whose vertices correspond to the set of bidders, and an edge (i,j) exists iff the bundles of bidder i and j are in conflict.

Locally Highest Bid (LHB) Mechanism – (Brief Description)

- First Phase
  - Prunes the bundle graph by greedily rejecting all but the local highest bid.
  - (one bid should only be considered once.)
- Second Phase
  - Translate the resulting hypergraph into a bipartite graph, and compute a maximum weight matching in this graph

- (NB: d represents the maximum number of items which a bundle includes)

Properties of LHB Mechanism

- **Lemma.** The conflict graph (in Phase 2) is a forest of path graphs.

- **Theorem.** The LHB mechanism guarantees a 2(d-1)-approximation.

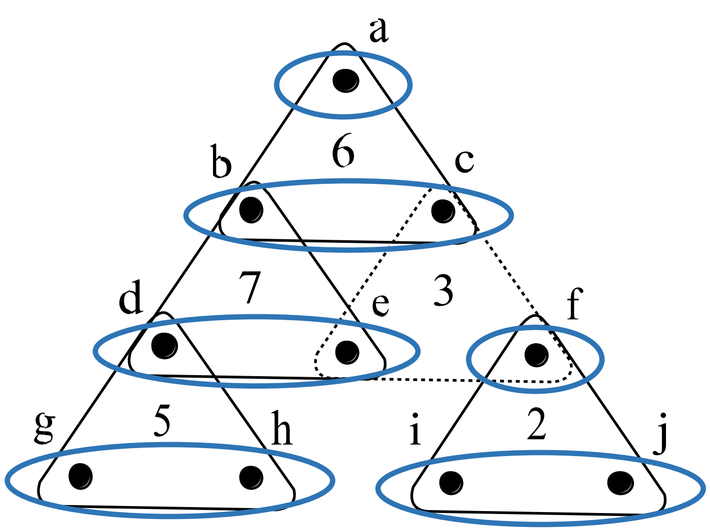
LHB Mechanism – (Detailed Algorithm)

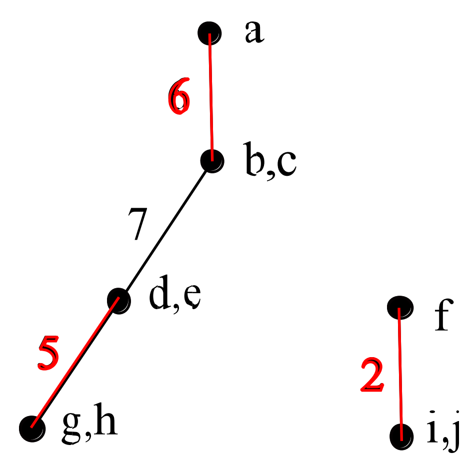
Algorithm 2: LHB mechanism

1 Let all the bids be initially unmarked, and let u be a pointer to an arbitrary item.
2 **if** *item u is not contained in any candidate bids* **then**
3   Point u to any other item that has not been pointed to before.
4 Reject all candidate bids that contain item u except the one with the highest value.
5 The bid b that was not rejected contains $q \leq d - 1$ new items.
6 **if** $q > 0$ **then**
7   Contract the q original items into one item and point u to this item[9].
8   Mark bid b and continue with Step 2.
9 **else if** *there exists some item that has not been pointed to* **then**
10   Point u to that item and continue with Step 2.
11 Let G_p be the partition graph induced by the first phase of the mechanism.
12 Accept the bids that correspond to the maximum weight matching of G_p.

Locally Highest Bid (LHB) Mechanism – (Toy Example)

$$bundle_1 = \{a, b, c\}, b_1 = 6$$
$$bundle_2 = \{b, d, e\}, b_2 = 7$$
$$bundle_3 = \{d, g, h\}, b_3 = 5$$
$$bundle_4 = \{c, e, f\}, b_4 = 3$$
$$bundle_5 = \{f, i, j\}, b_5 = 2$$





A DA Auction implementation of LHB Mechanism (1/2)

Algorithm 3: DA auction implementation of the first phase of the LHB mechanism

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1  Let all the bids be initially unmarked, and let u be a pointer to an arbitrary item.
2  if item u is not contained in any candidate bids then
3  |   Point u to any other item that has not been pointed to before.
4  while there exist more than one candidate bids containing item u do
5  |   The score of any candidate bid that does not contain u is equal to infinity.
6  |   The score of any candidate bid that contains u is equal to the value of its bid.
7  |   Reject the bid with the lowest score value.
8  The bid b that was not rejected contains q ≤ d − 1 new items.
9  if q > 0 then
10 |   Contract the q original items into one item and point u to this item.
11 |   Mark bid b and continue with Step 2.
12 else if there exists some item that has not been pointed to then
13 |   Point u to that item and continue with Step 2.
```

The DA auction implementation and the LHB mechanism provide the exactly the same outcomes in Phase 1.

A DA Auction implementation of LHB Mechanism (2/2)

- Solve the maximum weight matching problem by scoring functions.
- The score of bidder i is the ratio $b_i/[c_i(c_i+1)]$.
- (c_i represents the number of other active bids that it is conflict with)

c_i is at most two in the conflict graph, and hence yield a 2-approximation of the maximum weight matching in Phase 2 [Sakai et al. 2003].

Properties of DA Auction for Combinatorial Models

- Incentive compatibility
- Weakly group-strategyproof
- 4(d-1)-approximation guarantee in social welfare

Conclusion