Fair Allocation in Online Markets

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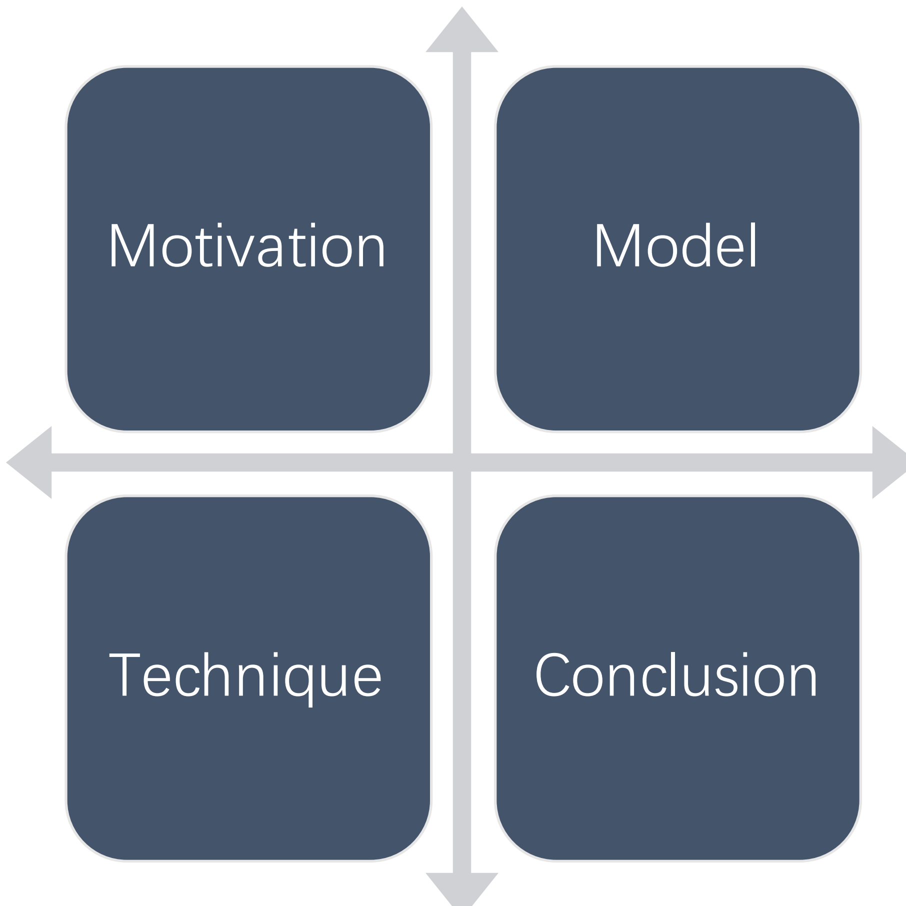
In CIKM 2014

CIKM: The Conference on Information and Knowledge Management

Outline



Motivation

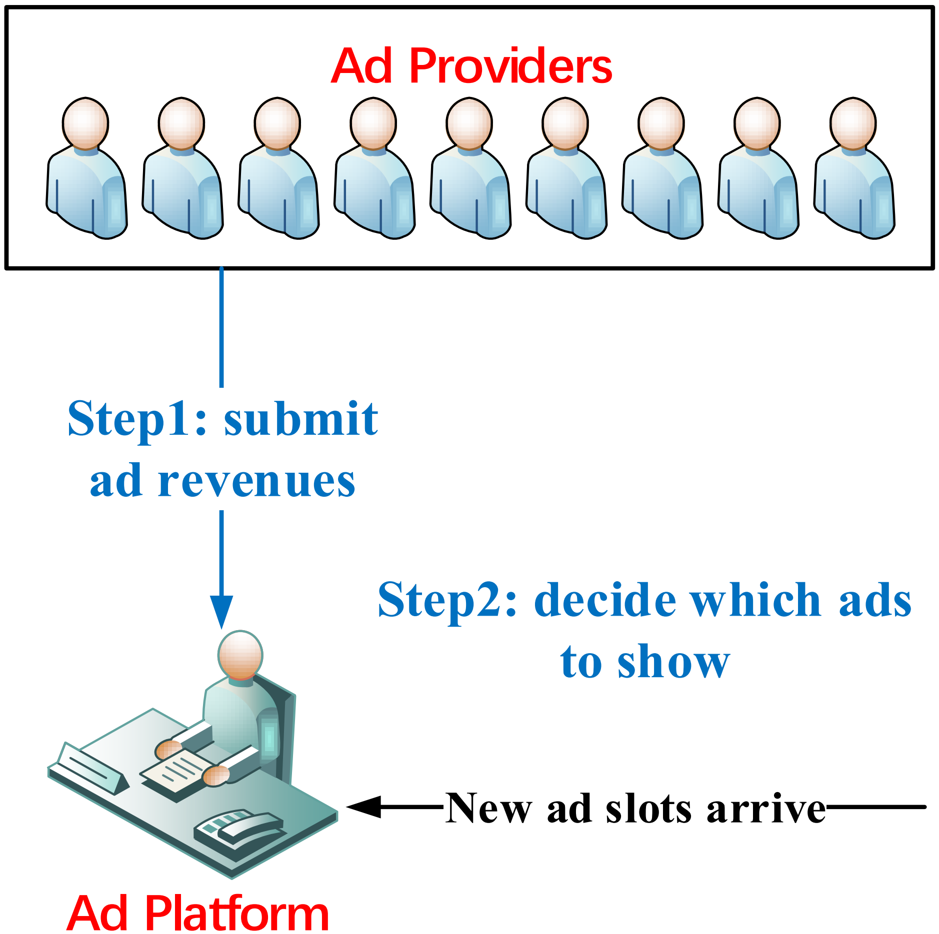
There have been many impressive work on understanding online markets in terms of revenue maximizing objectives

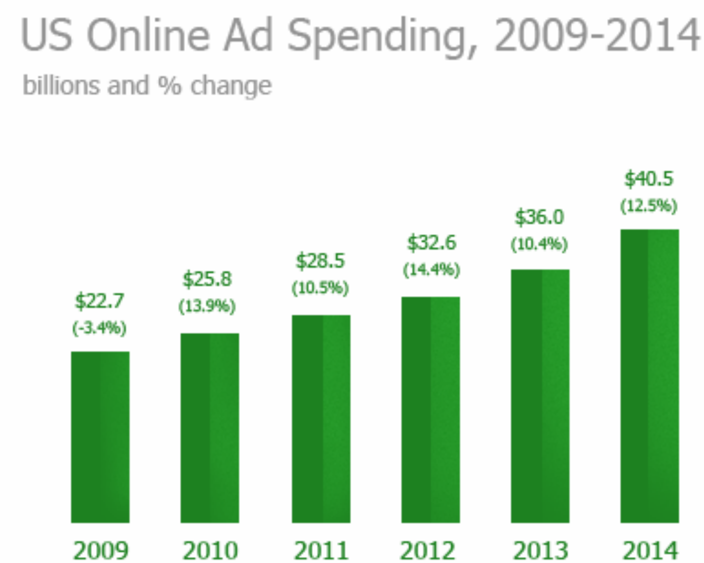
However, **fairness** have not received much attention

Fairness retains current participants and attracts new ones

Enhancing **the long-term performance** of online markets

Application: Online Advertising





Model

Input set D of n providers

For each provider $i \in D$

- Revenue target T_i
- Revenue budget B_i
- $T_i \leq B_i$

Input set Q of m requests (i.e., ad slots)

For each request $j \in Q$

- b_{ij}: the revenue when request j is allocated to provider i
- $Q(i)$: the set of requests allocated to provider i
- $d(j)$: the provider obtaining request j

Fraction coverage $c_i = \min\{1, (total\ revenue)/T_i\}$

Objective: Fairness

Each provider has revenue budget and target

Requests arrives in an online manner

The auctioneer aims to make more providers achieve or approach their revenue **targets**

$$maximize \min_{i \in D}\{c_i\}$$

Relaxed Model

ξ is the **index of the support** of the distribution drawn from unknown distribution

$p(\xi)$ denotes the **probability** any request having type ξ

$\omega_i(\xi)$ is the fraction to which a request is allocated to provider i

$$
\begin{array}{rcll}
\sum_{\xi} \boxed{p(\xi)} \cdot \boxed{w_i(\xi)} \cdot b_i(\xi) & \geq & \dfrac{c_{\mathrm{OPT}} T_i}{m} & \forall\, i \in D \\
\sum_{i \in S} w_i(\xi) & = & 1 & \forall\, i \in D, \xi \\
0 \leq w_i(\xi) & \leq & 1 & \forall\, i \in D, \xi
\end{array}
$$

LP relaxation of the MAX-MIN **problem**

Technique

The current achieved ratio c_i

Reward function:

$$\phi(k) = \left(\frac{\alpha \ln n}{c_{\mathrm{OPT}}}\right) \exp\left(-\alpha \cdot \frac{\boxed{k}}{c_{\mathrm{OPT}}} \cdot \ln n\right)$$

Remaining reward:

$$\bar{\Phi}(k) = \int_{j=k}^{\infty} \phi(j)\ dj$$

Reward of request *j* for provider *i*:

$$r_{ij} = \int_{k=c_i}^{c_i+b_{ij}/T_i} \phi(k)$$

Technique

Assumption:

$$\rho_{\mathrm{OPT}} \geq \frac{\beta\alpha \cdot \ln n \cdot (\max_{\xi, i \in D} b_i(\xi))}{2}$$

where $\rho_{\mathrm{OPT}} = c_{\mathrm{OPT}} \min_{i \in D} T_i$

The expected decrease in bar *Φ* for next item is at least

$$(1 - 1/\beta)\,\frac{\alpha \ln n}{m}\,\bar{\Phi}$$

The final expected value of bar *Φ* is at most

$$n^{1-(1-1/\beta)\alpha}$$

where $\alpha = \frac{c_{\mathrm{OPT}}}{n \ln n} \quad \beta = 1/\epsilon$

Technique

Prove the competitive ratio by **contradiction**.

Suppose the algorithm is not $1-\epsilon$ competitive ratio, the total remaining reward of all providers might be

$$\bar{\Phi} \geq \bar{\Phi}_{i_{\min}} > \frac{c_{\mathrm{OPT}}}{\alpha \ln n} \cdot n^{-\alpha(1-\epsilon)} = n^{1-(1-1/\beta)\alpha}$$

which contradicts the proved upper bound of bar Φ

Conclusion

Learn how to model the problem focusing on fairness in online stochastic model

Recall the general primal-dual framework in online algorithm analysis

See a new direction to analyze by introducing auxiliary function instead of directly manipulating primal and dual variable.