Follow the Money: How the Wealth Distribute in a P2P System

“Wealth Condensation in a Barabasi-Albert Network,”

J. Vazquez-Montejo, R. Huerta-Quintanilla and M. Rodriguez-Achach,

Physica A, 2010

You Might Have Heard That Repeatedly …

- “The rich get richer and the poor get poorer”
- -- Sounds like a law of **nature**?
- -- Perhaps some physical or mathematical **rule**?
- -- ‘Who’ get richer is not what we care
- -- To find out what decide the distribution

The Price is Right

- The economy being simulated in mathematical models is a rather special one:
  - Pure, Free Market Trading
  - No Production of New Wealth
  - No Consumption
- Wealth is like Momentum or Energy:
  - Total amount never changes
  - One person can get richer only if another grows poorer

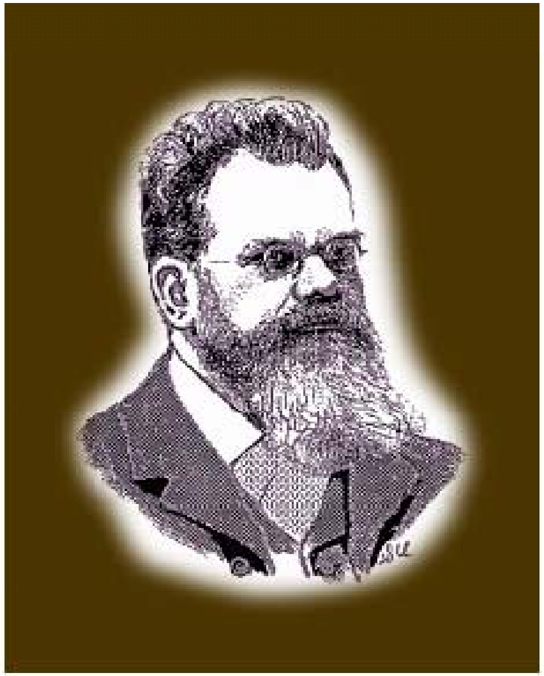
Why Price?

- **Yard-Sale**

- **Departures from true value bring dynamics**

“Money, it’s a gas” – Pink Floyd

Boltzmann-Gibbs versus Pareto distribution



Ludwig Boltzmann (1844-1906)

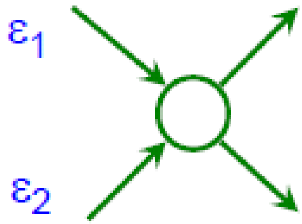
Vilfredo Pareto (1848-1923)

Boltzmann-Gibbs probability distribution $P(\varepsilon) \propto \exp(-\varepsilon/T)$, where ε is energy, and $T = \langle \varepsilon \rangle$ is temperature.

Pareto probability distribution $P(r) \propto 1/r^{(\alpha+1)}$ of income r.

Boltzmann-Gibbs probability distribution of **money**

Collisions between atoms



$\varepsilon_1' = \varepsilon_1 + \Delta\varepsilon$

$\varepsilon_2' = \varepsilon_2 - \Delta\varepsilon$

Conservation of energy:

$\varepsilon_1 + \varepsilon_2 = \varepsilon_1' + \varepsilon_2'$

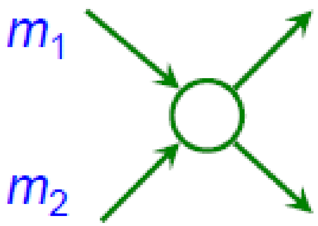
Detailed balance:

$\cancel{w_{12\to1'2'}}P(\varepsilon_1)\,P(\varepsilon_2) = \cancel{w_{1'2'\to12}}P(\varepsilon_1')\,P(\varepsilon_2')$

Boltzmann-Gibbs probability distribution $P(\varepsilon) \propto \exp(-\varepsilon/T)$ of energy ε, where $T = \langle\varepsilon\rangle$ is temperature. It is **universal** – independent of model rules, provided the model belongs to thetime-reversal symmetry class.

Boltzmann-Gibbs distribution maximizes entropy $S = -\Sigma_\varepsilon P(\varepsilon)\ln P(\varepsilon)$ under the constraint of conservation law $\Sigma_\varepsilon P(\varepsilon)\,\varepsilon = \text{const.}$

Economic transactions between agents



$m_1' = m_1 + \Delta m$

$m_2' = m_2 - \Delta m$

Conservation of money:

$m_1 + m_2 = m_1' + m_2'$

Detailed balance:

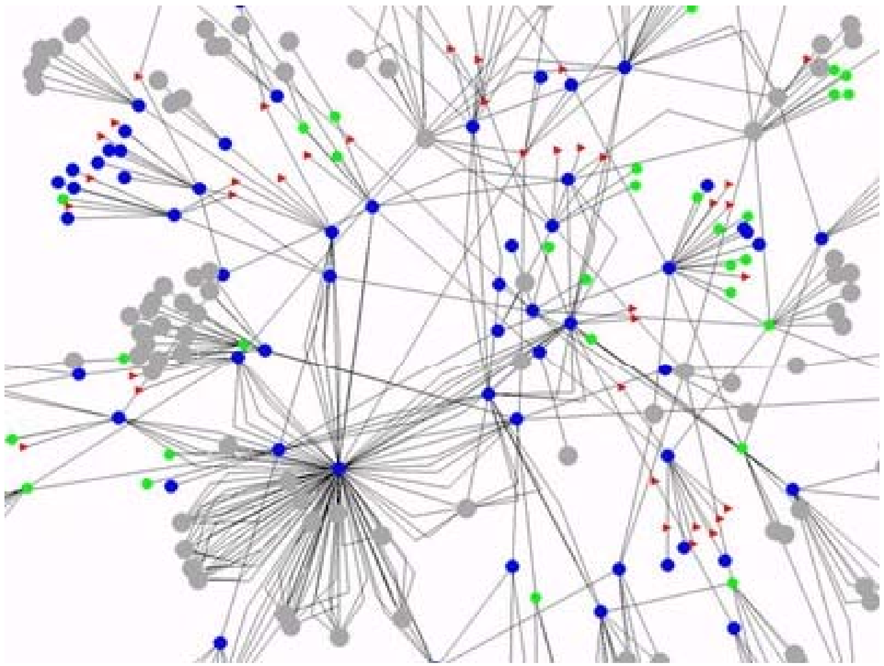
$w_{12\to1'2'}P(m_1)\,P(m_2) = w_{1'2'\to12}P(m_1')\,P(m_2')$

Boltzmann-Gibbs probability distribution $P(m) \propto \exp(-m/T)$ of money$m$, where $T = \langle m\rangle$ is the money temperature.

Gives Hints for P2P System Modeling and…

$$\begin{cases} W_i(t+1) = W_i(t) + \Delta W \\ W_j(t+1) = W_j(t) - \Delta W \end{cases}$$

- How to Decide ΔW ?



The Myth of ΔW -- Fair

$$\begin{cases} w_i(t+1) = \lambda_i w_i(t) + \varepsilon_{ij}[(1-\lambda_i)w_i(t) + (1-\lambda_j)w_j(t)] \\ w_j(t+1) = \lambda_j w_j(t) + (1-\varepsilon_{ij})[(1-\lambda_i)w_i(t) + (1-\lambda_j)w_j(t)] \end{cases}$$

- $0 \le \varepsilon_{ij} \le 1$ • Random factor
- $0 \le \lambda_i \le 1$ • Saving factor

The Myth of ΔW -- Fair

- Here begins with the technical analysis part

$$
\begin{aligned}
(\Delta w_{ij})_{t+1} &= (w_i - w_j)_{t+1} \\
&= \left(\frac{\lambda_i + \lambda_j}{2}\right)(\Delta w_{ij})_t + \left(\frac{\lambda_j - \lambda_i}{2}\right)(w_i + w_j)_t \\
&+ (2\varepsilon_{ij} - 1)[(1 - \lambda_i)w_i(t) + (1 - \lambda_j)w_j(t)]
\end{aligned}
$$

The Myth of ΔW -- Biasing

- **Poorer** has a probability of **p** to gain a fraction of its total wealth **f**

$$W_i(t+1) = W_i(t) \cdot (1+f)^p (1-f)^{1-p}$$

Justifications in Budget-Based P2P Systems

- Behavior of the Rich and the Poor
- Budgets → Strategy
  - The **poor** need to earn money rather than go shopping
  - The **rich** can buy what he wants, care less about price, which is encouraged

Justifications in Budget-Based P2P Systems

- Relationship between **Earning** and **Budget Level**
- Budgets → Trading Amount
  - The **poor** can only earn a small amount of money by time
  - The **rich** are able to invest for big money

1929, The Great Crisis