The Cross-ISP Traffic and System Performance Tradeoff in VoD System

May 24, 2011

Questions Seeking to Answer through Modeling

- In P2P VoD systems, if we limit the number of peers' cross-ISP links, how much inter-ISP traffic will be reduced, what is the impact on system performance? *e.g.* How many server capacities are added for ISPs with few peer resources or reduced for ISPs with many peer resources to ensure smooth playback by the inter-ISP traffic reduction? How does the loss rate of chunk requests change in different ISPs if we fix the server capacity?
- Based on the peer resources in different ISPs, to what extents can peers in different ISPs limit the number of cross-ISP links to achieve a win-win strategy?

VoD System Model

- There are M ISPs: ISP 1, ISP 2, ISP 3,..., ISP M.
- The peer number and average peer upload bandwidth in different ISPs:
    - the peer number: We use the ON-OFF model. There are totally N_i peers (who have installed the software for VoD streaming) in ISP i. A part of N_i peers stays offline. The probability that a peer stays offline is π_0. The probability that there are $N_i^{off} = m_i^0$ peers offline is $P(N_i^{off} = m_i^0) = C_{N_i}^{m_i^0} \pi_0^{m_i^0} (1 - \pi_0)^{N_i - m_i^0}$.
    - The online peers are downloading chunks and uploading chunks in the system.
    - the average peer upload bandwidth in ISP i is U_i.
    - A total of J constant-length chunks to be shared in the VoD system: $\mathcal{C} = \{c_1, c_2, ..., c_J\}$, $|\mathcal{C}| = J$.
    - Peer p has a local cache $\mathcal{S}_p$. The cache of a peer can store B chunks, $B << J$. A peer will be able to serve requests for content c for all $c \in \mathcal{S}_p$.

Chunk Demand in ISP i

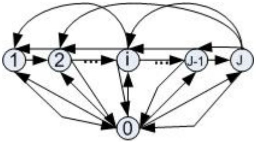
- A total of J constant-length chunks to be shared in the VoD system: $\mathcal{C} = \{c_1, c_2, ..., c_J\}$, $|\mathcal{C}| = J$.
- In ISP i, at time t, there are m_i^j peers downloading chunk j, and there are m_i^0 offline peers. $m_i^0 + m_i^1 + ... + m_i^J = N_i$. Say peers are in state $0, 1, 2, ..., J$ as peers are offline, downloading chunk 1, downloading chunk 2,...,downloading chunk J. The probability that peers are in some state is π_j for state j. $\sum_{j=0}^{J} \pi_j = 1$.
- The probability that there are $m_i^j (0 \leq j \leq J)$ peers in state j is $P(m_i^0, m_i^1, ..., m_i^J) = N_i! \frac{\pi_0^{m_i^0}}{m_i^0!} ... \frac{\pi_J^{m_i^J}}{m_i^J!}$.
- The probability that there are m_i^j peers downloading chunk $j (1 \leq j \leq J)$ is $P(m_i^j) = C_{N_i}^{m_i^j} \pi_j^{m_i^j} (1 - \pi_j)^{N_i - m_i^j}$.

Chunk Demand in ISP i

- Every peer in state j send a request for chunk j. So, the number of requests, k is the same as the number of peers in state j, m_i^j. It is a random variable of binomial distribution, $P(Req = k) = C_{N_i}^k \pi_j^k (1-\pi_j)^{N_i - k}$. For large N_i, the binomial distribution can be approximated by the Poisson distribution, $P(Req = k) = \frac{\lambda_{i,j}^k}{k!} e^{-\lambda_{i,j}}$, $\lambda_{i,j} = N_i \times \pi_j$. So, the requests for chunk j in a time slot is a random variable of Poisson distribution, $P(Req = k) = \frac{\lambda_{i,j}^k}{k!} e^{-\lambda_{i,j}}$, $\lambda_{i,j} = N_i \times \pi_j$.
- Request rate for chunk 1,...J are $\lambda_{i,1}$,...,$\lambda_{i,J}$ respectively.

Chunk Popularity in VoD System

transition for user behavior.jpg



- User behaviors can be modeled by the state transition of peers. Based on the state transition model for user behavior, we can get the stationary state distribution for peer state, $(\pi_0, \pi_1, ..., \pi_{J-1}, \pi_J)$. We can get the chunk popularity from this.
  (This is the same user behavior model proposed by Yipeng Zhou in an infocom2011 paper)
- User behaviors: Joining, Departures, Random seek.

Define Performance Metrics

- The resource used to serve chunk j is from peers or from servers.
- The request rate for chunk j is $\lambda_{i,j}$. At a time slot, m_i^j peers demand chunk j, the probability that the peer requesting chunk j has cached it is D_j. Let w_j denote the copies of chunk j that online peers can upload.
- The needed server capacity to satisfy the demand for chunk j is $U_{sj} = max\{(1 - D_j) \cdot m_i^j - w_j, 0\}$. The total needed server capacity is $U_s = \sum_{j=1}^{J} U_{sj}$.
- If the server capacity is given as S, the rate of chunk loss for streaming is $P = \frac{\sum_{j=1}^{J} U_{sj} - S}{N_i - m_i^0}$.

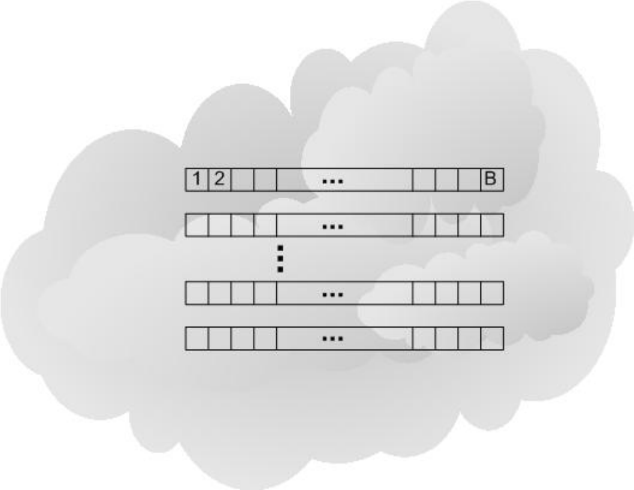
Chunk Distribution in Peers' Cache

- The cache of a peer can store B chunks, a total of J chunks shared in the VoD system.
- The Least-Recently Used Caching replacement algorithms (LRU) are adopted.
- At each time slot, the cache is kept as a sequence of chunks arranged in order of latest access time. At time slot t, the state of the cache is $\mathcal{S}(t) = (c_t^{(1)}, c_t^{(2)}, ..., c_t^{(B)})$.
- When the last accessed chunk is $c_t^{(1)} = w_t$, the cache state at time slot t is
  - $\mathcal{S}(t) = (w_t, c_{t-1}^{(1)}, c_{t-1}^{(2)}, ..., c_{t-1}^{(B-1)})$, if $w_t \notin \mathcal{S}(t-1)$
  - $\mathcal{S}(t) = (w_t, c_{t-1}^{(1)}, ..., c_{t-1}^{(j-1)}, c_{t-1}^{(j+1)}, ..., c_{t-1}^{(B)})$, if w_t is in j-th position of the cache.

Chunk Distribution in Peers' Cache

- The LRU caching model is a Markov Chain with $B!C_J^B$ states. This Markov Chain is irreducible and aperiodic. By standard Markov Chain theory, the long run stationary probability of cache states is well defined. But the computation complexity is $\frac{J^B}{B!}$.
  (W.F.King, Analysis of Demand Paging Algorithms, IFIP Congress Aug. 1971)

Chunk Distribution in Peers' Cache



Chunk Requests and Service

attempt to use loss network model
(F.Kelly, Loss networks, The Annals of Applied Probability, 1991)

Chunk Requests and Service

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- The chunk request rates for chunk 1, chunk 2,..., chunk J that need to be served by other peers or servers are $\nu_{i,1}$, $\nu_{i,2}$,...,$\nu_{i,J}$. A chunk request needs a service from other peers in the system. Assume the mean downloading time for a chunk is unit. The vector of numbers n_c requests for chunk c is $\mathbf{n} = \{n_c\}_{c \in \mathcal{J}}$.
- The vector of request numbers is feasible iff there exist non-negative integers z_{cp}(number of concurrent downloads of chunk c from peer p)
  $\sum_{p:c_j \in \mathcal{S}_p} z_{jp} = n_j, \forall c_j \in \mathcal{J}$
  $\sum_{j:c_j \in \mathcal{S}_p} z_{jp} \leq U, \forall p \in \mathcal{P}$
- By an application of Hall's theorem, the above two constraints can be written as:
  $\forall \mathcal{A} \subseteq \mathcal{C}, \sum_{c_j \in \mathcal{A}} n_j \leq U \cdot |\{p \in \mathcal{P} : \mathcal{A} \cap \mathcal{S}_p \neq \emptyset\}|$

Chunk requests and service

- The vector $\mathbf{n}$ of requests under service is a particular instance of a general stochastic process known as a loss network model. the stochastic process is reversible and admits a closed-form stationary distribution:
  $\pi(\mathbf{n}) = \frac{1}{G} \prod \frac{\nu_j^{n_j}}{n_j!}$
  $\mathbf{n}$ satisfies the constraint in the previous slide.

Chunk requests and service

- The loss rate for chunk c_j is L_j, $1 - L_j = \frac{E[n_j]}{\nu_j}$.
- Kelly shows that the stationary distribution concentrates around its mode $\mathbf{n}^*$, such concentration suggests the following approach which is the premise of the 1-point approximation: Compute the mode $\mathbf{n}^* = (n_j^*)$ of the distribution and use n_j^* as an approximation for $E[n_j]$, $1 - L_j = \frac{n_j^*}{\nu_j}$.
- The mode $\mathbf{n}^*$ is a solution of the following optimization problem.
  $maximize \sum_j n_j log \nu_j - log n_j!$
  $over \forall \mathcal{A} \subseteq \mathcal{C}, \sum_{c_j \in \mathcal{A}} n_j \leq U \cdot |\{p \in \mathcal{P} : \mathcal{A} \cap \mathcal{S}_p \neq \emptyset\}|$
- By Stirling's approximation, $log n_j! = n_j log n_j - n_j + O(log n_j)$. Use this and ignore the $O(log n_j)$ term to replace $log n_j$ in the optimization problem.
- The optimization problem is convex. The strong duality holds.

Locality

- We assume every peer keeps x inter-ISP neighbors. The request rate for chunk j in ISP i is $\lambda_{i,j}$.
- As the number of inter-ISP links is limited, the request rate for chunk j in ISP i are mainly served by peers in the same ISPs. When every peer's inter-ISP links is x, the average request rate for chunk j from ISP i that is allocated to other ISPs is $\frac{x}{N_i+x}\lambda_{i,j}$.
- Requests for chunk j to ISP i from other ISPs: the request rate for chunk j from ISP k to ISP i is $\frac{N_i}{N-N_k}\frac{x}{N_k+x}\lambda_{k,j}$. So, the requests rate for chunk j from other ISPs to ISP i is $\sum_{k=1,k!=i}^{k=M}\frac{N_i}{N-N_k}\frac{x}{N_k+x}\lambda_{k,j}$.

Next-step Work

- Calculate the system performance based on the above model.
- Derive the relationship between system performance and inter-ISP neighbor size x.