Characterizing Cascade Dynamics in A Microblogging System

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Diffusion in Social Networks

A fundamental process in social networks: behaviors that cascade from node to node

- News, opinions, rumors,...
- Virus, disease propagation
- Localized effects: riots

Microblogging Changes How People Discover and Consume Information Online

Post

Case Study: Gangnam Style

Katy Perry @katyperry

Help, I'm in a gangnam style k hole: bit.ly/PVPJ4p

12K reposts

Britney Spears @britneyspears

I am LOVING this video - so fun! Thinking that I should possibly learn the choreography. Anybody wanna teach me?! haha britspears.me/159zje

4K reposts

T-Pain @TPAIN

Words cannot even describe how amazing this video is... youtube.com/watch?v=9bZkp7...

2K reposts

Motivation

Study the temporal dynamics of an information cascade in a microblogging system

- The number of users influenced at any given time

Related Work

- Epidemic model:
  - SIS model
  - SIIRP model
- Independent Cascade (IC) Model
- Linear Threshold (LT) Model
- Linear Influence Model

Data-driven Approach: Measurement Study

Tencent Weibo.

- 0.5B users - one of the largest social network services in China

A sample of video sharing in 20 days

- 1M users - social relation, behaviors
- 2M entries - each entry corresponds to one post or repost
- 350K video links - 5 video sharing websites, 14 categories

A Glance of Microblogging Diffusion

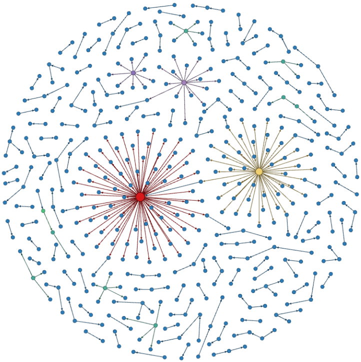


Figure: Example diffusion cascades in Tencent Weibo.

Power-law Distributions of the Number of Followers and the Number of Reposts

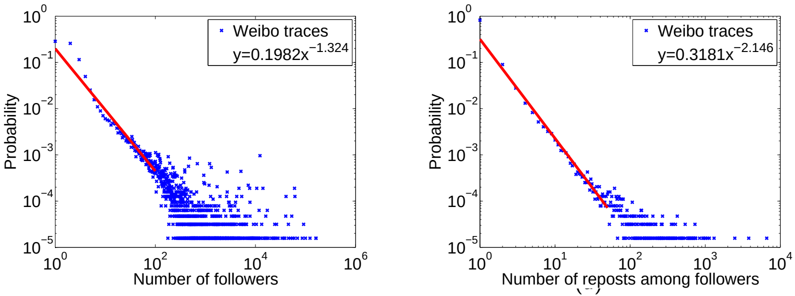


Figure: Distribution of the number of followers of users, and the number of reposts to their microblogs.

Evolution of Cascade Size

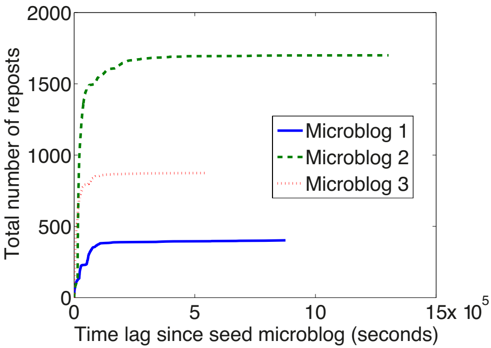


Figure: The total number of reposts versus the time lag since when the seed microblogs are posted.

Gamma Distribution of the Response Delays

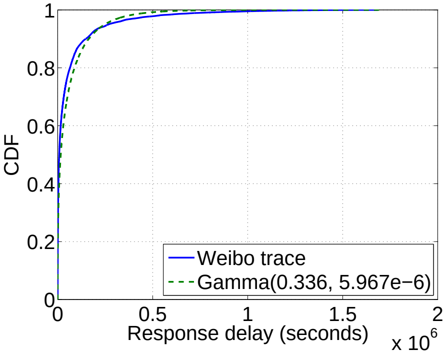


Figure: CDF of response delays of all reposts in our traces.

Objective

How many users in total are expected to have reposted the microblog after a certain time t?

Branching Process

Branching process

- Each individual gives birth to a random number of offsprings independently according to a certain distribution

Age-dependent branching process

- The lifetimes of individuals are considered based on a lifetime distribution

Mapping

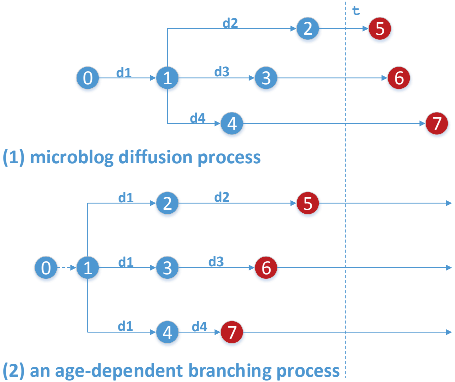


Figure: Mapping between the microblog diffusion cascade and an age-dependent branching tree.

Basic Notations

- $X(t)$: the number of inactive nodes in a branching tree at time t
- $Y(t)$: the total number of nodes in a branching tree at time t
- $Z(t)$: the number of active nodes in a branching tree at time t

Degree Distribution

- R: the random variable of the number of offsprings of a node
- $p(R = k) = p_k$: the probability density function of the number of offsprings of a node in the branching tree
- $\mu = \sum_k^\infty p_k k$: the reproductive number of a node in the branching process

Response Delay

- $G(\tau)$: the cumulative distribution function of the lifetimes of nodes in a branching process

The Probability $P(Z(t) = k)$

Through decomposing in accordance with the lifetime and the number of successors, we have:

$$
\begin{aligned}
P(Z(t) = k) = & [1 - G(t)]\delta_{1k} \\
& + \int_0^t dG(\tau) \sum_{j=0}^{\infty} p_j P^{*j}(Z(t-\tau) = k),
\end{aligned}
$$

where P^{*j} is the j-fold convolution of P, and δ_{1k} is the Kronecker delta.

Probability Generating Function $F(s,t)$

The probability generating functionof $Z(t)$ is

$$\begin{aligned} F(s,t) =& [1-G(t)]\sum_{k=0}^{\infty} s^k \delta_{1k} \\ &+ \int_0^t dG(\tau) \sum_{j=0}^{\infty} p_j \sum_{k=0}^{\infty} P^{*j}(Z(t-\tau)=k)s^k. \end{aligned}$$

Noting that $\sum_{k=0}^{\infty} P^{*j}(Z(t-\tau)=k)s^k = F^j(s,t-\tau)$ and $\sum_{k=0}^{\infty} s^k \delta_{1k} = s$, we have

$$F(s,t) = s[1-G(t)] + \int_0^t h[F(s,t-\tau)]dG(\tau). \tag{1}$$

Since $F(s,t)$ is a convergent power series for $|s| < 1$, we differentiate both sides of (1) over s and derive

$$\frac{\partial F(s,t)}{\partial s} = [1 - G(t)] + \int_0^t h^{'}[F(s,t-\tau)] \frac{\partial F(s,t-\tau)}{\partial s} dG(\tau).$$

We could prove that $z(t)$ is the bounded limit of $\frac{\partial F(s,t)}{\partial s}$ as s approaches 1. Hence, taking limit $s \to 1$, we obtain

$$z(t) = [1 - G(t)] + \mu \int_0^t z(t-\tau) dG(\tau). \tag{2}$$

Total Number of Nodes in a Branching Tree

Similarly, we can obtain

$$y(t) = 1 + \mu \int_0^t y(t-\tau) dG(\tau). \tag{3}$$

Based on Renewal Theory, we can get the solutions for $z(t)$ and $y(t)$, as follows:

$$z(t) = [1 - G(t)] * U(t), \tag{4}$$

and

$$y(t) = U(t), \tag{5}$$

where $U(t) = \sum_{n=0}^{\infty} \mu^n G^{*n}(t)$.

Thus,

$$x(t) = y(t) - z(t) = G(t) * U(t). \tag{6}$$

Denote the Laplace transform of $G(t)$ as $H(s)$, we can calculate the Laplace Transform of $x(t)$ as

$$L(s) = \frac{H(s)}{1 - \mu s H(s)}. \tag{7}$$

Through inverse Laplace transform, we can get the analytic form of $x(t)$.

Overall Size of a Microblog Cascade

Since the expected number of direct reposts from the seed post is μ, we can derive that the overall size of a microblog cascade is

$$\tilde{x}(t) = \sum_{k=0}^{\infty} p_k k x(t) + 1 = \mu x(t) + 1, \tag{8}$$

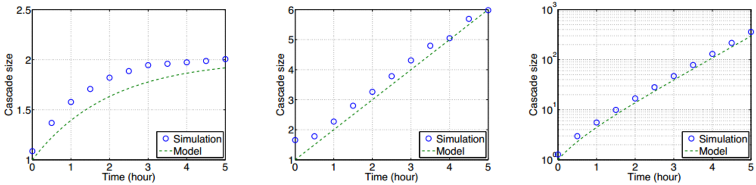
where 1 corresponds to the seed post.

Experiment Setup

Simulate a microblogging network

- The number of followers and response delays follow the same distributions in the measurement study
- Run 10^4 times for every set of parameters

Evolution of Cascade Size



(a) $\mu = 0.5$. (b) $\mu = 1$. (c) $\mu = 2$.

Figure: Comparison of the evolution of cascade sizes generated by simulations and our model.

Cascade Size over Time with Two-stage Degree Distributions

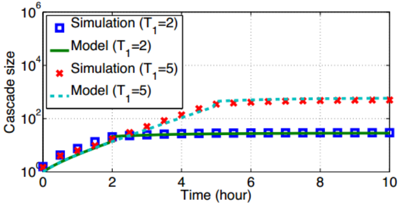


Figure: Comparison of the evolution of cascade sizes generated by simulations and our model: two-stage μ.

Final Cascade Size

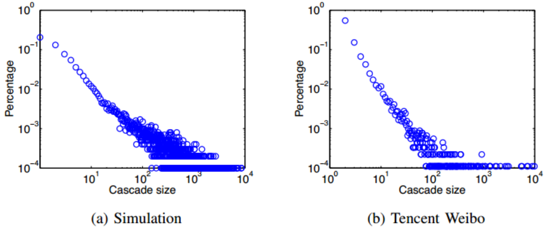


Figure: Distribution of final cascade sizes.

Summary

- A large-scale measurement study reveals several facts on microblog propagation
- Detailed mathematical derivation of the expected cascade size at any time during a microblog diffusion process is given
- Trace-based simulation experiments demonstrate the effectiveness of our model

Thanks!