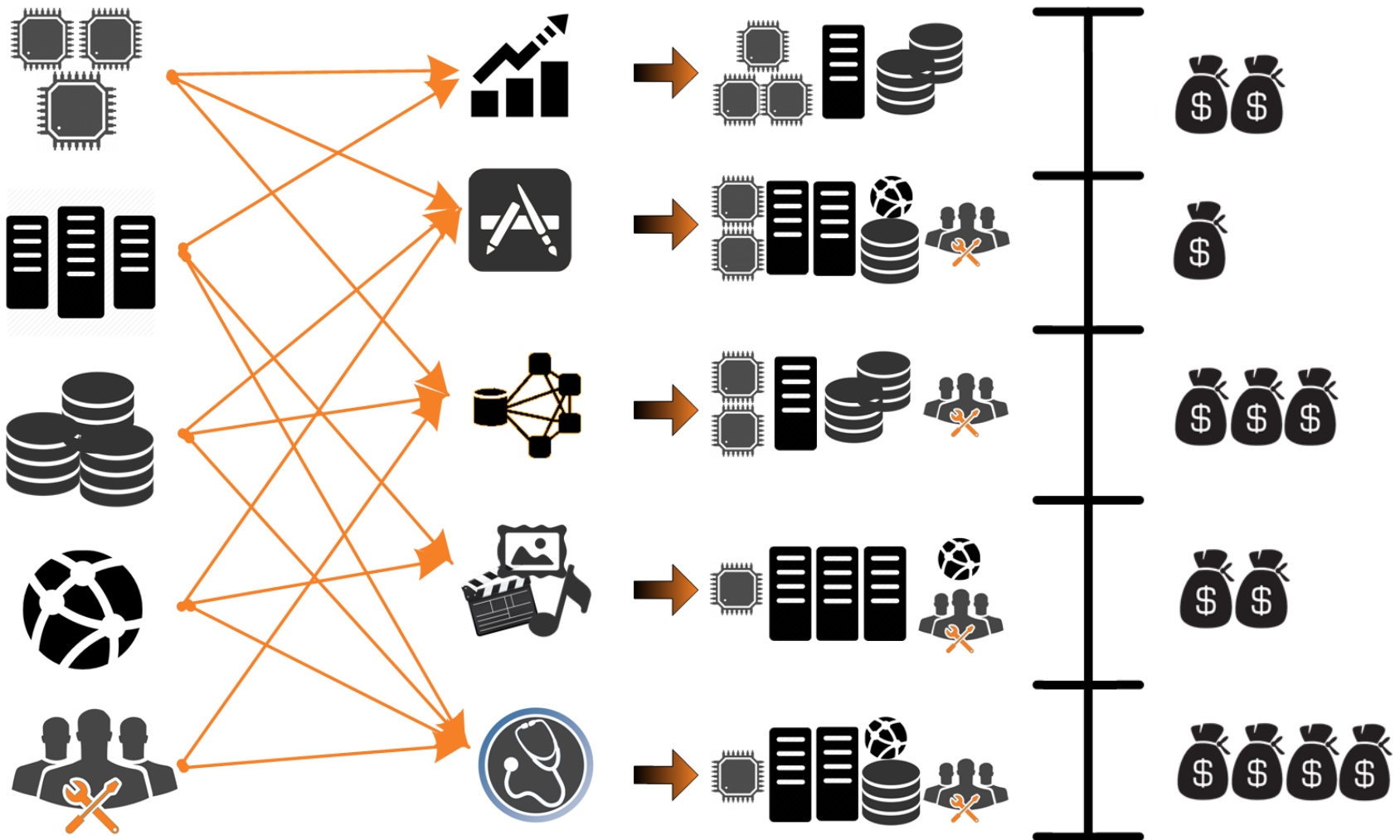


# **How to Bid the Cloud**

SIGCOMM 2015

# Cloud Resource Allocation and Pricing



# Cloud pricing

## Usage-based cloud pricing






## Auction-based cloud pricing



**Amazon's Elastic Compute Cloud (EC2)  
spot instance**

# Spot instance

## Step 1: Choose the instance type

 Amazon Linux Free tier eligible	<input type="checkbox"/>	Memory optimized	r3.large	2	15	1 x 32 (SSD)	-	Moderate
	<input type="checkbox"/>	Memory optimized	r3.xlarge	4	30.5	1 x 80 (SSD)	Yes	Moderate
 Red Hat Free tier eligible	<input type="checkbox"/>	Memory optimized	r3.2xlarge	8	61	1 x 160 (SSD)	Yes	High
	<input type="checkbox"/>	Memory optimized	r3.4xlarge	16	122	1 x 320 (SSD)	Yes	High
 SUSE Linux Free tier eligible	<input type="checkbox"/>	Memory optimized	r3.8xlarge	32	244	2 x 320 (SSD)	-	10 Gigabit

## Step 2: Configure the instance details

– Number of instances & bid price

Number of instances ⓘ

Purchasing option ⓘ ☒ Request Spot Instances

Current price ⓘ

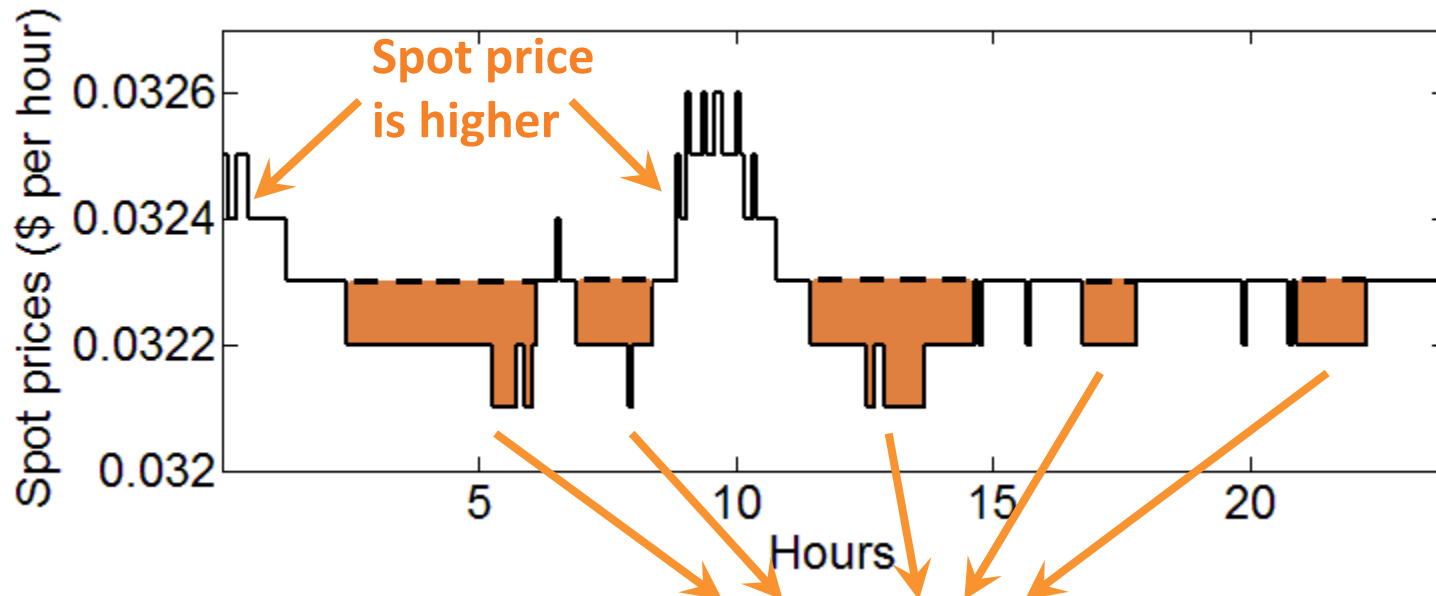
us-east-1a	0.1582
us-east-1b	0.1587
us-east-1d	0.1821
us-east-1e	0.1856

Maximum price ⓘ \$

**Name the price  
YOURSELF!**

# Spot pricing

Spot price history for an r3.xlarge instance in the US Eastern region on September 09, 2014



Two questions arise.

Spot instance  
can run

# Our questions

- Question #1

How might the cloud provider set the price?

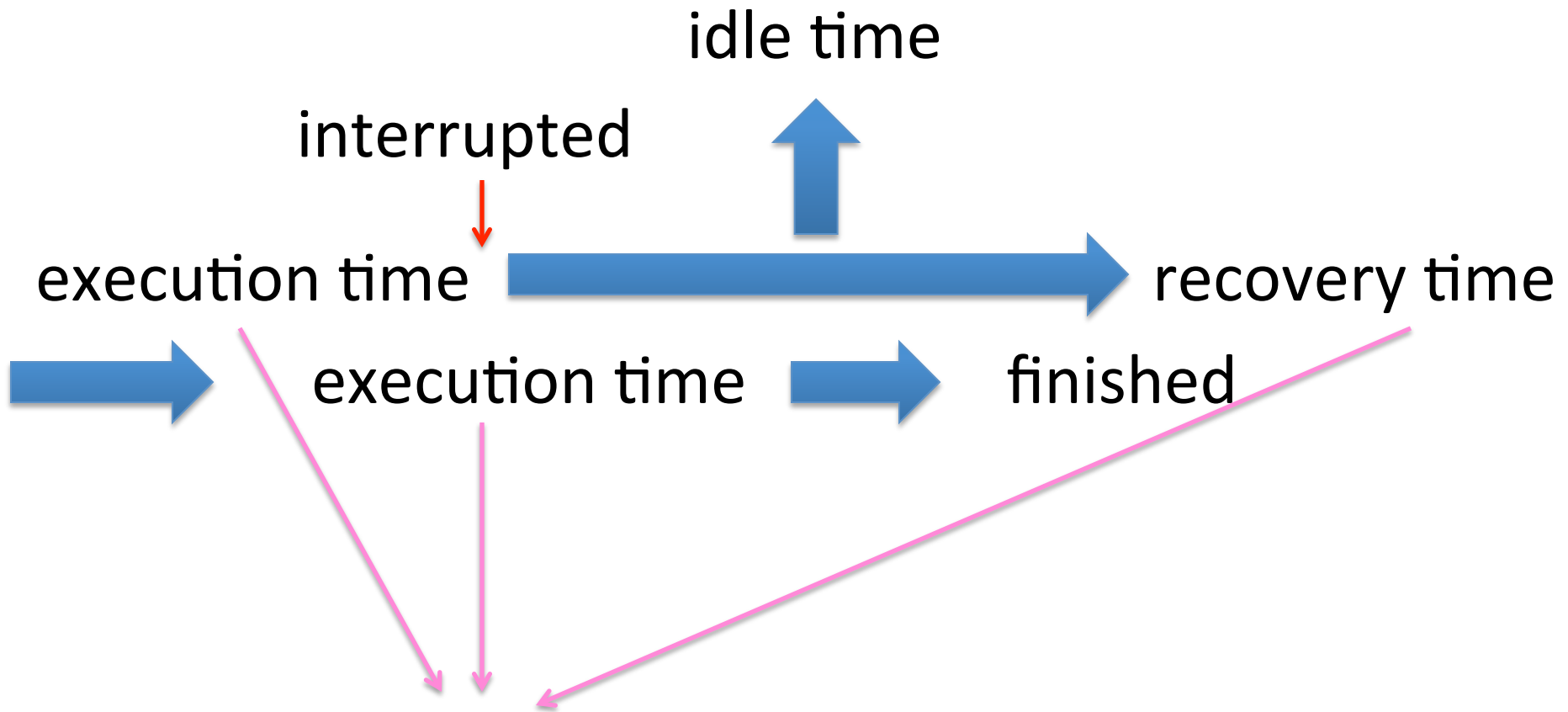
- Question #2

What prices should users bid?

# Differences

1. User-oriented
2. Jobs could be interrupted
3. Bid prices are dependent on spot prices
4. Goals: satisfy interruptibility requirement  
minimize user cost (payment);

What is user cost? Runtime \* spot price



The user is get charged in job's running time



# Outline

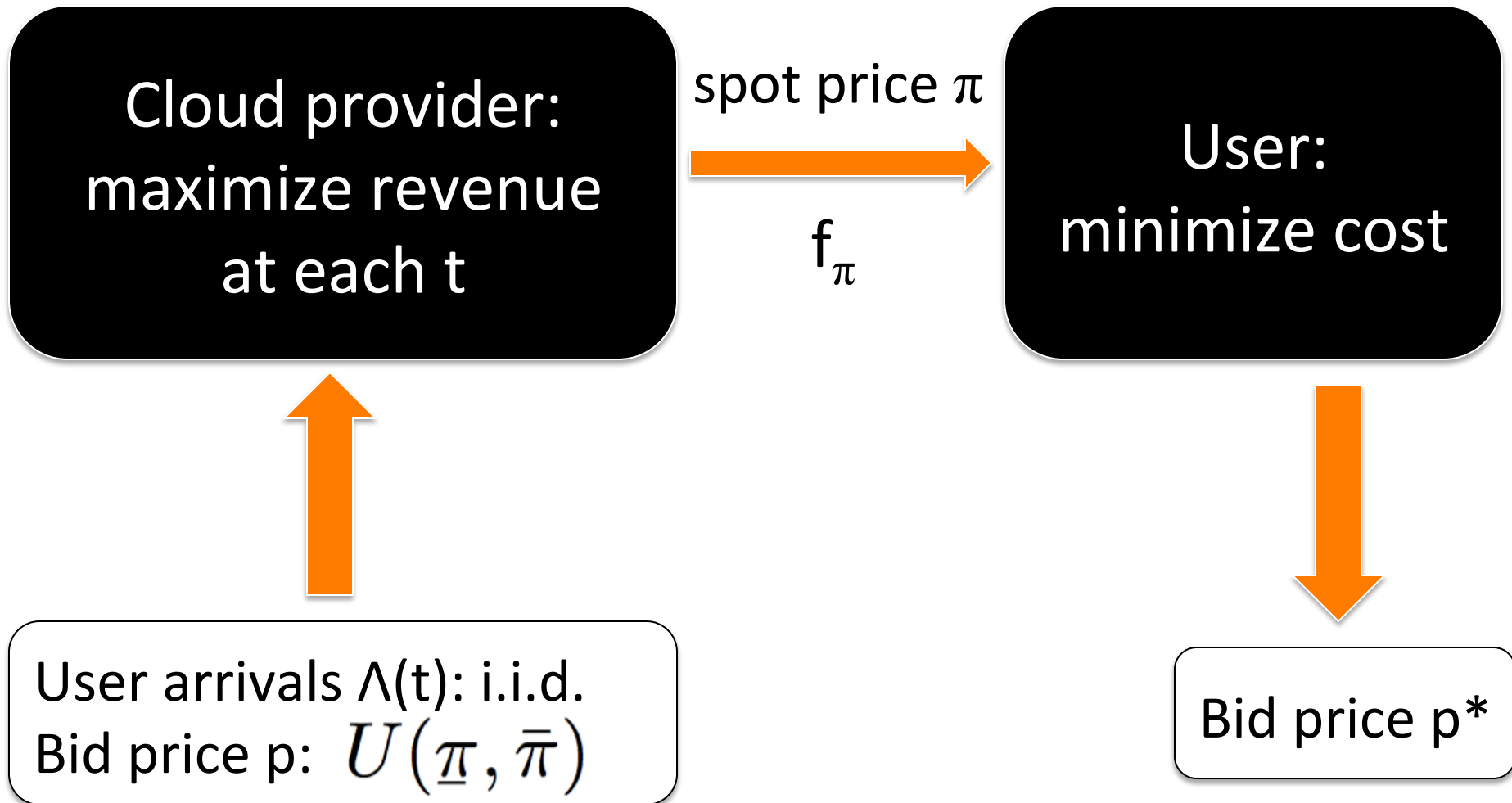
Extract pattern of  
spot prices in  
real systems

$f_{\pi}$

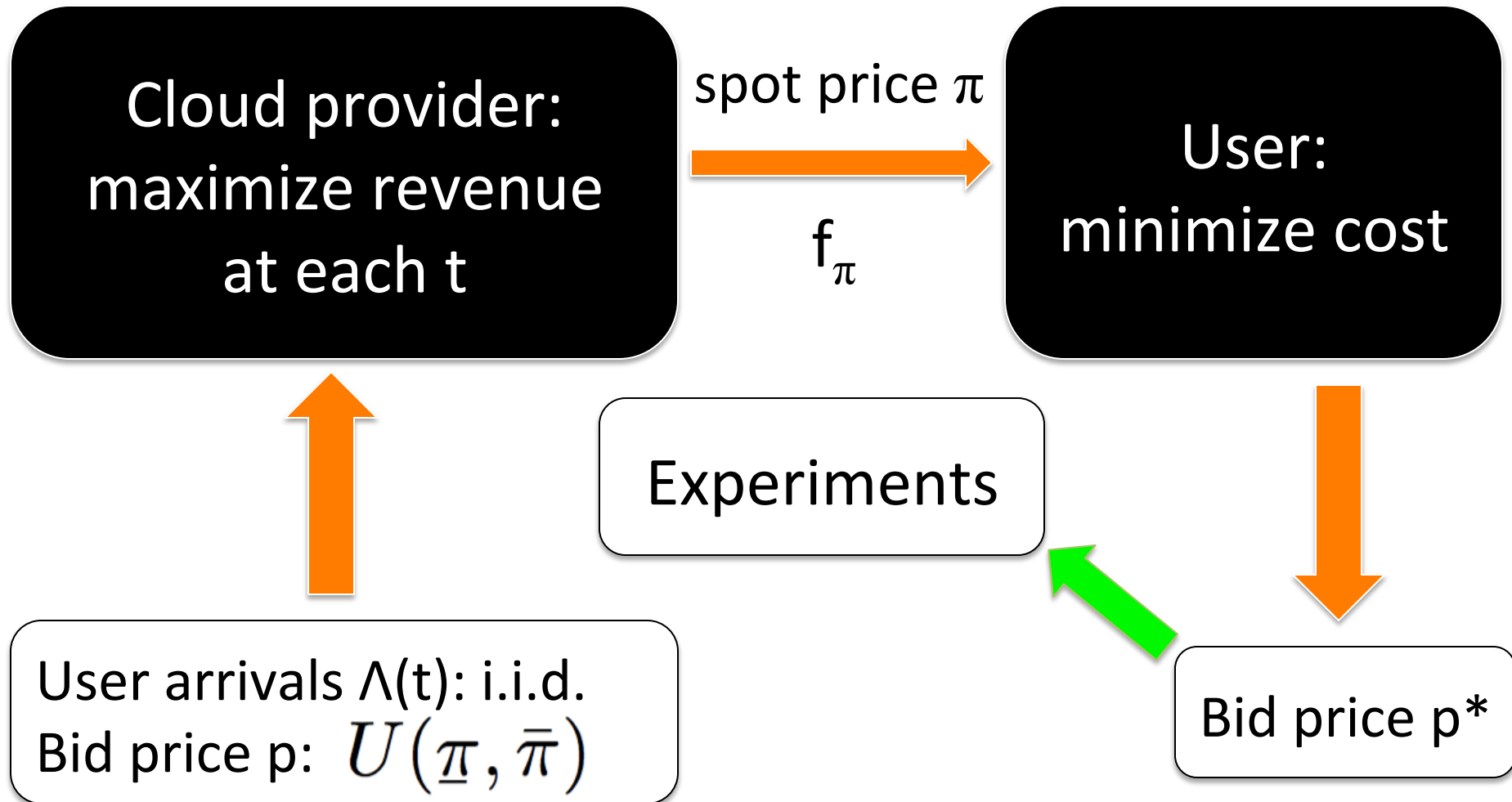
User:  
minimize cost

Bid price  $p^*$

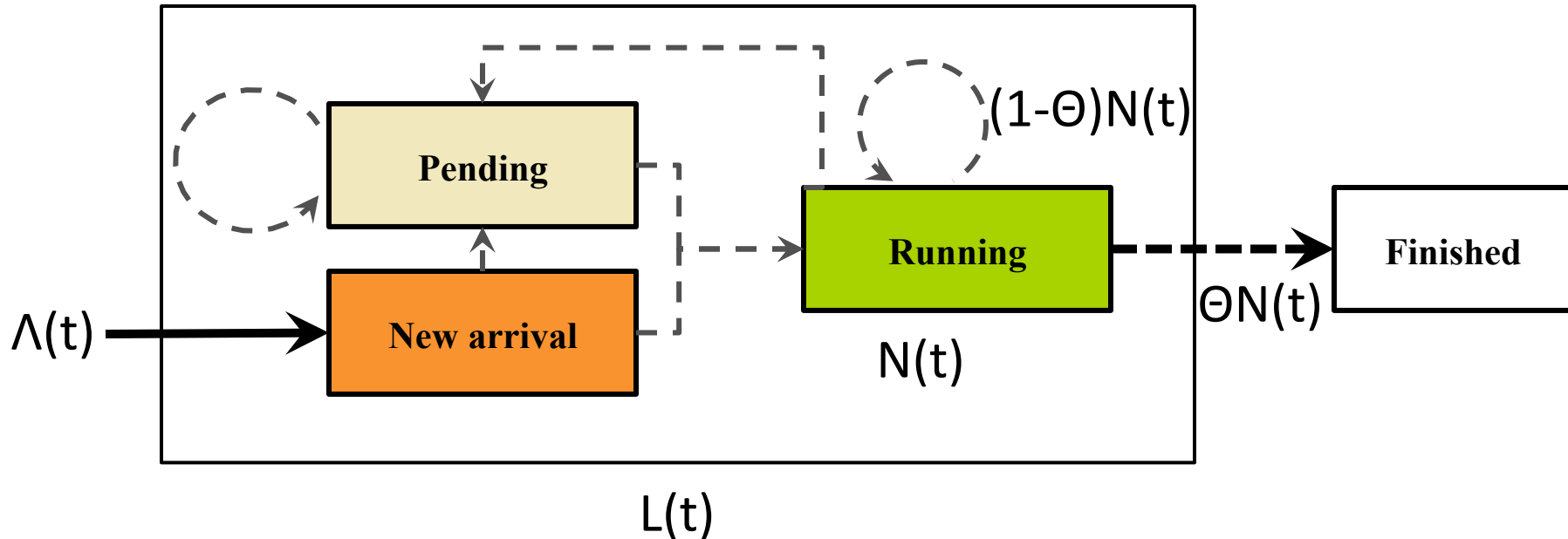
# Outline



# Outline



# Cloud provider model



$$f_p(x) = 1/(\bar{\pi} - \underline{\pi}) \longrightarrow N(t) = L(t) \frac{\bar{\pi} - \pi(t)}{\bar{\pi} - \underline{\pi}}$$

# Cloud provider revenue maximization

$$f_p(x) = 1/(\bar{\pi} - \underline{\pi}) \longrightarrow N(t) = L(t) \frac{\bar{\pi} - \pi(t)}{\bar{\pi} - \underline{\pi}}$$

$$\begin{aligned} \underset{\pi(t)}{\text{maximize}} \quad & \beta \log \left( 1 + \boxed{L(t) \frac{\bar{\pi} - \pi(t)}{\bar{\pi} - \underline{\pi}}} \right) \\ & + \pi(t) L(t) \frac{\bar{\pi} - \pi(t)}{\bar{\pi} - \underline{\pi}} \end{aligned}$$

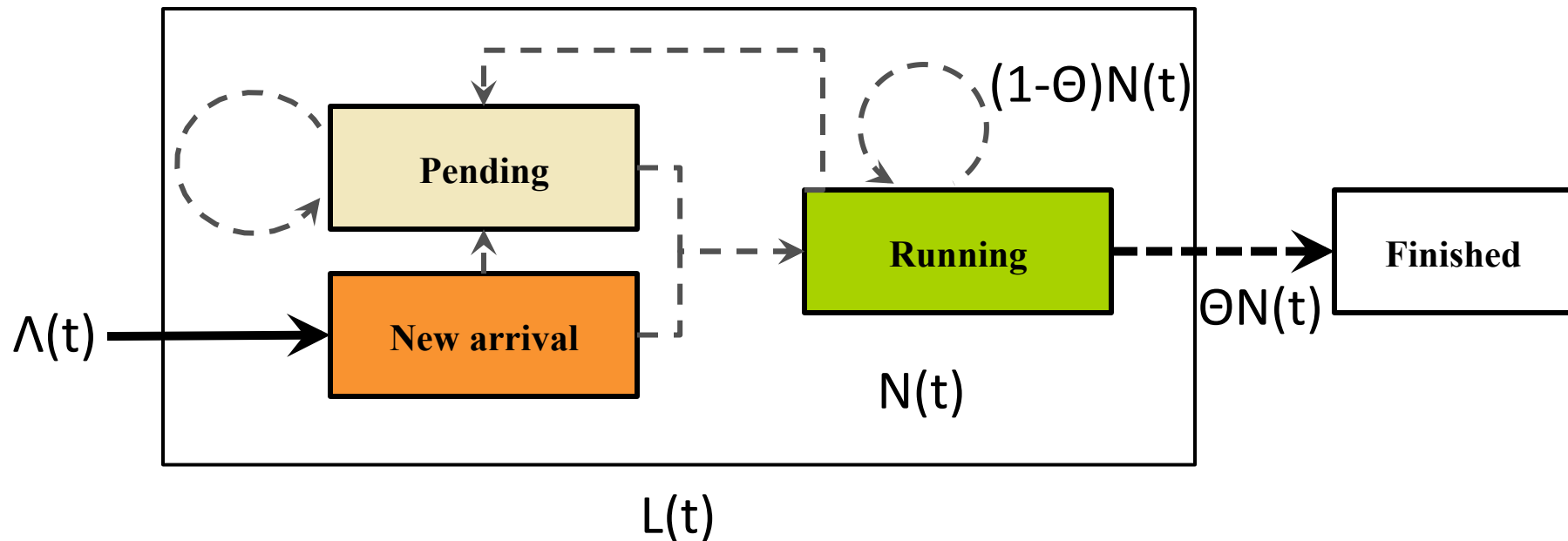
$$\text{subject to } \underline{\pi} \leq \pi(t) \leq \bar{\pi}.$$

$$\begin{aligned} \longrightarrow \pi^*(t) = \max & \left\{ \underline{\pi}, \frac{3}{4}\bar{\pi} + \frac{1}{2}(\bar{\pi} - \underline{\pi}) \frac{1}{L(t)} \right. \\ & \left. - \frac{1}{4} \sqrt{\left( \bar{\pi} + 2(\bar{\pi} - \underline{\pi}) \frac{1}{L(t)} \right)^2 + 8\beta(\bar{\pi} - \underline{\pi}) \frac{1}{L(t)}} \right\} \end{aligned}$$

# Stable job queues

$$f_p(x) = 1/(\bar{\pi} - \underline{\pi}) \longrightarrow N(t) = L(t) \frac{\bar{\pi} - \pi(t)}{\bar{\pi} - \underline{\pi}}$$

$$L(t+1) = \left(1 - \theta \frac{\bar{\pi} - \pi^*(t)}{\bar{\pi} - \underline{\pi}}\right) L(t) + \Lambda(t).$$



# Stable job queues

Proposition 1. The time-averaged queue size at any time  $t$  is uniformly bounded.

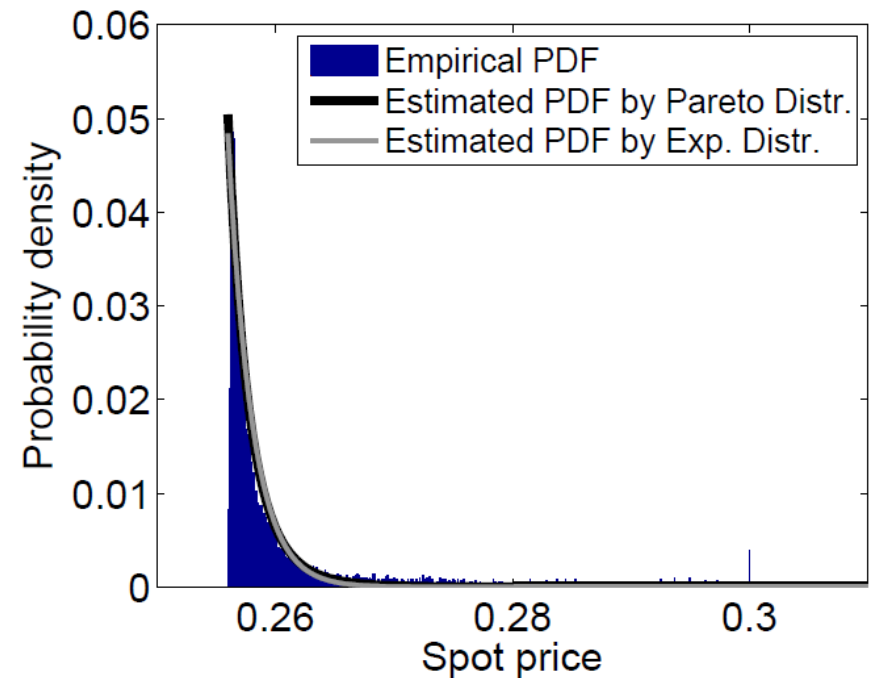
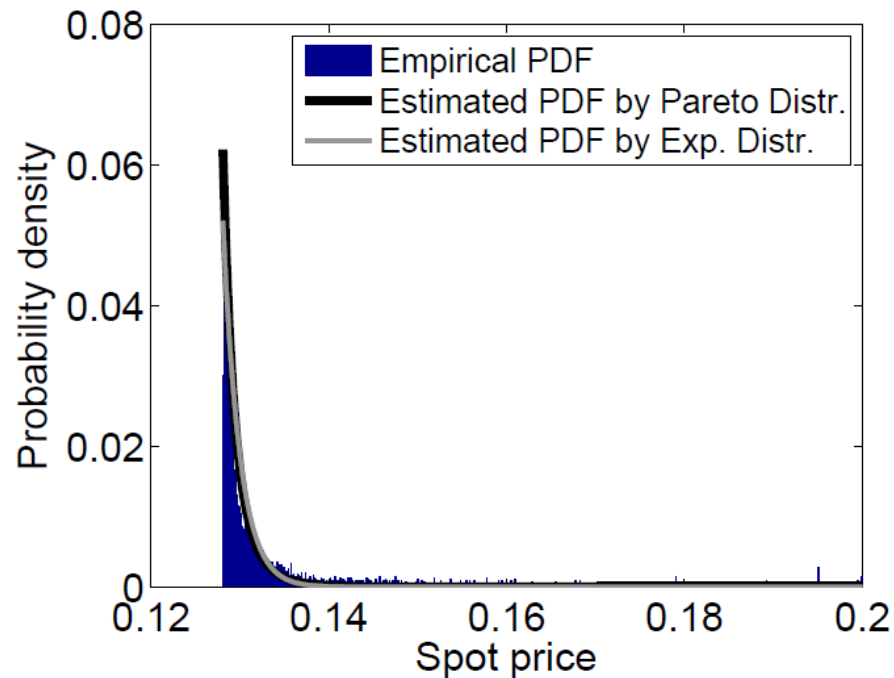
Proposition 2. Let  $L(t)=L(t+1)$ , the optimal spot prices should satisfy:

$$\pi^*(t) = h(\Lambda(t)) = \frac{1}{2} \left( \bar{\pi} - \frac{\beta}{1 + \frac{1}{\theta} \Lambda(t)} \right). \quad (6)$$

Proposition 3. The predicted PDF of the spot price is:

$$f_{\pi}(\pi) \simeq f_{\Lambda}(h^{-1}(\pi))$$

# Validation from historical spot prices



$$f_{\Lambda}(\Lambda) = \frac{\alpha \Lambda_{min}^{\alpha}}{\Lambda^{\alpha+1}}, \quad \text{for } \Lambda \geq \Lambda_{min}, \quad f_{\Lambda}(\Lambda) = \frac{1}{\eta} e^{-\frac{1}{\eta} \Lambda}, \quad \text{for } \Lambda \geq 0.$$



# Bid types

- One-time user bid

- Exit the system once they fall below spot price.

- Persistent user bid

- Resubmitted until job finishes or is manually terminated by the user.

- Longer completion time.

# Bid types

- One-time user bid

How to finish the job before it exits?

How to reduce cost?

- Persistent user bid

How to reduce cost with acceptable completion time?

# Bid types

- Job completion time

= running time ( $p \geq \pi$ ) + idle time

- One-time bid

running time = execution time

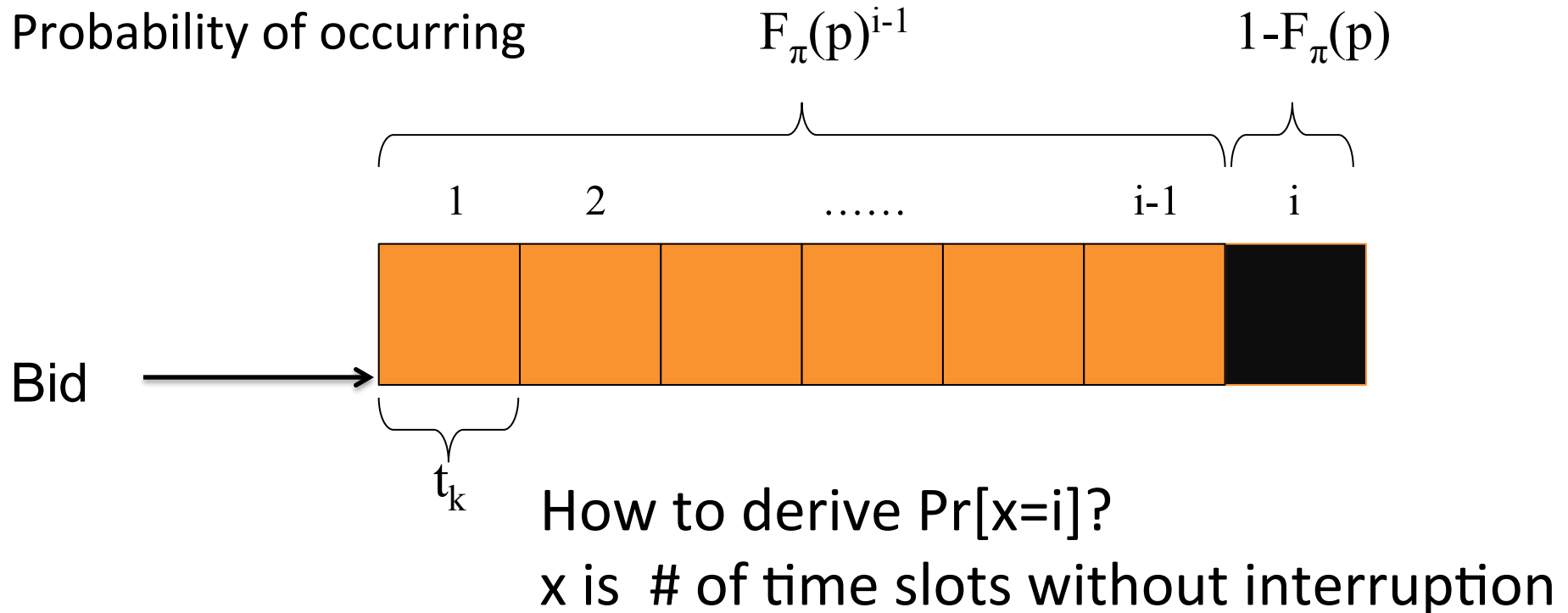
- Persistent bid

running time = execution time + recovery time

# Notations

- $f_{\pi}(\pi) \rightarrow f_{\pi}$
- $F_{\pi}(p) = \Pr[p \geq \pi(t)]$
- $\underline{\pi}, \bar{\pi}$

# Placing one-time bids



Expected length of time without interruption =

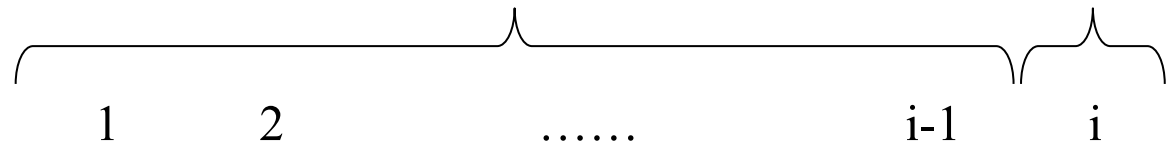
$$t_k \sum_{i=1}^{\infty} i F_{\pi}(p)^{i-1} (1 - F_{\pi}(p)) = \frac{t_k}{1 - F_{\pi}(p)}.$$

# Placing one-time bids

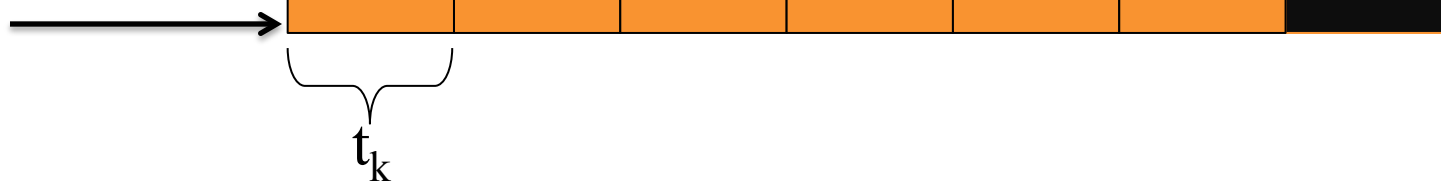
Probability of occurring

$$F_{\pi}(p)^{i-1}$$

$$1 - F_{\pi}(p)$$



Bid



Execution time  $t_s \leq$

$$\frac{t_k}{1 - F_{\pi}(p)}$$

Expected length of time  
without interruption =

$$t_k \sum_{i=1}^{\infty} i F_{\pi}(p)^{i-1} (1 - F_{\pi}(p)) = \frac{t_k}{1 - F_{\pi}(p)}.$$

# Cost minimization for one-time jobs

execution time

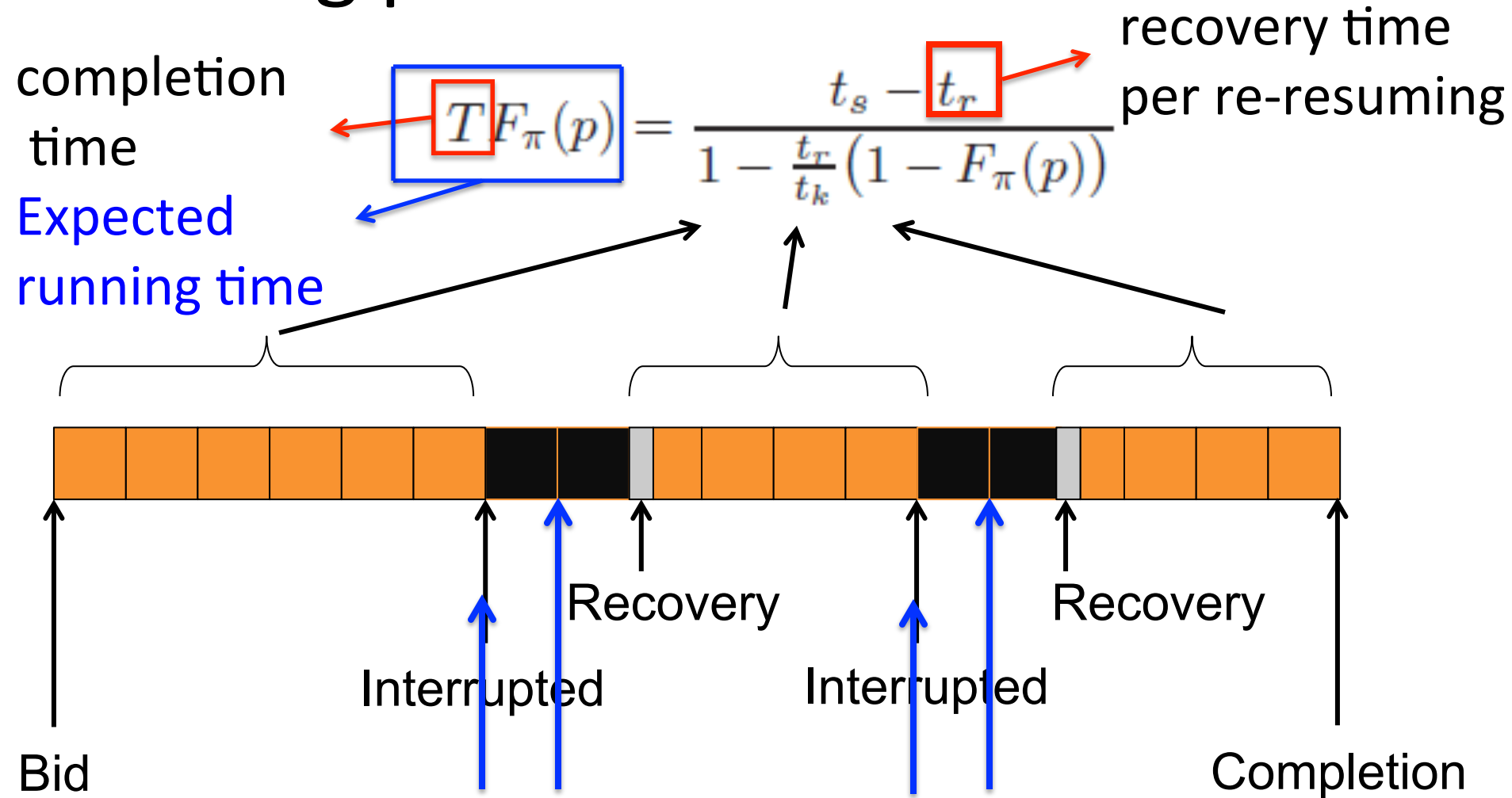
$$\begin{aligned} \underset{p}{\text{minimize}} \quad & \Phi_{so}(p) = t_s \mathbb{E}(\pi \mid \pi \leq p) = \frac{t_s \int_{\underline{\pi}}^p x f_{\pi}(x) dx}{F_{\pi}(p)} \\ \text{subject to} \quad & \Phi_{so}(p) \leq t_s \bar{\pi}, \quad t_s \leq \frac{t_k}{(1 - F_{\pi}(p))}, \quad \underline{\pi} \leq p \leq \bar{\pi}. \end{aligned}$$

execution time  $\leq$  expected running time without interruption

The optimal bid price: 
$$p^* = \max \left\{ \underline{\pi}, F_{\pi}^{-1} \left( 1 - \frac{t_k}{t_s} \right) \right\}.$$

Conclusion:  $t_s/t_k \uparrow \rightarrow p^* \uparrow$

# Placing persistent bids





# Placing persistent bids

The expected number of interruptions is:

$$\begin{aligned}
 & \mathbb{E} \left( \frac{1}{2} \sum_{k=0}^{T/t_k-1} \left( \boxed{\mathbb{I}_{\pi}(\pi(t))} - \mathbb{I}_{\pi}(\pi(t+1)) \right)^2 \right) \\
 & \stackrel{(a)}{=} \frac{T}{t_k} \left( \mathbb{E} \left( \mathbb{I}_{\pi}(\pi(t)) \right) - \mathbb{E} \left( \mathbb{I}_{\pi}(\pi(t)) \mathbb{I}_{\pi}(\pi(t+1)) \right) \right) \\
 & = \frac{T}{t_k} F_{\pi}(p) (1 - F_{\pi}(p)),
 \end{aligned}$$

$\left. \begin{array}{l} =1, \text{ if } p \geq \pi(t) \\ =0, \text{ otherwise} \end{array} \right\}$

# Placing persistent bids

The expected # of interruptions =  $\frac{T}{t_k} F_{\pi}(p) (1 - F_{\pi}(p))$

Expected running

time  $TF_{\pi}(p) = \left( \frac{T}{t_k} F_{\pi}(p) (1 - F_{\pi}(p)) - 1 \right) t_r + t_s$

constraint:

$$TF_{\pi}(p) = \frac{t_s - t_r}{1 - \frac{t_r}{t_k} (1 - F_{\pi}(p))}, \quad \text{with } 1 - \frac{t_r}{t_k} (1 - F_{\pi}(p)) > 0 \quad \text{constraint: } t_r < \frac{t_k}{1 - F_{\pi}(p)}$$

# Cost minimization (persistent bids)

Expected running time,  $TF_{\pi}(p)$

$E[\pi | P \geq \pi]$

$$\begin{aligned} &\underset{p}{\text{minimize}} && \Phi_{sp}(p) = \frac{t_s - t_r}{1 - \frac{t_r}{t_k} (1 - F_{\pi}(p))} \frac{\int_{\underline{\pi}}^p x f_{\pi}(x) dx}{F_{\pi}(p)} \\ &\text{subject to} && \Phi_{sp}(p) \leq t_s \bar{\pi}, \quad t_r < \frac{t_k}{1 - F_{\pi}(p)}, \quad \underline{\pi} \leq p \leq \bar{\pi}, \end{aligned}$$

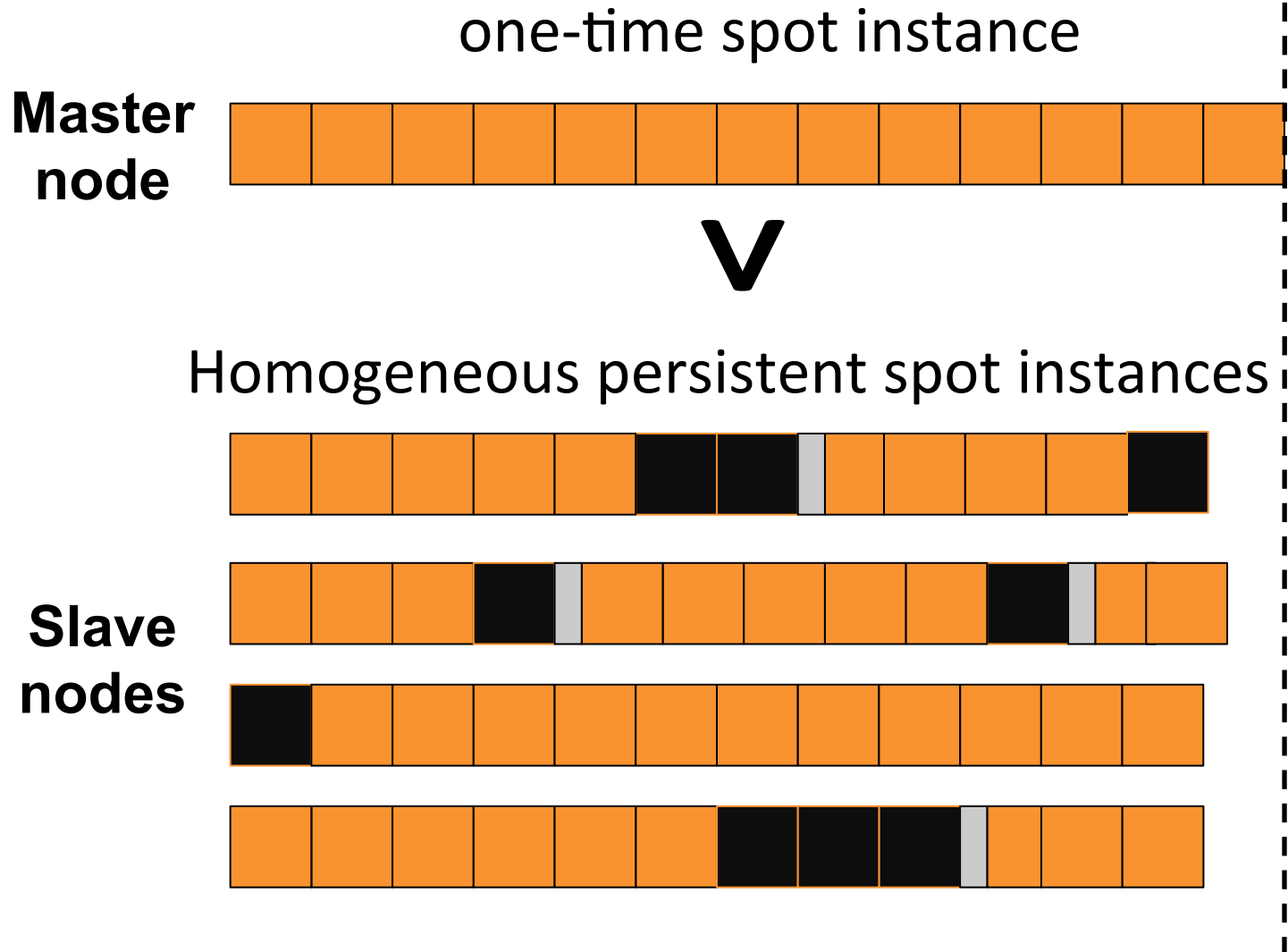
recovery time should be short

Optimal bid price:  $p^* = \psi^{-1} \left( \frac{t_k}{t_r} - 1 \right)$

where  $\psi^{-1}(\cdot)$  is the inverse function of

$$\psi(p) = F_{\pi}(p) \left( \frac{\int_{\underline{\pi}}^p x f(x) dx}{\int_{\underline{\pi}}^p (p - x) f(x) dx} - 1 \right)$$

# Bidding MapReduce jobs



# Experiment setup ( single-instance bids)

They simulate an one hour running time of a spot instance by creating Amazon Machine Image (AMI)

$t_k = 5 \text{ min}$

Spot price history: 2 months

Job:  $t_s = 1 \text{ hour}$ ,  $t_r = 10 \text{ s}$  /  $t_r = 30 \text{ s}$

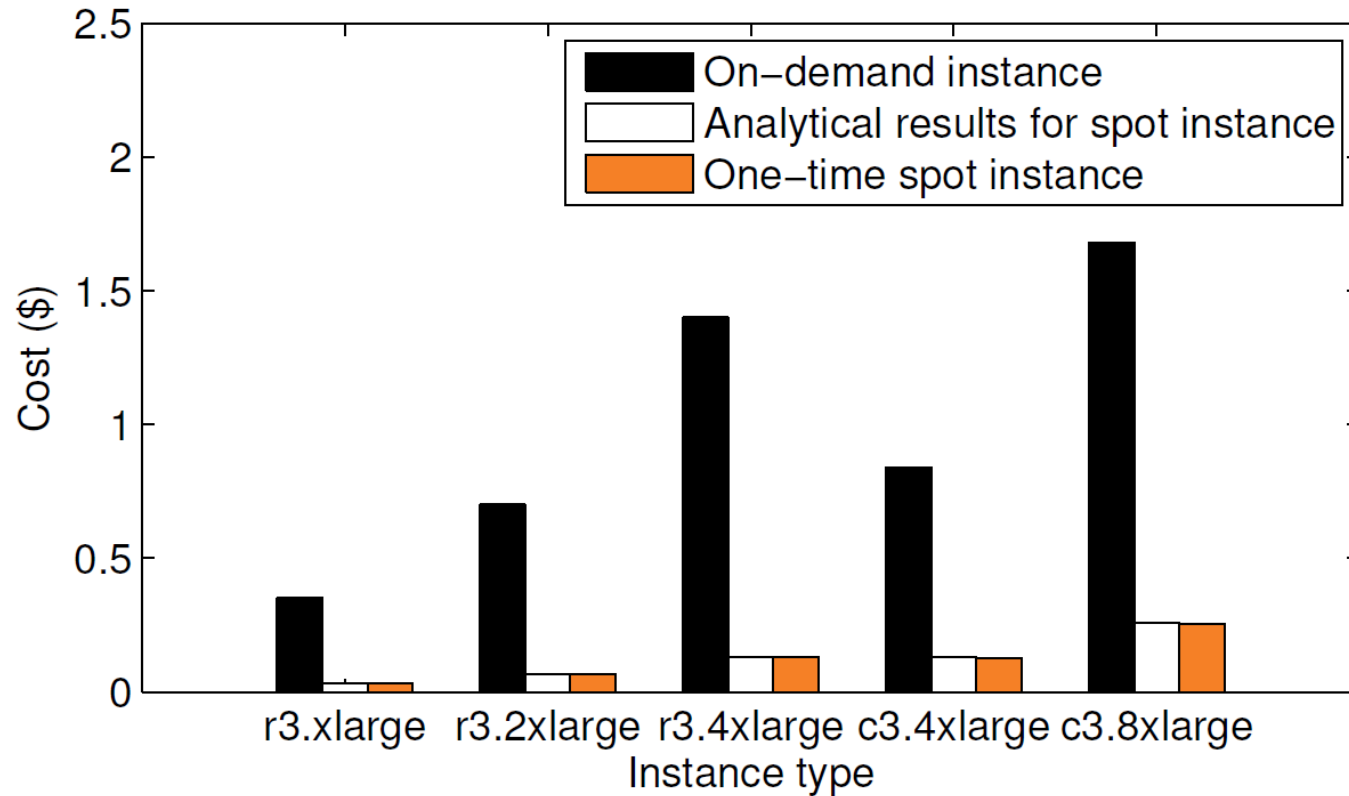
Compared heuristic: bid price is the minimum that consistently exceeds the spot prices for the past 1 hour

# Single-instance one-time bids

Optimal bid prices for one-time bids that run for one hour.

Instance type	On-demand price	One-time bid		
		Optimal bid price $p^*$	Offline retrospective price	Actual price $E[\pi   \pi < p^*]$
<b>r3.xlarge</b>	\$0.35	\$0.0374	\$0.0324	\$0.033
<b>r3.2xlarge</b>	\$0.70	\$0.0795	\$0.0644	\$0.066
<b>r3.4xlarge</b>	\$1.40	\$0.1430	\$0.1304	\$0.130
<b>c3.4xlarge</b>	\$0.84	\$0.1669	\$0.1324	\$0.128
<b>c3.8xlarge</b>	\$1.68	\$0.2903	\$0.2640	\$0.256

# One-time bids



**User costs are reduced by up to 91%, without any interruptions.**

# Single-instance persistent bids

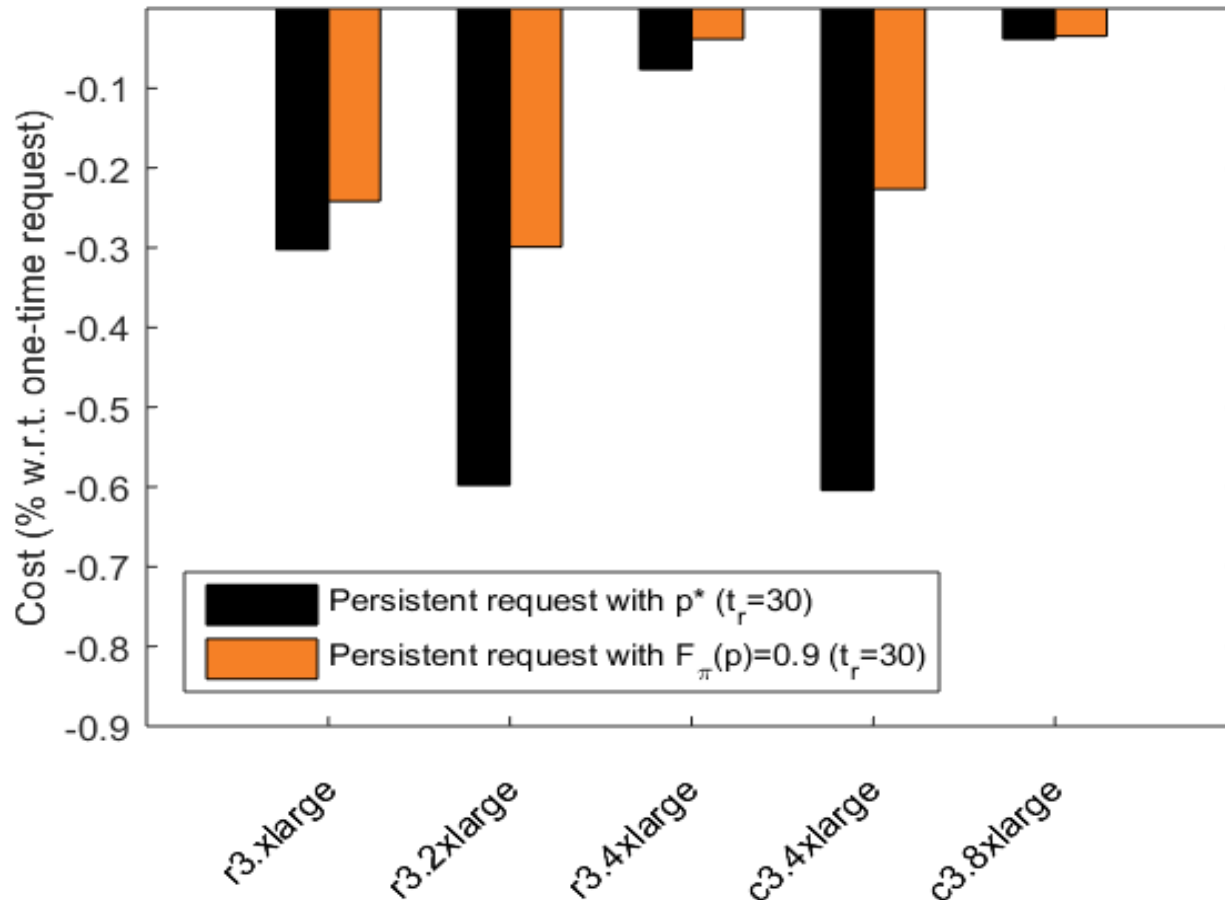
Optimal bid prices with different recovery times.

Instance type	On-demand price	Persistent bid	
		Optimal price $p^*$ ( $t_r=10s$ )	Optimal price $p^*$ ( $t_r=30s$ )
r3.xlarge	\$0.35	\$0.0332	\$0.0355
r3.2xlarge	\$0.70	\$0.0661	\$0.0711
r3.4xlarge	\$1.40	\$0.1327	\$0.1422
c3.4xlarge	\$0.84	\$0.1322	\$0.1413
c3.8xlarge	\$1.68	\$0.2648	\$0.2831

**Longer recovery times yield higher bid prices.**



# Single-instance persistent bids

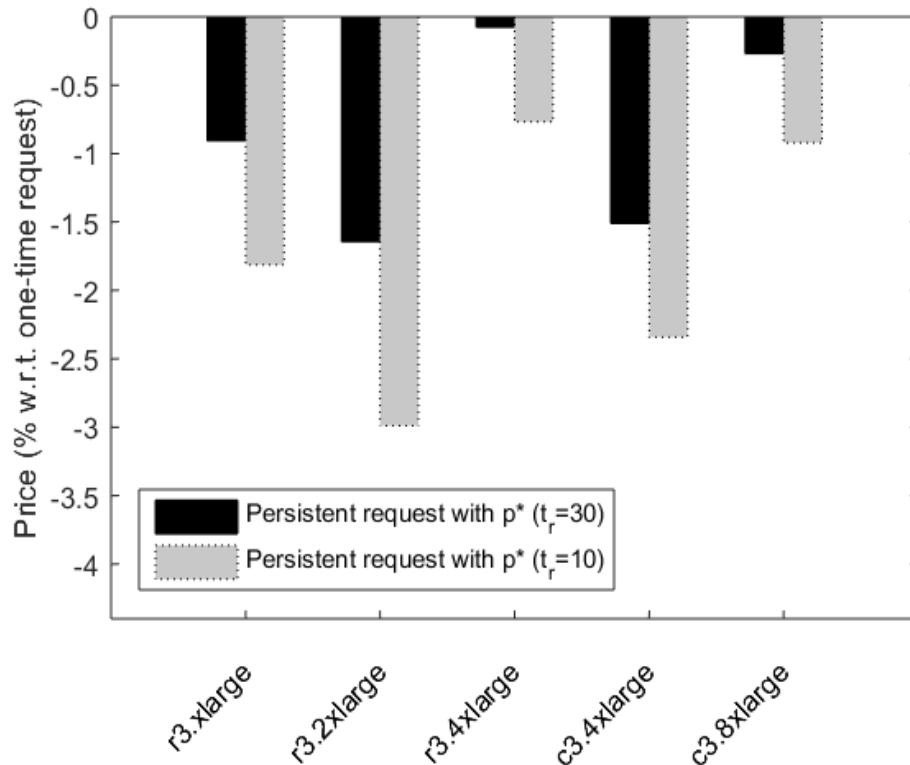


**Our bid prices are optimal for minimizing users' costs.**

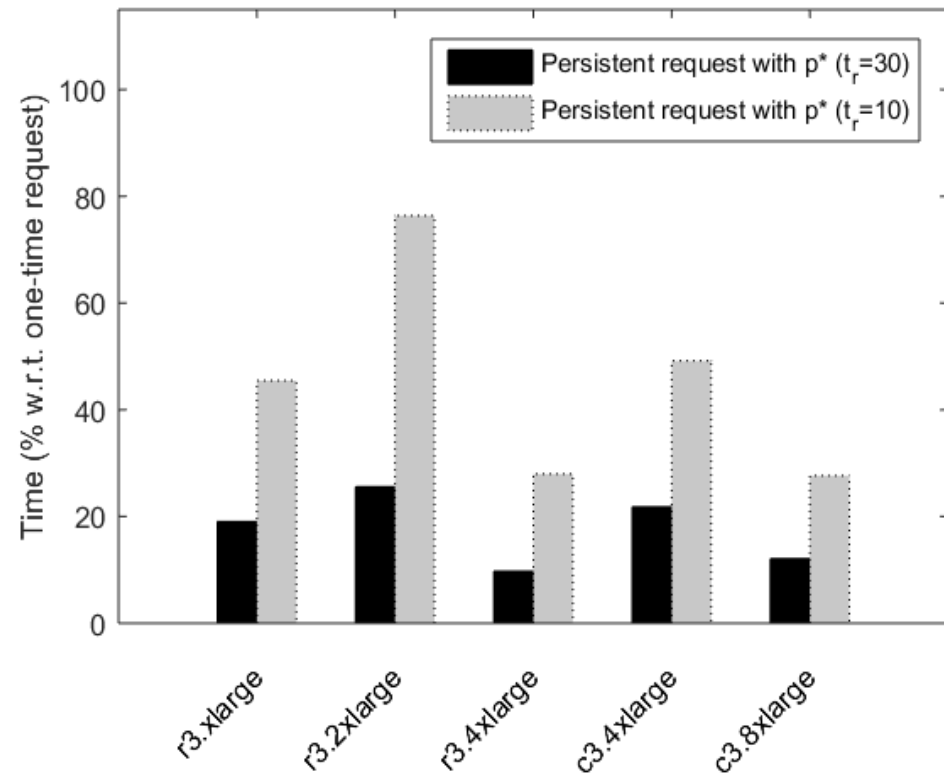
# Persistent bids

bid price (time) of persistent bids – bid price (time) of one-time bids

bid price (time) of one-time bids

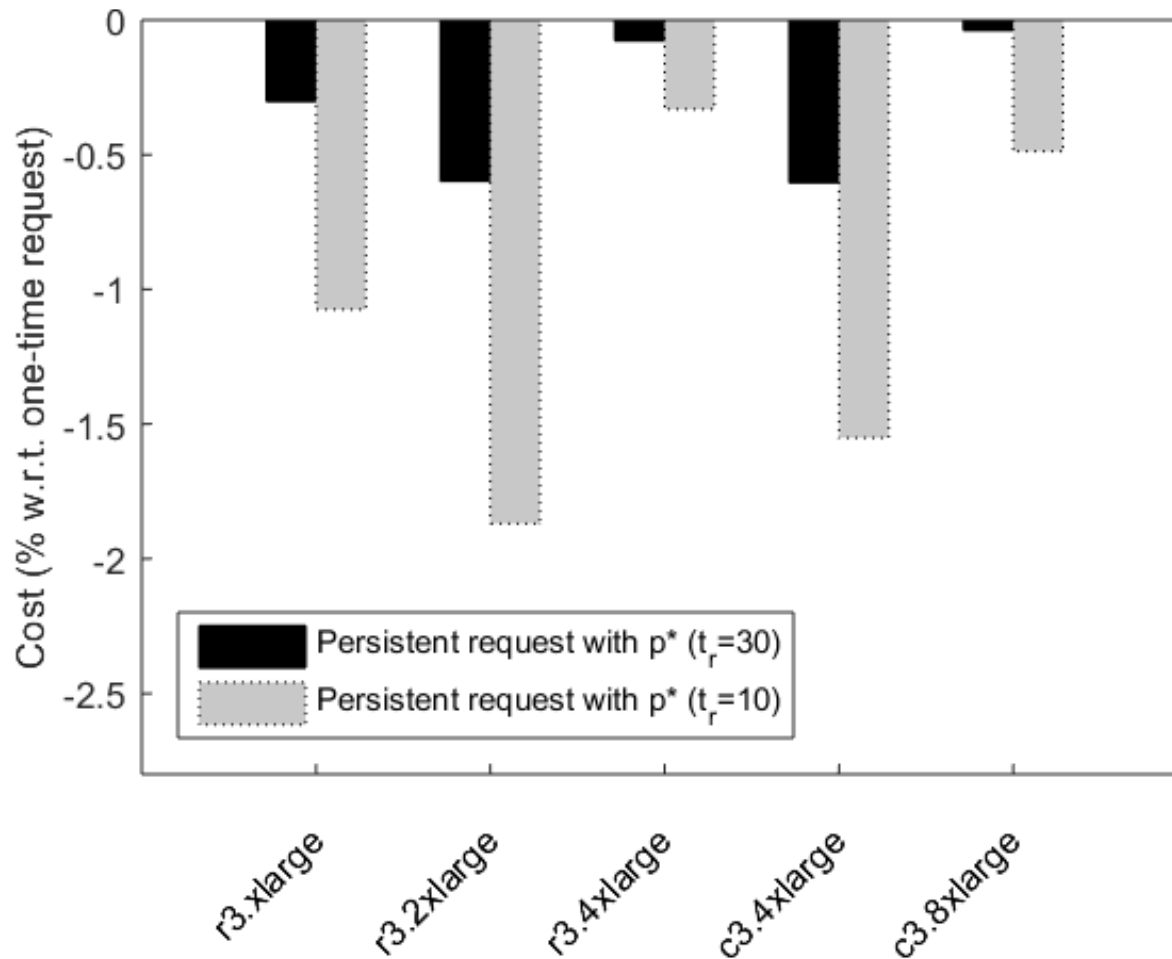


**A lower optimal bid price.**



**A longer completion time.**

# Persistent bids



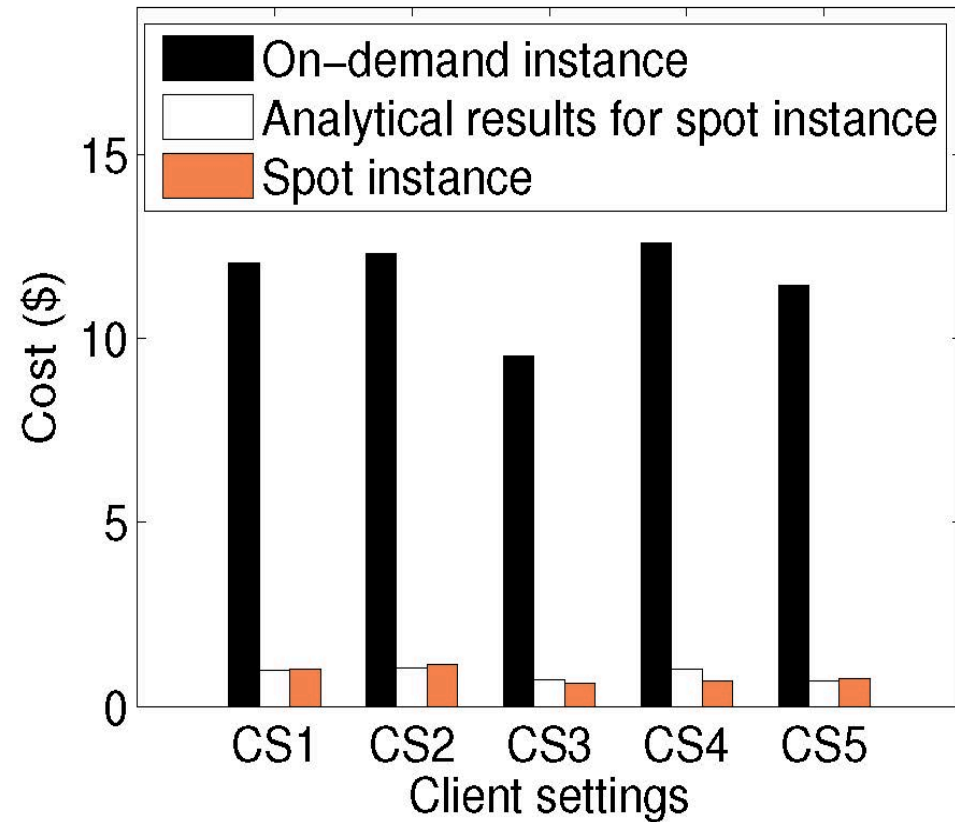
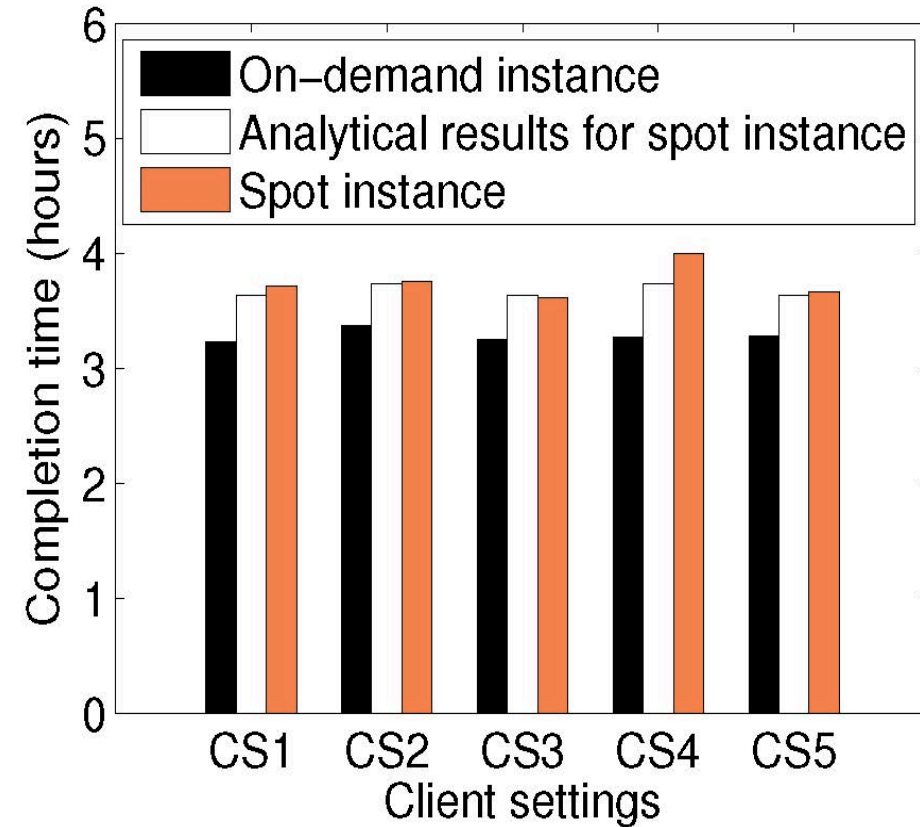
**The overall costs are further reduced.**

# Experiment setup ( MapReduce jobs)

**Job:** run Hadoop MapReduce jobs (count the frequency of words) on the Common Crawl Dataset using Amazon Elastic MapReduce (EMR)

$t_r = 10s$ ;  $t_0 = 60s$

# MapReduce jobs



**The cost is reduced by up to 92.6% with just a 14.9% completion time increase.**

# Conclusion

- Model for cloud provider's setting of the spot prices.
- Bidding strategies: tradeoff between prices and runtimes
  - One-time bids: bidding higher prices to avoid interruptions .
  - Persistent bids: bidding lower prices to save money.
- Application to the MapReduce jobs.

# Discussion

- Risk-awareness of users.
- Social welfare maximization in predicting the spot price.
- Adaptation to an online algorithm with good competitive ratio if possible.

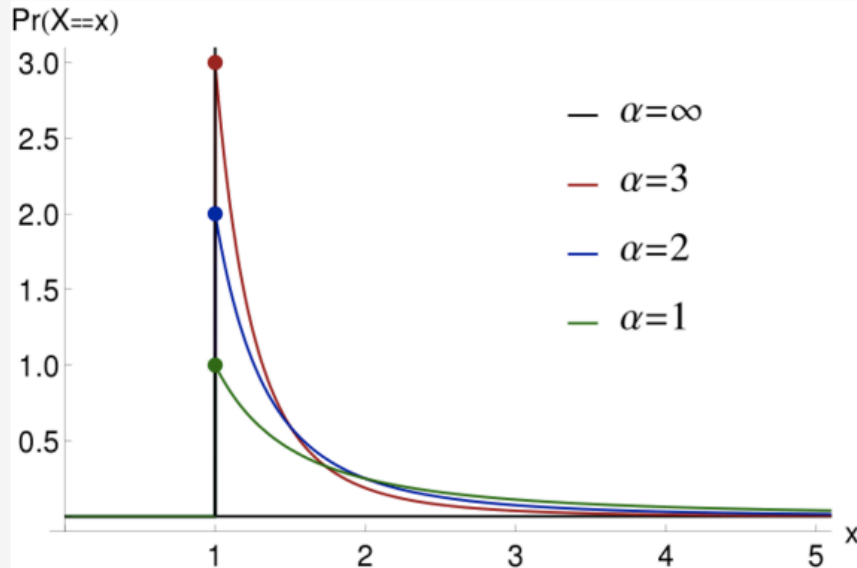
**Thank you**  
**Q&A**

Jan. 21, 2016



# Validation from historical spot prices

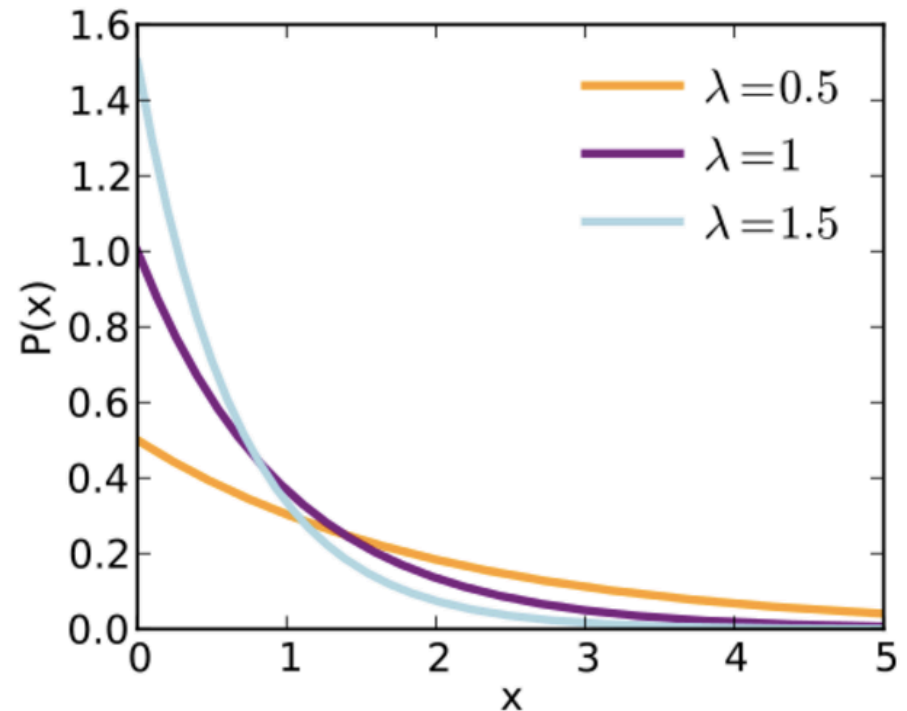
Probability density function



Pareto Type I probability density functions for various  $\alpha$  with  $x_m = 1$ . As  $\alpha \rightarrow \infty$  the distribution approaches  $\delta(x - x_m)$  where  $\delta$  is the Dirac delta function.

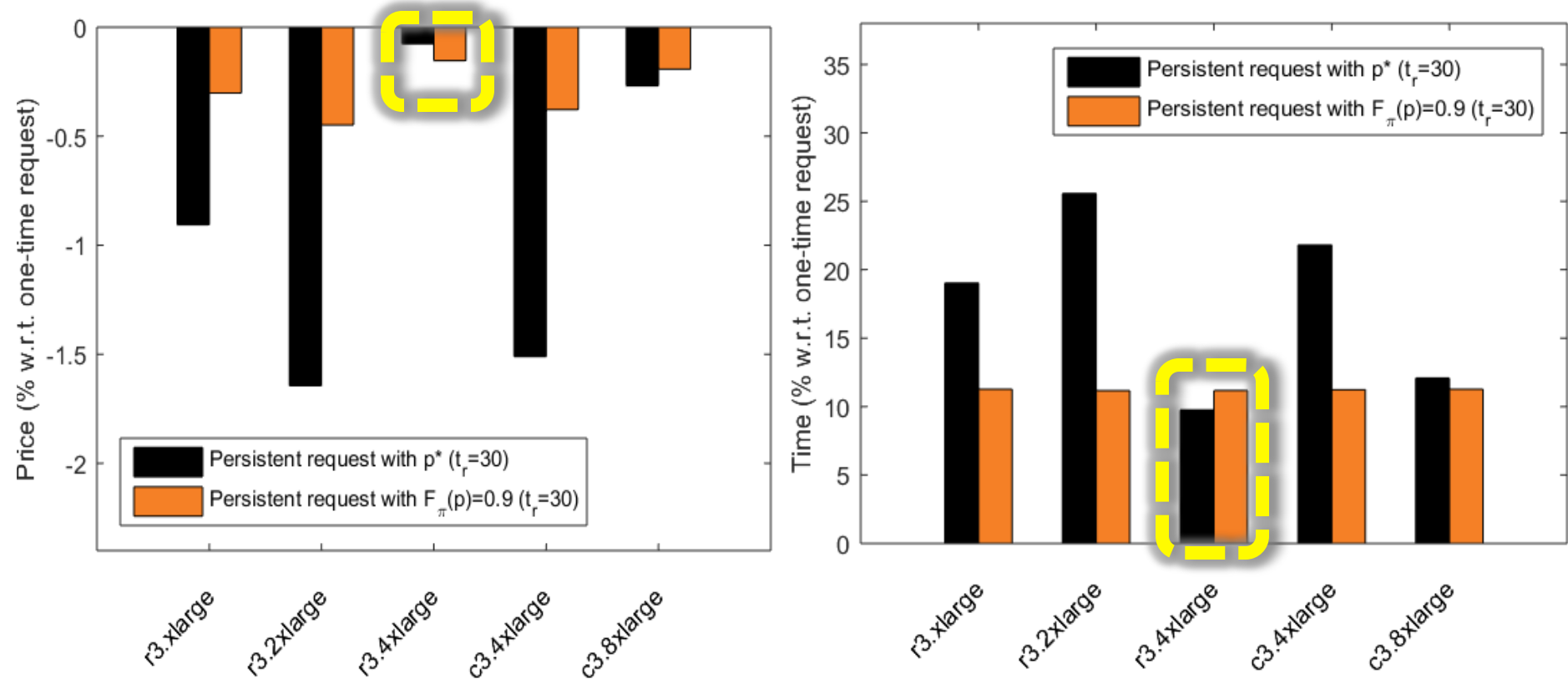
PDF of Pareto distribution

Probability density function



PDF of exponential distribution

# Single-instance persistent bids



**Bidding at the 90<sup>th</sup> percentile price yields either higher bid prices and lower completion times or lower bid prices and longer completion times.**