Weekly Report (2009-11-08)

Hongxing Li

1. Problem Model

Recall that last time we tried to formulate our issue as an optimization problem which aims to maximize the amount of data aggregated to the sink within one time-unit with one unit of energy.

We define our problem as an optimization problem as follows.

$$\begin{aligned} &minimize && T \times P \\ &s.t. && \exists S = \{S_1, S_2, ..., S_T\} \\ &&& i \neq j \Rightarrow T(S_i) \cap T(S_j) = \emptyset \\ &&& i < j \Rightarrow T(S_i) \cap R(S_j) = \emptyset \\ &&& \cup_i T(S_i) = V \\ &&& R(S_T) = \{V_0\} \\ &&& P = \sum_{V_i \in V} P_i \\ &&& SINR_i \geq \beta, (i = 1, 2, ..., n) \end{aligned}$$

Here, T is the aggregation latency and P is the overall energy-consumption. S is the link schedule and S_i is the set of links, which is represented as a two-tuple of transmitter and receiver, scheduled in time i. $T(S_i)$ is the set of transmitters in S_i and $R(S_i)$ is the set of receivers in S_i.

But it is hard to combine the tree construction and link scheduling as constraint functions in one single optimization. And it is not appropriate to express the constraint functions as logic ones. This week I did a quick view on optimization problems in network society, especially Dr. Yi Cui and Prof. Mung Chiang's work, and conducted some revision to our model.

Intuitively, our problem should be NP-hard. So it may not be able to get an analytical result as a single optimization problem. At current stage, we firstly divide the problem into two subproblems: aggregation tree construction and link scheduling.

1) **Aggregation tree construction:** Given with a matrix $D \in R^n \times R^n$. $D(i,j)$ represents the Euclidean distance between node i and node j. We need to get an aggregation tree E, which is also represented in form of matrix. And E has the property that

$$\begin{aligned} &\forall i, j \in [1, n], E^i(j, j) = 0 \\ &E \times 1^T = 1^T \end{aligned}$$

   In other words, the graph is Directed Acyclic Graph (DAG) and any node can only be linked to exact one node. We still have no clear understanding on which kind of tree property is desirable for our optimization, so I just present constraint functions here and leave the objective function of tree construction for further exploration.

2) **Link scheduling:** Given with the aggregation tree E and distance matrix D, we should give a sequence of schedule $S = \{S_1, ..., S_T\}$, which are also in the form of matrix. And the constraint on S is that

$$\begin{aligned} &\sum_{i=1}^{T} S_i = E \\ &\forall i < j, S_j \times S_i = 0 \end{aligned}$$

   In other words, each link is scheduled exactly once and no link can be scheduled before all its children have been scheduled.

Note that the domain of elements in above matrixes must be $\{0, 1\}$.

2. Discussion

The objective is to minimize $f_0(T, P) = T \times P$. As the constraint functions are too complicated. I firstly conducted some analysis on different values of T to get some intuitive understanding of the problem.

1) $T = 1$**:** This case is only possible when SIC is applied and each node transmit directly to the sink node in one hop.

$$
\begin{aligned}
(i) \quad & \frac{P_1/d_{11}^{\alpha}}{N_0} \geq \beta \\
& \Rightarrow P_1 \geq d_{11}^{\alpha} N_0 \beta \\
(ii) \quad & \frac{P_2/d_{22}^{\alpha}}{N_0 + P_1/d_{11}^{\alpha}} \geq \beta \\
& \Rightarrow P_2 \geq d_{22}^{\alpha} N_0 \beta (1+\beta) \\
(iii) \quad & \frac{P_3/d_{33}^{\alpha}}{N_0 + P_1/d_{11}^{\alpha} + P_2/d_{22}^{\alpha}} \geq \beta \\
& \Rightarrow P_3 \geq d_{33}^{\alpha} N_0 \beta (1+\beta)^2 \\
& \ldots\ldots \\
(n) \quad & \frac{P_n/d_{nn}^{\alpha}}{N_0 + \sum_{j<n} P_j/d_{jj}^{\alpha}} \geq \beta \\
& \Rightarrow P_n \geq d_{nn}^{\alpha} N_0 \beta (1+\beta)^{n-1}
\end{aligned}
$$

It is not hard to get the following by induction,

$$f_0(T, P) = f_0(1, P) = P = N_0 \beta \sum_{i=1}^{n} d_{ii}^{\alpha} (1+\beta)^{i-1}$$

2) $T = n$**:**

- **Single Hop:** Each node is directly linked to the sink node in one hop. But each link will be scheduled individually and sequentially. Thus, n timeslots are required in total and SIC is not applied. And the transmission power of each node fulfills the following constraint.

$$\frac{P_i/d_{ii}^{\alpha}}{N_0} \geq \beta$$

To minimize the power consumption, we just need a SINR value of β. So we have $P_i = d_{ii}^{\alpha} N_0 \beta$. Then the objective function will be,

$$f_0(T, P) = f_0(n, P) = n N_0 \beta \sum_{i=1}^{n} d_{ii}^{\alpha} \tag{1}$$

- **Multiple Hop:** In this case, the network is constructed into a tree. As $T = n$, each node will still be scheduled individually and sequentially. And SIC is not applied either. So the result for objective function is the same with above case.

$$f_0(T, P) = f_0(n, P) = n N_0 \beta \sum_{i=1}^{n} d_{ii}^{\alpha} \tag{2}$$

However, we should be noted that in multiple-hop topology, $\forall i \in [0, n], d_{ii}$ is no larger than previous case. As a result, the objective function value in this case is no larger than that of previous case.

3) $1 < T < n$**:**

- **Single Hop:** SIC must be applied here. Suppose SIC is conducted m times and for the i^{th} time there are k_i links transmitting concurrently. Then there should be $T - m$ links which are scheduled individually. Besides, we should have the constraint that $n - T = \sum_{i=1}^{m} k_i - m$. Then the objective function should be,

$$f_0(T, P) = T \times N_0 \beta (\sum_{i=1}^{m} \sum_{j=1}^{k_i} d_{i_j i_j}^{\alpha} (1+\beta)^{j-1} + \sum_{l=1}^{T-m} d_{ll}^{\alpha}) \tag{3}$$

- **Multiple Hop:** This case is most complicated. SIC may be applied to reduce T. But it is also possible that multiple links, which share no common receiver, are scheduled concurrently causing no failure.
  This case may be the focus in coming weeks.

Compare formula (1) and (2). If the link length of each link is similar to each other, i.e. $d_{ii} \approx d_{jj} \approx d$, we can transfer (1) and (2) into following

$$
\begin{aligned}
(1): \quad & f_0(T,P) \approx d^\alpha N_0 \beta \sum_{i=1}^{n} (1+\beta)^{i-1} = d^\alpha N_0 \beta \times \frac{(1+\beta)^n - 1}{\beta} \\
(2): \quad & f_0(T,P) \approx d^\alpha N_0 \beta \times n
\end{aligned}
$$

As $\beta > 1$, it is not hard to see that $\frac{(1+\beta)^n-1}{\beta} > n$. So *single-hop with* $T = 1$ is not preferable.

Now considering formula (3), we see that the Non-SIC portion is reduced as $T < n$ compared to formula (2). However, the SIC portion is increased regarding formula (1). More important, it should be noticed that later cancelled signal requires an exponentially ($(1+\beta)^{i-1}$) increasing transmission power. So if we need a smaller value for f_0 in the *signal-hop with* $1 < T < n$ case, the power requirement for signal participating SIC should be exponentially smaller than that of signals scheduled individually.

The other cases will be even more complicated. Despite the choice between SIC and Non-SIC, we also need to face the interference caused by concurrent transmitting links in Non-SIC cases or different subtrees in SIC cases.

In conclusion, we may get two guidelines in future analysis or algorithm design:

1) If the aggregation tree is given, SIC may be more profitable in cases where link diversity is high. And short links should be cancelled later in the cancellation sequence.
2) It may be better to avoid SIC in a subtree with dense node distribution. As SIC will exponentially increase the transmission power of later cancelled signals, the interference brought into neighborhood will also increase exponentially. In future analysis, we may use a matrix I to quantify the interference caused by each node i to another node j. We set that $I(i,j) = \frac{\beta}{d_{ij}^\alpha d_{jj}^\alpha}$ which means that if node i increase the transmission power by one unit, node j should increase its transmission power by $\frac{\beta}{d_{ij}^\alpha d_{jj}^\alpha}$ units to alleviate the interference.