Combinatorial Auctions with Restricted Complements

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- Model
- For planar graph, $(1+\varepsilon)$-approximate truthful auction
- For hypergraph-r valuation, r-approximate algorithm
- For hypergraph-r valuation with m items, $O(\log^r m)$-approximate truthful auction

Model

- A set M of m items
- n users
- Valuation $v_i(S)$, for $S \subseteq M$

Hypergraph-r Valuation

- Complement items
- Hypergraph H=(M,E)
- Every edge e in E contains at most r items, with valuation w_e
- r : rank of the hypergraph

$$v(S) = \sum_{e : e \subseteq S} w_e$$

Planar Graph Valuation

- Hypergraph-2 (normal graph)
- Planar graph (can be extended to any graph)

Tree Decomposition

- G=(M,E) has a tree decomposition with width k if:
- Tree (X,T), X={X_1,X_2...} are bags, T is a tree of bags

$$\bigcup X_i = M$$

$$\max |X_i| \leq k+1$$

- For every edge, there exists a bag contains it
- If X_i,X_j,X_l are bags, and X_l is on the path from X_i to X_j, then $X_i \cap X_j \subseteq X_l$

Tree Decomposition

- Treewidth of a graph G: the smallest k for G to have a decomposition
- Linear time algorithm

- The valuation of user i $v_{i,}$ is a subgraph of G

Welfare Maximizing Algorithm

- Time complexity $n^{O(k)}$
- DP

Auction 1

- $k = 2 / \varepsilon$
- Divide G into k+1 parts P_0, P_1,… P_k.
- $M_i = M \setminus P_i$ has treewidth 3k
- Compute optimal allocation for M_i
- Pick the best allocation among k+1 options
- Use VCG payment

Auction 1

- Why efficient?
- Why truthful?
  - MIR allocation (maximal-in-range)

Algorithm 2

$$\max \quad \sum_{i=1}^{n} \left(\sum_{e \in E_i} w_{ie} z_{ie} \right)$$

subject to:

$$\begin{aligned}
\textstyle\sum_{i=1}^{n} x_{ij} = 1 \quad & \text{for every good } j. \\
z_{ie} \le x_{ij} \quad & \text{for every player } i\text{, edge } e \in E_i, \\
& \text{and good } j \in e. \\
x_{ij} \ge 0 \quad & \text{for every player } i \text{ and good } j \in M \\
z_{ie} \ge 0 \quad & \text{for every player } i \text{ and edge } e \in E_i
\end{aligned}$$

Algorithm 2

- Poly-time, r-approximate algorithm by random rounding

- While exists unallocated item
  - Choose user randomly
  - Allocate item with prob. x^*_{ij}

Auction 3

- Demand oracle:

- Given price p_j for each good j, find the optimal set of items S, to maximize $v(S) - \sum_{j \in S} p_j$
- By supermodular property

Auction 3

- Duplicate each item B times

$$B = \Omega(\log m)$$

- Adjust valuations accordingly

$$v_i'(S) = \sum_{e\,:\,e \subseteq S} \frac{w_{ie}}{B^{|e|}}$$

Auction 3

$$
\begin{array}{rll}
\textbf{max} & f(\mathbf{y}) = \sum_{i,S\neq\emptyset} v_i'(S) y_{i,S} & \\
\textbf{subject to:} & \sum_{S\neq\emptyset} y_{i,S} \leq 1 & \text{for every player } i. \\
& \sum_i \sum_{S|j\in S} y_{i,S} \leq B & \text{for every good } j \in M. \\
& y_{i,S} \geq 0 & \text{for every player } i \text{ and bundle } S \subseteq M
\end{array}
$$

Auction 3

- Solve the LP, obtain optimal fractional solution
- Decompose into a combination of feasible integer solutions. (Use ellipsoid method and the oracle)
- Randomly pick allocation

- MIDR

Thank you!