Some Preliminary Conditions on Analysis of Queueing Networks

I. DEFINITIONS

Network J_N, N is the number of peers in the network, and M denotes the number of total budgets.

$p_{ij,N}$ stands for the probability of the transition of a budget from peer i to peer j, so that a finite Markov chain can be defined with a probability routing matrix $P_N = \{p_{ij,N}\}_{i,j=1}^{N}$.

$\mu_{i,N}$ is the service rate at node i in J_N, and the average budgets amount per peer $\lambda = \frac{M}{N}$.

$r_{i,N}$ is *relative utilization* that can be computed by the above two matrices P_N and $\mu_{i,N}$. Then, a normalizing constant $Z_{M,N}$ can be calculated with all above, the details can refer to [1], [2].

Now we can get the factorized probability $\mathcal{P}_{M,N}$ at equilibrium for queue length :

$$\mathcal{P}_{M,N} = \frac{1}{Z_{M,N}} \prod_{i=1}^{N} r_{i,N}^{n_i}, \quad 0 \le n_i \le M, i = 1, \cdots, N, \tag{1}$$

where n_i is the number of budgets that peer i has.

With the probability, we can compute the mean number of budgets at peer i: $m_{i,M,N}$.

II. SOME CONCLUSIONS WHICH MIGHT BE USEFUL

A. The condition that NO condensation happens

If

$$\lambda < \lim_{z \to 1-} \int_0^1 \frac{r}{1 - zr} dI(r)$$

,

where I() is a Borel measure over [0,1], [3]

Then,

$$Z_N \sim \frac{1}{\sqrt{2\pi N \big(-\lambda \ln z - \int_0^1 \ln(1 - zr) dI(r) \big)''}} \frac{1}{z_0} exp\big(N(-\lambda \ln z_0 - \frac{1}{N} \sum_{i=1}^{N} \ln(1 - z r_{i,N})) \big)$$

So that there are no condensation happens.

More properties can be explored later.

B. The condition that condensation occurs

If

$$\lambda > \lim_{z \to 1-} \int_0^1 \frac{r}{1 - zr} dI(r)$$

,

Then at least one peer will have budgets $\to \infty$

Proof of the above two subsections take too many pages.

III. Implication to Budget-based P2P System

We can adjust λ to control the budget distribution in P2P networks.

References

[1] J. Jackson, "Jobshop-like Queueing Systems," *Management Science*, no. 10, pp. 131–142, 1963.

[2] W. Gordon and G. Newell, "Closed Queueing Systems with Exponential Servers," *Operation Research*, no. 15, pp. 254–265, 1967.

[3] M. E. Munroe, *Introduction to Measure and Integration.* Addison Wesley, 1953.