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# Model Draft

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#### Abstract

#### I. MODEL OVERVIEW

The model consists of a set of geographically diverse cloud clusters F, a set of videos O and a set of clients D. ( $D^{(f)}$  denotes the consolidated viewer group within domain of cloud cluster f ( $f \in F$ ).) Without loss of generality, I will first assume all the videos have unit length. (Extend this when the model is done)

#### A. Alphabet Soup

- 1) Each cloud cluster is assigned a storage capacity,  $S_f$ .
- 2) Each cloud cluster has a bandwidth capacity,  $\mu_f$ . Here I plan to borrow the idea from our ICDCS paper's model *i.e.*, bandwidth will be abstracted into a VM instance, which will actually provide the bandwidth.
- 3)  $x_{jf}^{(o)}$  denotes the variable indicating whether the request for video o issued from viewer j will be directed to cloud cluster f.
- 4)  $y_f^{(o)}$  denotes the variable indicating whether to store a copy of video o at the cloud cluster f.
- 5)  $c_f$  denotes the storage cost of cloud cluster f.
- 6)  $v_f$  denotes the transferring cost from viewer j's location to cloud cluster f.
- 7)  $R_{jf}$  denotes the transferring latency from viewer j's location to cloud cluster f. It can be assigned with value of RTT between these two geographical regions.

There should be a mapping function to map user j to a location f. For simplicity, we can denote it as  $D^{-1}(j)$ . In that way, we can represent  $x_{jf}^{(o)}$ ,  $c_{jf}$  and  $R_{jf}$  as  $x_{D^{-1}(j)f}^{(o)}$ ,  $c_{D^{-1}(j)f}$  and  $R_{D^{-1}(j)f}$  respectively. For clearness, I will use j afterwards.

#### B. Objective function

To minimize the operation cost, based on the premise that the expected average global latency should be bounded below some tolerant value.

$$\min \sum_{o \in O} \sum_{f \in F} y_f^{(o)} \times c_f + \sum_{o \in O} \sum_{f \in F} \sum_{j \in D^{(f)}} x_{jf}^{(o)} \times v_f$$

#### C. Constraints

1) Storage  $\sum_{o \in O} y_f^{(o)} \le S_f , \forall f \in F$ 

2) VM capacity 
$$\sum_{o \in O} \sum_{j \in D} x_{jf}^{(o)} \leq \mu_f, \forall f \in F$$

3) Placement 
$$\sum_{f \in F} y_f^{(o)} \ge 1, \forall o \in O$$

- 4) Latency guarantee  $\frac{\sum_{o \in O} \sum_{f \in F} \sum_{j \in D^{(f)}} x_{jf}^{(o)} \times R_{jf}}{|D|} \leq R_{threshold}, \text{ where } R_{threshold} \text{ is an input into the system.}$
- 5) Variable constraint  $y_f^{(o)} \in \{0,1\}, x_{jf}^{(o)} \in \{0,1\}$ 6)  $x_{jf}^{(o)} \leq y_f^{(o)}, \, \forall j \in D$

### II. ALTERNATIVE LP (RELAXATION)

Obviously, the optimization in Sec. I is an integer problem. Here we want to make an intuitive relaxation. The reason is two-fold. First, the number of users makes the optimization problem too large to solve. Second, we want to transform the original problem into a more tractable one.

#### A. Consolidate users

As what we have assumed, at any time, each user can at most view one video. So we can treat the users within one specific region f ( $f \in F$ ) as one, which will make our optimization much slimmer. Based on that, we are able to eliminate all the variables  $x_{jf}^{(o)}$  and consolidates them as one viewer. Suppose the total user set at time slot T is represented as  $D_T$ , so the viewer set in region f at that time slot is  $D_T^{(f)}$ . We introduce a new variable  $\alpha_{jf}^{(o)}$ , which denote the portion of request for video o issued from the aggregate user f to cloud cluster f. To note that,  $\alpha_{jf}^{(o)}$  is a fractional variable. Due to our charging mode, the storage deployment is scheduled at a larger time scale while the VM rental is done at a smaller one, i.e. T. From above, the original ILP has only one type of integer variable  $y_f^{(o)}$  and the original optimization problem is changed to,

$$\min \sum_{o \in O} \sum_{f \in F} y_f^{(o)} \times c_f + \sum_{o \in O} \sum_{f \in F} \sum_{j \in F} \alpha_{jf}^{(o)} \times D_T^{(f)} \times v_f$$

#### B. Constraints

- 1) Storage  $\sum_{o \in O} y_f^{(o)} \le S_f , \forall f \in F$
- 2) VM capacity  $\sum_{o \in O} \sum_{j \in F} \alpha_{jf}^{(o)} \times D_T^{(f)} \leq \mu_f, \forall f \in F$
- 3) Placement  $\sum_{f \in F} y_f^{(o)} \ge 1, \forall o \in O$
- 4) Latency guarantee  $\frac{\sum_{o \in O} \sum_{f \in F} \sum_{j \in F} \alpha_{jf}^{(o)} \times D_T^{(f)} \times R_{jf}}{|D|} \leq R_{threshold}, \text{ where } R_{threshold} \text{ is an input into the system.}$
- 5) Variable constraint  $y_f^{(o)} \in \{0,1\}, \alpha_{jf}^{(o)} \in [0,1]$ 6)  $\alpha_{jf}^{(o)} \leq y_f^{(o)}, \, \forall j,f \in F$ 7)  $\sum_{f \in F} \alpha_{jf}^{(o)} = 1$

## C. How to solve?

If we relax the variable  $y_f^{(o)}$ , the optimization problem in Sec. II-A is a problem with complicating constraints and we can utilize an efficient dual decomposition to solve it.