Weekly Report (2010-03-18)

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I. Aggregation Tree Construction

For aggregation tree construction, we need two steps:

1) *Spanning Tree*: A spanning tree should be constructed first. Each node will know its level and parent in the tree.
2) *Connected Dominating Set*: Construct a connected dominating set based on the spanning tree.

As stated in last report, what we can do is in the spanning tree part. In order to compare the time complexity and energy complexity in data aggregation, we have following options to construct the spanning tree:

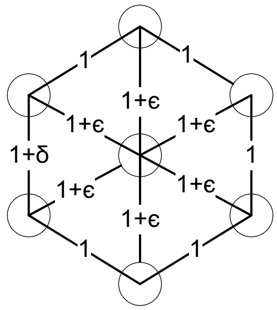
1) *Reduce time complexity:* Construct the spanning tree by node level labeling. Each node in level l will choose the nearest or smallest-ID node in level $l-1$. Now we have two choices:
   - Stop here and use the spanning tree as the aggregation tree. The advantage is that the tree depth is exactly the network diameter of the sink. However, the node degree in the tree is still bounded by Δ. So we do not choose this option.
   - Construct the backbone of aggregation tree with *Connected Dominating Set* based on the spanning tree. The dominated nodes can connect to their dominator directly or still maintain the structure in original spanning tree.
2) *Reduce energy complexity:* Construct the spanning tree as a minimum cost spanning tree. The cost of link i is proportional to d_i^α. However, the depth of this spanning tree cannot be bounded in $O(D)$. Fig. 1 gives an example. Fig. 1a shows the topology and the distance among all neighboring node pairs. It is trivial to get the minimum cost spanning tree as in fig. 1b. We can see that the depth of spanning tree has become 6 while the original network diameter $D = 1$.

   So we must use *CDS* as the backbone of aggregation tree to reduce the aggregation tree depth. We also have two choices here
   - Connect all dominated nodes to their dominator.
   - Maintain the *MST* structure for dominated nodes.

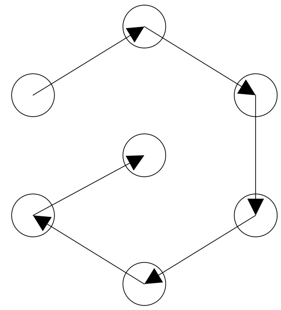
II. Energy Complexity

As analyzed in previous reports, the lower bound of energy complexity can be derived from the Minimum Cost Spanning Tree with link cost of d_i^α for any link i.

Obviously, the lower bound of general case is $nN_0\beta r^\alpha$ and the upper bound is nP_M, which are results not related with any topology information.



(a) Distance Information.



(b) Minimum Spanning Tree.

Fig. 1: An example of Minimum Spanning Tree.

Then, what is the lower bound for any given topology? Here we get a tighter lower bound related to network topology.

Definition 1 (Generalized Mean): If p is a non-zero real number, we can define the generalized mean with exponent p (or power mean with exponent p) of the positive real numbers $x_1, ..., x_n$ as

$$M_p(x_1, ..., x_n) = (\frac{1}{n} \cdot \sum_{i=1}^{n} x_i^p)^{1/p}$$

Theorem 1 (Generalized Mean Inequality): In general, if $p < q$ then $M_p(x_1, ..., x_n) \leq M_q(x_1, ..., x_n)$ and the two means are equal if and only if $x_1 = x_2 = ... = x_n$.

Lemma 1: $\forall \alpha \geq 1$, we have

$$\sum_{i=1}^{n} x_i^\alpha \geq \frac{(\sum_{i=1}^{n} x_i)^\alpha}{n^{\alpha-1}}$$

Proof: Since $\alpha \geq 1$, we know from theorem 1 that

$$M_1(x_1, ..., x_n) \leq M_\alpha(x_1, ..., x_n)$$

$$\Rightarrow \frac{\sum_{i=1}^{n} x_i}{n} \leq (\frac{1}{n} \sum_{i=1}^{n} x_i^\alpha)^{1/\alpha}$$

$$\Rightarrow \sum_{i=1}^{n} x_i^\alpha \geq \frac{(\sum_{i=1}^{n} x_i)^\alpha}{n^{\alpha-1}}$$

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Theorem 2 (Energy Complexity Lower Bound): Suppose that the size of maximum independent set of given network is n_{mis}, the energy complexity has a lower bound of $N_0 \beta \frac{(n_{mis} d_M)^\alpha}{n^{\alpha-1}}$.

Proof:

Suppose that the size of maximum independent set of given network is n_{mis} and the length of the uplink for each node v_i is x_i in the Minimum Cost Spanning Tree. Since the minimum distance between any two independent nodes is d_M, we know that the length of our Minimum Cost Spanning Tree is at least $\sum_{i=1}^{n} x_i \geq n_{mis} d_M$. Then we have that the power complexity is lower-bounded by $N_0\beta\sum_{i=1}^{n} x_i^\alpha \geq N_0\beta\frac{(n_{mis}d_M)^\alpha}{n^{\alpha-1}}$ according to lemma 1. ∎

Theorem 3 (Power Complexity Approximation Ratio): For any aggregation algorithm with connected dominating set as the backbone on any given network with maximum node degree of Δ, the approximation ratio of power complexity is bounded by $O(\Delta^{\alpha-1})$.

Proof: Suppose the size of maximum independent set of given network is n_{mis} and the size of minimum connected dominating set of given network is n_{cds}. According to [3], $n_{mis} \leq 3.8n_{cds} + 1.2$. Meanwhile, we have a distributed dominating set construction algorithm in [2] which derives a CDS with approximation ratio of 8. Suppose the constructed connected dominating set size is n_d. Then n_d can be bounded as $O(n_{mis})$.

It is easy to see that the energy complexity with the connected dominating set is at most $N_0\beta n_d d_M^\alpha$. Since $\Delta = O(n/n_{mis})$, we have that the approximation ratio of energy complexity is bounded by $\frac{N_0\beta n_d d_M^\alpha}{N_0\beta\frac{(n_{mis}d_M)^\alpha}{n^{\alpha-1}}} = O(n^{\alpha-1}/n_{mis}^{\alpha-1}) = O(\Delta^{\alpha-1})$. ∎

III. Link Scheduling

As stated in previous report, each pair of receiver should be separated by a constant range to ensure the safe transmission. I just found a paper in Infocom'20 [1] which also adopts this idea. However, their bound is not tight compared to ours while our bound is more difficult to derive.

References

[1] L. Fu, S.C. Liew and J. Huang, *Effective Carrier Sensing in CSMA Networks under Cumulative Interference*, In proceedings of INFOCOM'10, San Diego, USA, Mar. 15-19, 2010.

[2] P.-J. Wan, K.M. Alzoubi and O. Frieder, *Distributed Construction of Connected Dominating Set in Wireless Ad Hoc Networks*, In proceedings of

[3] W. Wu, H. Du, X. Jia, Y. Li and S.C.-H, Huang, *Minimum connected dominating sets and maximal independent sets in unit disk graphs*, In Theoretical Computer Science 352(1-3): 1-7 (2006).