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I. Difficulty in Primal-Dual Decomposition

For further decomposition, we put the problem mode in the form as follows,

$$\begin{aligned} minimize \quad & f_1(Y,S)+f_2(P,S) \\ s.t. \quad & f_3(S)=0 \\ & f_4(S)\leq 0 \\ & f_5(S,Y)\leq 0 \\ & f_6(P)\leq 0 \\ & f_7(P,S)\leq 0 \\ & f_8(P,S)\leq 0 \\ & s_{ijt}, y_t \in \{0,1\},\ \forall i,j,t\in[1,n] \end{aligned}$$

Our problem should be in the form as

$$\begin{aligned} minimize \quad & f_1(Y,S)+f_2(P,S) \\ s.t. \quad & Y\in C_1 \\ & P\in C_2 \\ & S\in C_3 \end{aligned}$$

to utilize primal decomposition.

Although f_1 and f_2 share the common variable S, we have complicating constraints $f_5,\ f_7,\ f_8$, which share S, Y and P. So primal decomposition may not be applicable to this problem. On the other hand, our problem should be in the form as

$$\begin{aligned} minimize \quad & f_1(Y)+f_2(P) \\ s.t. \quad & Y\in C_1 \\ & P\in C_2 \\ & H_1(Y)+H_2(P)\preceq 0 \end{aligned}$$

to utilize dual decomposition.

However, we have complicating variable S for both f_1 and f_2 and multiple complicating constraints (f_5, f_7 and f_8). So dual decomposition is also difficult to be applied for this problem.

Dual decomposition and subgradian method may lead to some distributed solution. But in our problem, global information is inherently required for the SINR inequality conditions (constraint f_8). That is the reason for the difficulty in implementing dual decomposition.

II. Dantzig-Wolfe Decomposition

Dantzig-Wolfe decomposition is suitable for optimization problems with complicating constraints of very large number of variables and constraints. Besides, it is very useful to handle multiple complicating constraints, which is the problem for our case.

The idea of *Dantzig-Wolfe decomposition* is to decompose the original problem into a set of subproblems with no complicating constraints. Each subproblem is assigned with some predefined coefficients. We solve the subproblems and get the optimal value and the value for each complicating constraint. Then the optimal values and complicating constraints constitute a master problem. We iteratively update the solution for master problem and get the optimal solution for the original problem.

A. Our problem with Dantzig-Wolfe decomposition

Instead of decompose the objective function into f_1 and f_2, we should decompose it along the time-dimension, which is to reform it into $\sum_t (ay_t + b\sum_{i,j} P_{ijt})$. For each time slot t, we have the subproblem as

$$\begin{aligned}
minimize \quad & ay_t + b\sum_{i,j} P_{ijt} \\
s.t. \quad & \sum_{i,j} s_{ijt} \leq n^2 y_t \\
& 0 \leq P_{ijt} \leq P_{max} s_{ijt} \quad \forall i,j \in [1,n] \\
& G_{ij}P_{ijt} - \beta \sum_{u\neq i}\sum_{w\neq j} G_{uj}P_{uwt} - \beta \sum_{u>i} G_{uj}P_{ujt} - \beta N_0 \geq \Phi(s_{ijt}-1) \quad \forall i,j \in [1,n] \\
& s_{ijt}, y_t \in \{0,1\}, \ \forall i,j \in [1,n]
\end{aligned}$$

Let the optimal value for subproblem in time slot t to be z^t, we assign two coefficients c_t^1 and c_t^2 to each of them and get the master problem as

$$\begin{aligned}
minimize \quad & \sum_{s=1,2}\sum_t c_t^s z^t u_s \\
s.t. \quad & \sum_s r_i^t u_s = b_i : \lambda_i; \ i = 1,...,m \\
& \sum_s u_s = 1 : \sigma \\
& u_s \geq 0; \ s = 1,2
\end{aligned}$$

where the corresponding dual variables λ_i and σ are indicated and m is number of complicating inequality constraints (f_2). Then we update this master problem iteratively until it converges.

According to the classification in [1], our problem falls in its case 6: Binary complicating constraints and a Binary Mixed Integer Programming subproblem. Some related literatures use Branch-and-Price method implemented with Depth-First-Search without backtracking method to find the optimal solution. *Dantzig-Wolfe decomposition* can be applied to get the initial solution as the starting point and intermediate solutions for further branching.

B. Applications of Dantzig-Wolfe Decomposition

Since *Dantzig-Wolfe decomposition* have been proposed in 1960, a lot of works have been solved with it. The most related problem may be *Vehicle Routing Problem with Time Windows*: given a fleet of vehicles assigned to a single depot, the vehicle routing problem with time windows consists of determining a set of feasible vehicle routes to deliver goods to a set of customers while minimizing, first, the number of vehicles used and, second, total distance traveled. [2] and [3] are two representatives for *Dantzig-Wolfe decomposition*'s application in *Vehicle Routing Problem with Time Windows*. Branch-and-price method is utilized in [3]. And *Tabu* search is implemented as a neighborhood search method.

References

[1] R. Jans, *Classification of Dantzig-Wolfe reformulations for binary mixed integer programming problems*, in European Journal of Operational Research 204 (2010) 251-254.

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[3] E. Prescott-Gagnon, G. Desaulniers, L.-M. Rousseau, *A branch-and-price-based large neighborhood search algorithm for the vehicle routing problem with time windows*. in Networks 54 (4), 190-204.