## 1 Modeling of the P2P service migration problem

We suppose there are M videos, and N ISPs. There are one on-premise server and one cloud node in each ISP.

## 1.1 Optimization of the problem with Lyapunov optimization

This is a combination of optimization for one time deployment and time-average variables. The placement of content is one time deployment while the schedule is for time-average.

Notation definition:

 $B_s$ : storage capacity of the on-premise server

 $B_u$ : upload bandwidth capacity of the on-premise server

 $h_j$ : charging rate for storage on the cloud at the j-th ISP

 $k_i$ : charging rate for upload bandwidth on the cloud at the j-th ISP

 $s_m$ : storage of m - th video

 $x_m^j=\{0,1\}, m=1,...,M$ :  $x_m^j=1$  if the placement of the m-th video is on the on-premise server at the j-th ISP;  $x_m^j=0$  otherwise;

 $y_m^j=\{0,1\}, m=1,...,M\colon y_m^j=1$  if the placement of the m-th video is on the cloud at the j-th ISP;  $y_m^j=0$  otherwise;

 $D_s^{ji}$  is the delay from source j to on premise server i, and  $D_c^{ji}$  is the delay from source j to on cloud node i.

 $A_m^j(t)$ : at time slot t, number of requests of the m-th video generated from the j-th ISP.

 $r_m^j(t)$ : at time slot t, number of requests of the m-th video that are admitted into the system.  $r_m^j(t) \leq A_m^j(t)$ 

 $S_m^j(t)$ : at time slot t, number of requests for video m that are routed from region j to on-premise server i

 $C_m^{ji}(t)$ : at time slot t, number of requests for video m that are routed from region j to cloud node i

 $Q_m^j(t)$ : at time slot t, queues of requests from video m from ISP j.

Note: The queue update is:  $Q_m^j(t+1)=\max[Q_m^j(t)+r_m^j(t)-S_m^j(t)-\sum_{i=1}^N C_m^{ji}(t),0]$ 

Different from the previous sub section,  $S_m^j(t)$  and  $C_m^{ji}(t)$  is not a schedule of fraction of arrival rates for all time slots. Now they are schedule of number of requests (integers) for each time slot.

Note: minimize sum of:

- time average spending cost of upload bandwidth at cloud node
- spending cost of time average upload bandwidth at on premise server
- · cost of storage at cloud
- · cost of storage at on premise server
- time average weighted delay

$$\begin{aligned} & \text{maximize } g(\sum_{m=1}^{N} \sum_{j=1}^{j} \overline{v_{m}^{j}}) - \alpha_{1} \overline{\sum_{m=1}^{M}} \sum_{j=1}^{N} \sum_{i=1}^{N} (s_{m} C_{m}^{ij}(t) k_{i}) - \alpha_{2} \sum_{m=1}^{M} \sum_{j=1}^{N} \sum_{i=1}^{N} \overline{s_{m}} S_{m}^{i}(t) - \alpha_{3} \sum_{m=1}^{M} \sum_{j=1}^{N} (s_{m} y_{m}^{j} h_{j}) - \alpha_{4} \sum_{m=1}^{M} \sum_{j=1}^{N} (s_{m} x_{m}^{j}) - \alpha_{5} \sum_{j=1}^{N} \sum_{i=1}^{N} \sum_{m=1}^{M} s_{m} (C_{m}^{ii}(t) D_{s}^{ii} + S_{m}^{ii}(t) D_{s}^{ji}) \\ & \text{subject to:} \end{aligned} \\ & y_{m}^{j} = \{0,1\}, \forall j=1, \dots, N, \forall m=1, \dots M \\ & x_{m}^{j} = \{0,1\}, \forall j=1, \dots, N, \forall m=1, \dots M \\ & 0 \leq S_{m}^{j}(t) \leq S_{m}^{j}(t) x_{m}^{j}, \forall j=1, \dots, N, \forall i=1, \dots, N, \forall m=1, \dots, N, \forall t \end{aligned} \\ & \text{(instead, we can assume that on premise server keeps all videos)} \\ & 0 \leq C_{m}^{ii}(t) \leq C_{m}^{ii}(t) y_{m}^{t}, \forall j=1, \dots, N, \forall i=1, \dots, N, \forall m=1, \dots, N, \forall t \end{aligned} \\ & \sum_{m=1}^{M} \sum_{s=1}^{N} s_{m} x_{m}^{j} \leq \sum_{s=1}^{N} S_{m}^{j}(t) \leq \sum_{s=1}^{N} S_{m}^{j} + \sum_{i=1}^{N} C_{m}^{j} \end{aligned} \\ & 0 \leq C_{m}^{ii}(t) \leq C_{m}^{ii}(t) y_{m}^{t}, \forall j=1, \dots, N, \forall i=1, \dots, N, \forall m=1, \dots, N, \forall t \end{aligned} \\ & \sum_{m=1}^{M} \sum_{s=1}^{N} s_{m} x_{m}^{j} \leq \sum_{s=1}^{N} s_{m}^{j} \sum_{s=1}^{N} s_{m}^{j} + \sum_{s=1}^{N} c_{m}^{j} \end{aligned} \\ & \sum_{m=1}^{N} \sum_{s=1}^{N} s_{m} S_{m}^{j}(t) \leq B_{m}, \forall i=1, \dots, N, \forall t \end{aligned} \\ & \text{(on-premise server's upload bandwidth constraint)} \end{aligned} \\ & \text{Queues } Q_{m}^{j}(t) \text{ is stable, } \forall m, j, \text{ i.e., } \overline{v_{m}^{j}(t), D_{c}^{i}, D_{s}^{j}} \end{aligned} \\ & \text{Note:} \\ & \text{Nonv natues: } B_{s}, B_{u}, h_{j}, k_{j}, s_{m}, r_{m}^{j}(t), D_{c}^{i}, D_{s}^{j} \end{aligned} \\ & \text{optimization variables: } x_{m}^{j}, y_{m}^{j}, S_{m}^{j}(t), C_{m}^{i}(t) \end{aligned} \\ & \sum_{m=1}^{M} \sum_{j=1}^{N} \sum_{s=1}^{N} \sum_{s=1}^{N} \sum_{s=1}^{N} \sum_{s=1}^{N} \sum_{s=1}^{M} s_{m} C_{m}^{j}(t) D_{c}^{i} + S_{m}^{j}(t) D_{s}^{j} + \sum_{s=1}^{N} s_{m}^{j} \sum_{s=1}^{N} \sum_{s=1}^{N} \sum_{s=1}^{N} s_{m}^{j} S_{m}^{j}(t) + \sum_{m=1}^{N} \sum_{j=1}^{N} (s_{m} y_{m}^{j}) + \gamma \sum_{m=1}^{M} \sum_{j=1}^{N} (s_{m} y_{m}^{j}) + \gamma \sum_{m=1}^{M} \sum_{j=1}^{N} (s_{m} y_{m}^{j}) + \gamma \sum_{m=1}^{M} \sum_{s=1}^{N} (s_{m} y_{m}^{j}) + \gamma \sum_{m=1}^{M} \sum_{s=1}^{N} (s_{m} y_{m}^{j}) + \gamma \sum_{m=1}^{M} \sum_{s=1}^{N} (s_{m} y_{m}^{j}) + \gamma \sum_{$$

## 2 Extension

1. Add time average budget constraint 2. Add queueing delay