Weekly Report (2010-02-28)

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I. Theoretical comparison between aggregation with and without interference cancellation

The results in this section are all theoretical analysis while not based on any specific algorithm. We assume that each node has the same upper bound of transmission power as P_M, which means the network topology is multi-hop. And the minimum distance between any two nodes is $r \geq 1$.

A. Time complexity

Time complexity counts the number of time slots required to successfully schedule each node exactly once under a given interference model, which is SINR model here, and aggregate the data to the sink in the end.

1) Lower bound: In a multi-hop network, we assume the network diameter of the sink node is D and the maximum node degree in the network is Δ.

- *Without interference cancellation*: Since each node should aggregate its data to the sink in a hop-by-hop fashion, the time complexity cannot be less than D. Meanwhile, we need to avoid the primary interference, which means one node cannot send or receive at the same time. So the time complexity should be at least Δ. Then, we have the lower bound for time complexity as $\Omega(max\{D, \Delta\})$.
- *With interference cancellation*: Even with interference cancellation, the data from each node still needs to follow the hop-by-hop transmission in order to be aggregated to the sink. So the time complexity cannot be less than D. On the other hand, we can improve the latency on the node with maximum node degree with interference cancellation. Suppose the maximum number of links that can be recovered by interference cancellation with power upper bound P_M is X, we should have the following result.

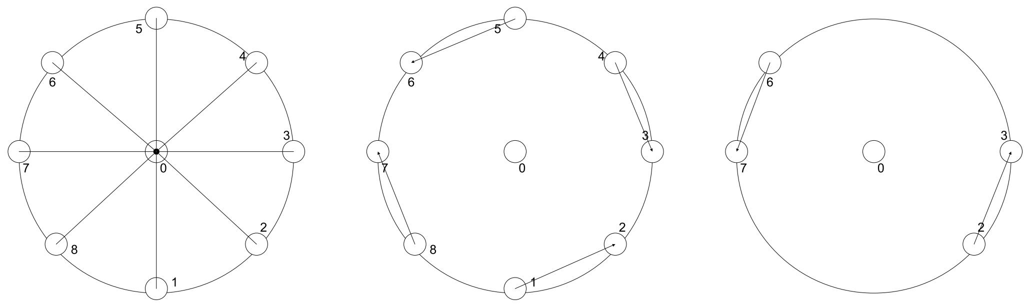
$$\frac{P_M}{r^\alpha} \geq N_0\beta(1+\beta)^{X-1}$$
$$\Rightarrow X = \lfloor \log_{1+\beta} \frac{P_M}{r^\alpha N_0 \beta} + 1 \rfloor$$

  So the lower bound for time complexity of aggregation with interference cancellation is

$$\Omega(max\{D, \Delta/(\lfloor \log_{1+\beta} \frac{P_M}{r^\alpha N_0 \beta} + 1 \rfloor)\})$$

In fact, the above bounds are not tight enough. Under the SINR model, we can further get some improvement on Δ.

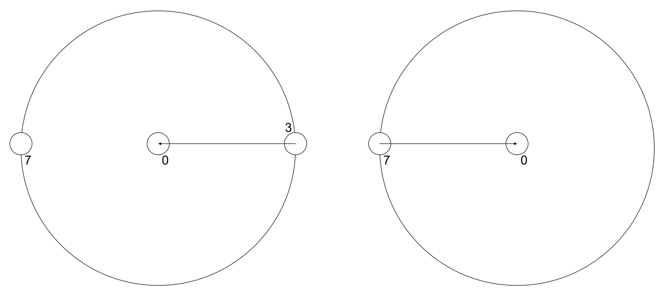
- $\Omega(\log \Delta)$: Fig. 1 is an example to reduce the latency from Δ to $\Omega(\log \Delta)$.
  In fig. 1, node 0 is the sink node with node degree of $\Delta = n$. That means each non-sink node can connect to the sink with one hop (fig. 1a). In graph interference model, it is clear that at least Δ time slots are required to



(a) Connect to the sink with one hop.

(b) Step 1 of multi-hop schedule.

(c) Step 2 of multi-hop schedule.



(d) Step 3 of multi-hop schedule.

(e) Step 4 of multi-hop schedule.

Fig. 1: Sink node 0 with maximum node degree of $\Delta = n$.

schedule all data to the sink. However, in SINR interference model, we can improve the result by constructing a multi-hop aggregation tree. In fig. 1a, we construct and schedule the links of $1 \to 2$, $4 \to 3$, $5 \to 6$ and $8 \to 7$. So in fig. 1b, we only have 4 non-sink nodes left and construct the links as $2 \to 3$ and $6 \to 7$. At last, in fig. 1d and fig. 1e, two time slots are used to individually schedule node 3 and 7 to the sink. To conclude, we can reduce the originally time complexity from Δ to $\Omega(\log \Delta)$.

I need to further prove that no algorithm can achieve a better time complexity than $\Omega(\log \Delta)$, which means the time complexity is

$$\Omega(max\{D, \log \Delta\})$$

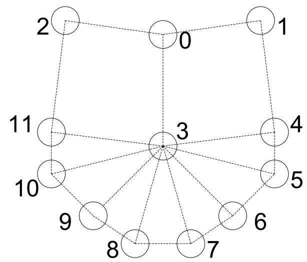
for aggregation without interference cancellation and

$$\Omega(max\{D, \log \Delta / (\lfloor \log_{1+\beta} \frac{P_M}{r^\alpha N_0 \beta} + 1 \rfloor)\})$$

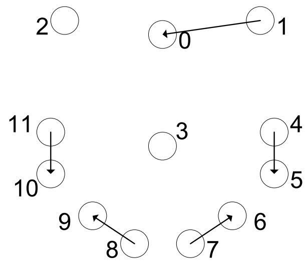
for that with interference cancellation. However, this is not a simple work since Ω may be larger than the actual lower bound as shown in fig. 2.

- *Possible better result than* $\Omega(\log \Delta)$:

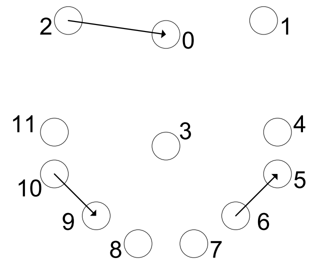
  Fig. 2 is an example when $\log \Delta$ may not be the lower bound of time complexity. Fig. 2a is the neighborhood relationship of the network. Each dashed line means that the connected vertexes are neighboring nodes. Node 0 is the sink and node 3 is the one with maximum node degree $\Delta = 8$. Fig. 2b to fig. 2f is the trace of



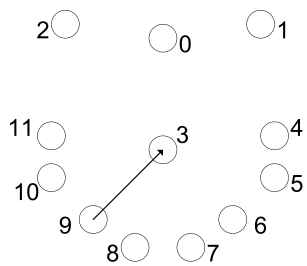
(a) Neighborhood relation graph.



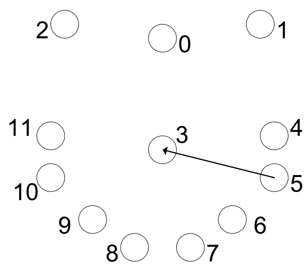
(b) Step 1 of multi-hop schedule I.



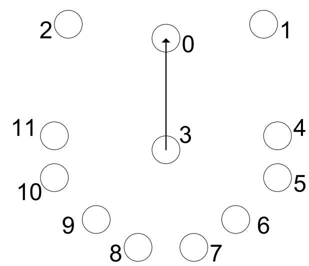
(c) Step 2 of multi-hop schedule I.



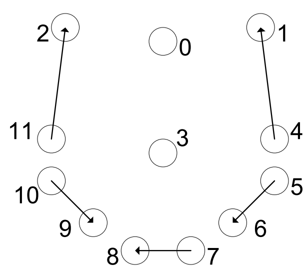
(d) Step 3 of multi-hop schedule I.



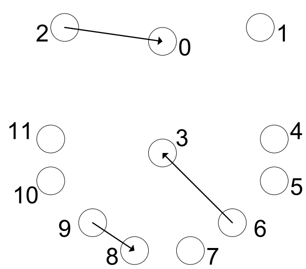
(e) Step 4 of multi-hop schedule I.



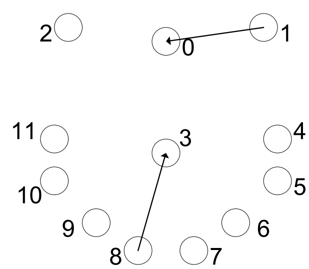
(f) Step 5 of multi-hop schedule I.



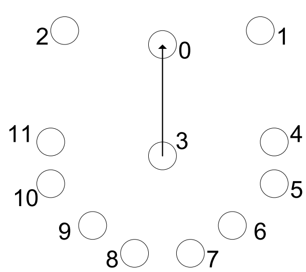
(g) Step 1 of multi-hop schedule II.



(h) Step 2 of multi-hop schedule II.



(i) Step 3 of multi-hop schedule II.



(j) Step 4 of multi-hop schedule II.

Fig. 2: Relay node 3 with maximum node degree of $\Delta = n$. Node 0 is the sink.

scheduling algorithm I which lets node 3 to be the parent for node 4 to 11. So the result is similar to that of fig. 1. We need 4 time slots, which is $\log \Delta + 1$ to complete the aggregation on node 3.

However, we have another option for this example. In fig. 2g to fig. 2j, I present the trace of scheduling algorithm II, which let node 4 (11) to choose node 1 (2) as the parent. In this way, we only need 3 time slots to complete the aggregation on node 3.

- *Network density* ρ: Different from the graph interference model, in which we only need local information to determine the interference, we have to get global information on the network. One important parameter may be the network density ρ. I have derived a loose lower bound with ρ as follows.

$$\Omega(C \cdot \frac{n}{(1+\sqrt{\frac{n}{\pi\rho}}/r)^{\alpha}})$$

here, $C = \frac{1}{N_0\beta^2(P_M/r^{\alpha}-N_0\beta)}$ is a constant. We can find that the aggregation latency will be larger if the network is more dense.

2) Upper bound: Since each non-sink needs to transmit exactly once, the upper bound for data aggregation will be n for both cases. Of course, the time complexity of aggregation without SIC cannot be better than that of aggregation with SIC. However, this bound is not good enough to compare the two cases.

B. Energy complexity

1) Lower bound: The energy complexity is the total amount of transmission energy consumed by all nodes in one aggregation. It is clear that the lower bound for energy complexity can be found using minimum spanning tree rooted at the sink as the aggregation tree. In each time slot, only one node is scheduled to transmit to its parent in the minimum spanning tree. Suppose for node $v_i \in V$, the distance between v_i and its parent in minimum spanning tree is d_{ip}. Then the lower bound of energy complexity can be computed as $\sum_i N_0\beta d_{ip}^{\alpha}$.

2) Upper bound: Since the maximum transmission power for each node is P_M and each node is only scheduled once, the the energy complexity is upper-bounded by $n \cdot P_M$ for both cases.

C. Message complexity

Each non-sink will only send exactly once for one aggregation. So both aggregation with and without interference cancellation will require an exactly n message complexity.

D. Approximation of Time and Energy complexities

We cannot get the individual approximation ratio for neither time nor energy complexity since there is a tradeoff between latency reduction and additional energy consumption. So I will investigate the combined approximation ratio for them. Suppose the approximation ratio of time complexity and energy complexity are $R_t(P_M)$ and $R_p(P_M)$ respectively, with power upper bound as P_M. Then, we will show the relation between $R_t(P_M)$ and $R_p(P_M)$ and how they are influenced by P_M.

II. Distributed Algorithm

A. Tree Construction

1) Drawbacks of previous algorithm: Although the previous tree construction algorithm can build a tree with maximum tree height of $2D$, which has a good and bounded hop spanning ratio, the maximum node degree in each level of the tree is still $O(\Delta)$. So the scheduling latency is still upper-bounded by $O(D \cdot \Delta)$.

2) Revised tree construction algorithm: In order to achieve $O(D+\Delta)$ aggregation latency, we need the resulting aggregation tree has two properties:

1) *Bounded hop spanning ratio*: A subgraph H of G has a bounded spanning ratio if for every pair of nodes u and v in H, the distance (counted by hop) of the shortest path connecting u and v in H is at most a constant times of the distance of the shortest path connecting them in original graph G.
2) *Constant bounded degree for internal nodes*: Let the maximum node degree for leaf node to be Δ while that of the internal nodes to be a constant value c.

If the aggregation tree has the above two properties, we can have a scheduling latency of $O(\Delta + D)$. The current best choice should be *Connected Dominating Set (CDS)* to be the backbone network.

We have two choices:

1) Use the most popular *CDS* construction algorithm proposed in [1].
2) Design our new *CDS* constructor adapted to interference cancellation.

B. Link Scheduling

We still have the requirement that each current scheduled receivers are separated with a distance of r_c as in previous report. But we need to divide the scheduling into two phases.

1) Schedule each non-dominator node to transmit to its parent in aggregation tree. This phase requires $O(\Delta)$ time complexity.
2) Schedule the *CDN* to aggregate data to the sink. This phase has a time complexity of $O(D)$.

References

[1] P.-J. Wan, K.M. Alzoubi and O. Frieder, *Distributed Construction of Connected Dominating Set in Wireless Ad Hoc Networks*, In proceedings of INFOCOM'02, 2002.