Modeling of the P2P service migration problem 1

We suppose there are M videos, and N ISPs. There are one on-premise server and one cloud node in each ISP.

1.1 Optimization of the problem without Lyapunov optimization

Notation definition:

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C_s^j: storage capacity of the on-premise server at the j-th ISP
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$$C_n^j$$
: upload bandwidth capacity of the on-premise server at the $j-th$ ISP

$$h_i$$
: charging rate for storage on the cloud at the $j-th$ ISP

$$k_j$$
: charging rate for upload bandwidth on the cloud at the $j-th$ ISP

$$s_m$$
: storage of $m - th$ video

$$x_m^j=\{0,1\}, m=1,...,M\colon x_m^j=1$$
 if the placement of the $m-th$ video is on the on-premise server at the $j-th$ ISP; $x_m^j=0$ otherwise;

$$y_m^j=\{0,1\}, m=1,...,M$$
: $y_m^j=1$ if the placement of the $m-th$ video is on the cloud at the $j-th$ ISP; $y_m^j=0$ otherwise;

$$r_m^j$$
 : request rate of the $m-th$ video from the $j-th$ ISP, i.e., the bandwidth demand is $s_m r_m^j$.

$$R_{ji}^m$$
: percentage of requests from j for video m is routed to on-premise server i

$$T_{ji}^m$$
: percentage of requests from j for video m is routed to cloud i

$$\min \sum_{m=1}^{M} \sum_{j=1}^{N} \sum_{i=1}^{N} (s_m r_m^j T_{ji} k + s_m h) y_m^j - \alpha \sum_{m=1}^{M} \sum_{j=1}^{N} s_m r_m^j (T_{jj} + R_{jj})$$
 (maximize local traffic, i.e., minimize delay)

subject to:
$$y_m^j = \{0,1\}, \forall j=1,...,N, \forall m=1,...M \\ x_m^j = \{0,1\}, \forall j=1,...,N, \forall m=1,...M$$

$$x_m^j = \{0, 1\}, \forall j = 1, ..., N, \forall m = 1, ...M$$

$$\sum_{i=1}^{N} (R_{ji}^{m} + T_{ji}^{m}) = 1, \forall j = 1, ..., N, \forall m = 1, ..., M$$

$$0 \leq R_{ji}^{m} \leq x_{m}^{i}, \forall j = 1, ..., N, \forall i = 1, ..., N, \forall m = 1, ..., N$$

$$0 \le T_{ji}^m \le y_m^i, \forall j = 1, ..., N, \forall i = 1, ..., N, \forall m = 1, ..., N$$

$$\sum_{i=1}^{M} s_m x_m^j < C_i^j, \forall i \text{ (on-premise server's storage constraint)}$$

$$\sum_{m=1}^{M} s_m x_m^j \leq C_s^j, \forall j$$
 (on-premise server's storage constraint) $\sum_{m=1}^{M} \sum_{j=1}^{N} s_m r_m^j R_{ji}^m \leq C_u^i, \forall i=1,...,N$ (on-premise server's upload bandwidth constraint)

known values: C_s^j , C_u^j , h_j , k_j , s_m , r_m^j optimization variables: $x_m^j, y_m^j, R_{ii}^m, T_{ii}^m$

1.2 Optimization of the problem with Lyapunov optimization

This is a combination of optimization for one time deployment and time-average variables. The placement of content is one time deployment while the schedule is for time-average.

Notation definition:

 C_s^j : storage capacity of the on-premise server at the j-th ISP

 C_u^j : upload bandwidth capacity of the on-premise server at the j-th ISP

 h_j : charging rate for storage on the cloud at the j-th ISP

 k_i : charging rate for upload bandwidth on the cloud at the j-th ISP

 s_m : storage of m - th video

 $x_m^j=\{0,1\}, m=1,...,M$: $x_m^j=1$ if the placement of the m-th video is on the on-premise server at the j-th ISP; $x_m^j=0$ otherwise;

 $y_m^j=\{0,1\}, m=1,...,M$: $y_m^j=1$ if the placement of the m-th video is on the cloud at the j-th ISP; $y_m^j=0$ otherwise;

 D_s^{ji} is the delay from source j to on premise server i, and D_c^{ji} is the delay from source j to on cloud node i.

 $r_m^j(t)$: at time slot t, number of requests of the m-th video generated from the j-th ISP.

 $R_m^{ji}(t)$: at time slot t, number of requests for video m that are routed from region j to on-premise server i

 $T_m^{ji}(t)$: at time slot t, number of requests for video m that are routed from region j to cloud node i

 $Q_m^j(t)$: at time slot t, queues of requests from video m from ISP j.

Note: The queue update is: $Q_m^j(t+1) = \max[Q_m^j(t) + r_m^j(t) - \sum_{i=1}^N R_m^{ji} - \sum_{i=1}^N T_m^{ji}, 0]$

Different from the previous sub section, $R_m^{ji}(t)$ and $T_m^{ji}(t)$ is not a schedule of fraction of arrival rates for all time slots. Now they are schedule of number of requests (integers) for each time slot.

Note: minimize sum of:

- time average spending cost of upload bandwidth at cloud node
- spending cost of time average upload bandwidth at on premise server
- · cost of storage at cloud
- · cost of storage at on premise server
- time average weighted delay

 $\begin{aligned} x_m^j &= \{0,1\}, \forall j=1,...,N, \forall m=1,...M\\ 0 &\leq R_m^{ji}(t) \leq R_m^{ji}(t) x_m^j, \forall j=1,...,N, \forall i=1,...,N, \forall m=1,...,N, \forall t\\ 0 &\leq T_m^{ji}(t) \leq T_m^{ji}(t) y_m^t, \forall j=1,...,N, \forall i=1,...,N, \forall m=1,...,N, \forall t\\ \sum_{m=1}^M s_m x_m^j &\leq C_s^j, \forall j \text{ (on-premise server's storage constraint)}\\ \sum_{m=1}^M \sum_{j=1}^N s_m R_m^{ji}(t) &\leq C_u^i, \forall i=1,...,N, \forall t \text{ (on-premise server's upload bandwidth constraint)}\\ \text{Queues } Q_m^j(t) \text{ is stable, } \forall m,j, \text{ i.e., } \overline{r_m^j(t)} \leq \overline{\sum_{i=1}^N R_m^{ji} + \sum_{i=1}^N T_m^{ji}}\\ Q_m^j(0) &= 0, \forall m,j \\ \text{Note:} \end{aligned}$

known values: $C_s^j, C_u^j, h_j, k_j, s_m, r_m^j(t), D_c^{ji}, D_s^{ji}$ optimization variables: $x_m^j, y_m^j, R_m^{ji}(t), T_m^{ji}(t)$