The draft for the cost minimization of a federated cloud participant

Abstract—**We plan to consider the placement and migration of virtual machines under the dynamic request arrival scenario in the cloud federation platform. The objective is to minimize the cost of an autonomy cloud.**

I. System Model

We study the problem that how an individual cloud provider in the federated clouds minimize its cost. The cost of an individual cloud provider includes: the operational cost related to the number of running servers; the cost of buying virtual machines from other providers; the cost of data transferring in other providers; the migration cost.

A. Cloud federation and job model

We consider a cloud federation with J cloud providers in total. N^j is the number of servers in cloud $j, 1 \leq j \leq J$. Each cloud in the federation provides M types of instances(i.e., virtual machines). Let $N^j_m(t)$ be the number of servers configured to provide type m instances at time t. Each server in cloud j can host H^j_m number of type m instances, $1 \leq m \leq M$. The price for type m instance of cloud j is p^j_m per unit time. The price for cloud j transferring data out or in is p^j_{do} or p^j_{di} per unit volume.

We consider a time slotted system. Let $t = 0, 1, 2, 3..., T$ be the time slots. We model users' requests for a bundle of instances for a fixed time size as jobs. Let $\mathcal{K}$ be the set of all job types. Job type k is denoted by a $(M+1)$-tuple $(s^k_1, ..., s^k_M, w^k)$. $s^k_m, 1 \leq m \leq M$ is the required number of type m instances by job type k. w^k is the required service time of job type k.

Let $\mathcal{A}^j_k(t)$ be the set of type k jobs arriving at time slot t in cloud j, $|\mathcal{A}^j_k(t)| = A^j_k(t)$. Let $\mathcal{U}^j_k(t)$ denote the set of newly served type k jobs in cloud j, $|\mathcal{U}^j_k(t)| = U^j_k(t)$. Then, the number of type k jobs at cloud j newly served at time slot $t-w, 0 \leq w \leq w^k - 1$ is $U^j_k(t-w), 0 \leq w \leq w^k - 1$.

We use a queue Q^j_k to denote the workload of job type k at cloud j, which is the number of total processing time slots needed for type k jobs in cloud j. The cloud provider guarantees to serve the jobs within a delay of d. Hence, if the cloud provider can not serve some jobs within a delay of d, it drops $D^j_k(t)$ jobs.

The dynamic of Q^j_k is as follows:

$$Q^j_k(t+1) = \max\{Q^j_k(t) - \sum_{w=0}^{w^k-1} U^j_k(t-w) - w^k \cdot D^j_k(t), 0\} + w^k \cdot A^j_k(t), \quad 1 \leq j \leq J, 1 \leq k \leq K. \tag{1}$$

Each job is either served or dropped (subject to a penalty) before the maximum response delay d. (2)

$\mathcal{U}^j_k(t-w)$ type k jobs are newly served at time slot $t-w$. Index the $\mathcal{U}^j_k(t-w)$ jobs from 1 to $U^j_k(t-w)$. Let $\mathcal{L}^j$ be the set of all jobs from cloud j being served in the federation at one time slot. A job l in $\mathcal{L}^j$ can be denoted by a 3-tuple (k, t, h), which means the job is the h-th type k job that received service at time slot t. Let T^l be the traffic matrix of job l. The entry $T^l_{s_1,s_2}$ is the traffic from instance s_1 to instance s_2 in job l.

B. VM provision and placement

In a job, we use a 2-tuple to denote an instance s of job l, (m, a), which means the instance is the a-th one of job l's type m instances, $1 \leq a \leq s^{k_l}_m$. Let $\mathcal{S}^l$ be the set of all instances of job l.

Let $r^{ji}_m(t)$ denote the number of type m virtual machines that cloud j places in cloud i at time slot t.

The total type m instances running in cloud i satisfy the following two constraints:

$$\sum_{i=1}^{J} r^{ij}_m \leq N^j_m \cdot H^j_m, \quad 1 \leq j \leq J, 1 \leq m \leq M. \tag{3}$$

$$\sum_{k=1}^{K} \sum_{w=0}^{w^k-1} s^k_m \cdot U^j_k(t-w) \leq \sum_{i=1}^{J} r^{ji}_m \tag{4}$$

Let $I^i_{l,s}$ be the indicator whether the instance $s \in \mathcal{S}^l$ is placed in cloud i or not.

$$I^i_{l,s} = 1 \text{ if instance } s \text{ is placed in cloud } i$$
$$I^i_{l,s} = 0 \text{ if not.} \tag{5}$$

The number of type m instances that is placed by cloud j in cloud i is $\sum_{l \in \mathcal{L}^j} \sum_{s: m_s = m} I^i_{l,s}, 1 \leq i \leq J$.

$$\sum_{l\in\mathcal{L}^j}\sum_{s:m_s=m} I^i_{l,s} \le r^{ji}_m, 1 \le i \le J \tag{6}$$

C. Data traffic among VMs

Let us consider the data traffic among different cloud providers in the federation. There are two parts of traffic: one is the data transfer among different VMs of the same job; the other is the traffic induced by VM migrations.

First, we consider the traffic due to traffic among VMs of jobs. The traffic transferring out of cloud j due to jobs of cloud i is $\sum_{l\in\mathcal{L}^i}\sum_{s_1\in\mathcal{S}^l}\sum_{s_2!=s_1}[I^j_{l,s_1} - I^j_{l,s_2}]^+ \cdot T^l_{s_1,s_2}$.

The traffic transferring into cloud j due to jobs of cloud i is $\sum_{l\in\mathcal{L}^i}\sum_{s_1\in\mathcal{S}^l}\sum_{s_2!=s_1}[I^j_{l,s_2} - I^j_{l,s_1}]^+ \cdot T^l_{s_1,s_2}$

Next, let us consider the traffic due to migration. We assume the VM downtime during migration is short and can be ignored. We also assume the bandwidth used for migration is large, and the migration time can be ignore compared to the time slot. Let B_m be the bandwidth used for transferring instance type m. Let U_m be the size of transferred data for migrating a type m instance. We consider the transferred data due to the instance migration. The migration traffic out of cloud j due to migration of jobs in cloud i can be calculated as follows $\sum_{l\in\mathcal{L}^i}\sum_{s\in\mathcal{S}^l}[I^j_{l,s}(t-1) - I^j_{l,s}(t)]^+ \cdot U_{m_s}$

The migration traffic into cloud j due to migration of jobs in cloud i is $\sum_{l\in\mathcal{L}^i}\sum_{s\in\mathcal{S}^l}[I^j_{l,s}(t) - I^j_{l,s}(t-1)]^+ \cdot U_{m_s}$.

The data transfer should satisfy the bandwidth constraints:

$$\begin{aligned}
&\sum_{l\in\mathcal{L}}\sum_{s_1\in\mathcal{S}^l}\sum_{s_2!=s_1}[I^j_{l,s_1} - I^j_{l,s_2}]^+ \cdot T^l_{s_1,s_2} \\
&+\sum_{l\in\mathcal{L}}\sum_{s\in\mathcal{S}^l}[I^j_{l,s}(t-1) - I^j_{l,s}(t)]^+ \cdot B_{m_s} \le B^j_{out} \\
&\sum_{l\in\mathcal{L}}\sum_{s_1\in\mathcal{S}^l}\sum_{s_2!=s_1}[I^j_{l,s_2} - I^j_{l,s_1}]^+ \cdot T^l_{s_1,s_2} \\
&+\sum_{l\in\mathcal{L}}\sum_{s\in\mathcal{S}^l}[I^j_{l,s}(t) - I^j_{l,s}(t-1)]^+ \cdot B_{m_s} \le B^j_{in}
\end{aligned} \tag{7}$$

B^j_{out} is the upload bandwidth limit of cloud j. B^j_{in} is the download bandwidth limit of cloud j.

D. Cost minimization problem definition

We formulate the cost minimization problem for one cloud provider in the federation.

Let us first consider the cost at time slot t of cloud j in the federation. The total cost is the operational cost for running servers and transferring data plus the cost for buying VMs or transferring data from other cloud providers in the federation minus the income from other cloud providers for selling VMs or transferring other cloud providers' data.

The operational cost for running servers is related to the number of running servers:

$$C^j_s = \beta^j \cdot \sum_{m=1}^{M} N^j_m$$

The network cost related to transferring data out and in:

$$\begin{aligned}
C^j_d = \beta^j_{do} \cdot \{&\sum_{l\in\mathcal{L}}\sum_{s_1\in\mathcal{S}^l}\sum_{s_2!=s_1}[I^j_{l,s_1} - I^j_{l,s_2}]^+ \cdot T^l_{s_1,s_2} \\
&+\sum_{l\in\mathcal{L}}\sum_{s\in\mathcal{S}^l}[I^j_{l,s}(t-1) - I^j_{l,s}(t)]^+ \cdot U_{m_s}\} \\
+\beta^j_{di} \cdot \{&\sum_{l\in\mathcal{L}}\sum_{s_1\in\mathcal{S}^l}\sum_{s_2!=s_1}[I^j_{l,s_2} - I^j_{l,s_1}]^+ \cdot T^l_{s_1,s_2} \\
&+\sum_{l\in\mathcal{L}}\sum_{s\in\mathcal{S}^l}[I^j_{l,s}(t) - I^j_{l,s}(t-1)]^+ \cdot U_{m_s}\}
\end{aligned}$$

The cost of buying instances from other cloud providers minus the revenue by selling instances to other providers is:

$$C^j_{fs} = \sum_{i!=j}\sum_{m=1}^{M} p^i_m \cdot r^{ji}_m - \sum_{i!=j}\sum_{m=1}^{M} p^j_m \cdot r^{ij}_m$$

The cost of transferring data into or out of other cloud providers minus the revenue by transferring data of other providers is:

$$\begin{aligned}
C^j_{fd} = &\sum_{i!=j} p^i_{do}\{\sum_{l\in\mathcal{L}^j}\sum_{s_1\in\mathcal{S}^l}\sum_{s_2!=s_1}[I^i_{l,s_1} - I^i_{l,s_2}]^+ \cdot T^l_{s_1,s_2} \\
&+\sum_{l\in\mathcal{L}^j}\sum_{s\in\mathcal{S}^l}[I^i_{l,s}(t-1) - I^i_{l,s}(t)]^+ \cdot U_{m_s}\} \\
&+\sum_{i!=j} p^i_{di}\{\sum_{l\in\mathcal{L}^j}\sum_{s_1\in\mathcal{S}^l}\sum_{s_2!=s_1}[I^i_{l,s_2} - I^i_{l,s_1}]^+ \cdot T^l_{s_1,s_2} \\
&+\sum_{l\in\mathcal{L}^j}\sum_{s\in\mathcal{S}^l}[I^i_{l,s}(t) - I^i_{l,s}(t-1)]^+ \cdot U_{m_s}\} \\
&-\sum_{i!=j} p^j_{do}\{\sum_{l\in\mathcal{L}^i}\sum_{s_1\in\mathcal{S}^l}\sum_{s_2!=s_1}[I^j_{l,s_1} - I^j_{l,s_2}]^+ \cdot T^l_{s_1,s_2} \\
&+\sum_{l\in\mathcal{L}^i}\sum_{s\in\mathcal{S}^l}[I^j_{l,s}(t-1) - I^j_{l,s}(t)]^+ \cdot U_{m_s}\} \\
&-\sum_{i!=j} p^j_{di}\{\sum_{l\in\mathcal{L}^i}\sum_{s_1\in\mathcal{S}^l}\sum_{s_2!=s_1}[I^j_{l,s_2} - I^j_{l,s_1}]^+ \cdot T^l_{s_1,s_2} \\
&+\sum_{l\in\mathcal{L}^i}\sum_{s\in\mathcal{S}^l}[I^j_{l,s}(t) - I^j_{l,s}(t-1)]^+ \cdot U_{m_s}\}
\end{aligned}$$

Hence, the total cost at time slot t of cloud j is:

$$C^j = C^j_s + C^j_d + C^j_{fs} + C^j_{fd}$$

The time-averaged cost of cloud j for running instances is:

$$\overline{C^j} = \lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T} C^j(t) \tag{8}$$

The cost minimization problem at cloud j can be formulated as follows:

$$\min \overline{C^j}$$
$$\text{constraints } 1-7.$$

E. Social cost minimization

The social cost of the federation is the overall cost of the cloud federation:

$$\sum_{j=1}^{J}[C_s^j + C_d^j + C_{fs}^j + C_{fd}^j]$$

Since the income and expenditure due to buying/selling instances and data transferring among clouds cancel each other, the formula above equals to $\sum_{j=1}^{J}[C_s^j + C_d^j]$. The social cost minimization problem is:

$$\min \sum_{j=1}^{J}[C_s^j + C_d^j]$$
$$\textbf{constraints } 1-7.$$

The variables include $U_k^j(t)$, i.e., the number of new type k jobs served at time t, $I_{l,s}^j$, i.e., the instance s assignment indicator, N_m^j, i.e., the instance provisioning, r_m^{ij}, i.e., the number of type m VMs cloud i buys from cloud j.

We summarize important notations in Table I for ease of reference.

TABLE I
IMPORTANT NOTATIONS

J	total number of cloud providers in the federation.
N^j	number of servers in cloud j.
M	types of instances.
$N_m^j(t)$	number of servers running type m instances in cloud j.
H_m^j	number of type m instances a server in cloud j can host.
p_m^j	price of buying an instance m in cloud j.
p_{do}^j	price of transferring one volume data out of cloud j.
p_{di}^j	price of transferring one volume data into cloud j.
$\mathcal{K}$	set of all job types.
s_m^k	the number of type m instance required by a job k.
w^k	the service time of a job k.
$\mathcal{A}_k^j(t)$	set of type k jobs arriving at t in cloud j.
$A_k^j(t)$	number of type k jobs arriving at t in cloud j.
$\mathcal{U}_k^j(t)$	set of newly served type k jobs in cloud j.
$U_k^j(t)$	number of newly served type k jobs at t in cloud j.
Q_k^j	queue backlog for workload of type k jobs in cloud j.
$D_k^j(t)$	number of dropped type k jobs at t in cloud j.
d	maximum responsive delay.
$\mathcal{L}^j$	set of all jobs from cloud j being served.
T^l	the traffic matrix of job l.
$\mathcal{S}^l$	set of all instances of job l.
r_m^{ij}	number of type m instances cloud i buys from cloud j.
$I_{l,s}^i$	indicator whether instance s of job l is placed in cloud i or not.
B_m	the bandwidth required to migrate a type m instance.
U_m	the data size of migrating a type m instance.
B_{out}^j	upload bandwidth limit of cloud j.
B_{in}^j	download bandwidth limit of cloud j.
β^j	operational cost of running one server.
β_{do}^j	cost of transferring one volume of data out of cloud j.
β_{di}^j	cost of transferring one volume of data into cloud j.

II. METHOD

Let $\mathbf{Q}^j = (Q_1^j, Q_2^j, ..., Q_K^j)$ be the queue backlog vector at cloud j. To make the job scheduling satisfy delay constraint, we apply ϵ-persistent queue technique [], to define a virtual queue associated with each job queue.

$$Z_k^j(t+1) = \max\{Z_k^j(t) + 1_{Q_k^j(t)>0} \cdot [\epsilon_k - \sum_{w=0}^{w^k-1} \mu_k^j(t-w)] \\ -w^k \cdot D_k^j(t) - 1_{Q_k^j(t)=0} \cdot \mu_k^{max}, 0\} \tag{9}$$

Let $\Theta^j(t) = (\mathbf{Q}^j, \mathbf{Z}^j)$ be the vector of actual queues and virtual queues in cloud j. $\Theta(t) = (\Theta^1(t), \Theta^2(t), ..., \Theta^J(t))$. The Lyapunov function of $\Theta(t)$ is:

$$L(\Theta(t)) = \frac{1}{2}\sum_{j=1}^{J}\sum_{k=1}^{K}[Q_k^j(t)^2 + Z_k^j(t)^2] \tag{10}$$

The one slot drift is

$$\Delta(\Theta(t)) \leq B + \sum_{j=1}^{J}\sum_{k=1}^{K} Q_k^j(t) \cdot \\ [w^k \mathbb{E}[A_k^j(t)] - \sum_{w=0}^{w^k-1} U_k^j(t-w) - w^k \cdot D_k^j(t)] + \sum_{j=1}^{J}\sum_{k=1}^{K} Z_k^j(t) \cdot \\ [1_{Q_k^j(t)>0}(\epsilon_k - \sum_{w=0}^{w^k-1} U_k^j(t-w)) - w^k D_k^j(t) - 1_{Q_k^j(t)=0} U_k^{max}]$$

B **is a constant.**

The drift plus penalty is:

$$\Delta(\Theta(t)) + V \cdot \sum_{j=1}^{J}[C_s^j(t) + C_d^j(t)] \leq$$

$$B + \sum_{j=1}^{J}\sum_{k=1}^{K} Q_k^j(t) \cdot$$

$$[w^k \mathbb{E}[A_k^j(t)] - \sum_{w=0}^{w^k-1} U_k^j(t-w) - w^k \cdot D_k^j(t)] + \sum_{j=1}^{J}\sum_{k=1}^{K} Z_k^j(t) \cdot$$

$$[(\epsilon_k - \sum_{w=0}^{w^k-1} U_k^j(t-w)) - w^k D_k^j(t)]$$

$$+ V \cdot \sum_{j=1}^{J}[\beta^j \cdot \sum_{m=1}^{M} N_m^j + \beta_{do}^j \cdot \{\sum_{l \in \mathcal{L}} \sum_{s_1 \in \mathcal{S}^l} \sum_{s_2 != s_1} [I_{l,s_1}^j - I_{l,s_2}^j]^+ \cdot T_{s_1,s_2}^l$$

$$+ \sum_{l \in \mathcal{L}} \sum_{s \in \mathcal{S}^l} [I_{l,s}^j(t-1) - I_{l,s}^j(t)]^+ \cdot U_{m_s}\}$$

$$+ \beta_{di}^j \cdot \{\sum_{l \in \mathcal{L}} \sum_{s_1 \in \mathcal{S}^l} \sum_{s_2 != s_1} [I_{l,s_2}^j - I_{l,s_1}^j]^+ \cdot T_{s_1,s_2}^l$$

$$+ \sum_{l \in \mathcal{L}} \sum_{s \in \mathcal{S}^l} [I_{l,s}^j(t) - I_{l,s}^j(t-1)]^+ \cdot U_{m_s}\}]$$

We minimize the right-hand-side of the above drift-plus-penalty expression.

$$\begin{aligned}
\min & \sum_{j=1}^{J}\sum_{k=1}^{K}[Q_k^j(t) + Z_k^j(t)] \cdot \\
& [-\sum_{w=0}^{w^k-1} U_k^j(t-w) - w^k \cdot D_k^j(t)] \\
& + V \cdot \sum_{j=1}^{J}[\beta^j \cdot \sum_{m=1}^{M} N_m^j \\
& + \beta_{do}^j \cdot \{\sum_{l \in \mathcal{L}} \sum_{s_1 \in \mathcal{S}^l} \sum_{s_2 != s_1} [I_{l,s_1}^j - I_{l,s_2}^j]^+ \cdot T_{s_1,s_2}^l \\
& + \sum_{l \in \mathcal{L}} \sum_{s \in \mathcal{S}^l} [I_{l,s}^j(t-1) - I_{l,s}^j(t)]^+ \cdot U_{m_s}\} \\
& + \beta_{di}^j \cdot \{\sum_{l \in \mathcal{L}} \sum_{s_1 \in \mathcal{S}^l} \sum_{s_2 != s_1} [I_{l,s_2}^j - I_{l,s_1}^j]^+ \cdot T_{s_1,s_2}^l \\
& + \sum_{l \in \mathcal{L}} \sum_{s \in \mathcal{S}^l} [I_{l,s}^j(t) - I_{l,s}^j(t-1)]^+ \cdot U_{m_s}\}]
\end{aligned}$$

Constraints $3-7$.