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# Fair Pricing in the Sky: Truthful Frequency Allocation with Dynamic Spectrum Supply

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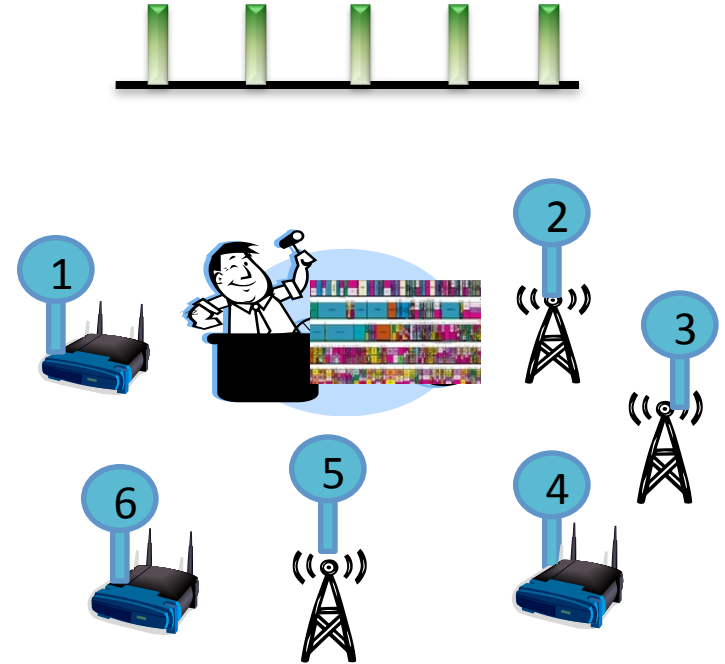
# Outline

- Motivations
- The Proposed Spectrum Auction Design
  - Single-unit case
  - Multi-unit case
- Evaluation
- Concluding Remarks

# Spectrum Auction: Addressing Inefficient Spectrum Distribution

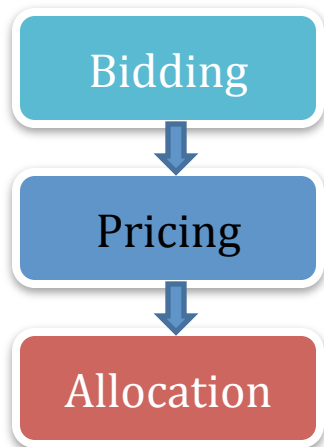
- Legacy wireless providers own the majority of spectrum
  - But cannot fully utilize it
- New wireless users are dying for usable spectrum
  - But have to crowd into limited unlicensed bands
- Market-based Spectrum Trading
  - On-demand spectrum auctions
    - Periodically auction spectrum based on user bids

## Spectrum Auctions



# Spectrum Auctions: A Overview

- Static auction model
  - Pre-determined set of users and channels
- Online auction model
  - Behaviors of spectrum users are dynamic
  - Number of available channels are fixed
- Pros and cons
  - Enable spatial reuse
  - Ensure truthfulness
  - Ignorance of dynamic nature of spectrum
  - Unfair pricing for homogeneous channels



# Objectives and Challenges

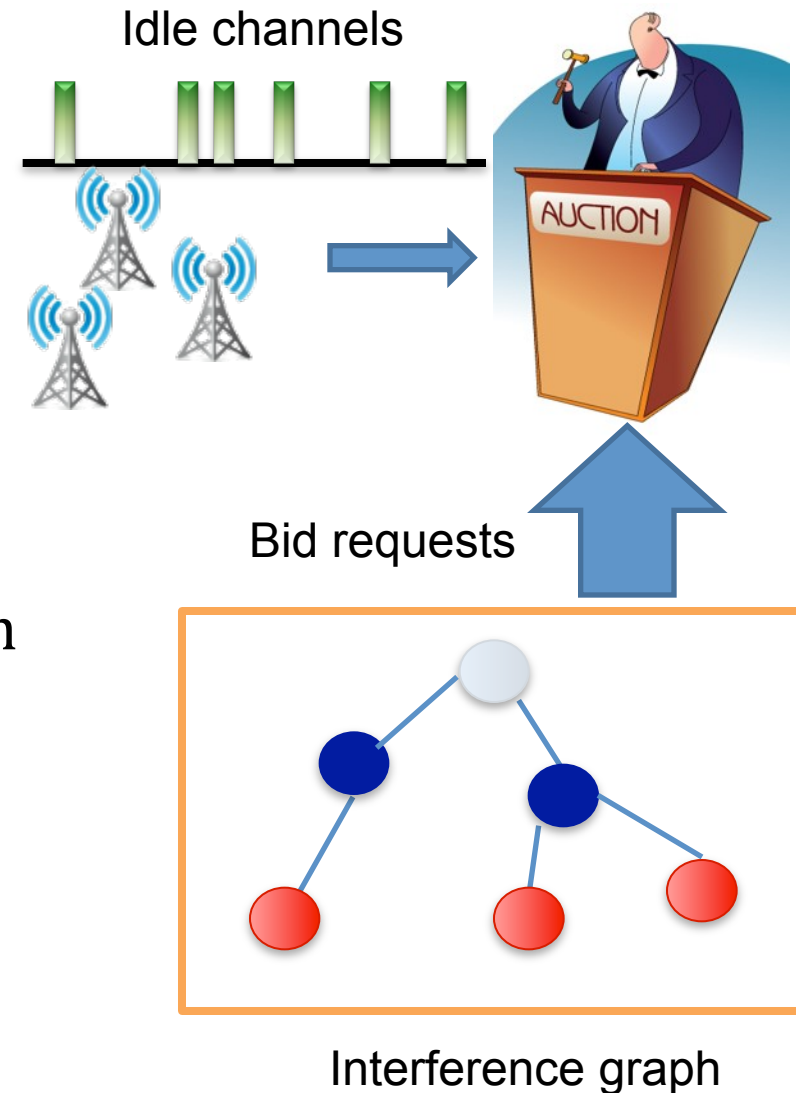
- Dynamic spectrum supply
  - Previously-occupied channels are dynamically released
  - Spectrum supply is unknown
- Design objectives and challenges
  - Achieve price fairness
  - Enable spatial reuse
  - Ensure truthfulness
  - Obtain performance bound

How to achieve all desirable properties simultaneously?



# Online Auction Model: Spectrum Auction with Dynamic Spectrum Supply

- **Users** request a number of channels when they need them
  - Interference condition among users is represented by a graph
- **Channel supply** is uncertain
  - Channels are dynamically occupied and released in one auction period
- **Auctioneers** periodically auction spectrum based on user bids
  - Do the allocation and pricing in an online manner

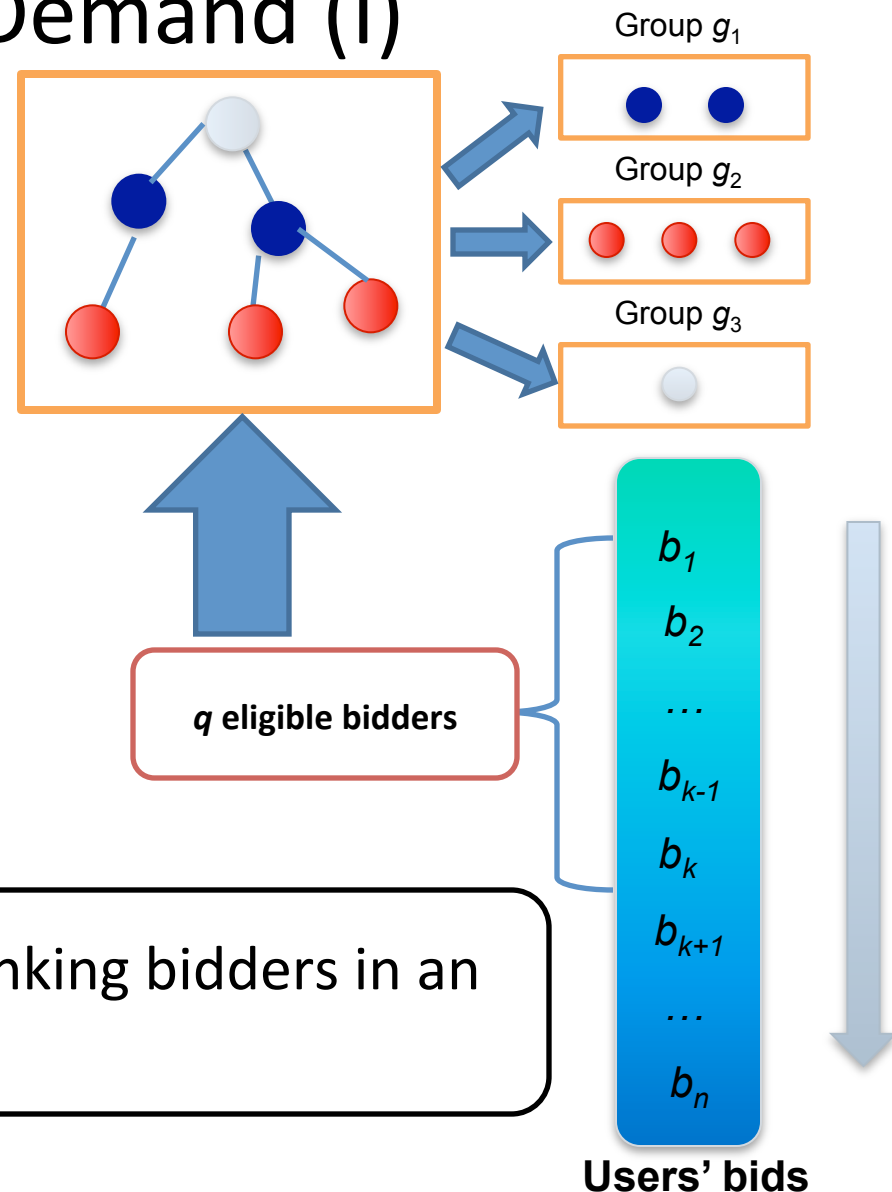


# Spectrum Auction with Dynamic Supply: Single-unit Demand (I)

- **Initialization**

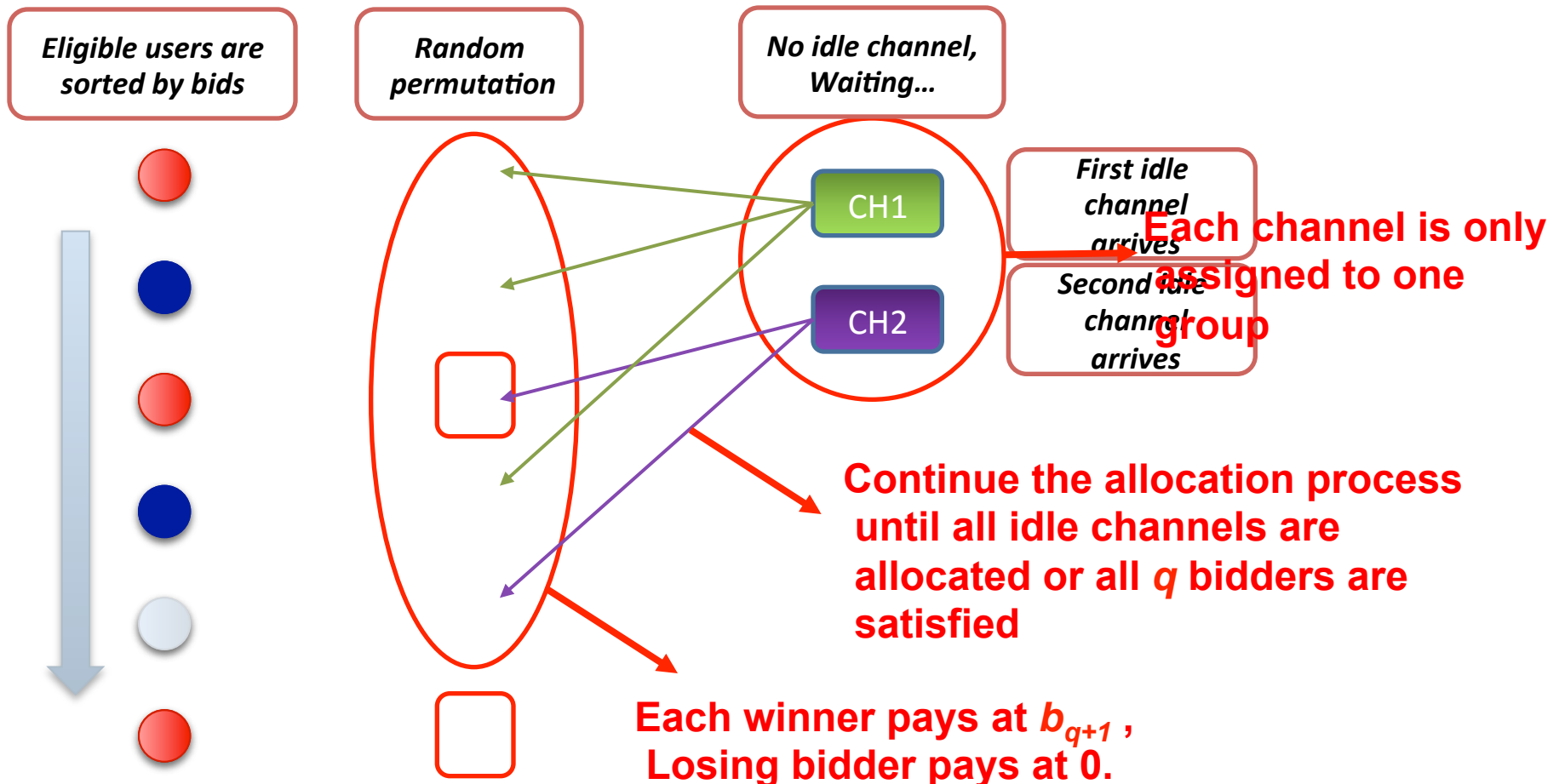
- Eligible bidders selection
  - Randomly select  $q$  from  $\{2^1, 2^2, \dots, 2^i, \dots, n\}$
  - Set the  $q$  top-ranking ones as eligible bidders
- **Bid-independent** group formation
- **Enable spatial reuse**

Assign idle channels to the  $q$  top-ranking bidders in an online manner.



# Spectrum Auction with Dynamic Supply: Single-unit Demand (II)

- Winner selection and pricing





# Ensuring Truthfulness

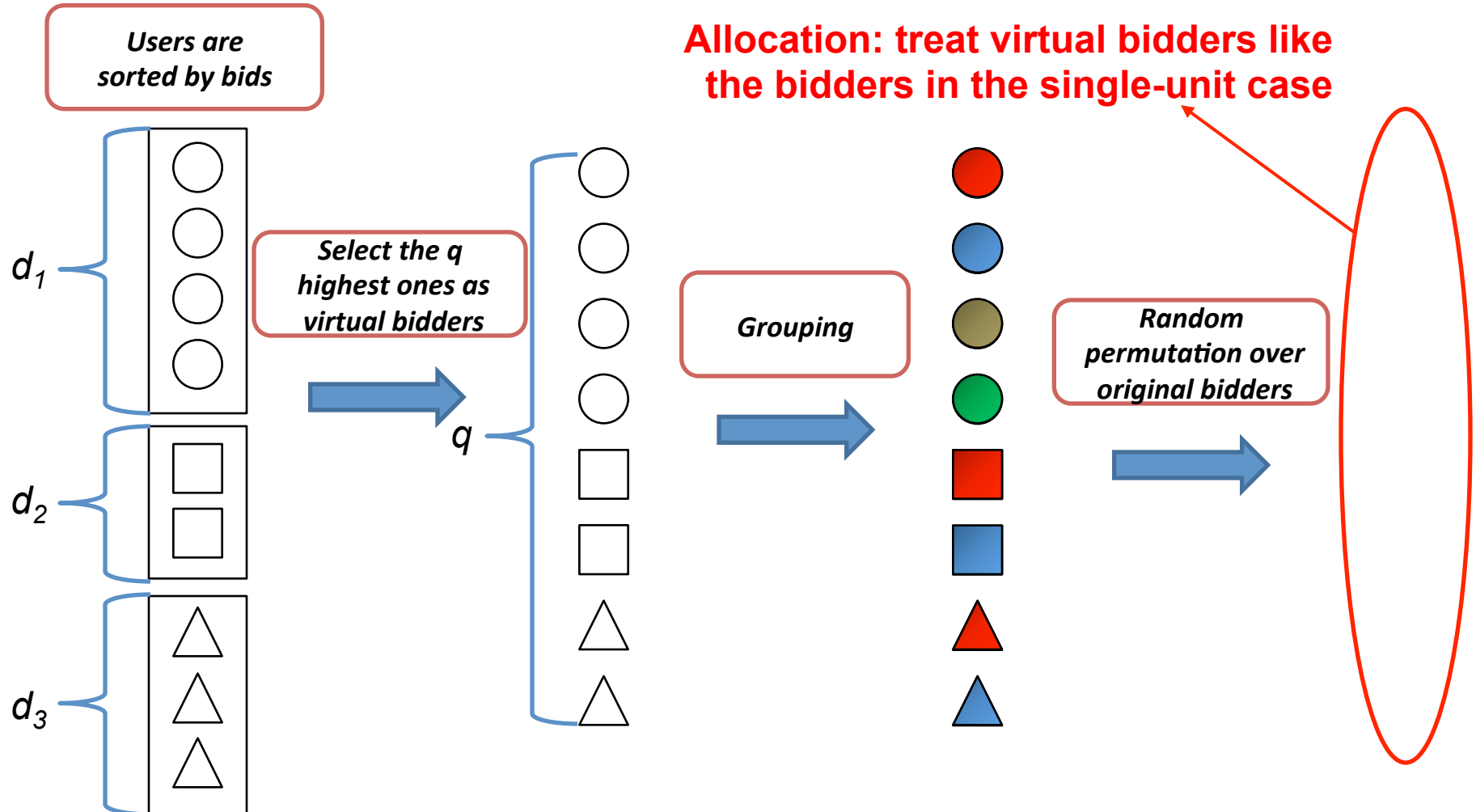
- **Theorem:** Under the dynamic channel supply, the proposed spectrum auction for the single-unit case is truthful
- Proof sketch
  - When all eligible bidders can be satisfied, any bidder cannot misreport the per-channel bid to increase its utility
  - When not all eligible bidders can be satisfied.....
  - No bidder has an incentive to lie

# Pricing Fairness and Efficiency

- Fair pricing
  - The per-channel payments are **the same** for all winners
  - No **price discrimination** for homogeneous channels
- Provable distance to the optimal auction efficiency
  - It is sufficient to analyze **two possible cases** to derive the approximation ratio
  - Compute the lower bound of the social welfare ***in expectation***
  - The auction design achieves a  **$\log n$**  approximation to the optimal social welfare

# Spectrum Auction with Dynamic Supply: Multi-unit Demand (I)

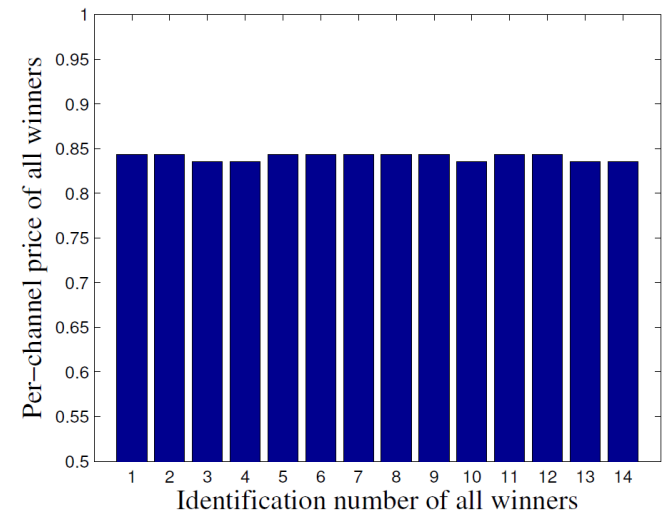
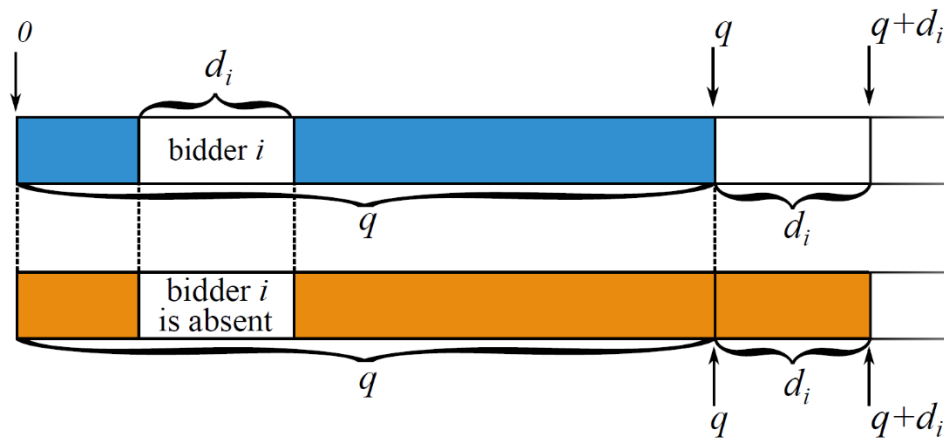
- An illustrating example



# Spectrum Auction with Dynamic Supply: Multi-unit Demand (II)

- Pricing before allocation
  - Compute the per-channel price for each winning original bidder

$$p_i = \frac{\sum_{j \neq i}^n x_j^{(-i)}(q) \cdot b_j - \sum_{j \neq i}^n x_j(q) \cdot b_j}{x_i(q)}$$



# Property Analysis

- Ensuring truthfulness (proof sketch)
  - Individual rationality
  - A fully-satisfied eligible bidder **cannot increase its utility by manipulating its bid**
  - A partially-satisfied eligible original bidder **cannot**.....
  - An ineligible bidder **cannot**.....
  - When not all eligible original bidders can be satisfied, an original bidder **cannot**.....
- Pricing fairness and efficiency
  - Per-channel prices for all winners are *almost the same*
  - Provable distance to the optimal auction efficiency

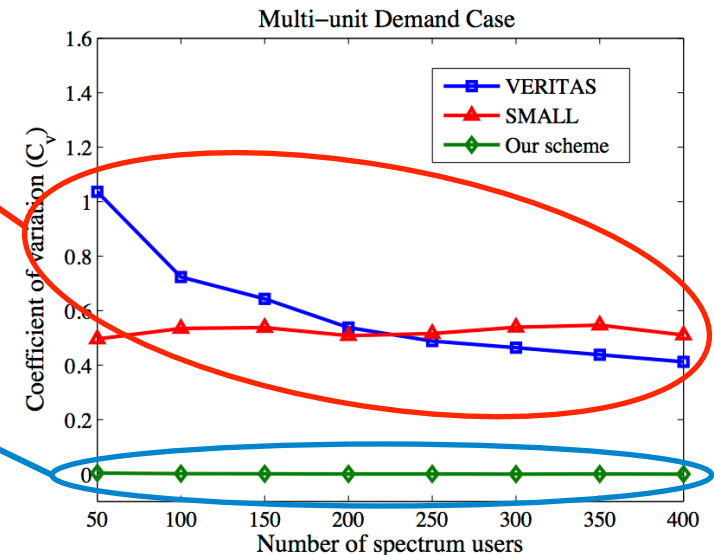
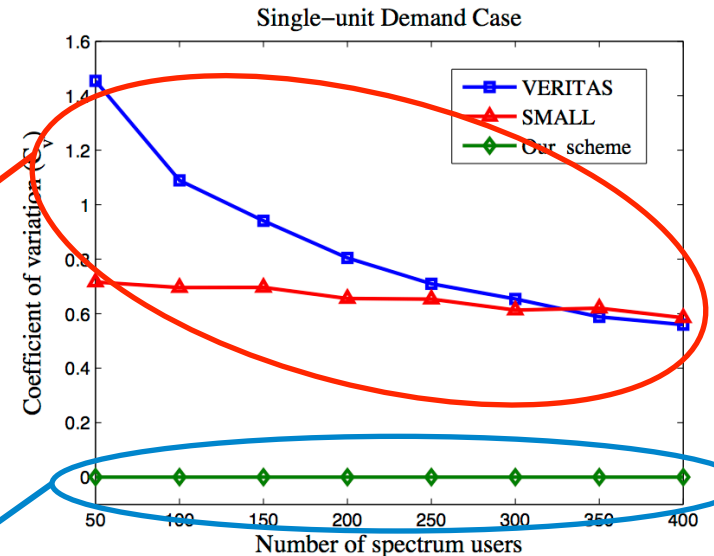
# Variation of per-channel price

Coefficient of variation represents the price distinction between winners

$$C_v = \frac{\sigma}{\mu}$$

In other mechanisms, there exist differences between winners' prices

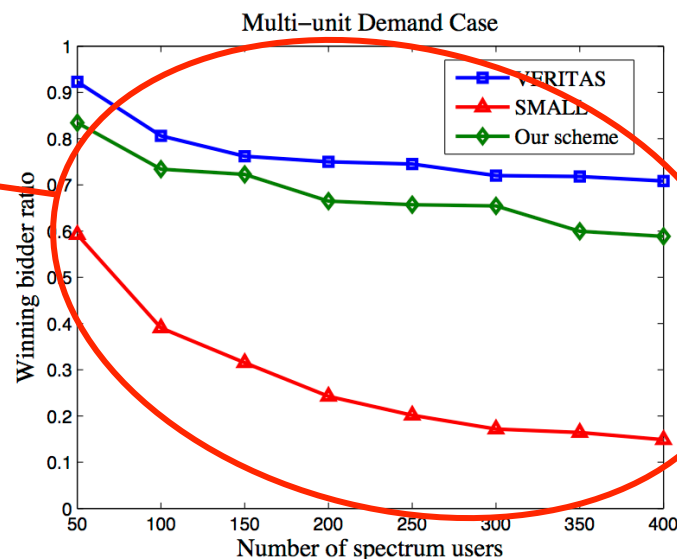
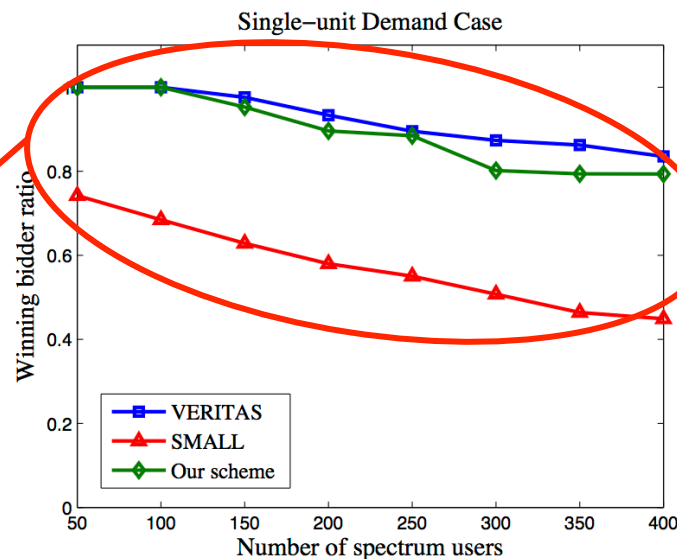
In our mechanism, the per-channel prices are absolutely or almost equal among winners



# Winning bidder ratio

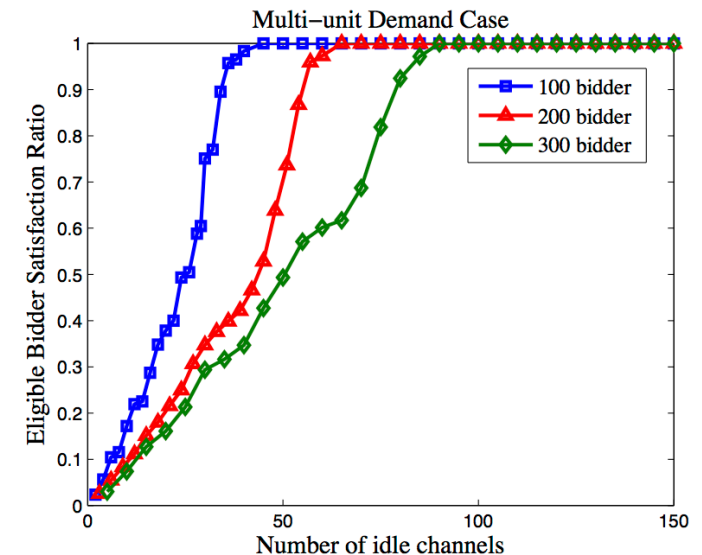
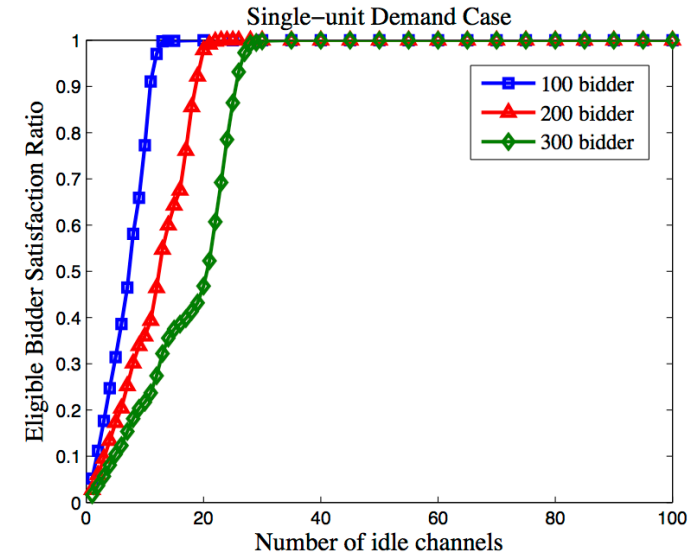
Winning bidder ratio represents the satisfaction of bidders with higher per-channel bids

Winning bidder ratio is decreasing with # of spectrum users.



# Eligible bidder satisfaction ratio

- Satisfaction ratio curve monotonically increases with # of idle channels
- The decrease of number of bidders can help satisfy eligible bidders





# Conclusion

- We formulate and investigate the problem of allocating channels to spectrum users with an unknown and dynamic supply
- We propose novel online spectrum auction schemes and show that all desirable properties can be achieved, including truthfulness, price fairness, efficiency
- We evaluate our spectrum designs show that they outperform the existing benchmarks by providing almost perfect price fairness

**Thank you for your attention!**