A Primal Dual Approach for inter-ISP Traffic Reduction in P2P VoD Streaming

TABLE I
IMPORTANT NOTATIONS

M	No. of ISPs
$\mathcal{N}_n(d)$	Peer d's total neighbor set in ISP n
$\mathcal{P}_m$	Peers in ISP m
$B(d)$	Peer d's upload bandwidth
$\mathcal{R}_{u\to d}(t)$	set of chunks peer d requests from peer u at time t
a_r	indicator of whether peer u transmits chunk c to peer d
v_d^c	valuation for peer d receiving chunk c
$w_{n\to m}$	transaction cost for transmitting a chunk from ISP n to ISP m

***Abstract*—**

I. ISP-AWARE P2P VOD MODEL

We consider a mesh-based P2P VoD streaming system deployed among M Internet Service Providers (ISPs). We assume the mesh topology is constructed and maintained by an independent module, which is orthogonal to our work. In the mesh construction module, a peer obtains from a tracker a set of neighbors with similar playback progresses upon joining the overlay.

Under the constructed mesh topology, we model the chunk scheduling and bandwidth allocation problem taking the ISP-awareness into consideration. Let $\mathcal{N}_n(d)$ denote the neighbor set of peer $d \in \cup_{m=1}^M \mathcal{P}_m$ in ISP n. We use $B(d)$ to denote the upload bandwidth of peer d. We assume the download bandwidth of peers are large enough to receive the playback rate video. The bandwidth bottleneck is at peers' upload bandwidth. When a peer is watching a specific position of a video, the peer does not need to request all its missing chunks of the video. Hence, peer d will request a chunk in an online way. A request r is denoted by a three-tuple (d, u, c), d is the requesting peer, u is the requested peer, c is the requested chunk. We consider a discrete time slot model for the system, $t = 0, 1, 2, \ldots, T$. Let $\mathcal{R}_{u\to d}(t)$ denote the set of chunks that peer d requests from neighbor u at time slot t. Let $\mathcal{N}_n(d, c)$ be the set of neighbors that peer d can download chunk c from. Let a_r be the indicator of whether request r obtained one unit bandwidth, i.e., $a_r = 1$ means request r obtains one unit bandwidth, $a_r = 0$ means request r does not obtain one unit bandwidth. Peer d's valuation for receiving chunk $c \in \mathcal{R}_{u\to d}(t), u \in \cup_{n=1}^M \mathcal{N}_n(d)$ is v_d^c. The transaction cost for peer d in ISP m receiving chunks from peers in ISP n is $w_{n\to m}$. The important notations are summarized in table I.

Hence, the total utility for chunk dissemination at time slot t is,

$$\max \sum_{d\in\cup_{m=1}^M \mathcal{P}_m} \sum_{u\in\cup_{n=1}^M \mathcal{N}_n(d)} \sum_{c\in\mathcal{R}_{u\to d}(t)} a_r(v_d^{(c)} - w_{n\to m}), \tag{1}$$

$$\text{s.t.} \sum_{d\in\cup_{n=1}^M \mathcal{N}_n(u)} \sum_{c\in\mathcal{R}_{u\to d}(t)} a_r \le B(u), \forall u \in \cup_{m=1}^M \mathcal{P}_m, \tag{2}$$

$$\sum_{r:r(u)\in\cup_{n=1}^M \mathcal{N}_n(d), c\in\mathcal{R}_{r(u)\to d}(t)} a_r = 1,$$
$$d \in \cup_{m=1}^M \mathcal{P}_m, c \in \cup\mathcal{R}_{u\to d}(t), \tag{3}$$

$$a_r \ge 0, d \in \cup_{m=1}^M \mathcal{P}_m, u \in \cup_{n=1}^M \mathcal{N}_n(d), c \in \mathcal{R}_{u\to d}(t). \tag{4}$$

Constraint (2) shows that peers upload chunks to its neighbors within its upload bandwidth limit. Constraint (3) shows that a peer will not download multiple copies of a chunk. Constraint (4) shows that the indicator is non-negative.

Next, let us see the dual problem of the optimization problem (1). The dual is as follows,

$$\min \sum_{u\in\cup_{m=1}^M \mathcal{P}_m} \lambda_u B(u) + \sum_{d\in\cup_{m=1}^M \mathcal{P}_m} \sum_{c\in\cup\mathcal{R}_{u\to d}(t)} \eta_d^{(c)}, \tag{5}$$

$$\text{s.t.} \lambda_u + \eta_d^{(c)} \ge v_d^{(c)} - w_{n\to m},$$
$$d \in \cup_{m=1}^M \mathcal{P}_m, u \in \cup_{n=1}^M \mathcal{N}_n(d), c \in \mathcal{R}_{u\to d}(t), \tag{6}$$

$$\lambda_u \ge 0, u \in \cup_{m=1}^M \mathcal{P}_m, \tag{7}$$

$$\eta_d^{(c)} unconstrained, d \in \cup_{m=1}^M \mathcal{P}_m, c \in \cup\mathcal{R}_{u\to d}(t). \tag{8}$$

II. A PRIMAL DUAL METHOD FOR THE PROBLEM

The bandwidth allocation to specific chunk requests takes the following two aspects into considerations: 1) the valuation of a peer receiving the requested chunk; 2) the transaction cost of sending a chunk across ISPs. We model the optimization problem in an online way as the requests are sent out gradually as time evolves.

Our primal dual algorithm maintains a single price $p(u_i), 1 \le i \le B(u)$ for each unit of peer u's upload bandwidth. The optimization problem in any time slot t can be solved through an auction-like primal-dual method,

Bidding at Peer d**:** After peer d exchanging the bitmap information with its neighbor $u \in \cup_{n=1}^M \mathcal{N}_n(d)$, it can bid for the chunks it does not have yet, i.e., $c \in \mathcal{R}_{u\to d}(t)$. Hence, we use (d, c) to denote one buyer who wants to buy one unit of bandwidth. Buyer (d, c) can buy one unit of bandwidth from anyone of its neighbors with chunk c. We use (u, i) to denote the ith unit of bandwidth that can be allocated.

The first problem is from which neighbor to bid for one unit of bandwidth. Buyer (d,c)'s current welfare for one unit of bandwidth from each unit bandwidth of its available neighbors is given by $v_d^{(c)} - w_{u\to d} - \min_{1\le i\le B(u)} p(u_i)$, $u \in \cup_{n=1}^{M}\mathcal{N}_n(d,c)$. Let $u^*(d,c)$ be the peer u that provides the largest welfare to buyer (d,c). (d,c) will submit the bid to peer $u^*(d,c)$.

The second problem is how much buyer (d,c) should bid. For (d,c), the maximum welfare for obtaining one unit of bandwidth is $\varphi(d,c,u^*) = v_d^{(c)} - w_{u^*\to d} - \min_{1\le i\le B(u^*)} p(u_i^*)$, the second best welfare offered by neighbors other than $u^*(d,c)$ is $\varphi(d,c,\hat{u}) = v_d^{(c)} - w_{\hat{u}\to d} - \min_{1\le i\le B(\hat{u})} p(\hat{u}_i)$. Buyer (d,c) bids $b(d,c,u^*,i^*) = \min_{1\le i\le B(u^*)} p(u_i^*) + \varphi(d,c,u^*) - \varphi(d,c,\hat{u}) = v_d^{(c)} - w_{u^*\to d} - \varphi(d,c,\hat{u})$ for bandwidth (u^*,i^*), here $i^* = argmin_{1\le i\le B(u^*)} p(u_i^*)$.

Bandwidth Assignment at peer u: Each peer u receives bids from its neighbors for the requested chunks. Upon receiving a bid $b(d,c,u,i)$, if $b(d,c,u,i) \le p(u_i)$, ignore, else, peer u allocates its ith bandwidth to transmit chunk c to peer d, update $p(u_i)$ to $b(d,c,u,i)$.

The algorithm is summarized as follows,

Algorithm 1 A Primal Dual Method for Bandwidth Allocation

1: **Initialization:** $p(u_i) = 0, \forall u, 1 \le i \le B(u)$
2: **Bidding at Peer d:**
3: **if** Peer d wants to download chunk c **then**
4: Peer d requests neighbors' price vector $p(u_i), 1 \le i \le B(u)$, $u \in \cup_{m=1}^{M}\mathcal{N}_m(d,c)$
5: Peer d calculates the welfare for one unit bandwidth from each neighbor, $v_d^{(c)} - w_{u\to d} - \min_{1\le i\le B(u)} p(u_i)$, select the neighbor that provides the maximum welfare, u^*, and i^* with the minimum $p(u_i^*)$.
6: Send the bid $b(d,c,u^*,i^*) = v_d^{(c)} - w_{u^*\to d} - \varphi(d,c,\hat{u})$ to neighbor u^*, $\varphi(d,c,\hat{u}) = v_d^{(c)} - w_{\hat{u}\to d} - \min_{1\le i\le B(\hat{u})} p(\hat{u}_i)$ is the second best welfare from peer d's neighbors.
7: **end if**
8: **Bandwidth Allocation at Peer u:**
9: **if** Receiving a bid $b(d,c,u,i) > p(u_i)$ **then**
10: Allocate its ith bandwidth to transmit chunk c to peer d
11: Update $p(u_i)$ to $b(d,c,u,i)$
12: **end if**

III. The Analysis of the Algorithm

We have the following theorem for the algorithm.

Theorem 1: With the assumption that the upload bandwidth is sufficient to support the video playback rate, the auction-based primal dual algorithm correctly solves the primal dual linear problem.

Proof: The proof consists of two parts: (1) The primal dual algorithm for one time slot terminates, and (2) Upon termination, the complementary slackness of the optimization problems is satisfied.

(1) The termination of the auction can be proved by way of contradiction. Suppose it never terminates. The units of allocated bandwidth grow, because one allocated unit of bandwidth remains allocated through the auction. The total units of allocated bandwidth is upper-bounded by total bandwidth demand from downloading peers in the streaming network. Under the assumption that the upload bandwidth is sufficient, there will be units of upload bandwidth that are never allocated. Therefore, as the algorithm does not terminate, we have a peer wanting to download a chunk bidding for one unit of allocated bandwidth whose price is growing unbounded, rather than bidding for an unallocated unit of bandwidth with price 0. This implies the valuation of downloading a chunk from the peer with price 0 is negative infinite, contradicting the fact that the valuation should be finite.

(2) We first list the complementary slackness conditions of the primal dual problem. The dual variable $\lambda(u) = \min_{1\le i\le B(u)} p(u_i)$, i.e., the minimum price for one unit of peer u's bandwidth.

$$\begin{cases} \lambda(u) > 0 \to \sum_{d\in\cup_{n=1}^{M}\mathcal{N}_n(u)} \sum_{c\in\mathcal{R}_{u\to d}(t)} a_r = B(u), \\ \qquad \forall u \in \cup_{m=1}^{M}\mathcal{P}_m; \\ a_r > 0 \to \lambda_u + \eta_d^{(c)} = v_d^{(c)} - w_{n\to m}, \\ \qquad d \in \cup_{m=1}^{M}\mathcal{P}_m, u \in \cup_{n=1}^{M}\mathcal{N}_n(d), c \in \mathcal{R}_{u\to d}(t). \end{cases}$$

The first condition means that a node u with non-zero price for its upload bandwidth must have all its bandwidth allocated. This is obviously true in our algorithm. As if there is one unit of bandwidth not allocated, the price of the unit will be zero, a node u will have a price $\lambda u = 0$.

The second condition means that when a request (d,c) obtains a unit of bandwidth from peer u, the optimal solution $\eta_d^{(c)}$ will be equal to $v_d^{(c)} - w_{n\to m} - \lambda_u$. This is true as $v_d^{(c)} - w_{n\to m} - \lambda_u$ will have the maximum value among all neighbors u that can provide chunk c to peer d.