Truthful Auction for Dynamic Spectrum Acces

December 7, 2011

Precious resource in wireless networks: Spectrum!

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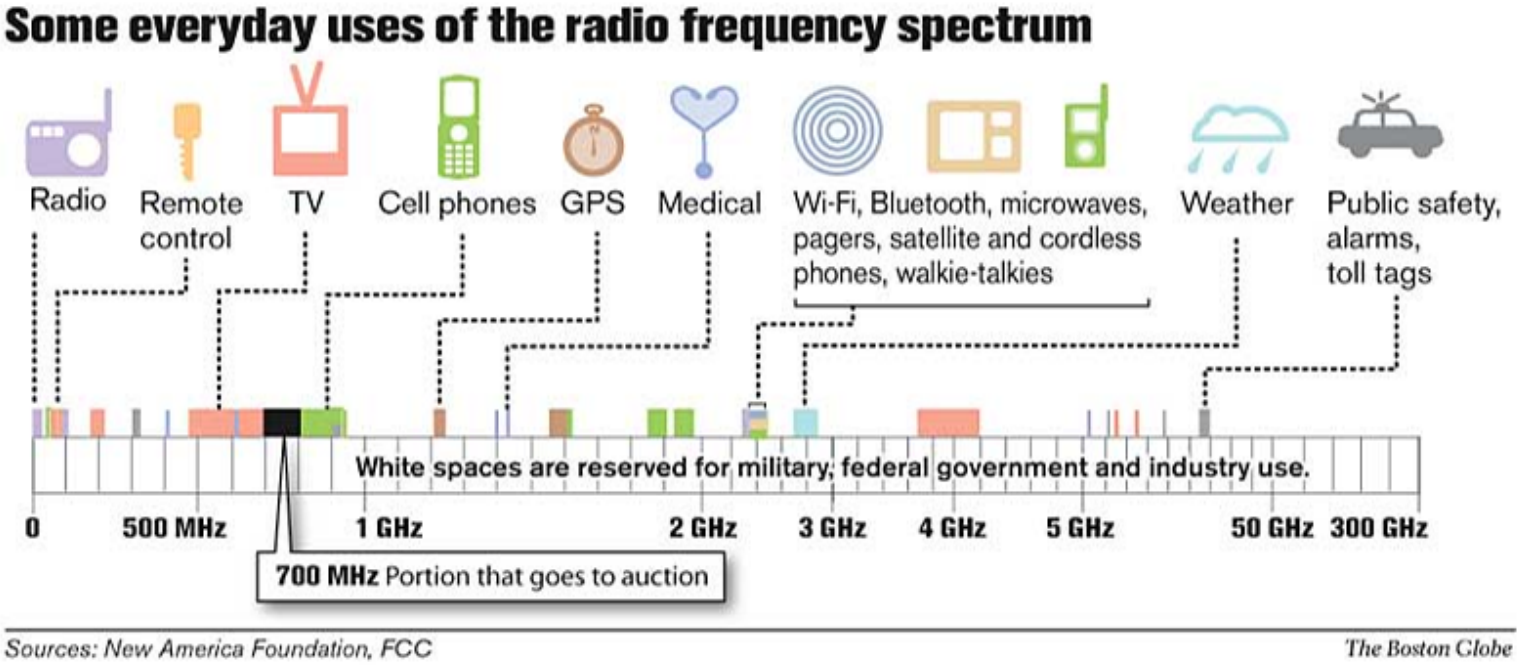


Figure: In 2007, $19 billion for 700 MHZ!

Precious resource in wireless networks: Spectrum!

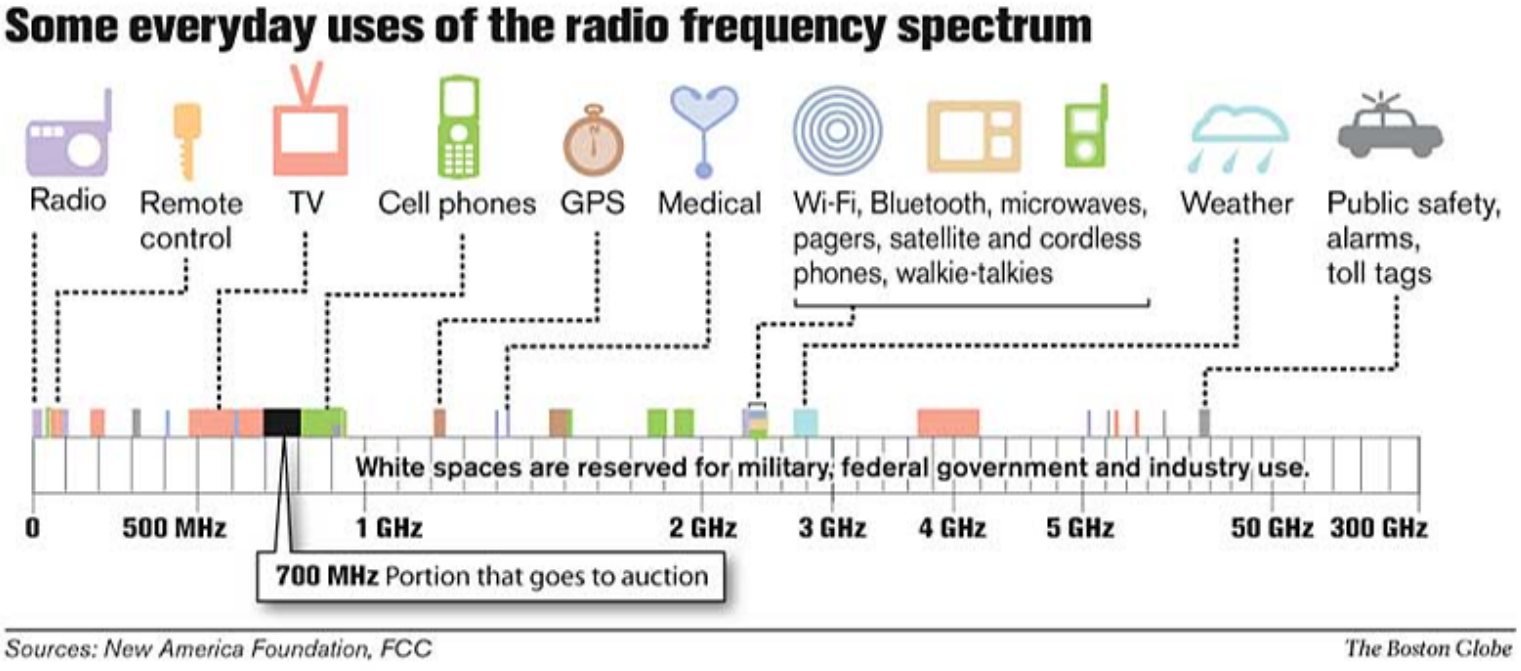


Figure: In 2007, $19 billion for 700 MHZ!

Expensive?

Precious resource in wireless networks: Spectrum!

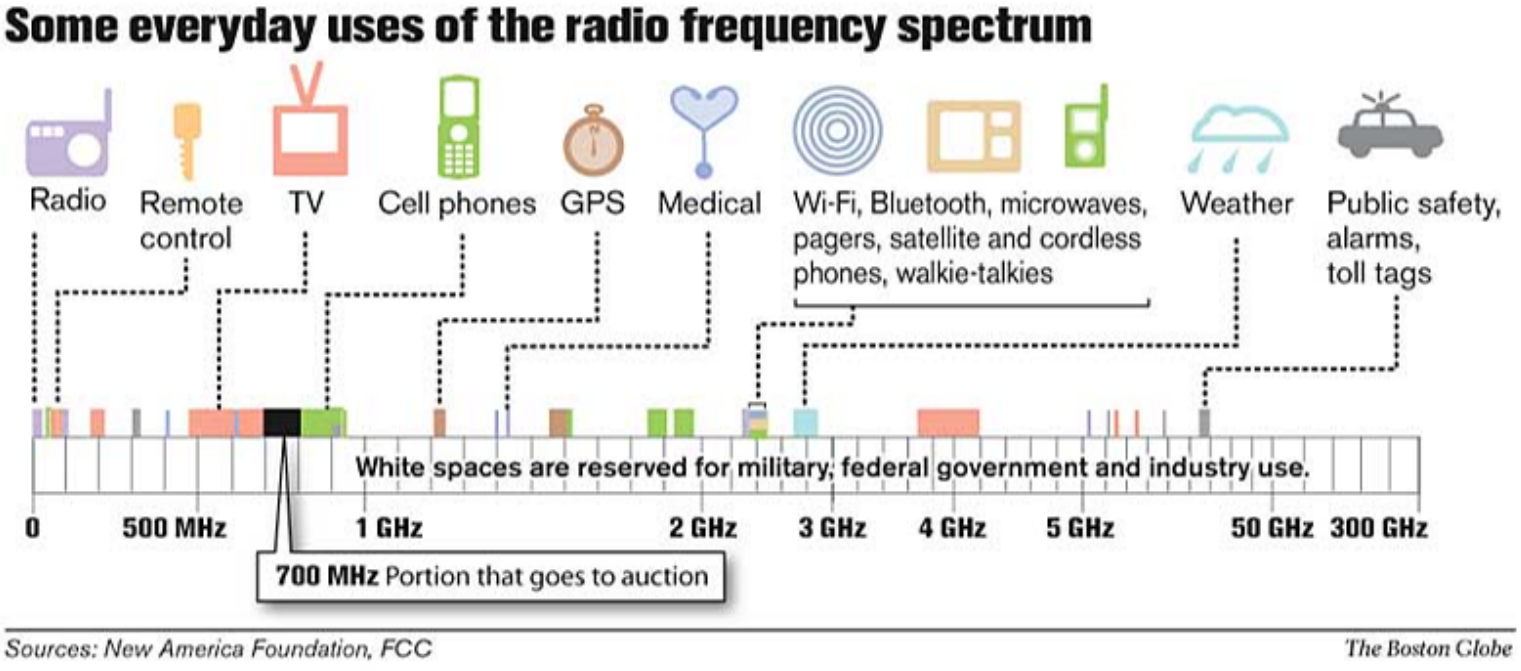
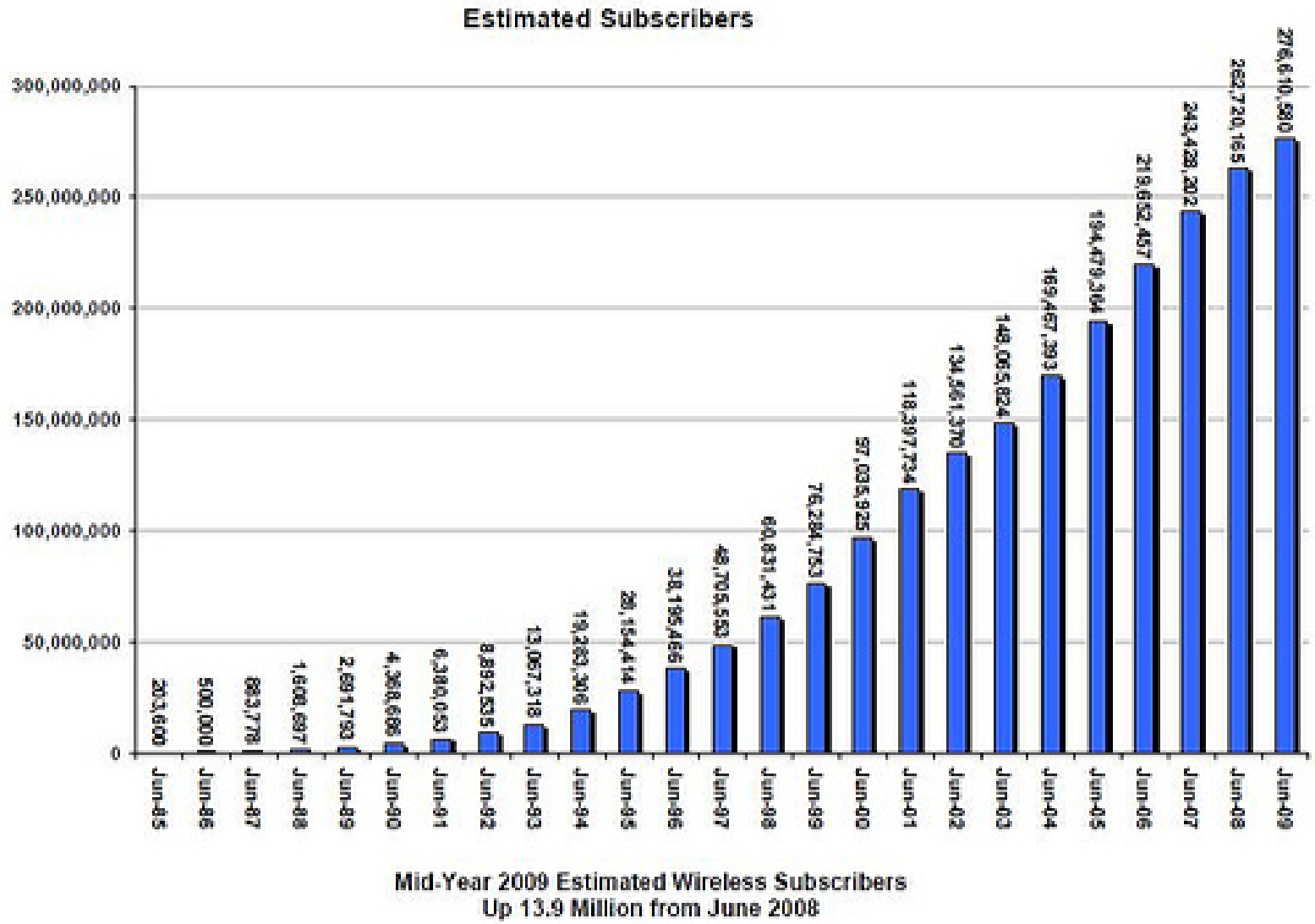


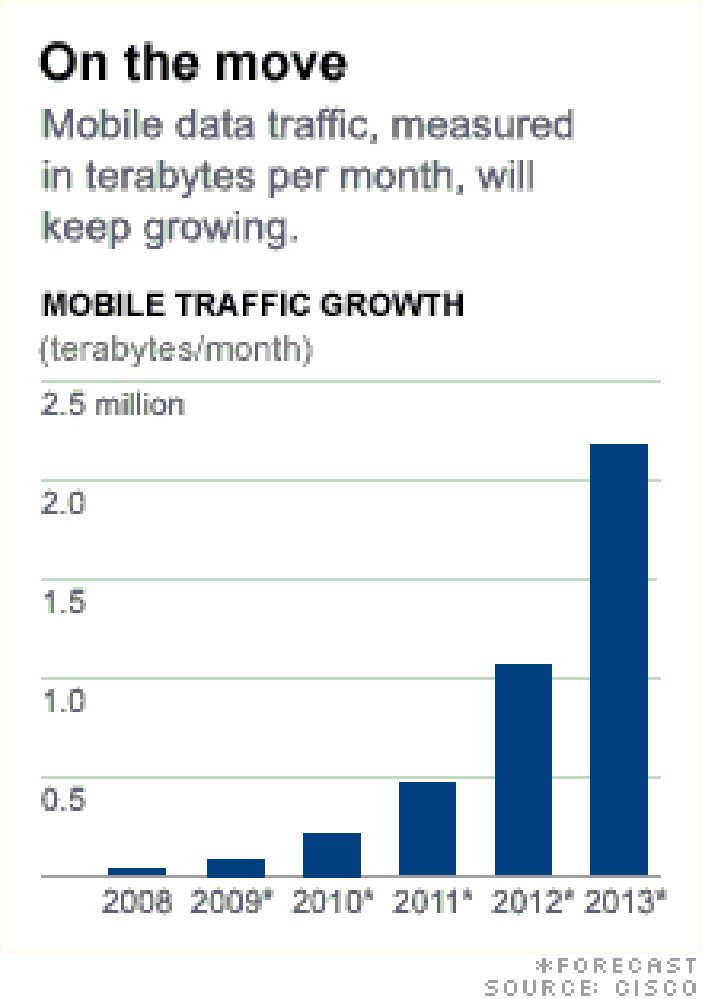
Figure: In 2007, $19 billion for 700 MHZ!

Expensive? Real bargin!
In 2010, $120 billion for 300 MHZ!

Why is spectrum so precious?



Why is spectrum so precious? (Cont.)



Static spectrum allocation

Seller:

Static spectrum allocation

Seller:

Buyer:

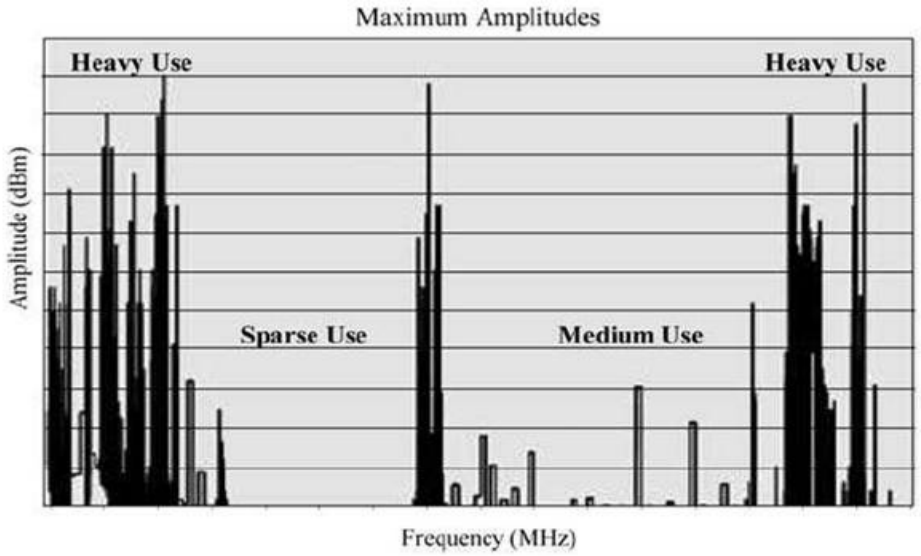


Figure: Under-utilized spectrum of licensed users.

Current status: static spectrum allocation into two types,

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Current status: static spectrum allocation into two types,

- Licensed spectrum, *e.g.*, reserved as military, TV and paging frequencies; Under-utilized!
- Unlicensed spectrum, *e.g.*, shared by computer networking, sensor networks, RFID and ultra-wideband devices. Congested!

Cognitive radio technology: dynamic spectrum access,

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- Let the secondary (unlicensed) user to get the licensed spectrum when the primary (licensed) users are idle;
- Increase the efficiency of spectrum.

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- Negotiated access: *e.g.*, auction-based.

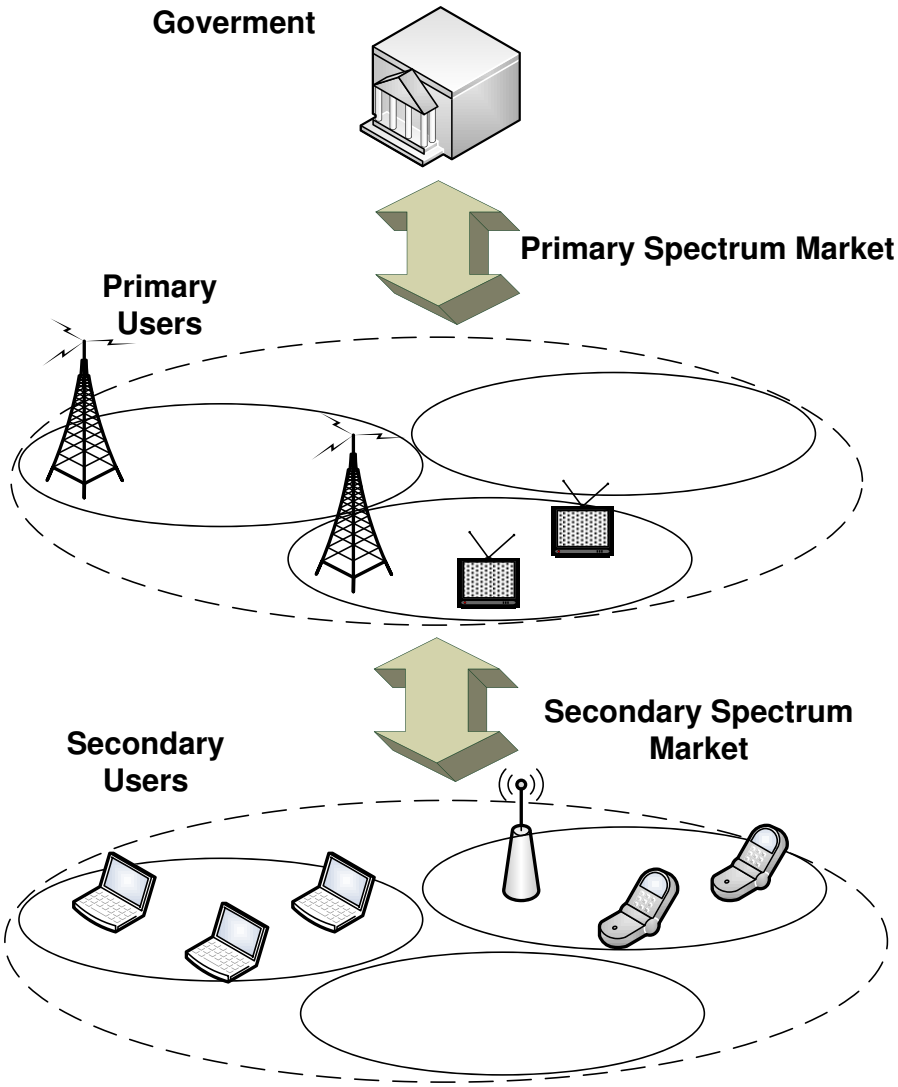
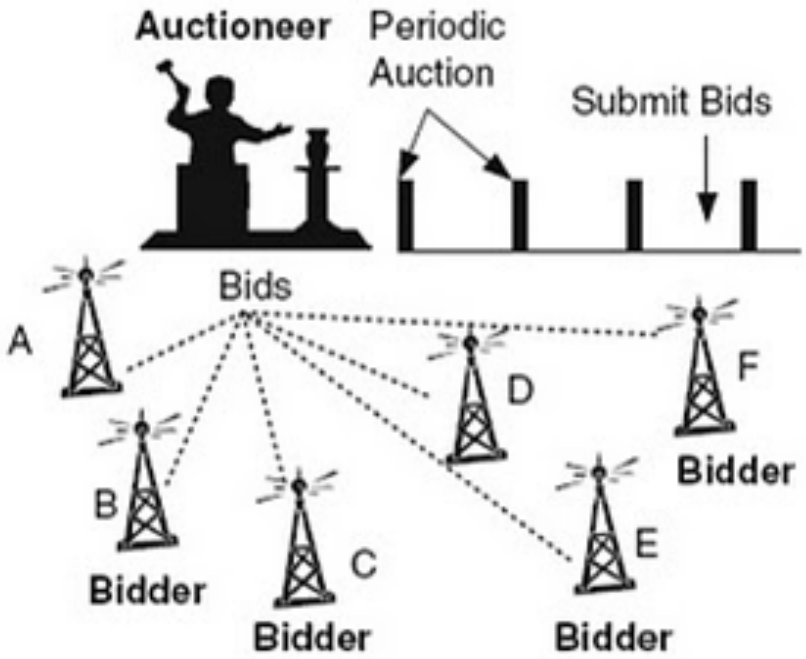


Figure: Spectrum Market Structure.

Spectrum auction



Spectrum auction:

- Incentives for primary users to share idle spectrum resource;

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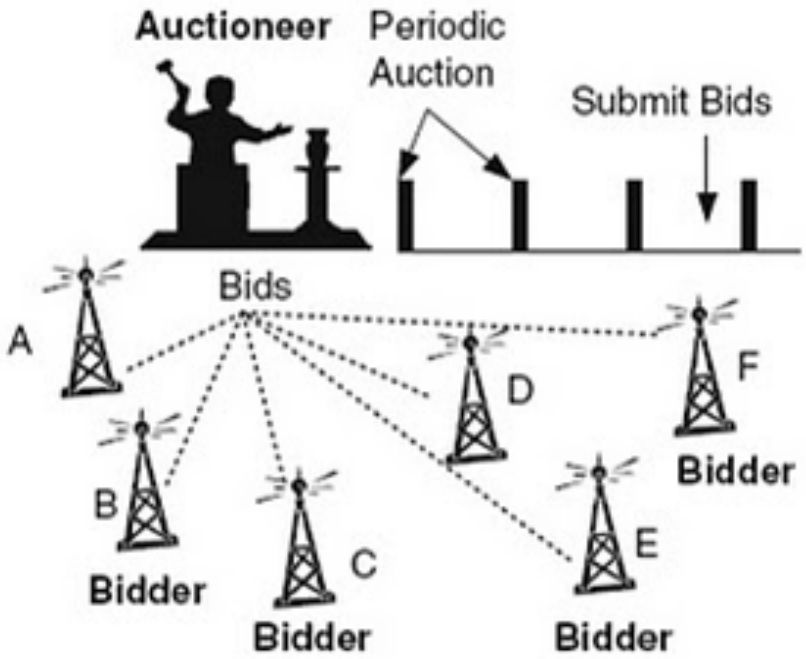

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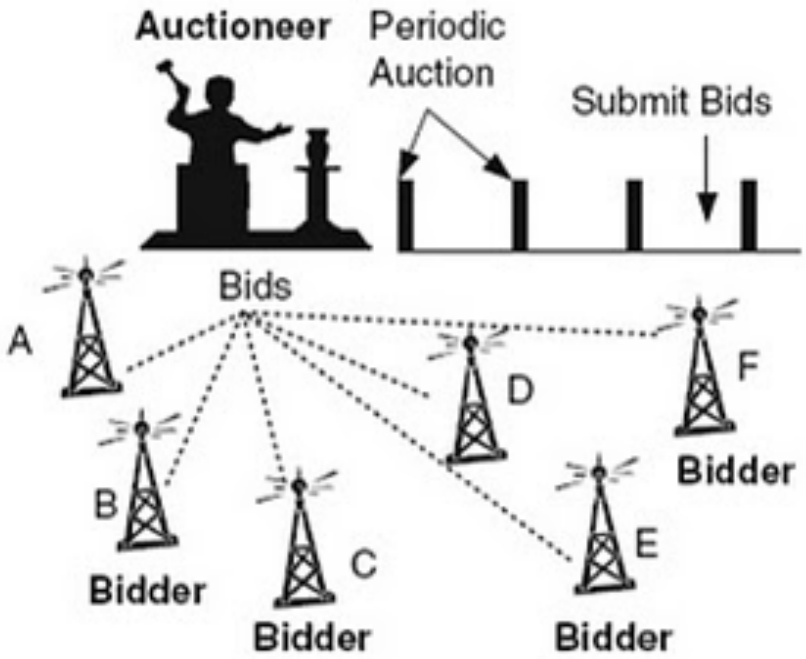

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- Incentives for primary users to share idle spectrum resource;
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- Efficient: allocate the good to who value them most.

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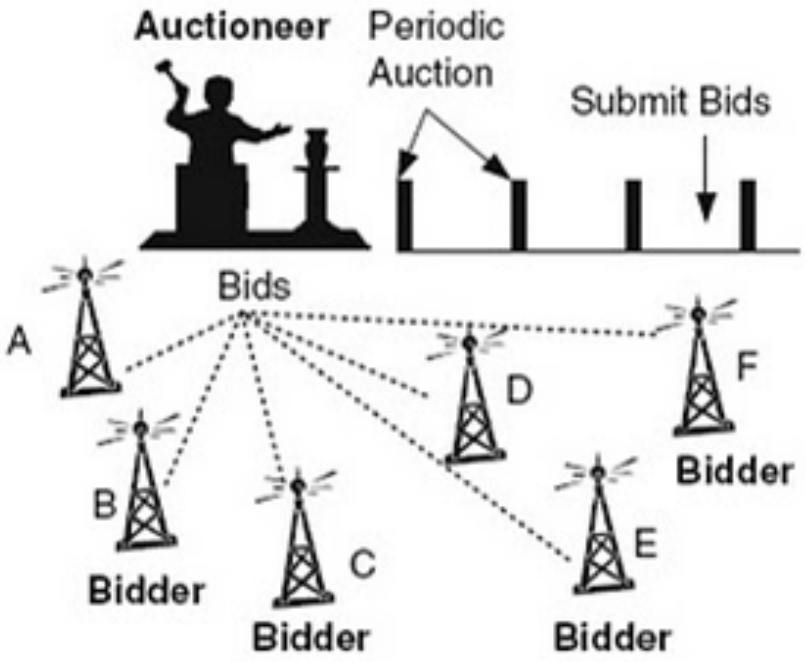

- Per-channel true value v_i

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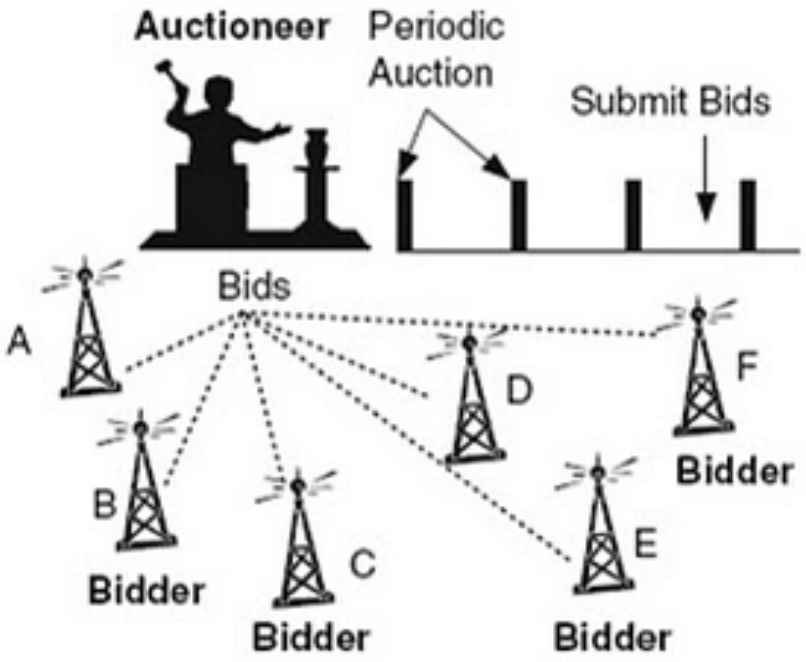
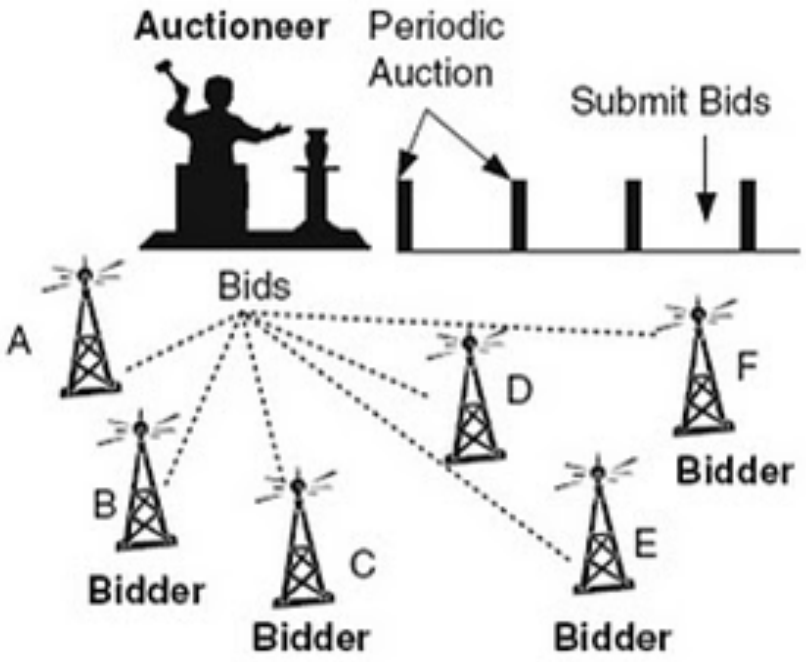


- Per-channel true value v_i
- Channel request d_i

Spectrum auction



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- Per-channel bid b_i

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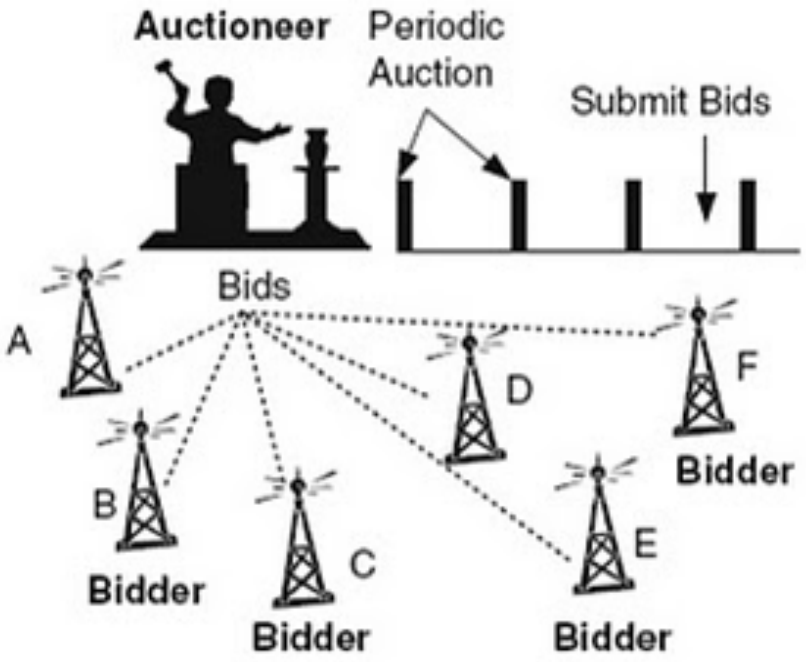

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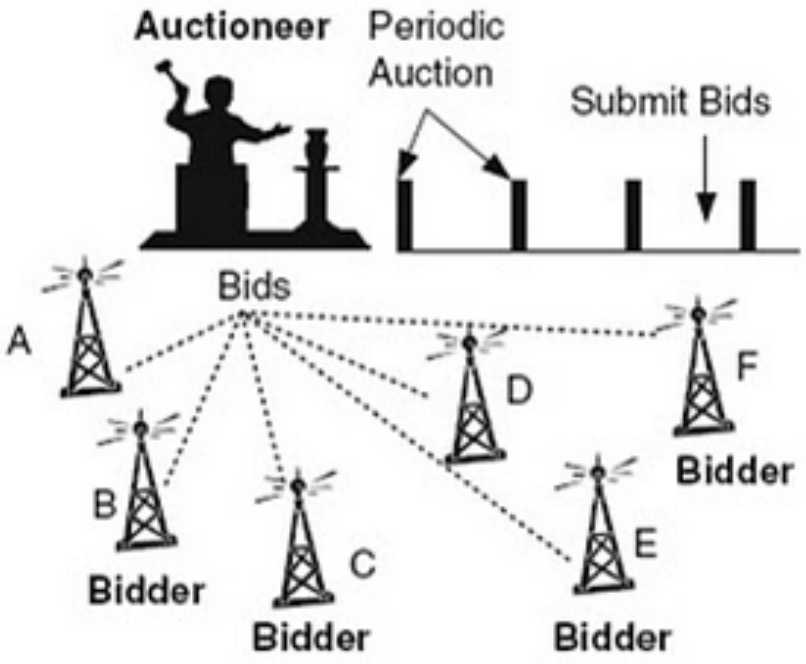

- Per-channel true value v_i
- Channel request d_i
- Per-channel bid b_i
- Clearing price p_i
- Bidder utility
  $u_i = v_i \cdot d_i - p_i$ if i win the auction, and 0 otherwise.

Definition (Truthful Auction)

A truthful auction is on in which no bidder i can obtain higher utility u_i by setting $b_i \neq v_i$.

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  - Prevent market manipulation.
- To auctioneer:
  - Encourage bidders to reveal their true values;
  - Increase its revenue by assigning spectrum to bidders valuing it most.

Conventional truthful auctions are not suitable:

- Fail to be truthful;
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Reason: unlike good (*e.g.*, paintings, bonds, electricity), spectrum is reusable among non-conflicting bidders. Optimal channel allocation is a maximum-weighted independent set problem (NP-hard).

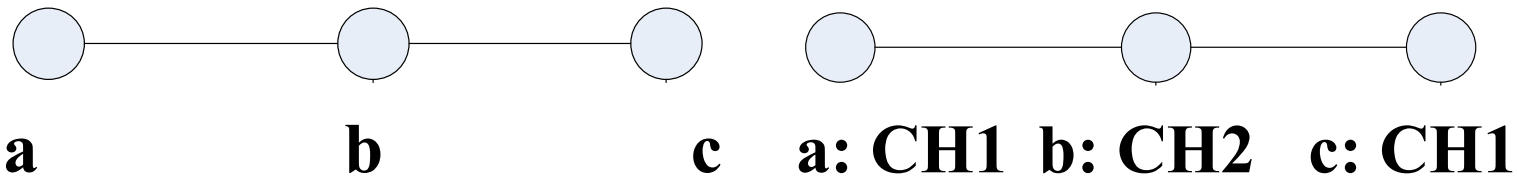

Figure: Example on spectrum allocations. (Left) The conflict graph with 3 bidders; (Right) The optimal spectrum allocation with 2 channels.

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  - Repeat 2 until all bidders are considered.
- Pricing for winner i: the highest price of its unallocated conflicting neighbors. 0 if there is no such neighbor.

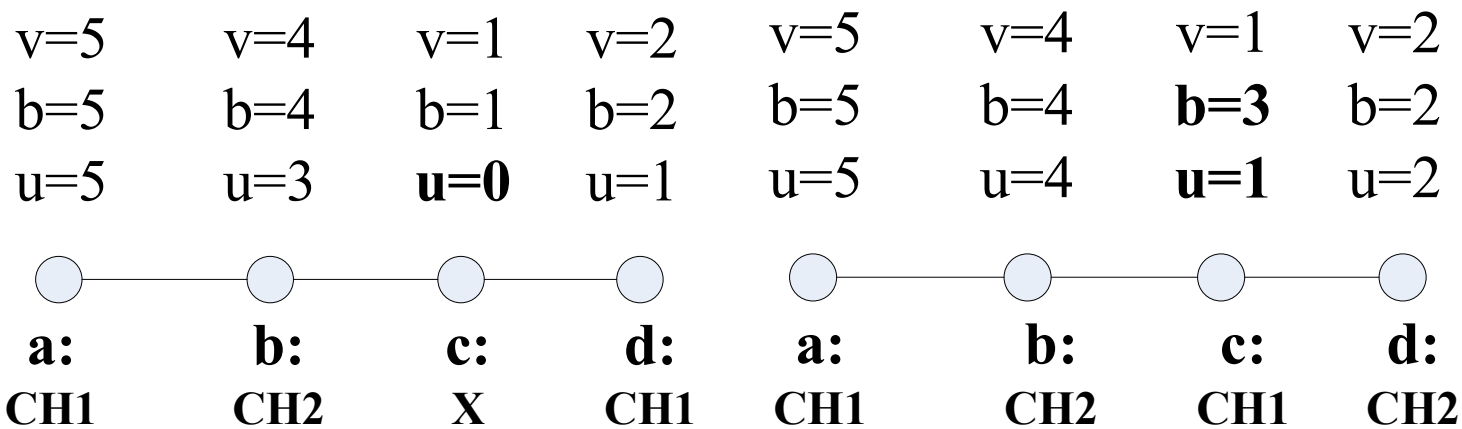

Figure: Example with secondary pricing. (Left) All bidders truthfully bid; (Right) Bidder c improves its utility by bidding higher than its true value.

VCG-style spectrum auction:

- Allocation: the same with seconday pricing spectrum auction.

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- Pricing for winner i: the bid of its first rejected neighbor who would have been allocated if i were absent from the auction. 0 if there is no such neighbor.

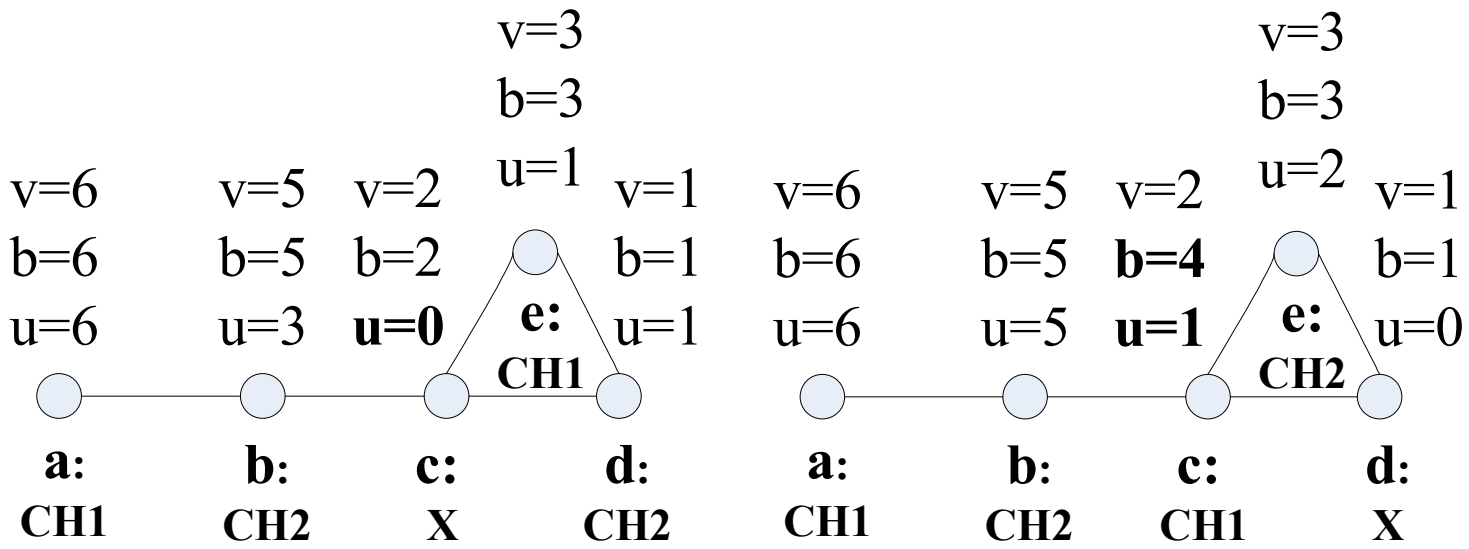

Figure: Example with VCG. (Left) All bidders truthfully bid; (Right) Bidder c improves its utility by bidding higher than its true value.

VERITAS [1]

- Allocation:

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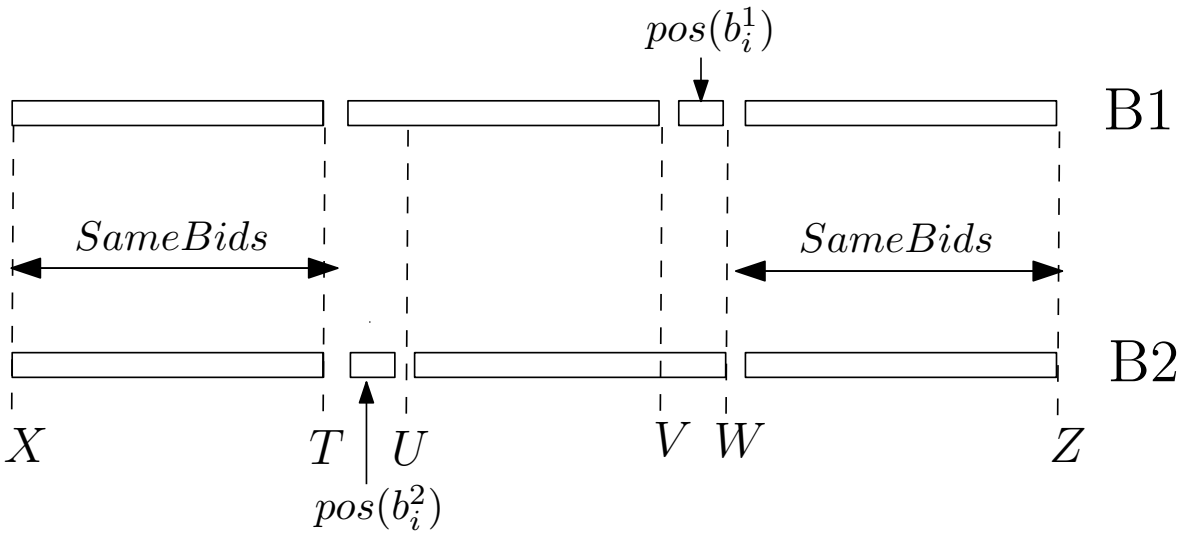
Definition (Critical Neighbor)

Given $\{B/b_i\}$[2], a critical neighbor $C(i)$ of bidder i is a bidder in $N(i)$ where if i bids lower than $C(i)$, i will not be allocated, and if i bids higher than $C(i)$, i will be allocated.

[2] $B = \{b_1, \ldots, b_n\}$ is the set of all bids.

Lemma (Monotonic Allocation 1)

If any bidder i is allocated by bidding b_i^1, it will also be allocated if it bids b_i^2, where $b_i^2 > b_i^1$ (provided all the other bids and channel demands remain the same).


$pos(b_i^1)$
B1
$SameBids$
$SameBids$
B2
X
T
U
V
W
Z
$pos(b_i^2)$

Lemma (Monotonic Allocation 2)

If any bidder i is rejected by bidding b_i^2, it will also be rejected if it bids b_i^1, where $b_i^2 > b_i^1$ (provided all the other bids and channel demands remain the same).

Lemma (Critical Neighbor/Value)

For any bidder i, if i would be rejected by bidding some value, then there exists a unique position in the sorted bid list, such that if i's bid is placed before that position i will win, and if i's bid is placed after that position it will lose. Moreover, that position is occupied by one of i's neighbors in $N(i)$[3].

[3] $N(i)$ is the neighbor of i in the conflict graph

Lemma

For each winner i in VERITAS, its clearing price is less than (or equal to) its submitted bid b_i multiplied by the number of requested channels d_i.

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Proof by two cases:

- i always win: charged by 0;
- Otherwise: charged by critical value, $\leq b_i$.

Theorem (Truthfulness)

VERITAS spectrum auction is truthful.

Proof.

Cases	1	2	3	4
i bids b_i	X	X	$\surd$	$\surd$
i bids v_i	X	$\surd$	X	$\surd$

Table: Four possible allocation results.

If $b_i > v_i$:

If $b_i < v_i$:

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$\square$

Summary

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- Critical Neighbor/ Value.

Thank You!

Q&A