4 Internet: Topology and Modeling

4.1 Introduction

The Internet was developed in the 1970-80s, initially for a small community of scientific researchers, which has literally become the biggest manmade infrastructure connecting about two billion users worldwide today.

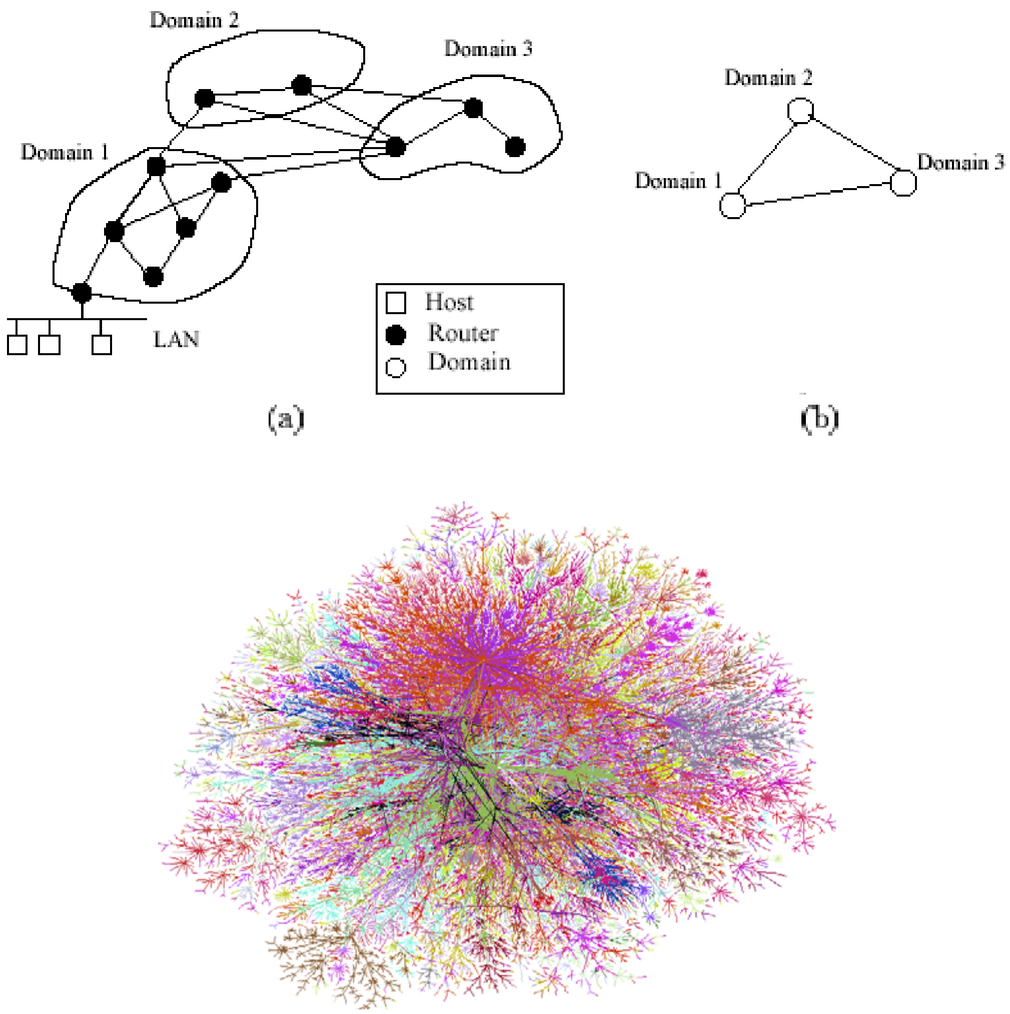
The Internet contains roughly a trillion of webpages, and transports about ten billion gigabytes of data every month in average, which was estimated to increase even to four times as much by year 2012, with applications ranging from online videos to e-commerce and to cloud-computing, not to mention the continuous transmissions of massive scientific datasets, among many others.

By its very nature, the Internet is a computer network, or more precisely a network of heterogeneous computer sub-networks, such as *Local Area Network* (LAN), *Metropolitan Area Network* (MAN), and *Wide Area Network* (WAN), which are mutually interconnected in many different ways. LANs are used to connect *hosts* (sets of computers) within a relatively small local area such as a building or a college department, and employ technologies like Ethernet and token rings. MANs and WANs, on the other hand, are used to connect hosts scattered over a regional area such as a large company or a college campus, and use optical fibers, long-distance landlines, and even wireless satellite transmission channels.

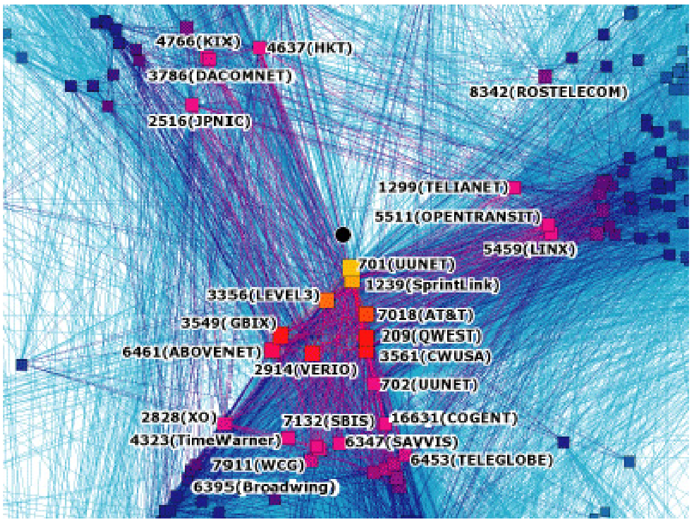
In the Internet, every Personal Computer (PC), router, host, LAN and even MAN, can be considered as a node, and every kind of connections among them, such as optical fiber, wire, or wireless channel, can be considered as an edge. The ensemble of all these nodes and edges compose a heterogeneous and self-organized huge network, the biggest artificial network of its kind in the world. On the software side, the Internet heterogeneity is reflected by its various protocols, such as Transmission Control Protocol (TCP) and Internet Protocol (IP protocol) suites consisting of a family of cooperative protocol software, and some others at the application level like Internet Control Message Protocol (ICMP), File Transfer Protocol (FTP), and Simple Mail Transfer Protocol (SMTP).

Although the Internet infrastructure operates and grows without any central management, it nevertheless has been built by many dependent or independent designers according to their engineering considerations, possibly subject to some local optimization and technical constraints. As result, the underlying structure of the entire Internet, or part of it, can be identified and described in an appropriate way. In fact, Internet modeling has gradually become a focal topic of the current scientific research. In order to predict and improve the performance of the Internet, it is very important to understand and to model its topology. For this purpose, one approach is to study the

Internet at the Autonomous Systems (AS) level, or in a much larger scale at the routers level, as illustrated by Fig. 4-1 [1]. At the routers level, each node is a router and each edge is a physical link (e.g., optical fiber); while at the AS level, each AS (also called domain) is a subnet within which the information is routed using an internal algorithm that may differ from the others used in other subnets. AS communicate each other using a specific routing algorithm—the Border Gateway Protocol (BGP). Thus, every AS approximately maps to an Internet Service Provider (ISP), which may consist of millions of routers linked together in some way.



(c) Illustration of an IP-address network at the router level



(d) Visualization of 12,979 AS nodes with 35,589 peering sessions

Fig. 4-1 Two layers of the Internet: (a) Routers level [1] (b) AS level [1]
(c) [William R. Cheswick, Lumeta Corporation, New Jersey, USA]
(d) [CAIDA Topology Mapping Analysis Team]

The Internet is a typical complex network with large numbers of nodes and of edges processing huge amounts of information data growing rapidly and continuously. To model this extremely huge and complex network, or part of it, Internet topology generators are usually used. In the past, the study of Internet topology generators has gone through three stages of development:

1. the first generation includes the random topology generators, invented in the 1980s, with the Waxman generator [2] as its representative;

2. the second generation includes the structural topology generators, developed in the 1990s, with Tiers [3] and Transit-stub [4] generators being the most typical ones that have prominent hierarchical structures;

3. the third generation has been evolving since year 2000, which are based on the node degrees, such as BRITE [5] and Inet [6], and then more recently several others based on small-world and, in particular, scale-free network models [7-12] (see also Table 1-3, Chapter1).

4.2 Topological Properties of the Internet

Real AS-level Internet data can be obtained from the website of the Oregon Router Views Project [13], which was managed by the National Laboratory for Applied Network Research (NLANR), and now managed by the Cooperative Association for Internet Data Analysis (CAIDA). This website is being updated within hours daily by taking snapshots from the routing tables of the Border Gateway Protocol (BGP). Figure 4-2 is the numbers of AS in the Internet from November 1997 to February 2002, from which one can see the trend of the AS expansion. Other useful information and data about the Internet, also at the AS level, mainly come from Skitter [15] and Whois [16] (see also [9-11]). Skitter provides the Internet topological measures by the CAIDA, using Traceroute (a computer network tool for determining the route taken by packets across an IP network), which is also continuously updated almost everyday. Whois is a domain search tool and data base, identifying the owners and IP addresses of all domains, but it is not automatically managed therefore information may not be updated timely. Most data shown in the discussions below were actually taken from Oregon rather than Skitter and Whois. It is noted that Réseaux IP Européens (RIPE) [17] provides another important BGP data source. Recently, the Internet Research Lab of UCLA merges the data from different sources such as Oregon Router Views, RIPE, Abilene, CERNET, Looking Glasses, and Route Servers, into one single overall topology [18].

4.2.1 Power-Law Node-Degree Distributions

Based on a careful analysis of the Internet statistical data from November 1997 to February 1998, for the first time in the literature the Faloutsos brothers observed some power-law types of distribution characteristics in 1999 [1] and then in 2003 using even more Internet data (from September 1997 to February 2002), both at the AS level [10]. They found that the Internet topology satisfies the following four kinds of power-law distributions:

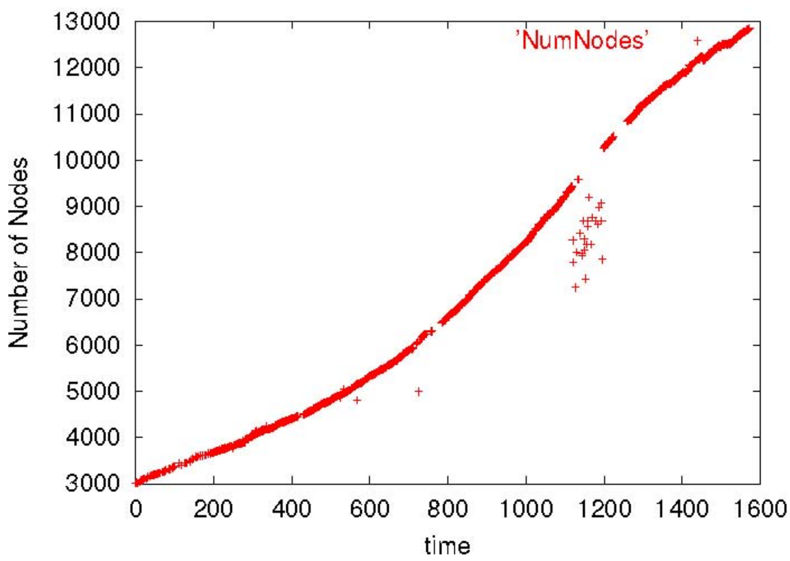


Fig. 4-2 Numbers of Internet AS (Nov. 1997 – Feb. 2002) [10]

Power law I: $d_v \sim r_v^{\,R}$, where d_v is the degree of node v, r_v is the index of node v in decreasing order of all node-degrees, and R is a rank constant exponent (which is negative as shown below).

Power law II: $D_d \sim d^D$, where D_d is the percentage of nodes with degrees larger than d, and D is a degree constant exponent satisfying $D = 1/R$ (so is negative).

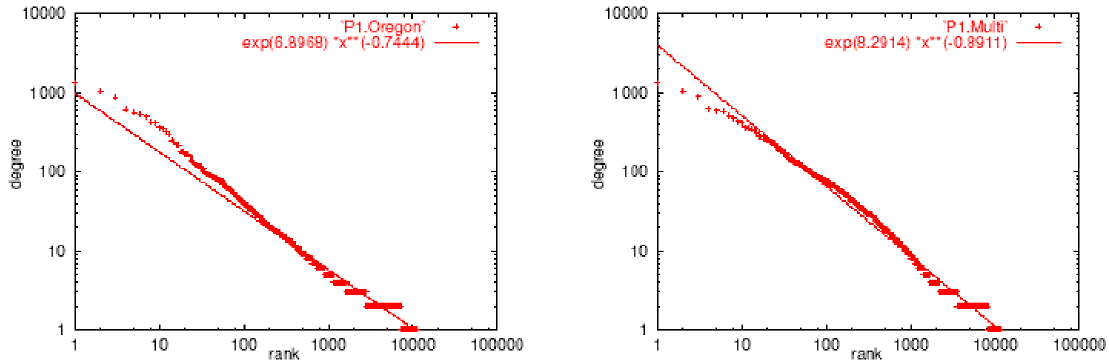
Power law III: $\lambda_i \sim i^{\varepsilon}$, where λ_i is the ith eigenvalue in decreasing order of the network connectivity matrix, and ε is the characteristic constant exponent, satisfying $\varepsilon \approx 0.5D$ (therefore is also negative).

Power law IV: $P(h) \sim h^H$ ($h << \delta$), where $P(h)$ is the number of node pairs of distance not larger than h, also referred to as the number of node pairs in the h-hop, including self-node pairs and counting twice of other node pairs, and H is a hop constant exponent satisfying

$$P(h) = \begin{cases} ch^H, & h << \delta \\ N^2, & h \geq \delta \end{cases}$$

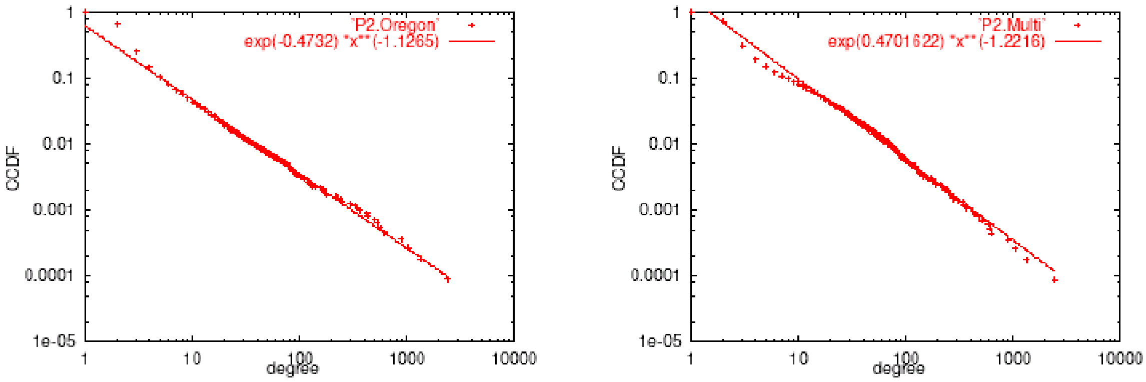
where $c = N + 2M$, N and M are the numbers of nodes and edges, respectively, and δ is the diameter of the network.

Figures 4-3 – 4-6 plot the resulting curves calculated based on the real Internet data on May 26, 2001, and Figure 4-7 displays the four power-law curves calculated based on the data from November 1997 to February 2002 [10], where (a), (b), (c) and (d) depict the evolution of the exponents R, D, ε and H, respectively. It shows that all these parameters are changing rather slowly although the size of the Internet grows very rapidly (for example, the total number of nodes within the six hops of the network increases from 3,000 on November 8, 1997 to 13,000 on February 28, 2002, as shown in Fig. 4-7 (d)).



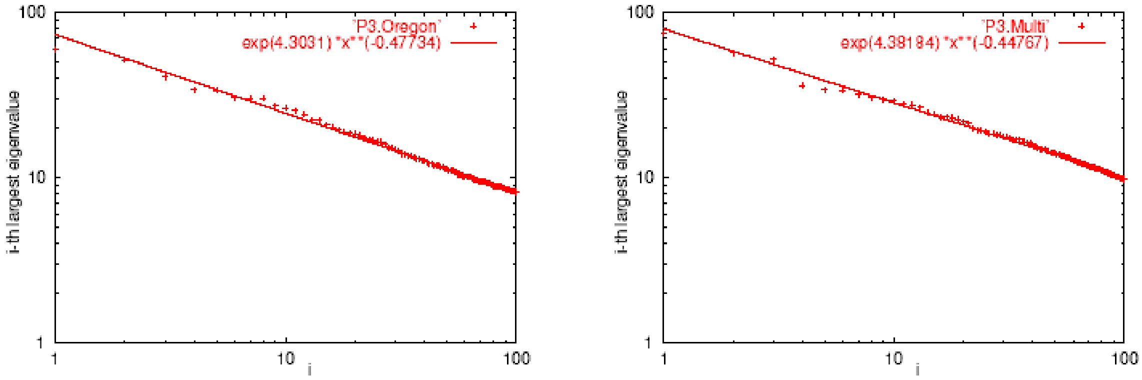
(a) Oregon (b) Multi

Fig. 4-3 The log-log distributions of (a) d_v and (b) r_v [10]



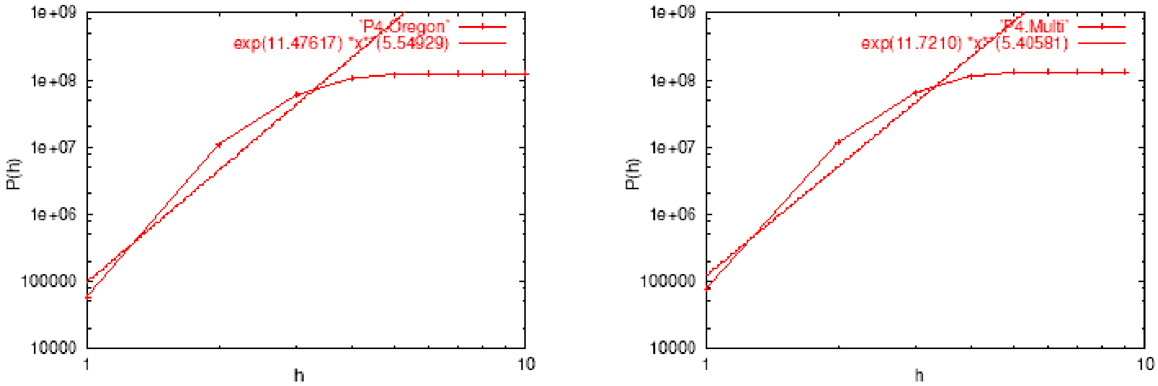
(a) Oregon (b) Multi

Fig. 4-4 The log-log distributions of (a) D_d and (b) d [10]



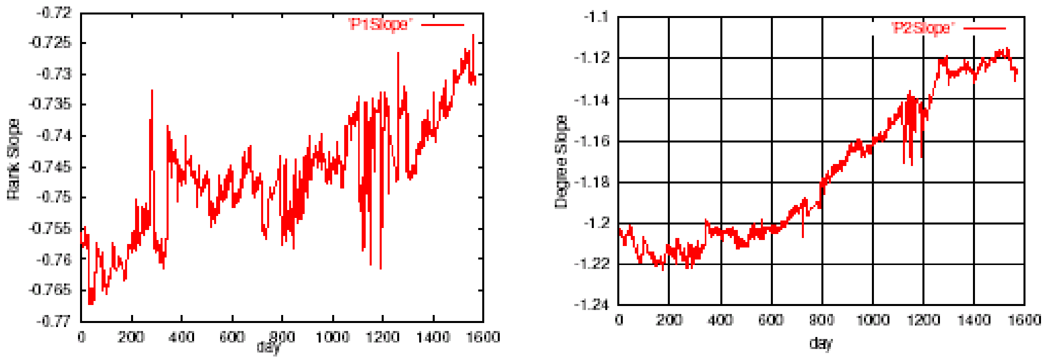
(a) Oregon (b) Multi

Fig. 4-5 The log-log distributions of (a) λ_i and (b) i [10]

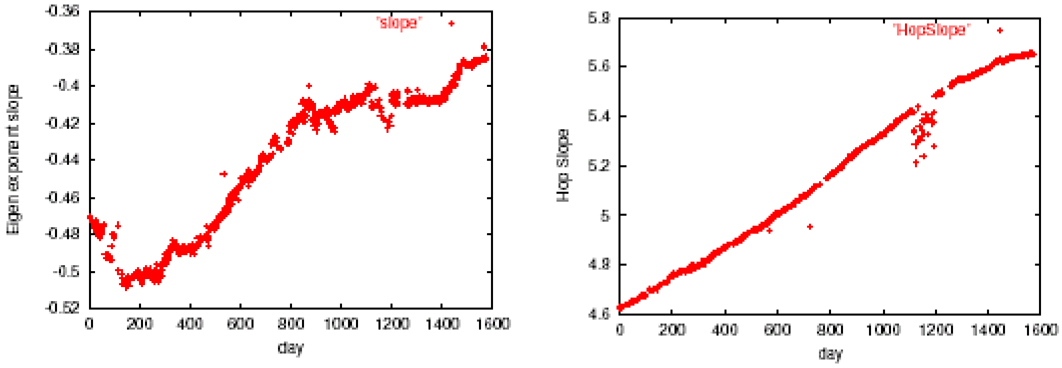


(a) Oregon (b) Multi

Fig. 4-6 The log-log distributions of (a) $P(h)$ and (b) h [10]



(a) (b)



(c) (d)

Fig. 4-7 Evolutions of the four power-law distributions [10]

4.2.2 Hierarchical Structures

The Internet consists of a large number of interconnected Autonomous Systems (AS). Each AS may be considered as a *Stub domain* or a *Transit domain*. A Transit domain can be a Metropolitan Area Network (MAN) or a Wide Area Network (WAN), typically a regional or even a national Internet Service Provider (ISP). A Stub domain usually only processes the information starting and ending inside the domain, while a Transit domain has no such restriction. In fact, a Transit domain is typically used to link many nearby Stub domains together, such that the Stub domains do not need to be linked directly. Typically, a Stub domain consists of campus networks or some other interconnected Local Area Networks (LAN), depending on the respective Transit domain or some parts of the domain, to carry out information processing and communications [19]. The structure of the Internet at the AS level is illustrated by Fig. 4-8 [12], and a simulated result is demonstrated in Fig. 4-9 [13].

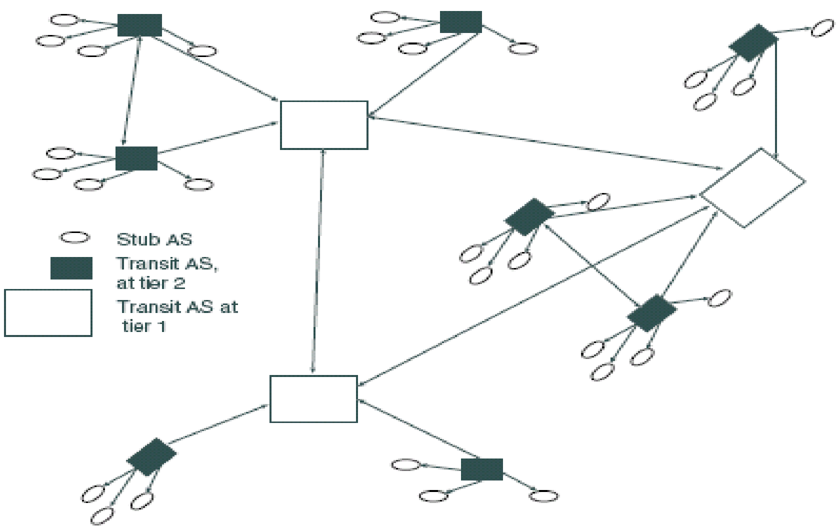


Fig. 4-8 Structure of the Internet at the AS level [12]

The concepts of Transit and Stub describes the structural inner-connections of the Internet at the AS level. A Transit as a node in the network connects one or more Stub nodes, where each path starting from or ending at a Stub node must go through those service-providing Transit nodes. With respect to other Transit domains, a Transit node can be a provider and also a customer; therefore, from such a provider-customer point of view, each AS on the Internet can be considered as some kind of *Tier*. An AS at the highest Tier belongs to the Transit domain, called Tier-1 provider. Those Transit and Stub domains at a lower Tier depend on the Transit nodes at a higher Tier to communicate with the other domains at their same level. On the other hand, a Transit domain can also communicate with other Transit domains through a certain peering relation at the same Tier, as illustrated by Fig. 4-10 [20], where the average numbers of AS in Tier-1 through Tier-4 are 614.29, 19.30, 6.93 and 4.30, respectively, as shown in Fig. 4-11 [12].

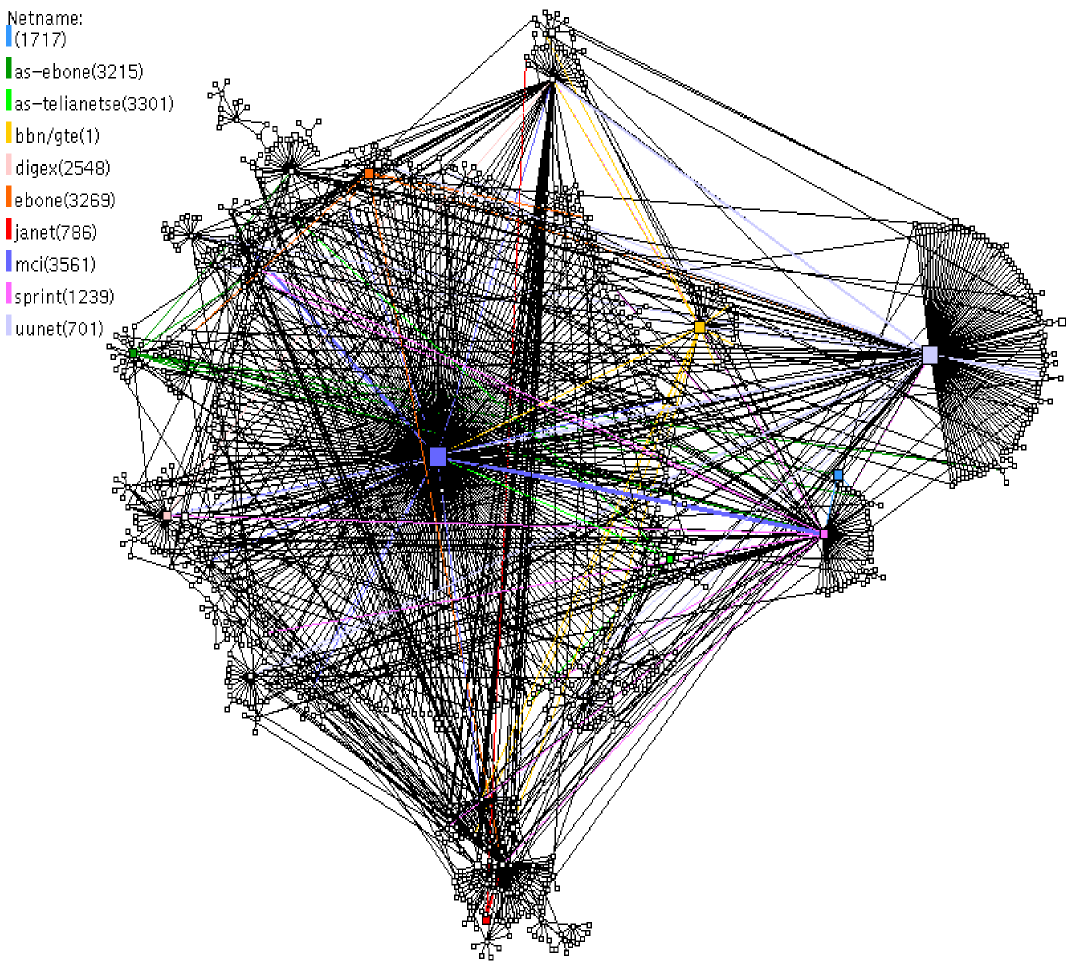


Fig. 4-9 Internet at the AS level on 3 December 1998 (generated by Skitter) [13]

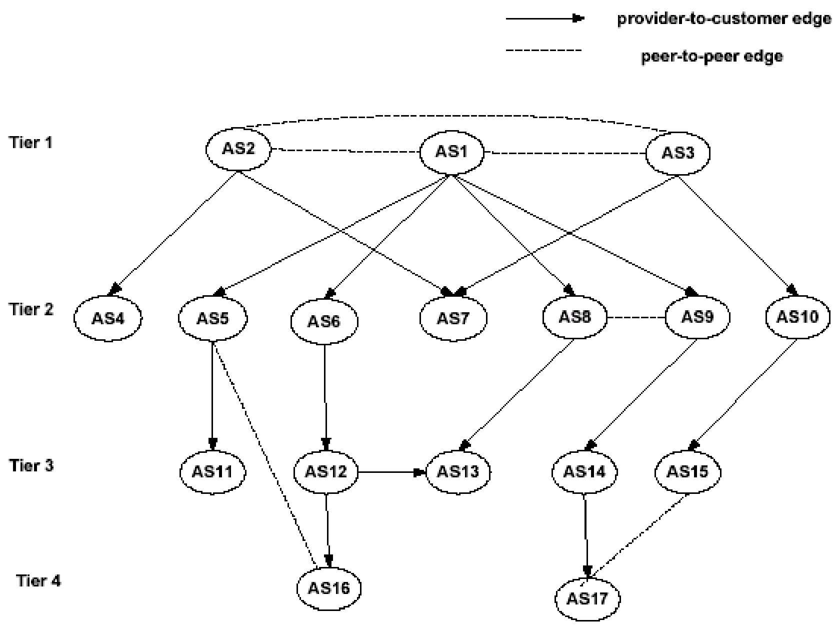


Fig. 4-10 Tier structure of the Internet [20]

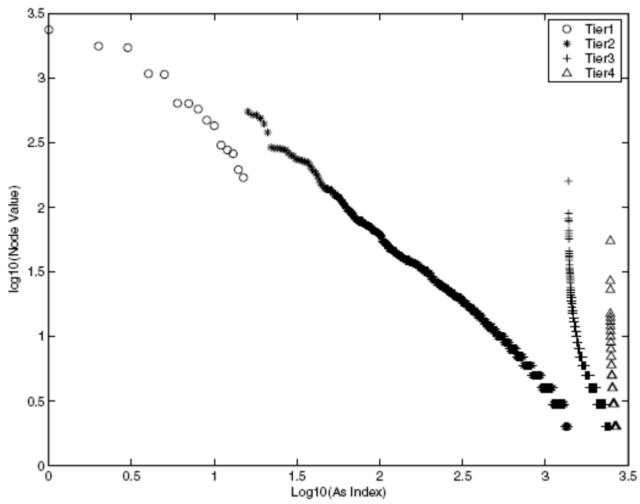


Fig. 4-11 Degree distributions of AS at different Tiers [12]

4.2.3 Rich-Club Structure

In the Internet, a few nodes have a large number of edges, called *hubs*, and they tend to connect to each other, as illustrated by Fig. 4-12, leading to a structure of so-called rich clubs [21]. A rich-club structure of the Internet at the AS level is illustrated by Fig. 4-13 [22].

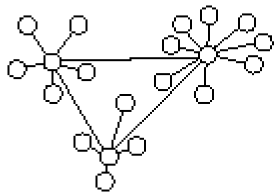


Fig. 4-12 Structure of rich club [21]

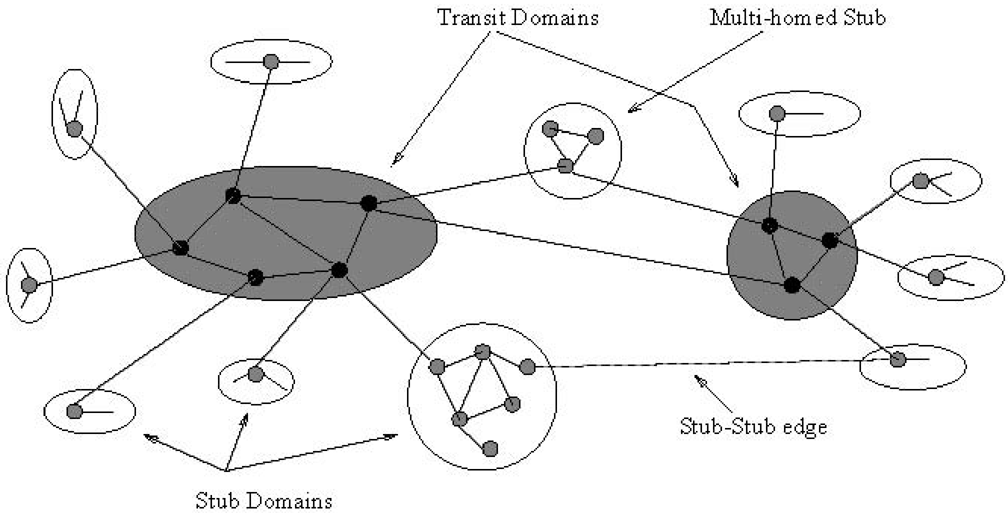


Fig. 4-13 Rich-club structure of the Internet at the AS level [22]

The rich-club phenomenon in an AS layer of size N can be described by the connectivity index $\Phi(r/N)$ of its first r biggest nodes defined by the ratio of the number M of their existing edges versus the number $r(r-1)/2$ of all possible edges among them; namely,

$$\Phi(r/N) = \frac{M}{r(r-1)/2} = \frac{2M}{r(r-1)}$$

If $\Phi(r/N)=1$, then the first r biggest nodes compose a connected sub-network, as shown by Fig. 4-14. Reportedly, some real data have verified that the connectivity index $\phi(r/N)$ follows a power-law form, $\Phi(r/N) \sim r^{-\gamma}$, with $\gamma = 1.1 \pm 0.2$ for the AS levels and $\gamma = 1.8 \pm 0.2$ for the router levels of the Internet [22].

The Internet contains many rich clubs, each has some high-degree hubs, and they are usually well connected as shown in Fig. 4-12. Moreover, as the Internet grows, more and more rich clubs are born and more and more rich clubs are being interconnected to each other, forming a truly heterogeneous network, as reflected also by the preferential attachment mechanism, which describes the familiar "rich gets richer" phenomenon of the Internet.

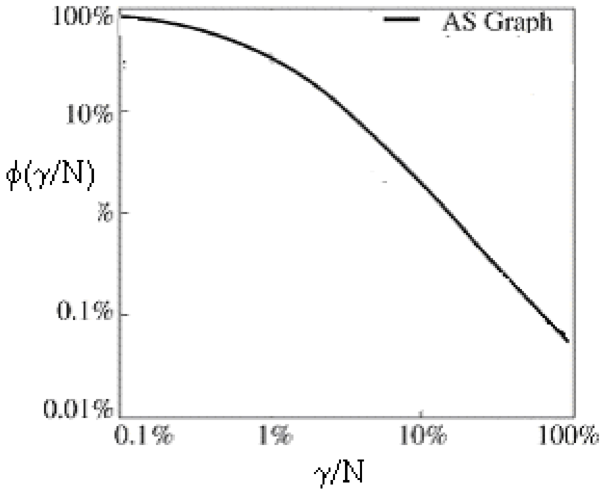


Fig. 4-14 Rich-club phenomenon of the Internet at the AS level [22]

4.2.4 Disassortative Property

As just mentioned, in the Internet hubs are well interconnected. Since hubs are big nodes of higher degrees, one is interested in figuring out what happens to the neighbors of such a hub.

It is quite phenomenal that most neighbors of a hub typically have small degrees. Analysis on the Internet data of April 2002, available at Traceroute [23], shows that nodes with degrees of 1, 2, and 3 were 26%, 38% and 14%, respectively, which sums up to about 80% of the whole network [24]. Quantitatively, Fig. 4-15 shows that the average neighboring connectivity of a node, defined by

$$k_{nn}(k) = \sum_{k'} k' P(k'|k) \tag{4-1}$$

where the conditional probability $P(k'|k)$ is the probability of nodes with degree k connects to nodes with degree k'.

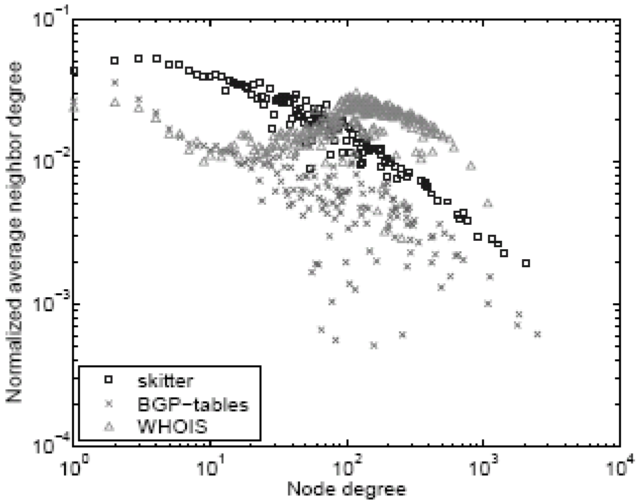


Fig. 4-15 Distribution of average neighboring connectivity [11]

The above phenomenon can be described by the so-called assortativity coefficient.

Definition 4-1 [25] The *assortativity coefficient* of a network is defined by

$$r = \frac{M^{-1}\sum_i j_i k_i - \left[M^{-1}\sum_i \frac{1}{2}(j_i + k_i) \right]^2}{M^{-1}\sum_i \frac{1}{2}(j_i^2 + k_i^2) - \left[M^{-1}\sum_i \frac{1}{2}(j_i + k_i) \right]^2} \qquad (4\text{-}2)$$

where k_i and j_i are the degrees of the end nodes of edge i, and M is the total number of edges in the network. If $r > 0$ then the network is *assortative*; if $r < 0$, then it is *disassortative*.

Geometrically, an assortative structure means hubs are mostly connected to hubs, while a disassortative structure reflects the opposite that hubs are mostly connected to small-degree nodes in a network.

Analysis of some real data from the Border Gateway Protocol (BGP), Skitter and Whois about the Internet at some AS levels shows that their assortativity coefficients on the Internet are -0.19, -0.24 and -0.04, respectively, implying that the Internet is disassortative [11].

In general, technological networks are disassortative, but social networks are assortative. This is probably due to the facts that social networks have prominent competition as well as cooperation behaviors, an interesting issue to be further studied.

4.2.5 Coreness and Betweenness

For graphs (networks), there are various measures of the *centrality* of a node within a graph, which determine the relative importance of the node in the graph; for example, it measures how important a person is within a social network, and how well a router is being used within a LAN, etc. In other words, a centrality of a node is a measure of the structural importance of the node in the network. These measures attempt to quantify the prominence of an individual node embedded in a network. Generally, a more "central" node has a stronger influence on other nodes in the same network.

Typical measures of the centrality include degree, betweenness, closeness, information, and flow-volume centralities. In network analysis and computation, the nodes may also be aggregated to obtain a group-level centralities. For example, centralization refers to the extent to which the network is concentrated on one group of nodes. For computational convenience, a network sometimes is reformulated to an equivalent one that has only one or a few nodes with considerably higher centrality values than the others in the network.

For the Internet topology, there are two particularly important centrality measures: coreness centrality and betweenness centrality.

Definition 4-2 [26,27]

1) The k*-core* in a graph is defined to be the remaining sub-graph after all the nodes with degrees $\le k-1$ have been removed successively, during which:
   (i) when a node is removed, all its adjacent edges will also be removed;
   (ii) after a node of degree $\le k-1$ is removed, in the remaining graph all the remaining nodes with a new degree $\le k-1$ also need to be removed.
2) If a node belongs to a k-core but not the $(k+1)$-core of the graph, this node is said to have *coreness* k.
3) The largest coreness in a graph is called the *coreness of the graph*.

Example 4-1 Some simple networks.

First, a single isolated node has coreness 0, and a fully connected network of size N has coreness $N-1$.

Then, consider a star-shaped network:

1) the 1-core of the network is the network itself;
2) all nodes, including the central node, have coreness 1;
3) the coreness of the network is 1.

Finally, consider a simple ring-shaped network:

1) the 1-core of the network is the network itself;
2) the 2-core of the network is the network itself;
3) all nodes have coreness 2;
4) the coreness of the network is 2.

The main purpose of introducing the concept of coreness is to reflect the fact that a higher core is more important than a lower core, and a higher-coreness node is more important than a lower-coreness node, in a network. This can reveal the hierarchical structure of a network, where higher cores and higher-coreness nodes belong to higher-levels of the hierarchical network. Clearly, the star-shaped and ring-shaped networks do not have prominent hierarchical structures.

The main implication of the concept of coreness, on the other hand, is that a network with a higher coreness will have better robustness against intentional attacks. Apparently, both star-shaped and ring-shaped networks are fragile to intentional attacks.

Example 4-2 In the network shown in Fig. 4-16, the 1-core of the network is the network itself; the 2-core of the network is Triangle 1-2-3. Moreover, Node 4 and Node 5 have coreness 1, while Nodes 1, 2 and 3 have coreness 2. The coreness of the whole network is 2.

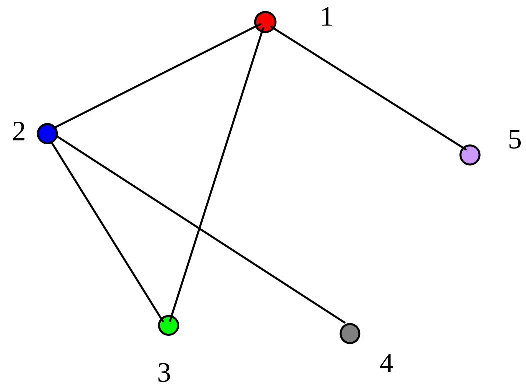


Fig. 4-16 An example for calculating coreness

Figure 4-17 shows the relations between node-degree and coreness of three sets of Internet data from Skitter, BGP and Whois: When the node-degree is relatively small, they have a power-law relation, with exponents 0.58, 0.68 and 1.07, respectively; while when the node-degree is larger than 100, their coreness values become saturated [11]. This implies that if those hub nodes continue to become bigger (with higher node degrees), it does not make them become more important (with higher coreness) in the Internet, nor make the Internet become more robust against intentional attacks.

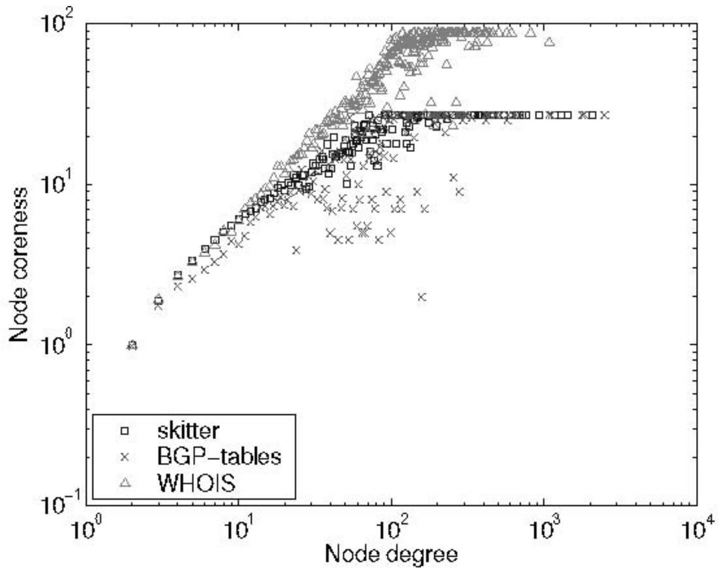


Fig. 4-17 Relations between node-degree and coreness [11]

Another important measure of the Internet topology is the betweenness centralities.

Definition 4-3 [26, 27] In a network of size n, the *node-betweenness* of node i is defined by

$$B(i) = \sum_{j \neq l \neq i} \frac{L_{jl}(i)}{L_{jl}} \tag{4-3}$$

where L_{jl} is the number of all existing shortest paths from node j to node l, and $L_{jl}(i)$ is the number of all shortest paths from node j to node l that actually pass through node i. The node-betweenness may be normalised by dividing with the total number of pairs of nodes not including node i, which is $(N-1)(N-2)/2$.

For an edge e_{ij} connecting node i and node j, the *edge-betweenness* of e_{ij} is similarly defined:

$$B(e_{ij}) = \sum_{(l,q) \neq (i,j)} \frac{\tilde{L}_{lq}(e_{ij})}{\tilde{L}_{lq}} \tag{4-4}$$

where $\tilde{L}_{lq}$ is the number of all existing shortest paths from node l to node q, and $\tilde{L}_{lq}(e_{ij})$ is the number of all shortest paths from node l to node q that actually pass through edge e_{ij}. The edge-betweenness can also be normalized, by dividing with the total number of edges not including e_{ij}, which is $\frac{1}{2}N(N-1)-1$.

Example 4-4 Consider the network shown in Fig. 4-18.

The betweenness of Node 1 is:

$$B(1) = \frac{(5,1,4)}{(5,1,4)} + \frac{(5,1,2)}{(5,1,2)+(5,3,2)} + \frac{(4,1,2,3)+(4,1,5,3)}{(4,1,2,3)+(4,1,5,3)} + \frac{(4,1,2)}{(4,1,2)}$$
$$= 1 + \frac{1}{2} + 1 + 1 = \frac{7}{2}$$

or $B(1) = \dfrac{7/2}{(N-1)(N-2)/2} = \dfrac{7}{12}$ after normalization, where (i,l,q,j) is the path from Node i to Node j passing though Node l and Node q successively.

The betweenness of Edge e_{12} is:

$$B(e_{12}) = \frac{(5,1,2)}{(5,1,2)+(5,3,2)} + \frac{(4,1,2,3)}{(4,1,2,3)+(4,1,5,3)} + \frac{(4,1,2)}{(4,1,2)} + \frac{(3,2,1)}{(3,2,1)+(3,5,1)} + \frac{(2,1)}{(2,1)}$$
$$= \frac{1}{2} + \frac{1}{2} + 1 + \frac{1}{2} + 1 = \frac{7}{2}$$

or $B(e_{12}) = \frac{7/2}{N(N-1)/2-1} = \frac{7}{18}$ after normalization.

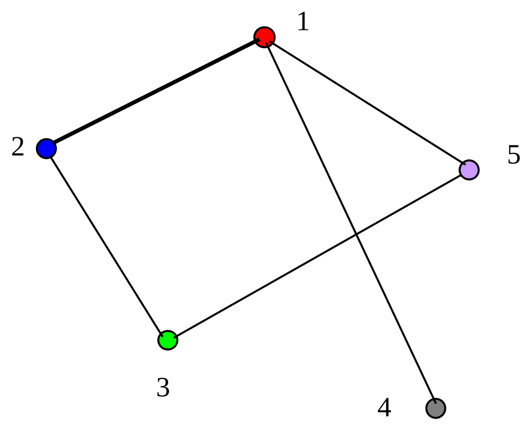


Fig. 4-18 An example for calculating betweenness

Figure 4-19 shows the relations between node-degree and betweenness of three sets of Internet data from Skitter, BGP and Whois: For those from Skitter and BGP, their relations follow prominent power-law distributions with components 1.35 and 1.17, respectively [11]. It shows that larger nodes have larger node-betweenness in general.

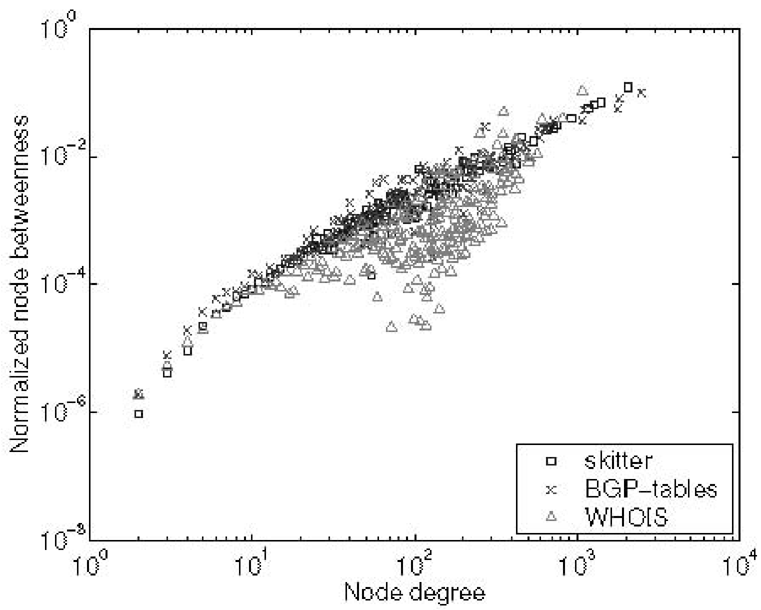


Fig. 4-19 Relations between node-degree and normalized betweenness [11]

The importance of the node- and edge-betweenness centralities can be easily understood from Fig. 4-20, where the bridging node has very small degree yet very large node- and edge-betweeness, while generally big nodes have large betweenness.

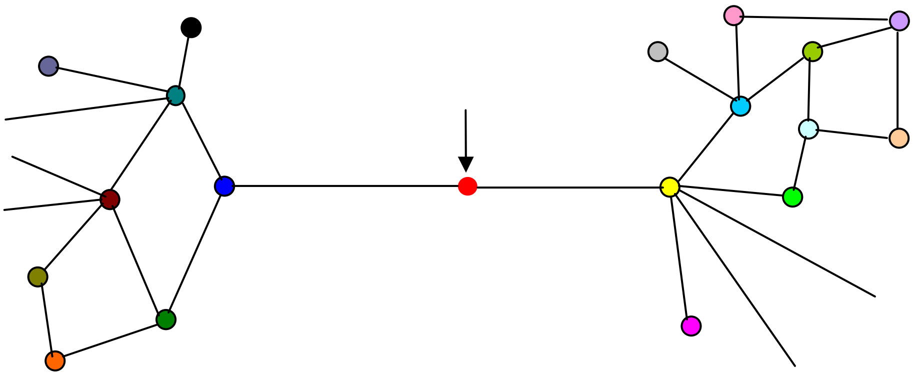


Fig. 4-20 The importance of a node with a large betweenness value

Definition 4-4 [29] The *information centrality* of a graph G of size N is defined to be the mean information flow rate over the network:

$$E(G) = \frac{\sum_{i \neq j \in G} \varepsilon_{ij}}{N(N-1)} = \frac{1}{N(N-1)} \sum_{i \neq j \in G} \frac{1}{d_{ij}} \tag{4-5}$$

where the *network efficiency* ε_{ij} of channel information transmission between node i and node j is defined to be inversely proportional to their shortest distance d_{ij}: $\varepsilon_{ij} = 1/d_{ij}$. If there are no edges in the graph, then $d_{ij} = \infty$, so $\varepsilon_{ij} = 0$.

The concept of information centrality is very useful in, for example, finding the community structure of a network [30].

4.2.6 Growth of the Internet

As seen above, there are some fundamental characteristics that are never changed, or basically do not change, during the evolution of the Internet, such as relatively small average path length, relatively large clustering coefficient, and power-law distribution of node degrees, and so on, in the AS level or router level of the Internet, or some regional and local portions of it. Of most interest is the fact that despite the above-mentioned inherent properties, the Internet is actually a dynamically evolving complex network, which is rapidly and continuously growing and restructuring.

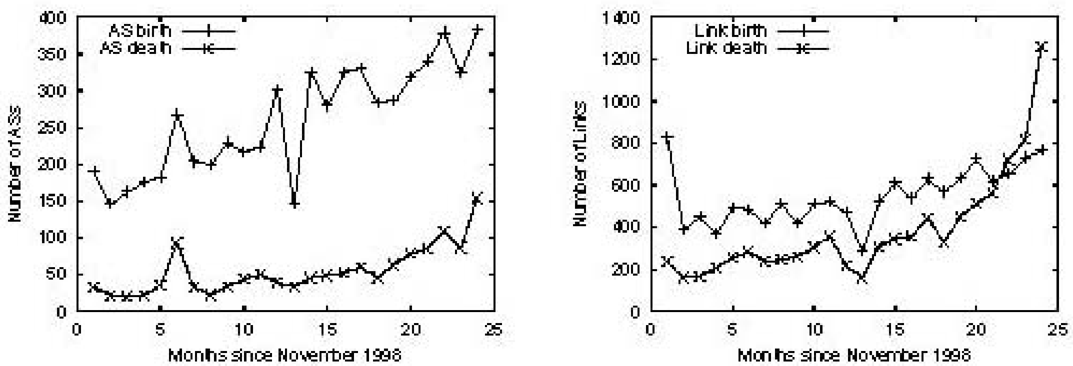
In this subsection, some historical data of the Internet will be examined, which were provided by the SCAN Project with a software named Mercator for the duration of October-November 1999 [31], by the Oregon router server (at the AS level), and by the Topology Project of the Computer Science Department of the Michigan University (at the Extended AS level), where the data obtained by the latter two projects were both from the same day of 26 May 2001 [32].

Table 4-1 shows the average measures of the Internet at the Internet Router (IR), Autonomous System (AS), and Extended Autonomous System (EAS) levels [19], respectively, where N is the number of nodes, E is the number of edges, $\langle k \rangle$ is the average node degree, C is the average clustering coefficient, L is the average shortest path lengths, and B is the average node-betweenness, of the network. It can be seen that $\langle k \rangle$ is small, implying that there are many nodes of small degrees in the network, and that L is small but C is large (relative to the ER random-graph model which typically has $C \approx 0.0001$), implying the small-world features of the Internet.

Table 4-1 Some average measures of the Internet at the IR, AS, and EAS levels [19]

Level	N	E	$\langle k \rangle$	C	L	B
IR	228,263	320,149	2.8	0.03	9.5	5.3
AS	11,174	23,409	4.2	0.30	3.6	2.3
EAS	11,461	32,730	5.7	0.35	3.6	2.3

The real data collected from Oregon router server, the Looking Glass site, and the Internet Routing Registry (IRR), reveal that the Internet continuously has addition (birth) and deletion (death) of AS and their peering relations, as shown in Fig. 4-21 for the duration from November 1998 to November 2000 [32]. It can be seen that the birth rates of nodes and edges are both larger than their death rates, at least during this period of time.



(a) Nodes (b) Edges

Fig. 4-21 Monthly numbers of birth and death of the Internet at the AS level [32]

It is also interesting to observe from Table 4-2 [32] that during the same period of time, small nodes (e.g., those connects to only one or two AS) have very high probabilities of changes (more additions and also more deletions) while giant nodes (e.g., those connects to more than five AS) are quite robust with no or very few changes. It is likely due to the fact that small nodes represent small companies (service providers), which are frequently born and connected to giant companies but they are also frequently die out of business.

Table 4-2 Total numbers of additions and deletions of the Internet at the AS level [32]

Degree	Number of Additions	Number of Deletions
1	5,591	1,184
2	816	204
3	23	22
4	4	6
5	1	4
6	1	1
7	1	1
9	0	1
10	1	0
11	1	0
12	0	1
14	1	0
48	0	1

On the other hand, Table 4-3 shows $E_{n,o}$, the numbers of new edge additions between new (incoming) nodes and old (existing) nodes, and $E_{o,o}$, the numbers of new edge additions between two old nodes, again at the AS level. It indicates that the Internet growth is significantly driven by the increase of wiring numbers (and bandwidths) among nodes, new or old, to meet the demand of continuously increasing data transmission.

Table 4-3 Monthly rate of new edges connecting old nodes to new and old ones [33]

Year	1998	1999
$E_{n,o}$	170	231
$E_{o,o}$	350	450
$E_{n,o}/E_{o,o}$	0.48	0.53

Another observation is that there is a prominent rewiring phenomenon within the Internet [32], meaning that there are some AS each shifts its connections from one existing AS to another existing one, from time to time during the evolution. This kind of rewiring not only change the figure of the existing edge addition $E_{o,o}$, but also implies the deletion (death) of some existing edges which, however, is not reflected by Table 4-3.

4.2.7 Router-Level Internet Topology

A common tool to represent the router-level Internet topology by a graph is the *traceroute* (Unix traceroute or Windows NT tracert.exe), or its IPv6 version, traceroute6 [34]. The traceroute uses hop-limited probe, which consists of a hop-limited IP (Internet Protocol) packet and the corresponding ICMP (Internet Control Message Protocol) response, to probe every possible IP address and record every reached router and the corresponding edges.

An earlier attempt in 1995 [35] was to use *traceroute* to trace 5,000 hosts, selected from a network accounting database. After the 5,000 destinations were selected, 11 of them were used as the new sources of routes to trace the remaining destinations. This eventually produced a graph of 3,888 nodes and 4,857 edges, excluding those routers that could not be traced due to transient routing or other technical problems. The analytical results show that more than 70% of the nodes have degree 1 or 2, and they do not belong to the core. The major limitation of this method is that it heavily depends on the choice of the destinations, namely, it needs to choose a certain number of destinations representing a subset of the Internet structure, to obtain the routing information before probing.

An intelligent heuristic technique was then introduced [36] to overcome this drawback, by using heuristic to decide whether the network includes a single node. This technique does not require an initial database of targets for exploring the network topology. Based on some careful analysis of the collected data, consisting of nearly 150,000 nodes (routers and interfaces) and almost 200,000 edges, it was found [37] that the degree distribution of nodes with degree less than 30 follows a power-law form. However, the distribution of nodes with degree larger than 30 turns out to be significantly different: it has a faster cut-off other than a power-law distribution, indicating that there may be another law governing the distribution of higher-degree nodes in the network. Moreover, it was found that the distribution of the numbers of node-pairs within a certain number of hops in the network follows neither exponential [38] nor power-law form. Some analysis on the real data collected during October and November of 1999 shows that the hierarchical characteristic basically does not exist in the router-level of the Internet topology [39], where the node-degree distribution has a power-law behavior which however is smoothed out by a clear exponential cut-off. Therefore, the Weibull distribution, instead of the power-law distribution, can better fit the collected data, agreeing with the result reported in [37]. However, this approach could not give a complete map of the Internet topology since it fails to represent the details of the Stub subnets although it can capture the topology of the Transit portion of the Internet. It is therefore suggested that probing from a large number of sources may be able to improve the performance regarding the completeness of the traceroute-style probes [40].

Recently, Border Gateway Protocol (BGP) routing tables were examined to determine the destinations of a traceroute [41]. A directed probing technique was used to interpret BGP tables thereby identifying relevant traceroutes and pruning the remainders [41]. A path reduction technique can also be used to identify redundant traceroutes, so as to generate a router-level Internet topology. An advantage of using these two techniques is that it can significantly reduce the number of required traces without sacrificing the accuracy. Actually, compared to the brute-force all-to-all approach, this method of combining the directed probing and the path-reduction techniques can reduce the number of required traces significantly by three orders in magnitude. Some analytical results on the real data collected during December 2001 to January 2002 show that the Weibull distribution can better fit the complementary cumulative distribution function of router out-degree than the Pareto (power-law) distribution [41].

In general, however, because most Internet Service Providers regard their router-level topologies as confidential, and there exist some technical problems such as multiple interfaces and hence multiple IP addresses for a single router, it is still a challenging task to build a relatively complete router-level Internet topology today.

4.2.8 Geographic Layout of the Internet

Due to the lack of topological information about the Internet with geographic layout of AS and routers, very little work has been done to explore the geometry of the Internet infrastructure to date.

One earlier work on this issue [42] used the NetGeo tool, developed by CAIDA [43], to identify the geographic coordinates of 228,265 routers of the *Mercator* map, aiming at investigating the fundamental driving forces that shape the Internet's evolution. The obtained Internet topology, embedded with geographic information of routers, allows one to analyze the physical layout of the Internet infrastructure. It is found that routers form a fractal set with fractal dimension $D_f = 1.5 \pm 0.1$, and that they strongly correlate with the population density around the world, as illustrated by Fig. 4-22, where (a) is the router distribution density of the geographic locations of 228,265 routers of the *Mercator* map, and (b) is the population density distribution calculated based on the CIESIN population data [44].

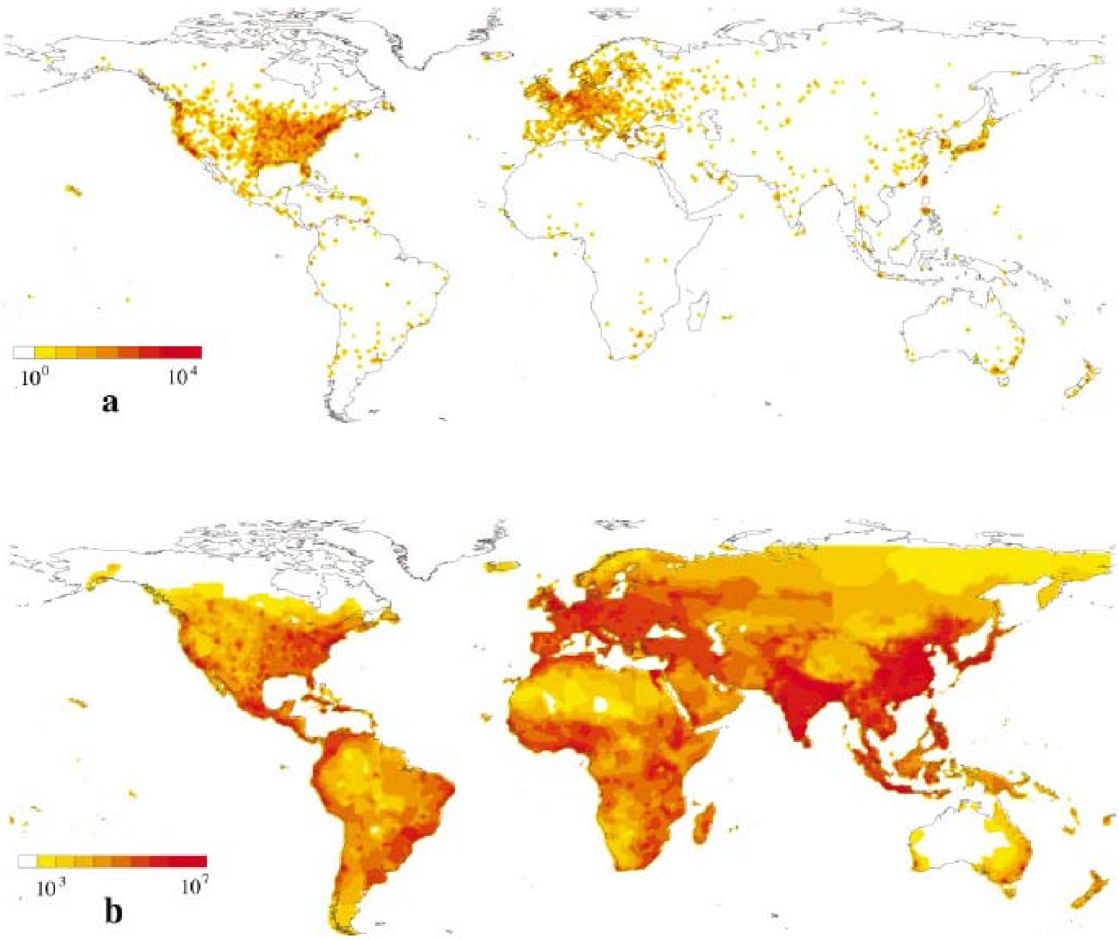


Fig. 4-22 (a) Router map density; (b) Population density map [42]

Two recent Internet maps are the *Mercator* map collected during August 1999, which consists of 268,382 routers and 320,149 edges, and the *Skitter* map collected during 26 December 2001 to 1 January 2002, which contains 704,107 routers and 1,075,454 edges [45]. The geographic information of routers in these two maps was obtained by using two geographic mapping tools: IxMapper [46] and EdgeScape [47]. Some analysis on these two topologies, with geographic coordinates of routers, indicates that there is a correlation between the router interface and the population density, as shown in Table 4.4 [46] for various geographic areas of the world, including both developed and developing regions.

It can be seen from Table 4.4 that penetration of Internet infrastructure varies dramatically over different areas, indicating that the interface density is strongly correlated with the population density. This co-incidence between the router interface and the population density is not surprising: higher population density in a wealthier region implies higher demands for Internet services, resulting in higher densities of routers and interfaces. More precisely, the correlation between the router or interface density R and the population density P are correlated as $R \sim P^{\alpha}$ with exponent $1.2 \le \alpha \le 1.7$, depending on the specific region of concern.

Table 4.4 Correlation between router interfaces and human population [46]

	Population (Millions)	Interface	People per interface
Australia	18	18,277	975
Japan	136	37,649	3,631
Mexico	154	4,361	35,534
USA	299	282,048	1,061
South America	341	10,131	33,752
W. Europe	366	95,993	3,817
Africa	837	8,379	100,011

By examining a BGP table that matches IP addresses to their corresponding AS numbers, one can label routers or interfaces with the AS number to which they belong. In this way, geographic information of the AS-level Internet infrastructure can be obtained. It is found that the number of distinct locations spanned by an AS is strongly correlated with the degree of the AS [46]. For a small AS, these locations show a wide variability in the geographic dispersal; however, for a large AS whose size exceeds a certain threshold, these locations are rapidly dispersed geographically.

Based on all the above observations and discussions, the geography-based BA model proposed in [42], referred to as the GeoBA model, can be summarized as follows:

1. Consider a network on a plane of linear size L, and divide it into squares of size $l \times l$ with $l << L$.
2. Assign to each square a population density $\rho(x, y)$ with a fractal dimension, $D_f = 1.5$.
3. At each step, place a new node i into the network in such a way that the probability of the node being placed at position (x, y) is linearly proportional to $\rho(x, y)$. This new node will bring in m new edges. The probability of a new edge connecting to an existing node j of degree k_j at geographical distance d_{ij} from node i is

$$\Pi(k_j, d_{ij}) \sim \frac{k_j^{\beta}}{d_{ij}^{\sigma}} \tag{4-6}$$

where β and σ are constant parameters.

Note that β and σ govern the preferential attachment and the penalty of the node-node distance, in the sense that increasing β favors the nodes with higher degrees and larger σ values discourage long-range connections. In comparison, L, l and m are less important parameters in the model.

4.3 Random-Graph Network Topology Generator

In the 1980s, the Internet started to be developed initially with a relatively small size. In 1988, Waxman proposed a simple model of the Internet [2], which was lately found to represent the Advanced Research Project Agency resource sharing computer network (ARPAnet) quite well.

The Waxman model is generated as follows:

1. Start with N nodes on a finite lattice.

2. At every step, one edge is being added to two randomly picked nodes, u and v, according to the following so-called *Waxman probability*:

$$\Pi(u,v) = \alpha\, e^{-d(u,v)/(\beta L_{\max})} \tag{4-7}$$

   where $0 < \alpha, \beta \leq 1$, $d(u,v)$ is the Euclidean distance between node u and node v, α is the average number of edges (after normalization), $L_{\max}$ is the diameter of the network, and β is a parameter determined by the average path length.

It is clear from formula (4-7) that if α is increasing, then the growth probability of edges, $P(u,v)$, is also increasing; if β is increasing, then in the end relatively long edges become more than relatively short edges in the resulting network. Figure 4-23 is a typical Waxman network, and Fig. 4-24 shows its degree distribution, where $N(k)$ (vertical axis) is the number of nodes of degree k (horizontal axis).

The Waxman network is a typical random graph, which had influence on some other random-graph-like models afterwards (for example, [48]). It can be seen that the probability of having an edge between two distant nodes is generally quite small, so that the probability of having a resulting connected graph is also small. Therefore, one usually only studies the biggest connected sub-graph of the Waxman network.

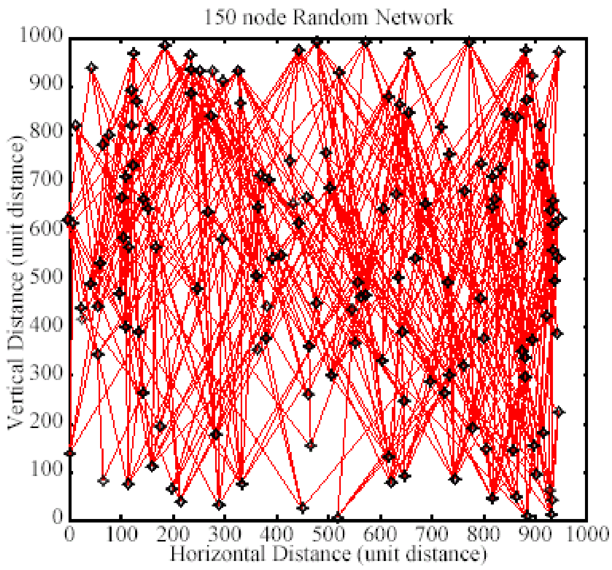


Fig. 4-23 A typical Waxman network of 150 nodes, with $\alpha = 0.25, \beta = 0.3$ [2]

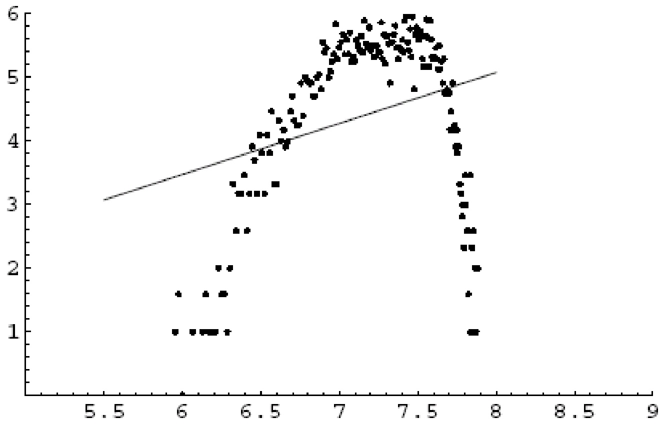


Fig. 4-24 Distribution of the Waxman network: $N(k)$ versus k (log-log plot) [49]

4.4 Structural Network Topology Generators

As the Internet becomes larger and more complex, and as the understanding of its topological features becomes deeper and more comprehensive, it has been realized that the Internet indeed is not a random graph in the classical sense but has prominent hierarchical structures (as discussed in Section 4.2.2 above) and many other features. The first generation of Internet topology generators in the mid-1990s was based on the belief that the hierarchical structure is the main characteristic of the Internet topology. Two representative hierarchical Internet models, the Tiers topology generator and the Transit-Stub topology generator, are briefly introduced here.

4.4.1 Tiers Topology Generator

This kind of topology generator is used to represent Wide Area Networks (WANs), Metropolitan Area Networks (MANs) and Local Area Networks (LANs), as well as their connectivities [3].

To generate a network topology by Tiers, it is required to pre-assign the numbers of MANs and LANs, where all LANs have to be in star-shape. The main model parameters are:

N_W – number of WANs (usually, set $N_W = 1$ for simplicity)

S_W – number of nodes in a WAN

N_M – number of MANs ($N_M \le S_W$, since every MAN connects to a node in a WAN)

S_M – number of nodes in a MAN

N_L – number of LANs ($N_L \le S_M$, since there is one MAN node for each LAN)

S_L – number of nodes in a LAN

Total number of nodes is $N = S_W + N_M S_M + N_M N_L S_L$ (a typical example for a corporate Internet has $N_W = 5$, $N_M = 10$ with $S_M = 10$, and $N_L = 5$ with $S_L = 50$, giving $N = 2{,}605$).

The degree of intranetwork redundancy for WAN, MAN and LAN, defined as the degree (i.e., number of directed edges) of connectivity from one node to another node of the same type (e.g., from LAN to LAN), is denoted by R_W , R_M and R_L , which are typically equal to 3, 2 and 1, respectively. The degree of intranetwork redundancy between a MAN and a WAN, or between a MAN and a LAN, is similarly defined, and is denoted by R_{MW} and R_{LW} , respectively.

The Tiers topology generator works as follows:

1) Generate one WAN:

    1. Randomly put some nodes on a finite-sized lattice; if the new one happens to be too close to an existing one then simply reject (ignore) the new one.

    2. After putting in all pre-assigned nodes on the lattice, connect them as a spanning tree (see Section 2.5, Chapter 2).

3. Check all nodes, in random order, to make sure that they all satisfy the redundancy $R_W = 3$ (if a node has more than R_W edges to its peer nodes then do nothing, but if it has less then add edges to its nearest nodes in the network, in increasing order of Euclidean distance).

2) Generate MANs:

The procedure is similar to that for WAN above, but now in a smaller scale, with the exception of not rejecting the new MAN when is too close to an existing one (since nodes in a MAN and in a LAN are much closer than those in a WAN), where typically $R_M = 2$, yielding a total of N_M MANs.

4) Generate LANs:

Randomly select a node in a LAN as the center of a star-shaped network, and then connect it to every other nodes of the LAN by a single edge, where usually $R_L = 1$ (if $R_L > 1$ then do nothing), which yields a total of N_L LANs.

4) Connect all the above-generated WAN, MANs and LANs together:

1. Connect MANs to the WAN: Randomly select a group, denoted by A, of N_M nodes from among S_W nodes in the generated WAN; then, connect one node in A to one randomly selected node X in each MAN (thus, each MAN is connected to the WAN via one edge).

2. If $R_{MW} > 1$ then connect one node in each MAN (which can be X again) to a node in A, which is nearest to the node that was already connected to X.

3. Similarly to the above procedure, connect all the LANs to the MANs, where the node X to be chosen from a LAN is always the center of the star-shaped network.

An illustrative example of Tiers topology so generated is shown in Fig. 4-25 [3].

4.4.2 Transit-Stub Topology Generator

The network structure generated by the Transit-Stub topology generator consists of three layers: the first one generated is the top layer, the Transit domain, followed by the Stub domain, and then by the LANs. The nodes on each layer are restricted on a rectangular region, where each layer has a different rectangle and the smallest one is for the LANs, as illustrated by Fig. 4-26 [4].

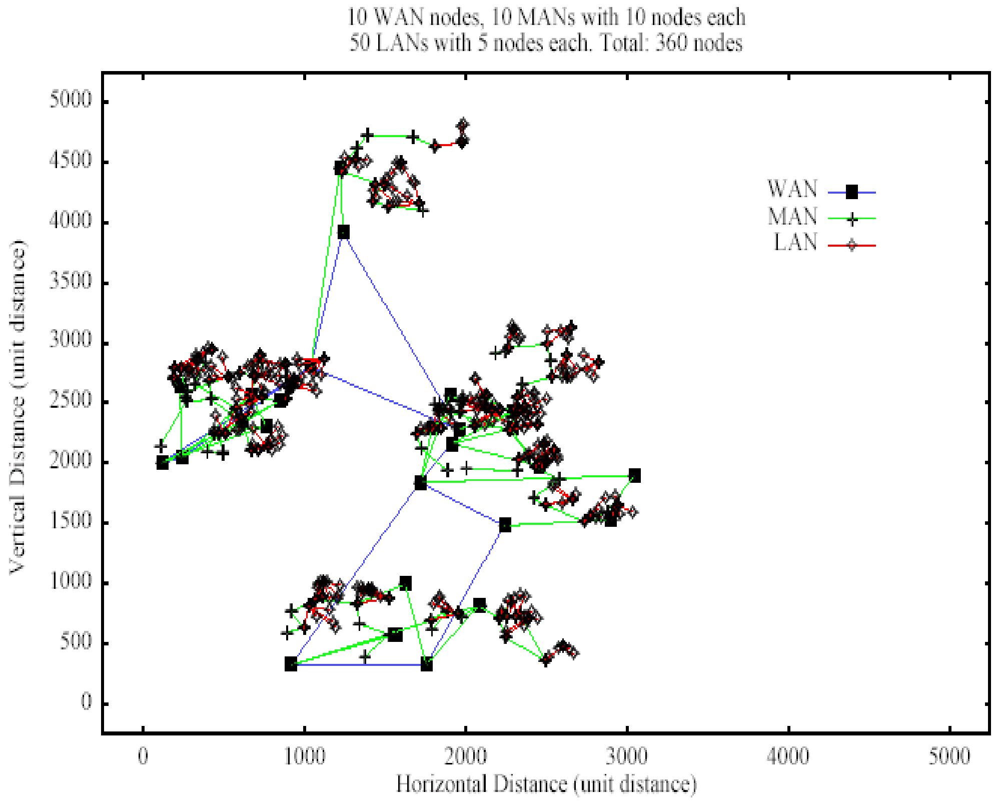


Fig. 4-25 A typical Tiers topology [3]

There are two groups of parameters that control the graph characteristics:

First group: This group of parameters controls the relative sizes of the domains.

T – number of Transit domains ($T \geq 1$)
N_T – average number of nodes in each Transit domain ($N_T \geq 1$)
S – average number of Stub domains in each Transit domain ($S \geq 1$)
N_S – average number of nodes in each Stub domain ($N_S \geq 1$)
L – average number of LANs in each Stub node ($L \geq 0$)
N_L – average number of hosts in each LAN ($N_L \geq 1$)
N_R – number of routers, satisfying $N_R = TN_T(1 + SN_S)$
N_H – number of hosts, satisfying $N_H = TN_T SN_S LN_L$

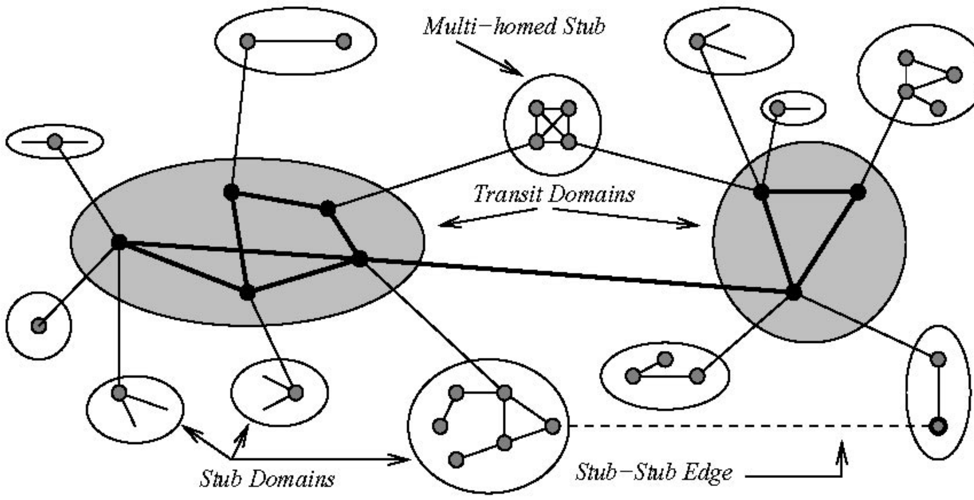


Fig. 4-26 Illustration of network structure from Transit-Stub topology generator [4]

Note that LANs are modeled as star-shaped networks with a router node at the center of each star and the host nodes each connected to the center router. This significantly reduces the number of edges in the generated network, and reflects the lack of physical redundancy in most LANs.

Second group: This group of parameters controls the connectivity of the domains.

E_T – average number of edges in each Transit domain ($E_T \geq 2$)
E_S – average number of edges in each Stub domain ($E_S \geq 2$)
E_{TT} – average number of edges in between Transit domains ($E_{TT} \geq 2$)
E_{ST} – average number of edges in between a Transit and a Stub domain ($E_{ST} \geq 1$)
E_{LS} – average number of edges in between a LAN and a Stub domain ($E_{LS} \geq 1$)

Note that E_T must be large enough so that the Transit domain is connected; E_S must be large enough so that the Stub domain is connected; E_{TT} must be large enough so that the Transit-Transit domains are connected; E_{ST} must be so large that every Stub connects to at least one Transit domain; finally, E_{LS} must be so large that every LAN connects to at least one Stub domain.

The Transit-Stub topology generator works as follows:

1. Generate all Transit domains within a desired region. To do so, any random-graph generation method may be used (usually, the Waxman algorithm is used). The resulting network must be connected, where each node represents a Transit domain.

2. Generate nodes in each Transit domain. To do so, place in N_T nodes around the Transit point, and then connect them with edges, where $N_T \geq 2$, $E_T \geq 2$.

3. Randomly select one node in each Transit domain and connect it to another Transit domain by one edge.

4. Generate Stubs for each Transit domain. To do so, select suitable locations for Stub domains, then generate N_S Stub domains in these locations, and finally connect them with edges.

5. Connect every Stub domain to a Transit domain. If $E_{ST} > 1$ then randomly select one node from a Stub domain and then connect this node to a Transit domain by an edge.

6. Generate LANs, all with star-shaped structures.

7. Connect every LAN to a Stub domain. If $E_{LN} \geq 1$ then connect the center of the LAN to a Stub domain.

Note that in the above modeling, typically Steps 1-5 are carried out while Steps 6-7 may be ignored.

An illustrative example of the out-degree (number of out-reaching edges) distribution of the resulting Transit-Stub network with 6660 nodes is shown in Fig. 4-27 [49].

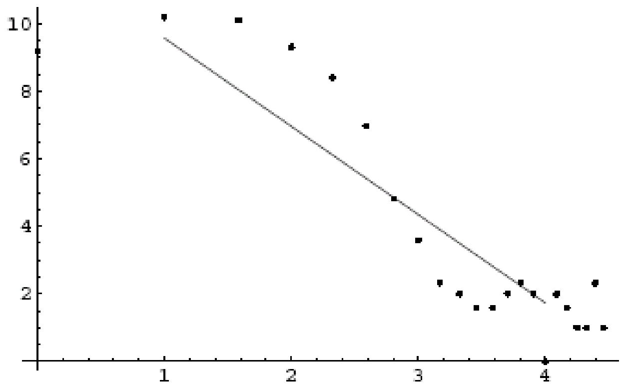


Fig. 4-27 Out-degree distribution of a Transit-Stub network (log-log plot) [49]

4.5 Connectivity-Based Network Topology Generators

In Section 4.4, two typical examples (Tiers and Transit-Stub models) of the first generation of Internet topology generators were introduced, which were developed based on the hierarchical structures of the Internet. Since the publication of the seminal paper [1], the main interest and emphasis have been gradually changed to the connectivity characteristics of the Internet. As a result, several connectivity-based Internet topology generators have been developed lately, which are briefly introduced here in this section.

4.5.1 Inet

To reflect the power-law out-degree distribution of the Internet, coined in [1], researchers in the University of Michgan proposed a connectivity-based Internet topology generatior, Inet 1.0, in year 2000. This model was then upgraded to Inet 2.0 and then Inet 3.0 [6]. The new version 3.0 works as follows, where all node degrees are out-degrees [50]:

1. From the real data set, find the ratio ρ of dgree-1 nodes over the total number of all nodes, which typically remains about $\rho \approx 30\%$.

2. Compute the number of months, t, over which the number of nodes of the Internet at the AS level has been increased from 3037 to N by using the following empirical formula:

$$N = \exp(0.0298 * t + 7.9842) \qquad (4\text{-}8)$$

   where according to the Oregon data [13] the number of AS nodes of the Internet in November 1997 was 3037 [50,51] (as shown also in Fig. 4-2). Clearly, there are a total of ρN nodes in V_1.

3. Let V_1 and V' be the set of degree-1 nodes and the set of the rest nodes, respectively, in the network. Use the t value obtained above in the following empirical formula to calculate the empirical complementary cumulative degree distribution of V':

$$f(d) = \sum_{i>d}^{\infty} k(i) = e^c d^{at+b} \qquad (4\text{-}9)$$

   where $k(i)$ is the degree of node i, and a, b, c are some known constants previously determined by historical data.

4. Generate a spanning tree consisting of nodes with degree larger than 1. To do so, let G be the graph to be generated, starting from empty initial conditions; then, a node i of degree larger than 1 located outside G is connected to a node j inside G according to the following (preferential attachment) probability:

$$\Pi(i, j) = \frac{w_i^j}{\sum_{k \in G} w_i^k} \qquad (4\text{-}10)$$

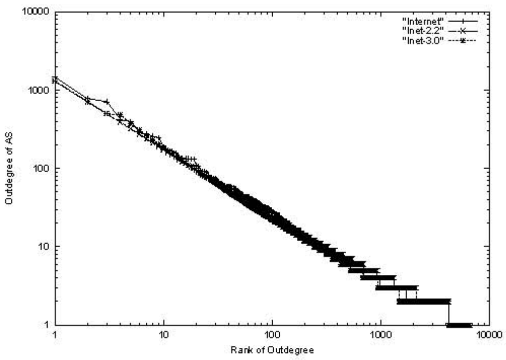
   where the Euclidean distance between two node degrees in the log-log scale of the d - $f(d)$ plot is used to set the weights, as

$$w_i^j = Max\left\{ 1,\ \sqrt{\left(\ln(d_i / d_j)\right)^2 + \left(\ln(f(d_i) / f(d_j)\right)^2} \right\} d_j \qquad (4\text{-}11)$$

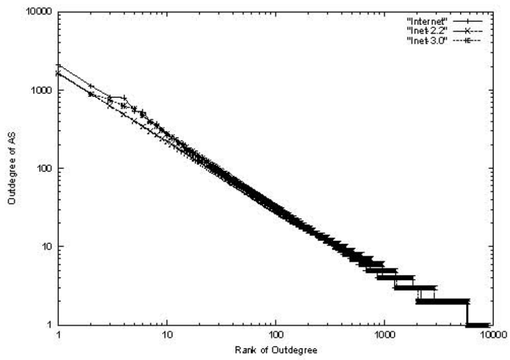
where d_i is the degree of node i, with $f(d_i)$ being the frequency (i.e., the total number of nodes) of degree d_i. Note that graph G grows in such a way that it includes more and more outside nodes as they are connected to it.

5. Connect all ρN nodes of V_1 to G, also according to the probability (4-10).

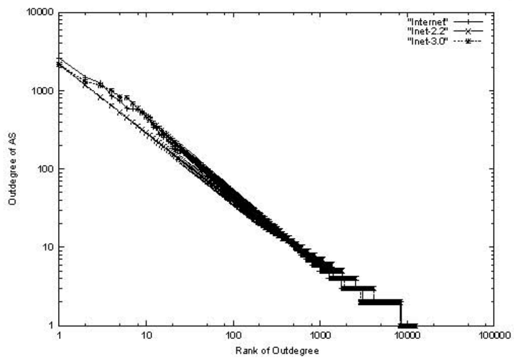
6. Connect high-degree nodes, in decreasing order, to those available nodes in G that do not have connections to V_1, also according to the probability (4-10).



a. February 01, 2000.



b. February 01, 2001.



a. February 01, 2002.

Fig. 4-28 Out-degree distributions of the Internet [6]

Figure 4-28 shows the power-law distributions of out-degrees, the simulation results obtained by using Inet 3.0, based on the AS-level Internet data of 1 February 2000, 1 February 2001, and 1 February 2002, with 6700, 8880, and 12700 nodes, respectively. Figure 4-29 further shows that the average path lengths of the Internet remained basically unchanged throughout the three years of time [6].

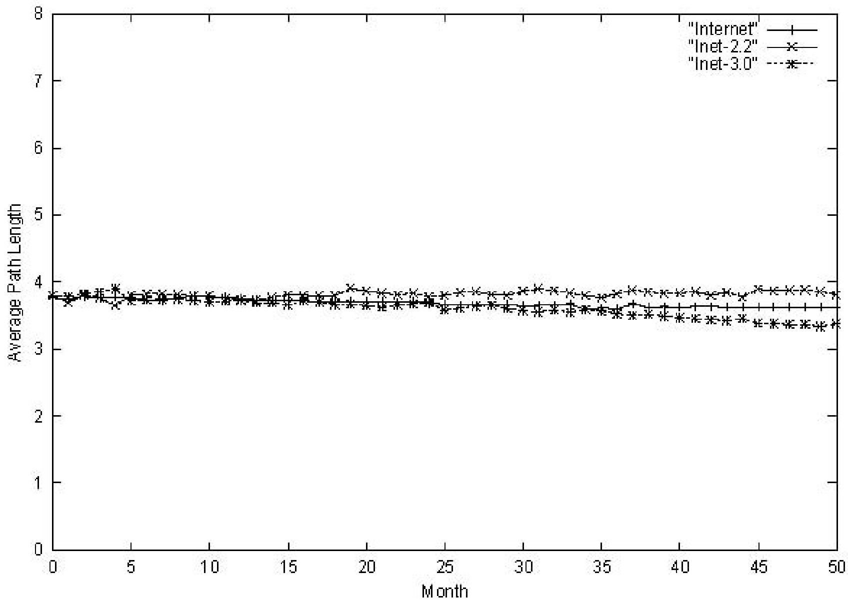


Fig. 4-29 Average path lengths of the Internet [6]

4.5.2 BRITE Model

BRITE (Boston university Representative Internet Topology gEnerator) [5] attempts to build a topology generation framework based on three basic design principles: representativeness, inclusiveness, and interoperability. Here, representativeness means it will synthesize a topology that can accurately reflect most important aspects of the real Internet topology (e.g. its hierarchical structure and node-degree distribution); inclusiveness tries to combine the advantages of many good topology generators; interoperability provides interfaces to widely-used simulation applications such as Network Simulator (NS) and Scalable Simulation Framework (SSF).

BRITE generates a network topology in the following steps:

1. Divide a planar region into $HS \times HS$ squares, and then further divide each square into $LS \times LS$ smaller squares.

2. According a uniform distribution, or a Pareto heavy-tailed distribution (with probability density function $f_k(x) = k\frac{x_m^k}{x^{k+1}}, x \ge x_m > 0$, where integer $k > 0$), determine the number of nodes to be placed in each large square. Then, in each large square, randomly pick a small square and assign at most one slot to a future node in the small square.

3. Now, place in nodes. Initially generate a random graph with m_0 nodes, and then add more nodes to the graph gradually. The way to connect nodes is determined by two parameters: Incremental Growth (*IG*) and Preferential Connectivity (*PC*). If $IG = 0$ then put m nodes onto the plane simultaneously, and randomly pick one node among them and then connect it to the other nodes; if $IG = 1$ then put one node onto the plane each time, and

connect this new node to m existing nodes in the network. The way to establish connections is based on the PC parameter value: if $PC = 0$ then follow the Waxman probability (4-7) to connect the new node to the existing nodes; if $PC = 1$ then follow the BA linear preferential attachment probability (3-19), Chapter 3; if $PC = 2$ then use the following weighted preferential attachment probability:

$$\Pi(k_i) = \frac{w_i k_i}{\sum_{j \in C} w_j k_j} \tag{4-12}$$

where k_i is the degree of node i, w_i is the Waxman probability (4-7), and C is the set of all m nodes being connected to node i.

Clearly, choosing different parameter values of IG and PC will generate different topologies. In particular, with $IG = PC = 0$, it generates the Waxman topology. Simulations show that with $IG = PC = 1$, BRITE generates a topology that is closest to the real Internet with similar characteristics such as power-law degree distribution and average path length, etc.

Figure 4-30 shows three curves of the Pareto distribution; Fig. 4-31 [52] shows two examples of different node placements: with uniform random distribution and with Pareto heavy-tailed distribution; Fig. 4-32 [53] shows a simulation example of 5,000 nodes generated by BRITE, where $d(v)$ is the node degree and $f(d)$ is the frequency (total number) of nodes.

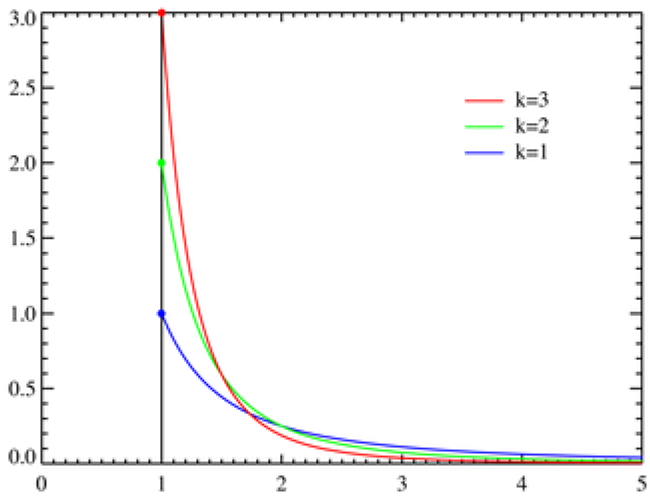
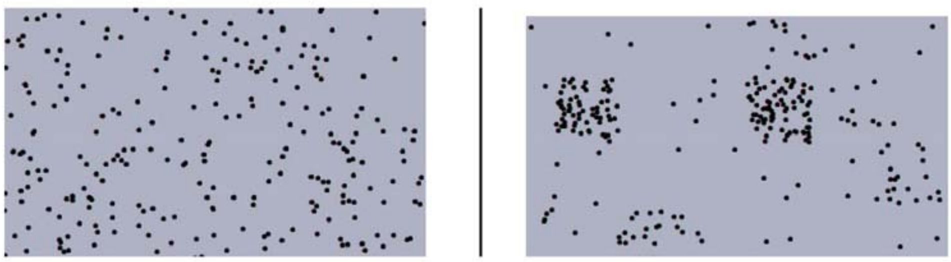


Fig. 4-30 Pareto distribution with $k = 1,2,3$, respectively



(a) (b)

Fig. 4-31 Node placements: (a) uniformly random (b) Pareto heavy-tailed [52]

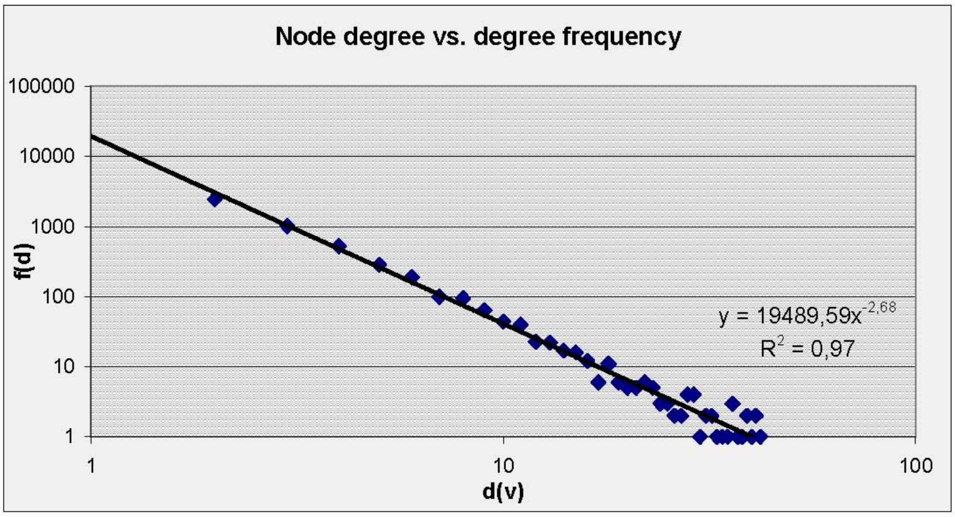


Fig. 4-32 Log-log plot of node-degree distribution: a 5000-node BRITE network [53]

4.5.3 GLP Model

The so-called Generalized Linear Preferential (GLP) model tries to further improve the BA model [54], which is formed in the following steps:

Start with a connected network of m_0 nodes and $m_0 - 1$ edges.

Perform one of the following two operations:

1. With probability p, $0 \le p \le 1$, the existing network receives m ($0 \le m \le m_0$) new edges; one end of each new edge is connected to node i according to the following probability:
$$\Pi(k_i) = \frac{k_i - \beta}{\sum_j (k_j - \beta)} \tag{4-13}$$
where k_i is the degree of node i, and $-\infty < \beta < 1$ is a tunable parameter, representing a certain bias in preferentially attaching to a more popular existing node, in which $\beta < 1$ gives degree-one nodes a chance to acquire new edges.

2. With probability $1 - p$, $0 \le p \le 1$, a new node is being added to the network, which brings in m ($0 \le m \le m_0$) new edges, each of which connects to node i according to probability (4-13).

Clearly, in the GLP model, the increments of new nodes and new edges are independent. The model generation procedure is illustrated by Fig. 4-33.

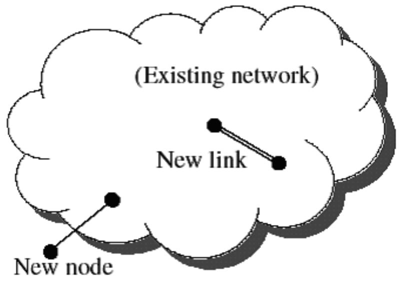
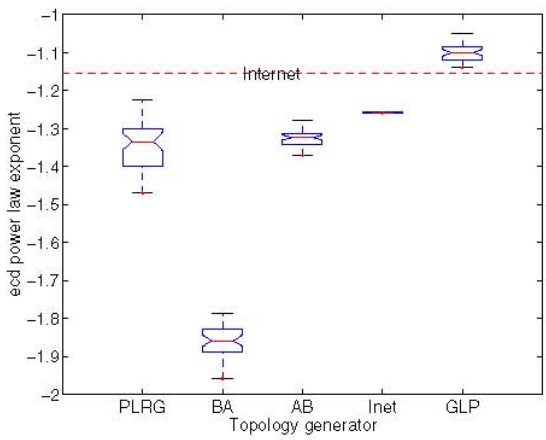


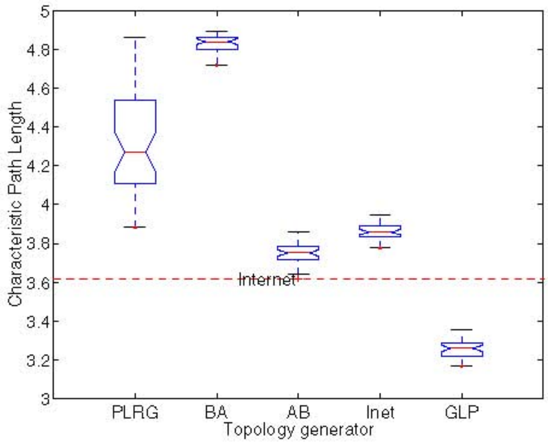
Fig. 4-33 Generation of the GLP model [54]

The above GLP model also has a power-law node-degree distribution [54], for which the verification is similar to that of the BA model.

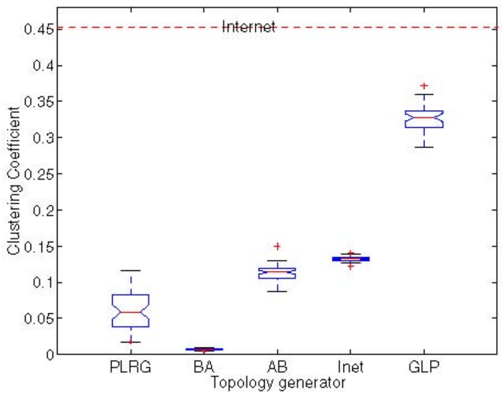
In September 2000, the Internet at the AS level had $N = 8{,}613$ nodes and $E = 18{,}346$ edges. Using this set of real data, with $m = 1.13$, $p = 0.4695$ and $\beta = 0.6447$ in the GLP model, a comparison on the power-law exponent, average path length and clustering coefficient of several modes is demonstrated in Fig. 4-34. This figure shows the average of 100 simulations, where PLRG refers to power-law random graph and AB is the EBA model of Albert and Barabasi [55], introduced in Section 3.5.3, Chapter 3.



(a) ecd power law exponents



(b) Characteristic Path Length



(c) Clustering Coefficient

Fig. 4-34 Comparison of several Internet AS models

4.5.4 PFP Model

The so-called Positive Feedback Preferential (PFP) model [24] is established based on the observation of the "rich club" phenomenon, discussed in Section 4.2.3 above. In this PFP model, the growths of new nodes and new edges are alternative, with a nonlinear preferential attachment scheme. In comparison to the so-called Interactive Growth (GI) model [56], it has a better match to the real Internet data.

In the PFP model, at each step of the model generation a new node is being added to an existing host in the network, and moreover some new edges are being added between the nodes in this host node and some other existing peer nodes. In looking for an existing node to attach, the following nonlinear preferential attachment probability is used:

$$\Pi(k_i) = \frac{k_i^{1+\delta \log_{10} k_i}}{\sum_j k_j^{1+\delta \log_{10} k_j}}, \qquad \delta \in [0,1] \tag{4-14}$$

More precisely, the model generation procedure is as follows:

1. With probability p, $0 \le p \le 1$, a new node is being added, which connects to a host; meanwhile, a new edge is being added to one node in the host to connect to another peer node existing in the network.

2. With probability q, $0 \le q \le 1 - p$, a new node is being added to a host node; meanwhile, two new internal edges are being added between the host node and two existing peer nodes.

3. With probability $1 - p - q$, a new node is being added to connect to two host nodes, respectively; meanwhile, a new edge is added to connect one host node and one peer node.

A comparison in the AS-level Internet of the PFP model with $p = 0.3$ and $q = 0.1$, against the IG and BA models, is shown in Table 4-5 [24].

Table 4-5 Comparison of network parameters among several models [24]

		AS graph	PFP model	IG model	BA model
Number of nodes	N	11122	11122	11122	11122
Number of links	L	30054	30151	33349	33349
Average degree	$\langle k \rangle$	5.4	5.4	6.0	6.0
Exponent of power law	γ	2.22	2.22	2.22	3
Rich-club connectivity	$\phi(r/N{=}0.01)$	0.27	0.30	0.32	0.045
Maximum degree	k_{max}	2839	2785	700	292
Degree distribution	$P(k{=}1)$	26%	28%	26%	0%
Degree distribution	$P(k{=}2)$	38%	36%	34%	0%
Degree distribution	$P(k{=}3)$	14%	12%	11%	40%
Characteristic path length	l^*	3.13	3.14	3.6	4.3
Average triangle coefficient	$\langle k_t \rangle$	12.7	12	10.4	0.1
Maximum triangle coefficient	$k_{t\ max}$	7482	8611	4123	64
Average quadrangle coefficient	$\langle k_q \rangle$	277	247	105.4	1.3
Maximum quadrangle coefficient	$k_{q\ max}$	9648	9431	8780	527
Average k_{nn}	$\langle k_{nn} \rangle$	660	482	103	20
Average betweenness	$\langle C_B^* \rangle$	4.13	4.14	4.6	5.3
Maximum betweenness	$C_{B\ max}^*$	3237	3419	1002	1064

4.5.5 T_{ANG} Model

The T_{ANG} model refers to the model generated by the Tel Aviv Network Generator, which combines the essential features of the Incremental Edge Addition (InEd) model and the Super-Linear Preferential Attachment (SLiP) model.

Start with a network of n_o nodes. At each step, one new node is being added along with m new edges connecting to m existing nodes, where the probability of an existing node i being chosen by the new node for connection is

$$\Pi(k_i) = \frac{k_i^{1+\varepsilon}}{\sum_j k_j^{1+\varepsilon}} \tag{4-15}$$

where $\varepsilon > 0$ is referred to as the super-linear preferential attachment parameter. Obviously, with $\varepsilon = 0$ the T_{ANG} model reduces to the BA model.

Figure 4-35 shows a comparison of the Complementary Cumulative Density Function (CCDF) of node degrees between the AS-level Internet and the T_{ANG} model with different values of ε [57].

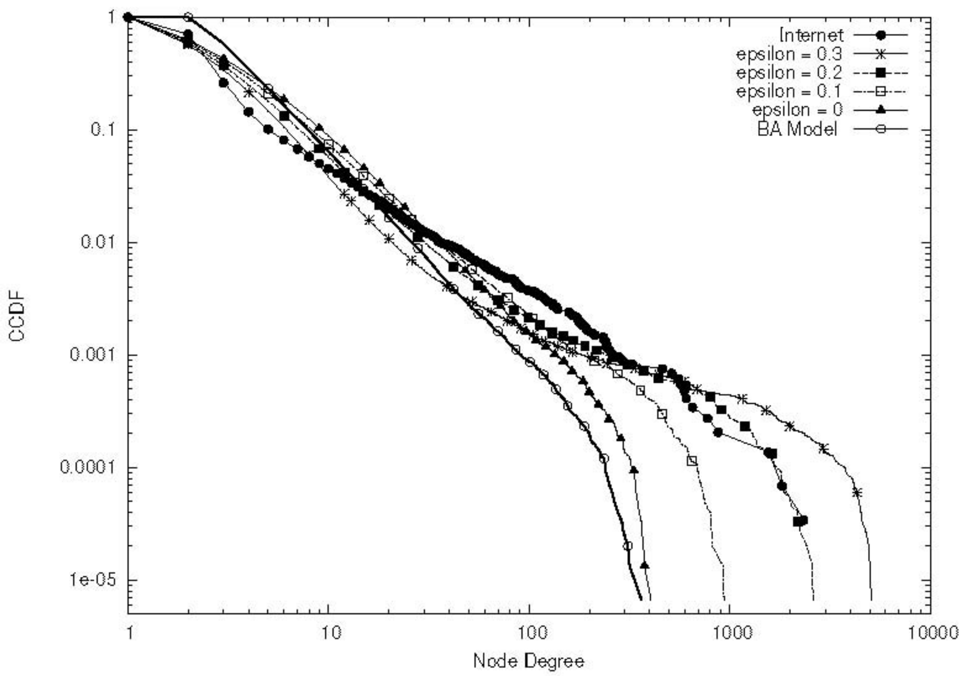


Fig. 4-35 CCDF of node degrees of the T_{ANG} model (log-log plot) [57]

Notice that connections in the Internet are generally not equivalent in the sense that more than 90% of connections represent customer-provider relationships, where information data flow from Internet Service Provider (ISP) to customers, therefore directed graphs are needed to describe the connections in reality. Based on geographic distribution of the Internet, the so-called G_DT_{ANG} model was suggested, with a generation procedure as follows [58].

Define several different geographic regions for the Internet model. At each step, one new (customer) node is being added to the network along with m directed edges pointing from customer nodes to ISP nodes. In doing so, randomly select a region and a node i in that region, and the new customer node is connected by a directed edge to an ISP node i according to the following probability:

$$\Pi(y_i) = \frac{y_i}{\sum_j y_j} \tag{4-16}$$

where y_i is the out-degree (the number of out-pointing edges) of node i.

Meanwhile, $m-1$ edges are being added among the existing nodes: a customer node l is being selected according to the following probability:

$$\Pi(k_l) = \frac{k_l}{\sum_j k_j} \tag{4-17}$$

where k_l is the in-degree (the number of in-pointing edges) of node l; while any ISP node is being selected according to the probability (4-16).

This G_DT_{ANG} model can generate various topological features closer to the Internet than the T_{ANG} model [58].

4.6 Multi-Local World Model

Observe that at the Inter-domain level, the actual Internet hierarchy can be schematically divided into international connections, national backbones, regional networks, and local area networks. The nodes in the regional networks are tightly connected, yielding very high clustering coefficients within the networks. These highly clustered regional networks are then interconnected sparsely by national backbones or international connections. This observation leads to an approach using the Multi-Local World (MLW) model, introduced in Section 3.5.4, Chapter 3, to describe the Internet structure [59].

4.6.1 Theoretical Considerations

When the MLW model is applied to model the Internet, some factors need to be carefully studied based on the collected Internet AS-level data.

(i) *Birth and Death of AS*

Define an AS as being “born” when a new ISP joins the Internet and as being “dead” after it is disappeared. It has been found that the Internet on the basis of AS-level granularity continues to grow after it was born. For example, according to the data collected by the Oregon router server [14], the number of AS in the Internet was increased from 4,320 in November 1998 to 9,520 in November 2000. During this period, although there were so many dead AS in each time duration (e.g., month or year), the number of dead AS was always small as compared to the number of newly born AS within the same time interval. For example, the newly born AS number was about 200 in November 1998 while less than 50 AS died at the same month. It clearly indicates that the birth rate of AS is larger than its death rate. Therefore, it is reasonable not to consider the event of death of AS in the MLW model for simplicity of modeling and analysis.

(ii) *Birth and Death of Edges*

When a new AS is being added into the network, it creates a certain number of edges connecting the existing nodes. On the other hand, there may also appear new interconnections between the already existing nodes due, for instance, to the consideration of having redundancy for having fault tolerance or for avoiding possible traffic congestions. This is referred to as the birth of an emerging edge. Meanwhile, an edge between two nodes may be disconnected, for many obvious reasons, which is called the death of the (deleted) edge. The analysis on the AS-level Internet topology data shows that the event of death of edges should not be neglected, because the death rate of edges is comparable with its birth rate in reality. In fact, for the real Internet, in some cases the number of dead edges can even be larger than that of the newly born edges. For example, in November 2000, the number of dead edges was about 1300, which was 800 more than the newly born edges in that month. Therefore, both birth and death of edges should be considered in a realistic model of the Internet, at least at the AS level.

(iii) *Rewiring*

An ISP in the Internet may rewire one of its edges to connect to another node with a higher node-degree in order to gain more benefits in, for example, reducing the distance from it to other peer nodes in the network. However, a recent study [9] shows that the rewiring mechanism may not be a significant factor in the evolution of the AS-level Internet topology. Therefore, the rewiring mechanism for an Internet model is not very necessary, which may be ignored in the modeling.

(iv) *Preferential Attachment*

When a new node joins the network, an existing node i receives an edge from the new node according to a certain probability, which may depend linearly on the degree of node i, k_i, as in the BA model, in the form

$$\prod(k_i) = \frac{k_i}{\sum_j k_j} \qquad (4\text{-}18)$$

or nonlinearly in the form

$$\Pi(k_i) = \frac{k_i^{\sigma}}{\sum_j k_j^{\sigma}} \tag{4-19}$$

where the parameter $\sigma > 0$.

A recent study on the real AS-level Internet topology data shows that the newly added nodes actually create new edges by a linear preferential attachment rule [33], which was lately confirmed by another study [42]. On the other hand, the edges attached to a node with a lower degree may more likely be removed, because ISPs always tend to delete those infrequently used edges so as to reduce the maintenance cost. Combining with the linear preferential attachment rule when adding an edge between two nodes, it is reasonable to assume that the probability of an edge attached to node i being deleted is

$$\Pi'(k_i) = \frac{1}{N(t)-1}(1-\Pi(k_i)) \tag{4-20}$$

where $N(t)$ is the number of nodes in the network at time step t. This term will be normalized such that $\sum_i \Pi'(k_i) = 1$.

(v) *Localization*

As mentioned, at the AS level, the Internet hierarchy can be schematically divided into international connections, national backbones, regional networks, and local area networks. The nodes in a regional network are tightly connected, yielding a high clustering coefficient within the network. These highly clustered regional networks are then interconnected sparsely by national backbones or international connections. This observation is supported by real Internet data, shown in Table 4-6, where it can be seen that the number of intradomain edges is much larger than that of the interdomain edges, which is a prominent localization effect of the real Internet.

Table 4-6 Interdomain and intradomain edges of the Internet [45]

	Number of Interdomain edges	Number of Intradomain edges
US	77,367	354,593
Europe	15,365	99,023
Japan	3,651	44,701
World	146,936	715,997

When a new node is to join a regional network, the nodes in other regional networks, even those with very large degrees, will have very little impact on the decision of receiving this new node. In other words, the ability that the node i in this regional network can capture a new edge from the newly added node may depend primarily on its position relative to the other nodes within the same regional network, but not to the entire multi-regional Internet. This regional network is referred to as a *local-world*, while the entire Internet can be considered as an ensemble of many local-worlds.

Combining the localization effect with the linear preferential attachment rule of the Internet, it is reasonable to assume that the probability with which a node i in a local-world Ω receives a new edge from the newly added node is in the form of

$$\prod(k_i) = \frac{k_i + \alpha}{\sum_{j \in \Omega}(k_j + \alpha)} \tag{4-21}$$

where Ω denotes the Ωth local-world in which node i locates, and the parameter $\alpha > 0$ represents the attractiveness of node i, which is used to govern the probability for “young” nodes to receive new edges.

Similarly, the probability of an edge attached to node i being deleted can be rewritten as

$$\Pi'(k_i) = \frac{1}{N_\Omega(t) - 1}(1 - \Pi(k_i)) \tag{4-22}$$

where $N_\Omega(t)$ represents the number of nodes within the Ωth local-world in the network, and $\Pi(k_i)$ is determined by (4-21).

All issues discussed in (i)-(v) above are then incorporated in the Multi-Local World (MLW) model, introduced in Section 3.5.4, Chapter 3, so as to describe the Internet structure [59]. The MLW modeling performance will be further discussed in the next subsection.

To this end, notice that the weight of each new incoming edge in the last step of the above MLW model is fixed to same value, say w_0, which is taken to be 1 in general. But, if the weights of the new edges between node i and node j, denoted w_{ij}, are considered to be variable:

$$w_{ij}(t+1) = w_{ij}(t) + \delta \frac{w_{ij}(t)}{k_{ij}(t)} \tag{4-23}$$

where $\delta \geq 0$ is a constant parameter, then an evolving weighted MLW model can be obtained [60,61], which can be applied to modeling the Internet even at the router level [61].

4.6.2 Numerical Results with Comparison

As discussed in the previous sections, many models have been proposed to describe the Internet topology. Therefore, it is quite natural to compare their performances in characterizing the Internet. To do so, some real Internet data are first reviewed.

Since the early 1990s, the number of hosts in the Internet continues to grow exponentially, as shown in Fig. 4-36 [62,63]. For example, there are 376,000 hosts in January 1991 but 43,200,504,230,200,504,000 hosts in January 1999. Six years later in January 2005, there were even 31,720,050,4646,200,504,084 hosts in the Internet, 634.3% increase comparing to that in January 1999.

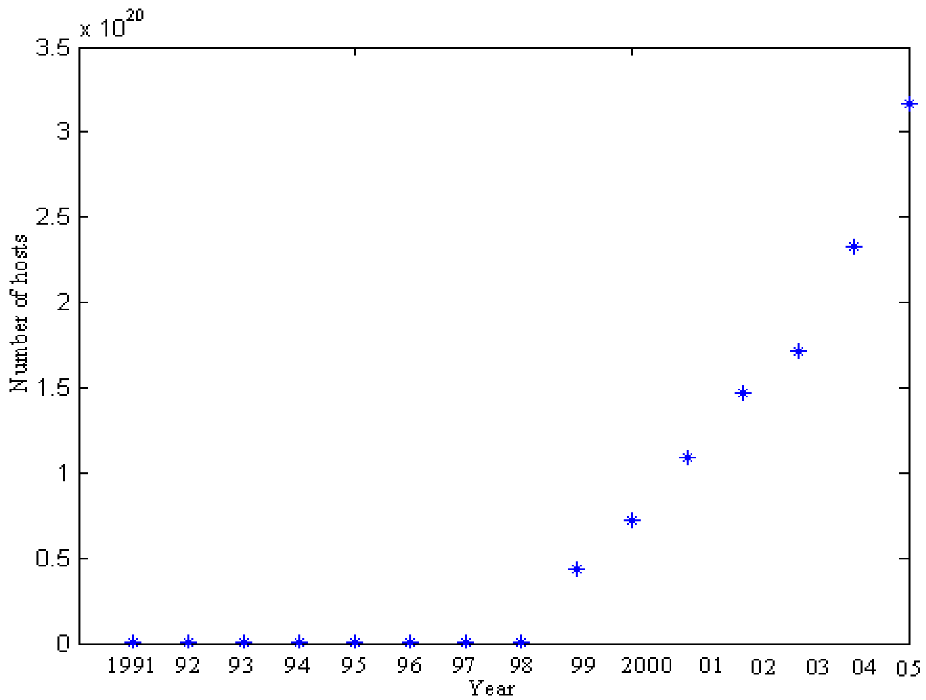


Fig. 4-36 Number of hosts in the Internet [62]

Although the number of hosts in the Internet increases exponentially, some statistical parameters of the Internet do not change at all, or only have very small changes. For example, at the AS-level it was observed that the degree distribution of the Internet follows a power-law form with degree exponent $\gamma = 2.2$ on 15 May 2002, as shown in Fig. 4-37 [62]. Yet, several years later, the degree distributions of the Internet, on 15 May 2004 and 15 May 2005, respectively, are still in the power-law form with the same degree exponent $\gamma = 2.2$ as shown in Fig. 4-38. Therefore, the AS-level Internet is indeed a scale-free network. This invalidates the Waxman and Transit-stub models in which the degree distribution follows a Poisson distribution.

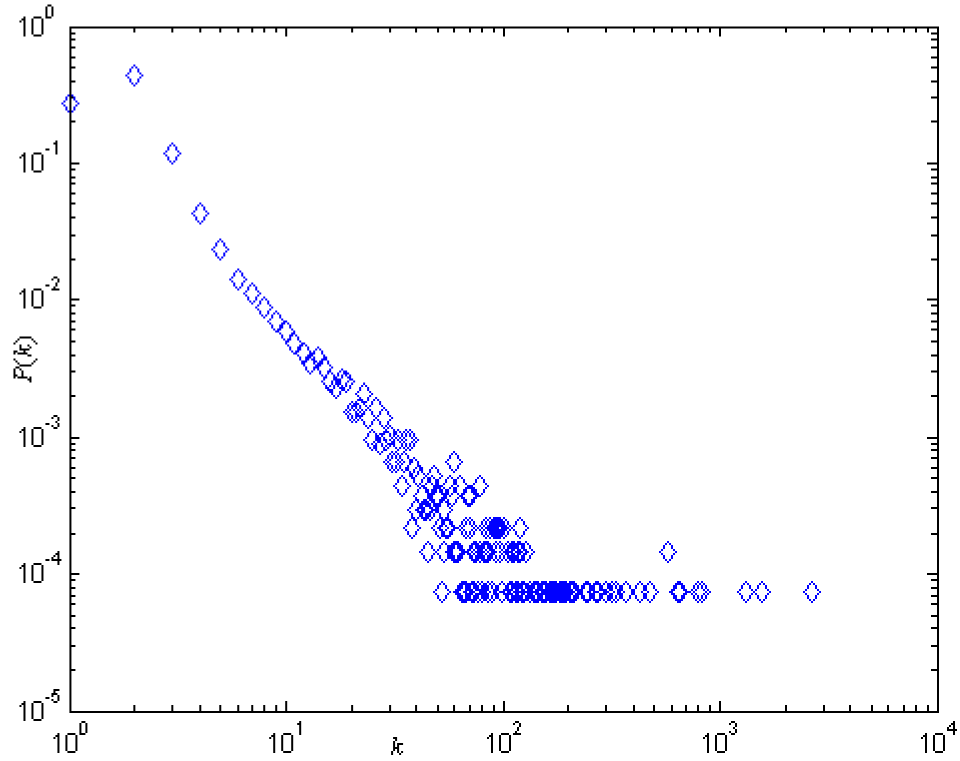


Fig. 4-37 Degree distribution of the Internet on 15 May 2002 [62]

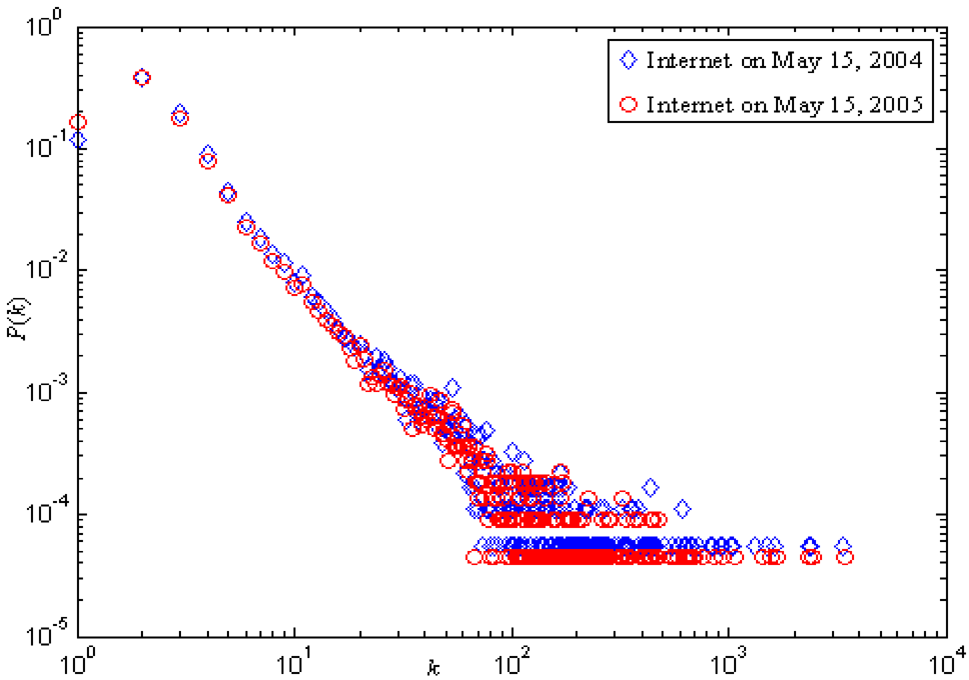


Fig. 4-38 Degree distributions of the Internet on 15 May 2004 and 15 May 2005 [62]

On the other hand, the number of AS in the Internet grows linearly despite the fact that the number of hosts increases exponentially. Figure 4-39 shows the number of AS viewed from AS16517 during 2001 to 2005 [62,63]. From this figure, one can see that the AS-level Internet is a linearly growing network. This observation is confirmed through analyzing the most completed datasets collected in [64].

Figure 4-40 shows the number of AS in the Internet from 1 January 2004 to 31 December 2005. There are 16,571 AS on 1 January 2004, increasing to 24,340 on 31 December 2005. However, due to some unknown technical reasons, the collected number of AS on 31 March 2005 decreases sharply. For this reason, this set of data will not be considered later when the Internet topology models are simulated.

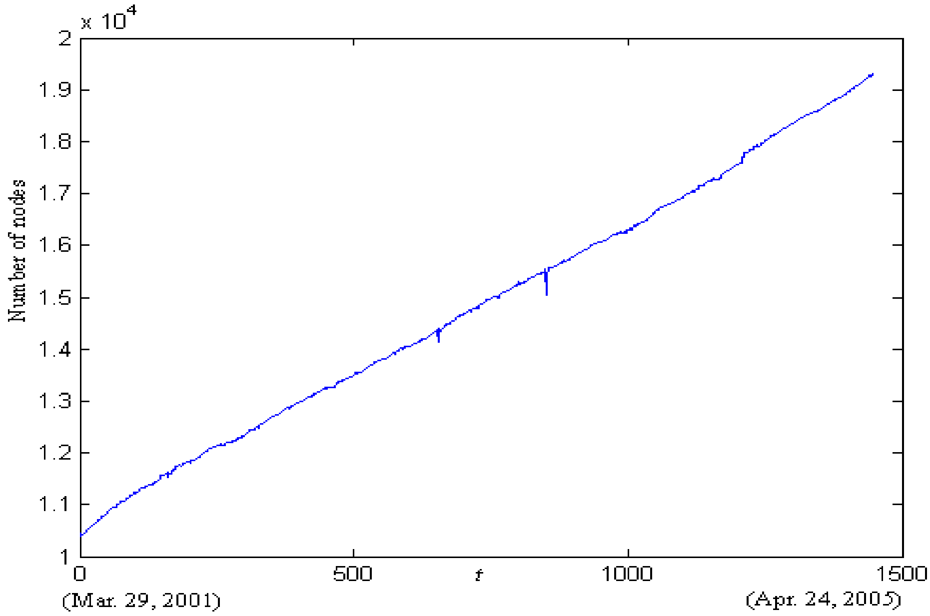


Fig. 4-39 Number of AS viewed from AS16517 [62]

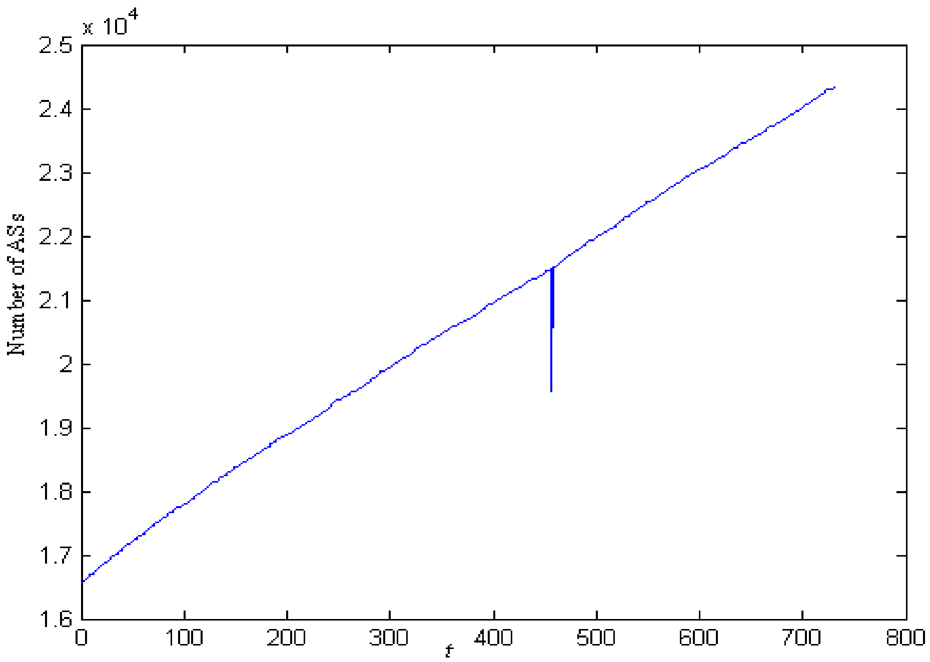


Fig. 4-40 Number of AS in the Internet
from 1 January 2004 to 31 December 2005 [62]

If one assumes that the Internet grows exponentially as in the so-called fluctuation-driven model [65], then the obtained number of AS will be much larger than the real number of AS, shown in Fig. 4-41 [62]. Thus, the fluctuation-driven model will not be compared below for performance evaluation on different models of the AS-level Internet topology.

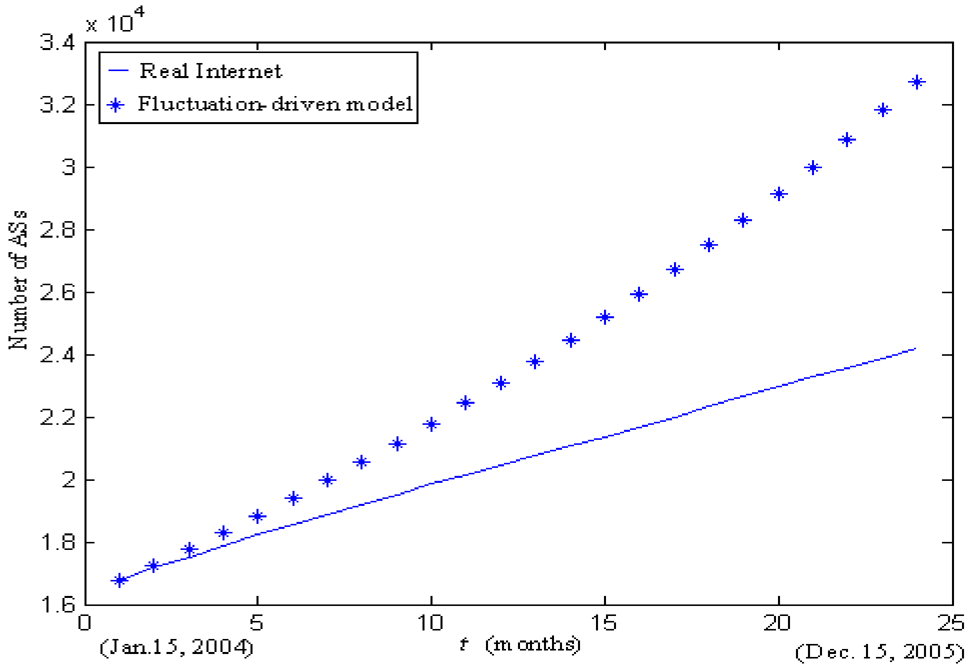


Fig. 4-41 Comparison of numbers of AS between the real Internet and the prediction of the fluctuation-driven model [62]

Therefore, only BA, EBA, Fitness, and MLW models will be further investigated as candidate models for the AS-level Internet topology, because they preserve at least two fundamental features, namely, the scale-free feature and the linear growth of the node numbers.

To compare the BA, EBA, Fitness, and MLW models, the real AS-level internet data collected on 15 May 2005 is applied to fit to these candidate models. On this snapshot of the Internet, it has 21,999 nodes and 85,762 edges.

For the BA model, the number of edges is $E \approx Nm$. Therefore, the parameters in the BA model are chosen as $m = m_0 = 4$.

For the EBA model, the number of nodes is $N = m_0 + (1 - p - q)t$, and the number of edges is $E = m(1-q)t - m/2$. Moreover, the degree exponent in the EBA model is $\gamma = 1 + \frac{2m(1-q) + 1 - p - q}{m}$. Therefore, in simulation, $\gamma = 2.2$ and $m = m_0 = 3$, which yield $p = 0.4$, and $q = 0.32$.

Similar to the BA model, the parameters in the Fitness model are selected to be $m = m_0 = 4$.

For the MLW model, it is quite natural to let a local-world in the model to represent a country that has connected to the global Internet in the world. From September 1991 to July 1997, the Internet had spread from 31 countries to 171 countries, as shown in Table 4-6 [62]. Although it was unknown as how many countries the Internet had spread in 2004, it is reasonable to assume that all countries in the world had been covered by the Internet due to the rapid growth of the Internet since 1999 as witnessed by Fig. 4-36. Therefore, the MLW model takes $m = 186$ local-worlds, which is equal to the number of countries in the world in 2004, with the parameter $p = 0$ in the model. For each local-world, in the simulation it is assumed that the initial local-world consists of only $m_0 = 2$ nodes and $e_0 = 1$ edge, for simplicity.

Table 4-7 Number of countries that the Internet covered from 1991 to 1997 [62]

Date	Number of Countries
09/91	31
12/91	33
02/92	38
04/92	40
08/92	49
01/93	50
04/93	56
08/93	59
02/94	62
07/94	75
11/94	81
02/95	86
06/95	96
06/96	134
07/97	171

On the other hand, assume that when a new AS joined the Internet, it had one or two edges to the existing nodes in most cases. Also, the nodes with degree 1 or 2 were removed from the network with higher probability than the others. Thus, one may reasonably set $m_1 = 2$ and $m_3 = 2$, and for simplicity also set $m_2 = m_4 = 2$, in the MLW model.

In the MLW model, the number of nodes, on average, is

$$N = mm_0 + qt \tag{4-24}$$

and the number of edges, on average, is

$$E = me_0 + t(pe_0 + qm_1 + rm_2 - sm_3 + um_4) \tag{4-25}$$

On the other hand, the degree exponent in the MLW model is

$$\gamma = 1 + 1/a \tag{4-26}$$

where

$$a = \frac{qm_1}{c} + \frac{rm_2(q + m_0 p - p)}{(q + m_0 p)c} + \frac{sm_3 p}{(q + m_0 p)c} + \frac{2um_4}{c},$$

$$c = 2(pe_0 + qm_1 + rm_2 - sm_3 + um_4) + q\alpha .$$

Let the degree exponent be $\gamma = 2.2$, as in the real data. Then, one has $s = 0.04$ and $u = 0.57$ by combining the (4-24)-(4-26) together under the setting of $\alpha = 0$, $q = 0.28$ and $r = 0.11$.

Figure 4-42 [62] shows the comparison of the degree distributions between the real Internet and the four (BA, EBA, Fitness, and MLW) scale-free models with the parameters given above.

Obviously, the BA model is not a good candidate to model the AS-level Internet topology since it can only generate a scale-free network with the degree exponent exactly being 3. Yet it is not surprising to see the EBA and MLW models generated scale-free networks with the precise degree exponent $\gamma = 2.2$, identical to the actual exponent value of the Internet.

The basic small-world features exist in all the models, since the average shortest-path lengths are all small compared to their sizes. For example, in the MLW model, the average shortest-path length is only 3.4, which is very small comparing to its size $N = 21{,}999$. However, the distance distributions of these models are quite different, as shown in Fig. 4-42 [62].

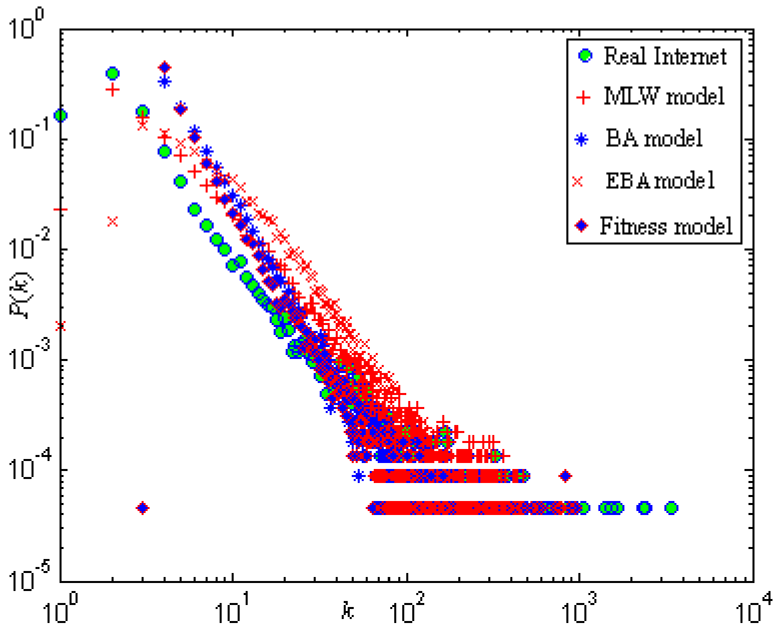


Fig. 4-42 Comparison of the degree distributions between the real Internet and the four scale-free models [62]

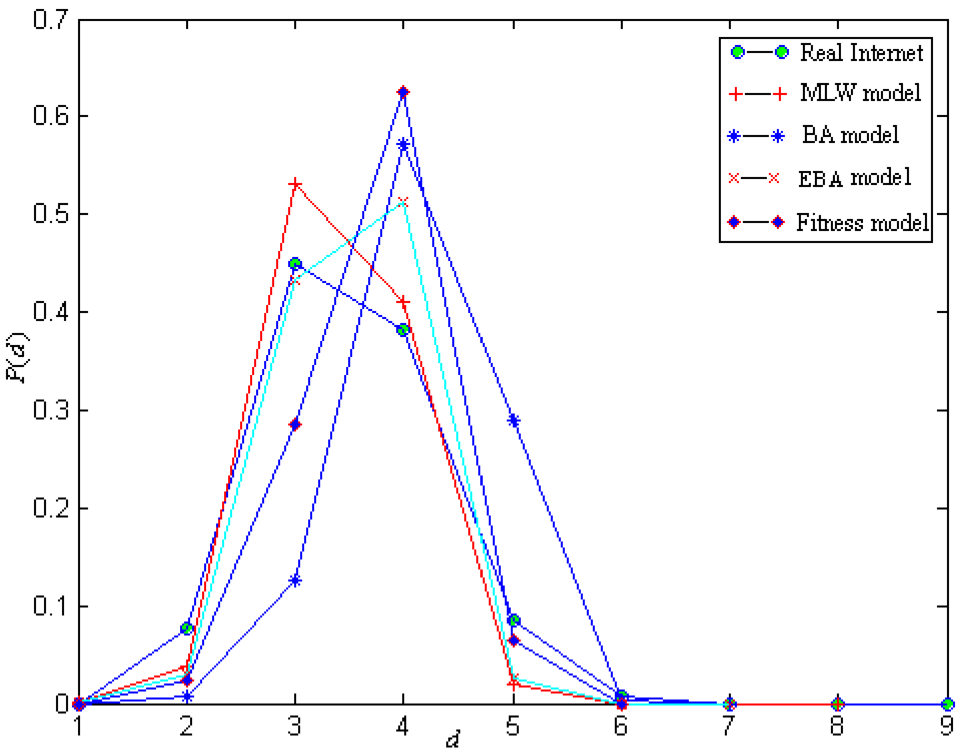


Fig. 4-43 Distance distributions of the Internet and the four scale-free models [62]

Some finer results at the distance distributions of the Internet and the BA, EBA, Fitness and MLW models are shown in Fig. 4-43 [62]. From this figure, one can see that the BA, EBA and Fitness models have similar distance distributions. In these models, most nodes are separated by a distance d_0, which is near the average shortest-path length. Once the distance between a pair of nodes exceeds this value, the probability $P(d)$ with which a randomly selected pair of nodes is separated by the distance d will decrease dramatically. For example, in the Fitness model, 62.5% of nodes are separated by distance 4, which is near the average shortest-path length 3.7. However, only 6.5% of nodes are separated by distance 5. In the real Internet and the MLW model, most nodes are separated by a distance d_0, which is slightly smaller than the average distance. When the distance between any pair of nodes exceeds d_0, the probability $P(d)$ decreases slowly. For instance, in the MLW model, 53.1% of nodes are separated by distance 3 while 41% of nodes are separated by distance 4, which is quite different from the BA, EBA, and Fitness models.

On the other hand, the clustering coefficients in the BA, EBA, and Fitness models are very small. For example, the value of the clustering coefficient is 0.009 in the EBA model. However, the clustering coefficient in the real Internet is very large although it is still much smaller than 1. Among the BA, EBA, Fitness, and MLW models, only the MLW model can reproduce a large clustering coefficient, comparable to that of the real Internet. More importantly, the MLW model can capture the essential feature of the distribution of clustering coefficients of the Internet, while the BA, EBA and Fitness models fail, as shown in Fig. 4-44 [62].

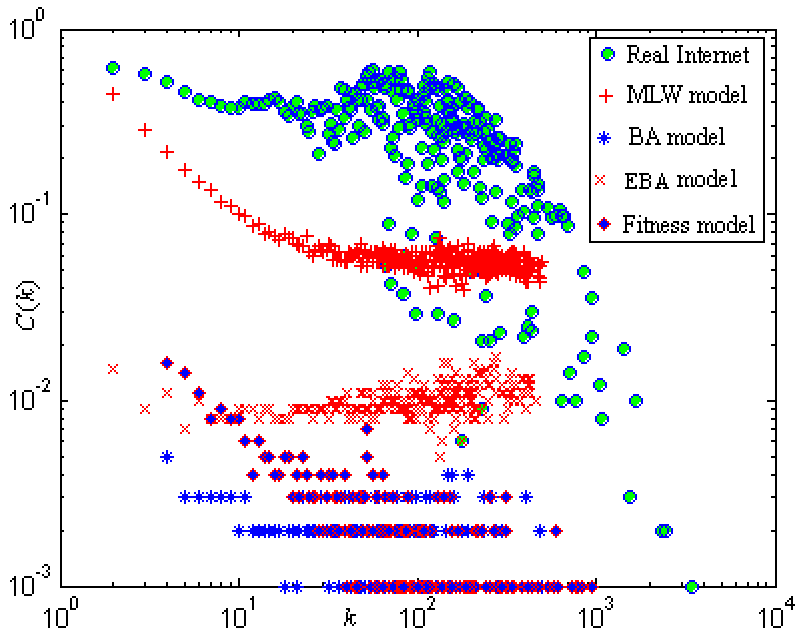


Fig. 4-44 Clustering coefficients $C(k)$ as functions of the degree k for the real Internet and the four models [62]

Therefore, after all, the MLW model is better to fit the Internet topology data than the BA, EBA, and Fitness models in terms of essential small-world features of the generated networks.

A detailed comparison of some statistical results obtained from the BA, EBA, Fitness, and MLW models against the real AS-level Internet topology is summarized in Table 4-8 [62], where N is the number of nodes, C the average clustering coefficient, and L the average shortest-path length.

Table 4-8 Comparison results for the four models against the real Internet [62]

	BA Model	EBA model	Fitness model	MLW Model	Real Internet (15 May 2005)
N	21,999	21,999	21,999	21,999	21,999
C	0.003	0.009	0.012	0.238	0.457
L	4.154	3.533	3.731	3.410	3.493
γ	3	2.2	2.26	2.2	2.2

One can clearly see from Table 4-8 that the BA model cannot be used to describe the AS-level Internet topology since they cannot reproduce the same scale-free features as the real Internet. The EBA and Fitness models can capture the power-law characteristic of the real Internet, but they do not satisfy several basic statistical properties. Fox example, for the average clustering coefficient, the EBA model gives 0.009 while the MLW model gives 0.238, which has a much smaller error in comparison against 0.457 of the real Internet. Thus, in summary, the MLW model is better than the BA, EBA, and Fitness models in representing the Internet AS-level topology, since it can capture both the scale-free and small-world features of the real Internet.

4.6.3 Performance Comparison

Although it has been shown, in the above, that the MLW model is better than the BA, EBA, and Fitness models in capturing the structural features of the Internet, one may still want to know what would happen to the performances of the MLW model in comparing to the real Internet and the other models, particularly in terms of the important and interesting “robust yet fragile” characteristics of the Internet.

In the Internet, local failures and errors often occur due to some hardware and software problems. Therefore, the ability to resist random failures or errors is quite important for a reliable Internet. Here, the “robustness yet fragility” property of the Internet is used as an evaluating indicator to compare the MLW model with the BA, EBA, and Fitness models against the real Internet topology.

To investigate the robustness and fragility of a network, it is natural to study S_f, the size of the largest component after a fraction of nodes, f, in the network are randomly removed from the network, such that the remaining network is undamaged (e.g., remains to be connected). Therefore, the ratio S_f / S_0 measures the capability of the network in which nodes can still communicate each other after the f portion of nodes has been randomly selected and removed. Obviously, if $S_f / S_0 \approx 1$, it means that the remaining network can still function as the original network; but if $0 < S_f / S_0 << 1$, then it indicates that the network has been broken into several clusters while a giant component still exists; yet, if $S_f / S_0 \approx 0$, it means that the network has been broken into several components and each component contains no meaningful networked nodes, namely, the network has collapsed.

Another metric to characterize the Internet's tolerance to random failures is L_g, the average shortest-path length of the largest component after the f portion of nodes has been randomly removed.

Figure 4-45 shows the ratios S_f / S_0, as functions of the portions of the randomly removed nodes, f, for different models. From the figure, one can clearly see that the Internet is quite resilient to random damages. For example, when 30% of nodes in the Internet have been randomly removed, the giant component of the network has a size close to 56.2% of that of the original network; even when 70% of nodes in the network have been randomly removed, the giant component still contains 15.7% of nodes of the original network.

Figure 4-46 shows the change of the average shortest-path length in the giant component of the Internet as a function of the portion of randomly removed nodes. In the beginning, with the increase of the portion of randomly removed nodes, the average shortest-path length in the Internet increases slightly because only some nodes and edges attached to these nodes in the Internet are removed but the Internet can still maintain a large component. With the further increase of the portion of randomly removed nodes, the Internet is finally broken into several small clusters, and consequently the average shortest-path length is drastically decreased.

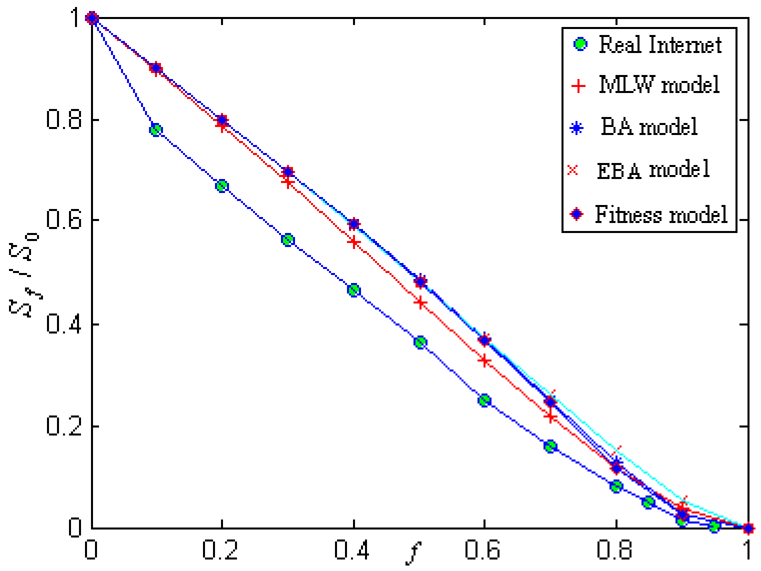


Fig. 4-45 Comparison of the ratios S_f / S_0

for the Internet and the four models [62]

Clearly, one can see from Figs. 4-45 and 4-46 that the MLW model yields a smallest error in comparison with other models against the real Internet. For example, when 70% of nodes have been removed from the network, the size of the resulting giant component relative to the size of the original undamaged network is 21.6% while the average shortest-path length is 4.01 in the MLW model. However, the corresponding values are 24.9% and 6.3 in the giant component of the BA model, which gives larger errors than the MLW model against the real Internet.

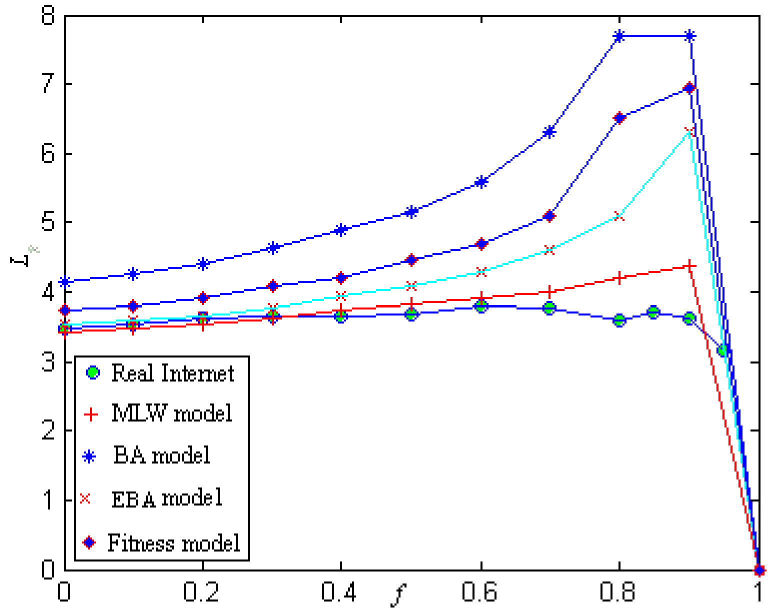


Fig. 4-46 Comparison of the average shortest-path lengths in the giant component for the real Internet and the four models [62]

In summary, one may conclude that the MLW model is the best as compared to the BA, EBA, and Fitness models in describing the AS-level Internet topology, since it can capture the basic structural features of the Internet and can also maintain the robust performance of the Internet in resisting random failures.

4.7 HOT Model

It has been observed lately that the preferential attachment scheme, which leads to power-law degree distributions of scale-free network models, is consistent with the favorable *cooperation* behaviors among the nodes in networks. When *competition* dominates a network, however, preferential attachment becomes less convincing. As it is intuitively clear, competitors usually try to optimize their own advantages and strengths in competitions and do not prefer to attach to giant nodes who have already had most benefits that may actually become threats to new comers and small entities (nodes) of the network.

With such a view in mind, an Internet model by Fabrikant *et al.* [67], called the *heuristically optimized trade-off* (HOT) model, found its place to step in. This model elaborates on the highly optimized tolerance mechanism for power-law distributions in designing a network, proposed by Carlson and Doyle [68], and suggests a new way to produce power laws by an intrinsic trade-off mechanism. More precisely, in designing a network, typically there will be some conflicting objectives; therefore, trade-off among them is usually necessary, while various optimizations toward the objectives are being performed.

Taking this approach, the HOT model suggests a growing network in which, at every step of its growth, a new node is being added and placed at a randomly selected position on a unit square in the partition of a planar region. The new node i is connected to an existing node j according to the minimization of a cost function, say of the form

$$\Phi(i, j) = a(N) d_E(i, j) + \phi(j) \tag{4-27}$$

where $a(N)$ is a constant depending only on the network size N, $d_E(i, j)$ is the Euclidean distance between node i and node j, and $\phi(j)$ is a measure of a certain centrality of node j such as one of the following [67]:

1. the average shortest-path length from j to all the other nodes in the network;
2. the maximum shortest-path length from j to any other node in the network;

3. the shortest-path length from j to a fixed "central" node.

Clearly, the first term in (4-27) aims to limit the cost of establishing the physical connection between the new node i and the existing node j by minimizing their Euclidean distance, while the second term there attempts to minimize the hop distance of node j to the "centrally located" node, so as to maximize the information transmission efficiency.

Figure 4-47 shows a representative simulated network [19], generated from a simulation of the HOT model with $N = 1{,}000$ and $a(N) = 25$, where the "central" node is the one near the center of the figure. This simulated example of the HOT network has a power-law degree distribution, with $\gamma = 1.8$ [19].

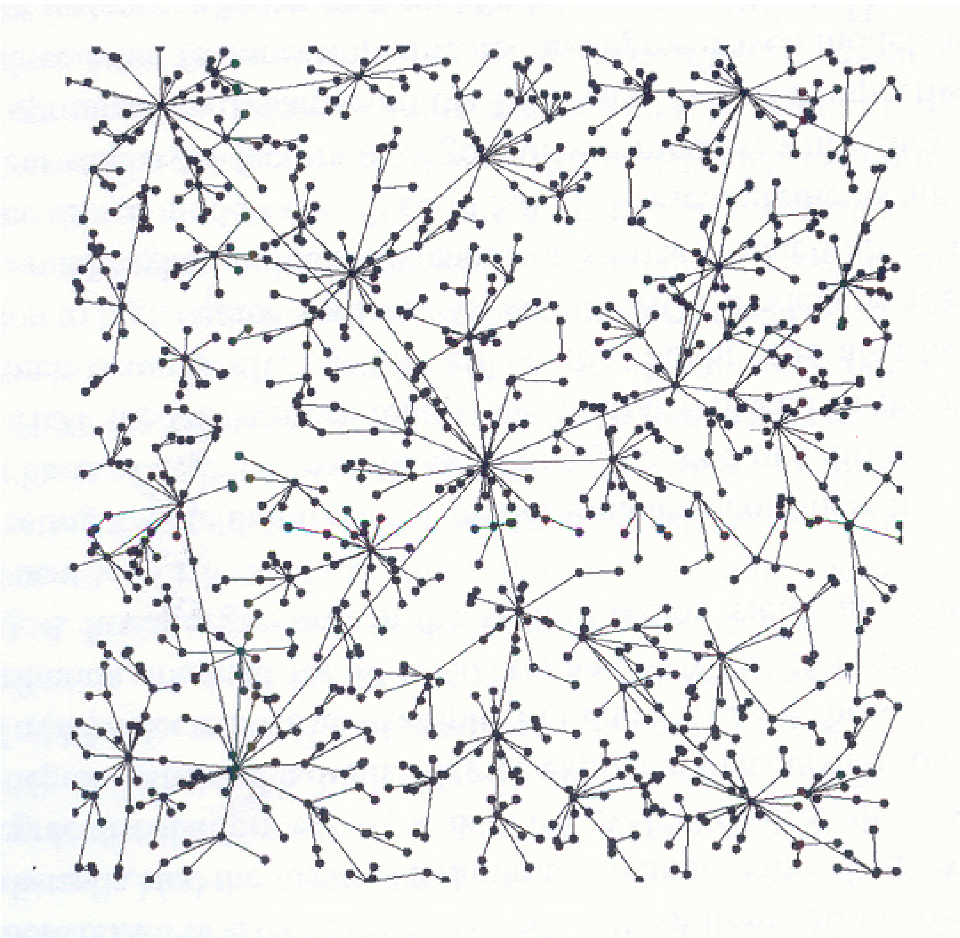


Fig. 4-47 A representative simulated HOT network [19]

The Abilene backbone network, illustrated by Fig. 4-48 [69], was often used for illustration in the discussion of the HOT model (see, for example, [70-72]).

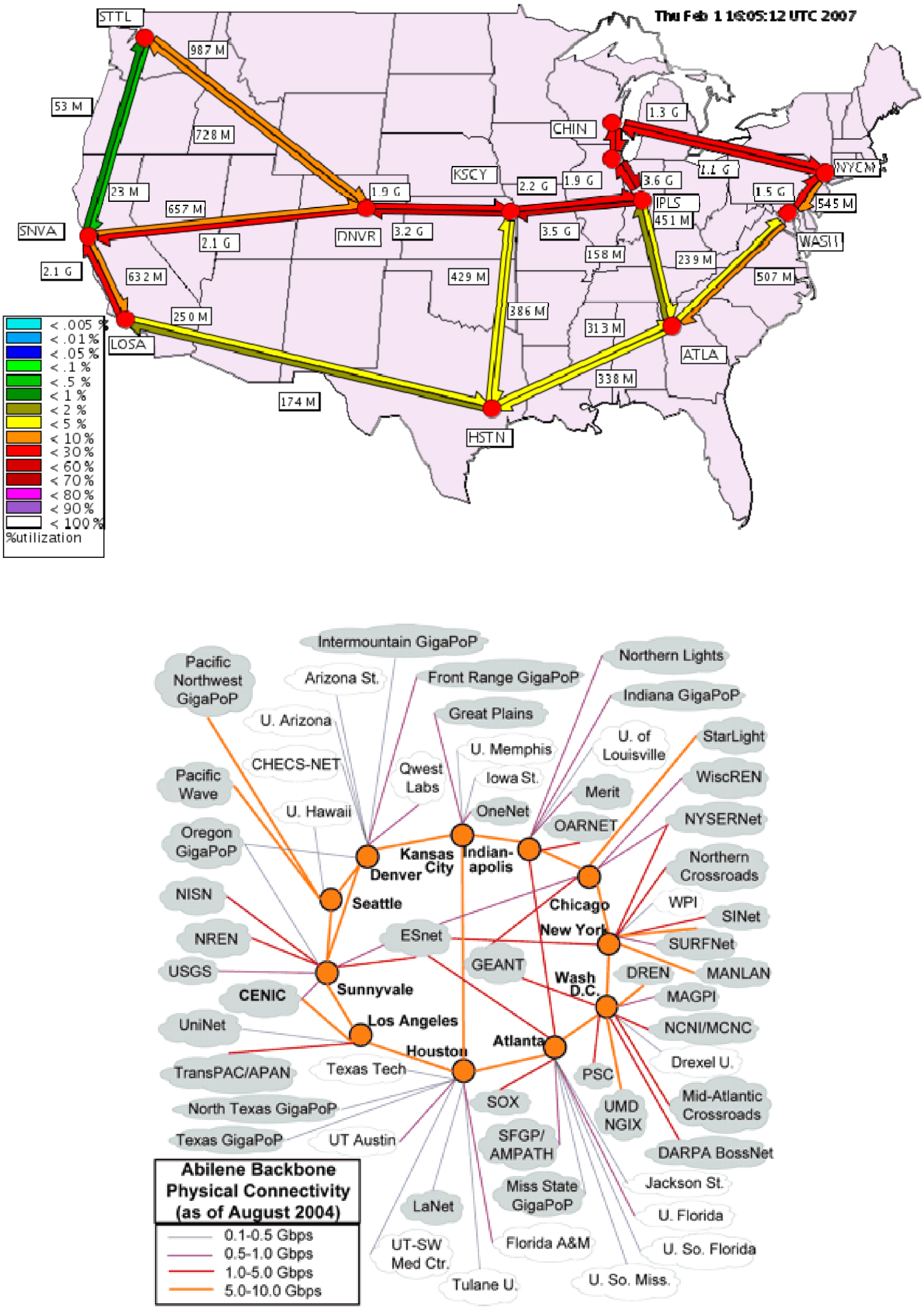


Fig. 4-48 The Abilene network [69]

Finally, there is a recent survey on Internet topology generators at the AS level [73].

Problems

P4-1 In the networks shown in Fig. 4-48 (a) and (b), calculate the k-cores for $k=1,2,\cdots$; the coreness of the each node; the node-betweenness of Node A, and the edge-betweenness of Edge e, respectively.

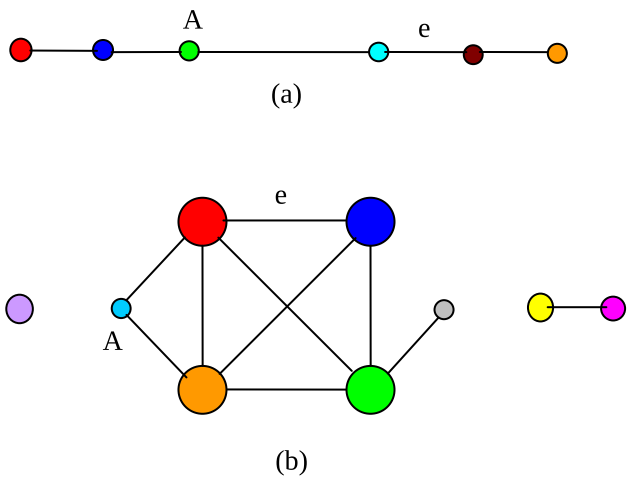


Fig. 4-48 Networks for calculating coreness and betweenness

P4-2 Internet Modeling:

- *Step 1* (Initialization) Start with a large-sized tree.
- *Step 2* (Preferential Attachment) Pick up every possible pair of nodes from the tree. If this pair of nodes is directly connected already, do nothing; if this pair of nodes is not directly connected, then with a probability proportional to the degree of the larger node, add an edge between them.
- *Step 3* (End) After every possible pair of nodes has been operated once, and once only, stop.

  Is this resultant network a good model for the Internet? If you think so, state three major advantages of this model; if you don't think so, state three major disadvantages of this model.

P4-3* Calculate the assortativity coefficient of the network shown in Fig. 4-48.

P4-4* Compare the pros and cons of the Internet topology generators discussed in this chapter.

P4-5* For computer programming exercises or research projects, there is a software: "Network Simulator ns-2: Topology Generation" available on the web [65]. Learn how to use this software.

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