Efficiency and Nash Equilibria in a Scrip System for P2P Networks

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The Free Rider Problem

Almost 70% of Gnutella users share no files and nearly 50% of responses are from the top 1% of sharing hosts

- Adar and Huberman '00

Approaches Taken

- Reputation Systems (Xiong and Liu ’02, Eigentrust: Kamvar et al. ’03, Gupta et al. ’03, Guha et al. 04, …)
  - Assume “good” vs “bad” agents
- Barter-like Systems (BitTorrent, Anagnostakis and Greenwald ’04, …)
  - No analogue of money – can only do swaps
  - Very good for popular files, not so good for rarer ones
- Scrip Systems (Karma: Vishnumurthy et al. ’03)
  - How to find the right amount of money
  - How to deal with new players

Outline

- **Model**
- Non-Strategic Play
- Strategic Play
- Designing a System

Model

- **n** agents
- In round **t**, agent $\mathbf{p_t}$ is chosen at random to make a request
- With probability β, an agent can satisfy the request, then decides whether to volunteer
- One volunteer $\mathbf{v_t}$ is chosen at random to satisfy the request
- For round **t**, $\mathbf{p_t}$ gets a payoff of **1** (if someone volunteered), $\mathbf{v_t}$ get a payoff of **–**α (a small cost)
- Total utility for a player is the discounted sum of round payoffs ($\mathbf{U_t}$):

$$\sum_{t=0}^{\infty} \delta^t u_t$$

Adding Scrip

- M dollars in the system
- Requests cost 1 dollar
- How to decide when I should satisfy a request?

Why Do I Want to Satisfy, or not?

Threshold Strategies

$\mathbf{S_k}$: Volunteer if the requester can pay me and I have less than k dollars

k is your “comfort level,” how much you want to have saved up for future requests

$\mathbf{S_0}$ corresponds to never volunteering

Main Results

- This game has a Nash equilibrium in which all agents play threshold strategies

- The optimal amount of money M (to maximize agent utilities) is mn for some average amount of money m.

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What happens if everyone uses the same strategy S_k?

- Models the system as a Markov chain
- States are vectors of the amount of money each agent has
- Transition probabilities are the probabilities of the relevant agents being chosen as requester and volunteer respectively

Theorem: Concentration Phenomenon

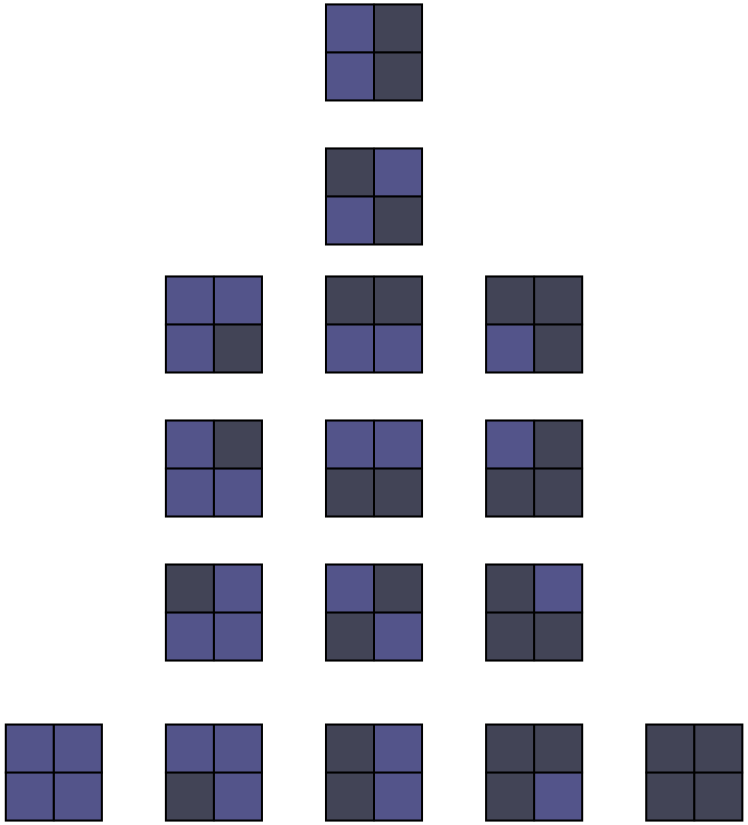
Remarkable property of this Markov Chain: For any amount of money, the number of people with that amount of money is essentially constant.

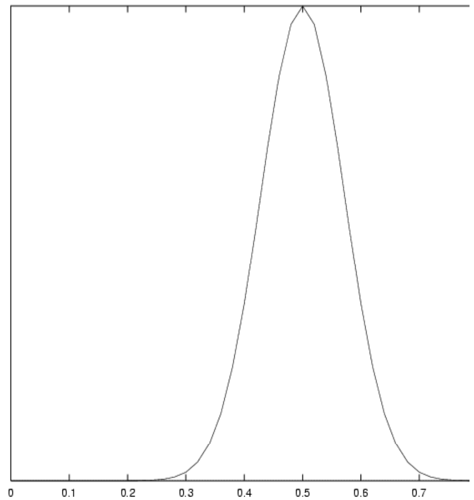
More formally, there exists a distribution:

$$(d(0),\ldots,d(k)) \in \Delta^{k+1}$$

(dependent on the parameters of the game) such that the fraction of people in the system with k dollars is within ε of d(k) with high probability.

Maximum Entropy





Maximum Entropy

We can express our constraints as:

$$1 = \sum_{j=0}^{k} d(j) \qquad\qquad M = \sum_{j=0}^{k} n \cdot d(j)\, j$$

There is a unique solution that maximizes entropy among all solutions that satisfy the constraints. This solution characterizes the number of people with each amount of money.

The system is stable: with high probability it will be in a state close to this distribution.

The system stays away from bad states: entropy is maximized when money is divided "as evenly as possible" subject to the constraints.

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Equilibria

Trivially, S_0 is always an equilibrium.

We would like to say:

"There is always a Nash equilibrium where all agents play S_k for some $k > 0$."

Two Problems:

- If δ is small, agents will be unwilling to do any work now for future payoff
- If n is small, the theorems do not apply (because each transaction causes a big shift, the system could get quite far away from the maximum entropy distribution)

THEOREM 4.1. *Fix a strategy S_γ and an agent i. There exists $\delta^* < 1$ and n^* such that if $\delta > \delta^*$, $n > n^*$, and every agent other than i is playing S_γ in game $G(n, \delta)$, then there is an integer k' such that the best response for agent i is $S_{k'}$. Either k' is unique (that is, there is a unique best response that is also a threshold strategy), or there exists an integer k'' such that $S_{\gamma'}$ is a best response for agent i for all γ' in the interval $[k'', k''+1]$ (and these are the only best responses among threshold strategies).*

There exist $\delta^* < 1$ and n^* such that for all systems with discount factor $\delta > \delta^*$ and $n > n^*$ agents, if every agent but i is playing S_k then there exists a best response for i of the form $S_{k'}$

Proof Sketch

If n is large enough, the actions of a single agent have essentially no impact on the system. Thus our calculations before make the problem a Markov Decision Problem (MDP).

Theorem: Existence of Equilibria

For all M, there exist $\delta^* < 1$ and n^* such that for all systems with discount factor $\delta > \delta^*$ and $n > n^*$ agents, there is a Nash equilibrium where all agents play S_k for some $k > 0$.

Experiments show that the system converges quickly to equilibrium when agents play equilibrium strategies.

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Choosing The Price of a Job

- The maximum entropy distribution only depends on the average amount of money **m** = **M** / **n** and the strategy **k**.
- The equilibrium solution depends only on the parameters of the game and the maximum entropy distribution.
- Therefore, the optimal value of **M** (to maximize expected utility of the agents) is **mn** for some average amount of money m.

Handling New Players

- Let new players join with 0 dollars
- Adjust the amount of money to maintain the optimal ratio M/n

Recap

- The long run behavior of a simple scrip system is stable.
- There is a nontrivial Nash equilibrium where all agents play a threshold strategy.
- There is an optimal average amount of money, which leads to maximum total utility. It depends only on the discount factor δ.
- The system is scalable; we can deal with new agents by adjusting the price of a job.

Open Problems

- Can we characterize the set of equilibria and the conditions under which there is a unique nontrivial equilibrium?
- Can we find an analytic solution for the best response function and the optimal pricing?
- What are the effects of sybils and collusion?