Optimal Power Cost Management Using Stored Energy in Data Centers

February 23, 2012

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- $30 - 50\%$ of all operational expenses.

Electricity supplier: power grid

Figure: Power grid

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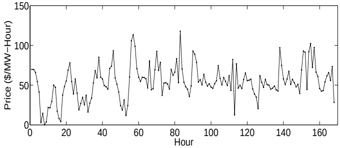

Figure 1: Avg. hourly spot market price during the week of 01/01/2005-01/07/2005 for LA1 Zone [1].

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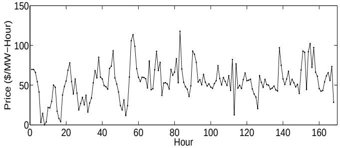

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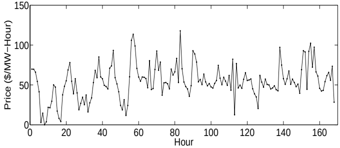

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Minimize energy consumption may not result in minimum electricity bill.

Power supply upon power grid failure

Figure: Diesel generator

Figure: Uninterrupted Power Supply (UPS), *e.g.*, Battery

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Motivation: store energy within the UPS when prices are low and discharge it when prices are high.

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- Tradeoff: optimality reduces as the storage capacity is increased.

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- Basic model: only delay intolerant workloads; served by a combination of power:
  - From power grid;
  - From battery.
- Extended model: both delay intolerant and delay tolerant workloads.

Workload model

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  - Independent of past control decisions;
  - $W(t) \leq W_{max}$.

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$$W(t) = P(t) - R(t) + D(t) \tag{1}$$

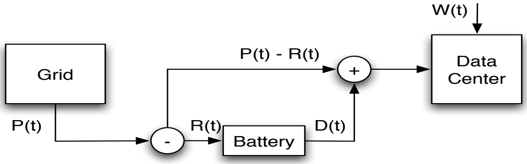

Figure 2: Block diagram for the basic model.

Battery model

Practical concerns:

- Useful lifetime v.s. how it is discharged/charged, *e.g.*, frequency and depth-of-discharge;
- Conversion loss: a portion of energy is lost upon discharging;
- Leaky battery: stored energy decreases over time.

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Assumptions in this paper:

- No conversion loss;
- Not leaky;
- Fixed cost (in dollars) C_{rc} and C_{dc} upon each recharging and discharging, respectively;
- Either recharged or discharged or neither in each slot:

$$R(t) > 0 \Rightarrow D(t) = 0, \; D(t) > 0 \Rightarrow R(t) = 0 \qquad (2)$$

Battery model (Cont.)

$Y(t)$: battery energy level in slot t:

$$Y(t+1) = Y(t) - D(t) + R(t) \tag{3}$$

and

$$Y_{min} \leq Y(t) \leq Y_{max} \tag{4}$$

- Y_{max}: maximum battery capacity;
- Y_{min}: minimum energy for the transition from power grid to DG upon failure.

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-

$$0 \leq R(t) \leq \min[R_{max}, Y_{max} - Y(t)] \tag{6}$$

$$0 \leq D(t) \leq \min[D_{max}, Y(t) - Y_{min}] \tag{7}$$

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$$C(t) = \hat{C}(S(t), P(t)) \in [C_{min}, C_{max}] \tag{8}$$

Nondecreasing function of $P(t)$; not necessarily convex or strictly monotonic or continuous.

Cost model (Cont.)

- Let P_{peak} be the peak power supply from power grid

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- Let $\chi_{min} > 0$ be a constant; $\forall P_1, P_2 \in [0, P_{peak}]$ and $P_1 \leq P_2$

$$P_1 \cdot (-\chi + \hat{C}(S, P_1)) \geq P_2 \cdot (-\chi + \hat{C}(S, P_2)) \ \forall S, \chi \geq \chi_{min}$$

Control objective

P1:

$$\begin{aligned} \min \quad & \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{P(\tau)C(\tau) + 1_R(t)C_{rc} + 1_D(\tau)C_{dc}\} \\ s.t. \quad & \text{Constraints 1,2,6,7,9.} \end{aligned}$$

where,

$$1_R(t) = \begin{cases} 1 & \text{if } R(t) > 0 \\ 0 & \text{else} \end{cases}$$

$$1_D(t) = \begin{cases} 1 & \text{if } D(t) > 0 \\ 0 & \text{else} \end{cases}$$

indicator variables for recharge or discharge operation in slot t.

P2:

$$\begin{aligned} \min \quad & \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{P(\tau)C(\tau) + 1_R(t)C_{rc} + 1_D(\tau)C_{dc}\} \\ s.t. \quad & \text{Constraints 1,2,5,9} \\ & \bar{R} = \bar{D}. \end{aligned}$$

where,

$$\bar{R} = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{R(\tau)\}, \;\; \bar{D} = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{D(\tau)\}$$

Auxiliary variable

"Queueing" state variable $X(t)$:

$$X(t) = Y(t) - V\chi_{min} - D_{max} - Y_{min}$$

with queueing dynamics:

$$X(t+1) = X(t) - D(t) + R(t)$$

Optimal control algorithm

P3:

$$\begin{aligned} \min \quad & X(t)P(t) + V[P(t)C(t) + 1_R(t)C_{rc} + 1_D(\tau)C_{dc}] \\ s.t. \quad & \text{Constraints 1,2,5,9} \end{aligned}$$

Resulting in the constraint on $P(t)$:

$$P_{low} \leq P(t) \leq P_{high}$$

where,

$$P_{low} = \max[0, W(t) - D_{max}], \; P_{high} = \min[P_{peak}, W(t) + R_{max}]$$

Then,

$$R^*(t) = \begin{cases} P^*(t) - W(t) & \text{if } P^*(t) > W(t) \\ 0 & \text{else} \end{cases}$$

$$D^*(t) = \begin{cases} W(t) - P^*(t) & \text{if } P^*(t) < W(t) \\ 0 & \text{else} \end{cases}$$

Intuition:

- Recharge the battery when $X(t)$ is negative and per unit cost is low;
- Discharge the battery when $X(t)$ is positive.

Solution to **P3** depends on the structure of unit cost function $\hat{C}(S(t), P(t))$. Two cases with closed form solutions:

- $\hat{C}(S(t), P(t))$ independent of $P(t)$: maximum weight scheduling problem;
- $\hat{C}(S(t), P(t))$ convex, increasing in $P(t)$: convex optimization with single variable $P(t)$.

Example

Let

- $W_{low} = 10$, $W_{mid} = 15$ and $W_{high} = 20$;
- $C_{low} = 2$, $C_{mid} = 6$ and $C_{high} = 10$;
- $R_{max} = D_{max} = 10$;
- $P_{peak} = 20$;
- $C_{rc} = C_{dc} = 5$;
- $Y_{init} = Y_{min} = 0$;
- $Y_{max} > R_{max} + D_{max}$;
- 5 slot in each frame.

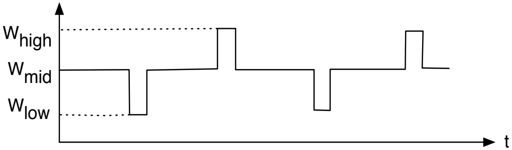

Figure 3: Periodic $W(t)$ process in the example.

Offline optimal solution:

- Recharge the battery as much as possible when $C(t) = C_{low}$;
- Discharge the battery as much as possible when $C(t) = C_{max}$.

- If $C(t) = C_{low}$, $W(t) = W_{low}$, then $P(t) = W_{low} + R_{max}$, $R(t) = R_{max}$, $D(t) = 0$;
- If $C(t) = C_{mid}$, $W(t) = W_{mid}$, then $P(t) = W_{mid}$, $R(t) = 0$, $D(t) = 0$;
- If $C(t) = C_{high}$, $W(t) = W_{high}$, then $P(t) = W_{high} - D_{max}$, $R(t) = 0$, $D(t) = D_{max}$.

Optimal cost with battery $Y_{max} > 10$: 87.0 dollars/slot;
Optimal cost without battery: 94.0 dollars/slot.

Y_{max}	20	30	40	50	75	100
V	0	1.25	2.5	3.75	6.875	10.0
Avg. Cost	94.0	92.5	91.1	88.5	88.0	87.0

Table 1: Average Cost vs. Y_{max}

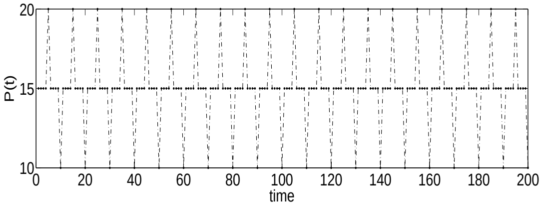

Figure 4: $P(t)$ under the offline optimal solution with $Y_{max} = 100$.

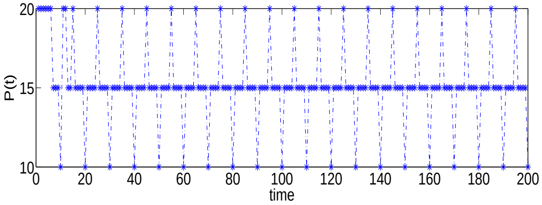

Figure 5: $P(t)$ under the Dynamic Algorithm with $Y_{max} = 100$.

Both delay intolerant and delay tolerant workloads:
$W(t) = W_1(t) + W_2(t)$,

- $W_1(t)$: delay tolerant workloads:
  - Finite average delay;
  - Bounded worst case delay;
- $W_2(t)$: delay intolerant workloads; must be served in current slot.

Let $\gamma(t) \in [0, 1]$ be the fraction of power to serve delay tolerant workload in slot t,

- Power for delay tolerant workload: $\gamma(t)(P(t) - R(t) + D(t))$;
- Power for delay intolerant workload: $(1 - \gamma(t))(P(t) - R(t) + D(t))$.

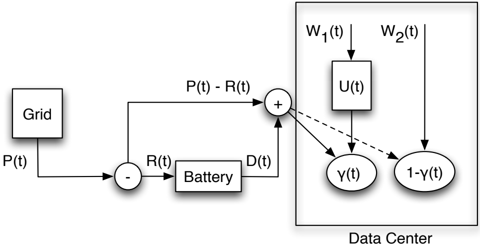

Figure 6: Block diagram for the extended model with delay tolerant and delay intolerant workloads.

Finite average delay

Define $U(t)$ as the unfinished work for delay tolerant workload in slot t:

$$U(t+1) = \max[U(t) - \gamma(t)(P(t) - R(t) + D(t)), 0] + W_1(t)$$

If $U(t)$ is stable, then we have finite average delay.

Bounded worst case delay

Despite queue $U(t)$, define $\epsilon-$persistece queue $Z(t)$:

$$Z(t+1) = \max[Z(t) - \gamma(t)(P(t) - R(t) + D(t)) + \epsilon 1_{U(t)}, 0]$$

where,

$$1_{U(t)} = \begin{cases} 1 & \text{if } U(t) > 0 \\ 0 & \text{O.W.} \end{cases}$$

Let $V_{max} \triangleq \frac{Y_{max} - Y_{min} - R_{max} - D_{max}}{\chi_{min} - C_{min}}$.

Theorem

Suppose the initial battery charge level Y_{init} *satisfies* $Y_{min} \leq Y_{init} \leq Y_{max}$. *Then, the dynamic algorithm with any fixed parameter* $0 \leq V \leq V_{max}$ *for all* $t \in \{0, 1, 2, \ldots\}$ *results in the following performance guarantees:*

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- $-V\chi_{min} - D_{max} \leq X(t) \leq Y_{max} - Y_{min} - D_{max} - V\chi_{min}$ *for all* t*;*

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- $Y_{min} \leq Y(t) \leq Y_{max}$ *for all* t;
- *All control decisions are feasible;*

- If $W(t)$ and $S(t)$ are i.i.d. over slots, then the time average cost under the dynamic algorithm is within B/V of the optimal:

$$\lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{P(\tau)C(\tau) + 1_R(\tau)C_{rc} + 1_D(\tau)C_{dc}\} \leq \phi_{opt} + B/V$$

where, $B = \frac{\max[R_{max}^2, D_{max}^2]}{2}$ and ϕ_{opt} is optimal solution to **P1**.

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- Smart application of Lyapunov theory: use battery power level $Y(t)$ to mimic the packet queue in traditional Lyapunov theory;
- Good presentation: justify each item in the model with references and practical meanings;
- Solid analysis: traditional Lyapunov theory; appears to be complicated; in fact, labor work.

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Apply Lyapunov theory on the problems with following properties:

- Dynamic system varies slot-by-slot;
- State variables depends on control decisions and evolves slot-by-slot, *e.g.*, packet queue or battery power level.

Thank You!

Q&A