Open Problems in Algorithmic Game Theory and Mechanism Design

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This is a note I took from ASI 2010: Theory and Applications on Algorithmic Game Theory. As the lectures do not provide slides currently, I just transfer the essential part of the slides into this note. The open problem session is given by Prof. Deng, Xiaotie from City University of Hong Kong.

I may not aim at solving the open problems now, but I think I could keep them in mind and think about their implications and applications, maybe some day when they are solved, we can use the results to some networking systems.

1^{st} Problem (Paul Spirakis)

Let G be an undirected graph of n vertices. Let A be its adjacency matrix. Let. Let $\lambda_1, \lambda_2, \cdots, \lambda_m$ be the positive eigenvalues of A.

(a) Prove or disprove:

$$\forall_i = 1, \cdots, m, \quad \lambda_j \leq \frac{n}{\sqrt{i}}$$

(b) Prove or disprove: if G is connected then,

$$\lambda_j \leq \frac{n}{i}$$

(I have some doubts on the notations on the slide, and at that time a professor from HKUST also raised his challenge, so we will check it later.)

(Also, I suspect the eigenvalue should be $\lambda_1 > \lambda_2 > \cdots > \lambda_m$).

2^{nd} Problem

Find a connected graph G for which $\sum_{i=1}^{m} \lambda_i/n$ is not $\Theta(1)$, where $\lambda_1, \lambda_2, \cdots, \lambda_m$ are the positive eigenvalues of the adjacency matrix A of G.

3^{rd} Problem

Find sufficient conditions for local optima (like he Karush Kuhn, Tucker points) for *cubic* programs under linear constraints. Are such conditions equivalent to the stationary points condition (Fermat's Rule)?

4^{th} Problem

Let $s = (\bar{x}, \bar{y}, \bar{a}, \bar{b})$ be a stationary point at $t = 1/2$ of the parameterized program $MS(t) : minimize (1-t)(a - x^T Ay) + t(b - x^T By)$ so that:

$Ay \leq a$
$B^T x \leq b$
$1^T x = 1$
$1^T y = 1$
$a, b \in \mathbb{R}, x, y \geq 0$

Find the t_1, t_2 so that s remains stationary point of $MS(t)$ for the whole interval $[t_1, t_2]$.

5^{th} Problem

Is there a way to force the regrets $a - x^T Ay$ and $b - x^T By$ to the almost equal for a $t > 1/5$ in the $MS(t)$ when they are evaluated at a stationary point of $MS(t)$?

6^{th} Problem

Is the random sampling auction for digital goods 4-competitive? Given values $v_1 \geq v_2 \cdots \geq v_n$ and benchmark $G(v) = \max_{i \geq 2} iv_i$ (the optimal single price revenue with at least two winners). An auction is β-competitive if its expected revenue, in worst case over v is at least $G(v)/\beta$.

The random sampling auction randomly partitions the bidders into two sets (with a fair coin for each bidder), computes the optimal sale price for each set, and offers each sale price to the opposite set, This auction is clearly no better than 4-competitive, *e.g.*, on $v = (1.1, 1)$.

Show it is exactly 4-competitive. Recently, Alaei *et. al.* (2009) showed that it is at worst 4.7-competitive but it is widely believed that the correct answer is 4. Prove or disprove.

7^{th} Problem

The following mathematical puzzle is related to designing deterministic and competitive auctions, see Aggarwal *et. al.* (2005). The "deterministic dice hat puzzle" is the following: There are n players indexed $1, \cdots, n$. Each player has a hat on their head the is a distinct shade of red. Each player can see the hats of all other players and but cannot see their own hat. Each player must simultaneously (without communication) choose one of k actions (*e.g.*, corresponding to sides of a k-sided die). The players win if among players with the m darkest hats at least m/k should chose each action, for all m.

Find an asymmetric deterministic strategy for any $k \geq 3$.

7^{th} and 8^{th} Problem will be Discussed Next Time

10^{th} Problem

What's the complexity of finding a Nash equilibrium in a tree, even when the degree is bounded?

11^{th} Problem

Theorem. (Lipton, Marakakis, Metha'06) For any $\varepsilon > 0$, computing a $(1+\varepsilon)$-approximate Nash equilibrium can be solved in time $N^{\log N}$, where N is the input size.

Can we convert it to a polynomial-time algorithm for any fixed $\varepsilon > 0$?

- The best known result is $\varepsilon = -.3393$ (Tsaknakis, Spirakis'07).

- Tow major challenges: $\varepsilon = 1/3$ and PTAS.

- [Daskalakis, Papadimitriou'08,'09] There is a PTAS for anonymous games and a class of games whose equilibria are guaranteed to have small value.

12^{th} Problem

Given a graph G, where each node represents a player and the utility of every player is only affected by his neighbors in G.

- more realistic in a non-cooperative and distributed setting.

- concise representation.

$13s^{th}$ Problem

There are n players and set R of resources.

Each resource r has cost function $f_r : \{1, \cdots, n\} \to R$.

The strategy set S_1 of each player is subset of resources, *i.e.*, $S_1 \in R$.

Given the selected strategies $s_i \in S_i$ of each player, the cost (*i.e.*, negative utility) of player i is

$$c_i(s_1 \cdots, s_n) = \sum_{r \in s_i} f_r(n_r)$$

where n_r is the number of players that select a containing r.

– I did not quite understand this problem, I will find a chance to ask the presenter.