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Research article

Volatility Analysis of Bitcoin Price Time Series

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Abstract: Bitcoin has the largest share in the total capitalization of cryptocurrency markets currently reaching above 70 billion USD. In this work we focus on the price of Bitcoin in terms of standard currencies and their volatility over the last five years. The average day-to-day return throughout this period is 0.328%, amounting in exponential growth from 6 USD to over 4,000 USD per 1 BTC at present. Multi-scale analysis is performed from the level of the tick data, through the 5 min, 1 hour and 1 day scales. Distribution of trading volumes (1 sec, 1 min, 1 hour and 1 day) aggregated from the Kraken BTCEUR tick data is provided that shows the artifacts of algorithmic trading (selling transactions with volume peaks distributed at integer multiples of BTC unit). Arbitrage opportunities are studied using the EUR, USD and CNY currencies. Whereas the arbitrage spread for EUR-USD currency pair is found narrow at the order of a percent, at the 1 hour sampling period the arbitrage spread for USD-CNY (and similarly EUR-CNY) is found to be more substantial, reaching as high as above 5 percent on rare occasions. The volatility of BTC exchange rates is modeled using the day-to-day distribution of logarithmic return, and the Realized Volatility, sum of the squared logarithmic returns on 5-minute basis. In this work we demonstrate that the Heterogeneous Autoregressive model for Realized Volatility Andersen et al. (2007) applies reasonably well to the BTCUSD dataset. Finally, a feed-forward neural network with 2 hidden layers using 10-day moving window sampling daily return predictors is applied to estimate the next-day logarithmic return. The results show that such an artificial neural network prediction is capable of approximate capture of the actual log return distribution; more sophisticated methods, such as recurrent neural networks and LSTM (Long Short Term Memory) techniques from deep learning may be necessary for higher prediction accuracy.

Keywords: bitcoin price; foreign exchange rate; volatility modeling; transaction volume distribution; artificial neural network; logarithmic return

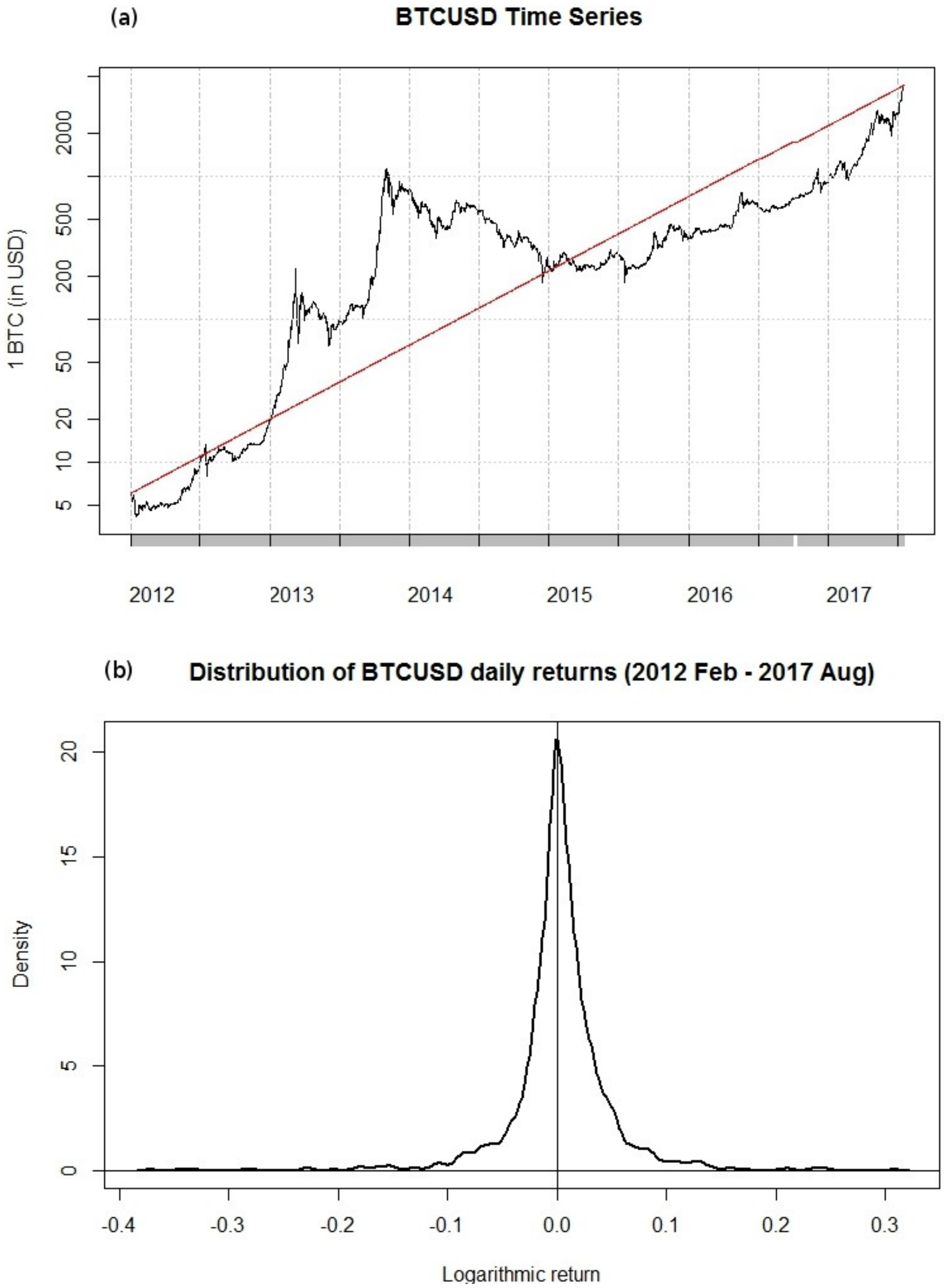

Figure 1. (a) BTCUSD time series for the past 5 years (data source: Investing.com) in logarithmic scale, and (b) The distribution of the corresponding daily logarithmic returns.

1. Introduction

The price of Bitcoin to US Dollar over the past five years is an example of the (super) exponential growth hardly seen in finance in any field except for the cryptocurrency markets. The data in Figure 1 (a) are shown in logarithmic scale, with the red line indicating the average daily return of 0.3283% per day. The time period shown comprises 2013 daily closing values with the logarithmic returns confined in the interval from -0.371564 (2012-08-19) to 0.308301 (2013-04-17), attesting to the large density and magnitude of extreme events as depicted in Figure 1 (b). The main source of Bitcoin demand and majority of trading volume comes from exchange markets in China. Worldwide, Bitcoin as the leading cryptocurrency aspires to become a rudimentary means of payment, being gradually accepted by online stores for payment of goods, cafes and restaurants, even some academic institutions for payments of tuition. Still, the share of Bitcoin payments per GDP does not reach even a single per cent in any country; the growth of its value is therefore propelled by risky speculation on its broader acceptance as a common means of payment. For instance, Estonia at present plans to introduce its own national cryptocurrency in line with the concept of electronic citizenship. Whether the cryptocurrencies prevail or not, and which one would eventually become a major standard is still an open question. Thus all the cryptocurrencies, including Bitcoin, are sensitive to major event news such as exchange market bankruptcy, fraud and occasional market crashes on the negative side, or the unheard of lucrative speculation opportunities on the positive side that result in herding behavior. In brief, extreme events are abundant in the cryptocurrency markets, Bitcoin being no exception here.

We have collected representative data sets from various Bitcoin exchanges as well as data from Bloomberg that aggregate major exchanges into relevant Bitcoin price indices, from the scale of tick events, through 1min, 5min, 1hour and 1day sampling resolution. In what follows, the distributions of logarithmic returns, trading volumes at various time scales, arbitrage opportunity windows, prediction of Realized Volatility and Bitcoin daily logarithmic return prediction by means of neural networks will be discussed, thus providing different angles of view on the extreme events in the Bitcoin market and its volatility. The paper is organized as follows. Section 2 provides a brief literature review of the still rather scarce but rapidly increasing research work on quantitative Bitcoin analysis. Data analysis methods are explained in Section 3, followed by Results and Discussions in Section 4. The paper is closed with Conclusion in Section 5. Exchange Rates are denoted as FX1 FX2 (1 unit of FX1 in terms of an amount in currency FX2). Same notation is applied to Bitcoin prices in standard currencies. We use the code of BTC for Bitcoin throughout although the notation of XBT is also common. Log returns are based on the natural logarithm.

2. Literature Review

The origins of Bitcoin date back to the end of October, 2008, when a developer using the pseudonym Satoshi Nakamoto published a paper entitled “Bitcoin: A peer-to-Peer Electronic Cash System” (https://bitcoin.org/bitcoin.pdf). The actual cryptocurrency software was released in the open source domain in January 2009. Since then Bitcoin established itself as the major cryptocurrency. Over the last five years Bitcoin price has increased more than 700 times, there is at least 35 Bitcoin exchange markets where Bitcoin prices are quoted in standard currencies, each with the daily transaction volume above 1 million USD. Bitcoin is increasingly accepted in real economy as a means of payment. The aspiration to become a major world’s means of payment is still far from accomplished, and investment in Bitcoin is a risky strategy. The number of research papers in major journals related to Bitcoin has been limited, and started to surge just recently as summarized in the following.

Balcilar et al. (2017) discuss the predictability of Bitcoin returns and volatility based on transaction volume, finding out that in the quantile range of 0.25 to 0.75, i.e., extreme events excluded, volume is an important predictor variable. Bariviera et al. (2017) study the stylized facts in

Bitcoin markets, showing that the Hurst exponents have undergone significant changes in the early years after Bitcoin introduction but stabilizes recently. Their multiscale analysis shows a self-similar process characteristics. The prospects of Bitcoin and the entire cryptocurrency markets are nicely summarized at the accessible level in the review work by Extance (2015). Econometric methods, in particular the GARCH model, are applied to volatility estimation of Bitcoin by Katsiampa (2017). Sentiment analysis using computational intelligence methods for Bitcoin fluctuation prediction based on user comments are applied in Kim et al. (2016). Kristoufek (2013) compares the Bitcoin phenomenon to other Internet phenomena of the present day; in a different paper Kristoufek (2015) also analyzes the main drivers of Bitcoin price, such as the demand in China, using wavelet coherence analysis.

There are also increasing more recent papers on the fundamental importance of Bitcoin and its security aspects. The role of Bitcoin in present day finance is questioned by Bouri et al. (2017b) and Dyhrberg (2016). Bitcoin market efficiency is studied by Urquhart (2016) with the conclusion that it is still transitioning to the regime of market efficiency. Bitcoin price clustering at round numbers is observed by Urquhart (2017). Price dynamics and speculative trading in Bitcoin is studied by Blau (2017) with the conclusion that speculative behavior cannot be directly linked to the unusual volatility of the Bitcoin market. Cheah and Fry (2015) also explore the role of speculation in the Bitcoin market from the viewpoint of Bitcoin's fundamental value. Dwyer (2015) examines the Bitcoin economy with the conclusion that Bitcoin is likely to limit government's revenue from inflation. Branvold et al. (2015) studies the role of various Bitcoin exchanges in the price discovery process, indicating that the information share is dynamic and significantly evolving over time. Security problems, inherent in the cryptocurrency world, are discussed by Bradbury (2013) for the case of Bitcoin. Analysis is also available for the period of crash in 2013 in the work of Bouri et al. (2017). There is a number of intriguing extreme events in the history of Bitcoin market which can be discussed from multi-disciplinary perspective, such as using the methods of Franzke (2012).

The existing research gap we are aiming at is the substantial lack of understanding of the Bitcoin price process dynamics and the mostly unexplored applicability of standard econometric, technical trading, and machine learning approaches.

The main contributions of the present paper, in view of the previous work, are as follows: (1) we provide theoretical and empirical bounds on Bitcoin arbitrage opportunities using different standard currency pairs, discovering major opportunity window at the Chinese market; (2) we show that the econometric HARRVJ model with adjusted parameters is well capable of capturing the dynamics of realized volatility time series, and (3) it is demonstrated that a feed-forward neural network architecture is capable of learning the statistical distribution of the logarithmic return but it exhibits rather limited prediction ability for the market trend and return magnitude on the daily sampling scale. In addition, we also provide a case study insight into the Kraken market liquidity and transaction volumes.

3. Data Analysis Methods

The Bitcoin prices in terms of a standard currency CRS, i.e. the BTCCRS time series, are denoted as B_i, assuming an equidistant time sampling represented by integer sequence i, i=1…n. In what follows we use the sampling frequencies of 1 second, 1 minute, 5 minutes, 1 hour, and 1 day.

3.1. Data transformation

The logarithmic return is defined as

$$R_i = \log\left(\frac{B_i}{B_{i-1}}\right). \tag{1}$$

In Eq. (1), B_i stands for the price of Bitcoin at time step i. The advantage of the logarithmic return over the prices is a symmetric representation of price increase and decrease by the same multiple, which differ only by the sign of the respective log return; constant price levels are represented by the zero return, and, importantly, unlike from the non-stationary price process, the time series of logarithmic return can often be approximated as stationary. The distribution of the daily log returns for BTCUSD time series is shown in Figure 1 (b), illustrating the approximate symmetry of the R-distribution. We notice, for instance, that whereas Figure 1 (a) shows a clear upward trend of the price process, Figure 1 (b) is approximately symmetric in regard to the change of the sign of the logarithmic return; the upward trend is a consequence of the small positive value of the mean of the distribution (exponential growth process shown as a red line in Figure 1 (a).

3.2. Data aggregation

In order to produce distribution of Bitcoin trading volumes, which is usually unavailable from the standard high frequency data sources, we use the application interface (API) of Kraken exchange market, collecting the last 5,000 transactions every minute. The data show the transaction time (resolved to 0.1 ms), price, volume (in BTC), and trade direction. Using the last preceding transaction available, we define the OHLCV data (Open, High, Low, Close prices and Volume of transactions) on 1 second grid, which are then aggregated for transformation to longer sampling periods.

3.3. Measuring arbitrage spread

Let us assume Bitcoin prices in two currencies, i.e. BTCFC1 and BTCFC2. A hypothetical arbitrage transaction can be defined by buying 1 BTC in currency FC1 (expense –BTCFC1), selling it in currency FC2 (revenue BTCFC2), then transforming the received cash back to currency FC1 (foreign exchange rate FC2FC1=1/FC1FC2). The profit rate (relative to the Bitcoin price BTCFC1) is then

$$\delta_{1,2} = \frac{-BTCFC1 + \frac{BTCFC2}{FC1FC2}}{BTCFC1} = \frac{BTCFC2}{BTCFC1}/FC1FC2 - 1. \tag{2}$$

In other words, the profit rate is taken as the ratio of the Bitcoin-implied foreign exchange (BTCFC2/BTCFC1≡FC1FC2(Bitcoin)) and the actual exchange rate, FC1FC2. Let us notice here that no transaction cost is assumed here; its incorporation is straightforward by distinguishing between the supply and demand part for each price or exchange rate in Eq. (2). The direction of the transaction, FC1-FC2 is important; it holds

$$\delta_{1,2} + \delta_{2,1} \leq 0. \tag{3}$$

3.4. Modeling realized volatility

Realized volatility, RV, is defined by

$$RV_i = \sum_{j\in\{i\}} R_j^2, \tag{4}$$

where the log returns R with index j are taken relative to a high-frequency sampling grid (5 minutes in our case) within the duration of the longer sampling period with index i (1 day for daily returns).

The regression equation for the Heterogeneous Autoregressive model for Realized Volatility RV including jumps Andersen (2007) applies the square root transform to the RV values, in particular

$$\sqrt{RV}_{i+1} = \beta_0 + \beta_1\sqrt{RV}_i + \beta_2\sqrt{RV}_{i-5} + \beta_3\sqrt{RV}_{i-10} + \beta_4\sqrt{J}_i + \beta_5\sqrt{J}_{i-5} + \beta_6\sqrt{J}_{i-10}, \tag{5}$$

where the values J_i are the jumps defined by Andersen (2007). The R-package highfrequency by Boudt et al. (2017) is used for implementation. The above model was developed in the series of papers by Andersen et al. (2000, 2001, 2007) and Barndorff-Nielsen and Shpephard (2004). Andersen et al. (2000, 2001, 2007) sample the past series of Realized Volatility back to one week (-5 days) and one month (-22 days); they also use a quadratic model of Realized Volatility rather than the square root, but suggest that the square root version or even a log (RV) version may be appropriate based on the process. In Eq. (5), the sampling horizons are selected as daily, 5 days back, and 10 days back; the change to 10 days back sampling rather than monthly sampling has been motivated by the observation that the coefficient β_3 becomes statistically significant for the -10 day sampling rather than -22 day sampling. Importantly, since the Bitcoin is traded 7 days a week, the -5 days period does not correspond to a full week trading; similarly, one month period would use -30 days rather than -22 days of sampling delay. Aiming at a reliable statistical estimate of the process in Eq. (5), and given the rather short time scale of the HARRVJ process specific to the Bitcoin market, we have therefore settled at the (1,5,10)-day parameter selection for the model.

3.5. Predicting daily log returns

Machine learning has been increasingly applied in the field of quantitative finance for prediction of prices or logarithmic returns. Here we briefly outline our method of choice. We adopt the feed-forward neural network in 2-hidden layer configuration, using the past 10-day moving window for daily log return sampling as predictors. The log returns are scaled to zero-mean and the standard deviation of 0.08 using the gradient vanish threshold of 0.005. The neural network is initialized at random, trained on the first two thirds of the BTCUSD data set shown in Figure 4, and tested for accuracy on the remaining part of the time series. The R-package neuralnet by Fritsch and Guenther (2016) is used for implementation.

4. Results and Discussions

4.1. Price and logarithmic return distribution

The BTCUSD price history over the past five years is depicted in Figure 1(a) with the red line showing the average logarithmic return (exponential explosion of prices with the exponent of 0.328 percent per day). The log return distribution in Figure 1 (b) shows the fat tail covering the extreme event region of bubbles and crashes. The fat tail distribution of the average logarithmic return is most elementary and universal financial time series characteristics. The price of Bitcoin is highly volatile and not supported by “fundamentals” that is, any real economy in behind of cryptocurrency, and may have random walk (martingale) property which is one of the stylized facts in financial time series.

Nevertheless the graph clearly demonstrates that the simple buy and hold strategy applied over several years is very lucrative. This may reflect the rising worldwide bets on the fulfillment of the major aspiration of Bitcoin, and other cryptocurrencies in general, to become the new prevailing ways of payment for the entire global market.

4.2. Distribution of trading volumes (Kraken, BTCEUR)

We have collected all transactions available from the online API of the Kraken Bitcoin market, which is a double auction market, and transformed them onto regular high frequency grids of 1 sec, 1 min, 1 hour and 1 day. The period covered consists of 88 days from 2017-05-17 to 2017-08-12. The resulting distribution of transaction volume is shown in Figure 2. The results in Figure 2 (d) are only tentative, since the number of data points is relatively small. There are small peaks in Figure 2 (a) at the volumes of 1, 2, 3 and 5 Bitcoins, which correspond to volume distribution for larger selling orders streamed into the market distributed into relatively small Bitcoin amounts.

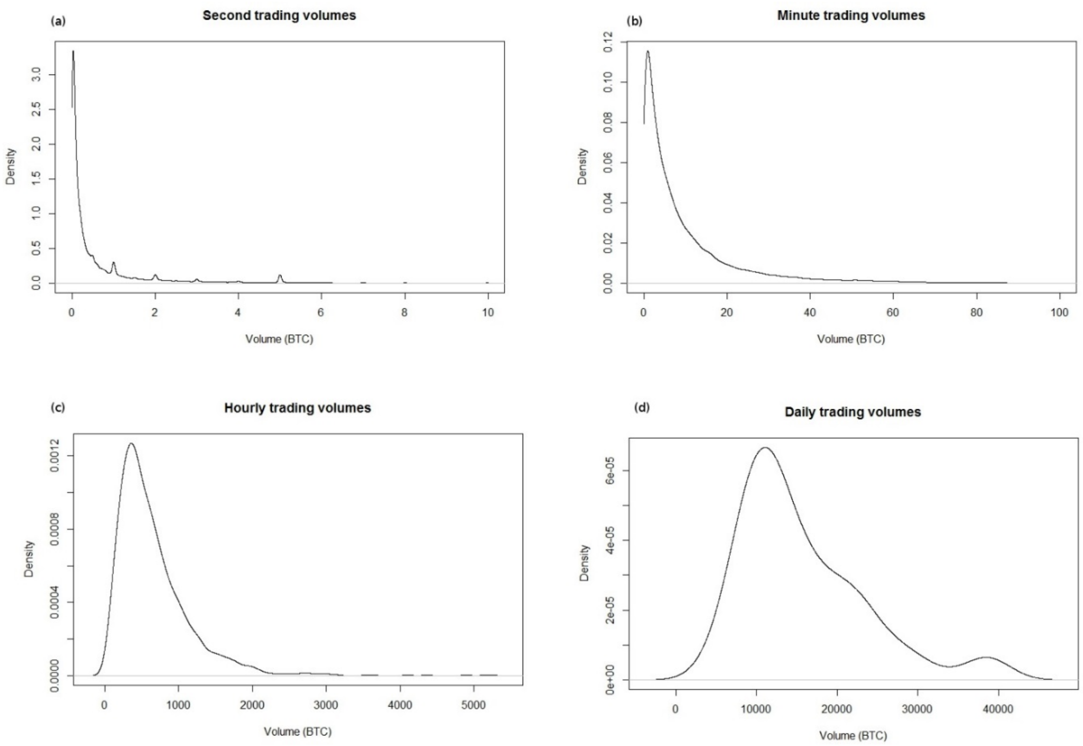

Figure 2. Aggregate trading volumes for all BTCEUR transactions at the Kraken market over the past 5 years on the (a) second, (b) minute, (c) hour, and (d) day time scales.

The volume distribution shown in the four panels of Figure 2 linearly scales with time to larger magnitudes, while the small peaks corresponding to distributed transaction packing on 1 sec scale disappear at larger, non-technical trading time scales of 1 min and 1 hour. Since there is yet very little quantitative research on the liquidity of Bitcoin markets, these data provide a rigorous insight into the typical aggregate volume of transactions that can be realized over multiscale time periods within a certain market such as Kraken.

According to the data.bitcoinity.org server (https://data.bitcoinity.org/markets/volume/30d?c=e&t=b) the daily trading volumes in the second half of 2017 rarely fall below 100 thousand BTC; the market share of Kraken in the trade volume is estimated to be about 8%.

We remark here that the Bitcoin price data from the Kraken market are unique in the sense that the time stamp available for all recorded transactions is resolved to 0.1 millisecond. Most of the Bitcoin transaction repositories collecting Bitcoin data, such as http://api.bitcoincharts.com/v1/csv/, contain transactions where the date and time is represented in the format of Unix time (integer indicating number of seconds elapsed from January 1st, 1970, 0:00 AM). As such, the trades are all resolved up to the unit of one second; consequently, at markets with frequent trading and high liquidity, it is not uncommon that several trades share the same time; thus the study of inter-arrival time distribution becomes difficult or impossible. To our knowledge, Kraken is the only market providing the data with 0.1 millisecond resolution using the online API. Although this had no impact on the 1 sec scale of the trade volume distribution, the data in principle allow for inter-arrival distribution fits such as the self-exciting process of Hawkes (1974).

4.3. Arbitrage opportunities at Bitcoin markets

The data sets used for the evaluation of arbitrage opportunities are XBTEUR, XBTUSD, XBTCNY and the foreign exchange rates EURUSD and USDCNY, all on 1 hour scale, from 2013/2/8 to 2017/4/7. Using the methods of Section 3.3, the arbitrage spread (transaction profit rate) is shown in Figure 3 (a) for BTCEUR-BTCUSD currency pair, and (b) for BTCUSD-BTCCNY currency pair. Notice the fat tail on the right-hand-side of the distribution in Figure 3 (b), showing substantial arbitrage windows. While both distributions in (a) and (b) are roughly similar in shape, the width is much larger when the BTCCNY market enters into the arbitrage transaction. Care should be taken, however, in regard to the interpretation of these distributions. First, it takes from 10 minutes to hours in extreme cases to record the new transaction in the blockchain. Thus, transactions across markets are excluded from the arbitrage opportunity windows. Second, given the exponential burst in Bitcoin prices, for most Bitcoin holders, who are not financial institutions specialized in technical trading, the most profitable strategy may simply be to hold the Bitcoin over time rather than to engage in trading which requires instant access to foreign exchange markets. Third, there is no transaction fees considered in Eq. (2). Nevertheless, Figure 3 (b) shows that arbitrage possibilities may, at least theoretically, exist even in case of substantial real transaction fees. It is a debatable issue whether the Bitcoin market is efficient. It calls for further consideration.

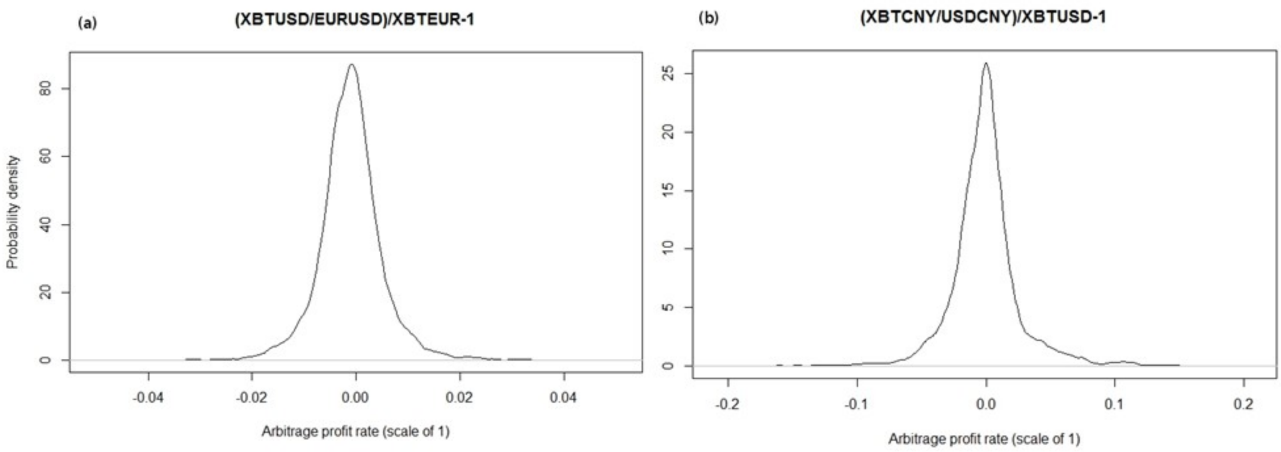

Figure 3. Bitcoin arbitrage spread (transaction costs excluded) as based on the 1-hour trading data (data source: Bloomberg) for (a) USD-EUR currency pair and (b) USD-CNY currency pair.

4.4. Model of realized volatility

The estimation of Realized Volatility using the HARRVJ model of Eq. (5) in Sec. 3.4 produces reasonably good results as shown in Table 1 and Figure 4. In Figure 4 (a), the 5-min time series of BTCUSD are shown that were used for the computation of realized volatility. Also, the lower panel of Figure 4 (a) depicts the corresponding logarithmic returns.

Table 1. HARRVJ regression coefficients for BTCUSD (5 min, 1 day).

Coef.	Estimate	Std. error	t-value	p-value	Signif.
beta0	0.010307	0.001459	7.065	3.22E-12	***
beta1	0.344821	0.05796	5.949	3.86E-09	***
beta2	0.517929	0.115008	4.503	7.57E-06	***
beta3	-0.22684	0.111337	-2.037	0.0419	*
beta4	-0.12326	0.077452	-1.591	0.1119	
beta5	-0.86087	0.147957	-5.818	8.27E-09	***
beta6	0.856319	0.160656	5.33	1.24E-07	***

The actual and predicted realized volatility are shown in Figure 4 (b). Table 1 gives the values and statistical significance of the regression coefficients. As mentioned in Sec. 3.4, the parameters of the process (1 day, 5 day, and 10 day sampling) were selected in order to obtain as many significant coefficients as possible. The choice of 10-day delay instead of 1 month delay has proven necessary to obtain the significant value of the coefficient beta 3. We could find no parameterization that would result in the significant value of coefficient beta 4 (for the jump at time *t*). It can be seen in Figure 4 that not all the peaks of the Realized Volatility distribution correspond to extreme values of daily log returns; some are due to large intraday volatility process instead. In brief, we consider the agreement of the observed and forecasted realized volatility in Figure 4 very good. It hereby appears that the HARRVJ process is well applicable to the Bitcoin market. This by far cannot be expected a priori and constitutes one of the empirical findings of this paper.

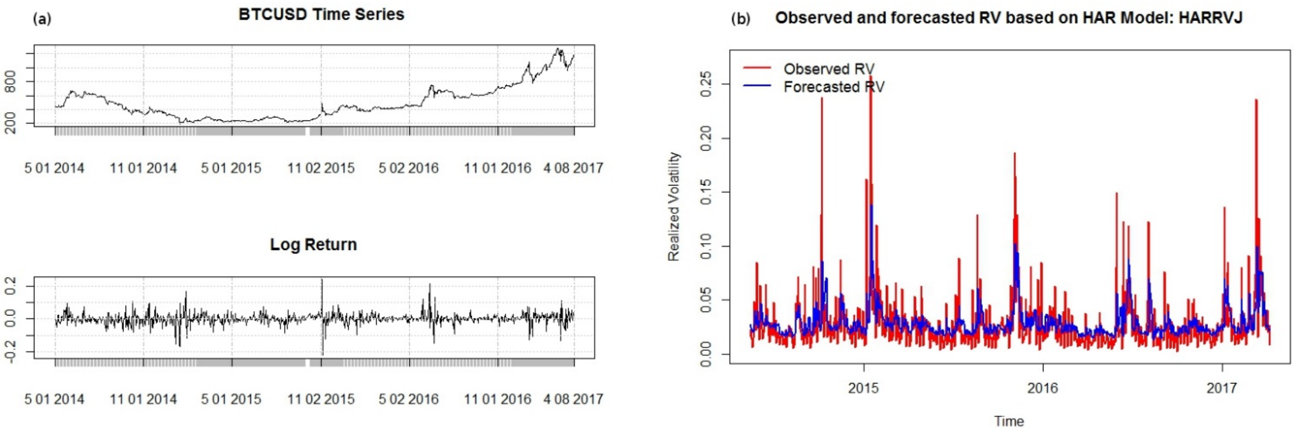

Figure 4. 5-min sampled BTCUSD time series in upper panel (a), the derived logarithmic returns on daily basis in lower panel (a), and the actual and HARRVJ model predicted realized volatility are shown in (b). Date format: (M) M DD YYYY.

4.5. Neural network prediction of log returns

Using the artificial neural network outlined in Sec. 3.5, we show that the method is roughly capable of capturing the clustering of extreme events in the market, where the absolute value of the log return is high, albeit there are some differences in Figure 5 (a). The reproduction of the density of the daily log return density in Figure 5 (b) is quite satisfactory. It is an open question whether more advanced machine learning methods can provide better results, or whether the market is relatively efficient, hence difficult to predict by any market-history based computational means.

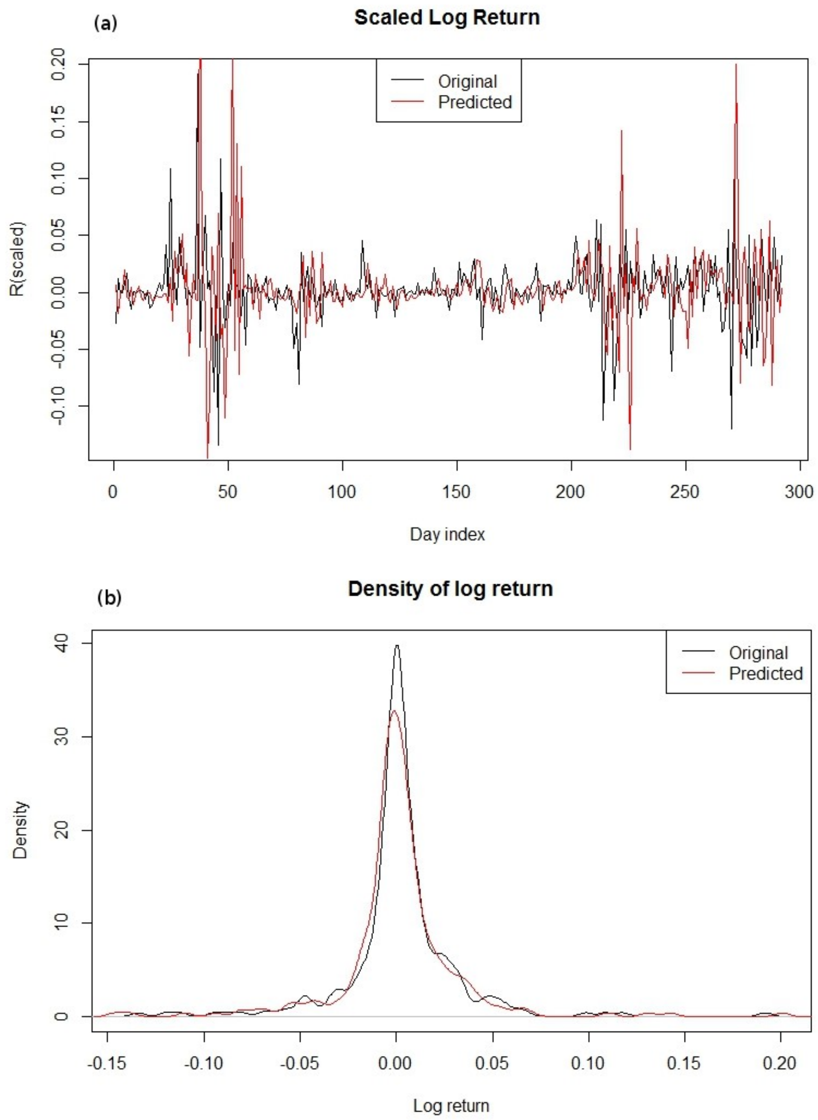

Figure 5. (a) Scaled daily logarithmic return as used for the neural network prediction on the data from Figure 4. The last one third of the time series is shown (testing dataset). (b) Comparison of the actual and predicted log return distribution.

In particular, a feed-forward neural network with two fully interconnected hidden layers, input layer of size 10, and output layer of size 1, when used as a statistical regression technique on scaled data of daily logarithmic volume, is capable of capturing the shape of the logarithmic return density distribution. The learning algorithm is the Backpropagation method explained in detail in Hsieh

(2009). A closer look at the comparison of the actual and predicted log returns shows a discrepancy in the peak location, sign of the logarithmic return, or the magnitude value. We find it interesting, nevertheless, that the overall statistical distribution can be learned by the neural network in this case. A detailed study of machine learning algorithms for Bitcoin price prediction will be deferred to a subsequent paper, since it requires more advanced network topologies than the one applied at the present work.

5. Conclusion

The present work is an empirical investigation into the properties of Bitcoin markets. Volatility of Bitcoin prices was studied from various viewpoints, ranging from the stylized features of logarithmic return distribution, transaction volume distribution at multiple time scales, arbitrage opportunities on 1-hour trading scale for the currency pairs of EUR-USD and USD-CNY, econometric analysis of the time series of Realized Volatitlity, and classical machine learning prediction of logarithmic returns for BTCUSD on daily time scale by means of the artificial neural network. The time series of Bitcoin prices are substantially more volatile than those of EURUSD exchange rates, for the sake of comparison, with market bubbles and crashes relatively abundant. Substantial arbitrage opportunities are available for currency USD or EUR currency pairs involving CNY. The HARRVJ model captures well the dynamics of daily Realized Volatility as aggregated on the 5-minute grid. Standard neural network prediction of daily logarithmic returns of BTCUSD time series is capable of reproducing the extreme event clustering feature and the shape of the distribution of logarithmic returns; more sophisticated methods will be applied in a future study to discover the ultimate prediction accuracy limits with deep learning algorithms.

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Conflict of Interest

All authors declare no conflict of interest.

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