Final Project

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Acknowledgements

Sincere thanks to Dr. Pazzula's patient and constructive guidance this semester. (Please forgive my clumsy English in writing an acknowledgement without GPT's help :) Your lesson is very meaningful to me both in theoretically introducing multiple common finance risk models and practically having us implement that. I seldom feel that a piece of homework is of so much importance and delicacy, which naturally helps with our understanding of risk models. To be honest, I kind of feel it difficult to catch up in class, where many concepts are first to me, and sorry to say that, but the distractions of devices from time to time make it harder. Your slides and homework are such an important part of my study that I decided to formally write this before any further answers to the project. So,
THANK YOU MANY A TIME!

(Some suggestions: ever think of breaking the homework down and add frequencies? Possibly more detailed slides? (I said it is delicate with almost no empty talk, but sometimes difficult to understand if neglect some minor part) Lecture guests speaking more realistic, business-world risk management? (Sadly, we lose one lecture this term)

Part 1

(For Part 1&2, I use the third method where consider rf as a factor)

1) CAPM regression

   An easy one using the built-in function of OLS. We Regress $Stock\_Excess\_Return$, which equals $daily\_stock\_return - rf\_rates$, on $Market\_Excess\_Return$, which equals $daily\_SPY\_return - rf\_rates$.

   Here are some of the regression summaries.

```
AAPL                          OLS Regression Results
==============================================================================
===
Dep. Variable:                   AAPL   R-squared:                       0.526
Model:                            OLS   Adj. R-squared:                  0.524
Method:                 Least Squares   F-statistic:                     273.9
Date:                Fri, 18 Apr 2025   Prob (F-statistic):           6.62e-42
Time:                        21:31:07   Log-Likelihood:                 829.43
No. Observations:                 249   AIC:                            -1655.
Df Residuals:                     247   BIC:                            -1648.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
===
               coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
---
const        0.0008      0.001      1.389      0.166      -0.000       0.002
SPY          1.1038      0.067     16.550      0.000       0.972       1.235
==============================================================================
===
Omnibus:                       35.915   Durbin-Watson:                   1.711
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              155.290
Skew:                          -0.446   Prob(JB):                     1.90e-34
Kurtosis:                       6.764   Cond. No.                         121.
==============================================================================
===

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is
correctly specified.
NVDA                          OLS Regression Results
```

```
==============================================================================
===
Dep. Variable:                   NVDA   R-squared:                       0.299
Model:                            OLS   Adj. R-squared:                  0.296
Method:                 Least Squares   F-statistic:                     105.2
...
==============================================================================
===
```

2) The next part to calculate the return & risk attribution is kind of tricky. Let's break it down.

   First, define a kt function that computes the portfolio's Carino k factor.

   Second, for each portfolio, keep track of the daily prices of its holding on each stock, (an easier way to calculate the weights)

   Third, return attribution is then out by sum the dot of the kt, weight, and systematic attribution for $stock_i$ (which equals $\beta_i\left(R_{SPY} - R_{rf}\right)$)

   Last, the risk attribution is even simpler. We do a regression for $R_p - R_{rf}$ on $R_{SPY} - R_{rf}$, (which I think could be approximated by the weighted beta of stocks, but I didn't do in that way). Then multiply it by σ_p, we would get our answer!

   Yet sadly, my answer is a bit off the right one provided on class.

   Here's the answer. (Note, I didn't do risk attribution to Rf asset, since I think it is kind of meaningless, and there are graphs showing individual stock's systematic or idiosyncratic contribution on its return in the notebook)

```
Portfolio A Results:
Stock                           Portfolio
Total Return                     0.136642
Systematic Contribution           0.194457
Rf Return                        0.054346
Idiosyncratic Contribution       -0.112161
Risk                             0.007418
Systematic Risk                   0.006303
```

```
Name: Total, dtype: object


Portfolio B Results:
Stock                        Portfolio
Total Return                  0.203526
Systematic Contribution        0.185674
Rf Return                     0.055949
Idiosyncratic Contribution    -0.038097
Risk                         0.006867
Systematic Risk               0.005269
Name: Total, dtype: object


Portfolio C Results:
Stock                        Portfolio
Total Return                  0.281172
Systematic Contribution        0.203779
Rf Return                     0.057783
Idiosyncratic Contribution     0.019611
Risk                         0.007924
Systematic Risk               0.007355
Name: Total, dtype: object


Portfolio Total Portfolio Results:
Stock                        Total Portfolio of A,B,C
Total Return                                0.204731
Systematic Contribution                      0.183558
Rf Return                                    0.05598
Idiosyncratic Contribution                  -0.034807
Risk                                        0.00709
Systematic Risk                             0.006013
Name: Total, dtype: object
```

3) The realized risk and return attribution splits portfolio performance into systematic and idiosyncratic contributions using the CAPM model. Systematic contribution reflects the portion of returns driven by market movements (e.g., SPY index), calculated using the beta coefficient. A higher systematic contribution indicates greater exposure to market risk, meaning the portfolio is more sensitive to market

fluctuations. For example, Portfolio A may have a high systematic contribution, showing strong correlation with the market.

Idiosyncratic contribution, on the other hand, represents returns driven by stock-specific factors, independent of the market. A higher idiosyncratic contribution suggests that individual stocks in the portfolio performed well due to unique factors, such as company performance or industry trends. For instance, Portfolio B might exhibit higher idiosyncratic contributions, indicating better diversification and reduced reliance on market movements.

Risk attribution shows that total portfolio risk is a combination of systematic and idiosyncratic risks. Portfolios with higher systematic risk are more exposed to market volatility, while those with higher idiosyncratic risk may benefit from diversification.

Part 2

1) Optimization using CAPM and Sharpe ratio. Note that a negative sharpe is applied to use the minimize function from scipy.

   Please refer to the jupyter notebook for specific weights.

2) Rerun part 1 on the optimal portfolios.

```
Optimal Portfolio A Results:
Stock                       Portfolio
Total Return                  0.288159
Systematic Contribution        0.222812
Rf Return                     0.057956
Idiosyncratic Contribution     0.007392
Risk                         0.008048
Systematic Risk               0.007761
Name: Total, dtype: object


Optimal Portfolio B Results:
Stock                       Portfolio
```

```
Total Return                   0.258774
Systematic Contribution          0.20778
Rf Return                       0.05726
Idiosyncratic Contribution     -0.006267
Risk                          0.007374
Systematic Risk                0.006446
Name: Total, dtype: object


Optimal Portfolio C Results:
Stock                        Portfolio
Total Return                   0.304962
Systematic Contribution         0.214734
Rf Return                      0.058335
Idiosyncratic Contribution      0.031893
Risk                          0.008234
Systematic Risk                0.007999
    Name: Total, dtype: object
```

3) The optimal portfolios were constructed to maximize the Sharpe ratio, balancing expected returns and risk. Comparing this to Part 1, where realized returns and risks were attributed, we observe differences between the expected and realized idiosyncratic contributions. The CAPM model in Part 1 assumes that idiosyncratic risk is uncorrelated with the market and diversifiable. However, in practice, realized idiosyncratic contributions may deviate due to unforeseen events, such as company-specific news or sector-wide shocks.

   For example, stocks with high expected idiosyncratic risk in Part 1 may have underperformed or outperformed in Part 2 due to these factors. The optimal portfolio in Part 2 likely reduced exposure to stocks with high idiosyncratic risk, favoring those with higher systematic contributions and better diversification. This highlights the importance of aligning portfolio construction with both expected and realized risk-return profiles for better performance.

Part 3

In quantitative finance, both the Normal Inverse Gaussian (NIG) and the Skew-Normal (SN) distributions extend the Gaussian framework by introducing tail-heaviness and asymmetry respectively, allowing for more realistic modeling of asset returns. The NIG is a four-parameter variance-mean mixture of a Normal with an Inverse Gaussian, featuring closed-form densities, affine-transformation closure, and infinite divisibility—properties that underpin its use in Lévy-driven asset-price models, GARCH extensions, Value-at-Risk (VaR) forecasting, and option valuation via Monte Carlo. The SN, introduced by Azzalini (1985), adds a single shape parameter to the Normal density to capture skewed return distributions without sacrificing tractability, and finds applications in portfolio allocation, skew-aware stochastic volatility, and explicit skew-normal option-pricing formulas. Both families have been adopted in this class to improve risk measures, calibrate to empirical return asymmetries, and compare against classical Gaussian assumptions.

Normal Inverse Gaussian (NIG) Distribution

Definition and Key Properties

The NIG distribution arises by mixing a Normal random variable with an Inverse Gaussian mixing law, yielding a four-parameter family $\boldsymbol{X} \sim \mathbf{NIG}(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\delta}, \boldsymbol{\mu})$ that controls tail decay (α), skew (β), scale (δ), and location (μ). As a special case of the generalized hyperbolic class, it admits a closed-form probability density function and moment-generating function, enabling explicit moment calculations and parameter estimation by EM algorithms. The NIG class is closed under affine transformations and convolution (when shape parameters match), and is infinitely divisible, making it well-suited for aggregating returns and defining Lévy-process-based price dynamics.

Applications in Finance

- **Asset Return Modeling**

  Barndorff-Nielsen first introduced NIG to finance in 1997 for modeling log-returns of stocks and interest rates, demonstrating superior fit to empirical heavy tails and skewness compared to the Gaussian.

- **VaR Forecasting & Risk Management**

  Employing NIG in dynamic conditional score frameworks yields improved Value-at-Risk forecasts, particularly under high-risk thresholds, by capturing evolving tail behavior more accurately than Gaussian-based models.

- **Option Pricing via Monte Carlo**

  Monte Carlo valuation methods incorporating NIG processes (with stratified sampling and bridges) allow realistic pricing of options under heavy-tailed returns, reducing mispricing seen in Gaussian models.

Skew-Normal (SN) Distribution

Definition and Key Properties

The univariate Skew-Normal distribution $SN(\boldsymbol{\xi}, \boldsymbol{\omega}, \boldsymbol{\alpha})$ augments the standard Normal by a shape parameter α, with density

$$f(x;\ \xi, \omega, \alpha) = \frac{2}{\omega} \phi!\left(\frac{x-\xi}{\omega}\right) \Phi!\left(\alpha\ \frac{x-\xi}{\omega}\right)$$

where ϕ and Φ are the N(0,1) PDF and CDF.

Setting α=0 recovers the Gaussian, while nonzero αinduces skewness. Multivariate and unified extensions (SUN family) allow correlated skewed vectors with tractable likelihoods and closed-form moments.

Applications in Finance

- **Stochastic Volatility**

  Skew-Normal shocks in time-varying SV or FSV models accommodate changing asymmetry in macroeconomic and financial volatility forecasting, delivering improved density and quantile forecasts cite turn1academia10 .

- **Risk Measures**

  Incorporating SN-based distributions into VaR and expected shortfall calculations corrects for skewness risk, addressing underestimation of downside tail probabilities inherent to symmetric models cite turn1search15 .

Part 4

1) Fit the stocks into models provided by scipy.stats (norm, skewnorm, norminvgauss are included, but generalized t not, I use jf_skew_t instead)
   The fitted models and their parameters are in the code file.
2) Use the CDF map for the raw return data and conduct spearman_rank on that. With the spearman_rank, use multivariate_normal to generate 1000 samples (not too many because running the next part ppf is time-consuming).
3) For the random_samples generated, which is the simulated quantile for the actual return. We then use the ppf function of fitted models to get the simulated returns.

PS: I ran into many crush due to unknow reasons for which the processor of ppf can't work and I have to use try and except the surpass that, which returns an zero array if encountered with the situation. (GPT says it is common in financial risk modelling)

4) After you get the simulated returns, it is very easy to calculate the VaR and ES. Simply compute the simulated portfolio value and find the smallest 5%. (I like these simulations more than delta normal and other pure-math based methods)

```
A current value: 295444.60820007324
1 Day 5% for Portfolio A VaR: 4088.77, ES: 5189.09
B current value: 280904.48240852356
1 Day 5% for Portfolio B VaR: 3532.09, ES: 4456.19
C current value: 267591.4399547577
1 Day 5% for Portfolio C VaR: 3464.35, ES: 4399.78
Total current value: 843940.5305633545
1 Day 5% for Portfolio Total VaR: 10534.53, ES: 13524.42
```

5) For the multivariate-normal simulation, I simply copy it from former homework, which requires you to find a nearest psd to do the Cholesky decomposition as an multiplier for the simulated n-d gaussian normal random variables.

```
1 Day 5% for Portfolio A VaR: 3626.53, ES: 4679.65
1 Day 5% for Portfolio B VaR: 3282.68, ES: 4240.80
1 Day 5% for Portfolio C VaR: 3376.62, ES: 4335.76

1 Day 5% for Portfolio Total VaR: 9783.77, ES: 12716.73
```

6) The VaR and ES using Gaussian Copulas is larger than the multivariate normal simulation, because the former fits many stocks into heavily tailed or skewed models. Yet the results don't vary greatly.
While considering the huge amount of time used to compute the ppf in the last step of gaussian copulas, it is really easy and fast to use a multivariate simulation.
A good approach is to look at the following 255 days, use the data as an historical simulation and compare their differences from the historical simulation.
Yet the stock market rages in 2024, so the var is even negative. (In worst situation it still profits for the total portfolio) So it is really hard to say which is a better approach.

Part 5

Thanks to Professor's patient explanation for this part.

1) I have defined detailed and readable functions in the notebook. So I will not expand too much here.
2) Basically, we need to minimize the $SSE = cov(CES)$, where $CES = w\# \frac{\delta ES}{\delta w}$, where the ES is the same using the simulated returns in Part 4, which is our objective

function. The constraints are simply $\Sigma w_i = 1$. After get the 'optimal' weight, do the same as Part 2 is a copy-paste thing.

Optimal weights: I wonder why it is almost same here. 😵

```
portfolio A least es : 0.010017981228249513
Optimal weights for Portfolio A: [0.03030303 0.03030303 0.03030303
0.03030303 0.03030303 0.03030303
 0.03030303 0.03030303 0.03030303 0.03030303 0.03030303 0.03030303
 0.03030303 0.03030303 0.03030303 0.03030303 0.03030303 0.03030303
 0.03030303 0.03030303 0.03030303 0.03030303 0.03030303 0.03030303
 0.03030303 0.03030303 0.03030303 0.03030303 0.03030303 0.03030303
 0.03030303 0.03030303 0.03030303]
portfolio B least es : 0.023318874542578555
Optimal weights for Portfolio B: [0.03170988 0.03170988 0.03170988
0.03170988 0.03170988 0.03170988
 0.03170988 0.03170988 0.03170988 0.03170988 0.03170988 0.03170988
 0.03170988 0.03170988 0.03170988 0.03170988 0.03170988 0.01699386
 0.03170988 0.03170988 0.         0.03170988 0.03170988 0.03170988
 0.03170988 0.03170988 0.03170988 0.03170988 0.03170988 0.03170988
 0.03170988 0.03170988 0.03170988]
portfolio C least es : 0.020836554482594635
Optimal weights for Portfolio C: [0.03030303 0.03030303 0.03030303
0.03030303 0.03030303 0.03030303
 0.03030303 0.03030303 0.03030303 0.03030303 0.03030303 0.03030303
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 0.03030303 0.03030303 0.03030303 0.03030303 0.03030303 0.03030303
    0.03030303 0.03030303 0.03030303]
```

Return & Risk Attribution:

```
Optimal Portfolio A Results:
Stock                          Portfolio
Total Return                    0.229236
Systematic Contribution          0.212938
Rf Return                       0.056568
Idiosyncratic Contribution        -0.04027
Risk                            0.008132
Systematic Risk                  0.007796
Name: Total, dtype: object
```

```
Optimal Portfolio B Results:
Stock                          Portfolio
Total Return                     0.26586
Systematic Contribution          0.193299
Rf Return                       0.057426
Idiosyncratic Contribution        0.015135
Risk                            0.00695
Systematic Risk                 0.005491
Name: Total, dtype: object
```

```
Optimal Portfolio C Results:
Stock                          Portfolio
Total Return                    0.397244
Systematic Contribution          0.229617
Rf Return                        0.06046
Idiosyncratic Contribution        0.107167
Risk                           0.008806
Systematic Risk                 0.009022
Name: Total, dtype: object
```

3) The most impressive must be the 39.7% profit for portfolio C. Yet the risk is also higher. And even the systematic risk attribution is higher than the portfolio risk, meaning that there is a β larger than 1 for the portfolio to the market. Yet on other hand, the results are nothing exciting that you can't find many out-liars, except fot the ES in the simulated case is smaller, to less than around 2%. (Even the real day market has a 0.25% ES given the big bull market.) For other discussion, I would really like to hear your instructive opinions!

Thanks for reading! Have a good day!