# High-temperature topological superconductivity in twisted double layer copper oxides

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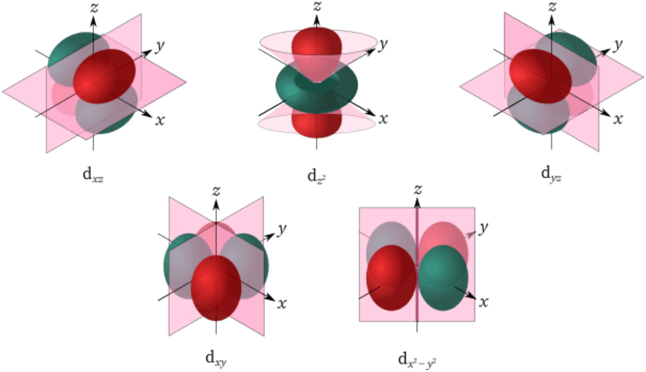
# Outline

1. Pairing Symmetry
2. Ginzburg Landau(GL) theory
3. Microscopic Theory
4. Lattice Model
5. Topological Phase
6. Experiment
7. Futher....

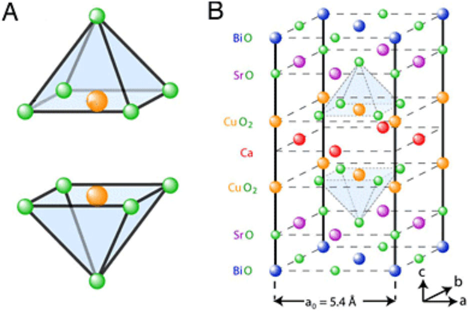
# Outline

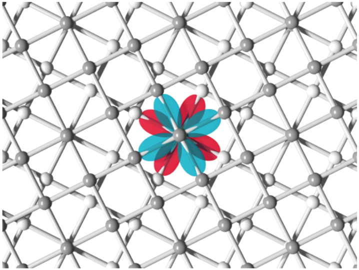
# D-orbit of Hydrogen




$$\Delta(\mathbf{k}) = \Delta_0(\cos k_x - \cos k_y) \tag{1}$$

# Twisted double layer copper-oxygen square lattices[1]



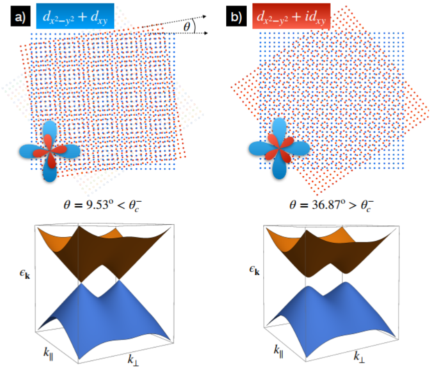



$$\Delta_{\mathbf{k}}^{\theta} = \Delta_0 \left[ \cos(2\theta)(\hat{k}_x^2 - \hat{k}_y^2) + \sin(2\theta) 2\hat{k}_x \hat{k}_y \right] \qquad (2)$$

The second term in Eq.(2) has the functional form of a $d_{xy}$ order parameter.

[1]Nature,575,156

# Twisted double layer copper-oxygen square lattices




$T$-breaking is evident because time reversal maps $d + id'$ to $d - id'$.

# Outline

# GL Free Energy Density

$$|\psi(\mathbf{r})|^2 = n_s(\mathbf{r}) \tag{3}$$

$$f_s = f_n + \alpha(T) n_s + \frac{1}{2}\beta(T) n_s^2 + \cdots \tag{4}$$

$$f_s = f_n + \alpha(T)|\psi(\mathbf{r})|^2 + \frac{1}{2}\beta(T)|\psi(\mathbf{r})|^4 \tag{5}$$

# GL Free Energy Density

$$\mathcal{F}[\psi_1,\psi_2] = f_0[\psi_1] + f_0[\psi_2] + A|\psi_1|^2|\psi_2|^2 + B(\psi_1\psi_2^* + c.c) + C(\psi_1^2\psi_2^{*2} + c.c) \tag{6}$$

where $\psi_{a=1,2}$ are complex order parameters of two layers and $f_0[\psi] = \alpha|\psi|^2 + \frac{1}{2}\beta|\psi|^4$ is the free energy of a monolayer.

If the two layers are physically identical, then the order parameter amplitudes must be the same and the most general solution (up to an overall phase) is

$$\psi_1 = \psi \qquad \psi_2 = \psi e^{i\varphi} \tag{7}$$

$$\psi_a \to -\psi_a \quad \text{Under rotation} \quad 90^\circ \tag{8}$$

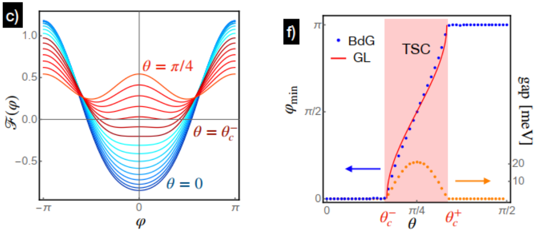
For the free energy $(6)$ to remain invariant, the parameter $B$ must also change sign

$$B = -B_0 \cos(2\theta) \tag{9}$$

# GL Free Energy Density

$$\mathcal{F}(\varphi) = \mathcal{F}_0 + 2B_0\psi^2\left[-\cos(2\theta)\cos\varphi + \mathcal{K}\cos(2\varphi)\right] \tag{10}$$

where $\mathcal{K} = C\psi^2/B_0$ and $\mathcal{F}_0$ contains all terms independent of $\varphi$.



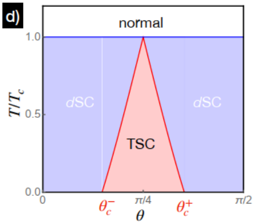

For $\theta \neq 45^\circ$, the free energy is minimized by

$$\varphi_{\min} = \arccos\left(\frac{\cos 2\theta}{4\mathcal{K}}\right) \tag{11}$$

Temperature dependence for the order parameter $\psi(T) = \psi_0\sqrt{1 - T/T_c}$, we can obtain

$$\tilde{T}_c(\theta) = T_c(1 - \frac{|\cos 2\theta|}{4\mathcal{K}_0}) \quad \theta_c^- \leq \theta \leq \theta_c^+ \tag{12}$$

where $\mathcal{K}_0 = C\psi_0^2/B_0$.



# Outline

$$\mathcal{H} = \sum_{\mathbf{k}\sigma a} \xi_{\mathbf{k}a} c^{\dagger}_{\mathbf{k}\sigma a} c_{\mathbf{k}\sigma a} + g \sum_{\mathbf{k}\sigma} (c^{\dagger}_{\mathbf{k}\sigma 1} c_{\mathbf{k}\sigma 2} + h.c) + \sum_{\mathbf{k}a} (\Delta_{\mathbf{k}a} c^{\dagger}_{\mathbf{k}\uparrow a} c^{\dagger}_{-\mathbf{k}\downarrow a} + h.c) - \sum_{\mathbf{k}a} \Delta_{\mathbf{k}a} \langle c^{\dagger}_{\mathbf{k}\uparrow a} c^{\dagger}_{-\mathbf{k}\downarrow a} \rangle \tag{13}$$

$$\Delta_{\mathbf{k}a} = \sum_{\mathbf{p}}{}' V^{(a)}_{\mathbf{kp}} \langle c_{-\mathbf{p}\downarrow a} c_{\mathbf{p}\uparrow a} \rangle \tag{14}$$

where $V^{(a)}_{\mathbf{kp}}$ denotes the interaction matrix element in layer a.

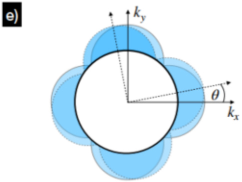
$$V^{(a)}_{\mathbf{kp}} = -2\mathcal{V} \cos(2\alpha_{\mathbf{k}}) \cos(2\alpha_{\mathbf{p}}) \tag{15}$$

where $\alpha_{\mathbf{k}}$ represents the polar angle of the vector $\mathbf{k}$.

Rotate the interaction in layer 2 by angle $\theta$

$$V_{\mathbf{kp}}^{(2)} = -2\mathcal{V}\cos(2\alpha_{\mathbf{k}} - 2\theta)\cos(2\alpha_{\mathbf{p}} - 2\theta) \tag{16}$$

Consider a circular Fermi surface generated by $\xi_{\mathbf{k}a} = \hbar^2 k^2/2m - \mu$ that remains invariant under rotation




$$\mathcal{H} = \sum_{\mathbf{k}} \Psi_{\mathbf{k}}^{\dagger} h_{\mathbf{k}} \Psi_{\mathbf{k}} + E_0 \tag{17}$$

$$h_{\mathbf{k}} = \begin{bmatrix} \xi_{\mathbf{k}} & \Delta_{\mathbf{k}1} & g & 0 \\ \Delta^*_{\mathbf{k}1} & -\xi_{\mathbf{k}} & 0 & -g \\ g & 0 & \xi_{\mathbf{k}} & \Delta_{\mathbf{k}2} \\ 0 & -g & \Delta^*_{\mathbf{k}2} & -\xi_{\mathbf{k}} \end{bmatrix} \tag{18}$$

and $E_0 = \sum_{\mathbf{k}} 2\xi_{\mathbf{k}} - \sum_{\mathbf{k}a} \Delta_{\mathbf{k}a} \langle c^\dagger_{\mathbf{k}\uparrow a} c^\dagger_{-\mathbf{k}\downarrow a} \rangle$.
The free energy of the system can be calculated from the standard expression

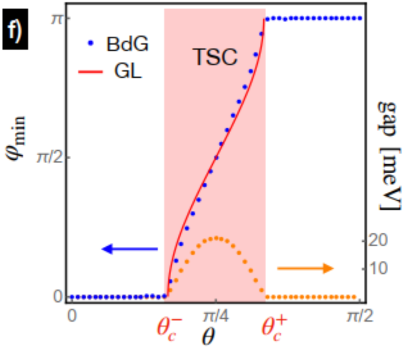
$$\mathcal{F}_{\mathrm{BdG}} = E_0 - 2k_B T \sum_{\mathbf{k}a} \ln\left[2\cosh(E_{\mathbf{k}\alpha}/2k_B T)\right] \tag{19}$$

$$E_{\mathbf{k}\alpha} = \sqrt{(\Delta^2_{\mathbf{k}1} + \Delta^2_{\mathbf{k}2})/2 + \xi^2_{\mathbf{k}} + g^2 + (-)^\alpha D_{\mathbf{k}}}$$
$$D^2_{\mathbf{k}} = (D^2_{\mathbf{k}1} - D^2_{\mathbf{k}2})^2/4 + g^2(\Delta^2_{\mathbf{k}1} + \Delta^2_{\mathbf{k}2} + 4\xi^2_{\mathbf{k}}) - 2g^2 \Delta_{\mathbf{k}1}\Delta_{\mathbf{k}2} \cos\varphi \tag{20}$$

# Microscopic Theory

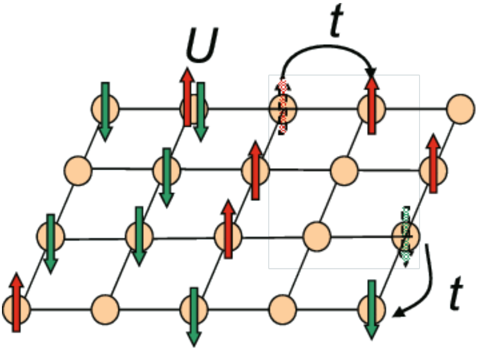
$$\Delta_{\mathbf{k}1} = \psi \cos(2\alpha_{\mathbf{k}}) \qquad \Delta_{\mathbf{k}2} = \psi \cos(2\alpha_{\mathbf{k}} - 2\theta) \tag{21}$$

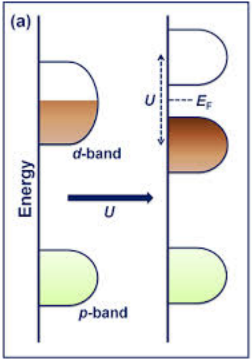
$$\mathcal{F}(\varphi) = \mathcal{F}_0 + 2B_0\psi^2 \left[-\cos(2\theta)\cos\varphi + \mathcal{K}\cos(2\varphi)\right] \tag{22}$$



# Outline

# Hubbard Model




$$H = \sum_{ij,\sigma} t_{ij} C^{\dagger}_{i\sigma} C_{j\sigma} + U \sum_{i\sigma} n_{i\sigma} n_{i\bar{\sigma}} \qquad n_{i\sigma} = C^{\dagger}_{i\sigma} C_{i\sigma} \tag{23}$$

# Hubbard Model

$$H = -\sum_{ij,\sigma a} t_{ij} c^\dagger_{i\sigma a} c_{j\sigma a} - \mu \sum_{i\sigma a} n_{i\sigma a} + \sum_{ij,a} V_{ij} n_{ia} n_{ja} - \sum_{ij\sigma} g_{ij} c^\dagger_{i\sigma 1} c_{j\sigma 2} \quad (24)$$

Mean field approximation

$$c_i c_j = \langle c_i c_j \rangle + (c_i c_j - \langle c_i c_j \rangle) \quad (25)$$

$$\begin{aligned} H = &-t \sum_{\langle ij \rangle \sigma a} c^\dagger_{i\sigma a} c_{j\sigma a} - t' \sum_{\langle\langle ij \rangle\rangle \sigma a} c^\dagger_{i\sigma a} c_{j\sigma a} - \mu \sum_{i\sigma a} n_{i\sigma a} \\ &+ \sum_{\langle ij \rangle a} (\Delta_{ij,a} c^\dagger_{i\uparrow a} c^\dagger_{j\downarrow a} + h.c) - \sum_{ij\sigma} g_{ij} c^\dagger_{i\sigma 1} c_{j\sigma 2} \end{aligned} \quad (26)$$

$$\Delta_{ij,a} = V \langle c_{i\uparrow a} c_{j\downarrow a} \rangle \quad (27)$$

$$S\left[\psi,\bar{\psi}\right]=\sum_{p}\bar{\psi}_{p\sigma}(-i\omega_n+\frac{p^2}{2m}-\mu)\psi_{p\sigma}+\frac{T}{2L^3}\sum_{pp'q}\bar{\psi}_{p+q\sigma}\bar{\psi}_{p'-q\sigma'}V(\mathbf{q})\psi_{p'\sigma}\psi_{p\sigma} \tag{28}$$

$$1\equiv\int D\phi\exp\left[-\frac{e^2\beta}{2L^d}\sum_{q}\phi_q V^{-1}(\mathbf{q})\phi_{-q}\right] \tag{29}$$

Employing the variable shift $\phi_q\rightarrow\phi_q+ie^{-1}V(q)\rho_q/\beta$, where $\rho_q\equiv\sum_p\bar{\psi}_{p\sigma}\psi_{p+q\sigma}$, $q=(\omega_m,\mathbf{q})$.

$$1=\int D\phi\exp\left[\frac{1}{L^d}\sum_{q}(-\frac{e^2\beta}{2}\phi_q V^{-1}(q)\phi_{-q}+ie\rho_q\phi_{-q}+\frac{1}{2\beta}\rho_q V(\mathbf{q})\rho_{-q})\right] \tag{30}$$

[2] Condensed Matter Field Theory(Alexander Altland,Ben Simons)

The field integral

$$\mathcal{Z} = \int D\phi \int D(\bar{\psi}_\sigma, \psi_\sigma) e^{-S[\phi, \bar{\psi}_\sigma, \psi_\sigma]} \tag{31}$$

$$\begin{aligned} S\left[\phi, \bar{\psi}_\sigma, \psi_\sigma\right] =& \frac{\beta}{8\pi L^d} \sum_q \phi_q \mathbf{q}^2 \phi_{-q} \\ &+ \sum_{pp'} \bar{\psi}_{p\sigma} \left[ (-i\omega + \frac{\mathbf{p}}{2m} - \mu)\delta_{pp'} + \frac{ie}{L^d} \phi_{p'-p} \right] \psi_{p'\sigma} \end{aligned} \tag{32}$$

# Hubbard-Stratonovich Transformation

After decoupling the attractive pairing potential in the Cooper channel with the Hubbard-Stratonovich transformation we obtain the action corresponding to the Hamiltonian $(26)$

$$S = \sum_{\mathbf{k}n} \Psi^{\dagger}_{\mathbf{k}n} \left[-i\omega_n + h_{\mathbf{k}}\right] \Psi_{\mathbf{k}n} + \frac{\beta\mathcal{N}}{V} \sum_{i,j\in u.c.a} \Delta_{ij,a}\Delta^{*}_{ij,a} \tag{33}$$

The effective action for $\Delta_{ij,a}$ can be determined by integrating out fermionic degrees of freedom

$$S_{\text{eff}} = -\sum_{\mathbf{k}n} \operatorname{Tr}\log\left[-\mathcal{G}(\mathbf{k}, i\omega)^{-1}\right] + \frac{\beta\mathcal{N}}{V} \sum_{i,j\in u.c.a} \Delta_{ij,a}\Delta^{*}_{ij,a} \tag{34}$$

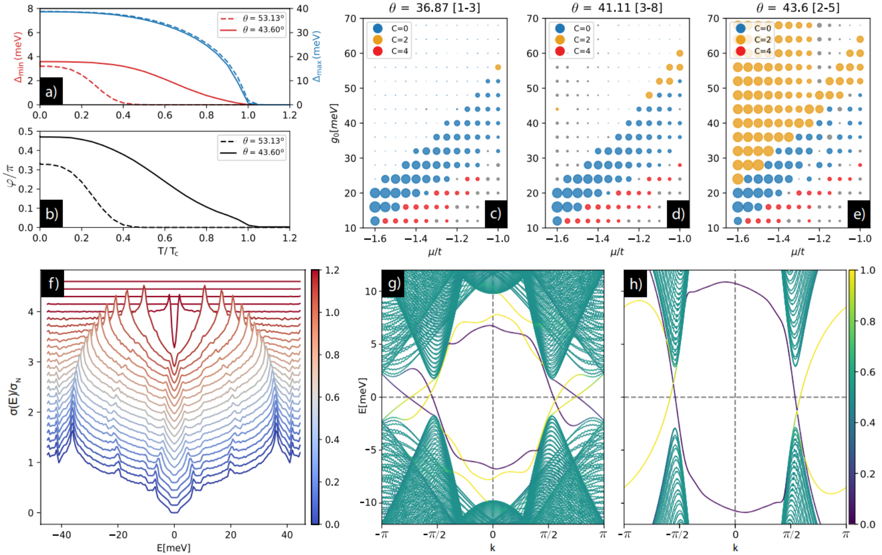
where $\mathcal{G}(\mathbf{k}, i\omega_n) = -(-i\omega_n + h_{\mathbf{k}})^{-1}$

Calculate the saddle point condition for this effective action $\partial S_{\mathrm{eff}}/\partial \delta^*_{ij,a} = 0$, employ the identity $\partial_\Delta (\mathrm{Tr}\log A) = \mathrm{Tr}\left[\partial_\Delta A A^{-1}\right]$. After performing the Matsubara sum

$$\Delta_{ij,a} = -\frac{V}{\mathcal{N}} \sum_{\mathbf{k}} \mathrm{Tr}\left[\frac{\partial h_{\mathbf{k}}}{\partial \Delta^*_{ij,a}} U_{\mathbf{k}} n_F(E_{\mathbf{k}}) U^\dagger_{\mathbf{k}}\right] \tag{35}$$

where $U^\dagger_{\mathbf{k}} h_{\mathbf{k}} U_{\mathbf{k}} = E_{\mathbf{k}}$ and $n_F(E_{\mathbf{k}})$ is Fermi function.

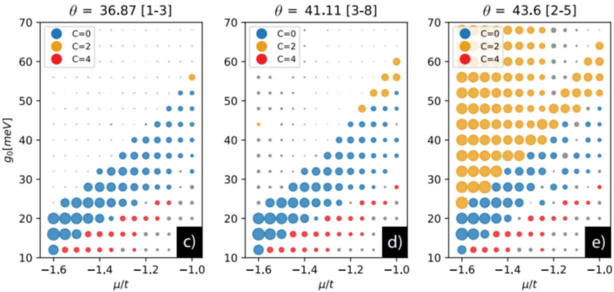
# Hubbard Model



# Outline

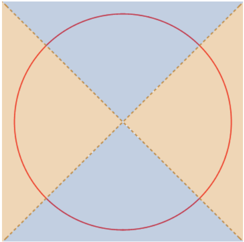
$$C = \frac{1}{2\pi}\int_{BZ}\Omega(\mathbf{k})d^2k \tag{36}$$

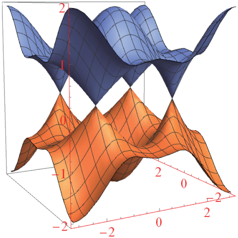
$$\Omega(\mathbf{k}) = \sum_{m\neq n}\frac{\langle n|\nabla_{\mathbf{k}}h_{\mathbf{k}}|m\rangle \times \langle m|\nabla_{\mathbf{k}}h_{\mathbf{k}}|n\rangle}{(E_m - E_n)^2} \tag{37}$$

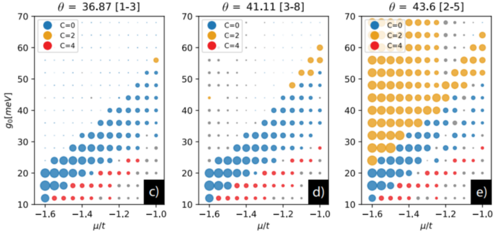


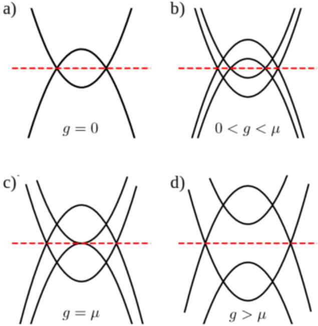
$$H = \sum_{\mathbf{k}\sigma} \epsilon(\mathbf{k}) d^\dagger_{\mathbf{k}\sigma} d_{\mathbf{k}\sigma} + \sum_{\mathbf{k}} \Delta(\mathbf{k}) (d^\dagger_{\mathbf{k}\uparrow} d^\dagger_{-\mathbf{k}\downarrow} + h.c) \tag{38}$$

where $\epsilon(\mathbf{k}) = -2t(\cos k_x + \cos k_y) - \mu$, $\Delta(\mathbf{k}) = \Delta_0(\cos k_x - \cos k_y)$





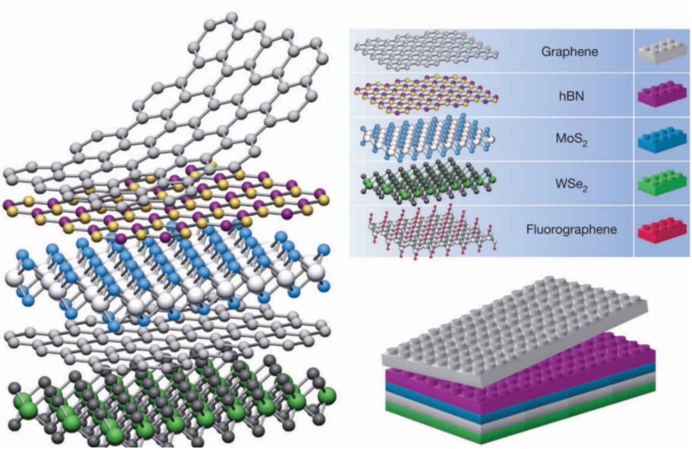



If the interlayer coupling g is strong enough to push one of the bands above the Fermi level, half of the Dirac cones disappear, leaving four Dirac cones behind.

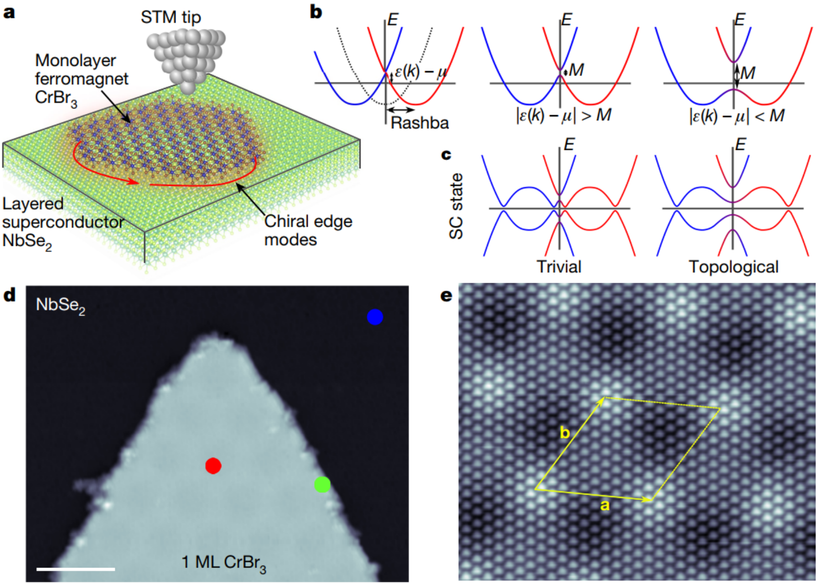
# Outline

# Van der Waals heterostructures[3]




[3] Nature,499,419-425(2013)

# Topological superconductivity[4]



# Quantum anomalous Hall insulators[5]

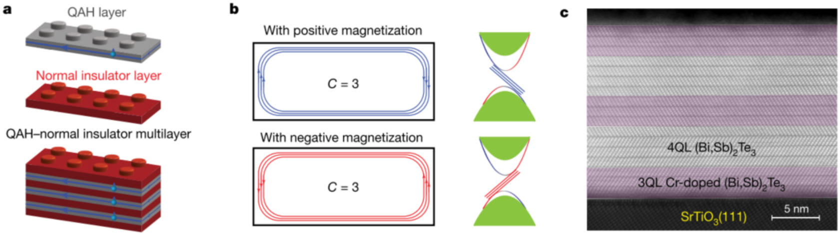



**Fig. 1 | The high-*C* QAH effect in magnetic–undoped topological insulator multilayer structures. a**, Schematics of the high-*C* QAH insulator in *C* = 1 QAH–normal insulator multilayer structures. The blue arrows illustrate the chiral edge channels of the QAH insulators. **b**, Schematics of the high-*C* QAH effect. We take a *C* = 3 QAH insulator as an example. Three chiral edge channels are shown in real space (left) and momentum space (right) for positive (top) and negative (bottom) magnetization. **c**, Cross-sectional STEM image of the *m* = multilayer structure grown on a $SrTiO_3$ substrate.

[5] Nature,588,419-423(2020)

# Outline

# Twisted

- arXiv:2012.07860v1(Magic angles and current-induced topology in twisted nodal superconductors)
- arXiv:2012.03986v1(Chiral $p$-wave superconductivity in a twisted array of proximitized quantum wires)