

# Parameterized Enriched Topoi, Geometric Cohomology and Algebra-Valued Geometry

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Math-Phys-Cat

March 24, 2023

About

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  - > on the relation between Logic, Geometry and Physics.

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  - > prof. Ivan Struchiner (IME-USP) provided support for bureaucratic matters

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  - > invite you to collaborate with us.

# The Talk



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## 1. Logic-Geometry-Physics

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$$\text{Logic} \xrightarrow{\text{SDG}} \text{Geom} \quad (1)$$

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$$\text{Logic} \xrightarrow{\text{Hilb}} \text{Phys} \quad (3)$$

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- > the elevation of Hilbert's 6th problem status to that of a genuine "Axiomatization Problem of Physics" was suggested by Hilbert, by Corry and also by the author
- > an attempt to a formal statement was presented by Bunge in his "Foundations of Physics" and generalized by the author in his PhD thesis.



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- > joining the previous diagrams one could try to use SDG to axiomatize Continuum Mechanics, as below.

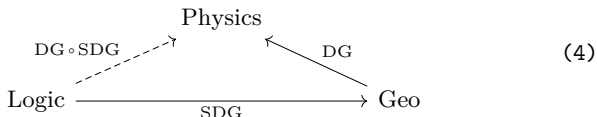
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- > in other words, the idea would be to take  $\text{Hilb} = \text{DG} \circ \text{SDG}$ .

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  - > such that Kock-Lawvere axiom is satisfied, i.e., certain canonical functor has a right-adjoint and the unit is an isomorphism.

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- > This leads us to ask:
  - > *is there an axiomatics to DG which is really model independent, being a natural candidate to attack Hilbert's sixth problem?*

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$$\begin{array}{ccc} & \text{Disc} & \\ & \curvearrowright & \\ \mathbf{H} & \begin{array}{c} \xleftarrow{\Gamma} \\ \xleftarrow{\text{LCst}} \end{array} & \begin{array}{c} \xrightarrow{\perp} \\ \xrightarrow{\perp} \\ \xrightarrow{\perp} \end{array} & \mathbf{Set} \\ & \curvearrowleft & \\ & \text{CoDisc} & \end{array} \quad (5)$$

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 \end{array}
 \qquad
 \begin{array}{ccc}
 & f & \\
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$$\text{Synt} : \mathbf{Cat} \rightleftarrows \mathbf{DType} : \text{Cntx} \quad (7)$$

- > for different classes of categories/logic this reduces to an 2-equivalence.

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 \mathbf{CohTopos} & \xrightarrow{U} & \mathbf{Topos} & \hookrightarrow & \mathbf{Cat}
 \end{array} \tag{8}$$



# Logic-Geometry-Physics

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 (1,1)\mathbf{Topos} & \hookrightarrow & (1,1)\mathbf{Cat} & \xrightarrow{\simeq} & \mathbf{Cat} \\
 \text{categ} \downarrow & & \text{categ} \downarrow & & \downarrow \text{categ} \\
 (2,1)\mathbf{Topos} & \hookrightarrow & (2,1)\mathbf{Cat} & \hookrightarrow & 2\mathbf{Cat} \\
 \text{categ} \downarrow & & \text{categ} \downarrow & & \downarrow \text{categ} \\
 \vdots & & \vdots & & \vdots \\
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 \tau_{\leq 2} \swarrow & & \searrow \tau_{\leq 2} \\
 (3, 1)\mathbf{Topos} & \hookrightarrow & (3, 1)\mathbf{Cat} \\
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(10)

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 (1, 1)\mathbf{Topos} & \xrightarrow{\quad} & (1, 1)\mathbf{Cat} \\
 (\infty, 1)\mathbf{CohTopos} & \xrightarrow{U} (\infty, 1)\mathbf{Topos} & \xrightarrow{\quad} & (\infty, 1)\mathbf{Cat} \\
 \text{Synt} \left( \begin{array}{c} \uparrow \\ \Leftarrow \\ \downarrow \end{array} \right)_{\text{Cntx}} & & \text{Synt} \left( \begin{array}{c} \uparrow \\ \Leftarrow \\ \downarrow \end{array} \right)_{\text{Cntx}} & & \text{Cntx} \left( \begin{array}{c} \uparrow \\ \Rightarrow \\ \downarrow \end{array} \right)_{\text{Synt}} \\
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(12)

- > thus, the initial "cohesive logic" could be obtained as a particular instance of this Cohesive HoTT.

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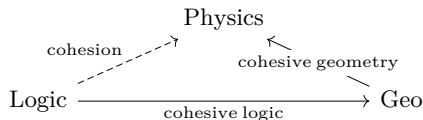
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(13)

- > but this language is not abstract enough, as we pass to discuss.

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    - > typically assumed a way to assign to each classical system a corresponding quantum system
    - > (called a *quantization process*)
    - > such that the axiomatization of quantum Physics factors through quantization.

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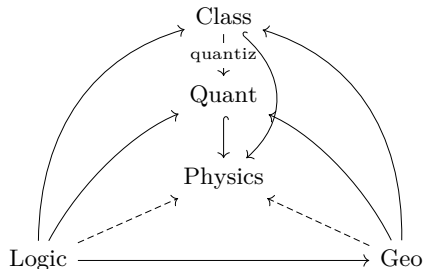
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  - > to provide a quantum version of Yang-Mills Theory (structured by any semi-simple Lie group  $G$ ) satisfying the mass-gap condition [?].

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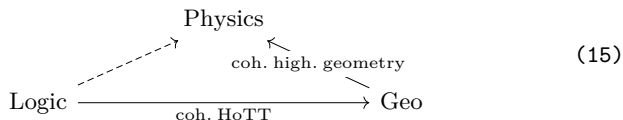
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- > Schreiber showed that cohesive HoTT can be used to formulate higher cohesive geometry:

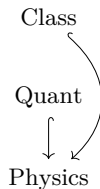
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(16)

$$\text{Logic} \xrightarrow{\text{coh. HoTT}} \text{Geo}$$

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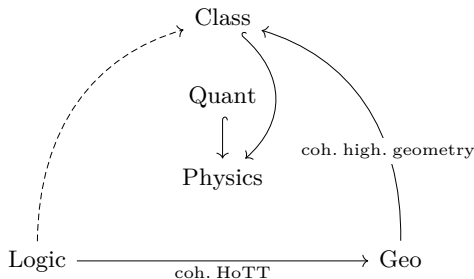
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(17)

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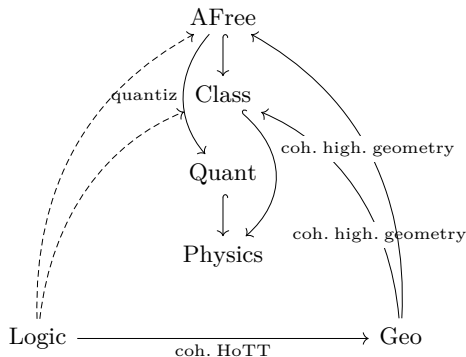
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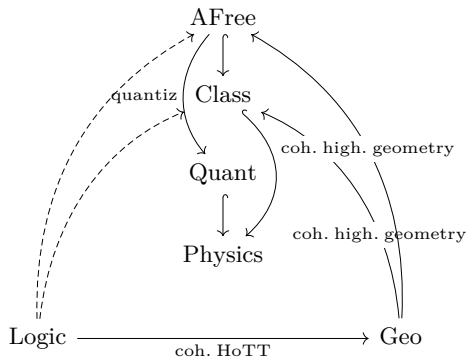
(18)



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(18)

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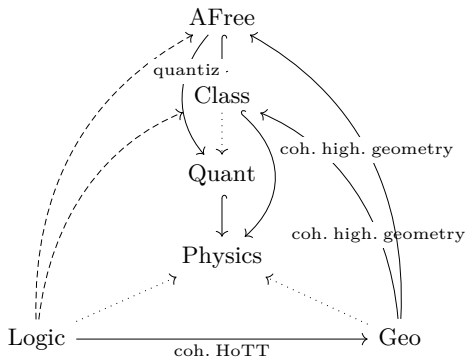
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(19)

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In few words



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- > hence to *integrated higher algebra parameterized physical systems*.

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- > furthermore, in these cases the fundamental physical entities
  - > are modeled by geometric quantities as  $G$ -connections and curvature
  - > so they are Lie algebra-valued differential forms
  - > and therefore are quantities take "take values in the algebraic object modeling infinitesimal symmetries".



# The Program

2. following a parameterized perspective

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## 3. using obstruction methods

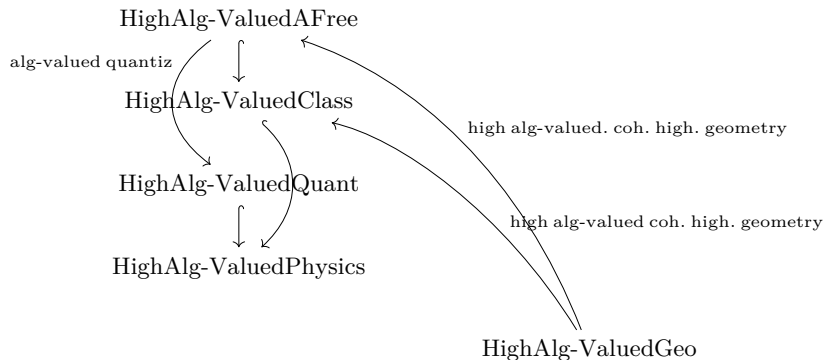
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    - > a version of higher algebra-valued cohomological quantization

# The Program

## 3. using obstruction methods



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# The Program

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- > for the class `HigherAlgebras` of higher algebras, let `HigherOperadAlgebras` be the class of all algebras over some higher operad that belongs to `HigherAlgebras`.



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- > the class HighAlg-Class is parameterized over HigherAlgebras
- > and one can restrict it to be parameterized over HigherOperadAlgebras

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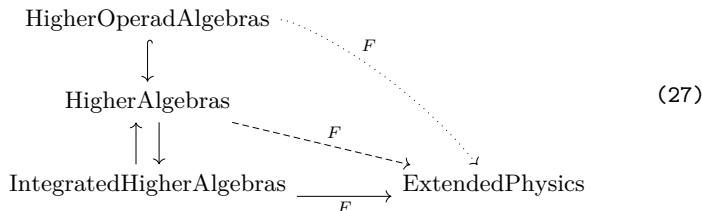
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- > for the class `HigherAlgebras` of higher algebras, let `HigherOperadAlgebras` be the class of all algebras over some higher operad that belongs to `HigherAlgebras`.
- > the class `HighAlg-Class` is parameterized over `HigherAlgebras`
- > and one can restrict it to be parameterized over `HigherOperadAlgebras`



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The diagram illustrates the relationships between various mathematical structures in the program. It consists of five nodes arranged in a vertical column on the left and one node on the right. The nodes are: HigherOperad, HigherOperadAlgebras, HigherAlgebras, IntegratedHigherAlgebras, and ExtendedPhysics. The connections are as follows: a solid vertical arrow points from HigherOperad to HigherOperadAlgebras; a solid vertical arrow points from HigherOperadAlgebras to HigherAlgebras; a solid vertical arrow points from HigherAlgebras to IntegratedHigherAlgebras, with a solid vertical arrow pointing back up; a solid horizontal arrow points from IntegratedHigherAlgebras to ExtendedPhysics, labeled with  $F$  below it; a wavy arrow points from HigherOperad to ExtendedPhysics, labeled with  $F|_{\mathcal{O}}$  next to it; a dotted arrow points from HigherOperadAlgebras to ExtendedPhysics, labeled with  $F$  next to it; and a dashed arrow points from HigherAlgebras to ExtendedPhysics, labeled with  $F$  next to it. The entire diagram is labeled with (29) on the right.

$$\begin{array}{ccc} \text{HigherOperad} & \xrightarrow{F|_{\mathcal{O}}} & \\ \downarrow & & \\ \text{HigherOperadAlgebras} & \xrightarrow{F} & \\ \downarrow & & \\ \text{HigherAlgebras} & \xrightarrow{F} & \text{ExtendedPhysics} \\ \uparrow \downarrow & \xrightarrow{F} & \\ \text{IntegratedHigherAlgebras} & & \end{array} \quad (29)$$

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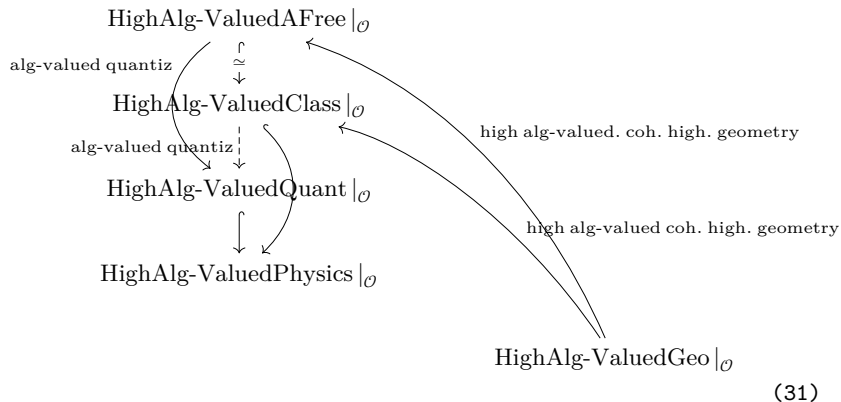
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- > this means exactly that:
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  - > instead of restricting the types of physical systems to ensure existing of quantization one can work with general types of physical systems, but restricted to specific kinds of infinitesimal symmetries.

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- > the problem is therefore about vanishing of characteristic classes, which is the natural context for obstruction theory.

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thanks =)