Axiomatic Foundations of Physics:

Problems and Ideas

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- The "axiomatization approach" was considered by *Hilbert's formalist school*;
- The problem of axiomatizing Mathematics was explicitly considered in his *Hilbert's Program* which is about finding a formalist approach to Mathematics through a consistent axiomatic language;
- Similarly, one could consider an *Axiomatization Program of Physics* as the search for a formalist approach to Physics through a consistent axiomatic language.

Plain

Part I: The Program

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1. Foundations of Mathematics

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1. what are “mathematical objects”?
2. are they “real”? Are they “abstract”?
3. what it means to “know” in mathematics?
4. when is a mathematical sentence true? In other words, what is “mathematical truth”?
5. what is the role of axioms?
6. what is their nature?

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2. **Formalism** (mainly by Hilbert)
3. **Intuitionism** (mainly by Brouwer)

Logicism

Mathematics can be reduced to philosophical logic.

Foundations of Mathematics | Logicism

Logicism

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- mathematical **ontology** reduces to logic.
- mathematical **epistemology** reduces to logic;

Formalism

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- none requirement on mathematical **ontology**;
- mathematical **epistemology** reduces to an axiomatic logic.

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Mathematics is fruit of our minds...

Foundations of Mathematics | Debates

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  - Fregue-Hilbert debate;
  - Brouwer-Hlbert debate.

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- In the Brouwer view, intuitionistic logics are allowed to be axiomatic, but they *avoid some axioms like Law of Excluded Middle* and Axiom of Choice.
- On the other hand, *these axioms are considered fundamental* in Hilbert's formalism.

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- **intuitionistic formalism**: is formalism for axiomatic intuitionistic logic, i.e., it is the claim that *mathematics can be formalized in an axiomatic intuitionistic logic*.

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- a family S of relations in $C \sqcup E$ defining the *structure* of $\mathcal{S}$.

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- Of course, this suggests a categorical interpretation of systemics.
- Curiously, Bunge does not even define what is a "morphism" between systems, although there is a natural notion of that.

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- Concerning axiomatization of Physics, I think that the above notion of system is not the correct formalization of "area of study"...
- ... we need "higher systems", so that "higher (or homotopical) systemics" (see Section 2.2 of the thesis).
- Thus, instead of fixing an *a priori* formalization to "area of study", we will first work *naively*.

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- thus, we can think of $\mathcal{O}$ as a "class of systems" of a specific type.

Formalism | Philosophical Foundations

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Formalism | Philosophical Foundations

Phil. Found. = Ontology + Epistemology + Logic + Phil. Mind

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- An *ontological theory* for $\mathcal{O}$ is a subclass $\mathcal{O}_{real} \subset \mathcal{O}$ of *real systems*.

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\begin{tikzcd}
\mathcal{O}_{real} \arrow[dr, hook] & & \mathcal{O}_{mind} \arrow[dl, hook'] \\
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  - a subclass $\mathrm{Know}(\mathcal{O}) \subset \mathrm{True}(\mathcal{O}) \cap \mathrm{Just}(\mathcal{O})$, describing *knowledge*.

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- thus we could consider analogous axiomatization programs to other areas of study $\mathcal{O}$.

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- They define the "physically interesting" Axiomatization Programs of Physics.

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Thus, given an area of study $\mathcal{O}$ we want to

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3. prove that the axiomatic language L satisfies additional properties, including consistency.

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  - step 3. consists in proving that such a morphism exist if L is suitable (e.g., semantically consistent).

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We presented three strategies, all of them based on the following additional epistemological requirement.

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- ... revealing *higher category theory as a natural language to discuss philosophy*.

Plain

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   a. the maps $Q_{ij} : (\mathrm{Ep}(\mathcal{O}_{cls,i}), L_{cls,i}) \Rightarrow (\mathrm{Ep}(\mathcal{O}_{qnt,j}), L_{qnt,j})$ are interpreted as *local axiomatic quantizations*
   b. the underlying $Q_{ij} : \mathrm{Ep}(\mathcal{O}_{cls,i}) \Rightarrow \mathrm{Ep}(\mathcal{O}_{qnt,j})$ are *local philosophical quantizations*

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- local/global axiomatic quantization depends on the axiomatic language
- **Ex.** Groenewold theorem establishes that there is no "suitable" axiomatic quantization, defined in any class of classical physical systems containing classical mechanics, relative to a certain axiomatic language...
- ...however it tells nothing about philosophical quantization or axiomatic quantization relative to other axiomatic languages.

Interpretation

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$$
\begin{array}{ccc}
\mathrm{pb}_{ijk} & \longrightarrow & \mathcal{O}_{qnt,j} \\
\downarrow & & \downarrow \\
\mathcal{O}_{cls,i} & \xrightarrow[PQ_{ik}]{} & \mathcal{O}_{prt,k}
\end{array}
$$

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   a if it has a section C_{kj} from the right (called *local philosophical completion*), we can define a local philosophical quantization by $Q_{ij} = C_{jk} \circ PQ_{ik}$.

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  c a subclass $\mathcal{O}_{cls,i} \subset \mathcal{O}_{cls}$ of physical systems is said to be *(j,k)-renormalizable* if the philosophical perturbative quantization onto a class of j-renormalizable k-perturbative systems is well-defined.

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4. Axiomatization Program of Physics

Part II: Towards an Approach

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Results | Preliminaries

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$$\begin{tikzcd}
 & \mathrm{Belief}(\mathcal{O}) & \\
\mathrm{Just}(\mathcal{O}) \arrow[ur, hook] & \mathrm{Know}(\mathcal{O}) \arrow[l, hook'] \arrow[u, hook] \arrow[r, hook] & \mathrm{True}(\mathcal{O}) \arrow[ul, hook']
\end{tikzcd}$$

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Results | Preliminaries

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- Let $\mathbf{H}$ be a category.An object $X \in \mathcal{H}$ is *injective* if every monomorphism $f : X \to Y$ is a split monomorphism.

Results | First Approach

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- then every abstraction process internal to $\mathbf{H}$ defines an "universal" axiomatic language L such that $(\mathrm{Ep}(\mathcal{O}', L)$ is a formalist approach for $\mathcal{O}'$ internal to $\mathbf{H}$.

Results | Example

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Corollary.

- Let $\mathcal{O}$ an area of study internal to ZFC or NBG. If there is some $\mathcal{O}_0 \subset \mathcal{O}$ which admits a formalist approach, then every abstraction process extends this formalist approach from $\mathcal{O}_0$ to $\mathcal{O}$.

Results | Physics

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- But gauge theories, supergravity and string theory are examples of classes of Physical systems in which“homotopies” play a fundamental role...
- ... which forces us to look for languages internal to more abstract categories than ZFC/NBG.

That is all...

A minha vó caiu
e quebrou o fêmur
Caiu o meu avô
e perdeu os dentes.

Espero que a chuva que cai lá fora
lave a culpa da ausência
e leve embora o desatino
que assola minh'alma fina.