# Parameterized Enriched Topoi, Geometric Cohomology and Algebra-Valued Geometry

Yuri Ximenes Martins

Math-Phys-Cat

March 24, 2023

# About

# About

- this talk is about a research program:

# About

- this talk is about a research program:
  - open to collaborations

# About

- this talk is about a research program:
  - open to collaborations
  - eager for collaborators

# About

- this talk is about a research program:
  - open to collaborations
  - eager for collaborators
  - maybe a bit ambitious

# About

- this talk is about a research program:
  - open to collaborations
  - eager for collaborators
  - maybe a bit ambitious
  - on the relation between Logic, Geometry and Physics.

# About

- a namesake project covering some aspects of the research program has been in preparation for a year

# About

- a namesake project covering some aspects of the research program has been in preparation for a year
- it is to be submitted to Fapesp as part of a proposal to Auxílio à Jovem Pesquisador

# About

- a namesake project covering some aspects of the research program has been in preparation for a year
- it is to be submitted to Fapesp as part of a proposal to Auxílio à Jovem Pesquisador
  - prof. Cristian Ortiz (IME-USP) will be the responsible researcher

# About

- a namesake project covering some aspects of the research program has been in preparation for a year
- it is to be submitted to Fapesp as part of a proposal to Auxílio à Jovem Pesquisador
  - prof. Cristian Ortiz (IME-USP) will be the responsible researcher
  - prof. Hugo Luiz Mariano (IME-USP) was the main encourager of the project

# About

- a namesake project covering some aspects of the research program has been in preparation for a year
- it is to be submitted to Fapesp as part of a proposal to Auxílio à Jovem Pesquisador
  - prof. Cristian Ortiz (IME-USP) will be the responsible researcher
  - prof. Hugo Luiz Mariano (IME-USP) was the main encourager of the project
  - prof. Ivan Struchiner (IME-USP) provided support for bureaucratic matters

# About

- Before starting this seminar properly I would like to:

# About

- Before starting this seminar properly I would like to:
  - thank Cristian, Hugo and Ivan for this opportunity

# About

- Before starting this seminar properly I would like to:
  - thank Cristian, Hugo and Ivan for this opportunity
  - invite you to collaborate with us.

# The Talk

# The Talk

1. Logic-Geometry-Physics

# The Talk

1. Logic-Geometry-Physics
2. The Program

## The Talk

1. Logic-Geometry-Physics
2. The Program
3. The Project

Logic-Geometry-Physics

# Logic-Geometry-Physics

Logic-Geometry

# Logic-Geometry-Physics

## Logic-Geometry

- the axiomatic aspects of geometry dates 300 BC, with Euclid's "Elements"

# Logic-Geometry-Physics

## Logic-Geometry

- the axiomatic aspects of geometry dates 300 BC, with Euclid's "Elements"
- was then reformulated by Hilbert in 1900s with his "The Foundations of Geometry"

# Logic-Geometry-Physics

## Logic-Geometry

- the axiomatic aspects of geometry dates 300 BC, with Euclid's "Elements"
- was then reformulated by Hilbert in 1900s with his "The Foundations of Geometry"
- a modern axiomatic approach for Differential Geometry (DG) was developed around 1970s by Lawvere, Kock, Dubuc, …

## Logic-Geometry

- the axiomatic aspects of geometry dates 300 BC, with Euclid's "Elements"
- was then reformulated by Hilbert in 1900s with his "The Foundations of Geometry"
- a modern axiomatic approach for Differential Geometry (DG) was developed around 1970s by Lawvere, Kock, Dubuc, …
- … this is *Synthetic Differential Geometry* (SDG).

# Logic-Geometry-Physics

## Logic-Geometry

- the axiomatic aspects of geometry dates 300 BC, with Euclid's "Elements"
- was then reformulated by Hilbert in 1900s with his "The Foundations of Geometry"
- a modern axiomatic approach for Differential Geometry (DG) was developed around 1970s by Lawvere, Kock, Dubuc, …
- … this is *Synthetic Differential Geometry* (SDG).

# Logic-Geometry-Physics

## Logic-Geometry

- the axiomatic aspects of geometry dates 300 BC, with Euclid's "Elements"
- was then reformulated by Hilbert in 1900s with his "The Foundations of Geometry"
- a modern axiomatic approach for Differential Geometry (DG) was developed around 1970s by Lawvere, Kock, Dubuc, …
- … this is *Synthetic Differential Geometry* (SDG).

$$\text{Logic} \xrightarrow{\text{SDG}} \text{Geom} \tag{1}$$

# Logic-Geometry-Physics

Geometry-Physics

# Logic-Geometry-Physics

## Geometry-Physics

- the role of Geometry in modeling Mechanics and Gravity is clear at least since 1600 with the works of Newton and Galileu

# Logic-Geometry-Physics

## Geometry-Physics

- the role of Geometry in modeling Mechanics and Gravity is clear at least since 1600 with the works of Newton and Galileu
- in 1915 Einstein proposed a unified approach to Mechanics and Gravitation in terms of DG (semi-Riemannian geometry)

# Logic-Geometry-Physics

## Geometry-Physics

- the role of Geometry in modeling Mechanics and Gravity is clear at least since 1600 with the works of Newton and Galileu
- in 1915 Einstein proposed a unified approach to Mechanics and Gravitation in terms of DG (semi-Riemannian geometry)
- in 1950s Yang and Mills introduced the Yang-Mills theories, which are a nonabelian version of electromagnetism fully modelled in terms of DG ($G$-principal bundles with connection)

## Geometry-Physics

- the role of Geometry in modeling Mechanics and Gravity is clear at least since 1600 with the works of Newton and Galileu
- in 1915 Einstein proposed a unified approach to Mechanics and Gravitation in terms of DG (semi-Riemannian geometry)
- in 1950s Yang and Mills introduced the Yang-Mills theories, which are a nonabelian version of electromagnetism fully modelled in terms of DG ($G$-principal bundles with connection)
- in the next decades it was observed that the other three "fundamental forces" acting between particles can be modeled as Yang-Mills theories, hence in terms of DG.

## Geometry-Physics

- the role of Geometry in modeling Mechanics and Gravity is clear at least since 1600 with the works of Newton and Galileu
- in 1915 Einstein proposed a unified approach to Mechanics and Gravitation in terms of DG (semi-Riemannian geometry)
- in 1950s Yang and Mills introduced the Yang-Mills theories, which are a nonabelian version of electromagnetism fully modelled in terms of DG ($G$-principal bundles with connection)
- in the next decades it was observed that the other three "fundamental forces" acting between particles can be modeled as Yang-Mills theories, hence in terms of DG.

## Geometry-Physics

- the role of Geometry in modeling Mechanics and Gravity is clear at least since 1600 with the works of Newton and Galileu
- in 1915 Einstein proposed a unified approach to Mechanics and Gravitation in terms of DG (semi-Riemannian geometry)
- in 1950s Yang and Mills introduced the Yang-Mills theories, which are a nonabelian version of electromagnetism fully modelled in terms of DG ($G$-principal bundles with connection)
- in the next decades it was observed that the other three "fundamental forces" acting between particles can be modeled as Yang-Mills theories, hence in terms of DG.

$$\mathrm{Geo} \xrightarrow{\mathrm{DG}} \mathrm{Phys} \tag{2}$$

# Logic-Geometry-Physics

Logic-Physics

## Logic-Physics

- the question of building a formal approach to Mechanics was considered by Newton in his "Principia"

# Logic-Geometry-Physics

## Logic-Physics

- the question of building a formal approach to Mechanics was considered by Newton in his "Principia"
- an ampler view on axiomatizing the entire Physics was considered as part of Hilbert's formalist school

# Logic-Geometry-Physics

## Logic-Physics

- the question of building a formal approach to Mechanics was considered by Newton in his "Principia"
- an ampler view on axiomatizing the entire Physics was considered as part of Hilbert's formalist school
- in 1900 he proved his famous list of 23 problems that would impact 20th century mathematics

# Logic-Geometry-Physics

## Logic-Physics

- the question of building a formal approach to Mechanics was considered by Newton in his "Principia"
- an ampler view on axiomatizing the entire Physics was considered as part of Hilbert's formalist school
- in 1900 he proved his famous list of 23 problems that would impact 20th century mathematics
- the 6th problem was about proving an "axiomatization for the existing Physics" at that time, being axiomatization of Statistical Mechanics, of Continuum Mechanics and the "continuum limit" the main issues

# Logic-Geometry-Physics

## Logic-Physics

- the question of building a formal approach to Mechanics was considered by Newton in his "Principia"
- an ampler view on axiomatizing the entire Physics was considered as part of Hilbert's formalist school
- in 1900 he proved his famous list of 23 problems that would impact 20th century mathematics
- the 6th problem was about proving an "axiomatization for the existing Physics" at that time, being axiomatization of Statistical Mechanics, of Continuum Mechanics and the "continuum limit" the main issues

# Logic-Geometry-Physics

## Logic-Physics

- the question of building a formal approach to Mechanics was considered by Newton in his "Principia"
- an ampler view on axiomatizing the entire Physics was considered as part of Hilbert's formalist school
- in 1900 he proved his famous list of 23 problems that would impact 20th century mathematics
- the 6th problem was about proving an "axiomatization for the existing Physics" at that time, being axiomatization of Statistical Mechanics, of Continuum Mechanics and the "continuum limit" the main issues

$$\text{Logic} \xrightarrow{\text{Hilb}} \text{Phys} \tag{3}$$

## Logic-Geometry-Physics

Logic-Physics

## Logic-Geometry-Physics

### Logic-Physics

- in 1930 the axiomatization of Probability Theory by Kolmogorov axioms was the first step towards the axiomatization of Statistical Mechanics

# Logic-Geometry-Physics

## Logic-Physics

- in 1930 the axiomatization of Probability Theory by Kolmogorov axioms was the first step towards the axiomatization of Statistical Mechanics
- the focus was then on Continuum Mechanics and on the "continuum limit"…

## Logic-Physics

- in 1930 the axiomatization of Probability Theory by Kolmogorov axioms was the first step towards the axiomatization of Statistical Mechanics
- the focus was then on Continuum Mechanics and on the "continuum limit"…
- … but the problem remains open since them

## Logic-Physics

- in 1930 the axiomatization of Probability Theory by Kolmogorov axioms was the first step towards the axiomatization of Statistical Mechanics
- the focus was then on Continuum Mechanics and on the "continuum limit"…
- … but the problem remains open since them
- part of its difficulty relies on the vagueness of the original statement.

# Logic-Geometry-Physics

## Logic-Physics

- in 1930 the axiomatization of Probability Theory by Kolmogorov axioms was the first step towards the axiomatization of Statistical Mechanics
- the focus was then on Continuum Mechanics and on the "continuum limit"…
- … but the problem remains open since them
- part of its difficulty relies on the vagueness of the original statement.
- the elevation of Hilbert's 6th problem status to that of a genuine "Axiomatization Problem of Physics" was suggested by Hilbert, by Corry and also by the author

## Logic-Physics

- in 1930 the axiomatization of Probability Theory by Kolmogorov axioms was the first step towards the axiomatization of Statistical Mechanics
- the focus was then on Continuum Mechanics and on the "continuum limit"…
- … but the problem remains open since them
- part of its difficulty relies on the vagueness of the original statement.
- the elevation of Hilbert's 6th problem status to that of a genuine "Axiomatization Problem of Physics" was suggested by Hilbert, by Corry and also by the author
- an attempt to a formal statement was presented by Bunge in his "Foundations of Physics" and generalized by the author in his PhD thesis.

# Logic-Geometry-Physics

Logic-Physics

### Logic-Physics

- joining the previous diagrams one could try to used SDG to axiomatize Continuum Mechanics, as below.

### Logic-Physics

- joining the previous diagrams one could try to used SDG to axiomatize Continuum Mechanics, as below.

## Logic-Physics

- joining the previous diagrams one could try to used SDG to axiomatize Continuum Mechanics, as below.

$$
\begin{tikzcd}
 & \text{Physics} & \\
\text{Logic} \arrow[ur, dashed, "\mathrm{DG} \circ \mathrm{SDG}"] \arrow[rr, "\mathrm{SDG}"'] & & \text{Geo} \arrow[ul, "\mathrm{DG}"']
\end{tikzcd}
\tag{4}
$$

- in other words, the idea would be to take $\mathrm{Hilb} = \mathrm{DG} \circ \mathrm{SDG}$.

# Logic-Geometry-Physics

Logic-Physics

### Logic-Physics

- in 1980s Lawvere proposed that SDG could really be used to axiomatize Continuum Mechanics

# Logic-Geometry-Physics

## Logic-Physics

- in 1980s Lawvere proposed that SDG could really be used to axiomatize Continuum Mechanics
- models for SDG are given by smooth toposes. These are:

## Logic-Physics

- in 1980s Lawvere proposed that SDG could really be used to axiomatize Continuum Mechanics
- models for SDG are given by smooth toposes. These are:
  - toposes $\mathcal{T}$

# Logic-Geometry-Physics

## Logic-Physics

- in 1980s Lawvere proposed that SDG could really be used to axiomatize Continuum Mechanics
- models for SDG are given by smooth toposes. These are:
  - toposes $\mathcal{T}$
  - endowed with:

# Logic-Geometry-Physics

## Logic-Physics

- in 1980s Lawvere proposed that SDG could really be used to axiomatize Continuum Mechanics
- models for SDG are given by smooth toposes. These are:
  - toposes $\mathcal{T}$
  - endowed with:
    - a distinguished object $k \in \mathcal{T}$ which is a commutative ring object

# Logic-Geometry-Physics

## Logic-Physics

- in 1980s Lawvere proposed that SDG could really be used to axiomatize Continuum Mechanics
- models for SDG are given by smooth toposes. These are:
  - toposes $\mathcal{T}$
  - endowed with:
    - a distinguished object $k \in \mathcal{T}$ which is a commutative ring object
    - a distinguished object $R \in \mathcal{T}$ which is a commutative $k$-algebra object

## Logic-Physics

- in 1980s Lawvere proposed that SDG could really be used to axiomatize Continuum Mechanics
- models for SDG are given by smooth toposes. These are:
  - toposes $\mathcal{T}$
  - endowed with:
    - a distinguished object $k \in \mathcal{T}$ which is a commutative ring object
    - a distinguished object $R \in \mathcal{T}$ which is a commutative $k$-algebra object
  - such that Kock-Lawvere axiom is satisfied, i.e., certain canonical functor has a right-adjoint and the unit is an isomorphism.

# Logic-Geometry-Physics

Logic-Physics

# Logic-Geometry-Physics

## Logic-Physics

- Notice that smooth toposes are toposes endowed with additional objects satisfying additional properties. This reveals a dependence on the language used to formalize the concept.

### Logic-Physics

- Notice that smooth toposes are toposes endowed with additional objects satisfying additional properties. This reveals a dependence on the language used to formalize the concept.
- Thus, considering SDG as a candidate to Hilbert's sixth problem we are trying to axiomatize Physics through a language whose syntax depends on the language used to build its models.

### Logic-Physics

- Notice that smooth toposes are toposes endowed with additional objects satisfying additional properties. This reveals a dependence on the language used to formalize the concept.
- Thus, considering SDG as a candidate to Hilbert's sixth problem we are trying to axiomatize Physics through a language whose syntax depends on the language used to build its models.
- This leads us to ask:

## Logic-Physics

- Notice that smooth toposes are toposes endowed with additional objects satisfying additional properties. This reveals a dependence on the language used to formalize the concept.
- Thus, considering SDG as a candidate to Hilbert's sixth problem we are trying to axiomatize Physics through a language whose syntax depends on the language used to build its models.
- This leads us to ask:
  - *is there an axiomatics to DG which is really model independent, being a natural candidate to attack Hilbert's sixth problem?*

Cohesion

### Cohesion

- in 1990s Lawvere proposed to look at the language whose models are not smooth toposes, but *cohesive toposes*

## Cohesion

- in 1990s Lawvere proposed to look at the language whose models are not smooth toposes, but *cohesive toposes*
- in other words, he suggested to focus on the so-called *Cohesive Geometry*.

## Cohesion

- in 1990s Lawvere proposed to look at the language whose models are not smooth toposes, but *cohesive toposes*
- in other words, he suggested to focus on the so-called *Cohesive Geometry*.
- roughly, a *cohesive topos* is given by:

## Cohesion

- in 1990s Lawvere proposed to look at the language whose models are not smooth toposes, but *cohesive toposes*
- in other words, he suggested to focus on the so-called *Cohesive Geometry*.
- roughly, a *cohesive topos* is given by:
  - a Grothendieck topos $\mathbf{H}$

## Cohesion

- in 1990s Lawvere proposed to look at the language whose models are not smooth toposes, but *cohesive toposes*
- in other words, he suggested to focus on the so-called *Cohesive Geometry*.
- roughly, a *cohesive topos* is given by:
  - a Grothendieck topos $\mathbf{H}$
  - such that the canonical geometric morphism $\mathbf{H} \to \mathbf{Set}$ has additional left and right adjoints

## Cohesion

- in 1990s Lawvere proposed to look at the language whose models are not smooth toposes, but *cohesive toposes*
- in other words, he suggested to focus on the so-called *Cohesive Geometry*.
- roughly, a *cohesive topos* is given by:
  - a Grothendieck topos $\mathbf{H}$
  - such that the canonical geometric morphism $\mathbf{H} \to \mathbf{Set}$ has additional left and right adjoints
  - and the adjoints preserve certain limits.

## Cohesion

- in 1990s Lawvere proposed to look at the language whose models are not smooth toposes, but *cohesive toposes*
- in other words, he suggested to focus on the so-called *Cohesive Geometry*.
- roughly, a *cohesive topos* is given by:
  - a Grothendieck topos $\mathbf{H}$
  - such that the canonical geometric morphism $\mathbf{H} \to \mathbf{Set}$ has additional left and right adjoints
  - and the adjoints preserve certain limits.
- the definition is purely in terms of categorical data

## Cohesion

- in 1990s Lawvere proposed to look at the language whose models are not smooth toposes, but *cohesive toposes*
- in other words, he suggested to focus on the so-called *Cohesive Geometry*.
- roughly, a *cohesive topos* is given by:
  - a Grothendieck topos $\mathbf{H}$
  - such that the canonical geometric morphism $\mathbf{H} \to \mathbf{Set}$ has additional left and right adjoints
  - and the adjoints preserve certain limits.
- the definition is purely in terms of categorical data
- it is, therefore, independent of the nature of the ambient where it was defined…

## Cohesion

- in 1990s Lawvere proposed to look at the language whose models are not smooth toposes, but *cohesive toposes*
- in other words, he suggested to focus on the so-called *Cohesive Geometry*.
- roughly, a *cohesive topos* is given by:
  - a Grothendieck topos $\mathbf{H}$
  - such that the canonical geometric morphism $\mathbf{H} \to \mathbf{Set}$ has additional left and right adjoints
  - and the adjoints preserve certain limits.
- the definition is purely in terms of categorical data
- it is, therefore, independent of the nature of the ambient where it was defined…

## Cohesion

- in 1990s Lawvere proposed to look at the language whose models are not smooth toposes, but *cohesive toposes*
- in other words, he suggested to focus on the so-called *Cohesive Geometry*.
- roughly, a *cohesive topos* is given by:
  - a Grothendieck topos $\mathbf{H}$
  - such that the canonical geometric morphism $\mathbf{H} \to \mathbf{Set}$ has additional left and right adjoints
  - and the adjoints preserve certain limits.
- the definition is purely in terms of categorical data
- it is, therefore, independent of the nature of the ambient where it was defined…

$$
\begin{tikzcd}[column sep=huge]
\mathbf{H} \arrow[r, bend left=10, "\Gamma" description] \arrow[r, bend right=45, "\mathrm{CoDisc}"'] & \mathbf{Set} \arrow[l, bend right=45, "\mathrm{Disc}"'] \arrow[l, bend left=10, "\mathrm{LCst}" description]
\end{tikzcd}
\tag{5}
$$

Cohesion

## Cohesion

- therefore, the presence of cohesion in a topos is about the extension of a adjoint pair to an adjoint 4-tuple between $\mathbf{H}$ and $\mathbf{Set}$

## Cohesion

- therefore, the presence of cohesion in a topos is about the extension of a adjoint pair to an adjoint 4-tuple between $\mathbf{H}$ and $\mathbf{Set}$
- but every adjoint 4-tuple between $\mathbf{C}$ and $\mathbf{D}$ induces an adjoint triple in $\mathbf{C}$.

## Cohesion

- therefore, the presence of cohesion in a topos is about the extension of a adjoint pair to an adjoint 4-tuple between $\mathbf{H}$ and $\mathbf{Set}$
- but every adjoint 4-tuple between $\mathbf{C}$ and $\mathbf{D}$ induces an adjoint triple in $\mathbf{C}$.
- in our context:

## Cohesion

- therefore, the presence of cohesion in a topos is about the extension of a adjoint pair to an adjoint 4-tuple between $\mathbf{H}$ and $\mathbf{Set}$
- but every adjoint 4-tuple between $\mathbf{C}$ and $\mathbf{D}$ induces an adjoint triple in $\mathbf{C}$.
- in our context:

## Cohesion

- therefore, the presence of cohesion in a topos is about the extension of a adjoint pair to an adjoint 4-tuple between $\mathbf{H}$ and $\mathbf{Set}$
- but every adjoint 4-tuple between $\mathbf{C}$ and $\mathbf{D}$ induces an adjoint triple in $\mathbf{C}$.
- in our context:

$$
\begin{tikzcd}
\mathbf{H} \arrow[r, bend left=50, "\mathrm{Disc}\circ\Gamma"] \arrow[r, "\mathrm{LCst}\circ\Gamma" description] \arrow[r, bend right=50, "\mathrm{LCst}\circ\mathrm{CoDisc}"'] \arrow[r, phantom, shift left=4, "\perp"] \arrow[r, phantom, shift right=4, "\perp"] & \mathbf{H}
\end{tikzcd}
\qquad
\begin{tikzcd}
\mathbf{H} \arrow[r, bend left=50, "\int" description] \arrow[r, "\flat" description] \arrow[r, bend right=50, "\sharp" description] \arrow[r, phantom, shift left=4, "\perp"] \arrow[r, phantom, shift right=4, "\perp"] & \mathbf{H}
\end{tikzcd}
\tag{6}
$$

## Logic-Geometry-Physics

Cohesion

### Cohesion

- we think of an endofunctor $F : \mathbf{H} \to \mathbf{H}$ (typically a monad) in a topos as an *internal modality*

## Cohesion

- we think of an endofunctor $F : \mathbf{H} \to \mathbf{H}$ (typically a monad) in a topos as an *internal modality*
- if $F$ happens to have a left adjoint $G$ we think of $G$ as the "dual modality"

## Cohesion

- we think of an endofunctor $F : \mathbf{H} \to \mathbf{H}$ (typically a monad) in a topos as an *internal modality*
- if $F$ happens to have a left adjoint $G$ we think of $G$ as the "dual modality"
- thus, "cohesion" in a given topos is given by the addition of three modalities $\flat, \int, \sharp$ such that $\int$ is dual to $\flat$ which is dual to $\sharp$

## Cohesion

- we think of an endofunctor $F : \mathbf{H} \to \mathbf{H}$ (typically a monad) in a topos as an *internal modality*
- if $F$ happens to have a left adjoint $G$ we think of $G$ as the "dual modality"
- thus, "cohesion" in a given topos is given by the addition of three modalities $\flat, \int, \sharp$ such that $\int$ is dual to $\flat$ which is dual to $\sharp$
- we say that:

## Cohesion

- we think of an endofunctor $F : \mathbf{H} \to \mathbf{H}$ (typically a monad) in a topos as an *internal modality*
- if $F$ happens to have a left adjoint $G$ we think of $G$ as the "dual modality"
- thus, "cohesion" in a given topos is given by the addition of three modalities $\flat, \int, \sharp$ such that $\int$ is dual to $\flat$ which is dual to $\sharp$
- we say that:
  - $\flat X$ is $X$ in the *flat modality*

## Cohesion

- we think of an endofunctor $F : \mathbf{H} \to \mathbf{H}$ (typically a monad) in a topos as an *internal modality*
- if $F$ happens to have a left adjoint $G$ we think of $G$ as the "dual modality"
- thus, "cohesion" in a given topos is given by the addition of three modalities $\flat, \int, \sharp$ such that $\int$ is dual to $\flat$ which is dual to $\sharp$
- we say that:
  - $\flat X$ is $X$ in the *flat modality*
  - $\int X$ is $X$ in the *shape modality*

## Cohesion

- we think of an endofunctor $F : \mathbf{H} \to \mathbf{H}$ (typically a monad) in a topos as an *internal modality*
- if $F$ happens to have a left adjoint $G$ we think of $G$ as the "dual modality"
- thus, "cohesion" in a given topos is given by the addition of three modalities $\flat, \int, \sharp$ such that $\int$ is dual to $\flat$ which is dual to $\sharp$
- we say that:
  - $\flat X$ is $X$ in the *flat modality*
  - $\int X$ is $X$ in the *shape modality*
  - $\sharp X$ is $X$ in the *sharp modality*

Cohesion

### Cohesion

- recall the relation between logic (dependent type theory) and category theory:

## Cohesion

- recall the relation between logic (dependent type theory) and category theory:
  - for every category $\mathbf{C}$ one assigns its internal logic $\mathrm{Synt}(\mathbf{C})$

## Cohesion

- recall the relation between logic (dependent type theory) and category theory:
  - for every category $\mathbf{C}$ one assigns its internal logic $\mathrm{Synt}(\mathbf{C})$
  - reciprocally, given a logic (dependent type theory) $L$ we build a locally cartesian closed category $\mathrm{Cntx}(L)$ of contexts, proving it a categorical semantics.

## Cohesion

- recall the relation between logic (dependent type theory) and category theory:
  - for every category $\mathbf{C}$ one assigns its internal logic $\mathrm{Synt}(\mathbf{C})$
  - reciprocally, given a logic (dependent type theory) $L$ we build a locally cartesian closed category $\mathrm{Cntx}(L)$ of contexts, proving it a categorical semantics.
  - the construction is functorial (2-functorial), providing adjoint functors

## Cohesion

- recall the relation between logic (dependent type theory) and category theory:
  - for every category $\mathbf{C}$ one assigns its internal logic $\mathrm{Synt}(\mathbf{C})$
  - reciprocally, given a logic (dependent type theory) $L$ we build a locally cartesian closed category $\mathrm{Cntx}(L)$ of contexts, proving it a categorical semantics.
  - the construction is functorial (2-functorial), providing adjoint functors

## Cohesion

- recall the relation between logic (dependent type theory) and category theory:
  - for every category $\mathbf{C}$ one assigns its internal logic $\mathrm{Synt}(\mathbf{C})$
  - reciprocally, given a logic (dependent type theory) $L$ we build a locally cartesian closed category $\mathrm{Cntx}(L)$ of contexts, proving it a categorical semantics.
  - the construction is functorial (2-functorial), providing adjoint functors

$$\mathrm{Synt} : \mathbf{Cat} \rightleftarrows \mathbf{DType} : \mathrm{Cntx} \tag{7}$$

- for different classes of categories/logic this reduces to an 2-equivalence.

Cohesion

### Cohesion

- as a formal language, the dependent type theory corresponding to a topos (or any locally cartesian closed category) $\mathbf{H}$ is intuitionistic (of Martin-Löf type)

### Cohesion

- as a formal language, the dependent type theory corresponding to a topos (or any locally cartesian closed category) $\mathbf{H}$ is intuitionistic (of Martin-Löf type)
- by the syntax-semantics relation, a modality in a topos $\mathbf{H}$ induces a modality in the underlying intuitionistic logic

## Cohesion

- as a formal language, the dependent type theory corresponding to a topos (or any locally cartesian closed category) $\mathbf{H}$ is intuitionistic (of Martin-Löf type)
- by the syntax-semantics relation, a modality in a topos $\mathbf{H}$ induces a modality in the underlying intuitionistic logic
- therefore, *cohesive logic* should be a flavor of modal Martin-Löf dependent type theory

## Cohesion

- as a formal language, the dependent type theory corresponding to a topos (or any locally cartesian closed category) $\mathbf{H}$ is intuitionistic (of Martin-Löf type)
- by the syntax-semantics relation, a modality in a topos $\mathbf{H}$ induces a modality in the underlying intuitionistic logic
- therefore, *cohesive logic* should be a flavor of modal Martin-Löf dependent type theory
- and one should have a diagram as follows…

## Cohesion

- as a formal language, the dependent type theory corresponding to a topos (or any locally cartesian closed category) $\mathbf{H}$ is intuitionistic (of Martin-Löf type)
- by the syntax-semantics relation, a modality in a topos $\mathbf{H}$ induces a modality in the underlying intuitionistic logic
- therefore, *cohesive logic* should be a flavor of modal Martin-Löf dependent type theory
- and one should have a diagram as follows…

## Cohesion

- as a formal language, the dependent type theory corresponding to a topos (or any locally cartesian closed category) $\mathbf{H}$ is intuitionistic (of Martin-Löf type)
- by the syntax-semantics relation, a modality in a topos $\mathbf{H}$ induces a modality in the underlying intuitionistic logic
- therefore, *cohesive logic* should be a flavor of modal Martin-Löf dependent type theory
- and one should have a diagram as follows…

$$
\begin{tikzcd}
\mathbf{CohType} \arrow[r, dashed, "U"] \arrow[d, dashed, bend left, "\mathrm{Cntx}"] \arrow[d, phantom, "\Leftarrow"] & \mathbf{MLDType} \arrow[r, hook] \arrow[d, bend left, "\mathrm{Cntx}"] \arrow[d, phantom, "\Leftarrow"] & \mathbf{DType} \arrow[d, bend right, "\mathrm{Cntx}"'] \arrow[d, phantom, "\Rightarrow"] \\
\mathbf{CohTopos} \arrow[r, "U"'] \arrow[u, dashed, bend left, "\mathrm{Synt}"] & \mathbf{Topos} \arrow[r, hook] \arrow[u, bend left, "\mathrm{Synt}"] & \mathbf{Cat} \arrow[u, bend right, "\mathrm{Synt}"']
\end{tikzcd}
\qquad (8)
$$

Higher Cohesion

## Higher Cohesion

- one of the advantage of working with a language whose models/contexts are independent of a background language (as happens for cohesive logic) is their contexts can be easily be abstracted:

## Higher Cohesion

- one of the advantage of working with a language whose models/contexts are independent of a background language (as happens for cohesive logic) is their contexts can be easily be abstracted:
  - applying categorication by internalization

# Logic-Geometry-Physics

## Higher Cohesion

- one of the advantage of working with a language whose models/contexts are independent of a background language (as happens for cohesive logic) is their contexts can be easily be abstracted:
  - applying categorication by internalization
  - iterating the process

## Higher Cohesion

- one of the advantage of working with a language whose models/contexts are independent of a background language (as happens for cohesive logic) is their contexts can be easily be abstracted:
  - applying categorication by internalization
  - iterating the process
  - passing the limit

## Higher Cohesion

- one of the advantage of working with a language whose models/contexts are independent of a background language (as happens for cohesive logic) is their contexts can be easily be abstracted:
  - applying categorication by internalization
  - iterating the process
  - passing the limit

## Higher Cohesion

- one of the advantage of working with a language whose models/contexts are independent of a background language (as happens for cohesive logic) is their contexts can be easily be abstracted:
  - applying categorication by internalization
  - iterating the process
  - passing the limit

$$\begin{tikzcd}
(1,1)\mathbf{Topos} \arrow[r, hook] \arrow[d, "\mathrm{categ}"'] & (1,1)\mathbf{Cat} \arrow[r, "\simeq"] \arrow[d, "\mathrm{categ}"'] & \mathbf{Cat} \arrow[d, "\mathrm{categ}"] \\
(2,1)\mathbf{Topos} \arrow[r, hook] \arrow[d, "\mathrm{categ}"'] & (2,1)\mathbf{Cat} \arrow[r, hook] \arrow[d, "\mathrm{categ}"'] & 2\mathbf{Cat} \arrow[d, "\mathrm{categ}"] \\
\vdots \arrow[d, "\mathrm{categ}"'] & \vdots \arrow[d, "\mathrm{categ}"'] & \vdots \arrow[d, "\mathrm{categ}"] \\
(\infty,1)\mathbf{Topos} \arrow[r, hook] & (\infty,1)\mathbf{Cat} \arrow[r, hook] & \infty\mathbf{Cat}
\end{tikzcd} \tag{9}$$

# Logic-Geometry-Physics

## Higher Cohesion

## Higher Cohesion

- another possibility is:

## Higher Cohesion

- another possibility is:
  - to take some "$(\infty, 1)$-categorification"

## Higher Cohesion

- another possibility is:
  - to take some "$(\infty, 1)$-categorification"
  - then apply truncations

## Higher Cohesion

- another possibility is:
  - to take some "$(\infty,1)$-categorification"
  - then apply truncations

## Logic-Geometry-Physics

### Higher Cohesion

- another possibility is:
  - to take some "$(\infty,1)$-categorification"
  - then apply truncations

$$
\begin{tikzcd}
(1,1)\mathbf{Topos} \arrow[r, hook] \arrow[d, mapsto, "(\infty{,}1)\,\mathrm{categ}"'] & (1,1)\mathbf{Cat} \arrow[d, mapsto, "(\infty{,}1)\,\mathrm{categ}"] \\
(\infty,1)\mathbf{Topos} \arrow[r, hook] \arrow[d, mapsto] \arrow[dd, bend right, "\tau_{\leq 3}"'] \arrow[ddd, bend right=50, "\tau_{\leq 2}"'] & (\infty,1)\mathbf{Cat} \arrow[d, mapsto] \arrow[dd, bend left, "\tau_{\leq 3}"] \arrow[ddd, bend left=50, "\tau_{\leq 2}"] \\
\vdots \arrow[d, mapsto] & \vdots \arrow[d, mapsto, "\tau_{\leq 3}"'] \\
(3,1)\mathbf{Topos} \arrow[r, hook] \arrow[d, mapsto, "\tau_{\leq 2}"] & (3,1)\mathbf{Cat} \arrow[d, mapsto, "\tau_{\leq 2}"'] \\
(2,1)\mathbf{Topos} \arrow[r, hook] & (2,1)\mathbf{Cat}
\end{tikzcd}
\tag{10}
$$

# Logic-Geometry-Physics

## Higher Cohesion

- one can then try to extend the syntax-semantics duality
  $\mathrm{Synt} : \mathbf{Cat} \rightleftarrows \mathbf{DType} : \mathrm{Cntx}$

# Logic-Geometry-Physics

## Higher Cohesion

- one can then try to extend the syntax-semantics duality $\mathrm{Synt} : \mathbf{Cat} \rightleftarrows \mathbf{DType} : \mathrm{Cntx}$
- in other words, one can ask for the syntax corresponding to the categorified semantics.

# Logic-Geometry-Physics

## Higher Cohesion

- one can then try to extend the syntax-semantics duality $\mathrm{Synt} : \mathbf{Cat} \rightleftarrows \mathbf{DType} : \mathrm{Cntx}$
- in other words, one can ask for the syntax corresponding to the categorified semantics.
- due works of Awodey, Joyal, Voevodsky, Kapulkin, Shulman, and others we know that in the presentation of $(\infty, 1)$-categories by quasicategories:

# Logic-Geometry-Physics

## Higher Cohesion

- one can then try to extend the syntax-semantics duality $\mathrm{Synt} : \mathbf{Cat} \rightleftarrows \mathbf{DType} : \mathrm{Cntx}$
- in other words, one can ask for the syntax corresponding to the categorified semantics.
- due works of Awodey, Joyal, Voevodsky, Kapulkin, Shulman, and others we know that in the presentation of $(\infty, 1)$-categories by quasicategories:
  - the type theory corresponding to locally cartesian closed $(\infty, 1)$-categories is intensional Martin-Löf dependent type theory (Homotopy Type Theory)

# Logic-Geometry-Physics

## Higher Cohesion

- one can then try to extend the syntax-semantics duality $\mathrm{Synt} : \mathbf{Cat} \rightleftarrows \mathbf{DType} : \mathrm{Cntx}$
- in other words, one can ask for the syntax corresponding to the categorified semantics.
- due works of Awodey, Joyal, Voevodsky, Kapulkin, Shulman, and others we know that in the presentation of $(\infty, 1)$-categories by quasicategories:
  - the type theory corresponding to locally cartesian closed $(\infty, 1)$-categories is intensional Martin-Löf dependent type theory (Homotopy Type Theory)
  - the internal type theory of $(\infty, 1)$-toposes is Homotopy Type Theory with Voevodsky's univalence axiom (Awodey's conjecture).

# Logic-Geometry-Physics

## Higher Cohesion

- one can then try to extend the syntax-semantics duality $\mathrm{Synt} : \mathbf{Cat} \rightleftarrows \mathbf{DType} : \mathrm{Cntx}$
- in other words, one can ask for the syntax corresponding to the categorified semantics.
- due works of Awodey, Joyal, Voevodsky, Kapulkin, Shulman, and others we know that in the presentation of $(\infty, 1)$-categories by quasicategories:
  - the type theory corresponding to locally cartesian closed $(\infty, 1)$-categories is intensional Martin-Löf dependent type theory (Homotopy Type Theory)
  - the internal type theory of $(\infty, 1)$-toposes is Homotopy Type Theory with Voevodsky's univalence axiom (Awodey's conjecture).

- one can then try to extend the syntax-semantics duality $\mathrm{Synt} : \mathbf{Cat} \rightleftarrows \mathbf{DType} : \mathrm{Cntx}$
- in other words, one can ask for the syntax corresponding to the categorified semantics.
- due works of Awodey, Joyal, Voevodsky, Kapulkin, Shulman, and others we know that in the presentation of $(\infty, 1)$-categories by quasicategories:
  - the type theory corresponding to locally cartesian closed $(\infty, 1)$-categories is intensional Martin-Löf dependent type theory (Homotopy Type Theory)
  - the internal type theory of $(\infty, 1)$-toposes is Homotopy Type Theory with Voevodsky's univalence axiom (Awodey's conjecture).

$$
\begin{tikzcd}
\mathbf{MLDType} \arrow[rr, hook] \arrow[d, bend left, "\mathrm{Cntx}"] \arrow[d, phantom, "\Longleftarrow"] & & \mathbf{DType} \arrow[d, bend right, "\mathrm{Cntx}"'] \arrow[d, phantom, "\Longrightarrow"] \\
(1,1)\mathbf{Topos} \arrow[rr, hook] \arrow[u, bend left, "\mathrm{Synt}"] \arrow[d, "(\infty{,}1)\,\mathrm{categ}"'] & & (1,1)\mathbf{Cat} \arrow[u, bend right, "\mathrm{Synt}"'] \arrow[d, "(\infty{,}1)\,\mathrm{categ}"] \\
(\infty,1)\mathbf{Topos} \arrow[rr, hook] \arrow[d, bend right, "\mathrm{Synt}"'] \arrow[d, phantom, "\Longleftarrow"] & & (\infty,1)\mathbf{Cat} \arrow[d, bend left, "\mathrm{Synt}"] \arrow[d, phantom, "\Longrightarrow"] \\
\mathbf{UnivHoTT} \arrow[rr, hook] \arrow[u, bend right, "\mathrm{Cntx}"'] & & \mathbf{HoTT} \arrow[u, bend left, "\mathrm{Cntx}"]
\end{tikzcd}
\tag{11}
$$

## Higher Cohesion

- Schreiber extended the axioms of cohesion from toposes to $(\infty, 1)$-toposes.

## Logic-Geometry-Physics

### Higher Cohesion

- Schreiber extended the axioms of cohesion from toposes to $(\infty, 1)$-toposes.
- he also suggested that the construction of a modal HoTT from any cohesive $(\infty, 1)$-topos

## Higher Cohesion

- Schreiber extended the axioms of cohesion from toposes to $(\infty, 1)$-toposes.
- he also suggested that the construction of a modal HoTT from any cohesive $(\infty, 1)$-topos
- this modal HoTT was constructed by Schreiber and Shulman, being called *Cohesive HoTT*

- Schreiber extended the axioms of cohesion from toposes to $(\infty, 1)$-toposes.
- he also suggested that the construction of a modal HoTT from any cohesive $(\infty, 1)$-topos
- this modal HoTT was constructed by Schreiber and Shulman, being called *Cohesive HoTT*
- they implicitly conjectured the extension of last diagram as follows

## Higher Cohesion

- Schreiber extended the axioms of cohesion from toposes to $(\infty, 1)$-toposes.
- he also suggested that the construction of a modal HoTT from any cohesive $(\infty, 1)$-topos
- this modal HoTT was constructed by Schreiber and Shulman, being called *Cohesive HoTT*
- they implicitly conjectured the extension of last diagram as follows

# Logic-Geometry-Physics

## Higher Cohesion

- Schreiber extended the axioms of cohesion from toposes to $(\infty,1)$-toposes.
- he also suggested that the construction of a modal HoTT from any cohesive $(\infty,1)$-topos
- this modal HoTT was constructed by Schreiber and Shulman, being called *Cohesive HoTT*
- they implicitly conjectured the extension of last diagram as follows

$$
\begin{tikzcd}
 & \mathbf{MLDType} \arrow[rr, hook] \arrow[d, bend left, "\mathrm{Cntx}"] \arrow[d, phantom, "\Longleftarrow"] & & \mathbf{DType} \arrow[d, bend left, "\mathrm{Synt}"] \arrow[d, phantom, "\Longrightarrow"] \\
 & (1,1)\mathbf{Topos} \arrow[rr, hook] \arrow[u, bend left, "\mathrm{Synt}"] \arrow[d, dashed, mapsto, "(\infty,1)\,\mathrm{categ}"] & & (1,1)\mathbf{Cat} \arrow[u, bend left, "\mathrm{Cntx}"] \arrow[d, mapsto, "(\infty,1)\,\mathrm{categ}"] \\
(\infty,1)\mathbf{CohTopos} \arrow[r, "U"] \arrow[d, dashed, bend left, "\mathrm{Cntx}"] \arrow[d, phantom, "\Longleftarrow"] & (\infty,1)\mathbf{Topos} \arrow[rr, hook] \arrow[d, bend left, "\mathrm{Cntx}"] \arrow[d, phantom, "\Longleftarrow"] & & (\infty,1)\mathbf{Cat} \arrow[d, bend left, "\mathrm{Synt}"] \arrow[d, phantom, "\Longrightarrow"] \\
\mathbf{CohHoTT} \arrow[r, "U"'] \arrow[u, dashed, bend left, "\mathrm{Synt}"] & \mathbf{UnivHoTT} \arrow[rr, hook] \arrow[u, bend left, "\mathrm{Synt}"] & & \mathbf{HoTT} \arrow[u, bend left, "\mathrm{Cntx}"]
\end{tikzcd}
\tag{12}
$$

- thus, the initial "cohesive logic" could be obtained as a particular instance of this Cohesive HoTT.

## Logic-Geometry-Physics

### (Higher) Logic-Physics

## Logic-Geometry-Physics

### (Higher) Logic-Physics

- let us return to the discussion on Hilbert sixth problem.

## (Higher) Logic-Physics

- let us return to the discussion on Hilbert sixth problem.
- recall that Lawvere introduced cohesive geometry as a possible alternative to SDG

## (Higher) Logic-Physics

- let us return to the discussion on Hilbert sixth problem.
- recall that Lawvere introduced cohesive geometry as a possible alternative to SDG
- in his "Axiomatic Cohesion" he presented some consequences of his axiomatics and implicitly suggested that the underlying "cohesive logic" should be a candidate to provide an axiomatization to Continuum Mechanics

## (Higher) Logic-Physics

- let us return to the discussion on Hilbert sixth problem.
- recall that Lawvere introduced cohesive geometry as a possible alternative to SDG
- in his "Axiomatic Cohesion" he presented some consequences of his axiomatics and implicitly suggested that the underlying "cohesive logic" should be a candidate to provide an axiomatization to Continuum Mechanics
- hence to attack the *original* version of Hilbert's sixth problem

## (Higher) Logic-Physics

- let us return to the discussion on Hilbert sixth problem.
- recall that Lawvere introduced cohesive geometry as a possible alternative to SDG
- in his "Axiomatic Cohesion" he presented some consequences of his axiomatics and implicitly suggested that the underlying "cohesive logic" should be a candidate to provide an axiomatization to Continuum Mechanics
- hence to attack the *original* version of Hilbert's sixth problem
- so, implicitly he proposed to consider $\mathrm{Hilb} = \mathrm{Cohesion}$.

## (Higher) Logic-Physics

- let us return to the discussion on Hilbert sixth problem.
- recall that Lawvere introduced cohesive geometry as a possible alternative to SDG
- in his "Axiomatic Cohesion" he presented some consequences of his axiomatics and implicitly suggested that the underlying "cohesive logic" should be a candidate to provide an axiomatization to Continuum Mechanics
- hence to attack the *original* version of Hilbert's sixth problem
- so, implicitly he proposed to consider $\mathrm{Hilb} = \mathrm{Cohesion}$.

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- let us return to the discussion on Hilbert sixth problem.
- recall that Lawvere introduced cohesive geometry as a possible alternative to SDG
- in his "Axiomatic Cohesion" he presented some consequences of his axiomatics and implicitly suggested that the underlying "cohesive logic" should be a candidate to provide an axiomatization to Continuum Mechanics
- hence to attack the *original* version of Hilbert's sixth problem
- so, implicitly he proposed to consider $\mathrm{Hilb} = \mathrm{Cohesion}$.

$$
\begin{tikzcd}
 & \mathrm{Physics} & \\
\mathrm{Logic} \arrow[ur, dashed, "\text{cohesion}"] \arrow[rr, "\text{cohesive logic}"'] & & \mathrm{Geo} \arrow[ul, "\text{cohesive geometry}"']
\end{tikzcd}
\tag{13}
$$

- but this language is not abstract enough, as we pass to discuss.

# Logic-Geometry-Physics

(Higher) Logic-Physics

## Logic-Geometry-Physics

### (Higher) Logic-Physics

- Physics deals with objects that exists in nature, that are "real"

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- Physics deals with objects that exists in nature, that are "real"
- the meaning of what is "real" is an additional layer of information that need to be given to provide an approach to the foundations of Physics

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- Physics deals with objects that exists in nature, that are "real"
- the meaning of what is "real" is an additional layer of information that need to be given to provide an approach to the foundations of Physics
- it corresponds to fixing an Ontological Theory.

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- Physics deals with objects that exists in nature, that are "real"
- the meaning of what is "real" is an additional layer of information that need to be given to provide an approach to the foundations of Physics
- it corresponds to fixing an Ontological Theory.
- there are different theories, but one typically suppose that Physics ontologically splits into two parts:

## Logic-Geometry-Physics

### (Higher) Logic-Physics

- Physics deals with objects that exists in nature, that are "real"
- the meaning of what is "real" is an additional layer of information that need to be given to provide an approach to the foundations of Physics
- it corresponds to fixing an Ontological Theory.
- there are different theories, but one typically suppose that Physics ontologically splits into two parts:
  - classical Physics

## (Higher) Logic-Physics

- Physics deals with objects that exists in nature, that are "real"
- the meaning of what is "real" is an additional layer of information that need to be given to provide an approach to the foundations of Physics
- it corresponds to fixing an Ontological Theory.
- there are different theories, but one typically suppose that Physics ontologically splits into two parts:
  - classical Physics
  - quantum Physics

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- Physics deals with objects that exists in nature, that are "real"
- the meaning of what is "real" is an additional layer of information that need to be given to provide an approach to the foundations of Physics
- it corresponds to fixing an Ontological Theory.
- there are different theories, but one typically suppose that Physics ontologically splits into two parts:
  - classical Physics
  - quantum Physics
- the axiomatization of Physics therefore involves building:

## (Higher) Logic-Physics

- Physics deals with objects that exists in nature, that are "real"
- the meaning of what is "real" is an additional layer of information that need to be given to provide an approach to the foundations of Physics
- it corresponds to fixing an Ontological Theory.
- there are different theories, but one typically suppose that Physics ontologically splits into two parts:
  - classical Physics
  - quantum Physics
- the axiomatization of Physics therefore involves building:
  - an axiomatization of classical Physics

## (Higher) Logic-Physics

- Physics deals with objects that exists in nature, that are "real"
- the meaning of what is "real" is an additional layer of information that need to be given to provide an approach to the foundations of Physics
- it corresponds to fixing an Ontological Theory.
- there are different theories, but one typically suppose that Physics ontologically splits into two parts:
  - classical Physics
  - quantum Physics
- the axiomatization of Physics therefore involves building:
  - an axiomatization of classical Physics
  - an axiomatization of quantum Physics

## (Higher) Logic-Physics

- Physics deals with objects that exists in nature, that are "real"
- the meaning of what is "real" is an additional layer of information that need to be given to provide an approach to the foundations of Physics
- it corresponds to fixing an Ontological Theory.
- there are different theories, but one typically suppose that Physics ontologically splits into two parts:
  - classical Physics
  - quantum Physics
- the axiomatization of Physics therefore involves building:
  - an axiomatization of classical Physics
  - an axiomatization of quantum Physics
  - a constrain between those axiomatizations…

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- Physics deals with objects that exists in nature, that are "real"
- the meaning of what is "real" is an additional layer of information that need to be given to provide an approach to the foundations of Physics
- it corresponds to fixing an Ontological Theory.
- there are different theories, but one typically suppose that Physics ontologically splits into two parts:
  - classical Physics
  - quantum Physics
- the axiomatization of Physics therefore involves building:
  - an axiomatization of classical Physics
  - an axiomatization of quantum Physics
  - a constrain between those axiomatizations…
    - typically assumed a way to assign to each classical system a corresponding quantum system

## (Higher) Logic-Physics

- Physics deals with objects that exists in nature, that are "real"
- the meaning of what is "real" is an additional layer of information that need to be given to provide an approach to the foundations of Physics
- it corresponds to fixing an Ontological Theory.
- there are different theories, but one typically suppose that Physics ontologically splits into two parts:
  - classical Physics
  - quantum Physics
- the axiomatization of Physics therefore involves building:
  - an axiomatization of classical Physics
  - an axiomatization of quantum Physics
  - a constrain between those axiomatizations…
    - typically assumed a way to assign to each classical system a corresponding quantum system
    - (called a *quantization process*)

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- Physics deals with objects that exists in nature, that are "real"
- the meaning of what is "real" is an additional layer of information that need to be given to provide an approach to the foundations of Physics
- it corresponds to fixing an Ontological Theory.
- there are different theories, but one typically suppose that Physics ontologically splits into two parts:
  - classical Physics
  - quantum Physics
- the axiomatization of Physics therefore involves building:
  - an axiomatization of classical Physics
  - an axiomatization of quantum Physics
  - a constrain between those axiomatizations…
    - typically assumed a way to assign to each classical system a corresponding quantum system
    - (called a *quantization process*)
    - such that the axiomatization of quantum Physics factors through quantization.

# Logic-Geometry-Physics

(Higher) Logic-Physics

### (Higher) Logic-Physics

- to axiomatize Physics is then to build a commutative diagram consisting of the continuum arrows below

## Logic-Geometry-Physics

### (Higher) Logic-Physics

- to axiomatize Physics is then to build a commutative diagram consisting of the continuum arrows below
- so that the dotted arrows follows from the universality of coproducts (ontological hypothesis)

## (Higher) Logic-Physics

- to axiomatize Physics is then to build a commutative diagram consisting of the continuum arrows below
- so that the dotted arrows follows from the universality of coproducts (ontological hypothesis)

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- to axiomatize Physics is then to build a commutative diagram consisting of the continuum arrows below
- so that the dotted arrows follows from the universality of coproducts (ontological hypothesis)

$$
\begin{tikzcd}
 & \text{Class} \arrow[d, dashed, "\text{quantiz}"] \arrow[dd, hook, bend left] & \\
 & \text{Quant} \arrow[d, hook] & \\
 & \text{Physics} & \\
\text{Logic} \arrow[uuur, bend left] \arrow[uur, bend left] \arrow[ur, dashed] \arrow[rr] & & \text{Geo} \arrow[uuul, bend right] \arrow[uul, bend right] \arrow[ul, dashed]
\end{tikzcd}
\tag{14}
$$

# Logic-Geometry-Physics

(Higher) Logic-Physics

### (Higher) Logic-Physics

- under this ontological assumption and following this approach, the Axiomatization Problem of Physics contains one of the Millennium Problems:

### (Higher) Logic-Physics

- under this ontological assumption and following this approach, the Axiomatization Problem of Physics contains one of the Millennium Problems:
  - to provide a quantum version of Yang-Mills Theory (structured by any semi-simple Lie group $G$) satisfying the mass-gap condition [?].

# Logic-Geometry-Physics

(Higher) Logic-Physics

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- if one tries to use cohesive logic to attack Axiomatization Problem of Physics we see that even for classical aspects of Yang-Mills theories we need to work in cohesive $(2, 1)$-toposes

## (Higher) Logic-Physics

- if one tries to use cohesive logic to attack Axiomatization Problem of Physics we see that even for classical aspects of Yang-Mills theories we need to work in cohesive $(2,1)$-toposes
- thus, cohesive logic is not abstract enough…

## (Higher) Logic-Physics

- if one tries to use cohesive logic to attack Axiomatization Problem of Physics we see that even for classical aspects of Yang-Mills theories we need to work in cohesive $(2,1)$-toposes
- thus, cohesive logic is not abstract enough…
- the natural candidate pass to be cohesive HoTT

## (Higher) Logic-Physics

- if one tries to use cohesive logic to attack Axiomatization Problem of Physics we see that even for classical aspects of Yang-Mills theories we need to work in cohesive $(2, 1)$-toposes
- thus, cohesive logic is not abstract enough…
- the natural candidate pass to be cohesive HoTT

### (Higher) Logic-Physics

- if one tries to use cohesive logic to attack Axiomatization Problem of Physics we see that even for classical aspects of Yang-Mills theories we need to work in cohesive $(2,1)$-toposes
- thus, cohesive logic is not abstract enough…
- the natural candidate pass to be cohesive HoTT

$$
\begin{tikzcd}
 & \text{Physics} & \\
\text{Logic} \arrow[ur, dashed] \arrow[rr, "\text{coh. HoTT}"'] & & \text{Geo} \arrow[ul, "\text{coh. high. geometry}"']
\end{tikzcd}
\tag{15}
$$

# Logic-Geometry-Physics

(Higher) Logic-Physics

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- Schreiber showed that cohesive HoTT can be used to formulate higher cohesive geometry:

### (Higher) Logic-Physics

- Schreiber showed that cohesive HoTT can be used to formulate higher cohesive geometry:

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- Schreiber showed that cohesive HoTT can be used to formulate higher cohesive geometry:

$$\begin{tikzcd}
& \mathrm{Class} \arrow[dd, hook, bend left] & \\
& \mathrm{Quant} \arrow[d, hook] & \\
& \mathrm{Physics} & \\
\mathrm{Logic} \arrow[rr, "\text{coh. HoTT}"'] & & \mathrm{Geo}
\end{tikzcd} \tag{16}$$

# Logic-Geometry-Physics

(Higher) Logic-Physics

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- in a sequence of papers Schreiber and his collaborators showed that the fundamental aspects of classical

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- in a sequence of papers Schreiber and his collaborators showed that the fundamental aspects of classical
  - gauge theories, as Yang-Mills Theories

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- in a sequence of papers Schreiber and his collaborators showed that the fundamental aspects of classical
  - gauge theories, as Yang-Mills Theories
  - higher gauge theories, as superstring theories, supergravity, M-Theory

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- in a sequence of papers Schreiber and his collaborators showed that the fundamental aspects of classical
  - gauge theories, as Yang-Mills Theories
  - higher gauge theories, as superstring theories, supergravity, M-Theory
- can be formalized internally to some cohesive $(\infty, 1)$-topos

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- in a sequence of papers Schreiber and his collaborators showed that the fundamental aspects of classical
  - gauge theories, as Yang-Mills Theories
  - higher gauge theories, as superstring theories, supergravity, M-Theory
- can be formalized internally to some cohesive $(\infty, 1)$-topos

## (Higher) Logic-Physics

- in a sequence of papers Schreiber and his collaborators showed that the fundamental aspects of classical
  - gauge theories, as Yang-Mills Theories
  - higher gauge theories, as superstring theories, supergravity, M-Theory
- can be formalized internally to some cohesive $(\infty, 1)$-topos

$$
\begin{tikzcd}
 & \text{Class} \arrow[dd, bend left] & \\
 & \text{Quant} \arrow[d, hook] & \\
 & \text{Physics} & \\
\text{Logic} \arrow[rr, "\text{coh. HoTT}"'] \arrow[uuur, dashed, bend left] & & \text{Geo} \arrow[uuul, bend right, "\text{coh. high. geometry}"']
\end{tikzcd}
\tag{17}
$$

# Logic-Geometry-Physics

## (Higher) Logic-Physics

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- Schreiber-Nuiten proved that a globally defined quantization process exists in those higher gauge theories which are "anomaly free"

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- Schreiber-Nuiten proved that a globally defined quantization process exists in those higher gauge theories which are "anomaly free"
- this is the so-called *cohomological/pull-push/motivic quantization*

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- Schreiber-Nuiten proved that a globally defined quantization process exists in those higher gauge theories which are "anomaly free"
- this is the so-called *cohomological/pull-push/motivic quantization*

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- Schreiber-Nuiten proved that a globally defined quantization process exists in those higher gauge theories which are "anomaly free"
- this is the so-called *cohomological/pull-push/motivic quantization*

$$
\begin{tikzcd}
& \mathrm{AFree} \arrow[d, hook] \arrow[dd, bend right, "\text{quantiz}"'] & \\
& \mathrm{Class} \arrow[dd, bend left] & \\
& \mathrm{Quant} \arrow[d, hook] & \\
& \mathrm{Physics} & \\
\mathrm{Logic} \arrow[uuuur, dashed, bend left] \arrow[uuur, dashed, bend left] \arrow[rr, "\text{coh. HoTT}"'] & & \mathrm{Geo} \arrow[uuuul, bend right, "\text{coh. high. geometry}"'] \arrow[uuul, bend right, "\text{coh. high. geometry}"']
\end{tikzcd}
\tag{18}
$$

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- Schreiber-Nuiten proved that a globally defined quantization process exists in those higher gauge theories which are "anomaly free"
- this is the so-called *cohomological/pull-push/motivic quantization*

$$
\begin{tikzcd}
 & \mathrm{AFree} \arrow[d, hook] \arrow[dd, bend right] & \\
 & \mathrm{Class} \arrow[dd, bend left] & \\
 & \mathrm{Quant} \arrow[d, hook] & \\
 & \mathrm{Physics} & \\
\mathrm{Logic} \arrow[uuuur, dashed, bend left, "\text{quantiz}"] \arrow[uuur, dashed, bend left] \arrow[rr, "\text{coh. HoTT}"'] & & \mathrm{Geo} \arrow[uuuul, bend right, "\text{coh. high. geometry}"'] \arrow[uuul, bend right, "\text{coh. high. geometry}"']
\end{tikzcd}
\tag{18}
$$

## Logic-Geometry-Physics

(Higher) Logic-Physics

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- the dream would be to extend this quantization to the whole Physics…

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- the dream would be to extend this quantization to the whole Physics…
- …providing an approach to the modern view of Hilbert's sixth problem

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- the dream would be to extend this quantization to the whole Physics…
- …providing an approach to the modern view of Hilbert's sixth problem

# Logic-Geometry-Physics

## (Higher) Logic-Physics

- the dream would be to extend this quantization to the whole Physics…
- …providing an approach to the modern view of Hilbert's sixth problem

$$
\begin{tikzcd}
 & \mathrm{AFree} \arrow[d, hook] \arrow[dd, bend right, "\text{quantiz}"'] & \\
 & \mathrm{Class} \arrow[d, dotted] \arrow[dd, hook, bend left] & \\
 & \mathrm{Quant} \arrow[d, hook] & \\
 & \mathrm{Physics} & \\
\mathrm{Logic} \arrow[uuuur, dashed, bend left] \arrow[uuur, dashed, bend left] \arrow[ur, dotted] \arrow[rr, "\text{coh. HoTT}"'] & & \mathrm{Geo} \arrow[uuuul, bend right, "\text{coh. high. geometry}"] \arrow[uuul, bend right, "\text{coh. high. geometry}"] \arrow[ul, dotted]
\end{tikzcd}
\tag{19}
$$

The Program

## The Program

In few words

# The Program

### In few words

- in this program we will try to attack the extension problem for cohomological quantization:

# The Program

### In few words

- in this program we will try to attack the extension problem for cohomological quantization:
  1. extending the domain of the "valid physical systems"

# The Program

## In few words

- in this program we will try to attack the extension problem for cohomological quantization:
  1. extending the domain of the "valid physical systems"
  2. following a parameterized perspective

# The Program

## In few words

- in this program we will try to attack the extension problem for cohomological quantization:
  1. extending the domain of the "valid physical systems"
  2. following a parameterized perspective
  3. using obstruction methods

# The Program

1. extending the domain of the "valid physical systems"

## The Program

1. extending the domain of the "valid physical systems"
   - in the traditional formulation of Physics, the physical systems are supposed to have

# The Program

1. extending the domain of the "valid physical systems"
   - in the traditional formulation of Physics, the physical systems are supposed to have
     - symmetries belonging to the algebraic structure of a (higher) group object

## The Program

1. extending the domain of the "valid physical systems"
   - in the traditional formulation of Physics, the physical systems are supposed to have
     - symmetries belonging to the algebraic structure of a (higher) group object
       - typically a (higher) Lie group

# The Program

1. extending the domain of the "valid physical systems"
   - in the traditional formulation of Physics, the physical systems are supposed to have
     - symmetries belonging to the algebraic structure of a (higher) group object
       - typically a (higher) Lie group
     - infinitesimal symmetries belonging to the algebraic structure of a (higher) Lie algebra object

# The Program

1. extending the domain of the "valid physical systems"
   - in the traditional formulation of Physics, the physical systems are supposed to have
     - symmetries belonging to the algebraic structure of a (higher) group object
       - typically a (higher) Lie group
     - infinitesimal symmetries belonging to the algebraic structure of a (higher) Lie algebra object
       - typically a (higher) Lie algebra

# The Program

1. extending the domain of the "valid physical systems"
   - in the traditional formulation of Physics, the physical systems are supposed to have
     - symmetries belonging to the algebraic structure of a (higher) group object
       - typically a (higher) Lie group
     - infinitesimal symmetries belonging to the algebraic structure of a (higher) Lie algebra object
       - typically a (higher) Lie algebra
   - we will extend the acceptable Physical systems by allowing them to have

# The Program

1. extending the domain of the "valid physical systems"
   - in the traditional formulation of Physics, the physical systems are supposed to have
     - symmetries belonging to the algebraic structure of a (higher) group object
       - typically a (higher) Lie group
     - infinitesimal symmetries belonging to the algebraic structure of a (higher) Lie algebra object
       - typically a (higher) Lie algebra
   - we will extend the acceptable Physical systems by allowing them to have
     - infinitesimal symmetries consisting more general algebraic entities

# The Program

1. extending the domain of the "valid physical systems"
   - in the traditional formulation of Physics, the physical systems are supposed to have
     - symmetries belonging to the algebraic structure of a (higher) group object
       - typically a (higher) Lie group
     - infinitesimal symmetries belonging to the algebraic structure of a (higher) Lie algebra object
       - typically a (higher) Lie algebra
   - we will extend the acceptable Physical systems by allowing them to have
     - infinitesimal symmetries consisting more general algebraic entities
       - as algebras over (higher) operads

# The Program

1. extending the domain of the "valid physical systems"
   - in the traditional formulation of Physics, the physical systems are supposed to have
     - symmetries belonging to the algebraic structure of a (higher) group object
       - typically a (higher) Lie group
     - infinitesimal symmetries belonging to the algebraic structure of a (higher) Lie algebra object
       - typically a (higher) Lie algebra
   - we will extend the acceptable Physical systems by allowing them to have
     - infinitesimal symmetries consisting more general algebraic entities
       - as algebras over (higher) operads
     - symmetries consisting a corresponding structure of "integrated higher algebraic object"

# The Program

1. extending the domain of the "valid physical systems"
   - in the traditional formulation of Physics, the physical systems are supposed to have
     - symmetries belonging to the algebraic structure of a (higher) group object
       - typically a (higher) Lie group
     - infinitesimal symmetries belonging to the algebraic structure of a (higher) Lie algebra object
       - typically a (higher) Lie algebra
   - we will extend the acceptable Physical systems by allowing them to have
     - infinitesimal symmetries consisting more general algebraic entities
       - as algebras over (higher) operads
     - symmetries consisting a corresponding structure of "integrated higher algebraic object"
       - with the "integration process" being part of the axiomatization.

# The Program

1. extending the domain of the "valid physical systems"
   - in the traditional formulation of Physics, the physical systems are supposed to have
     - symmetries belonging to the algebraic structure of a (higher) group object
       - typically a (higher) Lie group
     - infinitesimal symmetries belonging to the algebraic structure of a (higher) Lie algebra object
       - typically a (higher) Lie algebra
   - we will extend the acceptable Physical systems by allowing them to have
     - infinitesimal symmetries consisting more general algebraic entities
       - as algebras over (higher) operads
     - symmetries consisting a corresponding structure of "integrated higher algebraic object"
       - with the "integration process" being part of the axiomatization.

# The Program

1. extending the domain of the "valid physical systems"
   - in the traditional formulation of Physics, the physical systems are supposed to have
     - symmetries belonging to the algebraic structure of a (higher) group object
       - typically a (higher) Lie group
     - infinitesimal symmetries belonging to the algebraic structure of a (higher) Lie algebra object
       - typically a (higher) Lie algebra
   - we will extend the acceptable Physical systems by allowing them to have
     - infinitesimal symmetries consisting more general algebraic entities
       - as algebras over (higher) operads
     - symmetries consisting a corresponding structure of "integrated higher algebraic object"
       - with the "integration process" being part of the axiomatization.

$$\begin{array}{c} \text{HigherAlgebras} \\ \uparrow\downarrow \\ \text{IntegratedHigherAlgebras} \end{array} \tag{20}$$

# The Program

In few words

# The Program

### In few words

- in this program we will try to attack the extension problem for cohomological quantization:

# The Program

## In few words

- in this program we will try to attack the extension problem for cohomological quantization:
  1. extending the domain of the "valid physical systems"

# The Program

### In few words

- in this program we will try to attack the extension problem for cohomological quantization:
  1. extending the domain of the "valid physical systems"
  2. following a parameterized perspective

# The Program

2. following a parameterized perspective

## The Program

2. following a parameterized perspective
   - a particular physical system has a particular (higher) group of symmetries

# The Program

2. following a parameterized perspective
   - a particular physical system has a particular (higher) group of symmetries
   - however, there are physical systems which differ only by the representing group of symmetries…

# The Program

2. following a parameterized perspective
   - a particular physical system has a particular (higher) group of symmetries
   - however, there are physical systems which differ only by the representing group of symmetries…
   - …having the same "shape", being of the same "type"

# The Program

2. following a parameterized perspective
   - a particular physical system has a particular (higher) group of symmetries
   - however, there are physical systems which differ only by the representing group of symmetries…
   - …having the same "shape", being of the same "type"
   - this motivates us to organize the physical systems in "types"

# The Program

2. following a parameterized perspective
   - a particular physical system has a particular (higher) group of symmetries
   - however, there are physical systems which differ only by the representing group of symmetries…
   - …having the same "shape", being of the same "type"
   - this motivates us to organize the physical systems in "types"
   - in other word, motivate us to look at arrows

# The Program

2. following a parameterized perspective
   - a particular physical system has a particular (higher) group of symmetries
   - however, there are physical systems which differ only by the representing group of symmetries…
   - …having the same "shape", being of the same "type"
   - this motivates us to organize the physical systems in "types"
   - in other word, motivate us to look at arrows

# The Program

## 2. following a parameterized perspective

- a particular physical system has a particular (higher) group of symmetries
- however, there are physical systems which differ only by the representing group of symmetries…
- …having the same "shape", being of the same "type"
- this motivates us to organize the physical systems in "types"
- in other word, motivate us to look at arrows

$$\mathrm{HigherGroup} \xrightarrow{F} \mathrm{Physics} \tag{21}$$

- hence at parameterized families of physical systems.

# The Program

## 2. following a parameterized perspective

- a particular physical system has a particular (higher) group of symmetries
- however, there are physical systems which differ only by the representing group of symmetries…
- …having the same "shape", being of the same "type"
- this motivates us to organize the physical systems in "types"
- in other word, motivate us to look at arrows

$$\text{HigherGroup} \xrightarrow{F} \text{Physics} \tag{21}$$

- hence at parameterized families of physical systems.
- **Example:** one talk about Yang-Mills theories for any Lie group. In this perspective one would look at an arrow

2. following a parameterized perspective

- a particular physical system has a particular (higher) group of symmetries
- however, there are physical systems which differ only by the representing group of symmetries…
- …having the same "shape", being of the same "type"
- this motivates us to organize the physical systems in "types"
- in other word, motivate us to look at arrows

$$\mathrm{HigherGroup} \xrightarrow{F} \mathrm{Physics} \tag{21}$$

- hence at parameterized families of physical systems.
- **Example:** one talk about Yang-Mills theories for any Lie group. In this perspective one would look at an arrow

# The Program

## 2. following a parameterized perspective

- a particular physical system has a particular (higher) group of symmetries
- however, there are physical systems which differ only by the representing group of symmetries…
- …having the same "shape", being of the same "type"
- this motivates us to organize the physical systems in "types"
- in other word, motivate us to look at arrows

$$\mathrm{HigherGroup} \xrightarrow{F} \mathrm{Physics} \tag{21}$$

- hence at parameterized families of physical systems.
- **Example:** one talk about Yang-Mills theories for any Lie group. In this perspective one would look at an arrow

$$\mathrm{LieGroup} \xrightarrow{\mathrm{YM}} \mathrm{Physics} \tag{22}$$

2. following a parameterized perspective

# The Program

2. following a parameterized perspective
   - assuming that we extended the domain of the "valid physical systems"…

# The Program

2. following a parameterized perspective
   - assuming that we extended the domain of the "valid physical systems"…
   - …one should look at arrows

# The Program

2. following a parameterized perspective
   - assuming that we extended the domain of the "valid physical systems"…
   - …one should look at arrows

# The Program

2. following a parameterized perspective
   - assuming that we extended the domain of the "valid physical systems"…
   - …one should look at arrows

$$\text{IntegratedHigherAlgebras} \xrightarrow{\text{F}} \text{ExtendedPhysics} \tag{23}$$

# The Program

2. following a parameterized perspective
   - assuming that we extended the domain of the "valid physical systems"…
   - …one should look at arrows

$$\text{IntegratedHigherAlgebras} \xrightarrow{\text{F}} \text{ExtendedPhysics} \tag{23}$$

   - hence to *integrated higher algebra parameterized physical systems.*

# The Program

2. following a parameterized perspective

# The Program

2. following a parameterized perspective
   - for some kinds of physical theories (gauge theories as Yang-Mills theories):

# The Program

2. following a parameterized perspective
   - for some kinds of physical theories (gauge theories as Yang-Mills theories):
     - the dependence on the global symmetries are all induced by infinitesimal effects

# The Program

2. following a parameterized perspective
   - for some kinds of physical theories (gauge theories as Yang-Mills theories):
     - the dependence on the global symmetries are all induced by infinitesimal effects
     - meaning that the arrows defining a type of physical theory factors as follows

# The Program

2. following a parameterized perspective
   - for some kinds of physical theories (gauge theories as Yang-Mills theories):
     - the dependence on the global symmetries are all induced by infinitesimal effects
     - meaning that the arrows defining a type of physical theory factors as follows

# The Program

## 2. following a parameterized perspective

- for some kinds of physical theories (gauge theories as Yang-Mills theories):
  - the dependence on the global symmetries are all induced by infinitesimal effects
  - meaning that the arrows defining a type of physical theory factors as follows

$$\begin{tikzcd} \mathrm{LieAlgebras} \arrow[d, shift left] \arrow[drr, dashed, "F"] & & \\ \mathrm{IntegratedLieAlgebras} \arrow[u, shift left] \arrow[rr, "F"'] & & \mathrm{Physics} \end{tikzcd} \tag{24}$$

## The Program

2. following a parameterized perspective
   - for some kinds of physical theories (gauge theories as Yang-Mills theories):
     - the dependence on the global symmetries are all induced by infinitesimal effects
     - meaning that the arrows defining a type of physical theory factors as follows

$$
\begin{tikzcd}
\mathrm{LieAlgebras} \arrow[d, shift left] \arrow[drr, dashed, "F"] & & \\
\mathrm{IntegratedLieAlgebras} \arrow[u, shift left] \arrow[rr, "F"'] & & \mathrm{Physics}
\end{tikzcd}
\tag{24}
$$

   - furthermore, in these cases the fundamental physical entities

# The Program

## 2. following a parameterized perspective

- for some kinds of physical theories (gauge theories as Yang-Mills theories):
  - the dependence on the global symmetries are all induced by infinitesimal effects
  - meaning that the arrows defining a type of physical theory factors as follows

$$\begin{tikzcd}
\text{LieAlgebras} \arrow[d, shift right] \arrow[drr, dashed, "F"] & & \\
\text{IntegratedLieAlgebras} \arrow[u, shift right] \arrow[rr, "F"'] & & \text{Physics}
\end{tikzcd} \tag{24}$$

- furthermore, in these cases the fundamental physical entities
  - are modeled by geometric quantities as $G$-connections and curvature

# The Program

## 2. following a parameterized perspective

- for some kinds of physical theories (gauge theories as Yang-Mills theories):
  - the dependence on the global symmetries are all induced by infinitesimal effects
  - meaning that the arrows defining a type of physical theory factors as follows

$$\begin{tikzcd} \mathrm{LieAlgebras} \arrow[d, shift left] \arrow[drr, dashed, "F"] & & \\ \mathrm{IntegratedLieAlgebras} \arrow[u, shift left] \arrow[rr, "F"'] & & \mathrm{Physics} \end{tikzcd} \tag{24}$$

- furthermore, in these cases the fundamental physical entities
  - are modeled by geometric quantities as $G$-connections and curvature
  - so they are Lie algebra-valued differential forms

# The Program

## 2. following a parameterized perspective

- for some kinds of physical theories (gauge theories as Yang-Mills theories):
  - the dependence on the global symmetries are all induced by infinitesimal effects
  - meaning that the arrows defining a type of physical theory factors as follows

$$
\begin{tikzcd}
\mathrm{LieAlgebras} \arrow[d, shift left] \arrow[drr, dashed, "F"] & & \\
\mathrm{IntegratedLieAlgebras} \arrow[u, shift left] \arrow[rr, "F"'] & & \mathrm{Physics}
\end{tikzcd}
\tag{24}
$$

- furthermore, in these cases the fundamental physical entities
  - are modeled by geometric quantities as $G$-connections and curvature
  - so they are Lie algebra-valued differential forms
  - and therefore are quantities take "take values in the algebraic object modeling infinitesimal symmetries".

## The Program

2. following a parameterized perspective

## The Program

2. following a parameterized perspective
   - one should expect analogous behavior for the "extended physical systems":

# The Program

2. following a parameterized perspective
   - one should expect analogous behavior for the "extended physical systems":
     - certain integrated higher algebra parameterized physical systems (the analogous version of gauge theories) should factors as below

# The Program

2. following a parameterized perspective
   - one should expect analogous behavior for the "extended physical systems":
     - certain integrated higher algebra parameterized physical systems (the analogous version of gauge theories) should factors as below
     - being determined by higher algebra-valued geometric entities

# The Program

2. following a parameterized perspective
   - one should expect analogous behavior for the "extended physical systems":
     - certain integrated higher algebra parameterized physical systems (the analogous version of gauge theories) should factors as below
     - being determined by higher algebra-valued geometric entities
     - hence being viewed as "higher algebra-valued physical systems"

# The Program

2. following a parameterized perspective
   - one should expect analogous behavior for the "extended physical systems":
     - certain integrated higher algebra parameterized physical systems (the analogous version of gauge theories) should factors as below
     - being determined by higher algebra-valued geometric entities
     - hence being viewed as "higher algebra-valued physical systems"

# The Program

## 2. following a parameterized perspective

- one should expect analogous behavior for the "extended physical systems":
  - certain integrated higher algebra parameterized physical systems (the analogous version of gauge theories) should factors as below
  - being determined by higher algebra-valued geometric entities
  - hence being viewed as "higher algebra-valued physical systems"

$$
\begin{tikzcd}
\mathrm{HigherAlgebras} \arrow[d, shift left] \arrow[drr, dashed, "F"] & & \\
\mathrm{IntegratedHigherAlgebras} \arrow[u, shift left] \arrow[rr, "F"'] & & \mathrm{ExtendedPhysics}
\end{tikzcd}
\tag{25}
$$

# The Program

In few words

# The Program

### In few words

- in this program we will try to attack the extension problem for cohomological quantization:

# The Program

### In few words

- in this program we will try to attack the extension problem for cohomological quantization:
  1. extending the domain of the "valid physical systems"

# The Program

### In few words

- in this program we will try to attack the extension problem for cohomological quantization:
  1. extending the domain of the "valid physical systems"
  2. following a parameterized perspective

# The Program

### In few words

- in this program we will try to attack the extension problem for cohomological quantization:
  1. extending the domain of the "valid physical systems"
  2. following a parameterized perspective
  3. using obstruction methods

## The Program

3. using obstruction methods

# The Program

3. using obstruction methods
   - suppose we have a machine that take any semantic aspects of the first part of this talk and extend it to the "higher algebra-valued context"

# The Program

3. using obstruction methods
   - suppose we have a machine that take any semantic aspects of the first part of this talk and extend it to the "higher algebra-valued context"
   - this means to talk about:

## The Program

3. using obstruction methods
   - suppose we have a machine that take any semantic aspects of the first part of this talk and extend it to the "higher algebra-valued context"
   - this means to talk about:
     - "higher algebra-valued cohesive $(\infty, 1)$-toposes"

# The Program

3. using obstruction methods
   - suppose we have a machine that take any semantic aspects of the first part of this talk and extend it to the "higher algebra-valued context"
   - this means to talk about:
     - "higher algebra-valued cohesive $(\infty, 1)$-toposes"
     - "higher algebra-valued cohesive geometry"

# The Program

## 3. using obstruction methods

- suppose we have a machine that take any semantic aspects of the first part of this talk and extend it to the "higher algebra-valued context"
- this means to talk about:
  - "higher algebra-valued cohesive $(\infty, 1)$-toposes"
  - "higher algebra-valued cohesive geometry"
  - a higher algebra-valued geometric description of the higher algebra-valued physical systems above

# The Program

3. using obstruction methods
   - suppose we have a machine that take any semantic aspects of the first part of this talk and extend it to the "higher algebra-valued context"
   - this means to talk about:
     - "higher algebra-valued cohesive $(\infty, 1)$-toposes"
     - "higher algebra-valued cohesive geometry"
     - a higher algebra-valued geometric description of the higher algebra-valued physical systems above
       - a formalization of classical higher algebra-valued physical systems in terms of higher algebra-valued cohesive geometry

## The Program

### 3. using obstruction methods

- suppose we have a machine that take any semantic aspects of the first part of this talk and extend it to the "higher algebra-valued context"
- this means to talk about:
  - "higher algebra-valued cohesive $(\infty, 1)$-toposes"
  - "higher algebra-valued cohesive geometry"
  - a higher algebra-valued geometric description of the higher algebra-valued physical systems above
    - a formalization of classical higher algebra-valued physical systems in terms of higher algebra-valued cohesive geometry
    - a higher algebra-valued formalization of quantum physical systems

## The Program

### 3. using obstruction methods

- suppose we have a machine that take any semantic aspects of the first part of this talk and extend it to the "higher algebra-valued context"
- this means to talk about:
  - "higher algebra-valued cohesive $(\infty, 1)$-toposes"
  - "higher algebra-valued cohesive geometry"
  - a higher algebra-valued geometric description of the higher algebra-valued physical systems above
    - a formalization of classical higher algebra-valued physical systems in terms of higher algebra-valued cohesive geometry
    - a higher algebra-valued formalization of quantum physical systems
    - a version of higher algebra-valued cohomological quantization

3. using obstruction methods

$$
\begin{tikzcd}
\mathrm{HighAlg\text{-}ValuedAFree} \arrow[d, hook] \arrow[dd, bend right, "\text{alg-valued quantiz}"'] & & \\
\mathrm{HighAlg\text{-}ValuedClass} \arrow[dd, hook, bend left] & & \\
\mathrm{HighAlg\text{-}ValuedQuant} \arrow[d, hook] & & \\
\mathrm{HighAlg\text{-}ValuedPhysics} & & \\
& & \mathrm{HighAlg\text{-}ValuedGeo} \arrow[uuuull, bend right, "\text{high alg-valued. coh. high. geometry}"'] \arrow[uuull, bend right, "\text{high alg-valued coh. high. geometry}"']
\end{tikzcd}
\tag{26}
$$

## The Program

3. using obstruction methods

## The Program

3. using obstruction methods
   - for the class $\text{HigherAlgebras}$ of higher algebras, let $\text{HigherOperadAlgebras}$ be the class of all algebras over some higher operad that belongs to $\text{HigherAlgebras}$.

# The Program

3. using obstruction methods
   - for the class HigherAlgebras of higher algebras, let HigherOperadAlgebras be the class of all algebras over some higher operad that belongs to HigherAlgebras.
   - the class HighAlg-Class is parameterized over HigherAlgebras

## The Program

3. using obstruction methods
   - for the class HigherAlgebras of higher algebras, let HigherOperadAlgebras be the class of all algebras over some higher operad that belongs to HigherAlgebras.
   - the class HighAlg-Class is parameterized over HigherAlgebras
   - and one can restrict it to be parameterized over HigherOperadAlgebras

3. using obstruction methods
   - for the class HigherAlgebras of higher algebras, let HigherOperadAlgebras be the class of all algebras over some higher operad that belongs to HigherAlgebras.
   - the class HighAlg-Class is parameterized over HigherAlgebras
   - and one can restrict it to be parameterized over HigherOperadAlgebras

### 3. using obstruction methods

- for the class $\mathrm{HigherAlgebras}$ of higher algebras, let $\mathrm{HigherOperadAlgebras}$ be the class of all algebras over some higher operad that belongs to $\mathrm{HigherAlgebras}$.
- the class $\mathrm{HighAlg\text{-}Class}$ is parameterized over $\mathrm{HigherAlgebras}$
- and one can restrict it to be parameterized over $\mathrm{HigherOperadAlgebras}$

$$\begin{tikzcd}
\mathrm{HigherOperadAlgebras} \arrow[d, hook] \arrow[ddr, dotted, "F"] & \\
\mathrm{HigherAlgebras} \arrow[d, shift left] \arrow[dr, dashed, "F"] & \\
\mathrm{IntegratedHigherAlgebras} \arrow[u, shift left] \arrow[r, "F"'] & \mathrm{ExtendedPhysics}
\end{tikzcd} \tag{27}$$

## The Program

3. using obstruction methods

# The Program

3. using obstruction methods
   - in addition, there is the arrow assigning to each higher operad its algebras

## The Program

3. using obstruction methods
   - in addition, there is the arrow assigning to each higher operad its algebras

# The Program

## 3. using obstruction methods

- in addition, there is the arrow assigning to each higher operad its algebras

$$\text{HigherOperad} \to \text{HigherOperadAlgebras} \tag{28}$$

# The Program

## 3. using obstruction methods

- in addition, there is the arrow assigning to each higher operad its algebras

$$\mathrm{HigherOperad} \to \mathrm{HigherOperadAlgebras} \tag{28}$$

- composing with it one assign to each higher operad $\mathcal{O}$ the classical physical systems whose infinitesimal symmetries have the structure of $\mathcal{O}$-algebra.

# The Program

## 3. using obstruction methods

- in addition, there is the arrow assigning to each higher operad its algebras

$$\mathrm{HigherOperad} \to \mathrm{HigherOperadAlgebras} \tag{28}$$

- composing with it one assign to each higher operad $\mathcal{O}$ the classical physical systems whose infinitesimal symmetries have the structure of $\mathcal{O}$-algebra.

# The Program

## 3. using obstruction methods

- in addition, there is the arrow assigning to each higher operad its algebras

$$\mathrm{HigherOperad} \to \mathrm{HigherOperadAlgebras} \tag{28}$$

- composing with it one assign to each higher operad $\mathcal{O}$ the classical physical systems whose infinitesimal symmetries have the structure of $\mathcal{O}$-algebra.

$$\begin{tikzcd}
\mathrm{HigherOperad} \arrow[d] \arrow[drrr, squiggly, bend left, "F|_{\mathcal{O}}"] & & & \\
\mathrm{HigherOperadAlgebras} \arrow[d, hook] \arrow[ddrr, dotted, "F"] & & & \\
\mathrm{HigherAlgebras} \arrow[d, shift left] \arrow[drr, dashed, "F"] & & & \\
\mathrm{IntegratedHigherAlgebras} \arrow[u, shift left] \arrow[rr, "F"'] & & \mathrm{ExtendedPhysics} &
\end{tikzcd} \tag{29}$$

# The Program

3. using obstruction methods

## The Program

3. using obstruction methods
   - finally, one can then ask for which higher operads $\mathcal{O}$ the corresponding arrow is an isomorphism:

## The Program

3. using obstruction methods
   - finally, one can then ask for which higher operads $\mathcal{O}$ the corresponding arrow is an isomorphism:

## The Program

3. using obstruction methods
   - finally, one can then ask for which higher operads $\mathcal{O}$ the corresponding arrow is an isomorphism:

$$\begin{array}{c} \mathrm{HighAlg\text{-}ValuedAFree}\,|_{\mathcal{O}} \\ \big\downarrow \\ \mathrm{HighAlg\text{-}ValuedClass}\,|_{\mathcal{O}} \end{array} \tag{30}$$

3. using obstruction methods

- finally, one can then ask for which higher operads $\mathcal{O}$ the corresponding arrow is an isomorphism:

$$\begin{tikzcd} \mathrm{HighAlg\text{-}ValuedAFree}\,|_{\mathcal{O}} \arrow[d, hook] \\ \mathrm{HighAlg\text{-}ValuedClass}\,|_{\mathcal{O}} \end{tikzcd} \tag{30}$$

- this means exactly that:

# The Program

## 3. using obstruction methods

- finally, one can then ask for which higher operads $\mathcal{O}$ the corresponding arrow is an isomorphism:

$$\begin{array}{c} \mathrm{HighAlg\text{-}ValuedAFree}\,|_{\mathcal{O}} \\ \Big\downarrow \\ \mathrm{HighAlg\text{-}ValuedClass}\,|_{\mathcal{O}} \end{array} \tag{30}$$

- this means exactly that:
  - for those flavors of higher operads, there is a globally defined quantization process on the classical physical systems whose infinitesimal symmetries give the structure of algebra over them.

# The Program

3. using obstruction methods

- finally, one can then ask for which higher operads $\mathcal{O}$ the corresponding arrow is an isomorphism:

$$
\begin{array}{c}
\mathrm{HighAlg\text{-}ValuedAFree}\,|_{\mathcal{O}} \\
\big\downarrow \\
\mathrm{HighAlg\text{-}ValuedClass}\,|_{\mathcal{O}}
\end{array}
\tag{30}
$$

- this means exactly that:
  - for those flavors of higher operads, there is a globally defined quantization process on the classical physical systems whose infinitesimal symmetries give the structure of algebra over them.
- in other words:

# The Program

### 3. using obstruction methods

- finally, one can then ask for which higher operads $\mathcal{O}$ the corresponding arrow is an isomorphism:

$$\begin{array}{c} \text{HighAlg-ValuedAFree}\,|_{\mathcal{O}} \\ \Big\downarrow \\ \text{HighAlg-ValuedClass}\,|_{\mathcal{O}} \end{array} \tag{30}$$

- this means exactly that:
  - for those flavors of higher operads, there is a globally defined quantization process on the classical physical systems whose infinitesimal symmetries give the structure of algebra over them.
- in other words:
  - instead of restricting the types of physical systems to ensure existing of quantization one can work with general types of physical systems, but restricted to specific kinds of infinitesimal symmetries.

# The Program

## 3. using obstruction methods

$$
\begin{tikzcd}
\mathrm{HighAlg\text{-}ValuedAFree}\,|_{\mathcal{O}} \arrow[d, hook, dashed, "\simeq"] \arrow[dd, bend right, "\text{alg-valued quantiz}"'] & \\
\mathrm{HighAlg\text{-}ValuedClass}\,|_{\mathcal{O}} \arrow[d, dashed, "\text{alg-valued quantiz}"'] \arrow[dd, bend left] & \\
\mathrm{HighAlg\text{-}ValuedQuant}\,|_{\mathcal{O}} \arrow[d, hook] & \\
\mathrm{HighAlg\text{-}ValuedPhysics}\,|_{\mathcal{O}} & \\
& \mathrm{HighAlg\text{-}ValuedGeo}\,|_{\mathcal{O}} \arrow[uuuul, bend right, "\text{high alg-valued. coh. high. geometry}"'] \arrow[uuul, bend right, "\text{high alg-valued coh. high. geometry}"']
\end{tikzcd}
\tag{31}
$$

## The Program

3. using obstruction methods

# The Program

3. using obstruction methods
   - this involves obstruction techniques because:

3. using obstruction methods
   - this involves obstruction techniques because:
   - if a $(\infty, 1)$-topos is cohesive, its intrinsic nonabelian cohomology $H^1(X; G)$ admits a canonical differential refinement $H^1_{diff}(X; G)$

# The Program

3. using obstruction methods
   - this involves obstruction techniques because:
   - if a $(\infty, 1)$-topos is cohesive, its intrinsic nonabelian cohomology $H^1(X; G)$ admits a canonical differential refinement $H^1_{diff}(X; G)$
     - known as *differential nonabelian cohomology*

# The Program

3. using obstruction methods
   - this involves obstruction techniques because:
   - if a $(\infty, 1)$-topos is cohesive, its intrinsic nonabelian cohomology $H^1(X;G)$ admits a canonical differential refinement $H^1_{diff}(X;G)$
     - known as *differential nonabelian cohomology*
   - the geometric structure in cohesive geometry can be viewed as cocycles and characteristic classes for this cohomology

## The Program

3. using obstruction methods
   - this involves obstruction techniques because:
   - if a $(\infty,1)$-topos is cohesive, its intrinsic nonabelian cohomology $H^1(X;G)$ admits a canonical differential refinement $H^1_{diff}(X;G)$
     - known as *differential nonabelian cohomology*
   - the geometric structure in cohesive geometry can be viewed as cocycles and characteristic classes for this cohomology
   - in particular, gauge theories can be viewed as characteristic classes

## The Program

3. using obstruction methods
    - this involves obstruction techniques because:
    - if a $(\infty,1)$-topos is cohesive, its intrinsic nonabelian cohomology $H^1(X;G)$ admits a canonical differential refinement $H^1_{diff}(X;G)$
        - known as *differential nonabelian cohomology*
    - the geometric structure in cohesive geometry can be viewed as cocycles and characteristic classes for this cohomology
    - in particular, gauge theories can be viewed as characteristic classes
    - the class $\mathrm{AFree}$ of anomaly free classical physical systems can be characterized as those whose certain induced characteristic class in twisted differential cohomology vanishes for some coefficient

## The Program

### 3. using obstruction methods

- this involves obstruction techniques because:
- if a $(\infty, 1)$-topos is cohesive, its intrinsic nonabelian cohomology $H^1(X; G)$ admits a canonical differential refinement $H^1_{diff}(X; G)$
  - known as *differential nonabelian cohomology*
- the geometric structure in cohesive geometry can be viewed as cocycles and characteristic classes for this cohomology
- in particular, gauge theories can be viewed as characteristic classes
- the class $\mathrm{AFree}$ of anomaly free classical physical systems can be characterized as those whose certain induced characteristic class in twisted differential cohomology vanishes for some coefficient
- the problem is therefore about vanishing of characteristic classes, which is the natural context for obstruction theory.

# The Project

# The Project

Very briefly

## The Project

### Very briefly

- recall that one part of the steps of the program involves to build

# The Project

## Very briefly

- recall that one part of the steps of the program involves to build
  - "higher algebra-valued cohesive $(\infty, 1)$-toposes"

# The Project

## Very briefly

- recall that one part of the steps of the program involves to build
  - "higher algebra-valued cohesive $(\infty, 1)$-toposes"
  - "higher algebra-valued cohesive geometry"

# The Project

## Very briefly

- recall that one part of the steps of the program involves to build
  - "higher algebra-valued cohesive $(\infty, 1)$-toposes"
  - "higher algebra-valued cohesive geometry"
  - a higher algebra-valued geometric description of the higher algebra-valued physical systems above

# The Project

## Very briefly

- recall that one part of the steps of the program involves to build
  - "higher algebra-valued cohesive $(\infty, 1)$-toposes"
  - "higher algebra-valued cohesive geometry"
  - a higher algebra-valued geometric description of the higher algebra-valued physical systems above
    - a formalization of classical higher algebra-valued physical systems in terms of higher algebra-valued cohesive geometry

# The Project

## Very briefly

- recall that one part of the steps of the program involves to build
  - "higher algebra-valued cohesive $(\infty, 1)$-toposes"
  - "higher algebra-valued cohesive geometry"
  - a higher algebra-valued geometric description of the higher algebra-valued physical systems above
    - a formalization of classical higher algebra-valued physical systems in terms of higher algebra-valued cohesive geometry
    - a higher algebra-valued formalization of quantum physical systems

# The Project

## Very briefly

- recall that one part of the steps of the program involves to build
  - "higher algebra-valued cohesive $(\infty, 1)$-toposes"
  - "higher algebra-valued cohesive geometry"
  - a higher algebra-valued geometric description of the higher algebra-valued physical systems above
    - a formalization of classical higher algebra-valued physical systems in terms of higher algebra-valued cohesive geometry
    - a higher algebra-valued formalization of quantum physical systems
    - a version of higher algebra-valued cohomological quantization

# The Project

## Very briefly

- recall that one part of the steps of the program involves to build
  - "higher algebra-valued cohesive $(\infty, 1)$-toposes"
  - "higher algebra-valued cohesive geometry"
  - a higher algebra-valued geometric description of the higher algebra-valued physical systems above
    - a formalization of classical higher algebra-valued physical systems in terms of higher algebra-valued cohesive geometry
    - a higher algebra-valued formalization of quantum physical systems
    - a version of higher algebra-valued cohomological quantization
- the project to be submitted to Faspesp cover aspects of the first points above:

# The Project

## Very briefly

- recall that one part of the steps of the program involves to build
  - "higher algebra-valued cohesive $(\infty, 1)$-toposes"
  - "higher algebra-valued cohesive geometry"
  - a higher algebra-valued geometric description of the higher algebra-valued physical systems above
    - a formalization of classical higher algebra-valued physical systems in terms of higher algebra-valued cohesive geometry
    - a higher algebra-valued formalization of quantum physical systems
    - a version of higher algebra-valued cohomological quantization
- the project to be submitted to Faspesp cover aspects of the first points above:
  - "higher algebra-valued cohesive $(\infty, 1)$-toposes"

# The Project

## Very briefly

- recall that one part of the steps of the program involves to build
  - "higher algebra-valued cohesive $(\infty, 1)$-toposes"
  - "higher algebra-valued cohesive geometry"
  - a higher algebra-valued geometric description of the higher algebra-valued physical systems above
    - a formalization of classical higher algebra-valued physical systems in terms of higher algebra-valued cohesive geometry
    - a higher algebra-valued formalization of quantum physical systems
    - a version of higher algebra-valued cohomological quantization
- the project to be submitted to Faspesp cover aspects of the first points above:
  - "higher algebra-valued cohesive $(\infty, 1)$-toposes"
  - "higher algebra-valued cohesive geometry"

# The Project

Very briefly

# The Project

## Very briefly

- this in turn involves:

# The Project

## Very briefly

- this in turn involves:
  - to identify the correct *algebra-valued cosmoi* in which the theory will be constructed

# The Project

## Very briefly

- this in turn involves:
  - to identify the correct *algebra-valued cosmoi* in which the theory will be constructed
  - to define *algebra-valued topos* and construct its basic theory

# The Project

## Very briefly

- this in turn involves:
  - to identify the correct *algebra-valued cosmoi* in which the theory will be constructed
  - to define *algebra-valued topos* and construct its basic theory
  - to extend the cohesive modalities

# The Project

## Very briefly

- this in turn involves:
  - to identify the correct *algebra-valued cosmoi* in which the theory will be constructed
  - to define *algebra-valued topos* and construct its basic theory
  - to extend the cohesive modalities
  - to show that for the cohesive algebra-valued topoi there is a corresponding algebra-valued cohesive geometry

# The Project

## Very briefly

- this in turn involves:
  - to identify the correct *algebra-valued cosmoi* in which the theory will be constructed
  - to define *algebra-valued topos* and construct its basic theory
  - to extend the cohesive modalities
  - to show that for the cohesive algebra-valued topoi there is a corresponding algebra-valued cohesive geometry
  - to prove that in a algebra-valued cohesive topos there is a theory of geometric refinement for intrinsic cohomologies

# The Project

## Very briefly

- this in turn involves:
  - to identify the correct *algebra-valued cosmoi* in which the theory will be constructed
  - to define *algebra-valued topos* and construct its basic theory
  - to extend the cohesive modalities
  - to show that for the cohesive algebra-valued topoi there is a corresponding algebra-valued cohesive geometry
  - to prove that in a algebra-valued cohesive topos there is a theory of geometric refinement for intrinsic cohomologies
  - to show that the algebra-valued geometric entities can be represented by cocycles in geometrically refined cohomologies

# The Project

## Very briefly

- this in turn involves:
  - to identify the correct *algebra-valued cosmoi* in which the theory will be constructed
  - to define *algebra-valued topos* and construct its basic theory
  - to extend the cohesive modalities
  - to show that for the cohesive algebra-valued topoi there is a corresponding algebra-valued cohesive geometry
  - to prove that in a algebra-valued cohesive topos there is a theory of geometric refinement for intrinsic cohomologies
  - to show that the algebra-valued geometric entities can be represented by cocycles in geometrically refined cohomologies
  - …

# The Project

## Very briefly

- this in turn involves:
  - to identify the correct *algebra-valued cosmoi* in which the theory will be constructed
  - to define *algebra-valued topos* and construct its basic theory
  - to extend the cohesive modalities
  - to show that for the cohesive algebra-valued topoi there is a corresponding algebra-valued cohesive geometry
  - to prove that in a algebra-valued cohesive topos there is a theory of geometric refinement for intrinsic cohomologies
  - to show that the algebra-valued geometric entities can be represented by cocycles in geometrically refined cohomologies
  - …
- we plan to follow the project with the hypothesis that "algebra-valued" means "parameterized enriched"…

# The Project

## Very briefly

- this in turn involves:
  - to identify the correct *algebra-valued cosmoi* in which the theory will be constructed
  - to define *algebra-valued topos* and construct its basic theory
  - to extend the cohesive modalities
  - to show that for the cohesive algebra-valued topoi there is a corresponding algebra-valued cohesive geometry
  - to prove that in a algebra-valued cohesive topos there is a theory of geometric refinement for intrinsic cohomologies
  - to show that the algebra-valued geometric entities can be represented by cocycles in geometrically refined cohomologies
  - …
- we plan to follow the project with the hypothesis that "algebra-valued" means "parameterized enriched"…
- … in the sense of working with entities that are parameterized not on a fixed monoidal category, but on a family of them.

Opportunities

# The Project

### Opportunities

- together with the project we will ask for:

# The Project

## Opportunities

- together with the project we will ask for:
  - 1 PhD scholarship

# The Project

## Opportunities

- together with the project we will ask for:
  - 1 PhD scholarship
  - 1 Master's scholarship

# The Project

## Opportunities

- together with the project we will ask for:
  - 1 PhD scholarship
  - 1 Master's scholarship
- the PhD scholarship is to build a version of Giraud's theorem for parameterized enriched toposes.

# The Project

## Opportunities

- together with the project we will ask for:
  - 1 PhD scholarship
  - 1 Master's scholarship
- the PhD scholarship is to build a version of Giraud's theorem for parameterized enriched toposes.
- in previous works the author used the idea of parameterize a physical theory by the algebraic structure on its space of infinitesimal algebras to get topological and geometric obstructions on gravity.

# The Project

## Opportunities

- together with the project we will ask for:
  - 1 PhD scholarship
  - 1 Master's scholarship
- the PhD scholarship is to build a version of Giraud's theorem for parameterized enriched toposes.
- in previous works the author used the idea of parameterize a physical theory by the algebraic structure on its space of infinitesimal algebras to get topological and geometric obstructions on gravity.
- the Master's scholarship would be to apply the same techniques to get obstructions to the realization of other gauge theories, as Yang-Mills theories and supergravity theories.

thanks =)