

Axiomatic Foundations of Physics: Problems and Ideas

Yuri Ximenes Martins¹

Department of Mathematics, UFMG, Brazil
Math-Phys-Cat Group

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¹<https://math-phys.group/~yxmartins>

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- The “axiomatization approach” was considered by *Hilbert’s formalist school*;
- The problem of axiomatizing Mathematics was explicitly considered in his *Hilbert’s Program* which is about finding a formalist approach to Mathematics through a consistent axiomatic language;
- Similarly, one could consider an *Axiomatization Program of Physics* as the search for a formalist approach to Physics through a consistent axiomatic language.

Part I: The Program

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1. Foundations of Mathematics

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2. Formalism

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1. what are “mathematical objects”?
2. are they “real”? Are they “abstract”?
3. what it means to “know” in mathematics?
4. when is a mathematical sentence true? In other words, what is “mathematical truth”?
5. what is the role of axioms?
6. what is their nature?

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3. **Intuitionism** (mainly by Brouwer)

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- mathematical **ontology** reduces to logic.
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Mathematics is fruit of our minds...

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 - [Fregue-Hilbert](#) debate;
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- In the Brouwer view, intuitionistic logics are allowed to be axiomatic, but they *avoid some axioms like Law of Excluded Middle* and Axiom of Choice.
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- a family S of relations in $C \sqcup E$ defining the *structure* of \mathcal{S} .

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- Curiously, Bunge does not even define what is a “morphism” between systems, although there is a natural notion of that.

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- ... we need “higher systems”, so that “higher (or homotopical) systemics” (see Section 2.2 of the thesis).
- Thus, instead of fixing an *a priori* formalization to “area of study”, we will first work *naively*.

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- thus, we can think of \mathcal{O} as a “class of systems” of a specific type.

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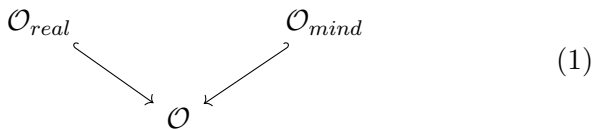
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Mind

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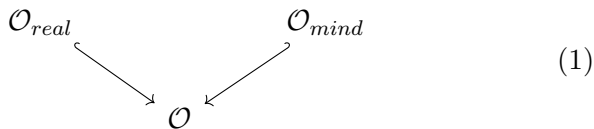
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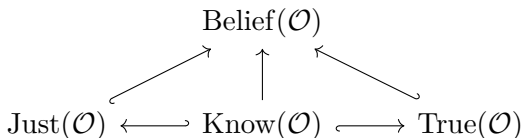
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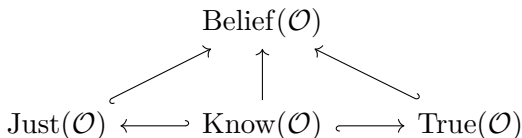
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Formalism | Epistemology

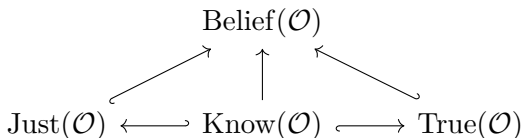
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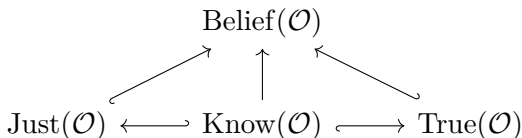
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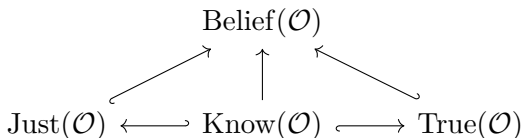
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 - a subclass $\text{Know}(\mathcal{O}) \subset \text{True}(\mathcal{O}) \cap \text{Just}(\mathcal{O})$, describing *knowledge*.



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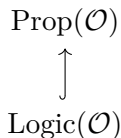
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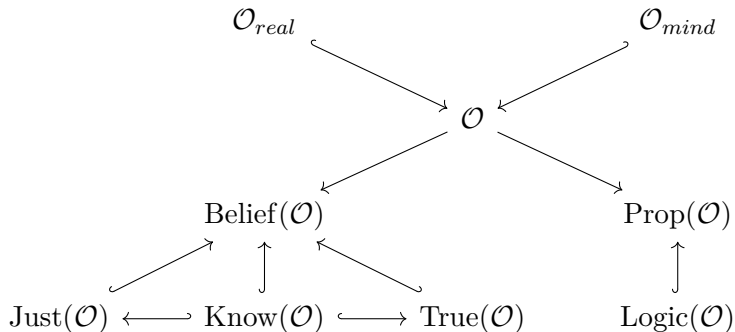
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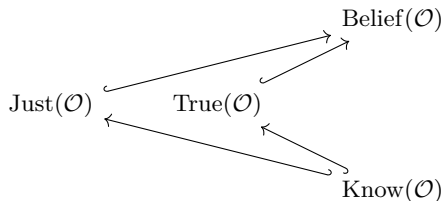
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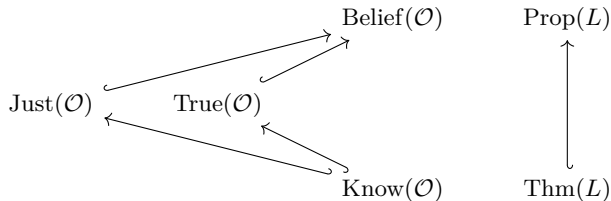
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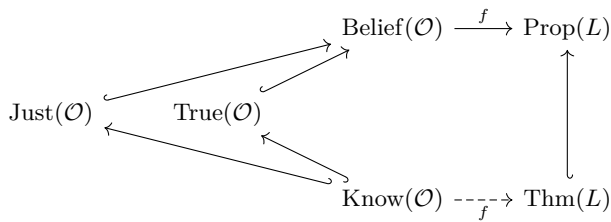
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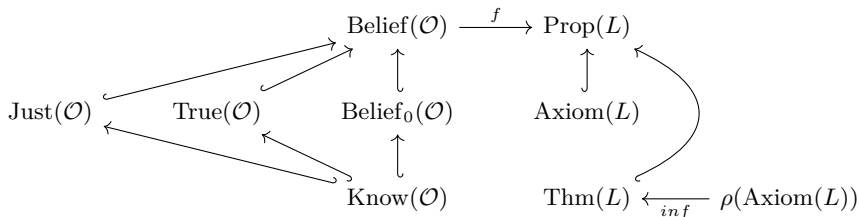
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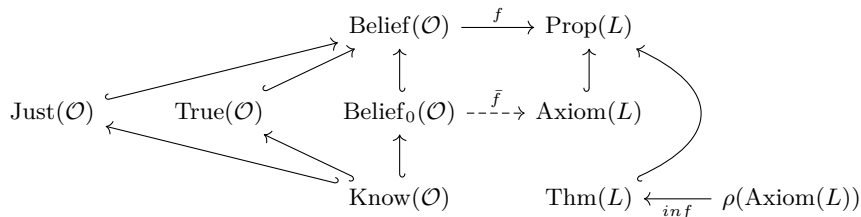
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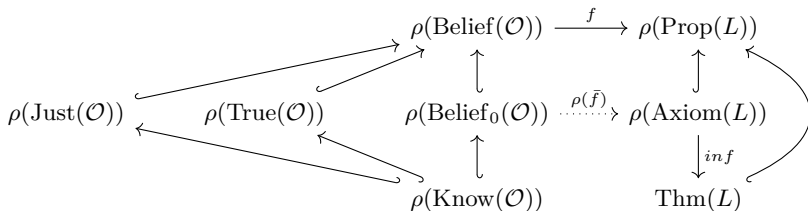
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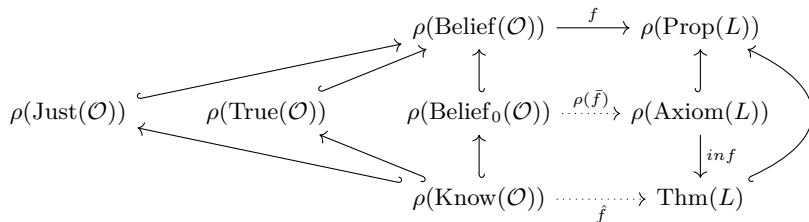
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Plain

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- thus we could consider analogous axiomatization programs to other areas of study \mathcal{O} .

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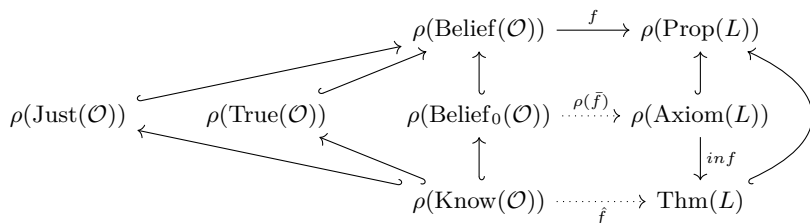
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We presented three strategies, all of them based on the following additional epistemological requirement.

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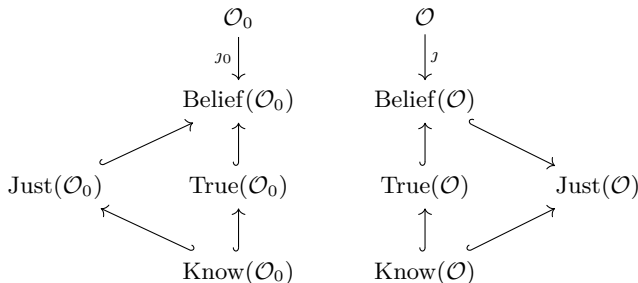
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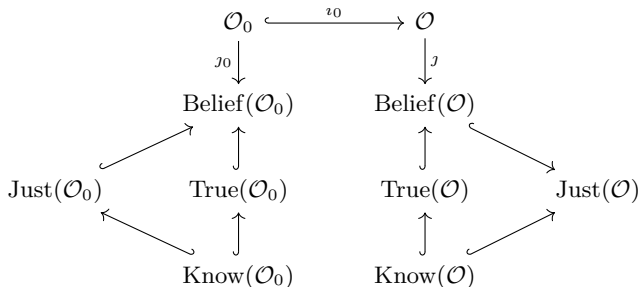


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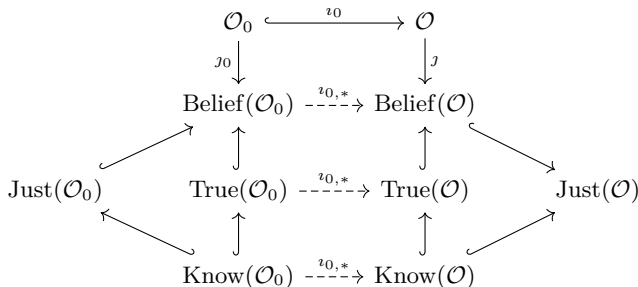


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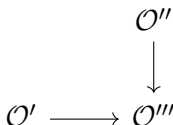
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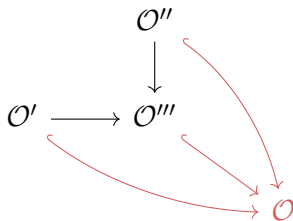
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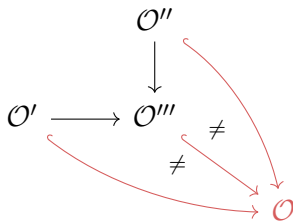
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- ...however it tells nothing about philosophical quantization or axiomatic quantization relative to other axiomatic languages.

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- b a subclass $\mathcal{O}_{prt,k} \subset \mathcal{O}_{prt}$ such that right-hand side section C_{kj} exist is said to be a class of *j-renormalizable k-perturbative systems*.

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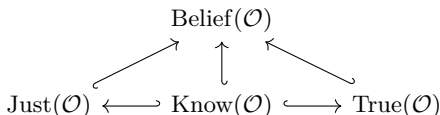
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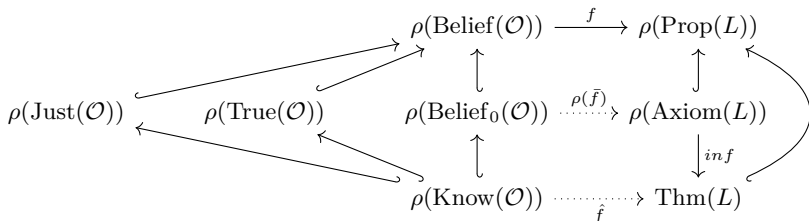
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- Let \mathbf{H} be a category. An object $X \in \mathcal{H}$ is *injective* if every monomorphism $f : X \rightarrow Y$ is a split monomorphism.

Results | First Approach

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 - the pair $(\text{Ep}(\mathcal{O}_0), L_0)$ is a formalist approach \mathcal{O}_0 internal to \mathbf{H} ,
- then every abstraction process internal to \mathbf{H} defines an “universal” axiomatic language L such that $(\text{Ep}(\mathcal{O}', L))$ is a formalist approach for \mathcal{O}' internal to \mathbf{H} .

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- Let \mathcal{O} an area of study internal to ZFC or NBG. If there is some $\mathcal{O}_0 \subset \mathcal{O}$ which admits a formalist approach, then every abstraction process extends this formalist approach from \mathcal{O}_0 to \mathcal{O} .

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- But gauge theories, supergravity and string theory are examples of classes of Physical systems in which “homotopies” play a fundamental role...
- ... which forces us to look for languages internal to more abstract categories than ZFC/NBG.

That is all...

A minha vó caiu
e quebrou o fêmur
Caiu o meu avô
e perdeu os dentes.

Espero que a chuva que cai lá fora
lave a culpa da ausência
e leve embora o desatino
que assola minh'alma fina.