Lie Algebroidal Categories

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Plain

1. Categorification
2. Lie Algebroidal Categories
3. Connections with Lie Algebroids
4. Speculations

Categorification

Intuition

- Roughly speaking:
  - **categorification** is a process that takes a set-theoretic concept and produces an analogous categorical-theoretic concept;
- There are at least two of such processes:
  - **vertical categorification** and
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Vertical Categorification

- Consists in two steps:
  1. to describe the set-theoretic concept to be categorified in terms of categorical structures in **Set**;
  2. to internalize the defining categorical structures in **Cat**.

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- in few words, we must replace:
  1. sets by categories;
  2. functions between sets by functors between categories;
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Examples

- A “monoid” is given by:
  1. a set X;
  2. a function $*: X \times X \to X$;
  3. a distinguished element $e \in X$;
  4. such that

$$x * (y * z) = (x * y) * z \tag{1}$$
$$x * e = x = e * x \tag{2}$$

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- **Remark:** 3. and 4. are not categorical!

Examples

- a "monoid" is given by:
  1. a set X;
  2. a function $* : X \times X \to X$;
  3. a function $e : 1 \to X$, where 1 is a singleton;
  4. commutative diagrams

$$\begin{tikzcd} X \times (X \times X) \arrow[r, "\simeq"] \arrow[d, "id \times *"'] & (X \times X) \times X \arrow[r, "* \times id"] & X \times X \arrow[d, "*"] \\ X \times X \arrow[rr, "*"'] & & X \end{tikzcd}$$

$$\begin{tikzcd} \mathbf{1} \times X \arrow[r, "\mathbf{1} \times id"] \arrow[dr, "\simeq"'] & X \times X \arrow[d, "*"] & X \times \mathbf{1} \arrow[l, "id \times \mathbf{1}"'] \arrow[dl, "\simeq"] \\ & X & \end{tikzcd}$$

Examples

- a categorified monoid ("monoidal category") is given by:
  1. a category $\mathbf{C}$;
  2. a functor $* : \mathbf{C} \times \mathbf{C} \to \mathbf{C}$;
  3. a functor $e : 1 \to \mathbf{C}$, where 1 is the categorical singleton;
  4. analogous commutative diagrams

$$\begin{tikzcd}
\mathbf{C} \times (\mathbf{C} \times \mathbf{C}) \arrow[r, "\simeq"] \arrow[d, "id \times *"'] & (\mathbf{C} \times \mathbf{C}) \times \mathbf{C} \arrow[r, "* \times id"] & \mathbf{C} \times \mathbf{C} \arrow[d, "*"] \\
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Examples

set-theoretic concept	vertical categorification
monoid	monoidal category

Table: Examples of Vertical Categorification

Examples

set-theoretic concept	vertical categorification
monoid	monoidal category
commutative monoid	braided monoidal category symmetric monoidal category
group	monoidal category s.t $\mathrm{Ob}(\mathbf{C}) \subset \mathrm{Pic}(\mathbf{C})$
free and fin. gen. monoid	rigid monoidal category
$\vdots$	$\vdots$

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Motivation

- why should an algebraist be interested in vertical categorification?
- **Folklore:** Let"P" be some algebraic concept which can be vertically categorified;
- then its category of representations $\mathbf{Rep}_P$ has the structure of a vertical categorification of "P";
- **Tannaka duality.**:*Let "P" be associative and suppose that it can be vertically categorified. If* $\mathbf{C}$ *is a category which realizes the vertical categorification of "P", then* $\mathbf{C} \simeq \mathbf{Rep}_P$.
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Lie algebra	??

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- A "Lie algebra[2]" is given by:
  1. an abelian group $\mathfrak{g}$;
  2. a bilinear map $[\cdot,\cdot] : \mathfrak{g} \times \mathfrak{g} \to \mathfrak{g}$;
  3. such that

$$[x, y] + [y, x] = 0 \tag{3}$$

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- Not only this: Jacobi identity is equivalently described by:
  1. the polynomial identity
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  2. the fact that the adjoint map $ad:\mathfrak{g}\rightarrow \mathrm{End}(\mathfrak{g})$ is an algebra morphism;
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- THUS...

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set-theoretic concept	**vertical categorification**
monoid	monoidal category
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- **Remark:** It seems (at least to me) that the corresponding categories **LieCatI**, **LieCatII** and **LieCatIII** are not equivalent.

Lie Algebroids

Horizontal Categorification

- Consists in taking a set-theoretic concept "P" and finding for a class of categories $\mathbf{Cat}_P$ such that categories $\mathbf{C} \in \mathbf{Cat}_P$ with a single object are equivalent to "P".
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Lie Algebroids

- A *Lie algebroid* over a manifold M is given by:
  1. a vector bundle $E \to M$;
  2. a Lie algebra structure on the space of gloal sections $\Gamma(E)$;
  3. a vector bundle morphism $\rho : E \to TM$ such that:
     3.1 it is a derivation relative to the action of the ring $C^\infty(M)$;
     3.2 the induced map $\Gamma(\rho) : \Gamma(E) \to \Gamma(TM)$ is a morphism of Lie algebras.
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Lie Algebroids

set-theoretic concept	**vertical categorification**
group	groupoid
Lie algebra	~~Lie algebroid~~

Table: Horizontal Categorification of Lie Algebra

From Lie Algebroids to Lie Algebras

- On the other hand...
- "Lie algebroid" over the point $M \simeq pt$ is equivalent to "Lie algebra".
- This motivates us to ask:

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- This motivates us to ask:
- **Question:** *Can we embed* $\mathbf{LieAlgd_M}$ *in some category of categories* $\mathbf{C} \subset \mathbf{Cat}$ *such that when regarded as object of this new category "Lie algebroid" becomes a horizontal categorification of "Lie algebra"?*

From Lie Algebroids to Lie Algebras

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- We propose a solution (at least after restriction to $\mathbb{Z}$):
- <u>**Theorem:**</u> *For every manifold M the category* $\mathbf{LieAlgd}^{\mathbb{Z}}_{\mathbf{M}}$ *of $\mathbb{Z}$-Lie algebroids over M can be fully embedded in* **LieCatIII**. *Furthermore, inside* **LieCatIII** *the one-object limit of* $\mathbf{LieAlgd}^{\mathbb{Z}}_{\mathbf{M}}$ *coincides with the one-point limit.*

Speculations

From Lie Algebras to Lie Algebroids

- The idea is to use the following steps:
    1. to consider a definition/result about Lie algebras;
    2. to redefine/reprove the result using only categorical structure, i.e, to show that it is internal to $\mathbf{LieAlg}_{\mathbb{Z}}$;
    3. to apply vertical categorification in order to get an analogous definition/result internal to **LieCatIII**;
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- **Example:** what is the Lie algebroid version of the classification of complex semisimple Lie algebras?
- OBS: a "semisimple Lie algebroid" should be a Courant algebroid fulfilling additional conditions...
- Example: what is representation theory for Lie algebroids??
- and so on.

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From Lie Algebroidal Categories to Lie Algebras

- by the very notion of vertical categorification, the category of representations of a Lie algebra should belong to **LieCatI**, **LieCatII** or **LieCatIII**.
- can we build some version of Tannaka duality for "nonassociative algebraic objects"?
- in affirmative case, any Lie algebroidal category should be equivalent (under Tannaka duality) to the category of representations of some Lie algebra.
- and so on.

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