1.

$$m(a+bX) = a + b \times m(X)$$

$$m(a+bX) = \frac{1}{N}\sum_{i=1}^{N}(a+bx_i) = \frac{1}{N}\sum_{i=1}^{N} a + \frac{1}{N}\sum_{i=1}^{N} bx_i$$

$$= a + b\left(\frac{1}{N}\sum_{i=1}^{N} x_i\right) = a + bm(X)$$

2. $\mathrm{cov}(X, a+bY) = b\,\mathrm{cov}(X, Y)$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))\big((a+by_i) - m(a+bY)\big)$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(a+by_i - (a+bm(Y)))$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(b(y_i - m(Y)))$$

$$= b \cdot \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(y_i - m(Y))$$

$$= b\,\mathrm{cov}(X, Y)$$

3. $\mathrm{cov}(a+bX, a+bX) = b^2\,\mathrm{cov}(X, X)$

$\mathrm{cov}(a+bX, a+bX) = b \cdot \mathrm{cov}(X, a+bX) = b \cdot (b \cdot \mathrm{cov}(X,X) = b^2 \cdot \mathrm{cov}(X, X)$

$\hookrightarrow \mathrm{cov}(a+bX, a+bX) = b^2\,\mathrm{cov}(X, X)$

$s^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))^2 = \mathrm{cov}(X, X) \Rightarrow \mathrm{cov}(X, X) = s^2$

therefore, $\mathrm{cov}(a+bX, a+bX) = b^2 s^2$ and $\mathrm{cov}(X, X) = s^2$

4. If median always increasing, then yes. But if decreasing, then not always because it can fail when there are flat parts. The quantile is only set inclusion if flat when decreasing but yes, if it is only increasing. The IQR can shrink when it is flat. For the Range, it is always like that.

5. No, $m(g(X)) \neq g(m(X))$ for a non-decreasing g. However, there are special cases.