Simulation Report

In this exercise, the distribution of the mean of exponential random variables with rate of 0.2 and sample size of 40 was analysed. The random variables were first simulated with the following codes and the means computed. 1000 simulation of sample size 40 was generated.

```
set.seed(1)
n <- 40; lambda <- 0.2
samples <- replicate(1000, rexp(n, lambda))
sample.means <- apply(samples, 2, mean)
```

Mean

The mean of sample means, mean of all the samples and the theoretical mean were then computed. The theoretical mean of the sample mean is also $1/\lambda$. It can also be estimated by the sample mean of all the samples.

```
mean.sample.mean <- mean(sample.means)
mean.sample <- mean(samples)
mu <- 1 / lambda
```

Sample mean of sample means: 4.99

Sample mean of all samples: 4.99

Theoretical mean: 5

Unsurprisingly, the sample means of the sample means and of all samples were identical. They were also very close to the theoretical mean.

Standard Deviation

The respective standard deviation were computed as followed. The theoretical standard deviation of the sample mean is $\sigma/\sqrt{n} = \frac{1/\lambda}{\sqrt{n}}$. It can also be estimated by $S/\sqrt{(n)}$, where S is the sample standard deviation of all samples.

```
sd.sample.mean <- sd(sample.means)
sd.sample <- sd(samples) / sqrt(n)
sigma <- 1 / lambda / sqrt(n)
```

Sample standard deviation of sample means: 0.7817

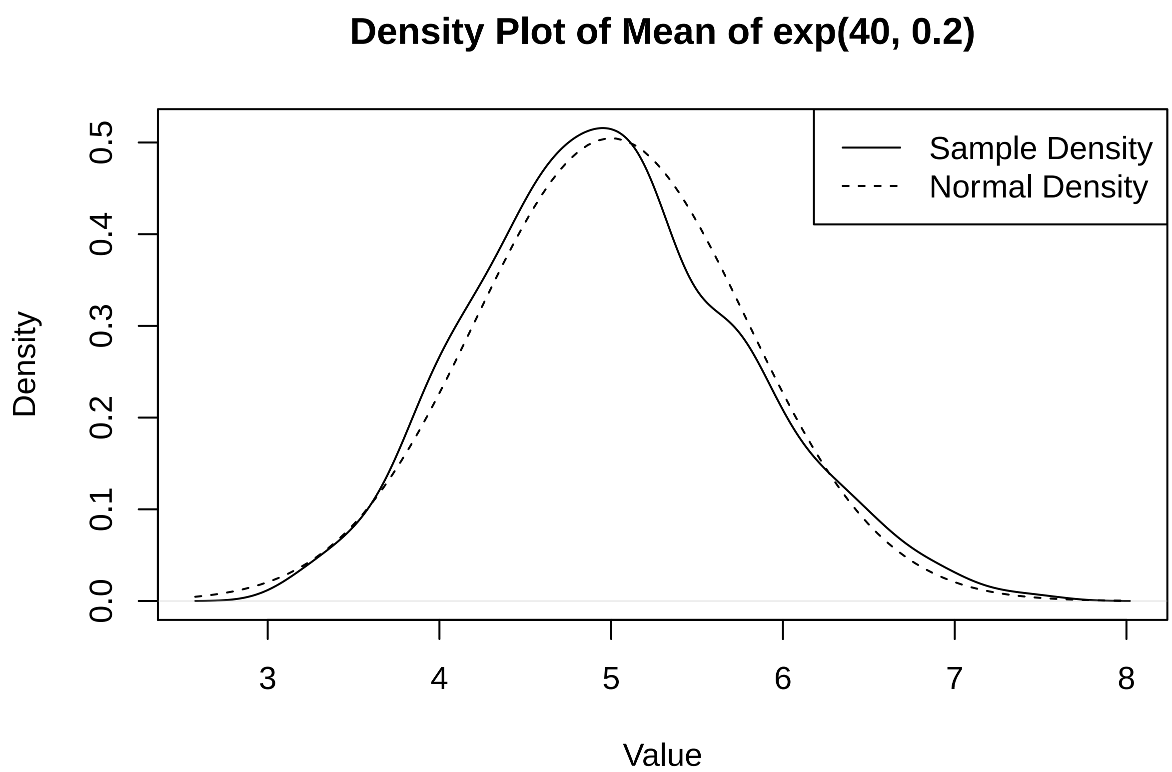
Sample standard deviation of all samples: 0.7913

Theoretical standard deviation: 0.7906

Compared to the theoretical standard deviation, the sample means showed less variation. However, the estimate from the standard deviation of all was very close to the theoretical value.

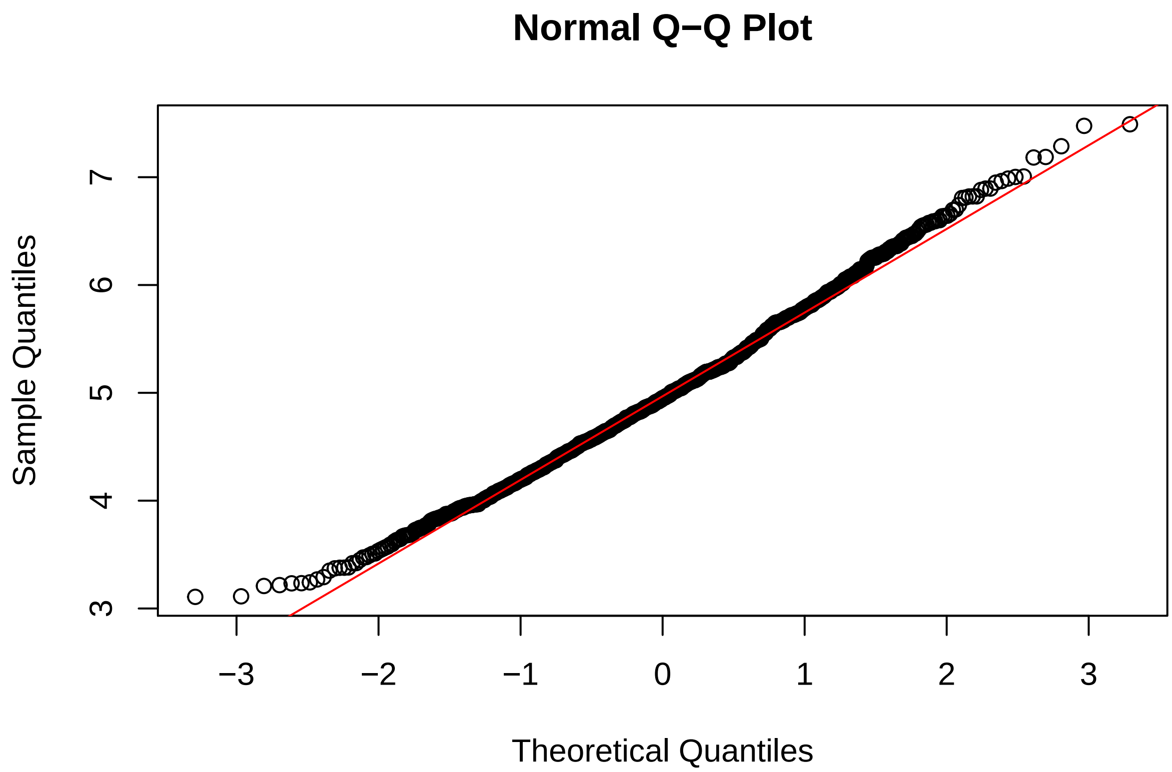
Normality

```r
plot(density(sample.means),
     main = "Density Plot of Mean of exp(40, 0.2)",
     xlab = "Value")
curve(dnorm(x, mu, sigma), add = T, lty = 2)
legend("topright", legend = c("Sample Density", "Normal Density"), lty = 1:2)
```



The plot above was the density plot of the sample means with the normal density of the same mean and standard deviation superimposed. The distribution of the sample means appeared to follow the normal distribution closely. This was due to the Central Limit Theorem, since a sample size of 40 was larger than the rule-of-thumb of 30. We could check how close the distribution was to normal with the QQ plot.

```r
qqnorm(sample.means)
qqline(sample.means, col = 2)
```



The center of the distribution followed the theoretical quantiles pretty closely, although the tails displayed some deviation.

Confidence Interval

The confidence interval of the mean was computed as followed. The coverage was close to the 95% expected from the normal distribution since the sample means was approximately normal.

```
CI <- mean.sample + c(-1,1) * sd.sample.mean * 1.96
mean(sample.means > CI[1] & sample.means < CI[2])
```

```
## [1] 0.948
```