

Name - YuniKa Upadhyaya

ID - 1001631183

Sandhya

Date

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1) Constants:  $A_1, A_2, A_3, C_1, C_2, C_3, \text{Boat}$

Predicates:

$\text{is-child}(x)$ :  $x$  is a child

$\text{is-adult}(x)$ :  $x$  is an adult.

$\text{in-leftBank}(x)$ :  $x$  is in left side of river.

$\text{in-rightBank}(x)$ :  $x$  is in right side of river.

$\text{boat-in-left}(\text{Boat})$ : Boat is in left side.

$\text{boat-in-right}(\text{Boat})$ : Boat is in right side.

Actions:

1) CA-MOVE-LR  $(x, y)$ :

Pre-condition:  $\text{in-leftBank}(x) \wedge \text{in-leftBank}(y) \wedge \text{is-child}(x) \wedge \text{is-adult}(y) \wedge \text{boat-in-left}(\text{Boat})$

Effect:  $\text{in-rightBank}(x) \wedge \text{in-rightBank}(y) \wedge \neg \text{in-leftBank}(x) \wedge \neg \text{in-leftBank}(y) \wedge \text{boat-in-right}(\text{Boat}) \wedge \neg \text{boat-in-left}(\text{Boat})$

2) AA-MOVE-LR  $(x, y)$ :

Pre-condition:  $\text{is-adult}(x) \wedge \text{is-adult}(y) \wedge \text{in-leftBank}(x) \wedge \text{in-leftBank}(y) \wedge \text{boat-in-left}(\text{Boat})$

Effect:  $\text{in-rightBank}(x) \wedge \text{in-rightBank}(y) \wedge \neg \text{in-leftBank}(x) \wedge \neg \text{in-leftBank}(y) \wedge \text{boat-in-right}(\text{Boat}) \wedge \neg \text{boat-in-left}(\text{Boat})$

3) AL-MOVE-RL  $(x, y)$

Pre-condition:  $\text{in-rightBank}(x) \wedge \text{is-adult}(x) \wedge \text{boat-in-right}(\text{Boat})$

Effect:  $\text{in-leftBank}(x) \wedge \text{boat-in-left}(\text{Boat}) \wedge \neg \text{boat-in-right}(\text{Boat}) \wedge \neg \text{in-rightBank}(x)$

$\Rightarrow$

Initial state:

$$\begin{aligned} & \text{in-leftBank}(A_1) \wedge \text{in-leftBank}(A_2) \wedge \text{in-leftBank}(A_3) \wedge \\ & \text{in-leftBank}(C_1) \wedge \text{in-leftBank}(C_2) \wedge \text{in-leftBank}(C_3) \wedge \\ & \text{boat-in-left}(\text{Boat}) \wedge \text{is-adult}(A_1) \wedge \text{is-adult}(A_2) \\ & \wedge \text{is-adult}(A_3) \wedge \text{is-child}(C_1) \wedge \text{is-child}(C_2) \wedge \\ & \text{is-child}(C_3) \end{aligned}$$

Goal-state:

$$\begin{aligned} & \text{in-rightBank}(A_1) \wedge \text{in-rightBank}(A_2) \wedge \text{in-rightBank}(A_3) \\ & \wedge \text{in-rightBank}(C_1) \wedge \text{in-rightBank}(C_2) \wedge \text{in-rightBank}(C_3) \\ & \wedge \text{boat-in-right}(\text{Boat}) \end{aligned}$$

$$2) \quad i) P(A=4 | B \neq 3) = 0.0575 + 0.0150 + 0.0028 + 0.0243 \\ = 0.0996 //$$

$$ii) P(A=1 \vee B=4) \\ = 0.0425 + 0.0702 + 0.0124 + 0.0370 + 0.0803 + 0.0031 + \\ 0.0652 + 0.0028 \\ = 0.3135 //$$

$$iii) P(A \neq B) \\ = 0.0695 + 0.0819 + 0.0575 + 0.0702 + 0.0595 + 0.0150 + \\ 0.0124 + 0.0575 + 0.0619 + 0.037 + 0.0031 + 0.0652 + \\ 0.0803 + 0.0745 + 0.0344 + 0.0243 \\ = 0.8042 //$$

iv) Conditions for total independence:

$$P(A|B) = P(A)$$

$$P(A, B) = P(A) \cdot P(B)$$

$$P(B|A) = P(B)$$

lets take value = 1

$$P(A=1 | B=1) = 0.0425$$

$$P(A=1) = 0.0425 + 0.0702 + 0.0124 + 0.0370 + \\ 0.0803 \\ = 0.2424$$

Since,

$$P(A=1 | B=1) \neq P(A=1)$$

we can say that, A and B are totally independent.



$$3) P(A=T | C=T, D=F) = \frac{P(A=T, C=T, D=F)}{P(C=T, D=F)}$$

Now,

Numerator:

$$P(A=T, C=T, D=F) = P(A=T, C=T, D=F, B=T) + P(A=T, C=T, D=F, B=F)$$

$$P(A=T) \cdot P(B=T) \cdot P(C=T | A=T, D=F) \cdot$$

$$= P(D=F | B=T) +$$

$$P(A=T) \cdot P(B=F) \cdot P(C=T | A=T, D=F) \cdot$$

$$P(D=F | B=F)$$

$$= (0.7 \times 0.4 \times 0.6 \times (1-0.9)) +$$

$$(0.7 \times 0.6 \times 0.6 \times (1-0.2))$$

$$= 0.0168 + 0.2016$$

$$= 0.2184$$

Denominator:

$$P(C=T, D=F)$$

$$= P(A=T, B=T, C=T, D=F) +$$

$$P(A=T, B=F, C=T, D=F) +$$

$$P(A=F, B=T, C=T, D=F) +$$

$$P(A=F, B=F, C=T, D=F)$$

$$= (0.7 \times 0.4 \times 0.6 \times 0.1) + (0.7 \times 0.6 \times 0.6 \times 0.8) + (0.7 \times 0.4 \times$$

$$0.2 \times 0.1) + (0.7 \times 0.6 \times 0.2 \times 0.8)$$

$$= 0.2912$$

Now,

$$P = \frac{0.2184}{0.2912} = 0.75$$

$$0.2912$$

//

4) Here, Given:

$$P(A_1, B_1) = P(A_2, B_2)$$

$$P(A_1, B_1) = P(A_1, B_1) \cdot P(B_1)$$

$$= \frac{P(B_1|A_1) \cdot P(A_1)}{P(B_1)} \times P(B_1)$$

$$= P(B_1|A_1) \cdot P(A_1)$$

Now, we can calculate the same for:

$$P(A_2, B_2) = P(A_2|B_2) \cdot P(B_2)$$

From the given true case:

$$P(A_1, B_1) = P(A_2, B_2)$$

$$\Rightarrow P(B_1|A_1) \cdot P(A_1) = P(A_2|B_2) \cdot P(B_2)$$

Here, the corresponding probability should be same in the above equation:

$$x_1 = P(B_2) = P(A_1) = 0.1208$$

$$P(A_2|B_2) = P(B_1|A_1)$$

So, we can write,

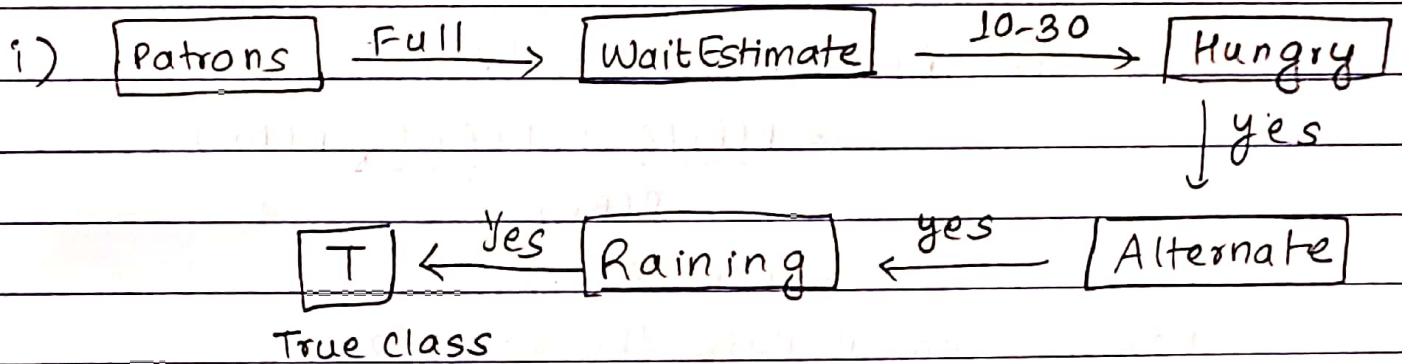
$B_2$	$P(A_2 B_2)$
T	$x_2 = 0.4308$
F	$x_3 = 0.5360$

$$\therefore x_1 = 0.1208$$

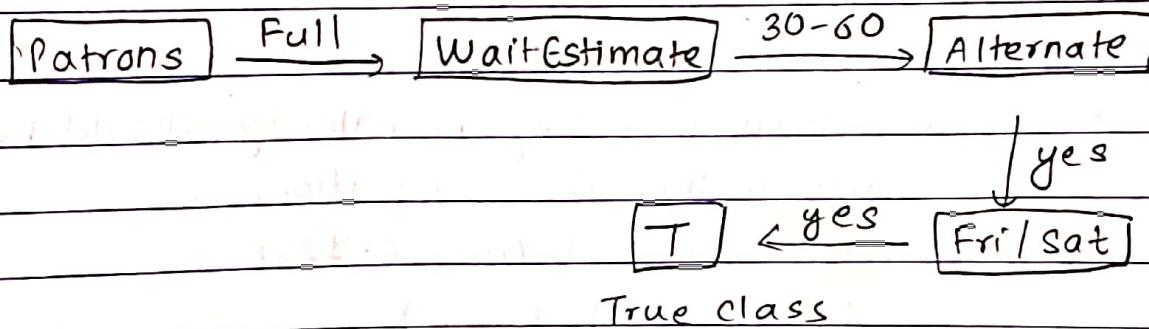
$$x_2 = 0.4308$$

$$x_3 = 0.5360$$

6)



ii)





7) Class X: 6

Class Y: 6

$$\text{Entropy (root)} = -\frac{6}{12} \log_2 \frac{6}{12} - \frac{6}{12} \log_2 \frac{6}{12}$$

$$= 1$$

For test A:

class	1	2	3
X	2	2	2
Y	2	2	2

$$\text{Entropy}(1) = -\frac{2}{4} \log_2 \frac{2}{4} - \frac{2}{4} \log_2 \frac{2}{4} = 1$$

$$\text{Entropy}(2) = -\frac{2}{4} \log_2 \frac{2}{4} - \frac{2}{4} \log_2 \frac{2}{4} = 1$$

$$\text{Entropy}(3) = -\frac{2}{4} \log_2 \frac{2}{4} - \frac{2}{4} \log_2 \frac{2}{4} = 1$$

$$\text{Information gain (A)} = 1 - \frac{4}{12} \times 1 - \frac{4}{12} \times 1 - \frac{4}{12} \times 1$$

$$= 0$$

For test B:

class	1	2	3
X	4	2	0
Y	0	2	4

$$\text{Entropy}(1) = -\frac{4}{4} \log_2 \frac{4}{4} - \frac{0}{4} \log_2 \frac{0}{4} = 0$$

$$\text{Entropy}(2) = -\frac{2}{4} \log_2 \frac{2}{4} - \frac{2}{4} \log_2 \frac{2}{4} = 1$$

$$\text{Entropy}(3) = -\frac{0}{4} \log_2 \frac{0}{4} - \frac{4}{4} \log_2 \frac{4}{4} = 0$$

$$\text{Information gain (B)} = 1 - \frac{4}{12} \times 0 - \frac{4}{12} \times 1 - \frac{4}{12} \times 0$$

$$= 0.667$$

For test C:

class	1	2	3
x	1	2	3
y	1	3	2

$$\text{Entropy}(1) = -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} = 1$$

$$\text{Entropy}(2) = -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} = 0.97$$

$$\text{Entropy}(3) = -\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5} = 0.97$$

$$\begin{aligned} \text{Information Gain}(C) &= 1 - \frac{2}{12} \times 1 - \frac{5}{12} \times 0.97 - \frac{5}{12} \times 0.97 \\ &= 0.025 \end{aligned}$$

Now,

$$\text{IG}(B) > \text{IG}(C) > \text{IG}(A)$$

Here, Since information gain of B is greatest, the best test to use at root is B.