

Alpha-Beta Search

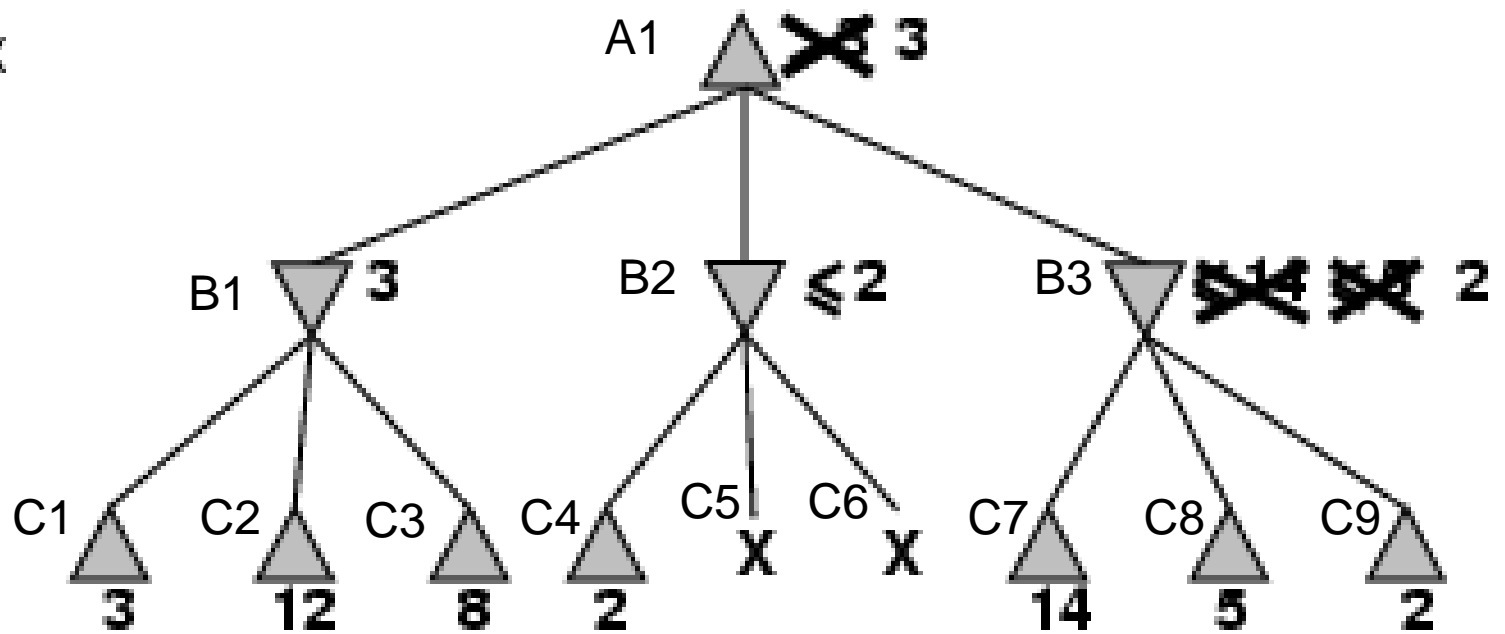
```
function AlphaBetaDecision(state) returns an action
    v = MaxValue(state, -infinity, +infinity)
    return the a in Actions(state) leading to a successor state with utility v.
```

```
function MaxValue(state, alpha, beta) returns a utility value
    if TerminalTest(state) then return Utility(state)
    v = -infinity
    for a, s in Successors(state) do
        v = Max(v, MinValue(s, alpha, beta))
        if v >= beta then return v
        alpha = Max(alpha, v)
    return v
```

```
function MinValue(state, alpha, beta) returns a utility value
    if TerminalTest(state) then return Utility(state)
    v = +infinity
    for a, s in Successors(state) do
        v = Min(v, MaxValue(s, alpha, beta))
        if v <= alpha then return v
        beta = Min(beta, v)
    return v
```

MAX

MIN



- $\text{MaxValue}(A1, -\infty, +\infty)$
TerminalState(A1)? No
 $v = -\infty$
 $\text{Successors}(A1) = \{B1, B2, B3\}$
 $s = B1$
 $\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B1, -\infty, +\infty)$

 $\text{state} = A1$
 $\alpha = -\infty$
 $\beta = +\infty$
 $v = -\infty$
 $s = B1$
- $\text{MinValue}(B1, -\infty, +\infty)$

 $\text{state} = B1$
 $\alpha = -\infty$
 $\beta = +\infty$

- $\text{MaxValue}(A1, -\text{infinity}, +\text{infinity})$
 $\text{TerminalState}(A1)?$ No
 $v = -\text{infinity}$
 $\text{Successors}(A1) = \{B1, B2, B3\}$
 $s = B1$
 $\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B1, -\text{infinity}, +\text{infinity})$
- $\text{MinValue}(B1, -\text{infinity}, +\text{infinity})$
 $\text{TerminalState}(B1)?$ No
 $v = +\text{infinity}$
 $\text{Successors}(B1) = \{C1, C2, C3\}$
 $s = C1$
 $\text{MaxValue}(s, \alpha, \beta) = \text{MaxValue}(C1, -\text{infinity}, +\text{infinity}) = 3$

$\text{state} = A1$
 $\alpha = -\text{infinity}$
 $\beta = +\text{infinity}$
 $v = -\text{infinity}$
 $s = B1$

$\text{state} = B1$
 $\alpha = -\text{infinity}$
 $\beta = +\text{infinity}$
 $v = +\text{infinity}$
 $s = C1$

- $\text{MaxValue}(A1, -\text{infinity}, +\text{infinity})$
 $\text{TerminalState}(A1)?$ No
 $v = -\text{infinity}$
 $\text{Successors}(A1) = \{B1, B2, B3\}$
 $s = B1$
 $\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B1, -\text{infinity}, +\text{infinity})$

$\text{state} = A1$
 $\alpha = -\text{infinity}$
 $\beta = +\text{infinity}$
 $v = -\text{infinity}$
 $s = B1$
- $\text{MinValue}(B1, -\text{infinity}, +\text{infinity})$
 $\text{TerminalState}(B1)?$ No
 $v = +\text{infinity}$
 $\text{Successors}(B1) = \{C1, C2, C3\}$
 $s = C1$
 $\text{MaxValue}(s, \alpha, \beta) = \text{MaxValue}(C1, -\text{infinity}, +\text{infinity}) = 3$
 $v = \text{Min}(v, 3) = \text{Min}(+\text{infinity}, 3) = 3$
 $v \leq \alpha? 3 \leq -\text{infinity}? \text{ No}$

$\text{state} = B1$
 $\alpha = -\text{infinity}$
 $\beta = +\text{infinity}$
 $v = 3$
 $s = C1$

- MaxValue(A1, -infinity, +infinity)
 TerminalState(A1)? No
 v = -infinity
 Successors(A1) = {B1, B2, B3}
 s = B1
 MinValue(s, alpha, beta) = MinValue(B1, -infinity, +infinity)

state = A1
 alpha = -infinity
 beta = +infinity
 v = -infinity
 s = B1

- MinValue(B1, -infinity, +infinity)
 TerminalState(B1)? No
 v = +infinity
 Successors(B1) = {C1, C2, C3}
 s = C1
 MaxValue(s, alpha, beta) = MaxValue(C1, -infinity, +infinity) = 3
 v = Min(v, 3) = Min(+infinity, 3) = 3
 v <= alpha? 3 <= -infinity? No
 beta = Min(beta, v) = min(+infinity, 3) = 3

state = B1
 alpha = -infinity
 beta = 3
 v = 3
 s = C1

- $\text{MaxValue}(A1, -\text{infinity}, +\text{infinity})$
 $\text{TerminalState}(A1)?$ No
 $v = -\text{infinity}$
 $\text{Successors}(A1) = \{B1, B2, B3\}$
 $s = B1$
 $\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B1, -\text{infinity}, +\text{infinity})$

$\text{state} = A1$
 $\alpha = -\text{infinity}$
 $\beta = +\text{infinity}$
 $v = -\text{infinity}$
 $s = B1$
- $\text{MinValue}(B1, -\text{infinity}, +\text{infinity})$
 $\text{TerminalState}(B1)?$ No
 $v = +\text{infinity}$
 $\text{Successors}(B1) = \{C1, C2, C3\}$
 $s = C2$
 $\text{MaxValue}(s, \alpha, \beta) = \text{MaxValue}(C2, -\text{infinity}, 3) = 12$
 $v = \text{Min}(v, 12) = \text{Min}(3, 12) = 3$
 $v \leq \alpha? 3 \leq -\text{infinity}? \text{ No}$
 $\beta = \text{Min}(\beta, v) = \text{min}(3, 3) = 3$

$\text{state} = B1$
 $\alpha = -\text{infinity}$
 $\beta = 3$
 $v = 3$
 $s = C2$

- $\text{MaxValue}(A1, -\text{infinity}, +\text{infinity})$
 $\text{TerminalState}(A1)?$ No
 $v = -\text{infinity}$
 $\text{Successors}(A1) = \{B1, B2, B3\}$
 $s = B1$
 $\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B1, -\text{infinity}, +\text{infinity})$

$\text{state} = A1$
 $\alpha = -\text{infinity}$
 $\beta = +\text{infinity}$
 $v = -\text{infinity}$
 $s = B1$
- $\text{MinValue}(B1, -\text{infinity}, +\text{infinity})$
 $\text{TerminalState}(B1)?$ No
 $v = +\text{infinity}$
 $\text{Successors}(B1) = \{C1, C2, C3\}$
 $s = C3$
 $\text{MaxValue}(s, \alpha, \beta) = \text{MaxValue}(C3, -\text{infinity}, 3) = 8$
 $v = \text{Min}(v, 8) = \text{Min}(3, 12) = 3$
 $v \leq \alpha? 3 \leq -\text{infinity}? \text{ No}$
 $\beta = \text{Min}(\beta, v) = \text{min}(3, 3) = 3$

$\text{state} = B1$
 $\alpha = -\text{infinity}$
 $\beta = 3$
 $v = 3$
 $s = C3$

- $\text{MaxValue}(A1, -\text{infinity}, +\text{infinity})$
TerminalState(A1)? No
 $v = -\text{infinity}$
 $\text{Successors}(A1) = \{B1, B2, B3\}$
 $s = B1$
 $\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B1, -\text{infinity}, +\text{infinity})$

state = A1
alpha = -infinity
beta = +infinity
v = -infinity
s = B1

- $\text{MinValue}(B1, -\text{infinity}, +\text{infinity})$
TerminalState(B1)? No
 $v = +\text{infinity}$
 $\text{Successors}(B1) = \{C1, C2, C3\}$
 $s = C3$
 $\text{MaxValue}(s, \alpha, \beta) = \text{MaxValue}(C3, -\text{infinity}, 3) = 8$
 $v = \text{Min}(v, 8) = \text{Min}(3, 12) = 3$
 $v \leq \alpha$? $3 \leq -\text{infinity}$? No
 $\beta = \text{Min}(\beta, v) = \text{min}(3, 3) = 3$

state = B1
alpha = -infinity
beta = 3
v = 3
s = C3

return v: returns 3

- $\text{MaxValue}(A1, -\infty, +\infty)$
 $\text{TerminalState}(A1)?$ No
 $v = -\infty$
 $\text{Successors}(A1) = \{B1, B2, B3\}$
 $s = B1$
 $\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B1, -\infty, +\infty) = 3$
 $v = \text{Max}(v, \text{MinValue}(B1, -\infty, +\infty)) = \text{Max}(-\infty, 3) = 3$

$\text{state} = A1$
 $\alpha = -\infty$
 $\beta = +\infty$
 $v = 3$
 $s = B1$

- $\text{MaxValue}(A1, -\infty, +\infty)$
 $\text{TerminalState}(A1)?$ No
 $v = -\infty$
 $\text{Successors}(A1) = \{B1, B2, B3\}$
 $s = B1$
 $\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B1, -\infty, +\infty) = 3$
 $v = \text{Max}(v, \text{MinValue}(B1, -\infty, +\infty)) = \text{Max}(-\infty, 3) = 3$
 $v \geq \beta?$ $3 \geq +\infty?$ No
 $\alpha = \text{Max}(\alpha, v) = \text{Max}(-\infty, 3) = 3$

state = A1
alpha = 3
beta = +infinity
v = 3
s = B1

- $\text{MaxValue}(A1, -\infty, +\infty)$
 $\text{TerminalState}(A1)?$ No
 $v = -\infty$
 $\text{Successors}(A1) = \{B1, B2, B3\}$
 $s = B2$
 $\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B2, 3, +\infty)$

 $\text{state} = A1$
 $\alpha = 3$
 $\beta = +\infty$
 $v = 3$
 $s = B2$

- MaxValue(A1, -infinity, +infinity)
TerminalState(A1)? No
v = -infinity
Successors(A1) = {B1, B2, B3}
s = B2
MinValue(s, alpha, beta) = MinValue(B2, 3, +infinity)

state = A1
alpha = 3
beta = +infinity
v = 3
s = B2
- MinValue(B2, 3, +infinity)

state = B2
alpha = 3
beta = +infinity

- $\text{MaxValue}(A1, -\text{infinity}, +\text{infinity})$
 TerminalState(A1)? No
 $v = -\text{infinity}$
 $\text{Successors}(A1) = \{B1, B2, B3\}$
 $s = B2$
 $\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B2, 3, +\text{infinity})$
- $\text{MinValue}(B2, 3, +\text{infinity})$
 TerminalState(B1)? No
 $v = +\text{infinity}$
 $\text{Successors}(B2) = \{C4, C5, C6\}$
 $s = C4$
 $\text{MaxValue}(s, \alpha, \beta) = \text{MaxValue}(C4, 3, +\text{infinity}) = 2$

$\text{state} = A1$
 $\alpha = 3$
 $\beta = +\text{infinity}$
 $v = 3$
 $s = B2$

$\text{state} = B2$
 $\alpha = 3$
 $\beta = +\text{infinity}$
 $v = +\text{infinity}$
 $s = C4$

- MaxValue(A1, -infinity, +infinity)
 TerminalState(A1)? No
 $v = -\text{infinity}$
 Successors(A1) = {B1, B2, B3}
 $s = B2$
 MinValue(s, alpha, beta) = MinValue(B2, 3, +infinity)

state = A1
 alpha = 3
 beta = +infinity
 $v = 3$
 $s = B2$
- MinValue(B2, 3, +infinity)
 TerminalState(B1)? No
 $v = +\text{infinity}$
 Successors(B2) = {C4, C5, C6}
 $s = C4$
 MaxValue(s, alpha, beta) = MaxValue(C4, 3, +infinity) = 2
 $v = \text{Min}(v, 3) = \text{Min}(+\text{infinity}, 2) = 2$
 $v \leq \text{alpha}$? $2 \leq 3$? **YES**
 return v: returns 2

state = B2
 alpha = 3
 beta = +infinity
 $v = 2$
 $s = C4$