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Assignment-4

Written Assignment - Planning, Probability and Bayesian Network

Task-1

Here,

CONSTANTS: $B_1, B_2, B_3, R_1, R_2, R_3, P_1, P_2$

Here, B_1, B_2, B_3 = Three blue marbles
R_1, R_2, R_3 = Three red marbles
P_1 = Bucket with red balls intially
P_2 = Bucket with blue balls initially

PREDICATES: IN-BUCKET (A,B), EMPTY (A), SAME-COLOR (A,B)

STATE: IN-BUCKET (B_1,P_2) ∧ IN-BUCKET (B_2,P_2) ∧ IN-BUCKET(B_3,P_2) ∧
IN-BUCKET (R_1,P_1) ∧ IN-BUCKET(R_2,P_1) ∧ IN-BUCKET (R_3,P_1)

ACTION:

→ MOVE-FROM-P_1-TO-P_2 (w,x,y,z)
PREDICATES: IN-BUCKET(w,y) ∧ IN-BUCKET(x,y) ∧ not SAME-COLOR (w,x)
EFFECT: not IN-BUCKET(w,y) ∧ not IN-BUCKET(x,y) ∧ IN-BUCKET(w,z) ∧
IN-BUCKET(x,z)

→ MOVE-FROM-P_2-TO-P_1 (w,x,y,z)
PREDICATES: IN-BUCKET (w,y) ∧ IN-BUCKET(x,y) ∧ SAME-COLOR (w,x)
EFFECT: not IN-BUCKET(w,y) ∧ NOT-IN-BUCKET (x,y) ∧ IN-BUCKET(w,z) ∧
IN-BUCKET(x,z)

GOAL: IN-BUCKET (B_1,P_1) ∧ IN-BUCKET (B_2,P_1) ∧ IN-BUCKET (B_3,P_1) ∧ IN-BUCKET
(R_1,P_2) ∧ IN-BUCKET(R_2,P_2) ∧ IN-BUCKET (R_3,P_2)

We can obtain the goal state by following actions:

a) MOVE-FROM-P_2-TO-P_1 (B_1, B_2, P_2, P_1)

→ satisfies PRED: IN-BUCKET(B_1, P_2) $\wedge$ IN-BUCKET(B_2, P_2) $\wedge$ SAME-COLOR(B_1, B_2)

EFFECT: not IN-BUCKET(B_1, P_2) $\wedge$ not IN-BUCKET(B_2, P_2) $\wedge$ IN-BUCKET$(B_1,$ $P_1)$ $\wedge$ IN-BUCKET(B_2, P_1)

b) MOVE-FROM-P_1-TO-P_2 (R_1, B_1, P_1, P_2)

→ satisfies PRED: IN-BUCKET(R_1, P_1) $\wedge$ IN-BUCKET(B_1, P_1) $\wedge$ not SAME-COLOR(R_1, B_1)

EFFECT: not IN-BUCKET(R_1, P_1) $\wedge$ not IN-BUCKET(B_1, P_1) $\wedge$ IN-BUCKET(R_1, P_2) $\wedge$ IN-BUCKET(B_1, P_2)

c) MOVE-FROM-P_2-TO-P_1 $(B_1, B_3, P_2 P_1)$

→ satisfies PRED: IN-BUCKET(B_1, P_2) $\wedge$ IN-BUCKET(B_3, P_2) $\wedge$ SAME-COLOR(B_1, B_3)

EFFECT: not IN-BUCKET(B_1, P_2) $\wedge$ not IN-BUCKET(B_3, P_2) $\wedge$ IN-BUCKET(B_1, P_1) $\wedge$ IN-BUCKET(B_3, P_1)

d) MOVE-FROM-P_1-TO-P_2 (B_1, R_2, P_1, P_2)

→ satisfies PRED: IN-BUCKET(B_1, P_1) $\wedge$ IN-BUCKET(R_2, P_1) $\wedge$ not SAME-COLOR(B_1, R_2)

EFFECT: not IN-BUCKET(B_1, P_1) $\wedge$ not IN-BUCKET$(R_2\ P_1)$ $\wedge$ IN-BUCKET$(B_1, P$

$\wedge$ IN-BUCKET(R_2, P_2)

e) MOVE-FROM-P_1-TO-P_2 (B_2, R_3, P_1, P_2)

→ satisfies PRED: IN-BUCKET(B_2, P_1) $\wedge$ IN-BUCKET(R_3, P_1) $\wedge$ not SAME-COLOR(B_2, R_3)

EFFECT: not IN-BUCKET(B_2, P_1) $\wedge$ not IN-BUCKET(R_3, P_1) $\wedge$ IN-BUCKET(B_2, P_2) $\wedge$ IN-BUCKET(R_3, P_2)

f) MOVE-FROM-P_2-TO-P_1 (B_1, B_2, P_2, P_1)

→ satisfies PRED: IN-BUCKET (B_1, P_2) $\wedge$ IN-BUCKET (B_2, P_2) $\wedge$ SAME-COLOR (B_1, B_2)

EFFECT: not IN-BUCKET (B_1, P_2) $\wedge$ IN-BUCKET (B_1, P_1) $\wedge$ not IN-BUCKET (B_2, P_2)
$\wedge$ IN-BUCKET (B_2, P_1)

Now,

Present successor state of P_1 : IN-BUCKET (B_1, P_1) $\wedge$ IN-BUCKET (B_2, P_1) $\wedge$
IN-BUCKET (B_3, P_1) $\wedge$ not IN-BUCKET (R_1, P_1)
$\wedge$ not IN-BUCKET (R_2, P_1) $\wedge$ not IN-BUCKET (R_3, P_1)

Present successor state of P_2 : IN-BUCKET (R_1, P_2) $\wedge$ IN-BUCKET (R_2, P_2) $\wedge$ IN-BUCKET (R_3, P_2
$\wedge$ not IN-BUCKET (B_1, P_2) $\wedge$ not IN-BUCKET (B_2, P_2)
$\wedge$ not IN-BUCKET (B_3, P_2)

Here, All positive literals of goal are present in the conjuction of state P_1 and P_2.

Thus, the present state entails the goal state //

Task-2:

Given word: JUNGLE

Number of constants = 5

Number of predicates = 4

Number of arguments each predicate can take = 3

For upper-bound:

Each predicate will take 3 arguments whose combination = 5^3

Since, we have four predicates, total combination = $5^3 \times 4 = 500$

Thus, Possible solutions = 2^{500}

For lower-bound:

Each predicate will take 1 argument whose combination = 5

Since, we have four predicates, total combination = $4 \times 5 = 20$

Thus, Possible solutions = 2^{20}

$\therefore$ Range of bound = 2^{20} to 2^{500}

Task-3

Part-a

We need to calculate P(Color is not Green | Vehicle is Truck).

Here,

$$P(\text{Not Green} \mid \text{Truck}) = \frac{P(\text{Not Green and Truck})}{P(\text{Truck})}$$

$$P(\text{Not Green} \mid \text{Truck}) = \frac{P(\text{Red}, \text{Truck}) + P(\text{Blue}, \text{Truck})}{P(\text{Truck})}$$

$$= \frac{0.0504 + 0.1032}{0.0504 + 0.0864 + 0.1032}$$

$$= \frac{0.1536}{0.24}$$

$$= 0.64$$

Part-b

Vehicle (v) and color (c) are totally independent if

$P(V \mid C) = P(V)$ or $P(C/V) = P(C)$ or $P(V,C) = P(V)\,P(C)$

for instance,

$$P(V=car \mid C=red) = \frac{P(V_{car} \wedge C_{red})}{C_{red}}$$

$$= \frac{0.0630}{0.0630 + 0.0441 + 0.0504 + 0.0525}$$

$$= \frac{0.0630}{0.21} = 0.3$$

$P(V_{car}) = 0.0630 + 0.1080 + 0.1290$

$= 0.3$

Since, $P(V_{car} | C_{red}) = P(V_{car})$,

Hence, Vehicle and Car are totally independent from each other.

Task-4

Part-a:

Possible values of $A = 8$

Possible values of B (B_1 to B_{10}) $= 5^{10}$

Possible value of $C = 6$

Thus, Total number of values we have to store for $P = 8 \times 5^{10} \times 6$

$= 4.69 \times 10^8$.

Part-b:

C is totally independent of A and B, number of values needed to be stored is reduced to $= 8 \times 5^{10}$

$= 7.81 \times 10^7$

Also, B is conditionally independent to each other,

$P(B_1, B2, \cdots B_{10}) = 5^{10} - 1$

Using chain rule,

$P(B1, B2, \ldots B_{10}) = P(B_2 \ldots, B_{10}) P(B_1 | B_2, \cdots, B_{10})$

$= P(B_1|B_2, \cdots, B_{10}) \; P(B_2|B_3, \cdots, B_{10}) \; P(B_3 \cdots B_{10}$

$= P(B_1|B_2, \cdots B_{10}) \; P(B_2|B_3, \cdots, B_{10}) \ldots P(B_8 | B_9, B_{10})$

$P(B_9)$

$= \ldots \ldots$

Thus, for each $P(B_i|A) \in$ chain rule,

Values stored $= 8 \times (5-1)$

$= 8 \times 4$

$= 32$ values

Again, In $P(B_1, \ldots B_{10})$:

For 10 of each conditionally independent B = 10×32

= 320 values

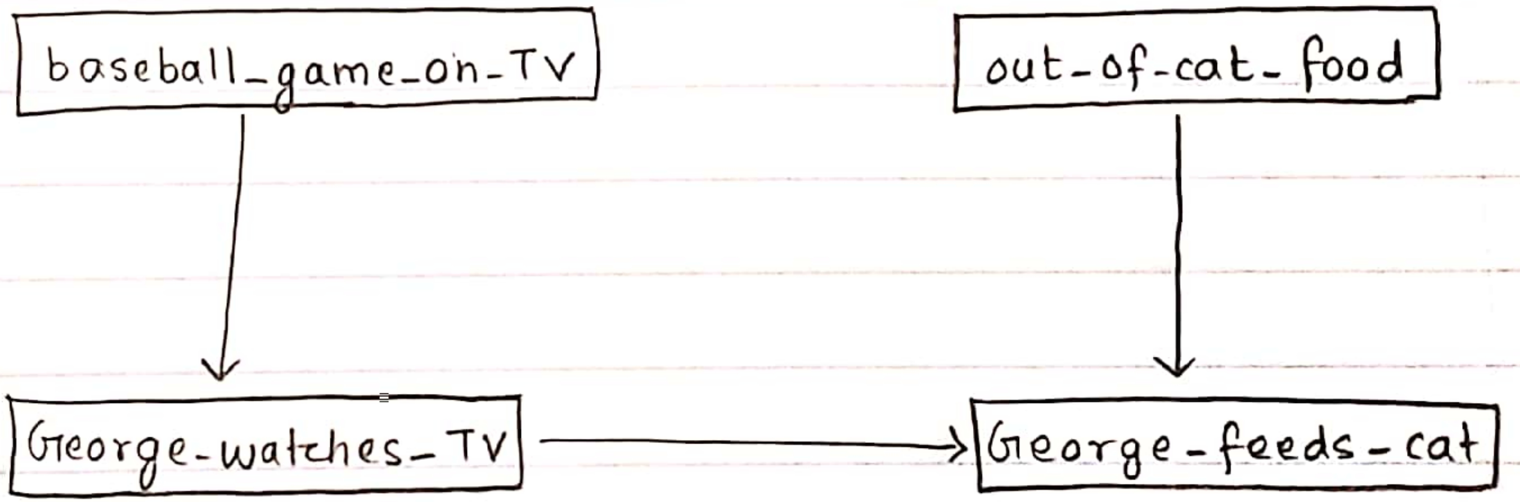
For A, we can store one value less = 8-1 = 7

∴ Total number of values that can be saved after optimization = 7+320

= 327 //

Task-5

Designing a Bayesian Network for modeling the relations between four events:



Task-6

B	P(G|B)
0	0.1181
1	0.9279

G	F	P(C|G, F)
0	0	0.9588
0	1	0.3158
1	0	0.7064
1	1	0.0417

Task-7

P(Baseball Game on TV | not George Feeds CAT)

$= P(B | \text{not } C)$

$= \dfrac{P(B, \text{not } C)}{P(\text{not } C)}$

For numerator:

$P(B, \text{not } C) = P(\text{not } C | G, F) \times P(G | B) \times P(F) \times P(B) + P(\text{not } C | \text{not } G, F) \times P(\text{not } G | B) \times P(F) \times P(B) + P(\text{not } C | G, \text{not } F) \times P(G | B) \times P(\text{not } F) \times P(B) + P(\text{not } C | \text{not } G, \text{not } F) \times P(\text{not } G | B) \times P(\text{not } F) \times P(B)$

[Values from Excel sheet] $= 0.9583 \times 0.9279 \times 0.1699 \times 0.3041 + 0.6842 \times 0.07207 \times 0.1698 \times 0.3041 + 0.2936 \times 0.9279 \times (1 - 0.1699) \times 0.3041 + 0.0412 \times 0.07207 \times (1 - 0.1699) \times 0.3041$

$= 0.11801$

For denominator:

$P(\text{not } C) = 1 - P(C)$

$= 1 - 0.7562 = 0.2438$

Thus, $P(B | \text{not } C) = \dfrac{0.11801}{0.2438}$

$= 0.484$

Task-8

Part-a :

Markov blanket of Node 'N':

a) Parent of the node : I

b) Children of the node : R, S

c) Other parents of the children : M, O

Part-b :

$$P(I,D) = p(I) * p(D)$$
$$= p(I|D) * p(D)$$
$$= 0.5 * 0.5$$

$\therefore$ $P(I,D) = 0.25$ //

Part-c :

Numerator:

$$p(M, not(C) | H) = p(M) * p(not\ C) * p(H)$$
$$= p(M|H) * p(not\ C) * p(H | not(C))$$
$$= 0.1 * (1-0.6) * 0.1$$

i.e $P(M, not(C)|H) = 0.004$

Denominator:

$$p(H) = p(H|C) + p(H + not C) = 0.6 + 0.1 = 0.7$$

i.e. $P(H) = 0.7$

Therefore, $p(M, not(C) | H) = \frac{0.004}{0.7} = 0.005714$ //