# AMATH 482/582: HOME WORK 1


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Abstract. In this project, we are given the noisy acoustic data in 3D of a submarine over 24 hours. We will use Fourier transform and design a filter to discriminate the spatial frequency the submarine emitted and denoise the data. We will use the denoised data to locate the path of the submarine during the 24 hour period. This document will present a brief definition of the terminologies we are using, the process of locating the submarine, and the path of the submarine in every half-hour interval.


## 1. Introduction and Overview

The submarine we are discussing uses a new technology that emits an unknown acoustic frequency. During the 24 hours recording period of the submarine's noisy acoustic data, the submarine is moving in a 3D coordinate with a uniform grid of size $64 \times 64 \times 64$. The acoustics data is recorded every half-hour in the 24 hours period, so we are getting a 4D $64 \times 64 \times 64 \times 49$ data. By Fourier transform, we can transform the signal that the submarine emitted recorded in time to spatical frequency. After designing a proper filter we can denoise the frequency after applying the Fourier transform and locate the position of the submarine in the period. The location at a different time and the path of its movement will be present as a graph in both 2D and 3D.

## 2. Theoretical Background

We use Fourier Transform by implementing the FFT algorithm to center the spatial frequency. The Fourier Transform is the limit of a Fourier series [1].

$$\hat{f(x)} = \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx$$

The Fast Fourier Transform (FFT) is an algorithm that computr Fourier Transform for a function $f(x)$ given a set of equidistance points [4]: let $f_n := f(x_n)$ for $n = 0, ..., N-1$, the FFT returen coefficients

$$\hat{c_k} = \sum_{n=0}^{N-1} f_n e^{-2\pi i \frac{xk}{N}}$$

By the FFT we can transform the acoustic signal recorded in time domain to spatial frequency. We will use FFT to each time the signal the submarine emitted to get the Fourier Transform to get the center frequency.

After we getting the center frquency, we will use a 3D Gaussian filter to denoised the data.

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By applying the filter to the center frequency, we can get the denoised the noise away from the center frequency to locate the position of the submarine at each time.

## 3. Algorithm Implementation and Development

We will use

- `numpy` [2] to implement the FFT algorithm `fftn` and `fftshift`
- `matplotlib` [5] to plot figures

The Gaussian filter [3] we will be using will be:

**Algorithm 1:** 3D Gaussian filter

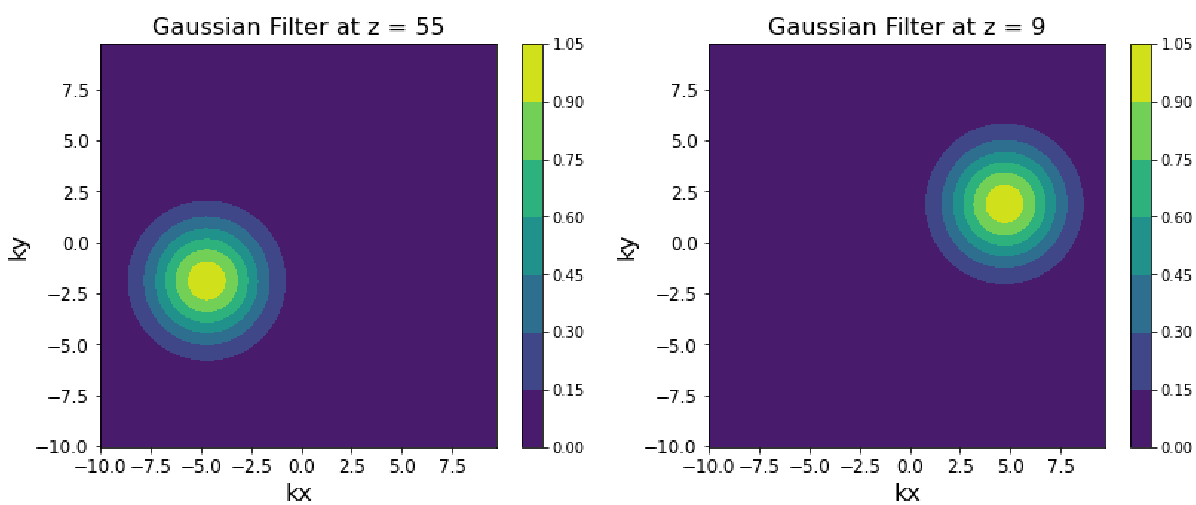
$\exp(-(((x - \texttt{shift of center frequency})^2 + (y - \texttt{shift of center frequency})^2 + (z - \texttt{shift of center frequency})^2)/(2\texttt{sigma}^2)))$

## 4. Computational Results

First we will use `fftn` and `fftshift` to transform the signal into spatial frequency. By averaging the frequency, we can find the indices of the center frequency which is

```
(17, 26, 55)
(47, 38, 9)
```

in 3D. The reason why we are gettiing 2 center frequency is that by the Fourier Transform, the frequency will be central symmetric at the index $(32, 32, 32)$. Since we are getting two center frequencies, we will define two Gaussian filter with different value of shifting corresponding to each point. The two Gaussian filter at different z slice will be:




(A) (B)

Figure 1. Two 2D Caussian Filters at slice z = 55 and at slice z = 9

Here is how the two Gaussian filter adding together looks like:

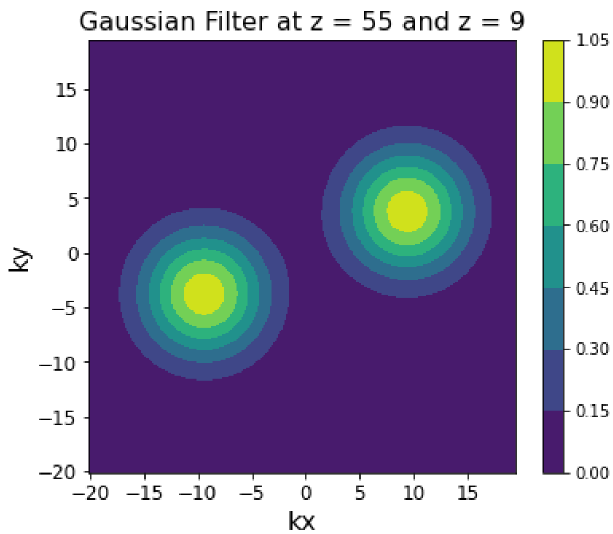
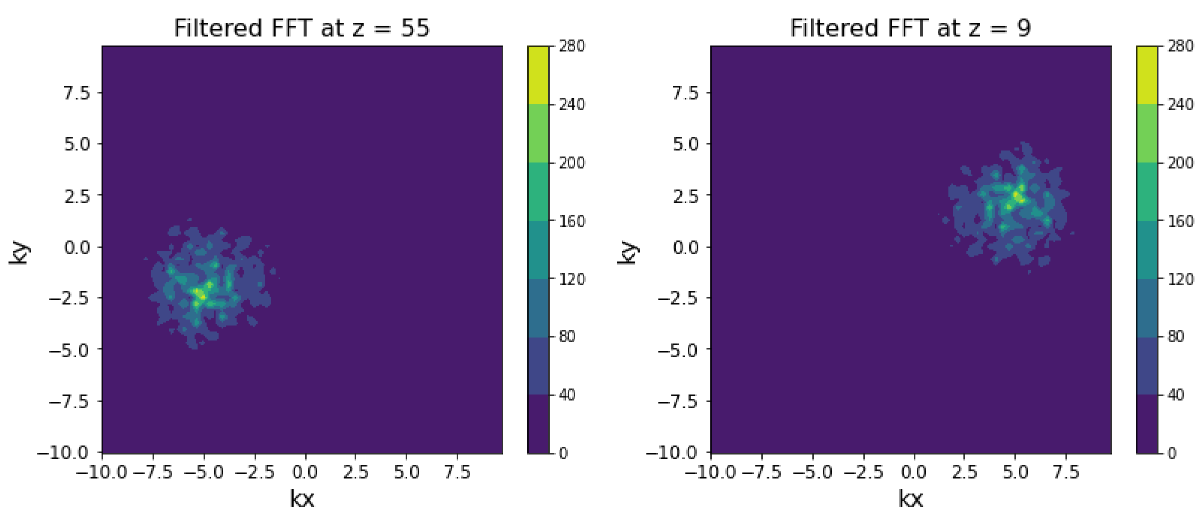



FIGURE 2. 2D Gaussian filter when adding the slices of z = 55 and z = 9

Now we can apply the Gaussian Filter to the frequency. For each time point, we will apply the FFT to the original signal and denoised with the Gaussian Filter. Here is the filtered frequency looks like at slice z = 55 and z = 9:




(A) (B)

FIGURE 3. Amplitude of frequency at slice z = 55 and at slice z = 9

Combining them together we will get a similar pattern as the Gaussian filter2:

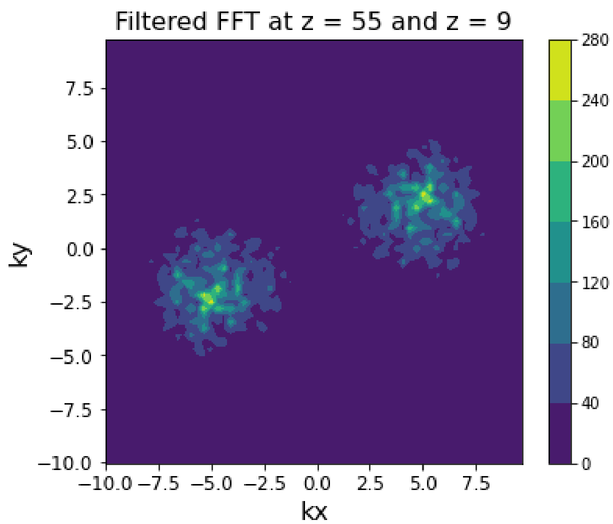



FIGURE 4. Filtered frequency when adding the slices of z = 55 and z = 9

Then use FFT to transform the spatial frequency back to the frequency in each time point. Record the index of maximum absolute value of the frequency in each time point and transform the index back to the location coordinates in 3D. Plot the location to form a path of the submarine:

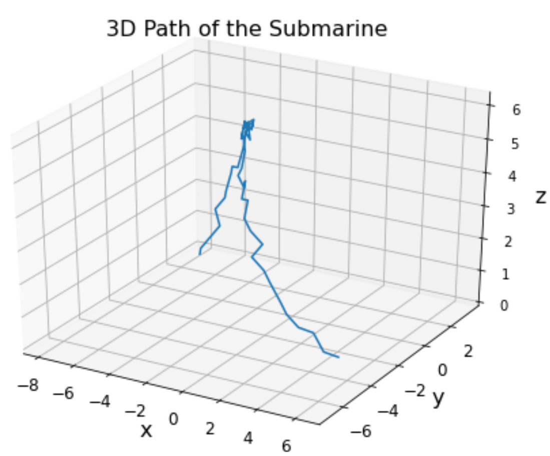



FIGURE 5. The path of the submarine in 3D

By looking at the $x, y$ coordinates we can see that the submarine start from `(-7.8125, 3.125)` to `(6.25, -5.0)` in the 24 hours period. Here is the visualization of the path in $x, y$ coordinates:

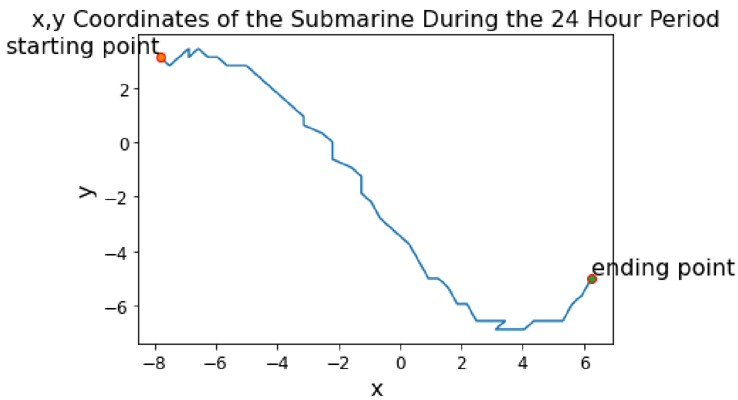



FIGURE 6. The 2D path of the submarine in x, y coordinates

In 3D coordinates, the submarine starts at `(-7.8125, 3.125, 0.0)` and ends at `(6.25, -5.0, 0.9375)` in the 24 hours period.

## 5. Summary and Conclusions

By averaging the Fourier Transform, we designed a Gaussian filter to filter the data. Applied the Gaussian filter to the acoustic frequency, we can denoise the frequency of the submarine and track its location during a given period. With the location of the submarine, we can get the path of the submarine in the 24 hours period. The path could help us track the submarine in the future.

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## References

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