Main Help

Bifurcation Calculations with AUTO

Running AUTO

Preparation

Before you can use AUTO, you must prepare your system for it. You must start your bifurcation analysis from either a fixed point of your model, a periodic orbit, or a boundary value. AUTO seems to work best when you start from a steady state, but I have had success starting at periodic orbits.

- If you want to start at a steady state, find one and integrate so that the system is at rest. Use the **Init.conds. Last** repeatedly.
- If you want to start at a periodic orbit, find one by integration and make sure you have converged to it. The set the total integration time is the ``period'' of your orbit. This is what the AUTO interface uses as an approximate starting period.
- If you want to use AUTO to solve a boundary value problem, you must do several things.
    1. First rescale everything so that the total length of the interval is 1.
    2. If the system is non-autonomous, define a new variable to make it autonomous.
    3. The number of ODEs must be the same as the number of boundary conditions.
    4. Compute a solution using XPP's boundary value solver or guessing a solution. (AUTO is pretty forgiving -- moreso than the XPP BVP solver.)
- Setting up for homoclinics is similar to BVPs. Make sure that you include the **hom_bcs** boundary conditions.

Choosing parameters

The first thing you should do is tell AUTO which parameters you might use in the bifurcation analysis. Up to 5 are allowed. Click on ``Parameter'' and a list of 5 parameters will appear. Type in the names of the parameters you want to use. The default is the first 5 or fewer parameters in your file. If you have fewer than 5 parameters, only the available ones will appear. *You can have as many parameters in your ODE file as you want. However, the AUTO interface allows only 5* ***hot*** *parameters.*

Diagram axes

Use this to set up the plotting of the diagram as well as to tell AUTO which parameters should be varied. This is also where you set up for two-parameter continuation. There are 8 choices

- **(H)i** This plots the maximum of the chosen variable.
- **(N)orm** This plots the *L_2* norm of the solution.
- **h(I)-lo** This plots both the max and min of the chosen variable (convenient for periodic orbits.)
- **(P)eriod** Plot the period versus a parameter
- **(T)wo par** Plot the second parameter versus the primary parameter for two-parameter continuations.
- **(Z)oom** Use the mouse to zoom in on a region.
- **last (1) par** Use the plot parameters from the last 1-parameter plot.
- **last (2) par** Use plot parameters from last 2-parameter plot.
- **(F)requency** Plot frequency versus a parameter
- **(A)verage** Plot the average over a period of a quantity

After clicking, a new window pops up with the following items:

- **Y-axis** This is the variable for the y-axis of the plot. For two-parameter and period plots, its contents is ignored.
- **Main Parm** This is the principal bifurcation parameter. It must be one of those you specified in the parameter window. The default is the first parameter in the parameter list.
- **2nd Parm** This is the other parameter for two-parameter continuations.
- **Xmin ... Ymax** The plotting dimensions of the diagram.

Once you press (OK) the axes will be redrawn and labeled.

Numerical parameters

You will often have to change these. In particular, you set the range of the parameters as well as the direction of bifurcation in this dialog.

- **Ntst** This is the number of mesh intervals for discretization of periodic orbits. If you are getting bad results or not converging, it helps to increase this. For following period doubling bifurcations, this is automatically doubled so you should reset it later.
- **Nmax** The maximum number of steps taken along any branch. If you max out, make this bigger.
- **Npr** Give complete info every Npr steps. Set this to a big number if you want to speed things along.
- **Ds** This is the initial step size for the bifurcation calculation. *The sign of Ds tells AUTO the direction to change the parameter.* Since stepsize is adaptive, (Ds) is just a ``suggestion."
- **Dsmin** The minimum stepsize (positive).
- **Dsmax** The maximum step size. If this is too big, AUTO will sometimes miss important points so if it seems to miss a stability transition, or if the diagram is jagged, decrease this.
- **Par Min** This is the left-hand limit of the diagram for the principle parameter. The calculation will stop if the parameter is less than this.
- **Par Max** This is the right-hand limit of the diagram for the principle parameter. The calculation will stop if the parameter is greater than this.
- **Norm Min** The lower bound for the L_2 norm of the solution. If it is less than this the calculation will stop.
- **Norm Max** The upper bound for the L_2 norm of the solution. If it is greater than this the calculation will stop.
- **Ncol** number of collocation points used. I never have changed this in 10 years of running AUTO.
- **EPSU,EPSS,EPSL** some sort of tolerances for AUTO. Keep 'em positive but make 'em smaller for better accuracy. 1e-7 seems to be a popular choice!

User functions

Suppose you want to get plots at specific values of parameters or at fixed periods of a limit cycle. Then you can click on ``User'' which produces a menu 0-9 asking you how many points you want to keep. Click on 0 for none or another number less than 10. A new window will appear with slots for 9 items. You can type in anything of the form:

```
parameter=value
```

or

```
T=value
```

AUTO will mark and save complete information for any point that satisfies either of these criteria. The second is used to indicate that you want to keep a point with a particular period, e.g., T=25 will save the any periodic orbit with period 25.

Running

Click on ``Run'' to run the bifurcation. Depending on the situation, a number of menus can come up. For initial exploration, there are four choices:

- starting at a new steady state
- starting at a periodic orbit
- starting at a boundary-value solution,
- starting at a homoclinic/heteroclinic orbit.

You must be sure that you are starting from a good initial solution or the continuation will fail. If successful, a nice diagram will show up and a bunch of points will move around in the stability circle. These indicate stability: for fixed points, they represent exponentials of the eigenvalues; for periodics, the Floquet multipliers. Thus those in the circle are stable and those out of the circle are unstable. Bifurcations occur on the circle. The outer eigenvalues are ``clipped'' so that they will always lie in the square, thus you can keep count of them.

Special Points

The diagram,itself, has two different lines and two different circles. Stable fixed points are thick lines, stable periodics are solid circles, unstable fixed points are thin lines, and unstable periodics are open circles. Additionally, there are crosses occasionally dispersed with numbers associated with them. These represent ``special'' points that AUTO wants to keep. There are several of them:

- **EP** Endpoint of a branch
- **LP** Limit point or turning point of a branch
- **TR** Torus bifurcation from a periodic
- **PD** Period doubling bifurcation
- **UZ** User defined function
- **MX** Failure to converge
- **BP** Bifurcation or branch point
- **HB** Hopf bifurcation point
- Output every N^{th} point.

Grabbing

You can use these special points to continue calculations with AUTO. The ``Grab'' item lets you peruse the diagram at a leisurely pace and to grab special points or regular points for importing into XPP or continuing a

bifurcation calculation. Click on ``Grab" and info appears in the info window and a cross appears on the diagram. Use the left and right arrow keys to cruise through the diagram. The right key goes forward and the left backward. At the bottom, information about the branch, the point number, the type of point, the AUTO label, the parameters, and the period are given. The points marked by crosses have lables and types associated with them. The type is one of the above. The label corresponds to the number on the diagram. If point is positive, it is an unstable solution and if it is negative it is stable. As you traverse the diagram, stability is shown in the circle.

You can traverse the diagram very quickly by tapping the (Tab) key which takes you the special points only. Type (Esc) to exit with no action or type (Enter) to grab the point. If it is a regular point (i.e., not special) then the parameters and the variables will be set to the values for that point within XPP. You can then integrate the equations or look at nullclines, etc. If you grab a special point, then you can use this as a restart point for more AUTO calculations, such as fixed period, two-parameter studies, and continuations. Then, you can run AUTO again. Bifurcation diagrams are cumulative unless you reset them in the ``File" menu. That is, new stuff is continually appended to the old. The only limit is machine memory.

If you grab a special point and click on ``Run" several possibilities arise depending on the point:

- **Regular Point** Reset the diagram and begin anew. You will be asked first if you want to reset the diagram first.
- **Hopf Point**
  - **Periodic** Compute the branch of periodics emanating from the Hopf point
  - **Extend** Continue the branch of steady states through this point.
  - **New Point** Restart whole calculation using this as a starting point
  - **Two Param** Compute a two parameter diagram of Hopf points.
- **Period doubling**
  - **Doubling** Compute the branch of period 2 solutions.
  - **Two-param** Compute two-parameter curve of period doubling points.
- **Limit point** Compute two parameter family of limit points (fixed points or periodic.)
- **Periodic point** The point is periodic so
  - **Extend** Extend the branch
  - **Fixed Period** Two parameter branch of fixed period points.
- **Branch point of periodic** Branch points of steady states are automatically followed by AUTO. If you click on a branch point for a periodic, the program traces out the new branch. Changing the sign of **Ds** will trace a different branch.
- **Torus point** Compute two-parameter family of torus bifurcations or extend the branch or compute two-parameter fixed period.

Before running, after a point is grabbed, be sure to set up the correct axes and ranges for the parameters.

Aborting

Any calculation can be gracefully stopped by clicking on the ``Abort" key. This produces a new end point from which you can continue. Note that if there are many branches, you may have to press "Abort" several times.

Clear clears the diagram and **reDraw** redraws it.

Saving diagrams

File allows you to do several things:

- **Import orbit** If the grabbed point is a special one and is a periodic orbit or BVP solution, this loads the orbit into XPP for plotting. This is useful for unstable orbits that can't be computed by integrating initial data.

- **Save diagram** Writes a file for the complete diagram which you can use later.
- **Load Diagram** Loads a previously saved one.
- **Postscript** This makes a hard copy of the bifurcation diagram
- **Reset diagram** This clears the whole thing.
- **Clear Grab** This clears the grab point.
- **Write Points** This makes an ascii file of the **(x,y)** coordinates of the diagram. Use this to make nicer figures with by importing the diagram into XPP
- **All info** writes all the fixed points, eigenvalues, etc for each point on the diagram.