Solve for x, y:

$$\begin{cases} \text{①}\ 3x - 2y = 5 \\ \text{②}\ x + 4y = 4 \end{cases}$$

① substitution
② elimination
③ matrices
④ cramer's rule

substitution ← isolate for a variable

$3x - 2y = 5$

$x + 4y = 4 \Rightarrow x = 4 - 4y$

① $3(\underbrace{4-4y}_{x}) - 2y = 5$

$12 - 12y - 2y = 5$

$12 - 14y = 5 \Rightarrow -14y = -7$

$y = \frac{-7}{-14} = \frac{1}{2}$

$x = 4 - 4\left(\frac{1}{2}\right)$

$= 4 - 2 = 2$

$x = 2$

$y = 1/2$

elimination

$3x - 2y = 5$

$x + 4y = 4$

$$\begin{array}{rcl} 6x - 4y &=& 2(5) \\ x + 4y &=& 4 \end{array}$$

$$7x + 0y = 2(5) + 4$$

$$7x = 14$$

$$x = 2$$

$$x + 4y = 4$$

$$2 + 4y = 4$$

$$4y = 2$$

$$y = \frac{1}{2}$$

$(2, \frac{1}{2})$

$x + 2y = 20$

$3x + 4y = 24$

sub

$x = 20 - 2y$

$3(20-2y) + 4y = 24$

$60 - 6y + 4y = 24$

$-2y = 24 - 60$

$-2y = -36$

$y = \frac{-36}{-2} = 18$

$x = 20 - 2(18)$

$= 20 - 36$

$x = -16$

elim

$-2x - 4y = -4$

$3x + 4y = 24$

$x = -16$

$-16 + 2y = 20$

$2y = 36$

$y = 18$

$= -16$

$3x - 2y = 5$

$x + 4y = 4$

variable side

side

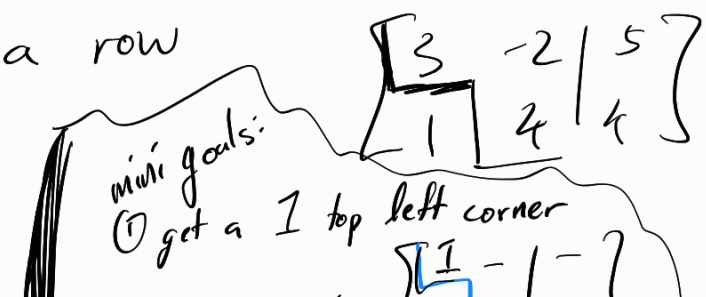
$$\left[\begin{array}{cc|c} 3 & -2 & 5 \\ 1 & 4 & 4 \end{array}\right]$$

row reduce (rubiks cube)

Goal: $\left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \end{array}\right]$ ← answers!!

3 Rules

1. switching rows is ok
2. you can add/subtract a row w/ another row
3. you can multiply/divide a row w/ a #



② everything below [illegible] must be 0.

Rule 1

$$\left[\begin{array}{cc|c} 3 & -2 & 5 \\ 1 & 4 & 4 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 4 & 4 \\ 3 & -2 & 5 \end{array}\right]$$

Rule 2

$$\left[\begin{array}{cc|c} 3 & -2 & 5 \\ 1 & 4 & 4 \end{array}\right] \xrightarrow{R_2 \to R_2 + R_1} \left[\begin{array}{cc|c} 3 & -2 & 5 \\ 1+3 & 4+-2 & 5+4 \end{array}\right]$$

Not ok

$$\left[\begin{array}{cc|c} 3 & -2 & 5 \\ 1 & 4 & 4 \end{array}\right] \xrightarrow{R_1 \to R_1 + 4} \left[\begin{array}{cc|c} 7 & 2 & 9 \\ 1 & 4 & 4 \end{array}\right]$$

$-2 - 3(4)$

$R_1 \to R_1 - 3R_2$

$$\left[\begin{array}{cc|c} 0 & -14 & 5-3(4) \\ 1 & 4 & 4 \end{array}\right]$$

$$\hookrightarrow \left[\begin{array}{cc|c} 0 & -14 & -7 \\ 1 & 4 & 4 \end{array}\right]$$

Rule 3

$k_1 \to \frac{R_1}{7}$

$$\left[\begin{array}{cc|c} 0 & -2 & -1 \\ 1 & 4 & 4 \end{array}\right]$$

$$1x + 2y = 20$$
$$3x + 4y = 24$$

using matrix

$$\left[\begin{array}{cc|c} 1 & 2 & 20 \\ 3 & 4 & 24 \end{array}\right]$$

make the 3 go away!

$R_2 \to R_2 - R_1$

$$\left[\begin{array}{cc|c} 1 & 2 & 20 \\ 0 & -2 & 24-60 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 2 & 20 \\ & & \end{array}\right]$$

$$\begin{bmatrix} 0 & -2 & -36 \end{bmatrix}$$

$R_2 / -2$

$$\begin{bmatrix} 1 & 2 & 20 \\ 0 & 1 & 18 \end{bmatrix}$$

$R_1 \to R_1 - 2R_2$

$20 - 2(18)$

$$\begin{bmatrix} 1 & 0 & 20 - 36 \\ 0 & 1 & -16 \\ & & 18 \end{bmatrix}$$