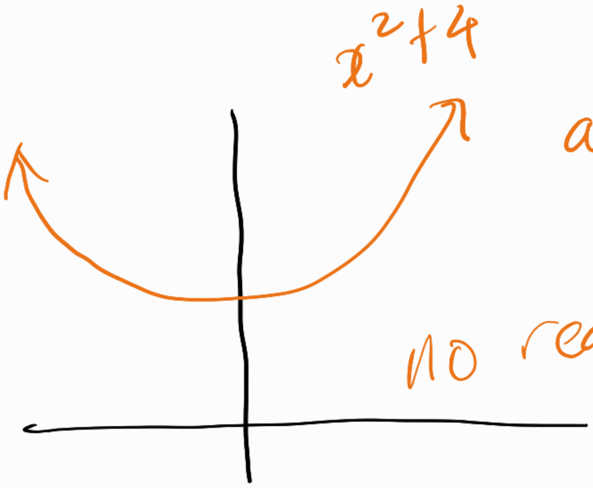
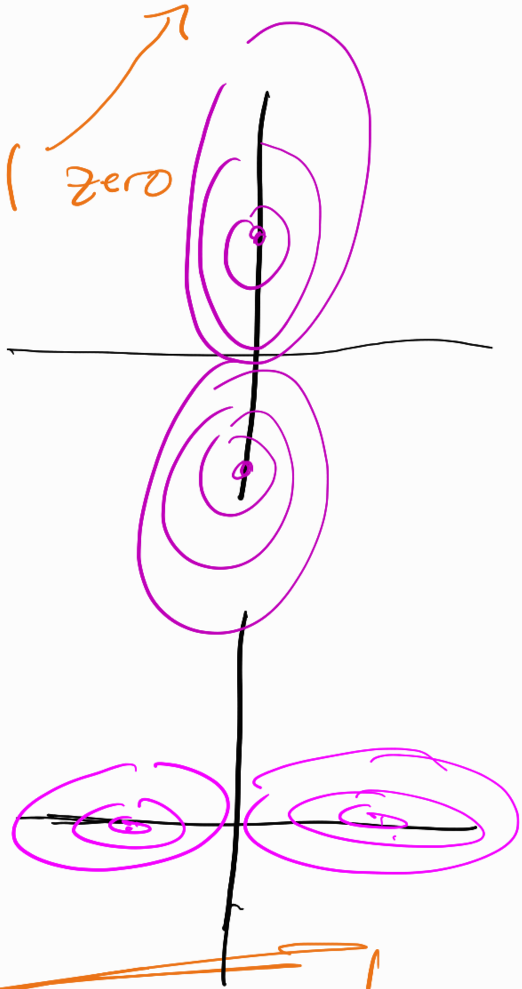
11/28/2023: $\boxed{\text{Fundamental Theorem of Algebra (FTA)}}$

$\boxed{\text{FTA}}$ a polynomial of degree n will have exactly n complex zeros

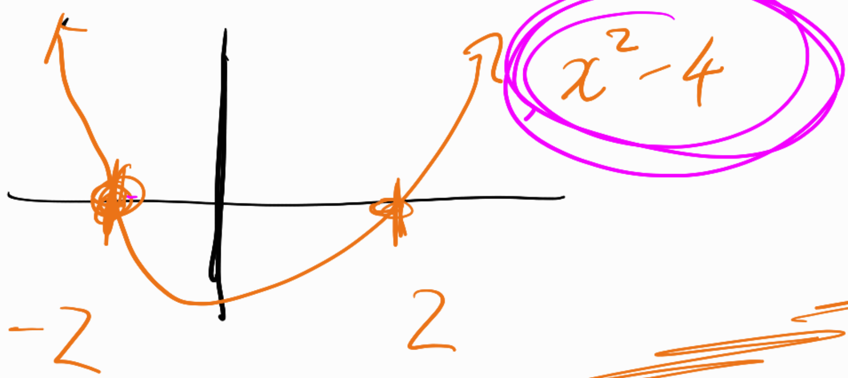
as opposed to real zero





no real zeros

no x-ints!



$\boxed{x = 2, -2 \text{ are zeros}}$

Consequences

- $x^{2023} = 0 \Rightarrow$ how many times will $x = 0$?

  2023 times!!

- $x^{2} = 1 \Rightarrow$ 2 answers deg $= 2$

- $x^{2023} = 3x \Rightarrow \underbrace{x^{2023} - 3x}_{} = 0$

  poly w/ deg 2023

  $\Rightarrow$ 2023 answers (possibly imaginary)

- $f(x), g(x)$ is a polynomial of deg up to 100

- $f(1) = 3$

  $f(2) = 3$

  $f(3) = 3$

$g(1) = 3$

$g(2) = 3$

$g(3) = 3$

$\vdots$

⋮

$f(100) = 3$

$g(100) = 3$

$g(101) = 3$

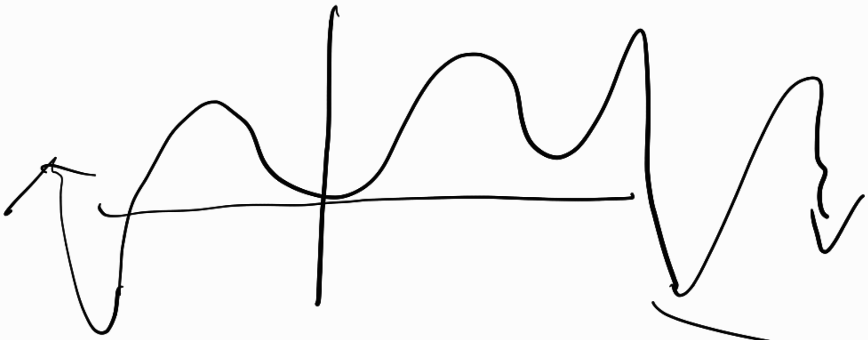
$\boxed{g(x) = 3}$

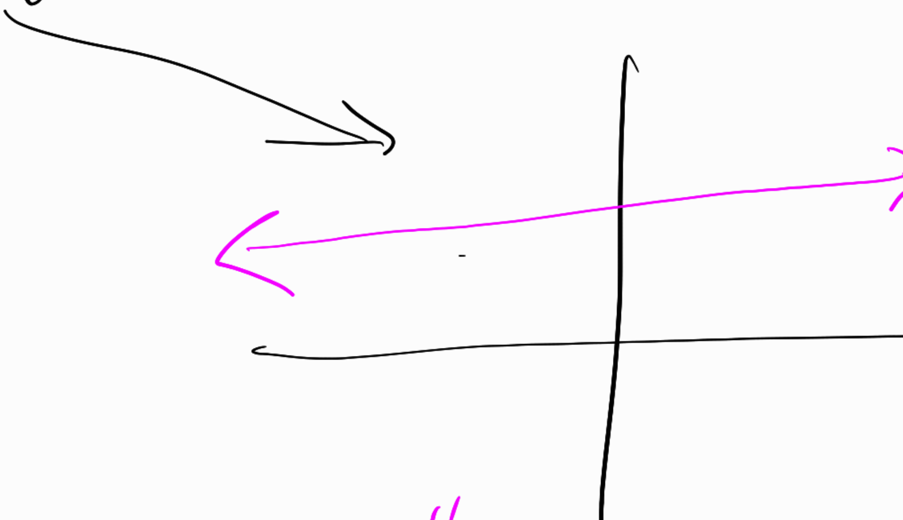
Do you know $f(x) = ?$

no

deg 20

21 pts specified as <u>same</u>





<u>n = degree</u>

<u>FTA</u>: ① guarantees exactly n answers

② polynomial $f(x) = 5$

at <u>most</u> n times

③ # of bumps is $\leq n-1$

Unfortunately FTA does not tell you what the answers are tho !!

Synthetic division

$$p(x) = 6 + 11x^2 - 8x^4 + 5x$$

quotient

remain

Q: Compute

$$\frac{14}{3} = 4 + \frac{2}{3}$$

$$\frac{p(x)}{x+2} = \text{quotient} + \frac{\text{rem}}{x+2}$$

rearrange

exp hi → low

$$p(x) = -8x^4 + 0x^3 + 11x^2 + 5x + 6$$

	x^4	x^3	x^2	x^1	x^0	
-2	-8	0	11	5	6	add

0	16	-32	42	-94
-8	16	-21	47	$\boxed{-88}$

rem

Quotient: $-8x^3 + 16x^2 + -21x + 47$

Rem = -88

$$\frac{3 + 2x + 4x^2 - 5x^4}{x - 2}$$

	x^4	x^3	x^2	x	const
2	-5	0	4	2	3
	↓	-10	-20	-32	-60
	-5	-10	-16	-30	-57

rem = -57

Quotient: $-5x^3 - 10x^2 - 16x - 30$

Q1 Find remainder of

$$\frac{x^4 - 2x^3 + 3x - 1}{x+2}$$

Q2 Find a if $x^3 - 2x + a$

divided by $x-2$ has rem $= 7$