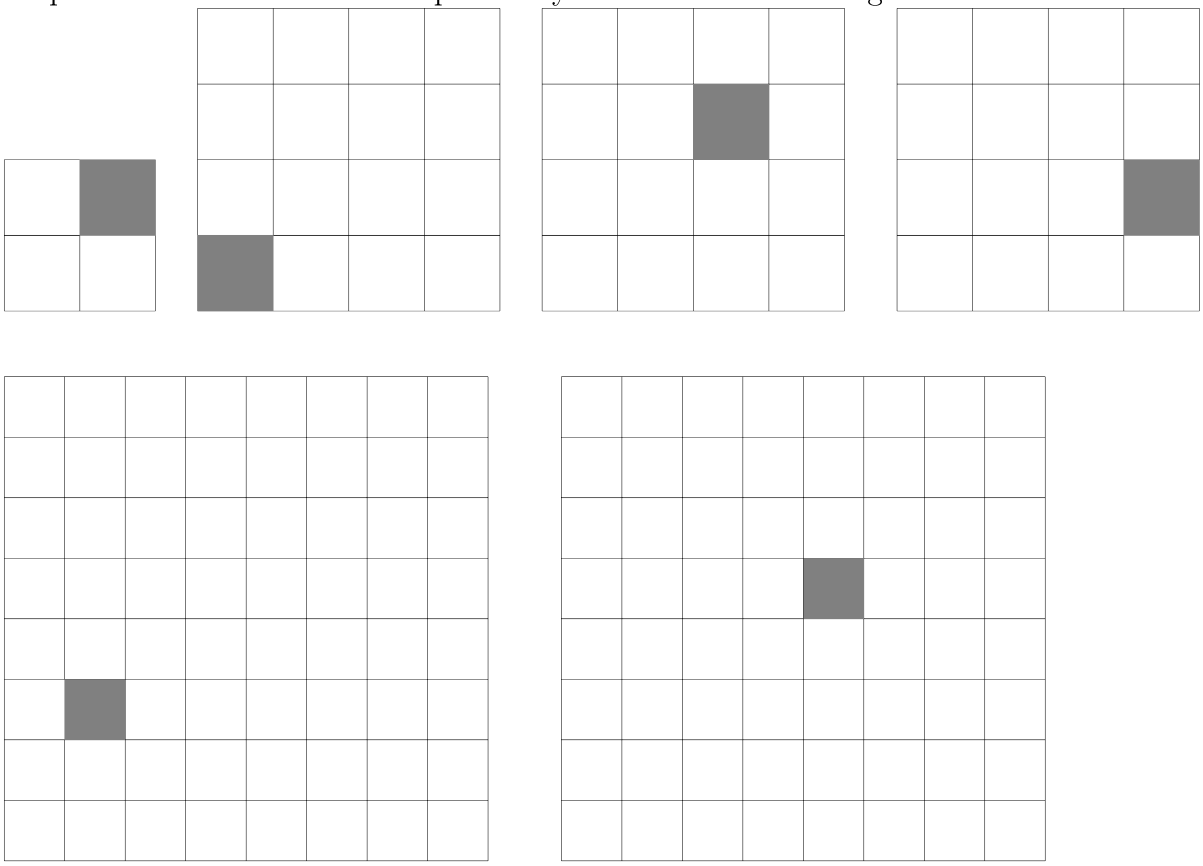
Elementary Number Theory
2024-25

Principle of Mathematical Induction

1 Game of Moose

A tromino is a shape created from 3 blocks. To win a game of Moose, played on a checkerboard, you must tile the checkerboard (except for 1 designated square) with L-shaped trominos with no overlaps. Can you Moose the following boards?



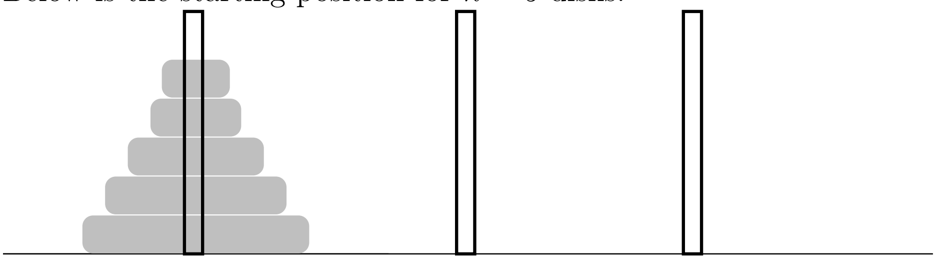
Prove that you can always Moose any $2^n \times 2^n$ board.

2 Tower of Hanoi

There are 3 sticks and n disks, each of different size. Initially, the disks are sorted and stacked on top of each other with the largest disk on the bottom leading up to the smallest at the top. Your goal is to move the disks from one stick to another stick following the rules:

1. You can only move one disk at a time
2. You can only select the topmost disk of any of the sticks
3. You may not place a larger disk on top of a smaller disk.

Below is the starting position for $n = 5$ disks.



1. Record the minimum number of moves it takes for you to win for small values of n:

n disks	# of moves to win
1	
2	
3	
4	
5	
6	

2. Prove that no matter how many disks you start with, the game can always be won (prove that there is a formula for the minimal number of moves!)
3. According to one legend, monks in a sacred temple move a tower of 64 disks, following the rules of the Tower of Hanoi game. The disks, which are delicate, require 1 second to move each disk from one peg to another. When the monks finish moving all 64 disks, the universe will come to an end. What is the minimal amount of time that it will take for the universe to end?