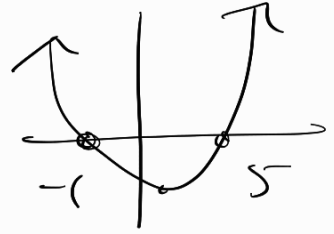
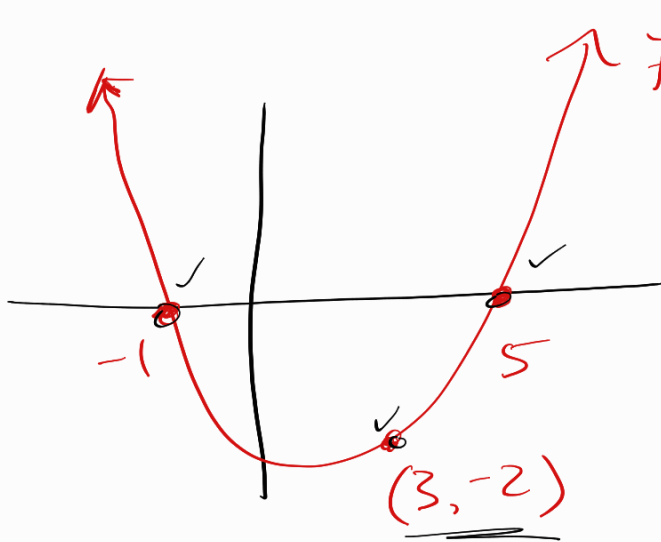


10/19/2023

$$f(x) = 3(x-5)(x+1) \longrightarrow$$



Now...



$$f(x) = \underline{a}x^2 + \underline{b}x + \underline{c}$$

what's a, b, c ?

x-int

$$x = \underline{-1}, \underline{5}$$

$$\Rightarrow f(x) = A(x - (-1))(x - 5)$$

\uparrow

some #

$$\Rightarrow f(x) = A(x+1)(x-5)$$

$$\bullet f(3) = -2$$

must be

$$f(3) = A(3+1)(3-5) \underline{\underline{= -2}}$$

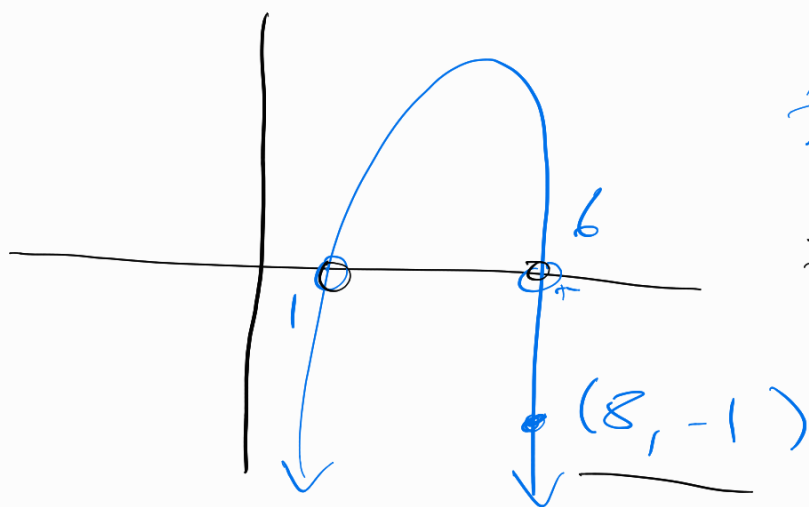
$$\begin{array}{c} | \\ = A(4)(-2) \end{array}$$

$$\Rightarrow \frac{-8A}{-8} \underline{\underline{= \frac{-2}{-8}}}$$

$$\Rightarrow A = \frac{1}{4}$$

$$f(x) = \frac{1}{4}(x+1)(x-5)$$

$$\begin{aligned}
 &= \frac{1}{4} (x+1)(x-5) \\
 &= \frac{1}{4} [x^2 - 4x - 5] \\
 &= \boxed{\frac{1}{4}x^2 - x - \frac{5}{4}}
 \end{aligned}$$



$$f(x) = ax^2 + bx + c$$

$$f(x) = a(x-1)(x-6)$$

\uparrow \nearrow
 flip sign!

$$\rightarrow f(8) = -1$$

$$\rightarrow f(8) = a(8-1)(8-6)$$

$$= a(7)(2)$$

$$= 14a$$

$$-1 = 14a \Rightarrow a = -\frac{1}{14}$$

$$f(x) = -\frac{1}{14} (x-1)(x-6)$$

$$= \frac{-1}{14} (x^2 - 7x + 6)$$

$$= -\frac{1}{14}x^2 + \frac{1}{2}x - \frac{3}{7}$$

Vertex form

- ① $ax^2 + bx + c \leftarrow$ "standard form"
- ② $(x-a)(x-b) \leftarrow$ factored form
- ③ $A(x-h)^2 + k \leftarrow$ "vertex form"

not every quadratic tho!!

constants

Example.

$$f(x) = 3(x - \underline{2})^2 - 1$$

vertex: $(2, -1)$

$$g(x) = -7(x + 5)^2 - 3$$

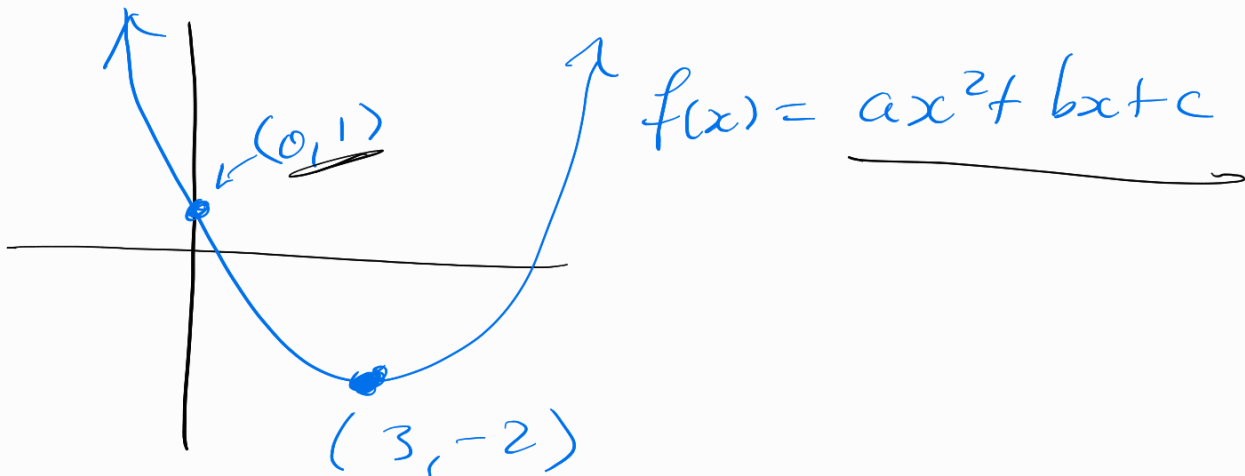
vertex: $(-5, -3)$

$$h(x) = 20(x - 10)^2 + 23$$

vertex: $(10, 23)$

$$Z(x) = \pi(x+7)^2 - 2023$$

vertex: $(-7, -2023)$



$$f(x) = A(x-3)^2 - 2$$

$$\begin{array}{c} \uparrow \\ f(0) = 1 = \underbrace{A(0-3)^2 - 2}_{A(-3)^2 - 2} \end{array}$$

\uparrow
graph

$$\Rightarrow 1 = 9A - 2$$

$$\Rightarrow 9A = 3$$

$$\Rightarrow A = \frac{1}{3}$$

$$f(x) = \frac{1}{3}(x-3)^2 - 2$$

$$= \frac{1}{3} \left[\underbrace{x^2 - 6x + 9}_{(x-3)^2} - 2 \right]$$

$$= \frac{1}{3} [x^2 - 6x + 7]$$

$$= \frac{1}{3}x^2 - 2x + \frac{7}{3}$$

How to get vertex form??

complete the \square

$$f(x) = x^2 + 10x + 21 \rightarrow a(x-h)^2 + k$$

$(x+3)(x+7) \leftarrow$ factored form

$\begin{matrix} \leftarrow & \leftarrow & \leftarrow \\ a & h & k \\ & \text{new #'s} & \end{matrix}$

$$\begin{aligned} (x+h)^2 &= (x+h)(x+h) \\ &= x^2 + 2hx + h^2 \end{aligned}$$

$$2hx = 10x$$

$$\hookrightarrow 2h = 10 \Rightarrow \underline{h = 5}$$

$$\underline{x^2 + 10x + 21} = \underbrace{(x + 5)^2}_{h} + k$$

$$\underline{(x^2 + 10x + 25)} + k$$

$$21 = 25 + k$$

$$k = -4$$

$$(x + 5)^2 - 4$$

vertex: $(-h, k)$

$$f(x) = x^2 - 16x + \underline{10} \rightarrow (x + h)^2 + k$$

$$(1) \text{ find } h = \frac{b}{2a} \quad \left| \quad h = -8 \right.$$

$$(2) \text{ compute } h^2 \quad \left| \quad h^2 = 64 \right.$$

$$(3) \quad c = h^2 + k \quad \begin{cases} (x - 8)^2 + k \\ x^2 - 16x + \underline{64} + k \end{cases}$$

$$10 = 64 + k$$

$$(x-8)^2 - 54$$

$$h = -54$$

vertex: $(8, -54)$

Find vertex of

Hint: $11^2 = 121$

$x^2 + 22x + 1$ by completing the square

① find $h = \frac{b}{2a}$ $h = 11$

② compute h^2 $h^2 = 121$

③ $c = h^2 + k$ solve for k $1 = 121 + k$
 $k = -120$

$$(x+11)^2 - 120$$

vertex: $(-11, -120)$

Quadratic equations

Solve for x in

$$x^2 + 10x + 21 = 0$$

2 methods: (1) completing the \square

$$h=5 \quad (x+5)^2 - 4 = 0$$

$$h^2 = 25$$

$$21 = 25 + k$$

$$\Rightarrow k = -4$$

Now reverse PEMDAS!

$$(x+5)^2 - 4 = 0 \implies (x+5)^2 = 4$$

$$\begin{array}{l} \sqrt{\text{both sides}} \\ \implies x+5 = 2 \quad \bigg| \quad x+5 = -2 \\ \implies \boxed{x = -3} \quad \bigg| \quad \boxed{x = -7} \end{array}$$

(2) factoring

$$(x+a)(x+b) \stackrel{\text{FOIL}}{=} x^2 + (a+b)x + ab$$

$$x^2 + 6x + 5 = (x+5)(x+1)$$

$\begin{array}{cc} \uparrow & \uparrow \\ a+b & ab \end{array}$

$$x^2 + 10x + 21 = \underline{(x+3)(x+7)}$$

Solve for x in

$$x^2 + 10x + 21 = 0$$

$$\underline{(x+3)} \underline{(x+7)} = 0$$

$$x+3 = 0 \Rightarrow x = -3$$

$$x+7 = 0 \Rightarrow x = -7$$

FACTOR

$$x^2 - 9 = (x+3)(x-3)$$

$$x^2 + 8x + \underline{12} = (x+6)(x+2)$$

$$x^2 - x - \underline{\underline{30}} = (x+5)(x-6)$$

