

11/28/23:

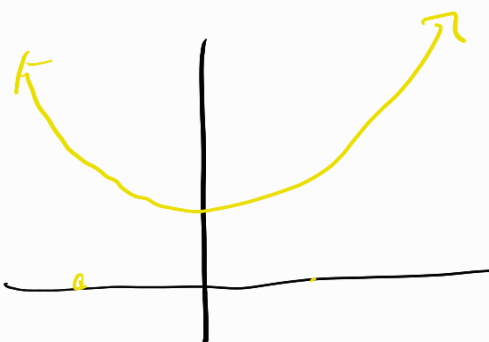
"Fundamental Theorem of Algebra" (FTA)

- combine algebra w/ geometric meanings!

FTA: every polynomial of degree n
has exactly n (complex) zeros

Real zero

x-int!!

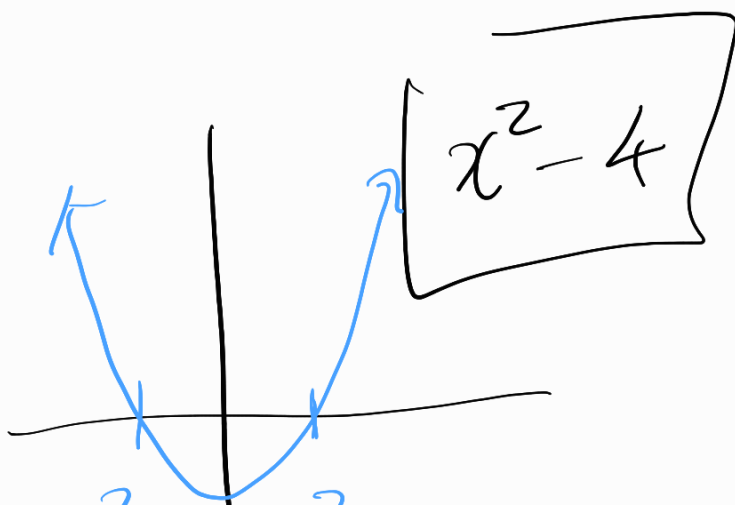
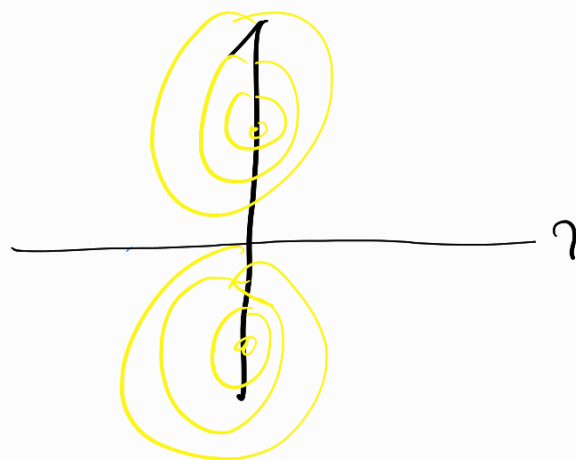


$$x^2 + 4$$

no real zeros !!

Complex zero

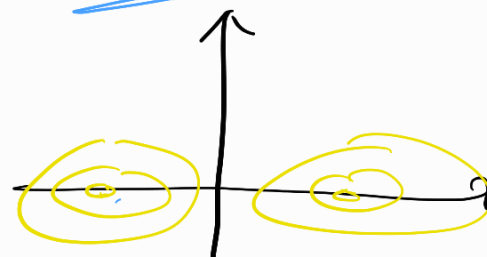
Focal points using computer



$$x^2 - 4$$

x-int at

$$x = \underline{2, -2}$$



Consequences of FTA

① $\deg(f) = 2023$

How many x values will

$$f(x) = 0$$

possibly none!!
up to 2023 real answers

2023 complex answers
exactly

②

$$f(x) = x^2 - 5$$

$$g(x) = 3x^{2023}$$

How many intersections of f and g
cross each other??

outputs

$$f = g \implies \underline{f - g = 0}$$

$$\underbrace{x^2 - 5 - 3x^{2023}} = 0$$

$$\text{deg} = \underline{2023}$$

2023 intersections!!

$$x^2 = 1 \Rightarrow x = 1, -1$$

$$x^4 = 1 \Rightarrow \underline{4 \text{ answers!}}$$

$$x = 1, -1, i, -i$$

Synthetic division

$$f(x) = 7 - 3x^2 + 2x^3$$

quotient

remainder

rem

14

2

$$\frac{f(x)}{x+3} = \text{quotient} + \frac{\boxed{3}}{x+3} - 4 + \frac{\boxed{3}}{x+3}$$

① rewrite $f(x)$ into standard form

$$\rightarrow 2x^3 + -3x^2 + 7$$

Setup:

	x^3	x^2	x	const	
$-3 \downarrow$	2	-3	0	7	↓ add
	0	-6	27	-21	
	2	-9	27	-74	

mult. ↗

quotient

$2x^2 - 9x + 27$

↑
remainder

quotient: $2x^2 - 9x + 27$

remainder: -74

$$\begin{array}{r} 10x^4 + 2x^3 - 1x + 5 \\ \hline x + 2 \end{array}$$

Find quotient & rem

	x^4	x^3	x^2	x^1	x^0
$-2 \mid$	10	2	0	-1	5
	0	-20	36	-72	146
	<hr/>				
	10	-18	36	-73	151

add

mult

quotient

$$10x^3 - 18x^2 + 36x - 73$$

rem

$$\begin{array}{r} 7 - 10x + 13x^2 - 3x^3 \\ \hline x + 1 \end{array}$$

$$\begin{array}{r} -1 \mid -3 \quad 13 \quad -10 \quad 7 \\ 0 \quad 3 \quad -16 \quad 26 \end{array}$$

$-3 \quad 16 \quad -26$ 33

$$-3x^2 + 16x - 26$$