ALGEBRA 2 HONORS
PROBLEM SET #2

DUE DATE: AUGUST 22, 2023

Question 1. Let (a_n) be a sequence with initial value $a_1 = 2$. Defining the recursive formula

$$a_{n+1} = 4a_n - 3,$$

compute a_2, a_3, a_4.

Question 2. Let (a_n) and (b_n) be the sequences given below:

n	1	2	3	4	5
(a_n)	-1	4	6	-8	10
(b_n)	1	-4	9	-16	25

Define a new sequence by the equation

$$(d_n) \overset{\text{def}}{=} a_n^2 + 2b_n$$

What are the values of d_1, d_2, d_3 going to be equal to?

Question 3. A *logistic map*, which models population given a reproduction rate r, is given by the recursive equation

$$x_{n+1} = rx_n(1 - x_n), n \geq 1, \quad 0 \leq x_1 \leq 1, \ r = \text{reproduction rate.}$$

John Von Neumann, a Hungarian-American mathematician who pioneered much of game theory, functional analysis, and quantum mechanics, thought up the logistic map $x_{n+1} = 4x_n(1 - x_n)$ as a random number generator (RNG).

(a) Using Von Neumann's RNG, what are the values of x_2, x_3, and x_4 if $x_1 = \dfrac{1}{2}$?

(b) If we use the initial condition $x_1 = 1$, what would be the values of x_2, x_3, and x_4 in the random number generator?