solve system of equations ← n variables
← n equations

"solving for 2023 variables req. 2023 equations"

methods

① substitution
② elimination
} Q1 & Q2 from HW

③ matrix row reduction ← 3 rules to solve a puzzle.

3 Rules

① swap rows

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & 5 & 6 & 7 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

② multiply/divide row w/ # ← nonzero

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \end{bmatrix} \xrightarrow{R_1 \times 2} \begin{bmatrix} 2 & 4 & 6 & 8 \\ 0 & 5 & 6 & 7 \end{bmatrix}$$

③ add/subtract row w/ another row

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 0 \\ 7 & 8 & 9 & 0 \end{bmatrix} \xrightarrow{R_3 \to R_2 - R_3} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 4-7 & 5-8 & 6-9 & 0-0 \\ 7 & 8 & 9 & 0 \end{bmatrix}$$

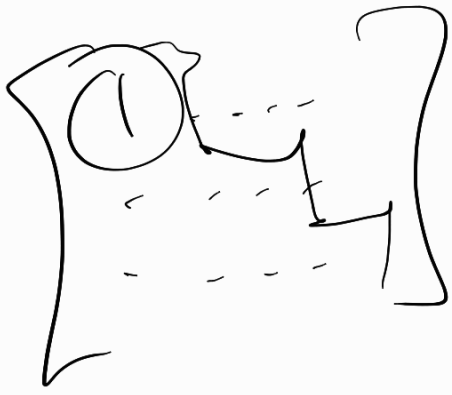
7 8 9 | 0

$$\left[\begin{array}{ccc|c} -3 & -3 & -3 & 0 \end{array}\right]$$

4-7 5-8 6-9

major goals

① get 1 in top left corner



② get staircase; everything below staircase needs to become a 0...

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & a \\ 0 & 4 & 5 & b \\ 0 & 0 & 1 & \frac{c}{6} \end{array}\right]$$

Cramer's Rule

Cramer's rule

determinant

1x1 $\det([2]) = 2$

$\det([5]) = 5$

$\det([-3.2243]) = -3.224$

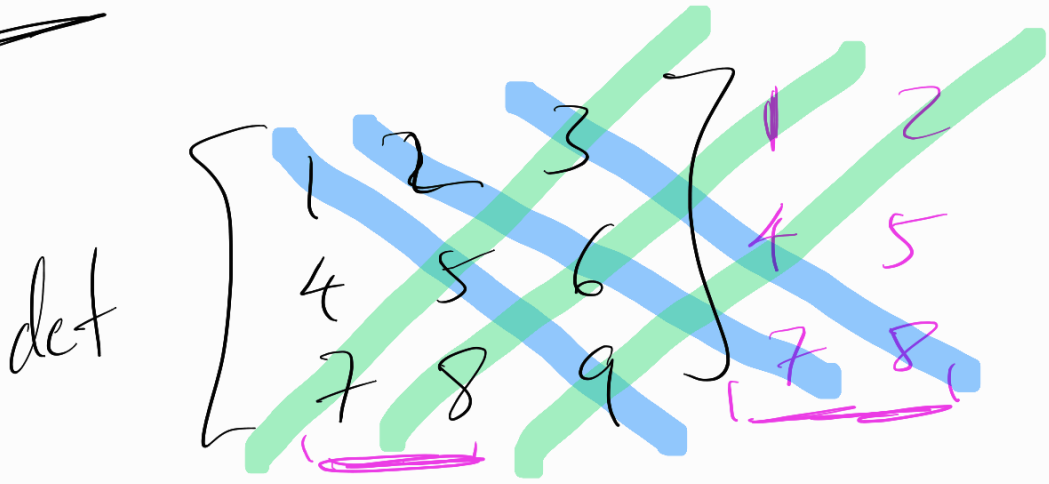
2x2 $\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$

$\det\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\right) = 4 - 6$

$= -2$

$\det\begin{bmatrix} 20 & 23 \\ 1 & 0 \end{bmatrix} = -23$

3x3



Steps ① copy first 2 columns

② paste first 2 columns to right of matrix

③ draw 3 diagonals up
3 diagonals down

result:

$(1 \cdot 5 \cdot 9) + (2 \cdot 6 \cdot 7) + (3 \cdot 4 \cdot 8)$

$= 7 \cdot 5 \cdot 3 - 1 \cdot 6 \cdot 8 - 9 \cdot 4 \cdot 2$