$X = \{1, 2, 3, 4043\}$

$f(x) = k \cdot x$ what is $k$?

$f(X) = 1$

$\hookrightarrow f(\{1, 2, 3, 4043\})$

$\hookrightarrow f(\{1\} \cup \{2\} \cup \{3\} \cup \{4043\})$

$\hookrightarrow f(1) + f(2) + f(3) + f(4043)$

$\hookrightarrow k + 2k + 3k + 4043k = 1$

$\hookrightarrow 4049k = 1$

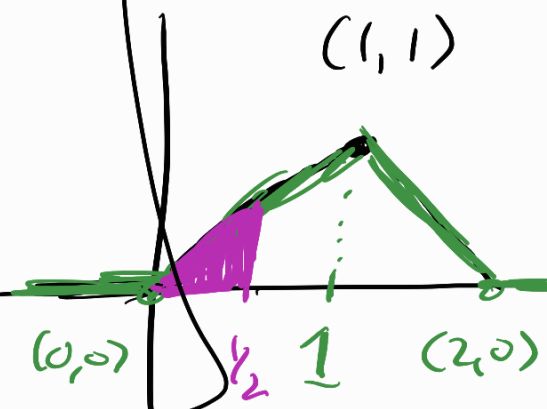
$\Longrightarrow \boxed{k = \frac{1}{4049}}$

$X = \{1, 2, 3, 4043\}$

$$f(x) = \frac{x}{4049}$$

| $x$ | $x=1$ | $x=2$ | $x=3$ | $x=4043$ |
|---|---|---|---|---|
| PMF | $\frac{1}{4049}$ | $\frac{2}{4049}$ | $\frac{3}{4049}$ | $\frac{4043}{4049}$ |
| CDF | $\frac{1}{4049}$ | $\frac{3}{4049}$ | $\frac{7}{4049}$ | $1$ |

$$f(x) = \begin{cases} x & \text{if } 0 \le x \le 1 \\ 2-x & \text{if } 1 < x \le 2 \\ 0 & \text{else} \end{cases}$$




$X = [0, 2]$

PDF

$$\text{Prob}\left(0 \le X \le \frac{1}{2}\right) = \frac{1}{2} \cdot \left(\frac{1}{2} \cdot \frac{1}{2}\right) = \frac{1}{8}$$

$$\int_0^x$$

$$CDF(x) = \int_{t=-\infty} PDF(t)$$

- $CDF(1)$ = Find area from $x = -\infty$ up to 1

PDF

$$f(x) = \begin{cases} x & \text{if } 0 \le x \le 1 \\ 2-x & \text{if } 1 < x \le 2 \\ 0 & \text{else} \end{cases}$$

$$CDF(x) = \begin{cases} c = 0 & \text{if } x < 0 \\ \frac{x^2}{2} + c \nearrow 0 & \text{if } 0 \le x \le 1 \\ 2x - \frac{x^2}{2} + c & \text{if } 1 < x \le 2 \\ c = 1 & \text{if } x \ge 2 \end{cases}$$

$4 - 2 + c = 1$

$$CDF(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{2} & 0 \le x \le 1 \\ 2x - \frac{x^2}{2} - 1 & 1 \le x \le 2 \\ 1 & x \ge 2 \end{cases}$$

goes up

$\frac{1}{2}$

Q3:

$$2x - \frac{x^2}{2} - 1 = \frac{3}{4}$$

$$CDF(x) = \frac{1}{2}$$

median

median: $x = 1$

solve for $x$

$$CDF(x) = \frac{1}{4}$$

$Q_1$

solve for $x$

$$x^2 = 1$$

2 4

$$x = \frac{1}{\sqrt{2}}$$

uniform distribution:

"roughly" uniform.

$$X = \{1, 2, 3, 4, 5, 6\}$$

- $f(x) = \frac{1}{6}$ ⟵ "fair die" uniform!!

- $g(x) = k \cdot x^2$ ⟵ find $k$

Probability measure on $X$

$X = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

$g(1) = k \cdot 1^2 \quad 1$

$g(2) = k \cdot 2^2 \quad 4$

$g(3) = 3^2 k \quad 9$

$g(4) = 4^2 k \quad 16$

$g(5) = 5^2 k \quad 25$

$g(6) = 6^2 k \quad 36$

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$91k = 1$

$\Rightarrow k = \frac{1}{91}$

$$g(6) = \frac{36}{91}$$

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$f(x) = 1/6$

| | $x=1$ | $x=2$ | $x=3$ | $x=4$ | $x=5$ | $x=6$ |
|---|---|---|---|---|---|---|
| $CDF_f$: | $1/6$ | $2/6$ | $3/6$ | $4/6$ | $5/6$ | $6/6$ |
| $CDF_g$: | $\frac{1}{91}$ | $\frac{5}{91}$ | $\frac{14}{91}$ | $\frac{30}{91}$ | $\frac{55}{91}$ | $\frac{91}{91}$ |

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$$g(x) = \frac{1}{91} x^2$$

says probabilities

$$\delta_{ij}(x) = \begin{cases} \infty & i = j \\ 0 & i \neq j \end{cases}$$