10/30/2023 : so far...

Quadratics

FOIL ← → FOIL

STANDARD ax^2+bx+c

$a>0$ ↑

$a<0$ ↓

VERTEX FORM $a(x-h)^2+k$

vertex will be at $(-h, k)$

FACTORED FORM $a(x+p)(x+q)$

x-intercepts $(-p, 0)$, $(-q, 0)$

Test

Factoring

"completing the square"

set $h = b/2a$

$k = c - h^2$

TODAY

Ex

$2(x-5)(2023-x)$ ← Factored

x-int: $(5, 0)$ and $(2023, 0)$

Not every quadratic is factorable!!

No factored form guaranteed

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The discriminant answers this!

$f(x) = ax^2 + bx + c$

Δ = discriminant

$b^2 - 4ac > 0$ — f has 2 x-int $\Rightarrow$ f has 2 factors

$b^2 - 4ac = 0$ — f has 1 x-int $\Rightarrow$ f has 1 factor (squared)

$b^2 - 4ac < 0$ — f has no x-int $\Rightarrow$ not factorable!!

$f(x) = x^2 + 3x - 4$

how many x-int? $b^2 - 4ac$

$9 - 4(1)(-4) = 9 + 16 > 0$

∴ 2 x-int

$g(x) = -2x^2 + 5x + 1$

how many x-int?

$b^2 + -4ac$

$25 + -4(-2)(1) \Rightarrow 25 + 8 = 33 > 0$

∴ 2 x-int.

$h(x) = -x^2 - 3$

$0^2 - 4(-1)(-3) = -12 < 0$

∴ no x-int

where are the x-int?

Find x-int of $f(x) = x^2 - x - 1$

how many x-int? $a = 1$

how many

$b = -1$

$c = -1$

$b^2 - 4ac \Rightarrow (-1)^2 - 4(1)(-1)$

memorise $1 - -4 > 0$

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Quadratic Formula

$ax^2 + bx + c = 0$

$$\Longrightarrow \boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$x^2 - x - 1 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1 - -4}}{} = \frac{1 \pm \sqrt{5}}{}$$

$$x = \frac{1+\sqrt{5}}{2}, \quad x = \frac{1-\sqrt{5}}{2}$$

Find x-int of $2x^2 + 2x - 4$

$a = 2$

$b = 2$

$c = -4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{4 - 4(2)(-4)}}{2(2)}$$

$$= \frac{-2 \pm \sqrt{4 + 32}}{4} = \frac{-2 \pm \sqrt{36}}{4}$$

$$= \frac{-2+6}{4}, \frac{-2-6}{4}$$

$$x = 1, -2$$

Find the x-int of

$$2x^2 + 4x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

BTW

$\sqrt{24} = \sqrt{4}\sqrt{6} = 2\sqrt{6}$

$$\Rightarrow x = \frac{-4 \pm \sqrt{16 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-4 \pm \sqrt{16 + 8}}{4}$$

$$= \frac{-4 \pm \sqrt{24}}{4} = \frac{-4 \pm \sqrt{4}\sqrt{6}}{4}$$

$$= \frac{-4 + 2\sqrt{6}}{4}, \frac{-4 - 2\sqrt{6}}{4}$$

$$x = \frac{-2 + \sqrt{6}}{2}, \frac{-2 - \sqrt{6}}{2}$$

24
4 6