Advanced Factoring

some polynomial

Factor Theorem: $(x+a)$ is a factor of $f(x)$ if $\frac{f(x)}{(x+a)}$ has rem $= 0$

Example. Factor fully

$$f(x) = x^7 - x^6 - 6x^5 + 6x^4 + 9x^3 - 9x^2 - 4x + 4$$

- $f(x)$ divided by $x-1$ ✓ multiplicity 2
  $x+1$ ✓ multiplicity 2
  $x-2$
  $x+2$
  leaves rem $= 0$

$1 \;|\; 1 \quad -1 \quad -6 \quad 6 \quad 9 \quad -9 \quad -4 \quad 4$

$\quad\quad 0 \quad 1 \quad 0 \quad -6 \quad 0 \quad 9 \quad 0 \quad -4$

$-1 \;|\; 1 \quad 0 \quad -6 \quad 0 \quad 9 \quad 0 \quad -4 \quad 0$

$\quad\quad 0 \quad -1 \quad 1 \quad 5 \quad -5 \quad -4 \quad 4$

$-1 \;|\; 1 \quad -1 \quad -5 \quad 5 \quad 4 \quad -4 \quad 0$

$\quad\quad 0 \quad -1 \quad 2 \quad 3 \quad -8 \quad 4$

$$\begin{array}{r|ccccc} -2 & 1 & -2 & -3 & 8 & -4 \\ & & 0 & 2 & 0 & -6 \\ \hline & 1 & -2 & -1 & 8 & -10 \end{array}$$

$$\begin{array}{r|ccccc} -2 & 1 & 0 & -3 & 2 & 0 \\ & & 0 & -2 & 4 & -2 \\ \hline & 1 & -2 & 1 & 6 & -2 \end{array}$$

$$\begin{array}{r|ccc} 1 & 1 & -2 & 1 \\ & & 1 & -1 \\ \hline & 1 & -1 & 0 \end{array}$$

x^2 x

$$x^2 - 2x + 1 = (x-1)^2$$

$$f(x) = x^7 - x^6 - 6x^5 + 6x^4 + 9x^3 - 9x^2 - 4x + 4$$

- $f(x)$ divided by $x-1$ ✓ (multiplicity 2), $x+1$, $x-2$, $x+2$

$$(x-1)^2(x-1)(x+1)^2(x-2)(x+2)$$

$$g(x) = x^4 + 6x^3 + 9x^2 - 4x - 12$$

then divided

leaves rem = 0 when

by $\cdot(x+2)$.

$\cdot(x+3)$

Example: $f(x) = x^4 - 7x^3 - 9x^2 + 7x + 8$

$f(x)$ leaves rem = 0 when divided by

$x+1$ and $x-1$.

Rational roots theorem

$$f(x) = 3x - 2 + 14x^{2023} - 5x^{25}$$

sussy figures: no x's $\longrightarrow$ -2 $\rightarrow$ $1, 2$

most x's $\longrightarrow$ 14 $\rightarrow$ $1, 2, 7, 14$

sus #'s

repeat

$$\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{1}{2}, \pm\frac{2}{2}, \pm\frac{1}{7}, \pm\frac{2}{7}, \pm\frac{1}{14}$$

$\pm\frac{2}{14}$ repeat

$f(x) = 3 - 15x^{10} + 2x^{20}$

Sussy #

$3 \to 1, 3$

$2 \to 1, 2$

$\Rightarrow \pm 1, \pm 3, \pm\frac{1}{2}$

$\pm\frac{3}{2}$