10/23/2023

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ ← conditional prob.

- A, B are mutually exclusive $A \cap B = \phi$

$$\Rightarrow Prob(A \cap B) = 0$$

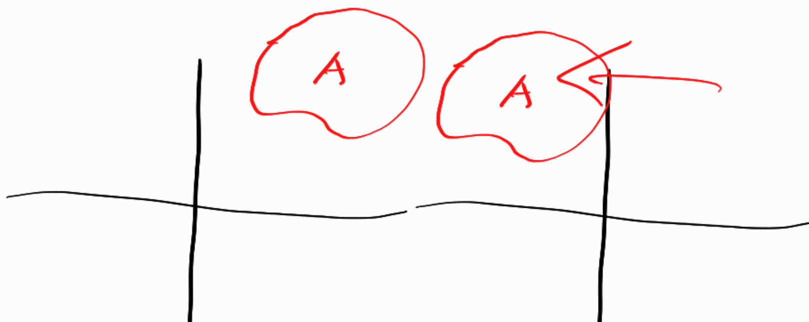
- A, B are independent if

$$P(A \mid B) = P(A)$$

B's outcome has no influence on A

- $P(A \cap B) = P(A) \cdot P(B)$

← A, B must be indep!!



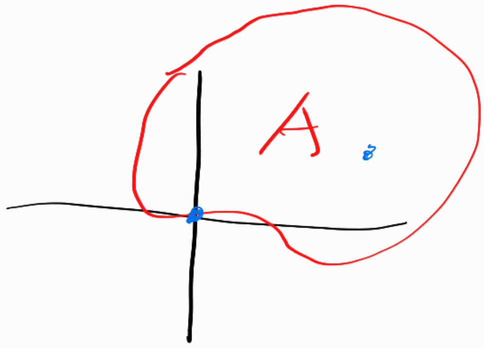
$X = \{3, 4, 2, 5\}$

① shifts

② dilations/ scaling

$$X+5=\{x+5 \mid x \text{ is in } X\}$$

$$=\{8,9,7,10\}$$



$$X+a=\{x+a \mid x \text{ is in } X\}$$

$$2X=\{2x \mid x \text{ is in } X\}$$

$$=\{6,8,4,10\}$$

- $X=\{2,4,3,5\}$
- $Y=\{-1,10\}$

$$X+Y=\left\{x+y \;\middle|\; \begin{matrix} x \text{ is in } X \\ y \text{ is in } Y \end{matrix}\right\}$$

$$=\left\{\begin{matrix} 1, & 3, & 2, & 4 \\ 12, & 14, & 13, & 15 \end{matrix}\right\}$$

$$X = \{2, 3, 4, 5\}$$

is $X + X = 2X$

$$\{x_1 + x_2 \mid x_1, x_2 \in X\} \qquad \{2x \mid x \in X\} = \{4, 6, 8, 10\}$$

$$= \{4, 5, 6, 7, 8, 9, 10\}$$

No not the same

X - random variable
- outcomes
- probability of each outcome

X = faces on fair die

$$= \{\underset{1/6}{1}, \underset{1/6}{2}, \dots, \underset{1/6}{6}\}$$

outcome

prob of

outcome

Expected Value $= \sum x \cdot p(x)$

$$1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + \dots + 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right)$$

$$= \frac{1}{6}(1+2+\dots+6) = 3.5$$

GAME

roll die:

roll 1, 2, 3 ..... lose \$ by amount rolled

roll 4, 5, 6 win \$ by amount rolled

$$E(X) = \frac{1}{6}\left[-1 + -2 + -3 + 4 + 5 + 6\right]$$

$$= \frac{1}{6}\left[9\right] = 3/2$$

roll die: roll 1, 2, 3 ..... lose \$ by amount rolled

using unfair die

roll 4, 5, 6 win \$ of amount rolled

die

rolling 1 = 1/2

2 = 1/4

3 = 1/8

4 = 1/16

5 = 1/32

6 = 1/32

$$E(X) = \frac{1}{2}(-1) + \frac{1}{4}(-2) + \frac{1}{8}(-3) + \frac{1}{16}(4) + \frac{1}{32}(5) + \frac{1}{32}(6)$$

$$= \boxed{-\frac{25}{32}}$$

$$E(X) = \begin{cases} \sum x P(x) & X\text{-discrete} \\ \int x P(x) & X\text{-cts} \end{cases}$$

interpret from graph

X - R.V. w/ $E(X) = 5$

$$Y = 3X + 2$$

Q: Does $E(aX+b) = aE(X) + b$

$$E(3X+2) \overset{??}{=} 3E(X) + 2$$

Proof

$$E(3X+2) = \sum_{x \in X} (3x+2)\, p(3x+2)$$

$\{3x+2 : x \in X\}$

$$= \sum_{x \in X} (3x+2) \cdot p(x)$$

$$= \sum_{x \in X} \left[3x\,p(x) + 2p(x) \right]$$

$$= \sum \left[3x\,p(x) \right] + \sum 2\,p(x)$$

$$\sum_{x\in X}\left[3x\,p(x)\right] + \sum_{x\in X} 2\,p(x)$$

$$= 3\underbrace{\sum_{x\in X} x\,p(x)} + 2\underline{\underline{\sum_{x\in X} p(x)}}$$

$$3E(X) + 2(1)$$

17

$$E(3X+2) = 3E(X)+2$$

$$\boxed{E(aX+b) = aE(X)+b}$$