MATH 11A WEEK 5
PROBLEM SOLVING DERIVATIVES

Announcements:

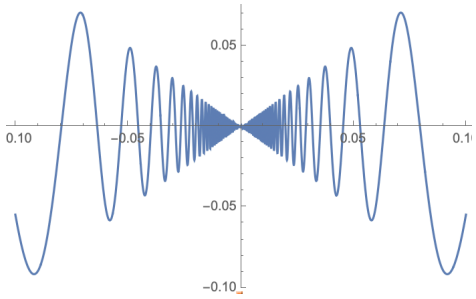
(1) Exam this week on WebAssign
(2) Office hours are now changing to be the hour immediately after the section, i.e.,
   - Tu 11:50PM - 1:00PM
   - Th 5:10- 6:00 PM
   - F 5:10 - 6:00 PM

Question 1. *(a) What makes a function $f(x)$ continuous at a point $x = a$?*
(b) What makes a function $f(x)$ differentiable at a point $x = a$?
(c) If $f(x)$ is a function continuous at a point $x = a$, is it differentiable at $x = a$? If so, why? If not, come up with an example of a function that is continuous at some point but not differentiable at the same point.
(d) If $f(x)$ is a function differentiable at a point $x = a$, is it continuous at $x = a$? If so, why? If not, come up with an example of a function that is differentiable at a point but not continuous at that point.

Question 2. *Consider the function*

$$f(x) = \begin{cases} x \sin\left(\dfrac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

which has the graph



(a) Is $f(x)$ continuous at $x = 0$?

(b) Does $f(x)$ have a derivative at $x = 0$? If so, what is the derivative? If not, explain why.

Question 3 (Practice your trig!)**.** *Recall that the derivative of a function $f(x)$ is given by*

$$\frac{\mathrm{d}f}{\mathrm{d}x} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

(a) Use the definition of the derivative and the trig identity

$$\sin(x+y) = \sin(x)\cos(y) + \sin(y)\cos(x)$$

to compute that

$$\frac{\mathrm{d}}{\mathrm{d}x} \sin(x) = \cos(x).$$

(b) Use the definition of the derivative and the trig identity

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

to compute that

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2} \sin(x) = -\sin(x)$$