AP STATISTICS
FREE RESPONSE QUESTIONS PRACTICE

Question 1. During a one week period in Dublin, CA, the average price of an orange was 50 cents with a standard deviation of 5.2 cents.

Find the probability that the average price per orange is 45 cents for a bag of 60 oranges.

Let $\overline{X}$ be the sampling distribution of sample average price of $n = 60$ oranges during a one week period in Dublin, CA. Since 60 oranges is a fairly large sample size, but is less than 10% of all oranges in Dublin, CA, the sampling distribution $\overline{X}$ will be approximately normal due to the Central Limit Theorem. From the CLT, we also get that the mean and standard deviation of this distribution, $\mu_{\overline{x}}$ and $\sigma_{\overline{x}}$, respectively, will be

$$\mu_{\overline{x}} = \mu \; = 50 \text{ cents} \qquad \text{and} \qquad \sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}} = \frac{5.2}{\sqrt{60}} \text{ cents} \approx 0.6713 \text{ cents.}$$

We can compute the probability that a sample average price for a bag of 60 oranges is $\overline{x} = 45$ cents using z-scores since $\overline{X}$ is normal by CLT. Compute that

$$z_{45} = \frac{45 - 50}{5.2/\sqrt{60}} = \frac{45 - 50}{0.6713} \approx -7.44...$$

which places us near the 0th percentile. So basically no shot of having an average price per orange of 45 cents for a bag of 60 oranges.

Question 2. In the Korean K-League, 75% of all players in the history of the K-league prefer to kick with their right leg.

(1) In a simple random sample of 20 players...
 (a) Find the probability that exactly 14 players prefer to kick with their right foot.
 Let X be the distribution of K-league players who prefer to kick with their right leg. This distribution is binomial distribution with $n = 20$ and $p = 0.75$.
 To compute the probability of getting exactly 14 players who prefer to kick with their right foot, compute that

$$Prob(X = 14) = \binom{20}{14}(0.75)^{14}(1 - 0.75)^{6} \approx 0.1686.$$

 So the probability of getting exactly 14 players out of 20 randomly selected K-league players who prefer to kick with their right foot is 0.1686.
 (b) Find the probability that no more than 2 players prefer to kick with their right foot.
 This is more annoying but this question is asking

$$Prob(X \leq 2) = Prob(X = 0) + Prob(X = 1) + Prob(X = 2)$$

$$= \binom{20}{0}(0.75)^0(0.25)^{20} + \binom{20}{1}(0.75)^1(0.25)^{19} + \binom{20}{2}(0.75)^2(0.25)^{18}$$
$$\approx 0.0000000016$$

So the probability of having a simple random sample of 20 K-league players with no more than 2 players who prefer to kick with their right foot is approximately 0.

(2) In a simple random sample of 1000 K-league players, find the probability that...

(a) Exactly 70% of the players prefer to kick with their right foot.

Let D be the sampling distribution of $n = 1000$ randomly selected K-league players. Letting p be the proportion of all K-league players who prefer to kick with their right foot, we get that $np = 1000(0.75) = 750 > 10$ and $n(1-p) = 1000(0.25) = 250 > 10$, from the Central Limit Theorem, we get that the sampling distribution D will be approximately normal and the sampling distribution D will have a mean $\mu_p = p = 0.7$ and $\sigma_p = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.7(0.3)}{1000}} = 0.0144914$. Since D is approximated by a continuous distribution, the probability of getting exactly $Prob(D = 0.7) = 0$.

(b) No more than 70% of the players prefer to kick with their right foot.

In this question, we are trying to compute $Prob(D \leq 0.7)$. We can compute this by computing the z-score of the sample proportion $\widehat{p} = 0.7$:

$$\underbrace{\frac{\widehat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}}_{\text{the } z\text{-score for proportions}} \qquad \Longrightarrow \frac{0.7 - 0.7}{0.0144914} = 0$$

which gives us 50th percentile; so the probability of obtaining a sample where no more than 70% of the players prefer to kick with their right foot is 50%.

Question 3. Let D be a fair die relabelled as $1, 1, 1, 2, 3, 4$. The die D is rolled 400 times. From the 400 times the die is rolled, find the probability that the average result of the die D is at least 2.2.
(Hint: You need to compute the mean and standard deviation of D to get the population parameters.)

The mean $\mu = 2$ and standard deviation $\sigma = 2\sqrt{\frac{2}{5}} \approx 1.265$.

Let $\overline{D}$ be the sampling distribution of the average value of D with $n = 400$ rolls. Since 400 is a lot of rolls, but less than the total number of rolls possible (basically infinite), we get that $\overline{D}$ is approximately normal by the Central Limit Theorem with with mean $\mu_{\overline{D}} = \mu = 2$ and $\sigma_{\overline{D}} = \frac{\sigma}{\sqrt{n}} = \frac{1.265}{\sqrt{400}} = 0.0632$.

Now we compute $Prob(D \geq 2.2)$ by

$$z_{2.2} = \frac{2.2 - \mu}{\sigma/\sqrt{n}} = \frac{2.2 - 2}{0.0632} = 3.16.$$

which places us in the 99.92nd percentile. Since we need $Prob(D \geq 2.2)$, we get that the probability of 0.08% chance of obtaining a sample average of at least 2.2 from 400 rolls.