LOGARITHMS AND EXPONENTIALS

Exponent Rules

$$a^1 = a \qquad \frac{a^m}{a^n} = a^{m-n}$$
$$a^0 = 1 \qquad a^n b^n = (ab)^n$$
$$a^{-m} = \frac{1}{a^m} \qquad \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$
$$a^m a^n = a^{m+n} \qquad a^{m/n} = \sqrt[n]{a^m}$$
$$(a^m)^n = a^{mn}$$

Logarithms are exponents. That is,

$$\log_b(x) = n \text{ means } \quad b^n = x.$$

A few rules for logarithms:

- Log of Product is Sum of Logs: $\log_b(xy) = \log_b(x) + \log_b(y)$
- Log of Quotient is Difference of Logs: $\log_b(\frac{x}{y}) = \log_b(x) - \log_b(y)$
- Power Rule: $\log_b(x^n) = n \log_b(x)$
- Log-in-Exp rule: $b^{\log_b(x)} = x$
- Change of Base: $\log_a(x) = \dfrac{\log_b(x)}{\log_b(a)}$

The following are common "rules" that students often use, but these are **WRONG**

- $\log(x+y) = \log(x)\log(y)$
- $\log(x+y) = \log(x) + \log(y)$
- $\dfrac{\log(x)}{\log(y)} = \log(\frac{x}{y})$
- $\dfrac{\log(x)}{\log(y)} = \log(x) - \log(y)$

$\log(x) = \log_{10}(x)$ and $\ln(x) = \log_e(x)$ where $e \approx 2.7182818284\ldots$