

ALGEBRA 2

PROBLEM SET 16

DUE DATE: NOVEMBER 7, 2023

Recall that we use the symbol $i = \sqrt{-1}$ with the understanding that $i^2 = -1$.

Question 1. Simplify each of the following in to $a + bi$ form.

(a) $(1 + i)^2$

(b) $(2i - 2) - (-1 + i)$

(c) $(2 + i)(3 + i)$

(d) $\frac{2 + i}{3 - i}$

Question 2. Assuming that x and y are real numbers, solve for x and y in each equation:

(a) $x + yi = 3i - 4$

$x = -4$ $y = 3$

(b) $x + yi = -7 + 2i$

$x = -7$, $y = 2$

(c) $x + yi = 0$

$x = 0$ $y = 0$

(d) $x + yi = (1 - i)^2$

$$\begin{aligned} (1 - i)(1 - i) &= 1^2 - 1i - 1i + i^2 \\ &= 1 - 1 - 2i \\ &= -2i \end{aligned}$$

(e) $(x + yi)^2 = 2i$

(f) $\frac{(x + yi)(2 - i)}{(2 - i)} = \frac{8 + i}{2 - i}$

Question 3. Solve for all x that satisfy the equation (real or complex)

$x + yi = \frac{8 + i}{2 - i} \left(\frac{2 + i}{2 + i} \right) = \frac{(8 + i)(2 + i)}{2^2 - i^2}$

$$= \frac{16-1}{5} + \frac{10}{5}i \Rightarrow \boxed{3+2i}$$

(a) $x^2 - 10x + 29 = 0$

(b) $22x^2 + 5 = 6x$

(c) $x^2 + 14x + 50 = 0$

(d) $2x^2 + 9 = 0$

$(a+b)^2 = a^2 + 2ab + b^2$

Question 4. Let $f(x) = 3x^2 + 2ix + i$. Compute $f(2+i)$

$3(2+i)^2 + 2i(2+i) + i$

$\hookrightarrow 3(4 + 4i + i^2) + 4i + 2i^2 + i$

Question 5. Recall that $a^3 = aaa$. Using the fact that $i^2 = -1$, compute each of the following, and plot them on the complex plane (you should see a pattern!)

(a) 1

(b) i

(c) $i^2 = -1$

(d) $i^3 = i^2 \cdot i = -1 \cdot i = -i$

(e) $i^4 = i^3 \cdot i = -i \cdot i = -i^2 = 1$

(f) $i^5 = i^4 \cdot i = 1 \cdot i = i$

(g) $i^6 = i^5 \cdot i = i \cdot i = i^2 = -1$

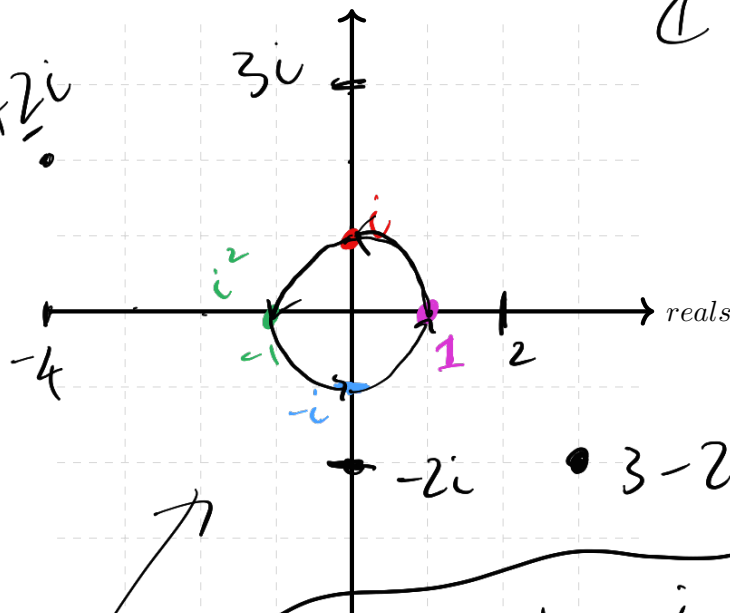
(h) $i^7 = i^6 \cdot i = -1 \cdot i = -i$

(i) $i^8 = i^7 \cdot i = -i \cdot i = -i^2 = 1$

imaginary

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$-4+2i$



$3-2i$

plot @ $(3, -2)$

multiplication by i rotates plane by 90° counter clockwise

rotates