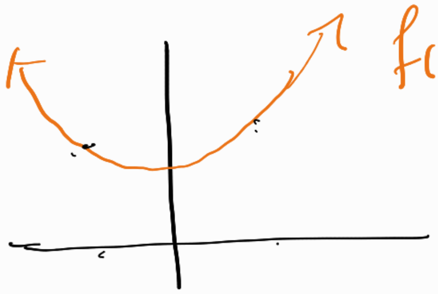
11/28/23 Fundamental theorem of Alg. and its consequences

FTA: Any polynomial w/ complex coefficients has at least one complex root (zero)

Domain = all possible inputs

Domain = real numbers $\mathbb{R}$

on number line



$f(x) = x^2 + 4$

no x-int !! no zeros

Domain = complex numbers $\mathbb{C}$

$f(x) = x^2 + 4$

has 2 zeros

"x-int"

$x = 2i, -2i$

deg = 7

$p(x) = 2x^2 - 3x^5 - 2023x^7$

# of zeros in $\mathbb{C}$: exactly 7

# of zeros in $\mathbb{R}$: up to ?

$x^2 = 1 \Rightarrow x^2 - 1 = 0$

$\deg = 2 \Rightarrow$ 2 answers

$x = \pm 1$

$x^3 = 1 \Rightarrow x^3 + 1 = 0$

$\Rightarrow$ 3 answers (not necessarily different)

$\underline{x = -1}$ has multiplicity 3

$x^{2023} + x^2 - 1 = 0$

$\Rightarrow$ 2023 complex answers

Consequences of FTA:

① $\deg(\text{polynomial}) = n$

$\Rightarrow n$ zeros (possibly repeating)

complex

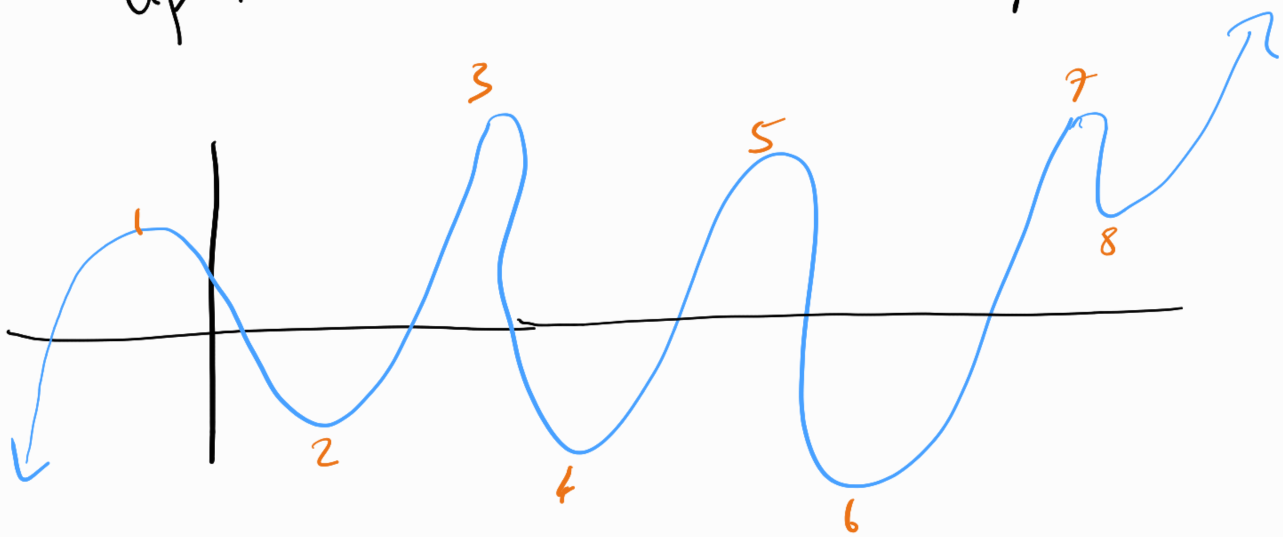
compl

(2)

$f(x) = 3x - 7 + 2x^{20}$

<u>how many times</u> will $f(x) = 5$

up to 20 times (exactly 20 times in complex plane)



$\deg \geq 8$

$$\boxed{\text{\# of bumps in poly graph} \leq \text{degree}}$$

if $f(x)$ is a polynomial of degree 2023

- can $f(x) = 7$ 2024 times? No
- can $f(x) = 7$ 2022 times? Yes

can $f(x) = 7$ 2023 times?

Yes

deg = 2

$f(x) = x^2 - 1$

$g(x) = 2x^3 + 1$

deg = 3

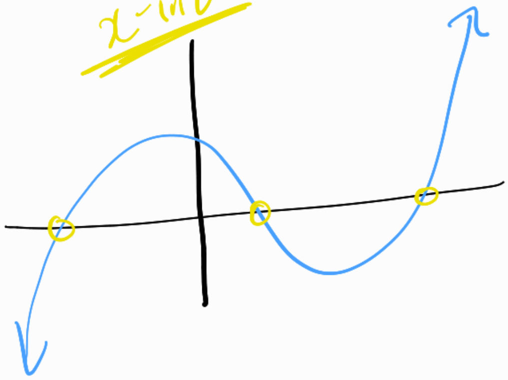
$f \cdot g = 0$

happens 5 times

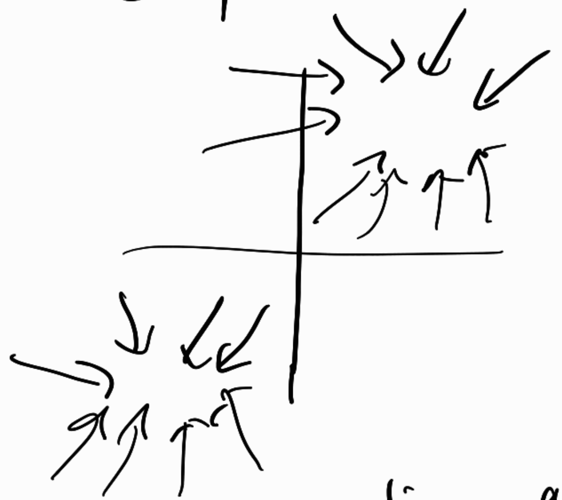
Warning: FTA does not tell you what answers are tho !!

Real zero vs Complex zero

x-int



3 real zeros



"complex function graph"

How to find these zeros??

① guess & check

② Factor & Remainder Theorem ← Today

Synthetic division

$f(x) = x^7 - 3x^2 + 6x^8 - 9 + 5x^4$

$f(2) = \text{rem} \Rightarrow \dfrac{f(x)}{x-2} = \text{Quotient} + \text{Remainder}$

① rewrite $f(x)$ into standard form:

$6x^8 + 1x^7 + 0x^6 + 0x^5 + 5x^4 + 0x^3 - 3x^2 + 0x - 9$

② set up

2	6	1	0	0	5	0	-3	0	-9
+	0	12	26	52	104	218	436	866	1732
	6	13	26	52	109	218	433	866	$\boxed{1723}$

x const

quotient $x^7 \quad x^6 \quad x^5 \quad x^4 \quad x^3 \quad x$

remainder

rem

$$\frac{f(x)}{x-2} = \underbrace{6x^7 + 13x^6 + 26x^5 + 52x^4 + 109x^3 + 218x^2 + 433x + 866}_{\text{quotient}} + \frac{1723}{x-2}$$

$$f(2) = 1723$$

$g(x) = 2x^3 - 20 + 3x$

$g(5) = ?$ using rem Thm

	x^3	x^2	x	const
5	2	0	3	-20
	0	10	50	265
	2	10	53	245

$g(5) = 245$

quotient: $2x^2 + 10x + 53$

① $\dfrac{x^4 - 2x^2 + 3x - 1}{x+2}$ has rem ___

What is $f(-2) =$

② Find a if

$\dfrac{x^3 - 2x + a}{(x-2)}$ has rem 7