Interpretable Hybrid-Rule Temporal Point Processes

Yunyang Cao, *Tongji University*
Juekai Lin*, *Tongji University*
Hongye Wang*, *Tongji University*
Wenhao Li, *Tongji University*, whli@tongji.edu.cn
Bo Jin, *Tongji University*, bjin@tongji.edu

APPENDIX

A. Introduction of Real-World Datasets

We choose the MIMIC-IV dataset as the real world dataset for our experiments. MIMIC-IV is a large, publicly available electronic health record database developed jointly by MIT and Beth Israel Deaconess Medical Center. It contains detailed clinical data from more than 40,000 adult ICU patients treated between 2008 and 2019. The database includes multiple data types such as demographics, clinical notes, nursing records, lab tests, vital signs, medications, and diagnosis codes. Based on this dataset, we created four distinct subsets for our study.

1) Key Tables Used from MIMIC-IV:

- **patients**: Contains demographic information for each patient, including age, gender, and death dates.
- **admissions**: Records hospitalization details such as admission type, admission and discharge times.
- **icustays**: Captures ICU stay episodes, including ICU admission and discharge times and length of stay.
- **chartevents**: Logs bedside vital signs and nursing charted observations with timestamps.
- **labevents**: Contains laboratory test results with numeric values and timestamps.
- **diagnoses_icd**: Lists patient diagnoses coded by ICD standards during hospital stays.
- **d_items**: Provides metadata and descriptions for clinical observation items recorded in other tables.
- **outputevents**: Records output events such as fluid outputs and ventilator settings during ICU stays.

2) Precautions for Processing Datasets: The dataset used in this study is derived from real clinical observations in hospitals, and all variables carry accurate timestamp information. The observation time points are automatically recorded by the monitoring equipment or manually recorded by clinical medical staff. Therefore, the data collection is irregular and the observation time is not continuously and uniformly distributed. Between two adjacent time records, some indicators may not have been collected, ostensibly presenting as missing values. However, since it was impossible to determine whether there were indeed missing observational data between these two records, we did not fill in or interpolate the missing values during data processing. Instead, we modeled and analyzed the MIMIC data as discrete time series data to more truly reflect the collection characteristics and temporal dynamic features of clinical data.

3) Key Variables in Real-world Datasets: The AKI dataset includes a selection of clinically relevant variables that are commonly used to monitor kidney function and overall patient status. Disease progression to Phase III serves as the target event. We use the creatinine and 12 consecutive hours of urine volume to determine whether the patient's disease has entered Phase III. The variables chosen encompass vital signs, blood gas measurements, metabolic panels, coagulation parameters, and inflammatory markers, specifically shown in Table I.

TABLE I
VARIABLES USED IN THE AKI DATASET.

Group	Variables
Vital Signs	Heart Rate, Arterial O_2 Saturation
Electrolytes	Sodium, Potassium, Chloride, Anion Gap
Renal Markers	BUN, Arterial CO_2 Pressure, Lactic Acid
Liver Function	ALT, AST, Total Bilirubin, Albumin
Coagulation	Prothrombin Time, Partial Thromboplastin Time
Blood Chemistry	Glucose, Hemoglobin, Arterial pH
Inflammation	White Blood Cell Count, C-Reactive Protein

The Stroke dataset focuses on variables that are closely related to cerebrovascular monitoring, vital organ function, and coagulation status. The target variable focuses on patients transitioning from moderate to severe conditions. We evaluate the patient's coma by calculating the Glasgow Coma Scale (GCS) score. When the GCS score is less than 8, it indicates that the patient is in a severe coma. The selected variables encompass a comprehensive set of clinical measurements across multiple physiological systems, shown in Table II.

* These authors contributed equally to this work.

TABLE II
VARIABLES USED IN THE STROKE DATASET.

Group	Variables
Vital Signs	Heart Rate, Respiratory Rate, Temperature, O_2 Saturation
Blood Pressure	Arterial Blood Pressure (Systolic, Diastolic, Mean)
Hematology	Hematocrit, Hemoglobin, Platelet Count, White Blood Cells
Electrolytes	Sodium, Potassium, Chloride, Bicarbonate
Renal Markers	Blood Urea Nitrogen (BUN), Creatinine, Anion Gap
Coagulation	Prothrombin Time, Partial Thromboplastin Time, International Normalized Ratio (INR)
Blood Gases	Arterial O_2 Pressure, Arterial CO_2 Pressure, Arterial Base Excess

The Sepsis dataset is constructed to capture the multi-organ dysfunction and systemic inflammatory response characteristics associated with sepsis. The occurrence of low urine output is tracked as a target of organ failure. We determine whether it is low urine output based on the urine output value in the past 6 hours. A wide range of clinical variables are selected to represent cardiovascular, renal, hepatic, hematologic, metabolic, and immune system functions, shown in Table III.

TABLE III
VARIABLES USED IN THE SEPSIS DATASET.

Group	Variables
Vital Signs	Heart Rate, Respiratory Rate, Temperature, SpO_2 Desaturation Limit
Blood Pressure	Arterial Blood Pressure (Systolic, Diastolic, Mean)
Electrolytes	Sodium, Potassium, Chloride, Bicarbonate, Ionized Calcium, Magnesium
Renal	Blood Urea Nitrogen, Creatinine
Liver Function	Albumin, Total Bilirubin, ALT, AST
Coagulation	Prothrombin Time, Partial Thromboplastin Time, International Normalized Ratio (INR), D-Dimer, Fibrinogen
Hematology	Hemoglobin, White Blood Cells, Platelet Count
Blood Gases	Arterial CO_2 Pressure, Venous O_2 Pressure, Arterial Base Excess, pH, Lactic Acid
Inflammation	C-Reactive Protein, HDL

The CAD dataset emphasizes variables relevant to cardiovascular function, metabolic regulation, hematology, and myocardial injury, shown in Table IV. These features are curated to support accurate modeling of cardiac health deterioration and coronary event risk. Patient mortality is defined as the target event.

TABLE IV
VARIABLES USED IN THE CAD DATASET.

Group	Variables
Vital Signs	Heart Rate, Respiratory Rate, Temperature, O_2 Saturation
Blood Pressure	Arterial Blood Pressure (Systolic, Diastolic, Mean)
Hematology	White Blood Cells, Hemoglobin, Hematocrit, Platelet Count
Renal	Creatinine, Blood Urea Nitrogen
Electrolytes	Sodium, Potassium, Calcium, Anion Gap
Liver	Total Bilirubin, Albumin, ALT, AST
Cardiac Biomarkers	CK, CK-MB Fraction, Brain Natriuretic Peptide, LDH
Metabolic and Lipids	Glucose, Cholesterol
Coagulation and Inflammation	International Normalized Ratio (INR), C-Reactive Protein

B. Interpretable Rule Forms in Baseline Rule Mining Methods

The rule form in HRTPP is a combination of temporal predicates and temporal relations. There are three types of temporal relations: *And*, *Before*, and *Equal*. The rule forms in the baseline methods are similar, shown as follows.

1) TELLER: The logic rule form in TELLER is regarded as a combination of temporal predicates and temporal relations. This method simplifies 13 temporal relations into a combination of two temporal relations: *Before* and *Equal*. The weight of a rule represents its influence on the overall intensity.

2) CLNN: The rule form in CLNN provides a detailed description of the event intervals between variables. For example, $c_{X_u} - c_{X_v} > t$ means that the latest event X_u occurs after the latest event X_v, and the time interval is greater than t. The weight of a rule represents its influence on the overall intensity.

3) CLUSTER: The logic rule form in TELLER is regarded as a combination of temporal predicates and temporal relations. There are four types of temporal relations: *None*, *Before*, *After*, and *Equal*. This method can get the weight of each rule, but this weight has no practical meaning, so it is not displayed.

C. Experimental Results of Real-World Datasets

1) AKI Dataset: The rules and reasoning assessments conducted by the TELLER, CLNN, CLUSTER, and HRTPP models on the AKI dataset are presented in Table V.

2) Stroke Dataset: The rules and reasoning assessments conducted by the TELLER, CLNN, CLUSTER, and HRTPP models on the Stroke dataset are presented in Table VI.

3) Sepsis Dataset: The rules and reasoning assessments conducted by the TELLER, CLNN, CLUSTER, and HRTPP models on the Sepsis dataset are presented in Table VII.

4) CAD Dataset: Due to space limitations, the rules of the CLUSTER method on CAD datasets are not fully listed in the main paper. They are supplemented in Table VIII.

TABLE V
RULES ON THE AKI DATASET.

Model	Rule	Weight	Correctness
TELLER	Hemoglobin Low $\rightarrow$ Phase III	0.4418	Correct
	BUN High $\rightarrow$ Phase III	0.2973	Incorrect
	Heart Rate High $\rightarrow$ Phase III	0.2923	Correct
	WBC High $\rightarrow$ Phase III	0.2002	Correct
	Glucoseserum High $\rightarrow$ Phase III	0.1545	Correct
CLNN	c_PTT High-c Potassiumserum High $>$ 0	0.73	Correct
	c_PTT High - c_ALT Low$>$ -0.05	0.72	Correct
	c_PTT High - c_C Reactive Protein CRP High $>$ -0.22	0.71	Correct
	c_BUN Low - c_Prothrombin time High $>$ 0.08	0.71	Incorrect
	c_BUN Low - c_Lactic Acid High $>$ 0	0.70	Correct
	c_Lactic Acid High - c_AST Low $>$ 0	0.70	Correct
CLUSTER	PTT High after ALT Low	-	Incorrect
	Anion gap High before Total Bilirubin High	-	Correct
	Arterial CO2 Pressure High before ALT Low	-	Correct
	Anion gap High after PH Arterial Low	-	Correct
	WBC High before PH Arterial Low	-	Correct
	PTT Low equal Albumin High	-	Incorrect
	Anion gap Low after C Reactive Protein CRP High	-	Correct
	PTT High before Prothrombin time Low	-	Incorrect
	Anion gap High after Heart Rate High	-	Correct
	PTT High equal BUN Low	-	Incorrect
	Sodiumserum Low before Hemoglobin High	-	Correct
	ALT High after BUN High	-	Correct
	PH Arterial Low before PTT Low	-	Correct
	Glucoseserum Low after BUN Low	-	Incorrect
	WBC High before BUN High	-	Correct
	Anion gap High equal Anion gap Low	-	Incorrect
	WBC High equal Potassiumserum High	-	Incorrect
	Heart Rate High after Total Bilirubin High	-	Correct
	Prothrombin time Low before ALT Low	-	Incorrect
	Hemoglobin Low before ALT Low	-	Incorrect
HRTPP	BUN High before Arterial O2 Saturation Low $\rightarrow$ Phase III	1.5478	Correct
	Prothrombin time High before ALT Low $\rightarrow$ Phase III	1.4121	Correct
	Prothrombin time High and Anion gap High $\rightarrow$ Phase III	0.8598	Correct
	PTT High before Chlorideserum High $\rightarrow$ Phase III	0.6883	Correct
	Prothrombin time High and ALT Low $\rightarrow$ Phase III	0.6129	Correct
	AST High $\rightarrow$ Phase III	0.5802	Correct
	Chlorideserum Low equal Prothrombin time High $\rightarrow$ Phase III	0.0572	Correct
	Anion gap Low and Prothrombin time High $\rightarrow$ Phase III	0.0555	Correct
	Sodiumserum High equal PTT High $\rightarrow$ Phase III	0.0176	Correct
	BUN High equal Hemoglobin Low $\rightarrow$ Phase III	0.0084	Correct

TABLE VI
RULES ON THE STROKE DATASET.

Model	Rule	Weight	Correctness
TELLER	Temperature Fahrenheit High → Highly unconscious	0.7287	Incorrect
	Respiratory Rate High → Highly unconscious	0.2787	Correct
	Hematocrit Low → Highly unconscious	0.1373	Correct
	Arterial Blood Pressure systolic High → Highly unconscious	-0.1965	Correct
	Temperature Fahrenheit High → Highly unconscious	-0.4720	Correct
CLNN	c_Respiratory Rate High - c_ Highly unconscious> -0.02	1.07	Correct
	c_Glucose High - c_Highly unconscious > -0.86	1.03	Correct
	c Glucose High - c_Anion gap Low > 1.05	1.03	Correct
	c_Arterial Blood Pressure systolic High - c_O2 saturation pulseoxymetry Low > 0.00	0.99	Correct
	c_Hematocrit High - c_Respiratory Rate High > 0.16	0.97	Incorrect
	c_BUN Low - c_Respiratory Rate High > 0.30	0.97	Incorrect
	c_Temperature Fahrenheit Low - c_Respiratory Rate High > 0.10	0.97	Correct
	c_Respiratory Rate High - c_ BUN Low > -0.76	0.96	Correct
	c_Temperature Fahrenheit High - c_Creatinine Low > -0.11	0.88	Correct
	c_Anion gap High - c_ Arterial Blood Pressure systolic High > 0.21	0.87	Incorrect
CLUSTER	Arterial Blood Pressure systolic High after INR High	-	Correct
	Chloride High equal BUN High	-	Correct
	O2 saturation pulseoxymetry Low after Arterial O2 pressure Low	-	Correct
	Arterial CO2 Pressure Low before Creatinine Low	-	Incorrect
	Arterial Blood Pressure diastolic High equal Hemoglobin High	-	Correct
	HCO3 Low after Potassium Low	-	Incorrect
	Hematocrit Low after Arterial Blood Pressure diastolic High	-	Incorrect
	Arterial Base Excess High equal Glucose Low	-	Correct
	Heart Rate High after Arterial Blood Pressure systolic Low	-	Incorrect
	White Blood Cells High equal Arterial Blood Pressure diastolic Hig	-	Correct
	Arterial CO2 Pressure Low before Sodium Low	-	Correct
	Respiratory Rate High equal Arterial CO2 Pressure High	-	Correct
	Glucose High before Arterial Base Excess High	-	Correct
	Respiratory Rate Low equal Chloride Low	-	Incorrect
	White Blood Cells Low after Hemoglobin High	-	Incorrect
	Chloride High equal HCO3 Low	-	Correct
	HCO3 Low before Heart Rate Low	-	Correct
	O2 saturation pulseoxymetry Low before Hematocrit Low	-	Correct
	Heart Rate High equal Hematocrit Low	-	Correct
	Platelet Count Low equal Potassium High	-	Incorrect
HRTPP	Temperature Fahrenheit High and Hemoglobin High → Highly unconscious	3.7855	Correct
	Temperature Fahrenheit High → Highly unconscious	1.5625	Correct
	HCO3 Low equal Temperature Fahrenheit High → Highly unconscious	1.2197	Correct
	Glucose High before Respiratory Rate Low → Highly unconscious	1.0801	Correct
	Arterial Blood Pressure mean Low and Arterial Blood Pressure systolic High →Highly unconscious	0.9993	Correct
	Arterial Blood Pressure systolic High before Creatinine High → Highly unconscious	0.7416	Correct
	Creatinine High equal PTT High → Highly unconscious	0.6306	Correct
	Temperature Fahrenheit High equal Chloride High → Highly unconscious	0.5012	Correct

TABLE VII
RULES ON THE SEPSIS DATASET.

Model	Rule	Weight	Correctness
TELLER	PTT High → Low Urine	0.0953	Correct
	Platelet Count High → Low Urine	0.0909	Incorrect
	Hemoglobin Low → Low Urine	0.0898	Correct
	Respiratory Rate High → Low Urine	0.0865	Correct
CLNN	c_Low Urine - c_Arterial Blood Pressure diastolic Low > 0	0.80	Correct
	c_Heart Rate High - c_Hemoglobin High > -0.03	0.78	Incorrect
	c_Arterial Blood Pressure diastolic Low - c_ALT High > -0.06	0.75	Correct
	c_Heart Rate High - c_Arterial Base Excess High > -0.09	0.75	Correct
	c_Arterial Blood Pressure diastolic Low - c_C Reactive Protein High > -0.13	0.74	Correct
	c_Arterial Blood Pressure diastolic Low - c_Hemoglobin High > -0.17	0.73	Incorrect
	c_BUN Low - c_Respiratory Rate High > 0.27	0.68	Correct
	c_Heart Rate High - c_C Reactive Protein High > -0.17	0.61	Correct
	c_Heart Rate High - c_Glucose Low > -0.18	0.52	Correct
CLUSTER	Arterial Blood Pressure diastolic High after Potassium High	-	Correct
	Total Bilirubin High equal Temperature Celsius Low	-	Incorrect
	SpO2 Desat Limit Low after Total Bilirubin Low	-	Correct
	HCO3 Low after Glucose Low	-	Correct
	Platelet Count Low after Sodium Low	-	Correct
	Respiratory Rate High before Arterial CO2 Pressure High	-	Incorrect
	Prothrombin time High before PTT Low	-	Correct
	Arterial CO2 Pressure Low before Venous O2 Pressure High	-	Correct
	Potassium High equal Hemoglobin High	-	Incorrect
	Hemoglobin High equal Total Bilirubin Low	-	Correct
	SpO2 Desat Limit Low before Hemoglobin Low	-	Incorrect
	ALT High before AST Low	-	Incorrect
	Hemoglobin Low before INR Low	-	Incorrect
	Arterial Blood Pressure mean High after BUN Low	-	Correct
	Total Bilirubin High before D-Dimer High	-	Incorrect
	HCO3 Low before Chloride Low	-	Correct
	SpO2 Desat Limit Low before SpO2 Desat Limit High	-	Correct
	Hemoglobin Low equal INR High	-	Incorrect
	SpO2 Desat Limit Low after Fibrinogen High	-	Correct
	Respiratory Rate Low equal Hemoglobin High	-	Incorrect
HRTPP	TPotassium Low equal Lactic Acid High → Low Urine	1.4875	Correct
	Respiratory Rate High and HCO3 Low → Low Urine	1.4736	Correct
	Lactic Acid High equal PH Low → Low Urine	1.3934	Correct
	PH Low before Potassium Low → Low Urine	1.221	Correct
	HCO3 Low equal ALT High → Low Urine	1.1394	Correct
	Heart Rate High equal Arterial Blood Pressure diastolic Low → Low Urine	1.0613	Correct
	UN Low and Respiratory Rate High → Low Urine	0.8878	Incorrect
	Fibrinogen Low equal Respiratory Rate High → Low Urine	0.7745	Correct
	Magnesium High before Respiratory Rate High → Low Urine	0.7687	Incorrect
	Lactic Acid High equal PTT High → Low Urine	0.5762	Correct

TABLE VIII
COMPLETE CLUSTER RULES ON THE CAD DATASET.

No.	Rule	Correctness
1	BUN Low equal Lactic Acid Low	Incorrect
2	Hemoglobin High equal Hematocrit High	Incorrect
3	Brain Natiuretic Peptide High after ALT Low	Correct
4	Heart Rate High before AST Low	Correct
5	Calcium ionized High equal Total Bilirubin High	Incorrect
6	CK-MB fraction High equal Calcium ionized Low	Correct
7	Calcium ionized Low equal Cholesterol High	Correct
8	Temperature Low equal Hemoglobin High	Correct
9	Hematocrit Low equal Total Bilirubin Low	Incorrect
10	Glucose Low before Anion gap Low	Incorrect
11	Temperature Low equal INR High	Correct
12	Heart Rate Low before CK-MB fraction High	Correct
13	Arterial Blood Pressure diastolic Low after Anion gap Low	Correct
14	AST Low equal INR Low	Incorrect
15	CK Low after CK High	Incorrect
16	Platelet Count High equal White Blood Cells Low	Correct
17	Heart Rate Low equal Lactic Acid Low	Incorrect
18	Potassium Low equal Total Bilirubin Low	Correct
19	O2 saturation pulseoxymetry High equal Albumin High	Incorrect
20	Temperature High before Cholesterol High	Correct

D. Ablation Experiment of the Two-Phase Mining Strategy

In the rule mining module, we propose a two-phase mining strategy to enhance the stability and accuracy of rule extraction. To assess its impact, we compare the accuracy of rule extraction with and without this strategy. Table IX illustrates the improvement in rule accuracy when the two-phase mining strategy is applied. The experimental results demonstrate that this strategy significantly enhances rule extraction precision, further validating its effectiveness in medical event modeling.

TABLE IX
RULE ACCURACY OF DIFFERENT MINING STRATEGIES.

	AKI	Stroke	Sepsis	CAD
HRTPP w/o two phase mining	80%	80%	71.42%	70%
HRTPP with two phase mining	**100%**	**100%**	**80%**	**100%**

The experimental results show that the two-phase mining strategy optimizes the precision of the rules in different disease scenarios, enhancing the interpretability of the model. Specifically, in the AKI, Stroke, and CAD datasets, rule accuracy reaches 100%, indicating that this strategy effectively improves the reliability of rule extraction. In the Sepsis dataset, although the accuracy improvement is relatively smaller (from 71.42% to 80%), it still confirms the applicability of this approach to complex medical conditions. These findings suggest that by leveraging point process modeling and an optimized rule mining strategy, we can develop a more stable and interpretable medical event prediction model, providing more reliable support for clinical decision-making.