Application of MATLAB in Complex Variable Function

December 21, 2022

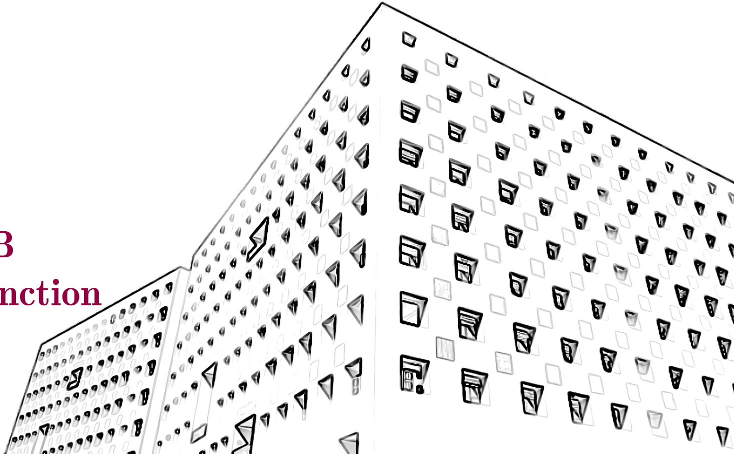

Table of Contents

1 Generation of complex numbers and complex matrices

Complex variable functions are deeply related to real variable functions. Many theorems and operation rules of complex variable functions promote the theory of real variable functions. Understanding this is very helpful for learning complex variable functions. However, complex variable functions have their own characteristics. Some operation rules come from the promotion of the operation rules of real variable functions, but they also have obvious characteristics different from real variable functions. This chapter focuses on the application of Maltab in complex variable functions. It is precisely because the complex variable function and the real variable function are so closely related that most Matlab commands dealing with complex variable functions and those dealing with real variable functions are the same command.

The complex number can be generated by the statement `z=a+b*i`, or it can be abbreviated as `z=a+bi`; Another statement for generating complex numbers is `z=r*exp(i*theta)`, which can also be abbreviated as `z=r*exp(theta i)`, where theta is the radian value of the argument of the complex number, and r is the module of the complex number.

Generation of complex matrices

There are two ways to create a complex matrix:

- Just like the general matrix, input the matrix in the ways described above.
  e.g., `A = [3+5*i, -2+3i, 9*exp(i*6), 23*exp(33i)]`
- Create a real matrix and a virtual matrix separately, and then write them in the form of sum.
  e.g., `re = rand(3,2); im = rand(3,2); com = re+i*im`
  The result is:

```
com =
      0.9501 + 0.4565i   0.4860 + 0.4447i
      0.2311 + 0.0185i   0.8913 + 0.6154i
      0.6068 + 0.8214i   0.7621 + 0.7919i
```

Table of Contents

Real and Imaginary Parts, Complex Conjugate, Modulus and Argument of Complex Numbers

- Real and Imaginary Parts

  `real(Z)` returns the real part of Z.

  `imag(Z)` returns the imaginary part of Z.

- Complex Conjugate

  `conj(Z)` returns the complex conjugate of Z.

- Modulus and Argument

  `abs(Z)` returns the complex magnitude of Z.

  `angle(Z)` returns the phase angle of Z.

Real and Imaginary Parts, Complex Conjugate, Modulus and Argument of Complex Numbers

Find the real and imaginary parts, complex conjugate, modulus and argument of the following complex number.

```
a=[1/(3+2i),1/i-3i/(1-i),(3+4i)*(2-5i)/2i,i^9-4*i^21+i]
R=real(a)
M=imag(a)
Con=conj(a)
Abs=abs(a)
Ang=angle(a)
```

Real and Imaginary Parts, Complex Conjugate, Modulus and Argument of Complex Numbers

The result is:

```
a = 0.23077 -  0.15385i    1.50000 -  2.50000i   -3.50000
    - 13.00000i    0.00000 -  2.00000i
R = 0.23077   1.50000  -3.50000   0.00000
M = -0.15385   -2.50000  -13.00000   -2.00000
Con = 0.23077 +  0.15385i    1.50000 +  2.50000i
    -3.50000 + 13.00000i    0.00000 +  2.00000i
Abs = 0.27735    2.91548   13.46291    2.00000
Ang = -0.58800  -1.03038  -1.83379  -1.57080
```

Multiplication and division, square root, power operation of complex numbers

- Multiplication and division of complex numbers are implemented by “/” and “*”.
- Square root of complex number
  `sqrt(Z)` returns the square root of Z.
- Power operation of complex number: `Z^n`

Exponential and logarithmic operations, roots of equations and trigonometric operations of complex numbers

- Exponential and logarithmic operations

  `exp(Z)` returns the exponential operation of Z.

  `log(Z)` returns the logarithmic operation of Z.

- Finding the root of complex equation or solving the complex root of equation can be solved by the function "solve".

  e.g., Find all roots of equation $x^3 + 8 = 0$.

```
roots=solve('x^3+8=0')
```

  The result is:

```
roots = [         -2]
        [ 1-i*3^(1/2)]
        [ 1+i*3^(1/2)]
```

- Trigonometric operations

函数名	函数功能	函数名	函数功能
sin(x)	返回复数x的正弦函数值	asin(x)	返回复数x的反正弦值
cos(x)	返回复数x的余弦函数值	acos(x)	返回复数x的反余弦值
tan(x)	返回复数x的正切函数值	atan(x)	返回复数x的反正切值
cot(x)	返回复数x的余切函数值	acot(x)	返回复数x的反余切值
sec(x)	返回复数x的正割函数值	asec(x)	返回复数x的反正割值
csc(x)	返回复数x的余割函数值	acsc(x)	返回复数x的反余割值
sinh(x)	返回复数x的双曲正弦值	coth(x)	返回复数x的双曲余切值
cosh(x)	返回复数x的双曲余弦值	sech(x)	返回复数x的双曲正割值
tanh(x)	返回复数x的双曲正切值	csch(x)	返回复数x的双曲余割值

Table of Contents

3 Operation of complex numbers

- Limit

  The command "limit()" is still used to calculate the limit of complex variable functions, but the conditions for the existence of the limit of complex variable functions are more stringent than those of real variable functions. The existence of limit of complex variable function requires that the real part and imaginary part of complex variable function have limit at the same time. The command format is as follows: `limit(F,x,a)`

  e.g., z is a complex number, there is a complex variable function $f(z) = z/(1+z)$, find the limit: $\lim\limits_{z \to 1+5i} f(z)$.

```
clear
syms z
f=z/(1+z);
limit(f,z,1+5*i)
```

- Derivative
  The command to calculate the derivative of a complex variable function is still "diff()". The specific format is: `diff(function, 'variable')`.
  e.g., Find the derivative of $\ln(1+\sin z)$ at $z=i/2$, and the derivative of $\sqrt{(z-1)(z-2)}$ at $z=3+i/2$.

```
syms z
f1=log(1+sin(z));
f2=sqrt((z-1)*(z-2));
df1=diff(f1,z)
df2=diff(f2,z)
vdf1=subs(df1,z,i/2)
vdf2=subs(df2,z,3+i/2)
```

The result is:

```
df1= cos(z)/(sin(z) + 1)
df2= (2*z - 3)/(2*((z - 1)*(z -2))
     ^(1/2))
vdf1= cosh(1/2)/(1 + sinh(1/2)*1i)
vdf2= (3/2 + 1i/2)/(7/4 +3i/2)^(1/2)
```

- Integral
  The definite integral of complex variable function is no different from the definite integral of real variable function in form, but the integral limit has changed from the original real number to a complex number. The specific format is: `int(function, variable, a, b)`, where function is the complex variable function expression to be integrated, variable is the integration variable, and a and b are the lower and upper limit of integration.
  e.g., Calculate definite integrals $\int_0^i z\cos z dz$ and $\int_0^i \frac{\ln(z+1)}{z+1} dz$.

```
syms z
f1=z*cos(z);
f2=log(z+1)/(z+1);
inf1=int(f1,z,0,i)
inf2=int(f2,z,0,i)
```

The result is:

```
inf1=cosh(1)-sinh(1)-1
inf2=1/2*log(1+i)^2-1/2*log(2)^2
```

Table of Contents

4 Taylor expansion of functions of complex variables

Taylor series expansion plays an important role in complex variable functions, such as the analyticity of complex variable functions. Taylor series of function $f(x)$ at the point $x = x_0$ is expanded as follows:

$$f(x) = x_0 + f(x_0)(x - x_0) + \frac{f'(x_0)(x - x_0)}{2!} + \frac{f''(x_0)(x - x_0)^2}{3!} + \cdots$$

In Matlab, it can be implemented by the "taylor" function. The specific format is: `taylor(f,varriable,a, 'PARAM1',val1,'PARAM2',val2,...)`, f is the function expression to be expanded, n represents the first n items of the output expansion, variable represents the expanded variable, and a represents the value point of variable derivation.

e.g., Expand function $\dfrac{1}{(1+z)^2}$ into power series of complex variable z.

```
syms z
f=1/(1+z)^2;
F=taylor(f,z,9,'Order',10)
```

The result is:

```
F=1-2*z+3*z^2-4*z^3+5*z^4-6*z^5+7*z^6-8*z^7+9*z^8-10*z
    ^9
```

Table of Contents

- Definition
  Let a be an isolated singular point of $f(z)$, and C be a closed path containing a point in a sufficiently small neighborhood of a, integral

$$\frac{1}{2\pi i}\oint_C f(z)dz$$

It is called the residue or residue of $f(z)$ at point a and recorded as $Res[f(z), a]$.

- Calculation

  Residue plays an important role in complex variable functions. It can be used to calculate the integral of complex variable functions. In the following, for the case that the numerator and denominator of the complex variable function are polynomials, the calculation method of residues is given. Matlab provides the command “residue()” to calculate residues, which is used to process complex variable functions whose numerator and denominator are both polynomial forms. The format of the command to calculate residues is as follows: `[r,p]=residue(B,A)` Parameter B is a vector composed of the coefficients of the numerator of a complex variable function, parameter A is a vector composed of the coefficients of the denominator of a complex variable function, and parameter r returns a residue, which is a vector composed of residues at different singular points. The parameter p returns the singular point, which is also a vector.

Computing residue of complex variable function $\dfrac{z}{z^4-1}$.

```
B=[1,0];
A=[1,0,0,0,-1];
[r,p]=residue(B,A)
```

The result is:

```
R=  0.2500
    0.2500
   -0.2500+0.0000i
   -0.2500-0.0000i
P=-1.0000
    1.0000
    0.0000+1.0000i
    0.0000-1.0000i
```

Compute residue of complex variable function $\dfrac{z^3+3z^2+2}{z^2+6z-1}$.

```
B=[1,3,0,2];
A=[1,6,-1];
[R,P]=residue(B,A)
```

The result is:

```
R=18.6706
    0.3294
P=-6.1623
    0.1623
```

5 Residue

Compute the integral $\oint_C \frac{z}{z^4-1}dz$, where C is the positive circumference, $|z|=2$.

```
B=[1,0];
A=[1,0,0,0,-1];
[R,P]=residue(B,A)

I=0;
for i = 1:length(P)
    if abs(P(i))^2 < 2
        I=I+R(i);
    end
end
I = 2*pi*i*I
```

The result is:

```
R= 0.2500
   0.2500
  -0.2500+0.0000i
  -0.2500-0.0000i
P=-1.0000
   1.0000
   0.0000+1.0000i
   0.0000-1.0000i
I=0
```

Table of Contents

6 Figure of complex variable function

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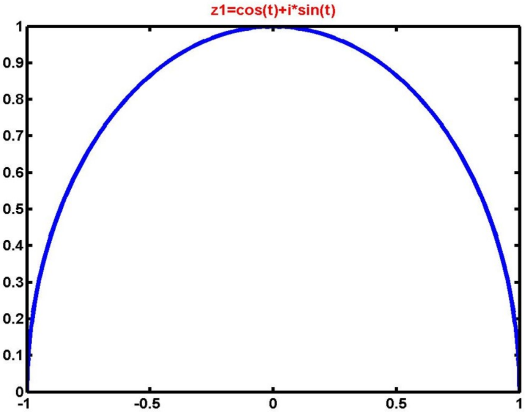
Draw the figures represented by the following complex numbers: $z = \cos t + i\sin t$ and $z = e^{it} + e^{-it}$, where t is a real number.

```
syms x y z t s
t=0:0.01*pi:pi;
x=cos(t);y=sin(t);
z1=x+i.*y
plot(z1)
title('z1=cos(t)+i*sin(t) ');
s=-5:0.01:5;
z2=exp(i.*s)+exp(-i.*s);
figure
plot(z2)
title('z2=exp(i*s)+exp(-i*s)')
```

6 Figure of complex variable function



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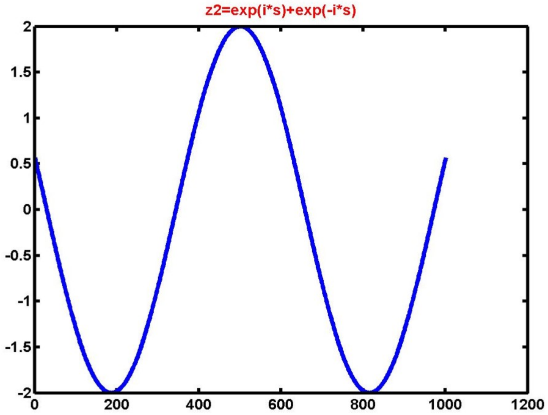

6 Figure of complex variable function

Draw the figure of $f(z) = z^3$.

```
colormap(hsv(64));
z = cplxgrid(30);
cplxmap(z, z.^3);
colorbar('vert');
```

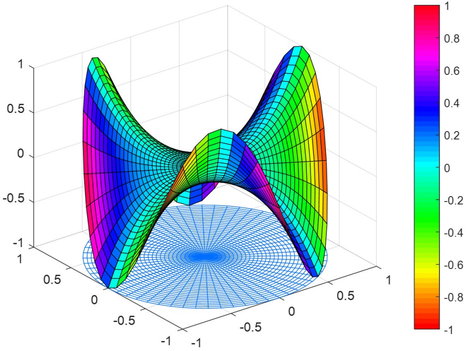

Table of Contents

Use Matlab to solve complex variable function problems

7 Experiment

1. Calculate the values of the following functions

   1) $\sqrt{1+i}$; 2) $\sqrt{-2+2i}$; 3) $\sqrt[6]{\sqrt{3}+\left(2\sqrt{3}-3\right)i}$.

2. Solve equation $\begin{cases} z_1+2z_2=1+i \\ 3z_1+iz_2=2-3i \end{cases}$.

3. Calculate limits: 1) $\lim\limits_{n\to\infty}\left(\frac{3+4i}{6}\right)^n$; 2) $\lim\limits_{n\to\infty}\left(n+\frac{n^2}{2}\right)^{\frac{1}{n}}$.

4. Calculate the first-order derivative of $f(z)=\left(z^2-1\right)^2\left(z^2+1\right)^2$ at the point $z=i/2$.

5. Calculate the integrals: 1) $\int_1^i \frac{1+\tan z}{\cos^2 z}dz$; 2) $\int_0^i (z-i)e^{-z}dz$.

Use Matlab to solve complex variable function problems

7 Experiment

6. Expand the top 10 Taylor items of the following expressions.

   1) $\sin(3+z)$; 2) $e^z \ln(1+z)$; 3) $\dfrac{2z^5+5z^3+z^2+2}{z^3+2z^2+3z+1}$.

7. Calculate the residue of $\dfrac{1+z^4}{z\left(z^2+1\right)^2}$ at its singular point.

8. Calculate the integral $\displaystyle\oint_{|z|=1} \frac{2z^2+1}{2z^3+z^2+4z-5}dz$.

9. Draw the figures represented by the following complex numbers, where t is a real number. 1) $z=t+it^2$; 2) $z=t+ie^t\sin t$.