

$$F = \{Z = a.Z + b.Y, Y = b.Y\}$$

We define an auxiliary guarded recursive specification $E' = \{Z_n = a.Z_{n+1} + b.Y \mid n \in \mathbb{N}\}$

and define a sequence of terms denoting a solution for E' :

Define $t_0, t_1, \dots, t_i, \dots$ ($i \in \mathbb{N}$) with induction on i by $t_0 \equiv X$ and $t_{i+1} \equiv t_i \triangleright b.Y$

We prove that $t_i = a.t_{i+1} + b.Y$ for all $i \in \mathbb{N}$ with induction on i :

If $i=0$, then $t_i \equiv t_0 = X = a.X \triangleright b.Y \stackrel{P_2}{=} a.(X \triangleright b.Y) + b.Y = a.t_1 + b.Y$

Let $i \in \mathbb{N}$ and suppose that $t_i = a.t_{i+1} + b.Y$ (IH). Then

$$\begin{aligned} t_{i+1} \equiv t_i \triangleright b.Y &\stackrel{IH}{=} (a.t_{i+1} + b.Y) \triangleright b.Y \stackrel{P_2}{=} a.(t_{i+1} \triangleright b.Y) + (b.Y \triangleright b.Y) \\ &\stackrel{D_2}{=} a.(t_{i+1} \triangleright b.Y) + b.Y + (b.Y \triangleright b.Y) \\ &= a.t_{i+2} + b.Y \end{aligned}$$

So t_0, t_1, t_2, \dots denotes a solution of E'

Define a sequence of terms denoting a solution of F :

Define $u_0, u_1, \dots, u_i, \dots$ ($i \in \mathbb{N}$) with induction on i by $u_0 \equiv Z$ and $u_{i+1} \equiv u_i$

We prove that $u_i = a.u_{i+1} + b.Y$ for all $i \in \mathbb{N}$ with induction on i :

If $i=0$, then $u_i \equiv u_0 = Z = a.Z + b.Y = a.u_1 + b.Y$

Let $i \in \mathbb{N}$ and suppose that $u_i = a.u_{i+1} + b.Y$ (IH). Then

$$u_{i+1} \equiv u_i + b.Y \stackrel{IH}{=} (a.u_{i+1} + b.Y) + b.Y = a.u_{i+1} + b.Y = a.u_{i+2} + b.Y$$

So u_0, u_1, u_2, \dots denotes a solution of E' .

Hence both t_0, t_1, \dots and u_0, u_1, \dots denote solution of E' , and therefore, in every model satisfying RSP we have that $\mu X.E = X \equiv t_0 = u_0 \equiv Z = \mu Z.F$