Process Algebra (2IMF10) — Assignment 3

Deadline: Friday June 16, 2023

This is the third assignment for the course on *Process Algebra* (2IMF10). Please submit your solutions via Canvas. The only accepted format for your document is PDF. As before, feel free to simplify notation and write X instead of $\mu X.E$, Y instead of $\mu Y.F$, etc.

Simulating infinitely branching behaviour in $\mathrm{TCP}_\tau(A,\gamma)$

Consider the (unguarded!) recursive specification E consisting of the following equations

$$\begin{aligned} X &= a.(X' \cdot X) \\ X' &= X' \cdot c.1 + b.1 \ . \end{aligned}$$

The goal of this assignment is to show that the behaviour associated with $\mu X.E$ by the operational semantics is finitely definable with abstraction in the process theory $\mathrm{TCP}_\tau(A,\gamma)$.

1. Sketch the transition system associated with $1 \cdot (\mu X.E)$ and notice that it is infinitely branching.
2. Sketch a *finitely branching* transition system that is rooted branching bisimilar to $\mu X.E$. Then define a guarded recursive specification F over $\mathrm{BSP}(A)$ including a variable Y such that

   $$\tau_{\{i\}}(\mu Y.F) \leftrightarroweq_{\mathrm{rb}} 1 \cdot (\mu X.E) \ .$$

   Prove the correctness of your specification F by defining a branching bisimulation that satisfies the root condition for $\tau_{\{i\}}(\mu Y.F)$ and $1 \cdot (\mu X.E)$.
3. Let G_1 be the recursive specification consisting of the following equations (which defines the behaviour of a counter that signals when it is zero):

   $$\begin{aligned} C &= \mathit{zero}.C' \cdot C \\ C' &= 1 + \mathit{inc}.C' \cdot \mathit{dec}.C' \ . \end{aligned}$$

   Declare a suitable set of actions A (which should at least include the actions a, b, c, zero, inc and dec, i, and possibly more), define a suitable communication function γ on A, define a suitable set $H \subseteq A$, and give a finite guarded recursive specification G_2 over $\mathrm{BSP}(A)$ including a definition for recursion variable Z such that

   $$\tau_{\{i\}}(\partial_H(\mu C.G_1 \parallel \mu Z.G_2)) \leftrightarroweq_{\mathrm{rb}} 1 \cdot (\mu X.E) \ ,$$

   and **prove that your answer is correct**.

   (Note that since $p \leftrightarroweq q$ implies $p \leftrightarroweq_{\mathrm{rb}} q$ and $\leftrightarroweq_{\mathrm{rb}}$ is compatible with $\tau_{\{i\}}$, by part 2 of the assignment it suffices to show that $\partial_H(\mu C.G_1 \parallel \mu Z.G_2) \leftrightarroweq \mu Y.F$.)