# Examination Process Algebra (2IMF10)

28 June 2022, 13:30 – 16:30

The number between parentheses in front of a problem indicates how many points you score with a correct answer to it. A partially correct answer is sometimes awarded with a fraction of those points. The grade for this examination will be determined by dividing the total number of scored points by 2.

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1. A word $\sigma \in A^*$ is a trace of a state $s$ in a transition-system space over the set of labels $A$ if, and only if, there exists a state $s'$ such that $s \xrightarrow{\sigma}^* s'$. We denote by $tr(s)$ the set of all traces of $s$. Two states are trace equivalent (notation: $s =_T t$) if they have the same sets of traces, i.e., if $tr(s) = tr(t)$.

(3) (a) Give an equation that is valid in $\mathbb{P}(\mathrm{MPT}(A))/_{=_T}$ but not in $\mathbb{P}(\mathrm{MPT}(A))/_{\underline{\leftrightarrow}}$. Support your answer with two proofs: one that the equation is valid in $\mathbb{P}(\mathrm{MPT}(A))/_{=_T}$ and one that the equation is not valid in $\mathbb{P}(\mathrm{MPT}(A))/_{\underline{\leftrightarrow}}$.

(2) (b) Is the process theory $\mathrm{MPT}(A)$ a ground-complete axiomatisation of $\mathbb{P}(\mathrm{MPT}(A))/_{=_T}$? (Motivate your answer.)

(1) 2. (a) Prove that $\mathrm{BCP}(A, \emptyset) \vdash x \parallel y = y \parallel x$.

Let $E$ be the recursive specification over $\mathrm{BCP}(A, \emptyset)$ consisting of the following equation:

$$X = 1 + a.(X \parallel b.1) \ .$$

(Throughout this exam, feel free to abbreviate $\mu X.E$ by simply writing $X$.)

(4) (b) Define an infinite guarded recursive specification $F$ over $\mathrm{BSP}(A)$ (i.e., not using parallel composition) defining a recursion variable $Y_0$ such that

$$\mathrm{BCP_{rec}}(A, \emptyset)\text{+RSP} \vdash \mu X.E = \mu Y_0.F$$

and prove that your answer is correct.

(3) (c) Does there also exist a finite guarded recursive specification $G$ over $\mathrm{BSP}(A)$ defining a recursion variable $Z$ such that

$$\mathrm{BCP_{rec}}(A, \emptyset)\text{+RSP} \vdash \mu X.E = \mu Z.G \ ?$$

Support your answer with a proof.

3. Let $E$ be the recursive specification over $\mathrm{TSP}(A)$ consisting of the following equation:

$$X = X \cdot a.1 + 1$$

(Throughout this exam, feel free to abbreviate $\mu X.E$ by simply writing $X$.)

(2) (a) Find two distinct closed $\mathrm{TSP_{rec}}(A)$-terms $p$ and $q$ such that $\mu X.E \xrightarrow{a} p$ and $\mu X.E \xrightarrow{a} q$, and prove the correctness of your answers by giving formal derivations of the transitions in accordance with the term deduction system (i.e., the structural operational semantics) for $\mathrm{TSP_{rec}}(A)$.

(1) (b) Sketch (a representative part of) the transition system associated with $\mu X.E$ by the term deduction system for $\mathrm{TSP_{rec}}(A)$.

(2) (c) Define a closed $\mathrm{TSP}_{rec}(A)$-term $r$ that denotes a solution for $E$ in $\mathbb{P}(\mathrm{TSP}_{\mathrm{rec}}(A))/_{\underline{\leftrightarrow}}$ but is not bisimilar to $\mu X.E$ (i.e., $r \not\underline{\leftrightarrow} \mu X.E$).

(2) (d) Let $F$ be the recursive specification consisting of the following equations:

$$\begin{array}{lcl} Y_0 & = & \tau.Y_1 + 1 \\ Y_{n+1} & = & \tau.Y_{n+2} + \tau.Y_n + a^{n+1}.1 \quad (n \in \mathbb{N}) \end{array}$$

(Here $a^n.1$ is inductively defined by $a^0.1 \equiv 1$ and $a^{n+1}.1 \equiv a.a^n.1$ for all $n \in \mathbb{N}$.)
Prove that $\mu Y_0.F \underline{\leftrightarrow}_b \mu X.E$.

(2) 4. Let $A$ be a set of actions that contains at least three distinct actions $a$, $b$ and $c$. Prove that there exists a communication function $\gamma$ on $A$ and a set $I \subseteq A$ such that

$$\mathrm{TCP}_\tau(A, \gamma) \not\vdash \tau_I(x \parallel y) = \tau_I(x) \parallel \tau_I(y) \ .$$