Process Algebra (2IMF10)

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Solution to Exercise 6.6.8

Let $E = \{X = X \cdot a.1 + a.1\}$. The standard solution for E in $\mathbb{P}(\mathrm{TSP}_{\mathrm{rec}}(A))/\underline{\leftrightarrow}$ assigns to X the element of $\mathbb{P}(\mathrm{TSP}_{\mathrm{rec}}(A))/\underline{\leftrightarrow}$ denoted by $\mu X.E$, i.e., $[\mu X.E]_{\underline{\leftrightarrow}}$. To see that $\mu X.E$ indeed denotes a solution for E, it suffices to note that, according to the operational rules for recursion, $\mu X.E \xrightarrow{\alpha} p'$ if, and only if, $\mu X.E \cdot a.1 + a.1 \xrightarrow{\alpha} p'$ and $\mu X.E\!\downarrow$ if, and only if, $(\mu X.E \cdot a.1 + a.1)\!\downarrow$, so $\mu X.E \;\underline{\leftrightarrow}\; \mu X.E \cdot a.1 + a.1$. (Define the bisimulation yourself!)

To define an alternative solution for E, consider the recursive specification

$$F = \left\{ \begin{array}{l} Y = Y \cdot a.1 + a.1 + b.Z \ , \\ Z = b.Z \end{array} \right\}$$

We argue that every solution for Y in F is also a solution for X in E. For this, it is convenient to first establish that

$$\mu Z.F = \mu Z.F \cdot a.1 \ . \tag{1}$$

To this end, note that

$$\mu Z.F \cdot a.1 \overset{\mathrm{Rec}}{=} (b.\mu Z.F) \cdot a.1 \overset{\mathrm{A10}}{=} b.(\mu Z.F \cdot (a.1)) \ ,$$

and hence, by RSP, $\mu Z.F = \mu Z.F \cdot a.1$. Now we have the following derivation

$$\begin{array}{lcl}
\mu Y.F & \overset{\mathrm{Rec}}{=} & (\mu Y.F) \cdot a.1 + a.1 + b.(\mu Z.F) \\
 & \overset{\mathrm{Rec}}{=} & ((\mu Y.F) \cdot a.1 + a.1 + b.(\mu Z.F)) \cdot a.1 + a.1 + b.(\mu Z.F) \\
 & \overset{(1)}{=} & ((\mu Y.F) \cdot a.1 + a.1 + b.(\mu Z.F)) \cdot a.1 + a.1 + b.(\mu Z.F) \cdot (a.1) \\
 & \overset{\mathrm{A1,A2,A4}}{=} & ((\mu Y.F) \cdot a.1 + a.1 + b.(\mu Z.F) + b.(\mu Z.F)) \cdot a.1 + a.1 \\
 & \overset{\mathrm{A3,\ Rec}}{=} & (\mu Y.F) \cdot a.1 + a.1
\end{array}$$

This derivation proves that the process denoted by $\mu Y.F$ denotes a solution for E too. Moreover, it is clear that $\mu X.E$ and $\mu Y.F$ denote distinct processes in $\mathbb{P}(\mathrm{TSP}_{\mathrm{rec}}(A))/\underline{\leftrightarrow}$, for $\mu Y.F \xrightarrow{b} \mu Z.F$, whereas $\mu X.E$ does not admit a b-transition and hence $\mu X.E \;\not\!\underline{\leftrightarrow}\; \mu Y.F$.

Figure 1 (on the next page) depicts the transition systems associated with X and Y. The picture is not part of the solution, but may give some intuition.

We proceed to prove that every solution for E in $\mathbb{P}(\mathrm{TSP}_{\mathrm{rec}}(A))/\underline{\leftrightarrow}$ is necessarily infinitely branching.

To this end, let s be any closed $\mathrm{TSP}_{\mathrm{rec}}(A)$-term that denotes a solution for E. We prove with induction on $n \in \mathbb{N}$ that there exists s' such that $s \xrightarrow{a} s'$ and $s' \;\underline{\leftrightarrow}\; (a.1)^n$.

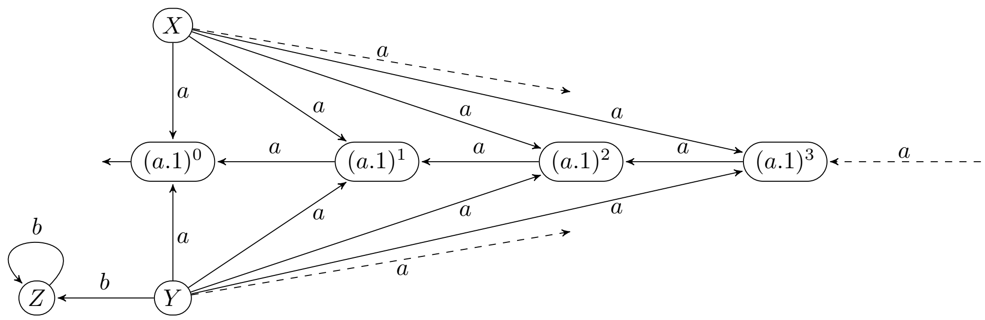

Figure 1: The transition systems associated with X, Y and Z by the structural operational semantics.

If $n = 0$, then, since s denotes a solution and hence $s \leftrightarroweq s \cdot (a.1) + a.1$, there exists s' such that $s \xrightarrow{a} s'$ and $s' \leftrightarroweq 1 = (a.1)^0$.

Let $n \geq 0$ and suppose that there exists s' such that $s \xrightarrow{a} s'$ and $s' \leftrightarroweq (a.1)^n$. Then, by the operational semantics, $s \cdot (a.1) + a.1 \xrightarrow{a} s' \cdot (a.1)$. Clearly, $s' \cdot (a.1) \leftrightarroweq (a.1)^{n+1}$, and since $s \leftrightarroweq s \cdot (a.1) + a.1$ it follows that there exists s'' such that $s \xrightarrow{a} s''$ and $s'' \leftrightarroweq (a.1)^{n+1}$.

Since $k \neq \ell$ implies $(a.1)^k \not\leftrightarroweq (a.1)^\ell$, it now follows that s is infinitely branching.