Process Algebra (2IMF10) — Assignment 1

Deadline: Friday May 12, 2023

This is the first assignment for the course on *Process Algebra* (2IMF10).

Submission is via Canvas. The only accepted format for your document is PDF.

You should do this assignment together with one or two fellow students and submit your solutions as a group via Canvas (clearly list the names and student numbers of all contributors). The only accepted format for your document is PDF. **We urge you, however, not to divide work between the two or three of you, but rather work on each part of the exercise together. Only then you will get the necessary skills for the exam!**

The goal of this assignment is, first of all, to stimulate you to think about the subject. The assignments may be challenging and you may need hints from the lecturer when solving them. You are actually encouraged to contact the lecturer when you are stuck. Also if you think that you have solved the problems, feel free to discuss your solutions with the lecturer before handing them in. In particular, the lecturer will give feedback on (the correctness of) operational rules and axioms if you show them to him, and help you with finding suitable axioms. Pay specific attention to how you write proofs; it is part of the assignment to learn to write formal arguments. Comment, also, on each other's formulations.

Theory of Sequential Processes with Disrupt in Simulation Semantics

The starting point of this assignment is the process theory $\mathrm{MPT_S}(A)$, which has the same signature as $\mathrm{MPT}(A)$ and the following axioms:

$$\begin{array}{lll}
x + y & = y + x & \text{A1} \\
(x + y) + z & = x + (y + z) & \text{A2} \\
x + x & = x & \text{A3} \\
x + 0 & = x & \text{A6} \\
a.(x + y) & = a.(x + y) + a.y & \text{S}
\end{array}$$

The axiom S is not valid in bisimulation semantics, but it is valid in *simulation semantics.*

Definition 1 (Simulation semantics)**.** *A binary relation R on the set of states S of a transition-system space is a* simulation relation *if the following condition holds:*

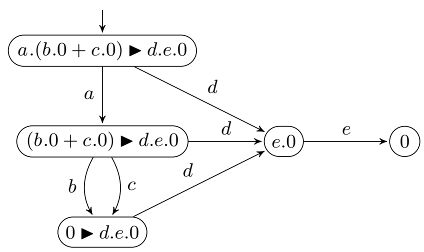
> *for all states $s, t, s' \in S$, whenever $(s, t) \in R$ and $s \xrightarrow{a} s'$ for some a in the set of labels A of the transition-system space, then there is a state $t' \in S$ such that $t \xrightarrow{a} t'$ and $(s', t') \in R$.*

Two states $s, t \in S$ are simulation equivalent *(notation: $s \leftrightarrows t$) if there exist simulation relations R_1 and R_2 such that sR_1t and tR_2s.*

The term deduction system for $\mathrm{MPT_S}(A)$ is the same as the term deduction system for $\mathrm{MPT}(A)$ (see Table 4.2 of [1]). Two closed $\mathrm{MPT_S}(A)$-terms p and q are simulation equivalent if they are simulation equivalent in the transition-system space associated to $\mathrm{MPT_S}(A)$ by the term deduction for $\mathrm{MPT_S}(A)$. In your solution to the assignment you may use that $\leftrightarrows$ is a congruence for $\mathrm{MPT_S}(A)$ and that the following theorem holds for the algebra of closed $\mathrm{MPT_S}(A)$-terms modulo simulation equivalence:

Theorem 2. *Simulation equivalence $\leftrightarrows$ is a congruence on the algebra $\mathbb{P}(\mathrm{MPT_S}(A))$ of closed $\mathrm{MPT_S}(A)$-terms, and the process theory $\mathrm{MPT_S}(A)$ is a sound and ground-complete axiomatisation of the algebra of closed $\mathrm{MPT_S}(A)$-terms modulo simulation equivalence.*

The assignment is about extending the theory $\mathrm{MPT_S}(A)$ with a binary *disrupt* operator $\blacktriangleright$. The process $p \blacktriangleright q$ executes p, but q may disrupt the execution of p by starting its own execution. Consider the following transition system, associated with the term $a.(b.0 + c.0) \blacktriangleright d.e.0$, which illustrates the behaviour of disrupt:



1. Extend the term deduction system for $\mathrm{MPT_S}(A)$ with operational rules for the disrupt operator to get a term deduction system for $\mathrm{MPT_S{+}D}(A)$.

2. One way to get confidence in the the correctness of your extended term deduction system is by showing that it indeed associates the transition system above with the closed $\mathrm{MPT_S{+}D}(A)$-term $a.(b.0 + c.0) \blacktriangleright d.e.0$. Present formal derivations for three of the transitions appearing in that transition system.

3. Prove that simulation equivalence is a congruence on $\mathbb{P}(\mathrm{MPT_S{+}D}(A))$, the algebra of closed $\mathrm{MPT_S{+}D}(A)$-terms.

   (In view of Theorem 2, you may assume that simulation equivalence is, indeed, an equivalence, and that it is compatible with the operations of $\mathrm{MPT_S}(A)$, and so you may focus on proving that simulation equivalence is compatible with disrupt. Provide a detailed argument that refers to the relevant definitions, and, in particular, the operational rules for disrupt.)

4. Propose a suitable set of axioms for the disrupt operator (characterising its interplay with the operations of $\mathrm{MPT_S}(A)$) and prove that your axioms are valid in the algebra of closed $\mathrm{MPT_S{+}D}(A)$-terms modulo simulation equivalence.

5. Prove that your axioms facilitate the elimination of the disrupt operator from closed $\mathrm{MPT_S{+}D}(A)$-terms, and conclude that $\mathrm{MPT_S{+}D}(A)$ is a ground-complete axiomatisation for the algebra of closed $\mathrm{MPT_S{+}D}(A)$-terms modulo simulation equivalence.

References

[1] J. C. M. Baeten, T. Basten, and M. A. Reniers. *Process Algebra: Equational Theories of Communicating Processes.* Number 50 in Cambridge Tracts in Theoretical Computer Science. Cambridge University Press, 2010.