Examination Process Algebra (2IMF10)

16 August 2022, 9:00 – 12:00

The number between parentheses in front of a problem indicates how many points you score with a correct answer to it. A partially correct answer is sometimes awarded with a fraction of those points. The grade for this examination will be determined by dividing the total number of scored points by 2.

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(1) 1. (a) Prove that $\mathrm{BCP}_{\mathrm{rec}}(A,\emptyset) \vdash x \parallel y = y \parallel x$ and $\mathrm{BCP}_{\mathrm{rec}}(A,\emptyset) \vdash 0 \parallel 0 = 0$.

Consider the recursive specification E consisting of the following equation:

$$X = a.(X \parallel X) + b.0 \ .$$

(Feel free to abbreviate $\mu X.E$ by simply writing X.) Furthermore, define closed $\mathrm{BCP}_{\mathrm{rec}}(A,\emptyset)$-terms p_i ($i \in \mathbb{N}$) inductively by

$$\begin{array}{lcl} p_0 & \equiv & \mu X.E \\ p_{i+1} & \equiv & p_i \parallel \mu X.E \quad (i \in \mathbb{N}) \ . \end{array}$$

(2) (b) Prove that $\mathrm{BCP}_{\mathrm{rec}}(A,\emptyset)$+RSP $\vdash p_i \parallel 0 = p_i$ for all $i \in \mathbb{N}$. (Suggestion: First establish $\mathrm{BCP}_{\mathrm{rec}}(A,\emptyset)$+RSP $\vdash \mu X.E \parallel 0 = \mu X.E$ and then prove the required result with induction on $i \in \mathbb{N}$.)

Let F be the recursive specification consisting of the following equations:

$$\begin{array}{lcll} Y_0 & = & a.Y_1 + b.0 & \\ Y_{i+1} & = & a.Y_{i+2} + b.Y_i & (i \in \mathbb{N}) \end{array}$$

(Feel free to abbreviate $\mu Y_i.F$ by simply writing Y_i.)

(2) (c) Prove that $\mathrm{BCP}_{\mathrm{rec}}(A,\emptyset)$+RSP $\vdash \mu X.E = \mu Y_0.F$.

(1) (d) Is the behaviour denoted by $\mu X.E$ finitely definable? (A simple 'Yes' or 'No' suffices.)

(2) 2. Let A be a set of actions that contains at least three distinct actions a, b and c.

Prove that there exist a communication function γ on A and a set $H \subseteq A$ such that

$$\mathrm{TCP}_{\tau}(A,\gamma) \nvdash \partial_H(x \parallel y) = \partial_H(x) \parallel \partial_H(y) \ .$$

(You need to define γ and H and then prove that $\partial_H(x \parallel y) = \partial_H(x) \parallel \partial_H(y)$ is not derivable from $\mathrm{TCP}_{\tau}(A,\gamma)$.)

3. A binary relation R on the set of states S of a transition-system space is a simulation relation if, and only if, the following condition holds:

- for all states $s, t, s' \in S$ and for every a in the set of labels of the transition system space, whenever $s \mathrel{R} t$ and $s \xrightarrow{a} s'$, then there exists a state $t' \in S$ such that $t \xrightarrow{a} t'$ and $s' \mathrel{R} t'$.

Two states $s, t \in S$ are simulation equivalent (notation: $s \leftrightarrows t$) if, and only if, there exist simulation relations R_1 and R_2 such that $s \mathrel{R_1} t$ and $t \mathrel{R_2} s$.

(2) (a) Prove that the equation $a.(x + y) = a.(x + y) + a.y$ is valid in $\mathbb{P}(\mathrm{MPT(A)})/_{\leftrightarrows}$.

(3) (b) Is the process theory $\mathrm{MPT}(A)$ a ground-complete axiomatisation of $\mathbb{P}(\mathrm{MPT}(A))/_{\leftrightarrows}$? Motivate your answer with a proof.

4. Consider the recursive specification E over TSP(A) consisting of the following equations:

$$\begin{array}{lcl} X & = & X \cdot Z + 1 \\ Y & = & b.Y \cdot Z + 1 \\ Z & = & a.1 + 1 \end{array}$$

(Feel free to abbreviate $\mu X.E$, $\mu Y.E$ and $\mu Z.E$ by simply writing X, Y and Z, respectively.)

(2) (a) Give formal derivations within the term deduction system (i.e., the structural operational semantics) for $\text{TSP}_{\text{rec}}(A)$ of two distinct transitions that have the closed term $\mu Y.E \cdot \mu Z.E$ as a source.

(2) (b) Sketch (representative parts of) the transition systems associated with $\mu X.E$ and $\mu Y.E$ by the term deduction system for $\text{TSP}_{\text{rec}}(A)$.

(3) (c) Does it hold that $\tau_{\{b\}}(\mu Y.E) \underline{\leftrightarrow}_b \mu X.E$? Motivate your answer with a proof.