Coalition Formation and Bidding Mechanism: Semi-reversible Agreements

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Abstract

This paper studies a non-cooperative coalitional bargaining game with externalities. I assume that agreements are semi-reversible, i.e., coalitions once formed are still active and can admit new members and merge with other coalitions if it invites at least one uncommitted player as a new member, but coalitions cannot break-up. A multi-bidding mechanism is employed to determine proposals. I show that if players preferences satisfy certain conditions for externalities and the discount factor is large, there exists a Markov perfect equilibrium with full dynamic efficiency property.

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1 Introduction

This paper studies a sequential coalition formation game with externalities. The existence of externalities is often said to be a source of inefficiency. Bloch (1996) studied a coalition formation game with externalities, where each player's payoff depends on an entire coalition structure or a partition. Bloch (1996) showed that a Markov perfect equilibrium (MPE) may fail to exist, or it can be inefficient.

Non-cooperative coalition formation games are intensively studied in the literature. Essential elements of the game are agreements and negotiation protocol. Players form a coalition by making an agreement. Which coalition is proposed is determined by the negotiation protocol. Agreements can be classified into two types, i.e., irreversible agreements and reversible agreements. Bloch (1996), Chatterjee et al. (1993), and Okada (1996) assumed that agreements are irreversible. Once a coalition forms, its members exit the game or become inactive and the game is played among the remaining players. It is often said that the irreversibility of agreements leads to an inefficient result. Chatterjee et al. (1993) showed that delay and the formation of inefficient subcoalitions can occur in equilibrium. Gomes and Jehiel (2005), Hyndman and Ray (2007) and Okada (2000) studied the game with reversible agreements. In the first two articles, agreements are entirely reversible, i.e., coalitions can admit new members, merge with other coalitions, and can break-up. Okada (2000) assumed that coalitions cannot break-up and only one coalition can be formed. Although Okada (2000) showed the existence of a renegotiation-proof solution, Gomes and Jehiel (2005) demonstrated that efficiency cannot be guaranteed.

In the coalition formation game, negotiation protocol chooses a proposer among players. Several negotiation protocols are studied in the literature. Bloch (1996) and Chatterjee et al. (1993) employed a fixed order protocol, which chooses a proposer according to a fixed order. Okada (1996) used a probabilistic protocol. At every period, a proposer is chosen among players with equal probability. Xue and Zhang (2012) studied the game with irreversible agreements and introduced a bidding mechanism constructed by Pérez-Castrillo and Wettstein

(2002) to determine proposals.[1] Players submit a vector of bids, each of which is for a coalition to be formed. The winning coalition and the associated vector of bids become the proposal. They showed the existence of a pure strategy MPE with full dynamic efficiency and the aggregate value monotonicity in time.

The present paper studies the implication of introducing the bidding mechanism in the game with *semi-reversible* agreements. If agreements are semi-reversible, coalitions once formed are still active and can admit new members and merge with other coalitions if it invites at least one uncommitted player as a new member, but coalitions cannot break-up. I show that if players preferences satisfy certain conditions for externalities and discount factor is large, there exists an MPE with full dynamic efficiency property.

The rest of this paper is organized as follows: Section 2 presents the model and Section 3 shows the result. Section 4 concludes and discusses future research.

2 Model

2.1 Basic Environment

Let $N = \{1, \dots, n\}$ be the set of players. I assume that $n \geq 3$. A coalition S is a nonempty subset of N. A partition π_N is a collection of coalitions such that $\pi_N = \{S_1, \dots, S_m\}$ such that $\cup_{i=1}^m S_i = N$ and $S_i \cap S_j = \emptyset$ for all $i \neq j$. Players form a coalition by concluding an agreement. Players who have made an agreement are called *committed players*, and the rest are called *uncommitted players*. Let C be the set of committed players and its complement $\bar{C} = N \setminus C$ be the set of uncommitted players. Also, let π_C be a partition of C and Π_C be the set of all partitions of C. I assume that players' payoffs depend on a partition π_C. Let u_i be player i's payoff function from Π_C to $\mathbb{R}$. I also assume that each player has complete information about players' payoff functions.

In this paper, I assume that agreements are *semi-reversible*, i.e., coalitions once formed

[1] Evans (1997) introduces a different bidding mechanism to determine proposers.

are still active and can admit new members and merge with other coalitions as long as it invites at least one uncommitted player as a new member, but coalitions cannot break-up. Consider the following example. $N = \{1,2,3,4,5\}$, $C = \{1,2,3,4\}$, and $\pi_C = \{\{1,2\},\{3,4\}\}$. If agreements are semi-reversible, $\{1,2,5\}$ and $\{1,2,3,4,5\}$ can be formed as a new coalition but $\{1,2,3,4\}$ cannot, since it must invite an uncommitted player 5.

2.2 Negotiation Process

Players sequentially form a coalition by making an agreement. This process is modeled as an infinite-horizon dynamic game. Time is discrete, and all players are free of commitments (i.e., exist as one-player coalitions) at the beginning of the game. A pair (C, π_C) is called a "state." Let $\mathcal{S}$ be the set of all states, i.e.,

$$\mathcal{S} = \{(C, \pi_C) \mid C \subset N, \pi \in \Pi_C\}$$

For any $a \in \mathcal{S}$, let $C(a)$ be the set of committed players at state a, and $\bar{C}(a)$ be the set of uncommitted players, i.e., $\bar{C}(a) = N \setminus C(a)$. Also, let $\pi_C(a)$ be the partition of committed players at state a.

In each period, players receive payoff determined by the current partition π_C. Players discount their future payoffs using a common discount factor $\delta \in (0,1)$.

Suppose that the current state is $a = (C, \pi_C)$. A *move* from the state $a = (C, \pi_C)$ to $b = (C \cup S, \pi_{C\cup S})$ induced by the formation of coalition S is denoted by $a \xrightarrow{S} b$, where

$$\pi_{C\cup S} = \Big[\pi_C \setminus \big\{T \in \pi_C \mid T \subset S\big\}\Big] \cup \{S\}.$$

Since agreements are semi-reversible, S must be a subset of uncommitted players or a union of a subset of uncommitted players and coalitions already formed. If a state can be reached from the state a by a formation of a new coalition, it is called an *achievable state* from a.

Let $F(a)$ be the set of achievable states from a, i.e.,

$$F(a) = \{b \mid a \xrightarrow{S} b, \text{ where } S = S_0 \cup (\cup_{T \in \hat{\pi}_C(a)} T) \text{ for some } S_0 \subset \bar{C}(a), S_0 \neq \emptyset$$
$$\text{and } \hat{\pi}_C(a) \subset \pi_C(a)\}.$$

A *path* is an infinite sequence of states. Let $P(a)$ be a path starting from a, i.e., $P(a) = \{a_0 = a, a_1, \ldots, a_k, \ldots\}$. $P(a)$ is *feasible path* if $a_k \in F(a_{k-1}) \cup \{a_{k-1}\}$ for all $k = 1, 2, \ldots$. Hence, if a_k has no uncommitted player for some k, then $a_{k'} = a_k$ for all $k' \geq k$. Let $\mathcal{P}(a)$ be the set of all feasible path starting from a. For $P(a) = \{a_0 = a, a_1, a_2, \ldots, a_k, \ldots, \} \in \mathcal{P}(a)$, let $U_i(P(a))$ be the present value of player i's stream of payoffs as determined by $P(a)$, i.e., $U_i(P(a)) = \sum_{t=0}^{\infty} \delta^t u_i(\pi_C(a_t))$.

2.2.1 Multi-bidding Mechanism

Following Xue and Zhang (2012), I use a *multi-bidding mechanism* constructed by Pérez-Castrillo and Wettstein (2002) to determine a proposal. This negotiation mechanism is composed of a bidding stage and a responding stage.

Bidding Stage

In this stage, a player submits a vector of bids, each of which is for a coalition to be formed. Since the formation of a new coalition leads to a corresponding achievable state, I can say that players bid for an achievable state instead of a coalition. Formally, let a be the current state. Each player i simultaneously submits a vector of bids, $[\beta_b^i(a)]_{b \in F(a)}$ and a message, $m^i(a) \in F(a)$. A message $m^i(a)$ is interpreted to mean that player i want to see a transition from the state a to b. If $\beta_b^i(a) \geq 0$, it implies that player i is willing to make a side-payment in the amount of $\beta_b^i(a)$ in order for the state transit from a to b. On the other hand, if $\beta_b^i(a) < 0$, player i demand compensation in the amount of $|\beta_b^i(a)|$ for a transition from a to b. Although a bid can be positive or negative, a player faces a budget constraint $\sum_{b \in F(a)} \beta_b^i(a) = 0$. By satisfying this constraint, player's expected payment is zero if there is a probability of $1/|F(a)|$ that each achievable state is proposed. Let $B(a,b)$ be the aggregate bid for state b submitted at a, i.e., $B(a,b) = \sum_{i=1}^{n} \beta_b^i(a)$.

Define $\mathcal{W}(a)$ and $\mathcal{W}(a,m)$ as follows:

$$\mathcal{W}(a) = \{d \in F(a) \mid B(a,d) \geq B(a,b) \; \forall b \in F(a)\},$$

$$\mathcal{W}(a,m) = \{b \in \mathcal{W}(a) \mid m^i(a) = b \text{ for some } i \in N\}.$$

$\mathcal{W}(a)$ is the set of states with the highest aggregate bid, and $\mathcal{W}(a,m)$ is the set of states with the highest aggregate bid that are selected by some player. A winning state is determined as follows. If $|\mathcal{W}(a,m)| \geq 1$, a winning state is randomly chosen with equal probabilities from $\mathcal{W}(a,m)$; if $\mathcal{W}(a,m) = \emptyset$, a winning state is randomly chosen with equal probabilities from $\mathcal{W}(a)$.

Responding Stage

Let q be the winning state at a and $W(a,q)$ be the proposed coalition. In this stage, each player in $W(a,q)$ sequentially respond to the winning proposal according to a fixed order. If there is unanimous consent, the proposed coalition forms, all the bids for it are paid, and the game proceeds to the next period. If the aggregate bid for state q is greater than zero, [2] this surplus is shared equally among all players. Hence, the final payment to player i is given by $-\beta_q^i(a) + B(a,q)/n$. If any agent rejects the proposal, no coalition forms, no bids are paid, and the game proceeds to the next period.

2.3 Strategies

A history h_t^i is a complete record of what has happened before player i moves in period t. Let H^i be the set of histories belonging to player i. For any $h_t^i \in H^i$, let $a(h_t^i)$ denote the

[2]The aggregate bid for the winning state cannot be negative since each player must satisfy his or her budget constraint $\sum_{b \in F(a)} \beta_b^i(a) = 0$.

current state in period t, and let $W(h_t^i)$ denote the proposed coalition in period t.

A pure strategy σ^i specifies an action of player i for each $h_t^i \in H^i$. Formally, in the bidding stage,

$$\sigma^i(h^i) = \big((\beta_b^i)_{b \in F(a(h^i))},\, m^i\big) \in A_i(h_t^i)$$

where

$$A_i(h^i) = \left\{ (\beta_b^i)_{b \in F(a(h^i))} \in \mathbb{R}^{|F(a(h^i))|} \;\middle|\; \sum_{b \in F(a(h^i))} \beta_b^i = 0 \right\} \times F(a(h^i)),$$

in the responding stage,

$$\sigma^i(h^i) \in A_i' \equiv \{\text{Yes, No}\}.$$

I limit my attention to Markov perfect equilibria. Consider history $h_t^i \in H^i$. A pure strategy σ^i is *Markovian* if $\sigma^i(h_t^i)$ depends on h_t^i only through the current state $a(h_t^i)$ and the ongoing proposal.

Given a Markovian strategy profile $\sigma = (\sigma^i)_{i=1}^n$, let v_i^σ be player i's value function. $v_i^\sigma(a)$ represents the present value for player i at state $a \in \mathcal{S}$ induced by σ. The value function for player i at state a can be written as $v_i^\sigma(a) = u_i(a) + \delta x_i^\sigma(a)$, where $x_i^\sigma(a)$ is the continuation value of player i at state a. Suppose that given σ, game transits from state a to state q with probability 1. If the winning proposal is rejected, the continuation value for each player i is $x_i^\sigma(a) = v_i^\sigma(a)$. In this case, I refer to the continuation value as the disagreement value.

A Markovian strategy profile σ specifies each player's actions at every state a. Hence, it determines a proposal $\Gamma_b^\sigma(a)$ for each state $b \in F(a)$ and players' responses to that. A proposal $\Gamma_b^\sigma(a)$ is said to be *acceptable* if there would be a unanimous consent if it is proposed. Otherwise, it is called *unacceptable*. A strategy profile $\sigma = (\sigma^i)_{i=1}^n$ is a *Markov perfect* *equilibrium* (MPE) if for each player i, σ^i is a Markovian strategy, and after every history, σ^i is a best-response for player i when other players $(-i)$ play according to σ^{-i}. Regarding a value function, σ is an MPE if, for each player i, σ^i is a Markovian strategy and $v_i^\sigma(a) \geq v_i^{(\sigma^{i'},\sigma^{-i})}(a)$ for all a and all $\sigma^{i'}$.

3 Result

In this section, I study the existence of MPE and its efficiency property. First, I define three concepts of efficiency following Xue and Zhang (2012).

Definition 1. *A state $\hat{e}$ is efficient if $\hat{e} \in \arg\max_{b \in \mathcal{S}} \sum_{i=1}^n u_i(b)$.*

Definition 2. *An MPE σ is dynamically efficient if there is no feasible path $P(a_0) \in \mathcal{P}(a_0)$ such that $\sum_{i=1}^n v_i^\sigma(a_0) < \sum_{i=1}^n U_i(P(a_0))$, where a_0 is the initial state of the game.*

Definition 3. *An MPE σ exhibits full dynamic efficiency if there is no state $a \in Z$ and a feasible path $P(a) \in \mathcal{P}(a)$ such that $\sum_{i=1}^n v_i^\sigma(a) < \sum_{i=1}^n U_i(P(a))$.*

Let $\pi_C^i = \{\{i\}, N \setminus \{i\}\}$ and $\pi_C^N = \{N\}$.

Assumption 1. *For all $i \in N$, payoff functions satisfy*

$$\sum_{k=1}^n u_k(\pi_C^N) > \sum_{k=1}^n u_k(\pi_C^i).$$

Assumption 2. *Let $d_i^j = u_i(\pi_C^N) - u_i(\pi_C^j)$. For all $i, j \in N$ with $i \neq j$, payoff functions satisfy*

$$\frac{1}{n-1} \sum_{k \neq i}^n d_k^j \geq d_j^j.$$

The following proposition provides conditions for the existence of an MPE with full dynamic efficiency. The proof is relegated to the appendix.

Proposition 1. *Under Assumptions 1 and 2, there exists* $\underline{\delta}$ *such that for any* $\delta > \underline{\delta}$*, there exists a pure strategy MPE with full dynamic efficiency property.* $\underline{\delta}$ *is given by*

$$\underline{\delta} = \max_{j \in N} \left(\frac{n d_j^j}{\sum_{k=1}^n d_k^j} - 1 \right).$$

4 Conclusion

In this paper, I study the sequential coalition formation game employing the bidding mechanism. Xue and Zhang (2012) also study the implications of introducing the bidding mechanism in a coalition formation game. However, they assumed that agreements are irreversible. An important feature of my model is that agreements are semi-reversible. Under this assumption, coalition structures can only grow coarser. I show that if players utility functions satisfy certain conditions for externalities and the discount factor is large, there exists a pure strategy MPE with full dynamic efficiency.

The present paper limits its attention to the game with semi-reversible agreements. It would be interesting to study the existence of MPE and its efficiency in the game with (entirely) reversible agreements as in Gomes and Jehiel (2005) and Hyndman and Ray (2007).

Appendix: Proof of Proposition 1

The proof consists of three steps. Step 1: construct a pure strategy profile σ; Step2: prove that σ is an MPE; Step 3: prove that σ exhibits full dynamic efficiency.

Step1

I construct a pure strategy profile recursively. I specify actions at a state using the values at its achievable states. The game ends when there are no uncommitted players left. Values at such states can be readily determined.

Consider an arbitrary state a with at least one uncommitted player. Without loss of generality, assume that player 1 is uncommitted if there is a single uncommitted player and that players 1 and 2 are uncommitted if there are more than one uncommitted players. Given σ, let $e^\sigma(a)$ be an *efficient achievable state* such that $e^\sigma(a) \in \arg\max_{b \in F(a)} \sum_{i=1}^n v_i^\sigma(b)$ and let S be the coalition such that $a \xrightarrow{S} e^\sigma(a)$. I construct strategies so that in equilibrium S forms and the game transit from a to $e^\sigma(a)$. I choose states $p_1(a)$, $p_2(a)$, and $p_3(a)$ such that $a \xrightarrow{T_i} p_i(a)$, $i = 1,\ 2,\ 3$, where T_i vary with S as specified in Table 1 and 3. Let $Q^\sigma(a) = F(a) \setminus \{e^\sigma(a)\} \setminus \{p_1(a)\} \setminus \{p_2(a)\} \setminus \{p_3(a)\}$.

I construct bids so that the net surplus when moving from the state a to $e^\sigma(a)$ is equally shared. To specify bidding strategies, define

$$A(a) = \frac{1}{n} \sum_{i=1}^n \Big[v_i^\sigma\big(e^\sigma(a)\big)(1-\delta) - u_i(\pi(a)) \Big]$$

$$E_i(a) = v_i^\sigma\big(e^\sigma(a)\big) - \frac{u_i(\pi(a)) + A(a)}{1-\delta}$$

$$M(a) = \max_{i \in N, b \in F(a)} \left[v_i^\sigma(b) - \frac{u_i(\pi(a))}{1-\delta} \right]$$

$$\bar{M}(a) = (n+1)M(a)$$

$$f = |F(a)|.$$

Players' bids and messages are specified in Tables 2, 4, and 5. When $\bar{C}(a) = \{1\}$, states in $F(a)$ are partitioned as in Table 1, where S' is an arbitrary coalition in $\pi_C(a)$.

Table 1: Partitioning $F(a)$ when $\bar{C}(a) = \{1\}$

	S	T_1
Case (1)	$\{1\}$	$\{1\} \cup S'$
Case (2)	Otherwise	$\{1\}$

Table 2: Bids and messages when $\bar{C}(a) = \{1\}$: Cases (1–2)

Player	$e(a)$	$p_1(a)$	$q \in Q(a)$	m_i
1	E_1	$(n-1)(f-2)\bar{M} - E_1$	$-(n-1)\bar{M}$	e
$j \neq 1$	E_j	$-(f-2)\bar{M} - E_j$	$\bar{M}$	e

When $\bar{C}(a) \supset \{1,2\}$, states in $F(a)$ are partitioned as in Table 3.

Table 3: Partitioning $F(a)$ when $\bar{C}(a) \supset \{1,2\}$

	S	T_1	T_2	T_3
Case (1)	$\{1\}$	$\{1,2\}$	$\{2\}$	
Case (2)	$\{2\}$	$\{1\}$	$\{1,2\}$	
Case (3)	$\{1,2\}$	$\{1\}$	$\{2\}$	
Case (4)	Otherwise	$\{1\}$	$\{2\}$	$\{1,2\}$

Table 4: Bids and messages when $\bar{C}(a) \supset \{1,2\}$: Cases (1–3)

Player	$e(a)$	$p_1(a)$	$p_2(a)$	$q \in Q(a)$	m_i
1	E_1	$[(n-2)(f-3)+1]\bar{M} - E_1$	$-\bar{M}$	$-(n-2)\bar{M}$	e
2	E_2	$-\bar{M} - E_2$	$\bar{M}$	0	e
$j \notin \{1,2\}$	E_j	$-(f-3)\bar{M} - E_j$	0	$\bar{M}$	e

Table 5: Bids and messages when $\bar{C}(a) \supset \{1,2\}$: Cases (4)

Player	$e(a)$	$p_1(a)$	$p_2(a)$	$p_3(a)$	$q \in Q(a)$	m_i
1	E_1	$[(n-2)(f-4)+2]\bar{M} - E_1$	$-\bar{M}$	$-\bar{M}$	$-(n-2)\bar{M}$	e
2	E_2	$-2\bar{M} - E_2$	$\bar{M}$	$\bar{M}$	0	e
$j \notin \{1,2\}$	E_j	$-(f-4)\bar{M} - E_j$	0	0	$\bar{M}$	e

I now construct response strategies. For a proposal $\Gamma_b(a) = (b, [\beta_b^i(a)]_{i=1}^n)$, the actions of each responder $j \in W(a, b)$ is given by

$$\sigma^j(a,\, \Gamma_b(a)) = \begin{cases} \text{“Yes”}, & \text{if } v_j^\sigma(b) - \beta_b^j(a) + \frac{B(a,b)}{n} \geq \frac{u_j(\pi(a))+\delta A(a)}{1-\delta} \\ \text{“No”}, & \text{if } v_j^\sigma(b) - \beta_b^j(a) + \frac{B(a,b)}{n} < \frac{u_j(\pi(a))+\delta A(a)}{1-\delta} \end{cases}$$

Step 2

Given σ I have constructed, all winning proposals are accepted. On the path determined by σ, all states beyond the initial one are efficient achievable states. Hence, player i's value function and the continuation value at state a can be written as

$$v_i^\sigma(a) = \frac{u_i(\pi(a)) + \delta A(a)}{1-\delta} \quad \text{and} \quad x_i^\sigma(a) = \frac{u_i(\pi(a)) + A(a)}{1-\delta}.$$

To show that σ is an MPE, I use the following lemmata.

Lemma 1 (Xue and Zhang, 2012)**.** *Let σ be the constructed strategy profile in Step 1.*

- $\Gamma_b^\sigma(a)$ *is unacceptable if there is a responder $j \in W(a, b)$ such that*

$$\beta_b^j(a) > M(a) = \max_{i \in N,\, b \in F(a)} \left[v_i^\sigma(b) - \frac{u_i(\pi(a))}{1-\delta} \right].$$

- *Suppose that proposal $\Gamma_b^\sigma(a)$ is unacceptable and would be rejected by some $j \in N$. A player $i \neq j \in N$ cannot benefit by turning state b into the solo winner and making the corresponding proposal acceptable if the following holds:*

$$\beta_b^i(a) \geq \begin{cases} -(n-1)\beta_b^j(a), & \textit{if there are only 2 achievable states,} \\ nM(a) - (n-1)\beta_b^j(a), & \textit{otherwise.} \end{cases}$$

Proof. Since $A(a) \geq 0$, $M(a) \geq v_j^\sigma(b) - u_j(\pi(a))/(1-\delta)$, and $\beta_b^j(a) > M(a)$, I have

$$v_j^\sigma(b) - \beta_b^j(a) < v_j^\sigma(b) - M(a) \leq \frac{u_j(\pi(a)) + \delta A(a)}{1-\delta}$$

Given σ, player j rejects $\Gamma_b^\sigma(a)$.

Suppose some player $i \in N$ can profitably deviate from σ^i by turning b into the solo winner and making the corresponding proposal acceptable. Given σ, the aggregate bids for all achievable states are zero. Hence, player i can easily turn b into the solo winner by increasing his or her bid for b by any $\varepsilon > 0$. However, in order to make player j accept the proposal, ε must satisfy the following inequality:

$$v_j^\sigma(b) - \beta_b^j(a) + \frac{\varepsilon}{n} \geq \frac{u_j(\pi(a)) + \delta A(a)}{1-\delta}. \tag{1}$$

Since it is beneficial for player i to increase his or her bid for b by ε, ε must also satisfy the following inequality:

$$v_i^\sigma(b) - \beta_b^i(a) - \varepsilon + \frac{\varepsilon}{n} > \frac{u_i(\pi(a)) + A(a)}{1-\delta}. \tag{2}$$

It is easy to verify that there is no ε satisfying (1) and (2) if

$$\beta_b^i(a) \geq \begin{cases} -(n-1)\beta_b^j(a), & \text{if there are only 2 achievable states,} \\ nM(a) - (n-1)\beta_b^j(a), & \text{otherwise.} \end{cases}$$

$\square$

Lemma 2 (Xue and Zhang, 2012)**.** *A strategy profile σ is an MPE if, at any state a with at least one uncommitted player, it satisfies the following six conditions:*

C1 the aggregate bid for every achievable state equals zero;

C2 every player submits the same state as her message: $m^i(a) = m^j(a) = e^\sigma(a)$ for all i, $j \in N$;

C3 *each player's bid for her intended state $e(a)$ guarantees at least her disagreement value :* $\beta^i_{e^\sigma(a)}(a) \leq v^\sigma_i(e^\sigma(a)) - v^\sigma_i(a)$ *for all* $i \in N$*;*

C4 *a player accepts a proposal if it offers her at least her disagreement value and reject it otherwise;*

C5 $\Gamma^\sigma_{e^\sigma(a)}(a)$ *is the most preferred proposal for every player among all the acceptable proposals determined by* σ *at state* a*;*

C6 *no player benefits by turning an unacceptable proposal (e.g.,* $\Gamma^\sigma_b(a)$*) into an acceptable one and turning the corresponding state into the solo winner.*

Proof. Suppose that there exists a Markovian strategy profile σ satisfying C1–C6. The winning proposal is $\Gamma^\sigma_{e^\sigma(a)}(a)$ according to C1 and C2. C3 and C4 together guarantee that $\Gamma^\sigma_{e^\sigma(a)}(a)$ is acceptable. There is no beneficial deviation in the responding stage by C4. No player benefits by changing only his or her message according to C1 and C5. By changing his or her bids, a player can make another state the solo winner. I show that no player benefits by turning any state into the solo winner. Suppose that player i increases his or her bid for a state b by ε and turns it into the solo winner. Let the new proposal associated with b be $\Gamma^d_b(a)$. If both $\Gamma^\sigma_b(a)$ and $\Gamma^d_b(a)$ are acceptable, the deviation is not beneficial by C5. Consider the case $\Gamma^\sigma_b(a)$ is unacceptable. If $\Gamma^d_b(a)$ is not acceptable, the deviation is not beneficial by C3. If $\Gamma^d_b(a)$ is acceptable, the deviation still cannot be beneficial by C6. Therefore, no player has incentives to change his or her bids and/or message σ. □

Suppose that $a = (N \setminus \{i\}, \pi^i_C)$ and let $b = (N, \pi^i_C)$. Under assumptions 1 and 2, C1–C4 and C6 are satisfied by construction. To satisfy C5, I must have

$$
\begin{aligned}
& v^\sigma_i(b) - \beta^i_b(a) < \frac{u_i(\pi^i_C) + \delta A(a)}{1-\delta} \\
\Longleftrightarrow\ & u_i(\pi^N_C) - u_i(\pi^i_C) < (1+\delta)A(a) \\
\Longleftrightarrow\ & \delta > \frac{n d^i_i}{\sum_{k=1}^n d^i_k} - 1.
\end{aligned}
$$

Hence, δ must be greater than

$$\max_{j \in N} \left(\frac{n d_j^j}{\sum_{k=1}^n d_k^j} - 1 \right).$$

For all other cases, it is easy to verify that σ satisfies C1–C6. Therefore, σ is an MPE.

Step 3

Finally, I prove that σ exhibits full dynamic efficiency. Let a be a state with the highest number of committed players whose continuation equilibrium is inefficient, i.e., there exists $P(a)$ such that $\sum_{i=1}^n v_i^\sigma(a) < \sum_{i=1}^n U_i(P(a))$. By construction of σ, $v_i^\sigma(a) \geq u_i(\pi(a)) + \delta v_i^\sigma(a)$, which implies that $\sum_{i=1}^n v_i^\sigma(a) \geq \sum_{i=1}^n u_i(\pi(a)) + \delta \sum_{i=1}^n v_i^\sigma(a)$. Thus, $P(a)$ must contain some state other than a. Let b be the first state on $P(a)$ that is different from a and let $P'(a)$ be the path obtained from $P(a)$ by removing all but one of the a's. Then I have $\sum_{i=1}^n v_i^\sigma(a) < \sum_{i=1}^n U_i(P'(a))$. Consider the unique continuation equilibrium path from a determined by σ, and let e be the state on this path that is adjacent from a. Then, e is an efficient achievable state from a, i.e., $\sum_{i=1}^n v_i^\sigma(e) \geq \sum_{i=1}^n v_i^\sigma(b)$. Let $P'(b)$ be the path obtained from $P'(a)$ by removing a. Since at a, the aggregate bid for e is zero, $\sum_{i=1}^n v_i^\sigma(a) < \sum_{i=1}^n U_i(P'(a))$ implies $\sum_{i=1}^n v_i^\sigma(e) < \sum_{i=1}^n U_i(P'(b))$. Thus, $\sum_{i=1}^n v_i^\sigma(b) < \sum_{i=1}^n U_i(P'(b))$ implying that the continuation equilibrium path from b is inefficient and b cannot be a state where all players are committed. Since $b \neq a$, b has strictly more committed players than a. This is a contradiction.$\square$

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