Bilateral Bargaining with a Biased Intermediary*

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Abstract

Bilateral bargaining is often mediated by a third party, called an "intermediary." In many cases, the intermediary shares some interest with one of the two parties in the negotiation and cannot commit to and enforce his decision. This paper studies the effect of such a "biased" intermediary without commitment and enforcement power in a fully general communication structure. I consider a bilateral trade model à la Myerson and Satterthwaite (1983) with binary valuations in which the intermediary offers a price to the traders. I focus on the set of sequential communication equilibria of the game, which characterizes what the players can achieve if they can freely communicate with each other. I show that the intermediary's bias is detrimental to efficiency, however small it is. I also characterize the second-best equilibrium and show that the expected trade surplus will decrease if the degree of bias exceeds a certain threshold.

Keywords: Bargaining, intermediary, bias, mediator, sequential communication equilibrium.

JEL Classification: C78, D82.

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1 Introduction

Bilateral bargaining is often mediated by a third party, called an "intermediary." He provides filtered communication and obfuscates the private information inherent in offers and counter-offers. For this reason, it is believed that he can mitigate strategic incentives in direct communication and hence help achieve efficient outcomes.

However, an intermediary is not necessarily benevolent and/or "omnipotent," which can adversely affect efficiency. In many cases, the intermediary shares some interest with one of the two parties in the negotiation and cannot commit to and enforce his decision. Such a "biased" intermediary can be found, for example, in international relations. When two countries A and B are in dispute, a third country, C, often acts as an intermediary to peacefully settle it.[1] C usually shares some interest with either A or B and cannot enforce its decision because A and B are sovereign entities. In addition, C sometimes cannot commit to its decision. A broker in the housing market also has some features of a biased intermediary; he is considered biased toward a seller because his compensation is usually tied to a sale price. He also cannot enforce trade without the consent of the traders.

The purpose of this paper is to study the effect of a biased intermediary without commitment and enforcement power in a fully general communication structure. In other words, my model can accommodate various possible communication structures between the players, while assuming no commitment power of the intermediary. As I discuss in Section 1.1, the effect of such a biased intermediary has not yet been studied in the literature.

To draw a connection between the literature on mechanism design, I study this effect in a bilateral trade model à la Myerson and Satterthwaite (1983) with binary valuations: with a seller, a buyer, and the intermediary. The seller owns a good and wants to trade it with the buyer. Each trader's valuation of the good is his private information and is independently and binary distributed. There is always gain from trade, except for the pair of high-type seller and low-type buyer. When there is trade, the seller gets his valuation minus the price, the buyer gets the price minus his valuation, and the intermediary gets a weighted average of the traders' payoffs. When

[1] In some cases, A and B accept C because they cannot reliably communicate with each other for some historical reason. In other cases, A and B can communicate directly with each other but find it beneficial to communicate through C. The latter scenario is related to Blume, Board, and Kawamura (2007).

there is no trade, all players get zero. I assume that the intermediary attaches a larger weight to the seller's payoff, which is interpreted as biased toward the seller. The underlying game I consider here proceeds as follows: first, the seller and the buyer observe the realizations of their types (valuations); second, the intermediary offers a price to the traders; and third, the traders simultaneously respond to the offer. If both traders accept it, trade occurs at that price; if at least one of them rejects it, no trade occurs.

The goal is to characterize what they can achieve in the underlying game if they can freely communicate with each other; that is, at any point in the game, they can send private or public messages to each other, toss coins, or build a correlation device. To this end, I assume the existence of "a fictitious mediator" and study the mediated bargaining game that proceeds as follows: first, the seller and the buyer observe the realizations of their types and confidentially report them to the mediator; second, the mediator confidentially gives action recommendations to each player—to the intermediary, a price to offer, and to each trader, a response to each possible price; third, after observing the recommendation, the intermediary offers a price to the traders; and last, the traders simultaneously respond to the offer. If both traders accept it, trade occurs at that price; if at least one of them rejects it, no trade occurs. Mapping from traders' reports to action recommendations is called a *mediation plan*. The mediation plan the mediator uses is common knowledge among the players.

The set of "incentive-compatible" mediation plans, known as the set of *sequential communication equilibrium (SCE)* of Myerson (1986b), fully characterizes what the players can achieve in the underlying game with some communication structure; that is, without a mediator. In my model, however, some SCE are not robust to the small mistakes of the players. Thus, I introduce a refinement *acceptable SCE (ASCE)* in the spirit of Myerson (1986a) and use it as the equilibrium concept.

I show that the intermediary's bias is detrimental to efficiency, however small it is. First, I assume that the intermediary is seller-biased and derive the necessary and sufficient condition for the existence of an ASCE that achieves an ex-post efficient outcome (FB-ASCE) (Proposition 2). This condition is independent of the degree of the intermediary's bias. In Appendix C.1.1, I show that a FB-ASCE exists under a weaker condition if the intermediary is unbiased (Proposition

4). Combined with Proposition 2, this implies that even a tiny bias can shrink the possibility of achieving the ex-post efficient outcomes.[2]

In contrast to the first-best case, a smaller bias is socially desirable in the second-best (SB) case. First, I characterize the SB-ASCE, the mediation plan that is an ASCE and maximizes the ex-ante expected trade surplus. Next, I provide the comparative static with respect to the degree of the intermediary's bias. I show that there exists a threshold value for the bias that determines SB-ASCE. All other things being equal, if the bias is originally below this threshold but increases above it, the seller will be better off in the SB-ASCE, the buyer will not, and the expected trade surplus will decrease.

1.1 Related literature

The first distinguishing feature of this paper is that the intermediary is biased and cannot commit to and enforce his decision. An intermediary, broadly defined, has been studied in the literature on mechanism design.[3] Myerson and Satterthwaite (1983) and Matsuo (1989) study an unbiased intermediary with commitment and enforcement power.[4] Note that mechanism design without enforcement power has also been studied in the literature (see Myerson (1982), for example). Note also that there is a growing literature on mechanism design with limited commitment (Bester and Strausz (2001, 2000, 2007); Doval and Skreta (2021); Lomys and Yamashita (2021), among others). Related to this literature, Eilat and Pauzner (2021) study bilateral trade with a benevolent intermediary without commitment power. Unlike this paper, Eilat and Pauzner (2021) directly study the game between the traders and the intermediary, in which the intermediary offers a mechanism.[5] Thus, the novelty of this paper relative to the literature is that I jointly drop unbiasedness and commitment and enforcement power. In particular, the intermediary's bias, combined with no commitment, gives him the incentive to offer as high a price as possible. This is the main driving force behind my results.

[2]In Appendix C.2, I also study the the mediated seller-offer bargaining game and show that FB-ASCE exists if and only if it exists in the seller-biased case. Thus, the seller-biased intermediary is of no help in achieving the ex-post efficient outcomes.

[3]An intermediary in this literature is typically called a *principal* or a *mechanism designer*.

[4]Unlike Myerson and Satterthwaite (1983), Matsuo (1989) assumes binary valuations as in this paper. The relationship with this paper is discussed in Appendix C.1.2.

[5]Note that the intermediary's role in my model is to offer a price. In this regard, Eilat and Pauzner (2021) allow more flexibility in the intermediary's ability.

The effect of intermediation has been studied in different contexts and models. Some papers in the bargaining literature have focused on the role of the intermediary as an information filter (see Čopič and Ponsatí (2008); Fanning (2021a,b); Jarque, Ponsatí, and Sákovics (2003)).[6] In contrast to these papers, I model an intermediary as an active player of the game, who has an objective to maximize and actions to choose from. This clarifies the effect of bias on the filtered communication he provides. Intermediation in international relations and cheap talk games have also been studied (see Ganguly and Ray (2012); Goltsman, Hörner, Pavlov, and Squintani (2009); Hörner, Morelli, and Squintani (2015); Kydd (2003, 2006), among others). The novelty of this paper is again the assumption on the intermediary: biased and without commitment and enforcement power.

Methodologically, I lean on the work of Forges (1986), Myerson (1986b), and Sugaya and Wolitzky (2021), who study multistage games with communication and establish the revelation principle for various equilibrium concepts. In line with these contributions, it is without loss of generality to focus on the direct communication between the players and the mediator in the mediated bargaining game described above.

The remainder of the paper is organized as follows. In Section 2, I describe the model and define the equilibrium concepts. Section 3 derives the necessary and sufficient condition for the existence of an equilibrium that achieves an ex-post efficient outcome. In Section 4, I characterize the second-best equilibrium and provide a comparative static with respect to the degree of the intermediary's bias. Section 5 concludes the paper.

2 The Model

There are three players: a seller, a buyer, and an intermediary. The seller owns a good and wants to trade it with the buyer. The seller's valuation of the good is high (s_H) with probability $\pi_S \in (0,1)$ and low (s_L) with probability $1-\pi_S$. Similarly, the buyer's valuation of the good is high (b_H) with probability $\pi_B \in (0,1)$ and low (b_L) with probability $1-\pi_B$. The valuations are independently distributed, and their realizations are the private information of the respective

[6]Gottardi and Mezzetti (2022) focus not only on this facilitative role but also on the evaluative role of the intermediary.

trader. Let $\Theta_S = \{s_H, s_L\}$ and $\Theta_B = \{b_H, b_L\}$ be the set of possible valuations of the seller and the buyer, respectively. I denote the pair of traders with valuations $s \in \Theta_S$ and $b \in \Theta_B$ by (s, b). I assume that there is always gain from trade except for the traders (s_H, b_L); that is, $s_L < b_L < s_H < b_H$. When (s, b) trade at the price p, the seller gets $p - s$ and the buyer gets $b - p$; when there is no trade, both get 0.

The intermediary's payoff is a weighted average of the traders' payoffs: when (s, b) trade at p, he gets $\lambda(p - s) + (1 - \lambda)(b - p)$; when there is no trade, he gets 0. The weight $\lambda \in [0, 1]$ captures the degree of the intermediary's bias toward the seller. The intermediary is *seller-biased* if $\lambda > 1/2$, *buyer-biased* if $\lambda < 1/2$, and *unbiased* if $\lambda = 1/2$. In what follows, I restrict my attention to the seller-biased case.[7] The buyer-biased case can be similarly analyzed.

The underlying game I consider here proceeds as follows: (i) the seller and the buyer observe the realizations of their types (valuations); (ii) the intermediary offers a price to the traders; and (iii) the traders simultaneously respond to the offer. If both traders accept it, trade occurs at that price; if at least one of them rejects it, no trade occurs.

2.1 The mediated bargaining game

I am interested in what the three players can achieve if they can freely communicate with each other (e.g., sending messages) at any point in the underlying game. To characterize it, I augment the model by adding a fictitious mediator. Between (i) and (ii), she collects reports from the traders and confidentially gives an action recommendation to each player depending on the reported types. She carries out such mediation by using a decision rule called a mediation plan. A *mediation plan* is a pair (q, r), where $q\colon \Theta_S \times \Theta_B \to \mathbb{R}_+$ is a *price-recommendation* and $r\colon \Theta_S \times \Theta_B \times \mathbb{R}_+ \to \{Y, N\}^2$ is a *response-recommendation*. For example, suppose that the mediator uses a mediation plan (q, r) and that the traders have reported $(s, b) \in \Theta_S \times \Theta_B$. Then, the mediator recommends the intermediary to offer $q(s, b) \in \mathbb{R}_+$. She also recommends each trader a response (acceptance, "Yes" (Y), or rejection, "No" (N)) to each possible price $p \in \mathbb{R}_+$; that is, she recommends a response-rule $r(\cdot \mid s, b)\colon \mathbb{R}_+ \to \{Y, N\}^2$ as a function of price.[8] Note that even though $r(\cdot \mid s, b)$ specifies both traders' responses to each possible price, she

[7] In Appendix C.1.1, I consider the unbiased case.

[8] I denote $r(s, b, p)$ by $r(p \mid s, b)$.

confidentially recommends it. In other words, what trader $i \in \{S,B\}$ observes is the corresponding coordinate of $r(\cdot \mid s,b)$, denoted by $r_i(\cdot \mid s,b) : \mathbb{R}_+ \to \{Y,N\}$. Let Q and R be the set of all price-recommendations and response-recommendations, respectively. In addition, let R_{marg} be the set of all functions from $\mathbb{R}_+$ to $\{Y,N\}$. For the greatest generality, I allow the mediator to use a mixed mediation plan, a probability distribution over $Q \times R$. Let $\Delta(X)$ denote the set of all probability distributions over the set X.

I consider the mediated bargaining game that proceeds as follows:

Time 1: The mediator publicly commits to a mediation plan $\mu \in \Delta(Q \times R)$ that she will use;

Time 2: The seller and the buyer privately observe the realizations of their types;

Time 3: The traders confidentially report their types $s \in \Theta_S$ and $b \in \Theta_B$ to the mediator;

Time 4: The mediator privately picks up a pure mediation plan $(q,r) \in Q \times R$ with probability $\mu(q,r)$. She confidentially recommends a price $q(s,b) \in \mathbb{R}_+$ to the intermediary and a response-rule $r_i(\cdot \mid s,b)$ to trader $i \in \{S,B\}$;

Time 5: The intermediary offers a price $p \in \mathbb{R}_+$;

Time 6: The traders simultaneously respond to p, by either acceptance (Y) or rejection (N). If both accept, trade occurs at the price p, and the payoffs are realized; otherwise, no trade occurs, and all players get 0.

As I discussed in Introduction, on top of bias, my intermediary features no commitment and no enforcement. Indeed, he does not commit to the price he offers; he updates his belief about the traders' types after he receives the recommendation and offers the optimal price for him given his belief. In contrast, the mediator can commit to the mediation plan she will use. Moreover, the traders' acceptances are necessary for trade to occur at the offered price.

2.2 Equilibrium concept

Note that each player can manipulate a mediation plan by disobeying recommendations, and the traders can also manipulate it by misreporting their types. In an equilibrium mediation plan defined later, none of such manipulations are profitable.

Communication equilibrium (CE) of Forges (1986) applies to the present framework. In CE, no player could ex-ante expect to gain by manipulation. In other words, no player could expect to gain by manipulation after any event that is observable to him and has a *positive probability* of occurring in equilibrium. In dynamic games with communication, however, the set of CE potentially includes unreasonable Nash equilibria of the game as the set of Nash equilibria in dynamic games with perfect information. Thus, I need a stronger concept, *sequential communication equilibrium (SCE)* of Myerson (1986b). In SCE, no player could expect to gain by manipulation after any event that is observable to him, including events that have *zero-probability* of occurring in equilibrium. In Appendix B.1, using the characterization of SCE (Myerson (1986b, Theorem 2)), I show that every CE is an SCE in the mediated bargaining game. Thus, it suffices to check the ex-ante incentives for manipulation in showing that a mediation plan is an SCE.

It is important to note that there is no loss of generality in the form of communication I assume (namely, Time 3 and 4 of the mediated bargaining game). Thanks to the revelation principle shown by Forges (1986); Myerson (1986b); Sugaya and Wolitzky (2021), any equilibrium outcome of the underlying game with any form of communication is attainable as an SCE of the mediated bargaining game. As such, the set of SCE outcomes fully characterizes what the three players can achieve with communication in the underlying game.

To formally define CE, let $v(p \mid s,b)$ be the intermediary's payoff when (s,b) trade at p; that is, $v(p \mid s,b) = \lambda(p-s) + (1-\lambda)(b-p)$. In addition, let $V(q,r)$ be his ex-ante expected payoff if the mediator uses a pure mediation plan (q,r) and all players are honest and obedient to the mediator:[9]

$$V(q,r) = \sum_{(s,b)\in\Theta_S\times\Theta_B} \Pr(s,b)\cdot v(q(s,b) \mid s,b)\cdot \mathbf{1}_{\{r(q(s,b)|s,b)=(Y,Y)\}},$$

where $\Pr(s,b)$ is the prior probability that the traders' types are (s,b), and $\mathbf{1}_{\{\cdot\}}$ is the indicator function. Similarly, for trader $i \in \{S,B\}$, let $U_i(q,r)$ be i's ex-ante expected payoff if the mediator

[9] To simplify the notations, I omit the dependence on the bias λ from $v(p \mid s,b)$ and $V(q,r)$.

uses a pure mediation plan (q,r) and all players are honest and obedient to the mediator:

$$U_S(q,r) = \sum_{(s,b)\in\Theta_S\times\Theta_B} \Pr(s,b)\cdot(q(s,b)-s)\cdot\mathbf{1}_{\{r(q(s,b)|s,b)=(Y,Y)\}},$$
$$U_B(q,r) = \sum_{(s,b)\in\Theta_S\times\Theta_B} \Pr(s,b)\cdot(b-q(s,b))\cdot\mathbf{1}_{\{r(q(s,b)|s,b)=(Y,Y)\}}.$$

Manipulation by the intermediary can be represented by a function $\alpha\colon \mathbb{R}_+ \to \mathbb{R}_+$. Let Σ_I be the set of all such functions. Manipulation by trader $i \in \{S,B\}$ can be represented by a pair of functions (β_i,γ_i), where $\beta_i\colon \Theta_i \to \Theta_i$ is a manipulation in reports, and $\gamma_i\colon \Theta_i \times R_{\text{marg}} \times \mathbb{R}_+ \to \{Y,N\}$ is a manipulation in responses. For $i \in \{S,B\}$, let Σ_i be the set of all such pairs of functions.

For any $\alpha \in \Sigma_I$, let $q\circ\alpha$ be such that $(q\circ\alpha)(s,b) = \alpha(q(s,b))$ for all $(s,b)\in\Theta_S\times\Theta_B$; that is, $q\circ\alpha$ gives the price that will be offered if the mediator uses q and the intermediary manipulates it by α while the traders are honest to the mediator. For any $(\beta_i,\gamma_i)\in\Sigma_i$, let $q\circ\beta_i$ and $r\circ(\beta_i,\gamma_i)$ be such that $(q\circ\beta_i)(\theta_i,\theta_j) = q(\beta(\theta_i),\theta_j)$ and $(r\circ(\beta_i,\gamma_i))(\cdot\mid\theta_i,\theta_j) = (\gamma_i(\theta_i,r_i,\cdot), r_j(\cdot\mid\beta(\theta_i),\theta_j))$ for all $(\theta_i,\theta_j)\in\Theta_i\times\Theta_j$, respectively; that is, $q\circ\beta_i$ gives the price that will be offered if the mediator uses q and trader i manipulates it by β_i while the other trader j and the intermediary are honest and obedient to the mediator. Similarly, $r\circ(\beta_i,\gamma_i)$ gives the responses that the traders will give if the mediator uses r and trader i manipulates it by (β_i,γ_i) while the other trader j and the intermediary are honest and obedient to the mediator. To simplify the notation, let $(q,r)\circ(\beta_i,\gamma_i)$ denote $(q\circ\beta_i, r\circ(\beta_i,\gamma_i))$. For any $\alpha\in\Sigma_I$ and $(\beta_i,\gamma_i)\in\Sigma_i$, the ex-ante expected payoffs of the intermediary and trader i when they use these manipulations, $V(q\circ\alpha,r)$ and $U_i((q,r)\circ(\beta_i,\gamma_i))$, respectively, are naturally defined.

Definition. A mediation plan $\mu\in\Delta(Q\times R)$ is a *communication equilibrium (CE)* if, for all $\alpha\in\Sigma_I$,

$$\sum_{(q,r)\in Q\times R} \mu(q,r)V(q,r) \geq \sum_{(q,r)\in Q\times R} \mu(q,r)V(q\circ\alpha,r), \tag{2.1}$$

and for all $(\beta_i, \gamma_i) \in \Sigma_i$ and all $i \in \{S, B\}$,

$$\sum_{(q,r)\in Q\times R} \mu(q,r)U_i(q,r) \geq \sum_{(q,r)\in Q\times R} \mu(q,r)U_i((q,r)\circ(\beta_i,\gamma_i)). \tag{2.2}$$

In the remainder of the paper, I consider either pure mediation plans or mixed mediation plans where the response-recommendation is fixed. I show in Appendix B.2 that the *ex-ante* IC constraints (2.1) and (2.2) are satisfied if and only if the *interim* IC constraints are satisfied; that is, (i) no trader has the incentive to manipulate after he learns his type (at Time 3) and (ii) the intermediary has no incentive to manipulate after he receives a recommendation (at Time 5). Accordingly, I consider the players' interim IC constraints in the proofs of the results.

3 First-Best Mediation Plan

In this section, I characterize the conditions under which an ex-post efficient outcome is achievable in an equilibrium of the game. A mediation plan is *first-best (FB)* if, assuming that the players are honest and obedient to the mediator, trade occurs if and only if the buyer has a higher valuation; that is, $\mu \in \Delta(Q\times R)$ is FB if for each $(q,r) \in \text{supp}(\mu)$, $r(q(s,b) \mid s,b) = (Y,Y)$ for all $(s,b) \neq (s_H, b_L)$ and $r(q(s_H,b_L) \mid s_H,b_L) \neq (Y,Y)$.

In what follows, I restrict my attention to the set of pure mediation plans $Q\times R$. For any price-recommendation $q \in Q$, let $r_q \in R$ be such that, for all $(s,b) \neq (s_H,b_L)$,

$$r_q(p \mid s,b) = \begin{cases} (Y,Y) & \text{if } p = q(s,b) \\ (N,N) & \text{otherwise} \end{cases}$$

and

$$r_q(p \mid s_H,b_L) = (N,N) \text{ for all } p \in \mathbb{R}_+.$$

In other words, when there is gain from trade, r_q recommends accepting only the on-path price $q(s,b)$. When there is no gain from trade, it always recommends rejection. Lemma 1 claims that for any price-recommendation q, it suffices to consider (q, r_q) to see if q can be a part of a FB mediation plan that is an SCE (FB-SCE).

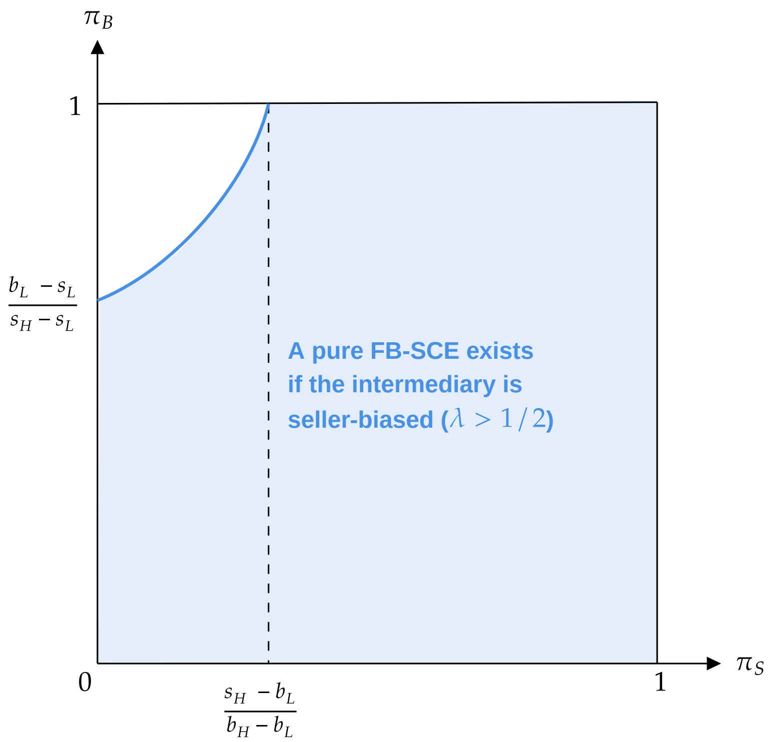

Figure 3.1: An illustration of Proposition 1: Let $\underline{\pi_S}(p) = \frac{p-b_L}{b_H-s_H+p-b_L}$ and $\overline{\pi_B}(p) = \frac{b_L-s_L}{s_H-p+b_L-s_L}$. The blue thick line is the locus of $\left(\underline{\pi_S}(p), \overline{\pi_B}(p)\right)$ as a function of $p \in (b_L, s_H)$. A pure FB-SCE exists if and only if (π_S, π_B) is in the south-east of $\left(\underline{\pi_S}(p), \overline{\pi_B}(p)\right)$ for some $p \in (b_L, s_H)$; that is, if and only if (π_S, π_B) belongs to the blue-shaded area.

Lemma 1. *For any price-recommendation $q \in Q$, if the mediation plan (q, r_q) is not an SCE, then any FB mediation plan (q, r) is not either.*

Proof. See Appendix A.1. □

Proposition 1 provides a necessary and sufficient condition for the existence of pure FB-SCE.

Proposition 1. *If the intermediary is seller-biased ($\lambda > 1/2$), then a pure FB-SCE exists if and only if*

$$\pi_S \pi_B b_H + (1-\pi_S) b_L \geq \pi_B s_H + (1-\pi_S)(1-\pi_B) s_L. \tag{3.1}$$

Proof. See Appendix A.2. □

In the proof of Proposition 1, I show that a pure FB-SCE exists if and only if there exists some $p \in (b_L, s_H)$ such that

$$\pi_S \geq \frac{p - b_L}{b_H - s_H + p - b_L} \text{ and } \pi_B \leq \frac{b_L - s_L}{s_H - p + b_L - s_L}. \tag{3.2}$$

I also show that this condition is equivalent to (3.1). Figure 3.1 illustrates this condition. To prove the if-part, I construct a particular price-recommendation q and show that the FB mediation plan (q, r_q) is an SCE if and only if (3.2) holds for some $p \in (b_L, s_H)$. To prove the only-if-part, I show the contraposition. A necessary condition for FB-SCE implies that in any pure FB-SCE, a price-recommendation q must fall into one of the following three cases.

Case 1: Recommend two different prices for $(s, b) \neq (s_H, b_L)$; that is, $q(s_H, b_H) = q(s_L, b_H)$ and $q(s_L, b_L)$;

Case 2: Recommend two different prices for $(s, b) \neq (s_H, b_L)$; that is, $q(s_H, b_H)$ and $q(s_L, b_H) = q(s_L b_L)$;

Case 3: Recommend three different prices for $(s, b) \neq (s_H, b_L)$; that is, $q(s_H, b_H)$, $q(s_L, b_H)$, and $q(s_L, b_L)$.

In each case, Lemma 1 allows me to focus on (q, r_q). I show that (q, r_q) cannot be an SCE if (3.2) does not hold for all $p \in (b_L, s_H)$, which in turn implies by Lemma 1 that no FB mediation plan (q, r) is an SCE.

3.1 Acceptable SCE

The proof of Proposition 1 crucially hinges on a particular property of r_q; that is, it recommends the traders $(s, b) \neq (s_H, b_L)$ to reject all off-path prices $p \neq q(s, b)$, even if they are mutually acceptable. Such responses are optimal for each trader, and hence they have no incentive to disobey r_q. However, the optimality of such responses is not robust to small mistakes of the players. For example, if there is a small probability that the intermediary offers $p > q(s, b)$ and the buyer of type b accepts it, the seller of type s has the incentive to disobey r_q and accept p.

To eliminate such non-robust SCE, I consider a refinement of SCE in the spirit of ε-correlated equilibrium and acceptable correlated equilibrium by Myerson (1986a) in static games. The aforementioned non-robustness does not arise if each trader accepts a price if and only if it guarantees him a nonnegative payoff. To capture this idea, let $r^A \in R$ be such that for all $(s,b) \neq (s_H, b_L)$,

$$r^A(p \mid s,b) = \begin{cases} (N,Y) & \text{if } p \in [0,s) \\ (Y,Y) & \text{if } p \in [s,b] \\ (Y,N) & \text{if } p \in (b,+\infty) \end{cases}$$

and

$$r^A(p \mid s_H,b_L) = \begin{cases} (N,Y) & \text{if } p \in [0,b_L] \\ (N,N) & \text{if } p \in (b_L,s_H) \\ (Y,N) & \text{if } p \in [s_H,+\infty). \end{cases}$$

A mediation plan $\mu \in \Delta(Q \times R)$ is an *acceptable SCE (ASCE)* if it is an SCE and $\mu(Q \times r^A) = 1$.[10] For $\mu \in \Delta(Q \times r^A)$, let $\mu(q)$ and $q \in \operatorname{supp}(\mu)$ denote $\mu(q, r^A)$ and $(q, r^A) \in \operatorname{supp}(\mu)$, respectively. Hereafter, I use ASCE as the equilibrium concept.

A natural question then arises: when does an ASCE that achieves a FB outcome (FB-ASCE) exist? To this end, I present a series of lemmas that help me establish the necessary and sufficient condition for its existence. Since I focus on pure mediation plans in Proposition 1, mixed FB-SCE may exist under a weaker condition. The lemmas show that it is without loss of generality to focus on pure mediation plans if I adopt ASCE as the equilibrium concept. Furthermore, they also show that I can focus on a particular pure mediation plan. Since a response-recommendation is fixed at r^A, this implies that there exists a price-recommendation $q^* \in Q$ to which I can restrict my attention.

Lemma 2 provides a necessary condition for a mediation plan to be an ASCE. This lemma will be useful later when I characterize the second-best ASCE in Section 4.

Lemma 2. *A mediation plan $\mu \in \Delta(Q \times r^A)$ is an ASCE only if it recommends either b_H or b_L whenever there is gain from trade; that is, $q(s,b) \in \{b_H, b_L\}$ for all $(s,b) \neq (s_H, b_L)$ and all $q \in \operatorname{supp}(\mu)$.*

[10] With slight abuse of notation, I write $Q \times r^A$ instead of $Q \times \{r^A\}$.

Table 3.1: The price-recommendation q^*.

	b_H	b_L
s_H	b_H	$q^*(s_H, b_L) \in (b_H, +\infty)$
s_L	b_L	b_L

Proof. See Appendix A.3. □

Note that by the definition of r^A, a mediation plan $\mu \in \Delta(Q \times r^A)$ is FB if $q(s,b) \in [s,b]$ for all $(s,b) \neq (s_H, b_L)$ and all $q \in \text{supp}(\mu)$. Lemma 3 provides a necessary condition for a FB-ASCE.

Lemma 3. *A FB mediation plan $\mu \in \Delta(Q \times r^A)$ is an ASCE only if it recommends b_H when the traders are (s_H, b_H), b_L when (s_L, b_H) or (s_L, b_L), and some prices not in $[s_H, b_H)$ when (s_H, b_L); that is, for all $q \in \text{supp}(\mu)$, $q(s_H, b_H) = b_H$, $q(s_L, b_H) = q(s_L, b_L) = b_L$, and $q(s_H, b_L) \notin [s_H, b_H)$.*

Proof. See Appendix A.4. □

Since Lemma 3 pins down $q(s,b)$ for all $(s,b) \neq (s_H, b_L)$, the price-recommendation can be random only for $q(s_H, b_L)$. Let $q^* \in Q$ be such that $q^*(s_H, b_H) = b_H$, $q^*(s_H, b_L) \in (b_H, +\infty)$, and $q^*(s_L, b_H) = q^*(s_L, b_L) = b_L$ (see Table 3.1).

Lemma 4 allows me to focus on q^*, which implies that the mediator does not need such randomization to achieve a FB outcome.

Lemma 4. *If the mediation plan (q^*, r^A) is not an ASCE, then any FB mediation plan $\mu \in \Delta(Q \times r^A)$ is not either.*

Proof. See Appendix A.5. □

Focusing on (q^*, r^A), I can establish the necessary and sufficient condition for the existence of FB-ASCE.

Proposition 2. *If the intermediary is seller-biased ($\lambda > 1/2$), then a FB-ASCE exists if and only if $\pi_B \leq \frac{b_L - s_L}{b_H - s_L}$.*

Proof. I show that the FB mediation plan (q^*, r^A) is an ASCE if and only if $\pi_B \leq \frac{b_L - s_L}{b_H - s_L}$. Consider the traders' incentives for manipulation in (q^*, r^A). It is easy to see that no manipulation by the high-type seller and both types of buyer are profitable. If the low-type seller s_L is honest and obedient, then he will get $b_L - s_L$. If he misreports his type, then he can get at most $\pi_B(b_H - s_L)$. Hence, no manipulation is profitable for him if and only if

$$b_L - s_L \geq \pi_B(b_H - s_L) \iff \pi_B \leq \frac{b_L - s_L}{b_H - s_L}.$$

Next, consider the intermediary's incentive for manipulation. When he receives b_H, he believes that the traders are (s_H, b_H), who accept an offer p if and only if $p \in [s_H, b_H]$. Since his payoff is increasing in price, following the recommendation is optimal for him. When he receives $q^*(s_H, b_L)$, he believes that the traders are (s_H, b_L), who accept no offer. Hence, following the recommendation is optimal for him. When he receives b_L, he believes that the traders are (s_L, b_H) with probability $\frac{(1-\pi_S)\pi_B}{(1-\pi_S)\pi_B+(1-\pi_S)(1-\pi_B)}$ and (s_L, b_L) with probability $\frac{(1-\pi_S)(1-\pi_B)}{(1-\pi_S)\pi_B+(1-\pi_S)(1-\pi_B)}$. Since the traders (s_L, b_H) (resp. (s_L, b_L)) accept an offer p if and only if $p \in [s_L, b_H]$ (resp. $p \in [s_L, b_L]$), following the recommendation is optimal for him if offering b_H is not profitable; that is,

$$\begin{aligned}
&(1-\pi_S)\pi_B v(b_L \mid s_L, b_H) + (1-\pi_S)(1-\pi_B) v(b_L \mid s_L, b_L) \geq (1-\pi_S)\pi_B v(b_H \mid s_L, b_H) \\
\iff &\pi_B \leq \frac{\lambda(b_L - s_L)}{\lambda(b_H - s_L) - (1-\lambda)(b_H - b_L)}. \qquad (3.3)
\end{aligned}$$

Note that the right side of (3.3) is strictly decreasing in λ when $\lambda \in (1/2, 1]$ and equals $\frac{b_L - s_L}{b_H - s_L}$ at $\lambda = 1$. Hence, $\pi_B \leq \frac{b_L - s_L}{b_H - s_L}$ implies (3.3) for any $\lambda \in (1/2, 1]$. Therefore, (q^*, r^A) is an ASCE if and only if $\pi_B \leq \frac{b_L - s_L}{b_H - s_L}$.

To prove the only-if-part, suppose $\pi_B > \frac{b_L - s_L}{b_H - s_L}$. Since (q^*, r^A) is not an ASCE, any FB mediation plan $\mu \in \Delta(Q \times r^A)$ is not either by Lemma 4. This completes the proof. □

The proposition is illustrated in Figure 3.2. A FB outcome can be achieved in ASCE of the game if (π_S, π_B) belongs to the orange-shaded area. Note that this condition is stronger than that for pure SCE: a FB outcome can be achieved in pure SCE of the game if (3.1) holds; that is, if (π_S, π_B) is in the south-east of the blue thick line in Figure 3.2. Note also that, as I discuss in Appendix C.1.1 and C.1.2, if the intermediary is unbiased (with or without commitment and enforcement power), then a FB outcome can be achieved under the same condition as in pure

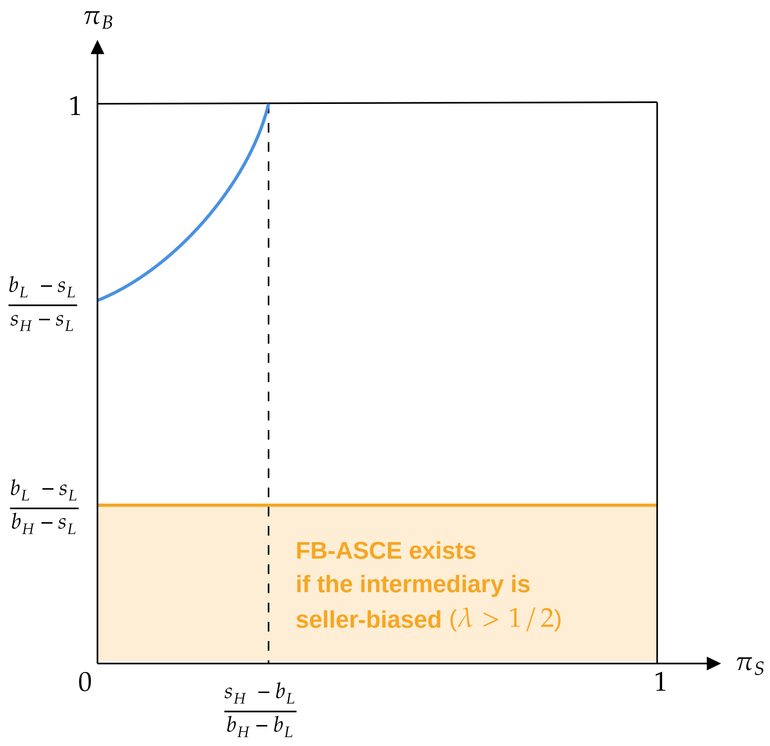

Figure 3.2: An illustration of Proposition 2: A FB-ASCE exists if and only if (π_S, π_B) belongs to the orange-shaded area. This area is strictly smaller than the blue one in Figure 3.1

SCE.[11] Therefore, introducing even a tiny bias shrinks the scope of the FB outcomes. This implies that the intermediary's bias is detrimental to efficiency, however small it is.

4 Second-Best Mediation Plan

In this section, I characterize the second-best mediation plan, an ASCE that maximizes the ex-ante expected trade surplus (SB-ASCE). Throughout this section, I assume $\pi_B > \frac{b_L - s_L}{b_H - s_L}$ (otherwise, the SB-ASCE coincides with the FB-ASCE by Proposition 2). If the mediator uses $\mu \in \Delta(Q \times r^A)$ and the players are honest and obedient to her, then the ex-ante expected trade surplus is

$$\sum_{(s,b)\in\Theta_S\times\Theta_B} \Pr(s,b) \sum_{q\in Q} \mu(q)\cdot(b-s)\cdot\mathbf{1}_{\{q(s,b)\in[s,b]\}},$$

[11]The case where the intermediary is unbiased and has commitment and enforcement power corresponds to the standard mechanism design setup studied in Matsuo (1989). See Appendix C.1.2 for further discussion.

where $\Pr(s,b)$ is the prior probability that the traders' types are (s,b), and $\mathbf{1}_{\{\cdot\}}$ is the indicator function. Thus, the SB-ASCE is characterized as a solution to the following optimization problem:

$$\max_{\mu\in\Delta(Q\times r^A)} \sum_{(s,b)\in\Theta_S\times\Theta_B} \Pr(s,b)\sum_{q\in Q}\mu(q)\cdot(b-s)\cdot\mathbf{1}_{\{q(s,b)\in[s,b]\}} \tag{4.1}$$

$$\text{subject to} \quad \mu \text{ is an ASCE.}$$

I express (4.1) as a linear programming problem. To this end, for any mediation plan $\mu\in\Delta(Q\times r^A)$, let

$$\begin{aligned}
x_{HH}(\mu) &= \sum_{q\in\{q'\in Q:\ q'(s_H,b_H)=b_H\}} \mu(q),\\
x_{HL}(\mu) &= \sum_{q\in\{q'\in Q:\ q'(s_H,b_L)=b_H\}} \mu(q),\\
x_{LH}(\mu) &= \sum_{q\in\{q'\in Q:\ q'(s_L,b_H)=b_H\}} \mu(q),\\
x_{LL}(\mu) &= \sum_{q\in\{q'\in Q:\ q'(s_L,b_L)=b_H\}} \mu(q).
\end{aligned}$$

For example, $x_{HH}(\mu)$ is the total probability that μ recommends the price b_H to the traders (s_H,b_H). For any price-recommendation $q\in Q$, let $q^{**}\in Q$ be such that (i) $q^{**}(s,b)=q(s,b)$ for all $(s,b)\neq(s_H,b_L)$; and (ii) $q^{**}(s_H,b_L)\in(b_H,+\infty)$. For any mediation plan $\mu\in\Delta(Q\times r^A)$, let $\mu^{**}\in\Delta(Q\times r^A)$ be such that $\mu^{**}(q^{**})=\mu(q)$ for all $q\in\operatorname{supp}(\mu)$. In other words, μ and μ^{**} can be different only in the price-recommendations to the traders (s_H,b_L). Note that $x_{HH}(\mu^{**})=x_{HH}(\mu)$, $x_{LH}(\mu^{**})=x_{LH}(\mu)$, and $x_{LL}(\mu^{**})=x_{LL}(\mu)$ by construction. Lemma 5 allows me to focus on the set of μ^{**}.

Lemma 5. *For any mediation plan $\mu\in\Delta(Q\times r^A)$:*

1. *the ex-ante expected trade surplus are the same in μ and μ^{**}; and*

2. *if μ^{**} is not an ASCE, then μ is not either.*

Proof. See Appendix A.6. □

In other words, μ^{**} is surplus-equivalent to μ and easier to sustain as an ASCE. In the proof of Lemma 5, I show that both the objective and the constraint of (4.1) are linear in $x_{HH}(\mu^{**})$, $x_{LH}(\mu^{**})$, and $x_{LL}(\mu^{**})$. Thus, combined with Lemma 2, this lemma implies that (4.1) reduces to a linear programming problem in these probabilities. Lemma 6 characterizes the solution to this problem.

Lemma 6. *Let* $x^{**} = \left(x^{**}_{HH}, x^{**}_{LH}, x^{**}_{LL}\right)$ *be the solution to the linear programming problem obtained from* (4.1). *Suppose* $\pi_B > \frac{b_L - s_L}{b_H - s_L}$. *If*

$$\begin{aligned}&\pi_S \pi_B \lambda (b_H - s_H) + (1-\pi_S)\pi_B(2\lambda - 1)(b_H - b_L)\\ &\quad - (1-\pi_S)(1-\pi_B)\lambda(b_L - s_L) \le \frac{\pi_S(1-\pi_B)\lambda(b_H - s_H)(b_L - s_L)}{b_H - b_L},\end{aligned} \tag{4.2}$$

then $x^{**} = x^{**1} = \left(\frac{(1-\pi_B)(b_L - s_L)}{\pi_B(b_H - b_L)}, 0, 0\right)$. *Otherwise,* $x^{**} = x^{**2} = (1, 1, 1)$.

Proof. See Appendix A.7. □

This lemma characterizes the SB-ASCE, but the condition (4.2) is hard to interpret as it is. By solving it with respect to λ, I can establish Proposition 3, which explains how the SB-ASCE and the associated expected trade surplus and the traders' expected payoffs change as the intermediary's bias λ changes.

Proposition 3. *Let* $x^{**} = \left(x^{**}_{HH}, x^{**}_{LH}, x^{**}_{LL}\right)$ *be the solution to the linear programming problem obtained from* (4.1).

1. *There exists* $\overline{\lambda}(s_H, s_L, b_H, b_L, \pi_S, \pi_B) < 1$ *such that the SB-ASCE is characterized by* $x^{**1} = \left(\frac{(1-\pi_B)(b_L - s_L)}{\pi_B(b_H - b_L)}, 0, 0\right)$ *if* $\lambda \in \left(1/2, \overline{\lambda}\right]$ *and* $x^{**2} = (1, 1, 1)$ *if* $\lambda \in \left(\overline{\lambda}, 1\right]$.
2. *All other things being equal, if the intermediary's bias increases from* $\lambda \in \left(1/2, \overline{\lambda}\right]$ *to* $\lambda' \in \left(\overline{\lambda}, 1\right]$*, then ex-ante, the seller will be better off but the buyer will not, and the expected trade surplus will decrease.*

Proof. By solving (4.2) with respect to λ, I obtain

$$\lambda \leq \frac{(1-\pi_S)\pi_B(b_H-b_L)^2}{[\pi_S(b_H-s_H)+(1-\pi_S)(b_H-b_L)]\underbrace{[\pi_B(b_H-b_L)-(1-\pi_B)(b_L-s_L)]}_{>0}+(1-\pi_S)\pi_B(b_H-b_L)^2}. \tag{4.3}$$

Let $\overline{\lambda}(s_H, s_L, b_H, b_L, \pi_S, \pi_B)$ be the right side of (4.3), which is smaller than 1. By Lemma 6, the SB-ASCE is characterized by $x^{**1} = \left(\frac{(1-\pi_B)(b_L-s_L)}{\pi_B(b_H-b_L)}, 0, 0\right)$ if $\lambda \in \left(1/2, \overline{\lambda}\right]$ and $x^{**2} = (1, 1, 1)$ if $\lambda \in \left(\overline{\lambda}, 1\right]$. This completes the proof of Part 1.

Note that $\overline{\lambda}$ is greater than $1/2$ if

$$\begin{aligned}
&(1-\pi_S)\pi_B(b_H-b_L)^2 \\
&\quad > [\pi_S(b_H-s_H)+(1-\pi_S)(b_H-b_L)][\pi_B(b_H-b_L)-(1-\pi_B)(b_L-s_L)] \\
\iff \pi_B &< \frac{(b_L-s_L)[\pi_S(b_H-s_H)+(1-\pi_S)(b_H-b_L)]}{\pi_S(b_H-s_H)(b_H-b_L)+(b_L-s_L)[\pi_S(b_H-s_H)+(1-\pi_S)(b_H-b_L)]}.
\end{aligned} \tag{4.4}$$

Let $\overline{\pi_B}(s_H, s_L, b_H, b_L, \pi_S)$ be the right side of (4.4). Since $\overline{\pi_B} \in \left(\frac{b_L-s_L}{b_H-s_L}, 1\right)$, the SB-ASCE can change as λ changes only if $\pi_B \in \left(\frac{b_L-s_L}{b_H-s_L}, \overline{\pi_B}\right)$. Otherwise, $\overline{\lambda} \leq 1/2$ and hence the SB-ASCE is characterized by x^{**2} for any $\lambda \in (1/2, 1]$.

Next, suppose $\pi_B \in \left(\frac{b_L-s_L}{b_H-s_L}, \overline{\pi_B}\right)$; that is, $\overline{\lambda} \in (1/2, 1)$. All other things being equal, if the bias increases from $\lambda \in \left(1/2, \overline{\lambda}\right]$ to $\lambda' \in \left(\overline{\lambda}, 1\right]$, then the SB-ASCE changes as the solution to the linear programming problem changes from x^{**1} to x^{**2}. In x^{**1}, ex-ante, the seller's expected payoff, the buyer's expected payoff, and the expected trade surplus are given by, respectively,

$$\begin{aligned}
\frac{\pi_S(1-\pi_B)(b_H-s_H)(b_L-s_L)}{b_H-b_L}+(1-\pi_S)(b_L-s_L),& \\
(1-\pi_S)\pi_B(b_H-b_L),& \\
\frac{\pi_S(1-\pi_B)(b_H-s_H)(b_L-s_L)}{b_H-b_L}+(1-\pi_S)\pi_B(b_H-s_L)+(1-\pi_S)(1-\pi_B)(b_L-s_L).&
\end{aligned}$$

In x^{**2}, those are given by, respectively,

$$\begin{aligned}&\pi_S\pi_B(b_H - s_H) + (1-\pi_S)\pi_B(b_H - s_L),\\ &0,\\ &\pi_S\pi_B(b_H - s_H) + (1-\pi_S)\pi_B(b_H - s_L).\end{aligned}$$

Since $(1-\pi_S)\pi_B(b_H - b_L) > 0$, the buyer will be worse off by this change. The change in the seller's expected payoff is

$$\begin{aligned}&\pi_S\pi_B(b_H - s_H) + (1-\pi_S)\pi_B(b_H - s_L)\\ &-\frac{\pi_S(1-\pi_B)(b_H - s_H)(b_L - s_L)}{b_H - b_L} - (1-\pi_S)(b_L - s_L)\\ =&\left[\frac{\pi_S(b_H - s_H)}{b_H - b_L} + (1-\pi_S)\right]\underbrace{[\pi_B(b_H - s_L) - (b_L - s_L)]}_{>0} > 0.\end{aligned}$$

Hence, in contrast to the buyer, the seller will be better off. Finally, the change in the expected trade surplus is

$$\begin{aligned}&\pi_S\pi_B(b_H - s_H) - \frac{\pi_S(1-\pi_B)(b_H - s_H)(b_L - s_L)}{b_H - b_L} - (1-\pi_S)(1-\pi_B)(b_L - s_L)\\ =&\frac{1}{b_H - b_L}\Big[\pi_S\pi_B(b_H - s_H)(b_H - b_L) - \pi_S(1-\pi_B)(b_H - s_H)(b_L - s_L)\\ &\qquad - (1-\pi_S)(1-\pi_B)(b_H - b_L)(b_L - s_L)\Big].\end{aligned}$$

This is strictly smaller than 0 because inside the brackets is negative if $\pi_B < \overline{\pi_B}$. Thus, the expected trade surplus will decrease by the change. This completes the proof. □

Proposition 3 implies that not every change in the intermediary's bias affects the SB-ASCE. Only those that cross the threshold $\overline{\lambda}$ change the SB-ASCE and hence affect the surplus and the traders' expected payoffs. As I show in the proof, such changes can happen only if $\overline{\lambda} \in (1/2, 1)$; that is, $\pi_B \in \left(\frac{b_L - s_L}{b_H - s_L}, \overline{\pi_B}\right)$. This is illustrated in Figure 4.1. Intuitively, when the probability of the high-type buyer is large enough ($\pi_B > \overline{\pi_B}$), the intermediary has a strong incentive to offer

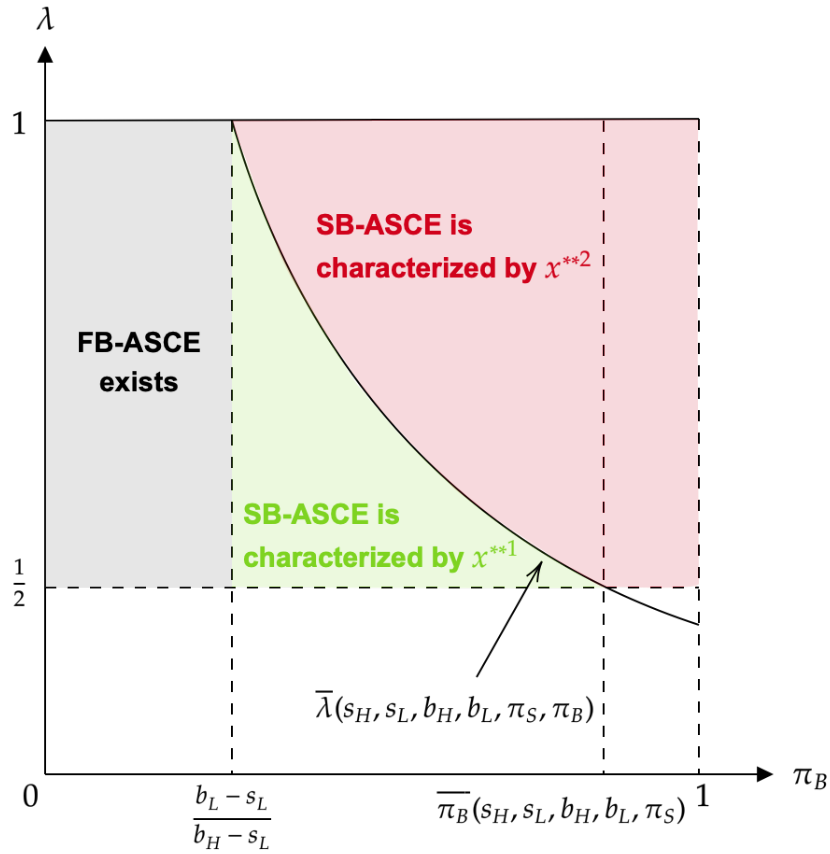

Figure 4.1: An illustration of Proposition 3: If $\pi_B < \overline{\pi_B}$, then $\overline{\lambda} \in (1/2, 1)$, and the SB-ASCE is characterized by x^{**1} if $\lambda \in \left(1/2, \overline{\lambda}\right]$ and by x^{**2} if $\lambda \in \left(\overline{\lambda}, 1\right]$. If $\pi_B \geq \overline{\pi_B}$, then $\overline{\lambda} \leq 1/2$ and hence the SB-ASCE is characterized by x^{**2} for any $\lambda \in (1/2, 1]$.

b_H to gain from the trade between (s_H, b_H). Thus, he always offers b_H, hence x^{**2}. When this probability is moderate ($\pi_B \in \left(\frac{b_L - s_L}{b_H - s_L}, \overline{\pi_B}\right)$), if the bias λ is not too large, then the intermediary can be incentivized to offer b_L with positive probability, hence x^{**1}. If the bias is too large, then the intermediary's payoff increases very fast in price, which makes it impossible to incentivize him to offer b_L, hence x^{**2}.

In terms of the expected surplus, this proposition suggests that if the prior probability of the high-type buyer is not too high ($\pi_B \in \left(\frac{b_L - s_L}{b_H - s_L}, \overline{\pi_B}\right)$), then a smaller bias is socially desirable. Otherwise, changes in the bias do not matter (either achieving a FB outcome is possible for any value of bias greater than 1/2, or the bias does not affect the SB outcome).

It is worth noting what happens on-path in the SB-ASCE. In x^{**1}, the intermediary offers b_H to the traders (s_H, b_H) with probability $\frac{(1-\pi_B)(b_L - s_L)}{\pi_B(b_H - b_L)} > 0$ and b_L with the complementary probability. The traders (s_H, b_H) only accept the former, and hence they do not trade with positive probability, even if they should in terms of efficiency. The intermediary always offers the price b_L to (s_L, b_H) and (s_L, b_L), and they accept this offer. Thus, there is no efficiency loss coming from them. This SB-ASCE is similar to the FB-ASCE (q^*, r^A) considered in the proof of Proposition 2. The only difference is that the traders (s_H, b_H) trade at b_H with probability 1 in the latter. The other SB-ASCE, characterized by x^{**2}, is very different from (q^*, r^A). Whenever there is gain from trade, the intermediary offers the price b_H, which is accepted by only (s_H, b_H) and (s_L, b_H). Thus, (s_L, b_L) do not trade with probability 1. Part 2 of the proposition suggests that the efficiency loss from them is larger than the gain from ensuring the trade between (s_H, b_H).

5 Conclusion

This paper has studied the effect of a biased intermediary without commitment and enforcement power in a fully general communication structure. In a canonical bilateral trade model with binary valuations, I have shown two key findings: first, the intermediary's bias is detrimental to efficiency, however small it is; and second, in the SB case, a smaller bias is socially desirable if the buyer is not very likely to highly value the good. It is worth mentioning that the first finding is closely related to the possibility result known in the standard mechanism design setup (see Matsuo (1989)); introducing even a tiny bias can shrink the scope of the efficient outcome compared to that case. Note also that changes in bias do not always affect the SB outcome: even when these changes can matter, only those that cross the threshold do.

Several properties of the players' payoffs are crucial for the results. In particular, first, the seller-biased intermediary's payoff is increasing in price; and second, the rejection gives both traders the payoff 0, which is preferred by the seller (resp. the buyer) to the trade at a price below (resp. above) his valuation. Thus, when at least one of these properties does not hold, which may be a reasonable assumption in some bargaining situations, further analysis is required to see the effect of a biased intermediary.

A Proofs

A.1 Proof of Lemma 1

First, I prove the following lemma, which provides the necessary conditions for FB-SCE.

Lemma 7. *A FB mediation plan $(q,r) \in Q \times R$ is an SCE only if*

(i) $q(s,b) \in [s,b]$ *for all* $(s,b) \neq (s_H, b_L)$*;*

(ii) $r(p \mid s,b) \neq (Y,Y)$ *for all* $p > q(s,b)$ *and all* $(s,b) \neq (s_H, b_L)$*;*

(iii)

$$r(q(s_H,b_L) \mid s_H,b_L) = \begin{cases} (N,Y) \text{ or } (N,N) & \text{if } q(s_H,b_L) \in [0,b_L) \\ (Y,N),(N,Y), \text{ or } (N,N) & \text{if } q(s_H,b_L) \in [b_L,s_H] \\ (Y,N) \text{ or } (N,N) & \text{if } q(s_H,b_L) \in (s_H,+\infty), \end{cases}$$

(iv) $r(p \mid s_H,b_L) \neq (Y,Y)$ *for all* $p > \frac{\lambda s_H - (1-\lambda) b_L}{2\lambda - 1}$.

Proof. I show the contraposition. Consider an arbitrary FB mediation plan (q,r). First, suppose that there exists some $(s,b) \neq (s_H,b_L)$ such that $q(s,b) \notin [s,b]$. Since (q,r) is FB, it recommends $r(q(s,b) \mid s,b) = (Y,Y)$. However, either the seller or the buyer will get a negative payoff if he accepts $q(s,b)$. Hence, this player has the incentive to disobey and reject $q(s,b)$.

Second, suppose that there exists some $(s,b) \neq (s_H,b_L)$ and $p > q(s,b)$ such that $r(p \mid s,b) = (Y,Y)$. If the intermediary is obedient and offers $q(s,b)$, then it will be accepted and he will get $v(q(s,b) \mid s,b)$. Since $v(\cdot \mid s,b)$ is strictly increasing, he has the incentive to disobey and offer p, which gives him $v(p \mid s,b) > v(q(s,b) \mid s,b)$.

Third, suppose that (iii) does not hold. Note that $r(q(s_H,b_L) \mid s_H,b_L) \neq (Y,Y)$ since (q,r) is FB. If $q(s_H,b_L) \in [0,b_L)$ and $r(q(s_H,b_L) \mid s_H,b_L) = (Y,N)$, then the low-type buyer b_L has the incentive to disobey and accept $q(s_H,b_L)$, which gives him $b_L - q(s_H,b_L) > 0$. If $q(s_H,b_L) \in (s_H,+\infty)$ and $r(q(s_H,b_L) \mid s_H,b_L) = (N,Y)$, then the high-type seller s_H has the incentive to disobey and accept $q(s_H,b_L)$, which gives him $q(s_H,b_L) - s_H > 0$.

Finally, suppose that there exists some $p > \frac{\lambda s_H - (1-\lambda) b_L}{2\lambda - 1}$ such that $r(p \mid s_H,b_L) = (Y,Y)$. If the intermediary is obedient and offers $q(s_H,b_L)$, then it will be rejected and he will get 0. Since

$v(p \mid s,b) > 0 \Leftrightarrow p > \frac{\lambda s_H-(1-\lambda)b_L}{2\lambda-1}$, he has the incentive to disobey and offer p.

Therefore, if (q,r) fails to satisfy any of (i)–(iv), then some player has the incentive to disobey, which implies that (q,r) is not an SCE. □

Note that if q satisfies condition (i) in Lemma 7, then no manipulation by the intermediary is profitable in (q,r_q); for $(s,b) \neq (s_H,b_L)$, any off-path offer $p \neq q(s,b)$ will be rejected and give him the payoff 0, while the on-path offer $q(s,b)$ will be accepted and give him a positive payoff $v(q(s,b) \mid s,b)$. For (s_H,b_L), no offer will be accepted and he will always get 0.

Let $r \in R$ be an arbitrary response-recommendation such that (q,r) is FB and satisfies the necessary conditions in Lemma 7 (otherwise, it is not an SCE and hence the statement of the lemma holds). Since (q,r) is FB, r must also satisfy $r(q(s,b) \mid s,b) = (Y,Y)$ for all $(s,b) \neq (s_H,b_L)$ and $r(q(s_H,b_L) \mid s_H,b_L) \neq (Y,Y)$. Hence, r can be different from r_q in; first, $r(p \mid s,b) \neq (N,N)$ for some $(s,b) \neq (s_H,b_L)$ and some $p \neq q(s,b)$; second, $r(q(s_H,b_L) \mid s_H,b_L) \neq (N,N)$; and third, $r(p \mid s_H,b_L) \neq (N,N)$ for some $p \neq q(s_H,b_L)$.

Case 1: Suppose that there exists some $(s,b) \neq (s_H,b_L)$ and $p \neq q(s,b)$ such that $r(p \mid s,b) \neq (N,N)$. The traders' IC constraints in (q,r) are the same as those in (q,r_q) because the off-path price p never appears in the traders' constraints. Note that condition (ii) in Lemma 7 ensures that p can be accepted only when $p \leq q(s,b)$. Hence, the intermediary can get at most $v(p \mid s,b) \leq v(q(s,b) \mid s,b)$ by manipulation, which implies that no manipulation is profitable as in (q,r_q).

Case 2: Suppose $r(q(s_H,b_L) \mid s_H,b_L) \neq (N,N)$. Note that the IC constraints for the high-type seller s_H and the low-type buyer b_L in (q,r) are the same as those in (q,r_q). This is because condition (iii) in Lemma 7 ensures that $r(q(s_H,b_L) \mid s_H,b_L) \neq (N,N)$ does not give rise to additional manipulations by them that can be profitable. Note also that no manipulation by the intermediary is profitable as in (q,r_q).

However, the IC constaints for the low-type seller s_L and the high-type buyer b_H can be more stringent than those in (q,r_q). If $q(s_H,b_L) \in (s_L,b_L)$ and $r(q(s_H,b_L) \mid s_H,b_L) = (N,Y)$, then s_L can have the incentive to misreport his type and accept $q(s_H,b_L)$, which gives him $q(s_H,b_L)-s_L > 0$. If $q(s_H,b_L) \in (s_H,b_H)$ and $r(q(s_H,b_L) \mid s_H,b_L) = (Y,N)$, then b_H can have the incentive to misreport his type and accept $q(s_H, b_L)$, which gives him $b_H - q(s_H, b_L) > 0$. Since there are now additional manipulations that must be detered, the IC constraints for them in (q, r) are at least as stringent as those in (q, r_q).

Case 3: Suppose that there exists some $p \neq q(s_H, b_L)$ such that $r(p \mid s_H, b_L) \neq (N, N)$. The traders' IC constraints in (q, r) are the same as those in (q, r_q) because the off-path price p never appears in the traders' constraints. Note that condition (iv) in Lemma 7 ensures that p can be accepted only when $p \leq \frac{\lambda s_H - (1-\lambda) b_L}{2\lambda - 1} \Leftrightarrow v(p \mid s, b) \leq 0$. Hence, the intermediary can get at most $v(p \mid s, b) \leq 0$ by manipulation, which implies that no manipulation is profitable as in (q, r_q).

In conclusion, the players' IC constraints in (q, r) are at least as stringent as those in (q, r_q). Therefore, if (q, r_q) is not an SCE, then (q, r) is not either.

A.2 Proof of Proposition 1

In what follows, I show that a pure FB-SCE exists if and only if there exists some $p \in (b_L, s_H)$ such that $\pi_S \geq \frac{p - b_L}{b_H - s_H + p - b_L}$ and $\pi_B \leq \frac{b_L - s_L}{s_H - p + b_L - s_L}$. By solving these inequalities with respect to p, I obtain

$$p \in \left[s_H - \frac{1 - \pi_B}{\pi_B}(b_L - s_L),\, b_L + \frac{\pi_S}{1 - \pi_S}(b_H - s_H) \right].$$

Since $s_H - \frac{1-\pi_B}{\pi_B}(b_L - s_L) < s_H$ and $b_L < b_L + \frac{\pi_S}{1-\pi_S}(b_H - s_H)$, such $p \in (b_L, s_H)$ exists if and only if

$$\begin{aligned} & s_H - \frac{1 - \pi_B}{\pi_B}(b_L - s_L) \leq b_L + \frac{\pi_S}{1 - \pi_S}(b_H - s_H) \\ \Longleftrightarrow\ & \pi_S \pi_B b_H + (1 - \pi_S) b_L \geq \pi_B s_H + (1 - \pi_S)(1 - \pi_B) s_L. \end{aligned}$$

Therefore, the condition is equivalent to (3.1).

Proof of the if-part: Let $q \in Q$ be such that $q(s_H, b_H) = s_H$, $q(s_H, b_L) \in (b_H, +\infty)$, $q(s_L, b_H) = p$ for some $p \in (b_L, s_H)$, and $q(s_L, b_L) = b_L$ (see Table A.1).

I show that the FB mediation plan (q, r_q) is an SCE if and only if $\pi_S \geq \frac{p - b_L}{b_H - s_H + p - b_L}$ and

Table A.1: The price-recommendation q.

	b_H	b_L
s_H	s_H	$q(s_H,b_L)\in(b_H,+\infty)$
s_L	$p\in(b_L,s_H)$	b_L

$\pi_B \leq \frac{b_L-s_L}{s_H-p+b_L-s_L}$, where $p\in(b_L,s_H)$. It is easy to see that no manipulation by the high-type seller s_H and the low-type buyer b_L is profitable. If the low-type seller s_L is honest and obedient, then he will get $\pi_B(p-s_L)+(1-\pi_B)(b_L-s_L)$. If he misreports his type, then he can get at most $\pi_B(s_H-s_L)$. Hence, he has no profitable manipulation if and only if

$$\pi_B(p-s_L)+(1-\pi_B)(b_L-s_L)\geq\pi_B(s_H-s_L) \iff \pi_B\leq\frac{b_L-s_L}{s_H-p+b_L-s_L}.$$

Similarly, if the high-type buyer b_H is honest and obedient, he will get $\pi_S(b_H-s_H)+(1-\pi_S)(b_H-p)$. If he misreports his type, then he can get at most $(1-\pi_S)(b_H-b_L)$. Hence, he has no profitable manipulation if and only if

$$\pi_S(b_H-s_H)+(1-\pi_S)(b_H-p)\geq(1-\pi_S)(b_H-b_L) \iff \pi_S\geq\frac{p-b_L}{b_H-s_H+p-b_L}.$$

By the same argument as in the proof of Lemma 1, the intermediary has no profitable manipulation in (q,r_q), as $q(s,b)\in[s,b]$ for all $(s,b)\neq(s_H,b_L)$. Thus, (q,r_q) is an SCE if and only if $\pi_S\geq\frac{p-b_L}{b_H-s_H+p-b_L}$ and $\pi_B\leq\frac{b_L-s_L}{s_H-p+b_L-s_L}$, where $p\in(b_L,s_H)$.

Proof of the only-if-part: First, I show the following lemma.

Lemma 8. *If $\pi_S\geq\frac{s_H-b_L}{b_H-b_L}$ or $\pi_B\leq\frac{b_L-s_L}{s_H-s_L}$, then there exists some $p\in(b_L,s_H)$ such that $\pi_S\geq\frac{p-b_L}{b_H-s_H+p-b_L}$ and $\pi_B\leq\frac{b_L-s_L}{s_H-p+b_L-s_L}$.*

Proof. Let $\underline{\pi_S}(p)=\frac{p-b_L}{b_H-s_H+p-b_L}$ and $\overline{\pi_B}(p)=\frac{b_L-s_L}{s_H-p+b_L-s_L}$. If $\pi_S\geq\frac{s_H-b_L}{b_H-b_L}$ and $\pi_B>\frac{b_L-s_L}{s_H-s_L}$ (e.g., point A in Figure A.1), then for any $p\geq p_B$, where p_B is such that $\overline{\pi_B}(p_B)=\pi_B$, I have $\pi_S\geq\underline{\pi_S}(p)$ and $\pi_B\leq\overline{\pi_B}(p)$. If $\pi_S<\frac{s_H-b_L}{b_H-b_L}$ and $\pi_B\leq\frac{b_L-s_L}{s_H-s_L}$ (e.g., point B in Figure A.1), then for any $p\leq p_S$, where p_S is such that $\underline{\pi_S}(p_S)=\pi_S$, I have $\pi_S\geq\underline{\pi_S}(p)$ and $\pi_B\leq\overline{\pi_B}(p)$. If $\pi_S\geq\frac{s_H-b_L}{b_H-b_L}$ and $\pi_B\leq\frac{b_L-s_L}{s_H-s_L}$ (e.g., point C in Figure A.1), then $\pi_S\geq\underline{\pi_S}(p)$ and $\pi_B\leq\overline{\pi_B}(p)$ for any $p\in(b_L,s_H)$. □

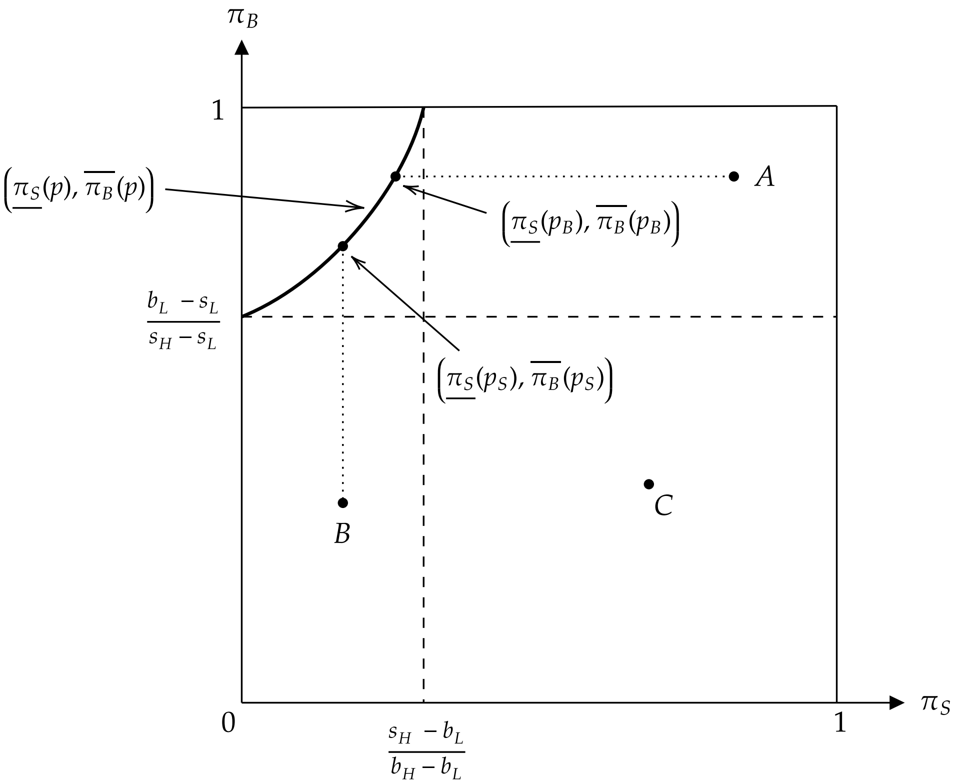

Figure A.1: An illustration of the proof of Lemma 8: The thick line is the locus of $\left(\underline{\pi_S}(p), \overline{\pi_B}(p)\right)$ as a function of $p \in (b_L, s_H)$. For point A (resp. B), any $p \geq p_B$ (resp. $p \leq p_S$) satisfies $\pi_S \geq \underline{\pi_S}(p)$ and $\pi_B \leq \overline{\pi_B}(p)$. For point C, any $p \in (b_L, s_H)$ satisfies the condition.

Table A.2: The price-recommendation q in Case 2.

	b_H	b_L
s_H	$p_H \in [s_H, b_H]$	$q(s_H, b_L) \in \mathbb{R}_+$
s_L	$p_H \in [s_H, b_H]$	$p_L \in [s_L, b_L]$

Suppose $\pi_S < \frac{p-b_L}{b_H-s_H+p-b_L}$ or $\pi_B > \frac{b_L-s_L}{s_H-p+b_L-s_L}$ for all $p \in (b_L, s_H)$. By Lemma 8, this implies $\pi_S < \frac{s_H-b_L}{b_H-b_L}$ and $\pi_B > \frac{b_L-s_L}{s_H-s_L}$. I show that no FB mediation plan is an SCE. By condition (i) in Lemma 7, any price-recommendation q that can constitute a FB-SCE with some response-recommendation must fall into one of the following three cases.[12]

Case 1: Recommend two different prices for $(s,b) \neq (s_H, b_L)$; that is, $p_H = q(s_H, b_H) = q(s_L, b_H)$ and $p_L = q(s_L, b_L)$;

Case 2: Recommend two different prices for $(s,b) \neq (s_H, b_L)$; that is, $p_H = q(s_H, b_H)$ and $p_L = q(s_L, b_H) = q(s_L, b_L)$;

Case 3: Recommend three different prices for $(s,b) \neq (s_H, b_L)$; that is, $p_{HH} = q(s_H, b_H)$, $p_{LH} = q(s_L, b_H)$, and $p_{LL} = q(s_L, b_L)$.

In each case, Lemma 1 allows me to restrict my attention to (q, r_q).

Case 1: Let $q \in Q$ be such that $q(s_H, b_H) = q(s_L, b_H) = p_H$ for some $p_H \in [s_H, b_H]$, $q(s_H, b_L) \in \mathbb{R}_+$, and $q(s_L, b_L) = p_L$ for some $p_L \in [s_L, b_L]$ (see Table A.2).

It is easy to see that no manipulation by both types of seller and the low-type buyer b_L is profitable. If the high-type buyer b_H is honest and obedient, then he will get $b_H - p_H$. If he misreports his type, then he can get at most $(1-\pi_S)(b_H - p_L)$. Hence, he has no profitable manipulation if and only if

$$b_H - p_H \geq (1-\pi_S)(b_H - p_L) \iff \pi_S \geq \frac{p_H - p_L}{b_H - p_L}.$$

By the same argument as before, the intermediary has no profitable manipulation in (q, r_q). Hence, (q, r_q) is an SCE if and only if $\pi_S \geq \frac{p_H-p_L}{b_H-p_L}$, which is least stringent at

[12]Note that recommending the same price to (s_H, b_H), (s_L, b_H), and (s_L, b_L) is impossible since the mutually acceptable prices for (s_H, b_H) are unacceptable for (s_L, b_L) and vice versa.

Table A.3: The price-recommendation q in Case 2.

	b_H	b_L
s_H	$p_H \in [s_H, b_H]$	$q(s_H, b_L) \in \mathbb{R}_+$
s_L	$p_L \in [s_L, b_L]$	$p_L \in [s_L, b_L]$

Table A.4: The price-recommendation q in Case 3.

	b_H	b_L
s_H	$p_{HH} \in [s_H, b_H]$	$q(s_H, b_L) \in \mathbb{R}_+$
s_L	$p_{LH} \in [s_L, b_H]$	$p_{LL} \in [s_L, b_L]$

$(p_H, p_L) = (s_H, b_L)$ and reduces to $\pi_S \geq \frac{s_H - b_L}{b_H - b_L}$. Since $\pi_S < \frac{s_H - b_L}{b_H - b_L}$, the mediation plan (q, r_q) cannot be an SCE.

Case 2 Let $q \in Q$ be such that $q(s_H, b_H) = p_H$ for some $p_H \in [s_H, b_H]$, $q(s_H, b_L) \in \mathbb{R}_+$, and $q(s_L, b_H) = q(s_L, b_L) = p_L$ for some $p_L \in [s_L, b_L]$ (see Table A.3).

It is easy to see that no manipulation by the high-type seller s_H and both types of buyer is profitable. If the low-type seller s_L is honest and obedient, then he will get $p_L - s_L$. If he misreports his type, then he can get at most $\pi_B(p_H - s_L)$. Hence, he has no profitable manipulation if and only if

$$p_L - s_L \geq \pi_B(p_H - s_L) \iff \pi_B \leq \frac{p_L - s_L}{p_H - s_L}.$$

By the same argument as before, the intermediary has no profitable manipulation in (q, r_q). Hence, (q, r_q) is an SCE if and only if $\pi_B \leq \frac{p_L - s_L}{p_H - s_L}$, which is least stringent at $(p_H, p_L) = (s_H, b_L)$ and reduces to $\pi_B \leq \frac{b_L - s_L}{s_H - s_L}$. Since $\pi_B > \frac{b_L - s_L}{s_H - s_L}$, the mediation plan (q, r_q) cannot be an SCE.

Case 3: Let $q \in Q$ be such that $q(s_H, b_H) = p_{HH}$ for some $p_{HH} \in [s_H, b_H]$, $q(s_H, b_L) \in \mathbb{R}_+$, $q(s_L, b_H) = p_{LH}$ for some $p_{LH} \in [s_L, b_H]$, and $q(s_L, b_L) = p_{LL}$ for some $p_{LL} \in [s_L, b_L]$, where $p_{HH} \neq p_{LH} \neq p_{LL}$ (see Table A.4).

If the high-type seller s_H is honest and obedient, then he wil get $\pi_B(p_{HH} - s_H)$. If he misreports his type, then he can get at most $\max\{\pi_B(p_{LH} - s_H), 0\}$. Hence, he has no profitable manipulation if and only if $p_{HH} > p_{LH}$. Similarly, the low-type buyer b_L has no profitable manipulation if and only if $p_{LH} > p_{LL}$. If the low-type seller s_L is honest and obedient, then he will get $\pi_B(p_{LH}-s_L)+(1-\pi_B)(p_{LL}-s_L)$. If he misreports his type, then he can get at most $\pi_B(p_{HH}-s_L)$. Hence, he has no profitable manipulation if and only if

$$\begin{aligned}&\pi_B(p_{LH}-s_L)+(1-\pi_B)(p_{LL}-s_L)\geq\pi_B(p_{HH}-s_L)\\ \iff &\pi_B\leq\frac{p_{LL}-s_L}{p_{HH}-p_{LH}+p_{LL}-s_L}.\end{aligned}$$

Similarly, the high-type buyer b_H has no profitable manipulation if and only if

$$\begin{aligned}&\pi_S(b_H-p_{HH})+(1-\pi_S)(b_H-p_{LH})\geq(1-\pi_S)(b_H-p_{LL})\\ \iff &\pi_S\geq\frac{p_{LH}-p_{LL}}{b_H-p_{HH}+p_{LH}-p_{LL}}.\end{aligned}$$

By the same argument as before, the intermediary has no profitable manipulation in (q,r_q). Hence, (q,r_q) is an SCE if and only if $p_{HH}>p_{LH}>p_{LL}$, $\pi_S\geq\frac{p_{LH}-p_{LL}}{b_H-p_{HH}+p_{LH}-p_{LL}}$, and $\pi_B\leq\frac{p_{LL}-s_L}{p_{HH}-p_{LH}+p_{LL}-s_L}$, which are least stringent at $(p_{HH},p_{LL})=(s_H,b_L)$ and reduce to $\pi_S\geq\frac{p_{LH}-b_L}{b_H-s_H+p_{LH}-b_L}$ and $\pi_B\leq\frac{b_L-s_L}{s_H-p_{LH}+b_L-s_L}$ for some $p_{LH}\in(b_L,s_H)$. Since $\pi_S<\frac{p-b_L}{b_H-s_H+p-b_L}$ or $\pi_B>\frac{b_L-s_L}{s_H-p+b_L-s_L}$ for all $p\in(b_L,s_H)$, the mediation plan (q,r_q) cannot be an SCE.

In conclusion, if $\pi_S<\frac{p-b_L}{b_H-s_H+p-b_L}$ or $\pi_B>\frac{b_L-s_L}{s_H-p+b_L-s_L}$ for all $p\in(b_L,s_H)$, then, for each possible q above, a FB mediation plan (q,r_q) is not an SCE. By Lemma 1, this in turn implies that no FB mediation plan (q,r) is an SCE.

A.3 Proof of Lemma 2

I show the contraposition. Consider an arbitrary mediation plan $\mu\in\Delta(Q\times r^A)$. Suppose that there exist some $q\in\operatorname{supp}(\mu)$ and some $(s,b)\neq(s_H,b_L)$ such that $q(s,b)\notin\{b_H,b_L\}$. When he receives $q(s,b)$, the intermediary's expected payoff from offering p is

$$\begin{cases} \xi(s_H,b_H \mid q(s,b))v(p \mid s_H,b_H) + \xi(s_L,b_H \mid q(s,b))v(p \mid s_L,b_H) & \text{if } p \in [s_H,b_H] \\ \xi(s_L,b_H \mid q(s,b))v(p \mid s_L,b_H) & \text{if } p \in (b_L,s_H) \\ \xi(s_L,b_H \mid q(s,b))v(p \mid s_L,b_H) + \xi(s_L,b_L \mid q(s,b))v(p \mid s_L,b_L) & \text{if } p \in [s_L,b_L] \\ 0 & \text{otherwise.} \end{cases}$$

where $\xi(\tilde{s},\tilde{b} \mid \tilde{p})$ is the intermediary's posterior belief that the traders are $(\tilde{s},\tilde{b})$ When he receives $\tilde{p}$, which is given by

$$\xi(\tilde{s},\tilde{b} \mid \tilde{p}) = \frac{\Pr(\tilde{s},\tilde{b}) \sum_{q\in\{q'\in Q:\, q'(\tilde{s},\tilde{b})=\tilde{p}\}} \mu(q)}{\sum_{(s,b)\in\Theta_S\times\Theta_B} \Pr(s,b) \sum_{q\in\{q'\in Q:\, q'(s,b)=\tilde{p}\}} \mu(q)}.$$

Since v is strictly increasing and $v(p \mid \tilde{s},\tilde{b})$ is positive if $p \in [\tilde{s},\tilde{b}]$, the intermediary has the incentive to disobey and offer either b_H or b_L. Thus, μ is not an ASCE.

A.4 Proof of Lemma 3

By Lemma 2 and the definition of FB, a FB mediation plan $\mu \in \Delta(Q \times r^A)$ is an ASCE only if, for all $q \in \text{supp}(\mu)$, $q(s_H,b_H) = b_H$, $q(s_L,b_H) \in \{b_H,b_L\}$, and $q(s_L,b_L) = b_L$. If the high-type buyer b_H is honest and obedient, then he can get $b_H - b_L$ when the seller is of low-type and the price b_L is offered. If he misreports his type, then he can get $b_H - b_L$ when the seller is of low-type, and $b_H - p > 0$ when the seller is of high-type and the price $p \in [s_H,b_H)$ is offered. Thus, he has no profitable manipulation if

$$\begin{aligned} &\sum_{q\in\{q'\in Q:\, q'(s_L,b_H)=b_L\}} \mu(q)(1-\pi_S)(b_H-b_L) \\ &\quad \geq \sum_{p\in[s_H,b_H)} \sum_{q\in\{q'\in Q:\, q'(s_H,b_L)=p\}} \mu(q)\pi_S(b_H-p) + (1-\pi_S)(b_H-b_L) \\ \iff &\sum_{p\in[s_H,b_H)} \sum_{q\in\{q'\in Q:\, q'(s_H,b_L)=p\}} \mu(q)\pi_S(b_H-p) \\ &\quad + \left(1 - \sum_{q\in\{q'\in Q:\, q'(s_L,b_H)=b_L\}} \mu(q)\right)(1-\pi_S)(b_H-b_L) \leq 0. \end{aligned}$$

This implies that μ is not an ASCE if there exists some $q \in \mathrm{supp}(\mu)$ such that $q(s_L, b_H) = b_H$ or $q(s_H, b_L) \in [s_H, b_H)$.

A.5 Proof of Lemma 4

Consider the traders' incentives for manipulation in (q^*, r^A). It is easy to see that no manipulation by the high-type seller s_H and both types of buyer is profitable. If the low-type seller s_L is honest and obedient, then he will get $b_L - s_L$. If he misreports his type, then he can get at most $\pi_B(b_H - s_L)$. Hence, the IC constraint for him is given by

$$b_L - s_L \geq \pi_B(b_H - s_L). \tag{A.1}$$

Consider an arbitrary FB mediation plan $\mu \in \Delta(Q \times r^A)$ that satisfies the necessary conditions in Lemma 3 (otherwise, μ is not an ASCE and hence the statement of the lemma holds). As in (q^*, r^A), no manipulation by the high-type seller s_H and both types of buyer is profitable. If the low-type seller s_L is honest and obedient, then he will get $b_L - s_L$. If he misreports his type, then he can get $b_H - s_L$ when the buyer is of high-type, and $p - s_L > 0$ when the buyer is of low-type and the price $p \in (s_L, b_L]$ is offered. Hence, the IC constraint for him is given by

$$b_L - s_L \geq \pi_B(b_H - s_L) + \sum_{p \in (s_L, b_L]} \sum_{q \in \{q' \in Q:\ q'(s_H, b_L) = p\}} \mu(q)(1 - \pi_B)(p - s_L),$$

which is at least as stringent as (A.1).

Next, consider the intermediary's incentive for manipulation. If $q(s_H, b_L) \notin \{b_H, b_L\}$ for all $q \in \mathrm{supp}(\mu)$, his posterior beliefs about the traders' types after each possible recommendation are the same as those in (q^*, r^A), which implies the same IC constraints for him. If there exists such $q \in \mathrm{supp}(\mu)$, then his posterior beliefs after the recommendations b_H and b_L put positive probabilities also on (s_H, b_L). However, since (s_H, b_L) accept no offer, the presence of such q does not affect the intermediary's IC constraints.

In conclusion, the players' IC constraints in μ are at least as stringent as those in (q^*, r^A). Therefore, if (q^*, r^A) is not an ASCE, then μ is not either.

A.6 Proof of Lemma 5

Consider an arbitrary mediation plan $\mu \in \Delta(Q \times r^A)$ that satisfies the necessary condition in Lemma 2 (otherwise, μ cannot be an ASCE). Note that μ and μ^{**} can be different only in the price-recommendation to the traders (s_H, b_L). Since they accept no offer, this difference does not affect the expected trade surplus. Indeed, the ex-ante expected trade surplus in μ and μ^{**} are the same and given by

$$\pi_S \pi_B x_{HH}(\mu^{**})(b_H - s_H) + (1-\pi_S)\pi_B(b_H - s_L) + (1-\pi_S)(1-\pi_B)(1 - x_{LL}(\mu^{**}))(b_L - s_L). \tag{A.2}$$

Next, I show that the players' IC constraints in μ are at least as stringent as those in μ^{**}. Consider the traders' incentives for manipulation in μ^{**}. If the high-type seller s_H is honest and obedient, then he will get $b_H - s_H$ when the buyer is of high-type and the price b_H is offered. If he misreports his type, then he can get $b_H - s_H$ when the buyer is of high-type and the price b_H is offered. Hence, he has no profitable manipulation if

$$\pi_B x_{HH}(\mu^{**})(b_H - s_H) \geq \pi_B x_{LH}(\mu^{**})(b_H - s_H). \tag{A.3}$$

If the low-type seller s_L is honest and obedient, then he will get $b_H - s_L$ when the buyer is of high-type and the price b_H is offered, and $b_L - s_L$ when the price b_L is offered whatever the buyer's type. If he misreports his type, then he can get $b_H - s_L$ when the buyer is of high-type and the price b_H is offered, and $b_L - s_L$ when the buyer is of high-type and the price b_L is offered. Hence, he has no profitable deviation if

$$\begin{aligned} &\pi_B(x_{LH}(\mu^{**})(b_H - s_L) + (1 - x_{LH}(\mu^{**}))(b_L - s_L)) + (1-\pi_B)(1 - x_{LL}(\mu^{**}))(b_L - s_L) \\ &\quad \geq \pi_B(x_{HH}(\mu^{**})(b_H - s_L) + (1 - x_{HH}(\mu^{**}))(b_L - s_L)). \end{aligned} \tag{A.4}$$

If the high-type buyer b_H is honest and obedient, then he can get $b_H - b_L$ when the seller is of low-type and the price b_L is offered. If he misreports his type, then he can get $b_H - b_L$ when the seller is of low-type and the price b_L is offered. Hence, he has no profitable deviation if

$$(1-\pi_S)(1-x_{LH}(\mu^{**}))(b_H-b_L) \geq (1-\pi_S)(1-x_{LL}(\mu^{**}))(b_H-b_L). \tag{A.5}$$

Since $q(s,b) \in \{b_H, b_L\}$ for all $(s,b) \neq (s_H, b_L)$ and all $q \in \operatorname{supp}(\mu^{**})$, the low-type buyer b_L has no profitable manipulation.

Consider the intermediary's incentive for manipulation in μ^{**}. When he receives b_H, he believes that the traders are (s_H, b_H) with probability $\frac{\pi_S \pi_B x_{HH}(\mu^{**})}{X(\mu^{**})}$, (s_L, b_H) with probablity $\frac{(1-\pi_S)\pi_B x_{LH}(\mu^{**})}{X(\mu^{**})}$, and (s_L, b_L) with probability $\frac{(1-\pi_S)(1-\pi_B)x_{LL}(\mu^{**})}{X(\mu^{**})}$, where $X(\mu^{**})$ is the total probability that the price b_H is recommended in μ^{**}; that is,

$$X(\mu^{**}) = \pi_S \pi_B x_{HH}(\mu^{**}) + (1-\pi_S)\pi_B x_{LH}(\mu^{**}) + (1-\pi_S)(1-\pi_B)x_{LL}(\mu^{**}).$$

Note that the traders $(s,b) \neq (s_H, b_L)$ accept an offer p if and only if $p \in [s,b]$. Hence, following the recommendation is optimal for him if offering b_L is not profitable; that is,

$$\begin{aligned}
&\pi_S \pi_B x_{HH}(\mu^{**}) v(b_H \mid s_H, b_H) + (1-\pi_S)\pi_B x_{LH}(\mu^{**}) v(b_H \mid s_L, b_H) \\
&\quad \geq (1-\pi_S)\pi_B x_{LH}(\mu^{**}) v(b_L \mid s_L, b_H) + (1-\pi_S)(1-\pi_B) x_{LL}(\mu^{**}) v(b_L \mid s_L, b_L).
\end{aligned} \tag{A.6}$$

Similarly, When he receives b_L, he believes that the traders are (s_H, b_H) with probability $\frac{\pi_S \pi_B (1-x_{HH}(\mu^{**}))}{1-\pi_S(1-\pi_B)-X(\mu^{**})}$, (s_L, b_H) with probablity $\frac{(1-\pi_S)\pi_B(1-x_{LH}(\mu^{**}))}{1-\pi_S(1-\pi_B)-X(\mu^{**})}$, and (s_L, b_L) with probability $\frac{(1-\pi_S)(1-\pi_B)(1-x_{LL}(\mu^{**}))}{1-\pi_S(1-\pi_B)-X(\mu^{**})}$. Hence, following the recommendation is optimal for him if offering b_H is not profitable; that is,

$$\begin{aligned}
&(1-\pi_S)\pi_B(1-x_{LH}(\mu^{**})) v(b_L \mid s_L, b_H) + (1-\pi_S)(1-\pi_B)(1-x_{LL}(\mu^{**})) v(b_L \mid s_L, b_L) \\
&\quad \geq \pi_S \pi_B (1-x_{HH}(\mu^{**})) v(b_H \mid s_H, b_H) + (1-\pi_S)\pi_B(1-x_{LH}(\mu^{**})) v(b_H \mid s_L, b_H).
\end{aligned} \tag{A.7}$$

When he receives $p \in (b_H, +\infty)$, he believes that the traders are (s_H, b_L), who accept no offer. Hence, following the recommendation is optimal for him.

Next, consider the traders' incentives for manipulation in μ. It is easy to see that the IC constraints for the high-type seller and the low-type buyer are the same as those in μ^{**}. If the low-type seller s_L is honest and obedient, then he will get the same expected payoff as in μ^{**}. If he misreports his type, then, in addition to the payoffs he can get when he does so in μ^{**}, he can also get $p - s_L > 0$ when the buyer is of low-type and the price $p \in (s_L, b_L]$ is offered. Hence, he has no profitable deviation if

$$\begin{aligned}
&\pi_B(x_{LH}(\mu^{**})(b_H - s_L) + (1 - x_{LH}(\mu^{**}))(b_L - s_L)) + (1 - \pi_B)(1 - x_{LL}(\mu^{**}))(b_L - s_L) \\
&\quad \geq \pi_B(x_{HH}(\mu^{**})(b_H - s_L) + (1 - x_{HH}(\mu^{**}))(b_L - s_L)) \\
&\qquad + \sum_{p \in (s_L, b_L]} \sum_{q \in \{q' \in Q:\ q'(s_H, b_L) = p\}} \mu(q)(1 - \pi_B)(p - s_L),
\end{aligned}$$

which is at least as stringent as (A.4). Similarly, if the high-type buyer b_H is honest and obedient, then he will get the same expected payoff as in μ^{**}. If he misreports his type, then, in addition to the payoffs he can get when he does so in μ^{**}, he can also get $b_H - p > 0$ when the seller is of high-type and the price $p \in [s_H, b_H)$ is offered. Hence, he has no profitable deviation if

$$\begin{aligned}
&(1 - \pi_S)(1 - x_{LH}(\mu^{**}))(b_H - b_L) \\
&\quad \geq (1 - \pi_S)(1 - x_{LL}(\mu^{**}))(b_H - b_L) \\
&\qquad + \sum_{p \in [s_H, b_H)} \sum_{q \in \{q' \in Q:\ q'(s_H, b_L) = p\}} \mu(q)\pi_S(b_H - p),
\end{aligned}$$

which is at least as stringent as (A.5).

Finally, by a similar argument as in the proof of Lemma 4, the intermediary's IC constraints in μ are the same as those in μ^{**}. Thus, the players' IC constraints in μ are at least as stringent as those in μ^{**}.

A.7 Proof of Lemma 6

By the proof of Lemma 5, the problem (4.1) reduces to maximizing (A.2) subject to (A.3) to (A.7). Simplifying the constraints, I obtain the following linear programming problem:

$$
\begin{aligned}
\max_{(x_{HH},x_{LH},x_{LL})\in\mathbb{R}^3_+} \quad & \pi_S\pi_B x_{HH}(b_H-s_H)+(1-\pi_S)\pi_B(b_H-s_L) \\
& +(1-\pi_S)(1-\pi_B)(1-x_{LL})(b_L-s_L) \\
\text{subject to} \quad & x_{HH}\le 1 \\
& x_{LH}\le 1 \\
& x_{LL}\le 1 \\
& x_{HH}\ge x_{LH} \\
& x_{LL}\ge x_{LH} \\
& (1-\pi_B)(1-x_{LL})(b_L-s_L)\ge \pi_B(x_{HH}-x_{LH})(b_H-b_L) \\
& \pi_S\pi_B x_{HH}\lambda(b_H-s_H)+(1-\pi_S)\pi_B x_{LH}(2\lambda-1)(b_H-b_L) \\
& \quad -(1-\pi_S)(1-\pi_B)x_{LL}\lambda(b_L-s_L)\ge 0 \\
& 0\ge \pi_S\pi_B(1-x_{HH})\lambda(b_H-s_H)+(1-\pi_S)\pi_B(1-x_{LH})(2\lambda-1)(b_H-b_L) \\
& \qquad -(1-\pi_S)(1-\pi_B)(1-x_{LL})\lambda(b_L-s_L).
\end{aligned}
$$

I consider the following dual problem:

$$
\begin{aligned}
\min_{\eta\in\mathbb{R}^8_+} \quad & (1-\pi_S)\pi_B(b_H-s_L)+(1-\pi_S)(1-\pi_B)(b_L-s_L) \\
& +\eta_1+\eta_2+\eta_3+(1-\pi_B)(b_L-s_L)\eta_6-\kappa\eta_8 \\
\text{subject to} \quad & \eta_1-\eta_4+\pi_B(b_H-b_L)\eta_6-\pi_S\pi_B\lambda(b_H-s_H)(\eta_7+\eta_8)\ge \pi_S\pi_B(b_H-s_H) \\
& \eta_2+\eta_4+\eta_5-\pi_B(b_H-b_L)\eta_6-(1-\pi_S)\pi_B(2\lambda-1)(b_H-b_L)(\eta_7+\eta_8)\ge 0 \\
& \eta_3-\eta_5+(1-\pi_B)(b_L-s_L)\eta_6+(1-\pi_S)(1-\pi_B)\lambda(b_L-s_L)(\eta_7+\eta_8) \\
& \ \ge -(1-\pi_S)(1-\pi_B)(b_L-s_L).
\end{aligned}
$$

where $\kappa=\pi_S\pi_B\lambda(b_H-s_H)+(1-\pi_S)\pi_B(2\lambda-1)(b_H-b_L)-(1-\pi_S)(1-\pi_B)\lambda(b_L-s_L)$. I solve this problem by the simplex method.

Step 0: I transform the problem into a standard form by introducing the slack variables η_9, η_{10}, and η_{11}:

$$\begin{aligned}
\min_{\eta\in\mathbb{R}^{11}_+}\ & (1-\pi_S)\pi_B(b_H-s_L)+(1-\pi_S)(1-\pi_B)(b_L-s_L) \\
& +\eta_1+\eta_2+\eta_3+(1-\pi_B)(b_L-s_L)\eta_6-\kappa\eta_8 \\
\text{subject to}\quad & \eta_1-\eta_4+\pi_B(b_H-b_L)\eta_6-\pi_S\pi_B\lambda(b_H-s_H)(\eta_7+\eta_8)-\eta_9=\pi_S\pi_B(b_H-s_H) \\
& \eta_2+\eta_4+\eta_5-\pi_B(b_H-b_L)\eta_6-(1-\pi_S)\pi_B(2\lambda-1)(b_H-b_L)(\eta_7+\eta_8)-\eta_{10}=0 \\
& \eta_3-\eta_5+(1-\pi_B)(b_L-s_L)\eta_6+(1-\pi_S)(1-\pi_B)\lambda(b_L-s_L)(\eta_7+\eta_8)-\eta_{11} \\
& \quad =-(1-\pi_S)(1-\pi_B)(b_L-s_L).
\end{aligned}$$

Step 1: If $\kappa\le\pi_S\pi_B\lambda(b_H-s_H)(1-\varepsilon)+(1-\pi_S)\pi_B(2\lambda-1)(b_H-b_L)$, then I choose η_5, η_6, and η_{11} as the basic variables and obtain the following dictionary:

$$\begin{aligned}
\min_{\eta\in\mathbb{R}^{11}_+}\ & \pi_S\pi_B(b_H-s_H)(1-\varepsilon)+(1-\pi_S)\pi_B(b_H-s_L)+(1-\pi_S)(1-\pi_B)(b_L-s_L) \\
& +\varepsilon\eta_1+\eta_2+\eta_3+(1-\varepsilon)(\eta_4+\eta_9) \\
& +\pi_S\pi_B\lambda(b_H-s_H)(1-\varepsilon)\eta_7+[\pi_S\pi_B\lambda(b_H-s_H)(1-\varepsilon)-\kappa]\,\eta_8 \\
\text{subject to}\quad & \eta_5=\pi_S\pi_B(b_H-s_H)-\eta_1-\eta_2+[\kappa+(1-\pi_S)(1-\pi_B)\lambda(b_L-s_L)](\eta_7+\eta_8)+\eta_9+\eta_{10} \\
& \eta_6=\frac{\pi_S(b_H-s_H)}{b_H-b_L}-\frac{\eta_1-\eta_4-\eta_9}{\pi_B(b_H-b_L)}+\frac{\pi_S\lambda(b_H-s_H)}{b_H-b_L}(\eta_7+\eta_8) \\
& \eta_{11}=(1-\pi_S)(1-\pi_B)(b_L-s_L)-\pi_S\pi_B(b_H-s_H)\varepsilon \\
& \qquad +\varepsilon(\eta_1-\eta_9)+\eta_2+\eta_3+(1-\varepsilon)\eta_4-\eta_{10}+[\pi_S\pi_B\lambda(b_H-s_H)(1-\varepsilon)-\kappa](\eta_7+\eta_8),
\end{aligned}$$

where $\varepsilon = \frac{\pi_B(b_H - s_L) - (b_L - s_L)}{\pi_B(b_H - b_L)}$. I can take a basic feasible solution η such that

$$\eta_5 = \pi_S \pi_B (b_H - s_H),$$
$$\eta_6 = \frac{\pi_S (b_H - s_H)}{b_H - b_L},$$
$$\eta_{11} = (1 - \pi_S)(1 - \pi_B)(b_L - s_L) - \pi_S \pi_B (b_H - s_H)\varepsilon,$$
$$\eta_i = 0 \text{ for all } i \notin \{5, 6, 11\}.$$

This is indeed feasible because η_5 and η_6 are clearly positive and

$$\eta_{11} \geq 0$$
$$\iff (1 - \pi_S)(1 - \pi_B)(b_L - s_L) - \pi_S \pi_B (b_H - s_H)[1 - (1 - \varepsilon)] \geq 0$$
$$\iff \kappa \leq \pi_S \pi_B \lambda (b_H - s_H)(1 - \varepsilon) + (1 - \pi_S)\pi_B (2\lambda - 1)(b_H - b_L).$$

If $\kappa \leq \pi_S \pi_B \lambda (b_H - s_H)(1 - \varepsilon)$, then all the coefficients of the nonbasic variables in the objective function are nonnegative, Thus, the solution to the dual problem is η^* such that

$$\eta_5^* = \pi_S \pi_B (b_H - s_H),$$
$$\eta_6^* = \frac{\pi_S (b_H - s_H)}{b_H - b_L},$$
$$\eta_{11}^* = (1 - \pi_S)(1 - \pi_B)(b_L - s_L) - \pi_S \pi_B (b_H - s_H)\varepsilon,$$
$$\eta_i^* = 0 \text{ for all } i \notin \{5, 6, 11\}.$$

At η^*, the objective function achieves the minimum value of

$$\pi_S \pi_B (b_H - s_H)(1 - \varepsilon) + (1 - \pi_S)\pi_B (b_H - s_L) + (1 - \pi_S)(1 - \pi_B)(b_L - s_L). \tag{A.8}$$

Step 2: If $\pi_S \pi_B \lambda (b_H - s_H)(1 - \varepsilon) < \kappa \leq \pi_S \pi_B \lambda (b_H - s_H)(1 - \varepsilon) + (1 - \pi_S)\pi_B (2\lambda - 1)(b_H - b_L)$, then the coefficient of η_8 in the objective function is negative. Thus, I choose η_8 as a new basic variable and rewrite the third constraint as

$$\eta_8 = \frac{1}{\kappa - \pi_S\pi_B\lambda(b_H - s_H)(1-\varepsilon)}\Bigg[(1-\pi_S)(1-\pi_B)(b_L - s_L) - \pi_S\pi_B(b_H - s_H)\varepsilon$$

$$+\varepsilon(\eta_1 - \eta_9) + \eta_2 + \eta_3 + (1-\varepsilon)\eta_4 - \eta_{10} - \eta_{11}\Bigg] - \eta_7.$$

Substituting this into the objective function and the other constraints, I obtain the second dictionary:

$$\min_{\eta\in\mathbb{R}^{11}_+} \pi_S\pi_B(b_H - s_H) + (1-\pi_S)\pi_B(b_H - s_L) + \kappa\eta_7 + \eta_9 + \eta_{10} + \eta_{11}$$

subject to $\eta_5 = \pi_S\pi_B(b_H - s_H)$

$$+\frac{\kappa + (1-\pi_S)(1-\pi_B)\lambda(b_L - s_L)}{\kappa - \pi_S\pi_B\lambda(b_H - s_H)(1-\varepsilon)}[(1-\pi_S)(1-\pi_B)(b_L - s_L) - \pi_S\pi_B(b_H - s_H)\varepsilon]$$

$$+\left[\frac{\varepsilon\{\kappa + (1-\pi_S)(1-\pi_B)\lambda(b_L - s_L)\}}{\kappa - \pi_S\pi_B\lambda(b_H - s_H)(1-\varepsilon)} - 1\right](\eta_1 - \eta_9)$$

$$+\left[\frac{\kappa + (1-\pi_S)(1-\pi_B)\lambda(b_L - s_L)}{\kappa - \pi_S\pi_B\lambda(b_H - s_H)(1-\varepsilon)} - 1\right](\eta_2 - \eta_{10})$$

$$+\frac{\kappa + (1-\pi_S)(1-\pi_B)\lambda(b_L - s_L)}{\kappa - \pi_S\pi_B\lambda(b_H - s_H)(1-\varepsilon)}[\eta_3 + (1-\varepsilon)\eta_4 - \eta_{11}]$$

$$\eta_6 = \frac{1}{\kappa - \pi_S\pi_B\lambda(b_H - s_H)(1-\varepsilon)}\Bigg[\pi_S(1-\pi_S)\pi_B(2\lambda - 1)(b_H - s_H)$$

$$+\frac{\pi_S\pi_B\lambda(b_H - s_H) - \kappa}{\pi_B(b_H - b_L)}(\eta_1 - \eta_9)$$

$$+\frac{\pi_S\lambda(b_H - s_H)}{b_H - b_L}(\eta_2 + \eta_3 - \eta_{10} - \eta_{11}) + \frac{\kappa}{\pi_B(b_H - b_L)}\eta_4\Bigg]$$

$$\eta_8 = \frac{1}{\kappa - \pi_S\pi_B\lambda(b_H - s_H)(1-\varepsilon)}\Bigg[(1-\pi_S)(1-\pi_B)(b_L - s_L) - \pi_S\pi_B(b_H - s_H)\varepsilon$$

$$+\varepsilon(\eta_1 - \eta_9) + \eta_2 + \eta_3 + (1-\varepsilon)\eta_4 - \eta_{10} - \eta_{11}\Bigg] - \eta_7.$$

I can take a basic feasible solution η such that

$$
\begin{aligned}
\eta_5 &= \pi_S\pi_B(b_H - s_H) \\
&\quad + \frac{\kappa + (1-\pi_S)(1-\pi_B)\lambda(b_L - s_L)}{\kappa - \pi_S\pi_B\lambda(b_H - s_H)(1-\varepsilon)}[(1-\pi_S)(1-\pi_B)(b_L - s_L) - \pi_S\pi_B(b_H - s_H)\varepsilon], \\
\eta_6 &= \frac{\pi_S(1-\pi_S)\pi_B(2\lambda - 1)(b_H - s_H)}{\kappa - \pi_S\pi_B\lambda(b_H - s_H)(1-\varepsilon)}, \\
\eta_8 &= \frac{(1-\pi_S)(1-\pi_B)(b_L - s_L) - \pi_S\pi_B(b_H - s_H)\varepsilon}{\kappa - \pi_S\pi_B\lambda(b_H - s_H)(1-\varepsilon)}, \\
\eta_i &= 0 \text{ for all } i \notin \{5, 6, 8\}.
\end{aligned}
$$

This is indeed feasible because η_6 is clearly positive and

$$
\begin{aligned}
&\kappa \le \pi_S\pi_B\lambda(b_H - s_H)(1-\varepsilon) + (1-\pi_S)\pi_B(2\lambda - 1)(b_H - b_L) \\
\iff &(1-\pi_S)(1-\pi_B)(b_L - s_L) - \pi_S\pi_B(b_H - s_H)\varepsilon \ge 0,
\end{aligned}
$$

which implies that η_5 and η_8 are nonnegative. Since all the coefficients of the nonbasic variables in the objective function are nonnegative, the solution to the dual problems is η^* such that

$$
\begin{aligned}
\eta_5^* &= \pi_S\pi_B(b_H - s_H) \\
&\quad + \frac{\kappa + (1-\pi_S)(1-\pi_B)\lambda(b_L - s_L)}{\kappa - \pi_S\pi_B\lambda(b_H - s_H)(1-\varepsilon)}[(1-\pi_S)(1-\pi_B)(b_L - s_L) - \pi_S\pi_B(b_H - s_H)\varepsilon], \\
\eta_6^* &= \frac{\pi_S(1-\pi_S)\pi_B(2\lambda - 1)(b_H - s_H)}{\kappa - \pi_S\pi_B\lambda(b_H - s_H)(1-\varepsilon)}, \\
\eta_8^* &= \frac{(1-\pi_S)(1-\pi_B)(b_L - s_L) - \pi_S\pi_B(b_H - s_H)\varepsilon}{\kappa - \pi_S\pi_B\lambda(b_H - s_H)(1-\varepsilon)}, \\
\eta_i^* &= 0 \text{ for all } i \notin \{5, 6, 8\}.
\end{aligned}
$$

At η^*, the objective function achieves the minimum value of

$$
\pi_S\pi_B(b_H - s_H) + (1-\pi_S)\pi_B(b_H - s_L). \tag{A.9}
$$

Step 1': If $\kappa > \pi_S\pi_B\lambda(b_H - s_H)(1-\varepsilon) + (1-\pi_S)\pi_B(2\lambda - 1)(b_H - b_L)$, then I choose η_1, η_5, and η_6 as basic variables and obtain the following dictionary:

$$
\begin{aligned}
\min_{\eta\in\mathbb{R}^{11}_{+}}\ & \pi_S\pi_B(b_H - s_H) + (1-\pi_S)\pi_B(b_H - s_L) + \kappa\eta_7 + \eta_9 + \eta_{10} + \eta_{11} \\
\text{subject to}\quad & \eta_1 = \pi_S\pi_B(b_H - s_H) \\
& \quad - \frac{1}{\pi_B(b_H - b_L) - (1-\pi_B)(b_L - s_L)} \\
& \quad \times \Bigg[(1-\pi_S)\pi_B(1-\pi_B)(b_H - b_L)(b_L - s_L) \\
& \quad + \pi_B(b_H - b_L)(\eta_2 + \eta_3 - \eta_{10} - \eta_{11}) + (1-\pi_B)(b_L - s_L)\eta_4 \\
& \quad - \pi_B\{\kappa(b_H - b_L) - \pi_S(1-\pi_B)\lambda(b_H - s_H)(b_L - s_L)\}(\eta_7 + \eta_8)\Bigg] + \eta_9 \\
& \eta_5 = \frac{1}{\pi_B(b_H - b_L) - (1-\pi_B)(b_L - s_L)} \\
& \quad \times \Bigg[(1-\pi_S)\pi_B(1-\pi_B)(b_H - b_L)(b_L - s_L) \\
& \quad + (1-\pi_B)(b_L - s_L)(\eta_2 + \eta_4 - \eta_{10}) \\
& \quad + \pi_B(b_H - b_L)(\eta_3 - \eta_{11}) \\
& \quad + (1-\pi_S)\pi_B(1-\pi_B)(1-\lambda)(b_H - b_L)(b_L - s_L)(\eta_7 + \eta_8)\Bigg] \\
& \eta_6 = \frac{1}{\pi_B(b_H - b_L) - (1-\pi_B)(b_L - s_L)} \\
& \quad \times \Bigg[(1-\pi_S)(1-\pi_B)(b_L - s_L) + \eta_2 + \eta_3 + \eta_4 - \eta_{10} - \eta_{11} \\
& \quad - \{\kappa - \pi_S\pi_B\lambda(b_H - s_H)\}(\eta_7 + \eta_8)\Bigg].
\end{aligned}
$$

I can take a basic feasible solution η such that

$$\begin{aligned}
\eta_1 &= \pi_S \pi_B (b_H - s_H) - \frac{(1-\pi_S)\pi_B(1-\pi_B)(b_H - b_L)(b_L - s_L)}{\pi_B(b_H - b_L) - (1-\pi_B)(b_L - s_L)}, \\
\eta_5 &= \frac{(1-\pi_S)\pi_B(1-\pi_B)(b_H - b_L)(b_L - s_L)}{\pi_B(b_H - b_L) - (1-\pi_B)(b_L - s_L)}, \\
\eta_6 &= \frac{(1-\pi_S)(1-\pi_B)(b_L - s_L)}{\pi_B(b_H - b_L) - (1-\pi_B)(b_L - s_L)}, \\
\eta_i &= 0 \text{ for all } i \notin \{1, 5, 6\}.
\end{aligned}$$

This is indeed feasible because η_5 and η_6 are clearly positive and

$$\eta_1 \geq 0 \iff \kappa \geq \pi_S \pi_B \lambda (b_H - s_H)(1-\varepsilon) + (1-\pi_S)\pi_B(2\lambda - 1)(b_H - b_L).$$

Thus, the solution to the dual problem is η^* such that

$$\begin{aligned}
\eta_1^* &= \pi_S \pi_B (b_H - s_H) - \frac{(1-\pi_S)\pi_B(1-\pi_B)(b_H - b_L)(b_L - s_L)}{\pi_B(b_H - b_L) - (1-\pi_B)(b_L - s_L)}, \\
\eta_5^* &= \frac{(1-\pi_S)\pi_B(1-\pi_B)(b_H - b_L)(b_L - s_L)}{\pi_B(b_H - b_L) - (1-\pi_B)(b_L - s_L)}, \\
\eta_6^* &= \frac{(1-\pi_S)(1-\pi_B)(b_L - s_L)}{\pi_B(b_H - b_L) - (1-\pi_B)(b_L - s_L)}, \\
\eta_i^* &= 0 \text{ for all } i \notin \{1, 5, 6\}.
\end{aligned}$$

At η^*, the objective function achieves the same minimum value as (A.9).

Next, I recover the solution to the primal problem.

- If $\kappa \leq \pi_S \pi_B \lambda (b_H - s_H)(1-\varepsilon)$, then by the complementary slackness conditions, I have

$$\begin{aligned}
&x_{LH}^{**} = x_{LL}^{**} \qquad \text{(A.10)} \\
&(1-\pi_B)(b_L - s_L)(1 - x_{LL}^{**}) = \pi_B(b_H - b_L)(x_{HH}^{**} - x_{LH}^{**}).
\end{aligned}$$

  These together implies

$$x_{HH}^{**} = \varepsilon x_{LL}^{**} + 1 - \varepsilon. \qquad \text{(A.11)}$$

  Since the objective function is linear in x_{HH} and x_{LL}, either $x = (1-\varepsilon, 0, 0)$ or $x = (1, 1, 1)$

is the solution to the primal problem. If I increase x_{LL} by one unit, then the net effect on the objective function is $-\eta_{11}^{*} = \pi_S \pi_B (b_H - s_H)\varepsilon - (1-\pi_S)(1-\pi_B)(b_L - s_L) < 0$. Thus, $x = (1-\varepsilon, 0, 0)$ is a candidate for the solution. Note that this candidate satisfies all the constraint of the primal problem and that the value of the objective function at $x = (1-\varepsilon, 0, 0)$ coincides with (A.8). Therefore, the solution to the primal problem is

$$x^{**} = (1-\varepsilon, 0, 0) = \left(\frac{(1-\pi_B)(b_L - s_L)}{\pi_B (b_H - b_L)}, 0, 0 \right).$$

- If $\pi_S \pi_B \lambda (b_H - s_H)(1-\varepsilon) < \kappa \leq \pi_S \pi_B \lambda (b_H - s_H)(1-\varepsilon) + (1-\pi_S)\pi_B (2\lambda - 1)(b_H - b_L)$, then by the complementary slackness conditions, I have (A.10), (A.11), and

$$\begin{aligned} &\pi_S \pi_B (1 - x_{HH}^{**})\lambda (b_H - s_H) + (1-\pi_S)\pi_B (1 - x_{LH}^{**})(2\lambda - 1)(b_H - b_L) \\ &- (1-\pi_S)(1-\pi_B)(1 - x_{LL}^{**})\lambda (b_L - s_L) = 0. \end{aligned}$$

  Substituting (A.10) and (A.11) into this, I get

$$[\kappa - \pi_S \pi_B \lambda (b_H - s_H)(1-\varepsilon)](1 - x_{LL}^{**}) = 0 \iff x_{LH}^{**} = x_{LL}^{**} = 1.$$

  Thus, $x = (1, 1, 1)$ is a candidate for the solution. Note that this candidate satisfies all the constraint of the primal problem and that the value of the objective function at $x = (1, 1, 1)$ coincides with (A.9). Therefore, the solution to the primal problem is $x^{**} = (1, 1, 1)$.

- If $\kappa > \pi_S \pi_B \lambda (b_H - s_H)(1-\varepsilon) + (1-\pi_S)\pi_B (2\lambda - 1)(b_H - b_L)$, then by the complementary slackness conditions, I have (A.10), (A.11), and $x_{HH}^{**} = 1$, which imply $x_{LH}^{**} = x_{LL}^{**} = 1$. By the same argument as in the previous case, the solution to the primal problem is $x^{**} = (1, 1, 1)$.

B More on CE and SCE

B.1 Equivalence between CE and SCE

The definition of SCE is involved, but Myerson (1986b, Theorem 2) shows that a mediation plan is an SCE if and only if it is a CE that would never recommend a "codominated action" to any player who has not lied to the mediator. Thus, it suffices to identify the set codominated actions and the set of CE.

To define codominated actions, I introduce some notations. Let C be a correspondence such that

$$
\begin{aligned}
C(h^0) &\subseteq \mathbb{R}_+,\\
C(\theta_i, p) &\subseteq \{Y, N\},\ \forall i \in \{S, B\}, \forall(\theta_i, p) \in \Theta_i \times \mathbb{R}_+,
\end{aligned}
$$

where h^0 denotes the initial history. For any such C, let $E(C)$ be the set of mediation plans that never recommends actions in $C(\theta_i, p)$ for all $i \in \{S, B\}$ and all $(\theta_i, p) \in \Theta_i \times \mathbb{R}_+$; that is,

$$E(C) = \{(q, r) \in Q \times R \colon \forall(s, b, p) \in \Theta_S \times \Theta_B \times \mathbb{R}_+,\ r_S(p \mid s, b) \notin C(s, p),\ r_B(p \mid s, b) \notin C(b, p)\}.$$

For any $p \in \mathbb{R}_+$, let $\phi^1(p)$ be the set of tuples consisting of a mediation plan and the traders' reports such that the mediator recommends p; that is,

$$\phi^1(p) = \{(q, r, s, b) \in Q \times R \times \Theta_S \times \Theta_B \colon q(s, b) = p\}.$$

For each $i \in \{S, B\}$, for any $a_i \in \{Y, N\}$ and any $(\theta_i, p) \in \Theta_i \times \mathbb{R}_+$, let $\phi^2(a_i, \theta_i, p)$ be the set of tuples consisting of a mediation plan, the traders' types, and a price such that i's type is θ_i and the mediator recommends a_i to i as a response to p if the traders do not lie; that is, for $i = S$,

$$\phi^2(a_S, \theta_S, p) = \{(q, r, s, b, p) \in Q \times R \times \Theta_S \times \Theta_B \times \mathbb{R}_+ \colon s = \theta_S,\ r_S(p \mid s, b) = a_S\},$$

and for $i = B$,

$$\phi^2(a_B, \theta_B, p) = \{(q, r, s, b, p) \in Q \times R \times \Theta_S \times \Theta_B \times \mathbb{R}_+ \colon b = \theta_B, r_B(p \mid s, b) = a_B\}.$$

Let $\phi^1(C)$ be the union of all sets $\phi^1(p)$ over all $p \in C(h^0)$ and $\phi^2(C)$ be the union of all sets $\phi^2(a_i, \theta_i, p)$ over all $i \in \{S, B\}$, $(\theta_i, p) \in \Theta_i \times \mathbb{R}_+$, and $a_i \in C(\theta_i, p)$; that is,

$$\phi^1(C) = \{(q, r, s, b) \in Q \times R \times \Theta_S \times \Theta_B \colon q(s, b) \in C(h^\emptyset)\},$$
$$\phi^2(C) = \{(q, r, s, b, p) \in Q \times R \times \Theta_S \times \Theta_B \times \mathbb{R}_+ : r_S(p \mid s, b) \in C(s, p) \text{ or } r_B(p \mid s, b) \in C(b, p)\}.$$

Now I can define codomination correspondence. C is a *codomination correspondence* if

1. for each $\tau^1 \in \Delta(Q \times R \times \Theta_S \times \Theta_B)$, if $\tau^1(E(C) \times \Theta_S \times \Theta_B) = 1$ and $\tau^1(\phi^1(C)) > 0$, then there exists some $p \in C(h^0)$ and some $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ such that

$$\begin{aligned}&\sum_{(q,r,s,b)\in\phi^1(p)} \tau^1(q, r, s, b) \cdot v(\alpha(p) \mid s, b) \cdot \mathbf{1}_{\{r(\alpha(p)|s,b)=(Y,Y)\}} \\ &> \sum_{(q,r,s,b)\in\phi^1(p)} \tau^1(q, r, s, b) \cdot v(p \mid s, b) \cdot \mathbf{1}_{\{r(p|s,b)=(Y,Y)\}}\end{aligned} \tag{B.1}$$

   and

2. for each $\tau^2 \in \Delta(Q \times R \times \Theta_S \times \Theta_B \times \mathbb{R}_+)$, if $\tau^2(\phi^2(C)) > 0$, then there exists some trader $i \in \{S, B\}$, some $(\theta_i, p) \in \Theta_i \times \mathbb{R}_+$, some $a_i \in C(\theta_i, p)$, and some $\gamma_i : \Theta_i \times \mathbb{R}_+ \to \{Y, N\}$ such that, if $i = S$,

$$\begin{aligned}&\sum_{(q,r,s,b,p)\in\phi^2(a_S,\theta_S,p)} \tau^2(q, r, s, b, p) \cdot (p - s) \cdot \mathbf{1}_{\{\gamma_S(s,p)=Y\cap r_B(p|s,b)=Y\}} \\ &> \sum_{(q,r,s,b,p)\in\phi^2(a_S,\theta_S,p)} \tau^2(q, r, s, b, p) \cdot (p - s) \cdot \mathbf{1}_{\{a_S=Y\cap r_B(p|s,b)=Y\}}\end{aligned} \tag{B.2}$$

or, if $i = B$,

$$\sum_{(q,r,s,b,p)\in\phi^2(a_B,\theta_B,p)} \tau^2(q,r,s,b,p)\cdot(b-p)\cdot\mathbf{1}_{\{\gamma_B(b,p)=Y\cap r_S(p|s,b)=Y\}}$$
$$> \sum_{(q,r,s,b,p)\in\phi^2(a_B,\theta_B,p)} \tau^2(q,r,s,b,p)\cdot(b-p)\cdot\mathbf{1}_{\{a_B=Y\cap r_S(p|s,b)=Y\}}. \tag{B.3}$$

Let D denote the union of all codomination correspondences.[13] The intermediary's offer $p \in \mathbb{R}_+$ is *codominated* if $p \in D(h^0)$. Trader i's response $a_i \in \{Y,N\}$ is *codominated* at the history $(\theta_i, p) \in \Theta_i \times \mathbb{R}_+$ if $a_i \in D(\theta_i, p)$. Intuitively, if codominated actions could be recommended with positive probability, then at least one player could expect to gain by manipulation after being told to use a codominated action.

Lemma 9 shows that no action is codominated in the mediated bargaining game, which implies that every CE is an SCE.

Lemma 9. *No action is codominated in the mediated bargaining game.*

Proof. I show that the only codomination correspondence in the mediated bargaining game is C such that

$$C(h^0) = \emptyset \tag{B.4}$$

$$C(\theta_i, p) = \emptyset,\ \forall i \in \{S,B\}, \forall(\theta_i,p) \in \Theta_i \times \mathbb{R}_+. \tag{B.5}$$

First, I show (B.5). Note that $C(\theta_i, p) \subseteq \{Y,N\}$ is either $\emptyset$, $\{Y\}$, $\{N\}$, or $\{Y,N\}$ for any $i \in \{S,B\}$ and any $(\theta_i,p) \in \Theta_i \times \mathbb{R}_+$. Fix arbitrary $(s,b,p) \in \Theta_S \times \Theta_B \times \mathbb{R}_+$.

1. If $C(s,p) = \{Y\}$ and $C(b,p) = \{Y\}$ or $\emptyset$, take $\tau^2 \in \Delta(Q \times R \times \Theta_S \times \Theta_B \times \mathbb{R}_+)$ such that $\tau^2(q,r,s,b,p) = 1$ for $q(s,b) = p$ and $r(p \mid s,b) = (Y,N)$. The seller is recommended an action in C but no manipulation is profitable for him, which violates (B.2).

2. If $C(s,p) = \emptyset$ and $C(b,p) = \{Y\}$, take $\tau^2 \in \Delta(Q \times R \times \Theta_S \times \Theta_B \times \mathbb{R}_+)$ such that $\tau^2(q,r,s,b,p) = 1$ for $q(s,b) = p$ and $r(p \mid s,b) = (N,Y)$. The buyer is recommended an action in C but no manipulation is profitable for him, which violates (B.3).

[13] In general, this union may not be well defined if type spaces or action spaces are not finite. Hhowever, there is the unique codomination correspondence in the present model (see Lemma 9). Thus, this concern has no bite.

3. In all the other cases except for $C(s,p) = C(b,p) = \emptyset$, take $\tau^2 \in \Delta(Q \times R \times \Theta_S \times \Theta_B \times \mathbb{R}_+)$ such that $\tau^2(q,r,s,b,p) = 1$ for $q(s,b) = p$ and $r(p \mid s,b) = (N,N)$. At least one trader is recommended an action in C but no manipulation is profitable for him, which violates (B.2) or (B.3).

4. If $C(s,p) = \emptyset$ and $C(b,p) = \emptyset$, then the definition is trivially satisfied.

Since I have chosen (s,b,p) arbitrarily, a codomination correspondence must satisfy $C(s,p) = \emptyset$ and $C(b,p) = \emptyset$ for all $(s,b,p) \in \Theta_S \times \Theta_B \times \mathbb{R}_+$.

Next, I show (B.4). Suppose $C(h^0) \neq \emptyset$. For each $p \in C(h^0)$, take $\tau^1 \in \Delta(Q \times R \times \Theta_S \times \Theta_B)$ such that $\tau^1(q,r,s,b) = 1$ for $q(s,b) = p$ and $r(p' \mid s,b) = (N,N)$ for all $p' \in \mathbb{R}_+$. The intermediary is recommended an action in C but no manipulation is profitable for him, which violates (B.1). Hence, $C(h^0)$ must be empty as well. Therefore, the correspondence C satisfying (B.4) and (B.5) is the only codomination correspondence in the mediated barganing game. □

B.2 Equivalence between ex-ante IC and interim IC

Thanks to Lemma 9, it suffices to check the players' ex-ante IC constraints, (2.1) and (2.2), to show that a mediation plan is an SCE. As I only consider pure mediation plans or mixed mediation plans in $\Delta(Q \times r^A)$, hereafter focus on mediation plans $\mu \in \Delta(Q \times r)$ for some arbitrary $r \in R$. I show that the ex-ante IC constraints are satisfied if and only if the interim IC constraints are satisfied.

For trader $i \in \{S,B\}$, let $U_i(q,r \mid \theta_i)$ be type θ_i's interim expected payoff if the mediator uses a pure mediation plan (q,r) and all players are honest and obedient to the mediator:

$$U_S(q,r \mid s) = \sum_{b \in \Theta_B} \Pr(b) \cdot (q(s,b) - s) \cdot \mathbf{1}_{\{r(q(s,b)|s,b)=(Y,Y)\}},$$
$$U_B(q,r \mid b) = \sum_{s \in \Theta_S} \Pr(s) \cdot (b - q(s,b)) \cdot \mathbf{1}_{\{r(q(s,b)|s,b)=(Y,Y)\}},$$

where $\Pr(\theta_i)$ is the prior probability that trader i is of type θ_i. Supose that the mediator uses a mediation plan $\mu \in \Delta(Q \times r)$. If all players are honest and obedient to the mediator, then trader i

of type θ_i expects to get

$$\sum_{q \in Q} \mu(q) U_i(q, r \mid \theta_i).$$

Let $V(\mu \mid p)$ be the intermediary's interim expected payoff if he receives the recommendation p and all players are honest and obedient to the mediator:

$$V(\mu \mid p) = \sum_{(s,b) \in \Theta_S \times \Theta_B} \xi(s, b \mid p) \cdot v(p \mid s, b) \cdot \mathbf{1}_{\{r(p|s,b)=(Y,Y)\}},$$

where $\xi(s, b \mid p)$ is the intermediary's posterior belief that the traders are (s, b) When he receives p, which is given by

$$\xi(s, b \mid p) = \frac{\Pr(s, b) \sum_{q \in \{q' \in Q:\ q'(s,b)=p\}} \mu(q)}{\sum_{(\tilde{s},\tilde{b}) \in \Theta_S \times \Theta_B} \Pr(\tilde{s}, \tilde{b}) \sum_{q \in \{q' \in Q:\ q'(\tilde{s},\tilde{b})=p\}} \mu(q)}.$$

For each recommendation p, the intermediary's interim manipulation is represented by the price $\hat{\alpha} \in \mathbb{R}_+$ he actually offers. For trader $i \in \{S, B\}$ of type $\theta_i \in \Theta_i$, his interim manipulation is represented by a pair $(\hat{\beta}_i, \hat{\gamma}_i)$, where $\hat{\beta}_i \in \Theta_i$ is a manipulation in a report and $\hat{\gamma}_i : R_{\text{marg}} \times \mathbb{R}_+ \to \{Y, N\}$ is a manipulation in responses. Let $\hat{\Sigma}_i$ be the set of all such pairs $(\hat{\beta}_i, \hat{\gamma}_i)$. For any $(q, r) \in Q \times R$ and $(\hat{\beta}_i, \hat{\gamma}_i) \in \hat{\Sigma}_i$, the interim expected payoff $U_i((q, r) \circ (\hat{\beta}_i, \hat{\gamma}_i) \mid \theta_i)$ of trader i of type θ_i when he manipulates (q, r) by $(\hat{\beta}_i, \hat{\gamma}_i)$ is naturally defined. If the intermediary receives the recommendation p and manipulates μ by $\hat{\alpha} \in \mathbb{R}_+$, then he expects to get

$$V(\mu \circ \hat{\alpha} \mid p) = \sum_{(s,b) \in \Theta_S \times \Theta_B} \xi(s, b \mid p) \cdot v(\hat{\alpha} \mid s, b) \cdot \mathbf{1}_{\{r(\hat{\alpha}|s,b)=(Y,Y)\}}.$$

The intermediary has no incentive to manipulate $\mu \in \Delta(Q \times r)$ after he receives a recommendation (at Time 5) if, for all $\hat{\alpha} \in \mathbb{R}_+$ and all $p \in \mathbb{R}_+$ such that $p \in \text{Range}(q)$ for some $q \in \text{supp}(\mu)$,

$$V(\mu \mid p) \geq V(\mu \circ \hat{\alpha} \mid p). \tag{B.6}$$

Trader $i \in \{S, B\}$ of type $\theta_i \in \Theta_i$ has no incentive to manipulate $\mu \in \Delta(Q \times r)$ after he learns his type (at Time 3) if, for all $(\hat{\beta}_i, \hat{\gamma}_i) \in \hat{\Sigma}_i$,

$$\sum_{q \in Q} \mu(q) U_i(q, r \mid \theta_i) \geq \sum_{q \in Q} \mu(q) U_i((q, r) \circ (\hat{\beta}_i, \hat{\gamma}_i) \mid \theta_i). \tag{B.7}$$

Lemma 10. *For a mediation plan $\mu \in \Delta(Q \times r)$ for some arbitrary $r \in R$, the ex-ante IC constraints* (2.1) *and* (2.2) *are satisfied if and only if the interim IC constraints* (B.6) *and* (B.7) *are satisfied.*

Proof. Suppose that (B.6) is not satisfied for $\mu \in \Delta(Q \times r)$; that is, there exists some recommendation $\hat{p} \in \mathbb{R}_+$ and the intermediary's manipulation $\hat{\alpha} \in \mathbb{R}_+$ such that

$$\begin{aligned} & V(\mu \circ \hat{\alpha} \mid \hat{p}) > V(\mu \mid \hat{p}) \\ \iff & \sum_{(s,b) \in \Theta_S \times \Theta_B} \xi(s, b \mid \hat{p}) \cdot v(\hat{\alpha} \mid s, b) \cdot \mathbf{1}_{\{r(\hat{\alpha} \mid s,b) = (Y,Y)\}} \\ & > \sum_{(s,b) \in \Theta_S \times \Theta_B} \xi(s, b \mid \hat{p}) \cdot v(\hat{p} \mid s, b) \cdot \mathbf{1}_{\{r(\hat{p} \mid s,b) = (Y,Y)\}}. \end{aligned} \tag{B.8}$$

Consider an ex-ante manipulation $\alpha \in \Sigma_I$ of the intermediary such that

$$\alpha(p) = \begin{cases} \hat{\alpha} & \text{if } p = \hat{p} \\ p & \text{if } p \neq \hat{p}. \end{cases}$$

In other words, the intermediary disobeys only When he receives $\hat{p}$. Let $Q_\mu(\hat{p})$ and $Q_\mu(s, b, \hat{p})$ be the set of price-recommendations such that $\mu \in \Delta(Q \times r)$ assings a positive probability and it recommends $\hat{p} \in \mathbb{R}_+$ for some pairs of types and for $(s, b) \in \Theta_S \times \Theta_B$, respectively; that is,

$$\begin{aligned} Q_\mu(\hat{p}) &= \{q \in Q : \mu(q) > 0 \text{ and } q(s, b) = \hat{p} \text{ for some } (s, b) \in \Theta_S \times \Theta_B\}, \\ Q_\mu(s, b, \hat{p}) &= \{q \in Q : \mu(q) > 0 \text{ and } q(s, b) = \hat{p}\}. \end{aligned}$$

For any $q \in Q$, let $\Theta_q(\hat{p})$ be the set of pairs of types for which q recommends $\hat{p}$; that is,

$$\Theta_q(\hat{p}) = \{(s, b) \in \Theta_S \times \Theta_B : q(s, b) = \hat{p}\}.$$

Then, I have $\sum_{q\in Q}\mu(q)V(q\circ\alpha,r)=\sum_{q\in Q_\mu(\hat{p})}\mu(q)V(q\circ\alpha,r)+\sum_{q\notin Q_\mu(\hat{p})}\mu(q)V(q,r)$. I can further decompose the first term in the right side as

$$
\begin{aligned}
&\sum_{q\in Q_\mu(\hat{p})}\mu(q)V(q\circ\alpha,r)\\
&=\sum_{q\in Q_\mu(\hat{p})}\mu(q)\Bigg[\sum_{(s,b)\in\Theta_q(\hat{p})}\Pr(s,b)\cdot v(\hat{\alpha}\mid s,b)\cdot\mathbf{1}_{\{r(\hat{\alpha}|s,b)=(Y,Y)\}}\\
&\qquad+\sum_{(s,b)\notin\Theta_q(\hat{p})}\Pr(s,b)\cdot v(q(s,b)\mid s,b)\cdot\mathbf{1}_{\{r(q(s,b)|s,b)=(Y,Y)\}}\Bigg]\\
&=\sum_{(s,b)\in\cup_{q\in Q_\mu(\hat{p})}\Theta_q(\hat{p})}\Pr(s,b)\Bigg[\sum_{q\in Q_\mu(s,b,\hat{p})}\mu(q)\cdot v(\hat{\alpha}\mid s,b)\cdot\mathbf{1}_{\{r(\hat{\alpha}|s,b)=(Y,Y)\}}\\
&\qquad+\sum_{q\in Q_\mu(\hat{p})\backslash Q_\mu(s,b,\hat{p})}\mu(q)\cdot v(q(s,b)\mid s,b)\cdot\mathbf{1}_{\{r(q(s,b)|s,b)=(Y,Y)\}}\Bigg]\\
&\quad+\sum_{(s,b)\notin\cup_{q\in Q_\mu(\hat{p})}\Theta_q(\hat{p})}\Pr(s,b)\sum_{q\in Q_\mu(\hat{p})}\mu(q)\cdot v(q(s,b)\mid s,b)\cdot\mathbf{1}_{\{r(q(s,b)|s,b)=(Y,Y)\}}.
\end{aligned}\tag{B.9}
$$

Since $\xi(s,b\mid\hat{p})$ is given by

$$
\xi(s,b\mid\hat{p})=\frac{\Pr(s,b)\sum_{q\in Q_\mu(s,b,\hat{p})}\mu(q)}{\sum_{(\tilde{s},\tilde{b})\in\Theta_S\times\Theta_B}\Pr(\tilde{s},\tilde{b})\sum_{q\in Q_\mu(\tilde{s},\tilde{b},\hat{p})}\mu(q)},
$$

$\xi(s,b\mid\hat{p})=0$ for all $(s,b)\notin\cup_{q\in Q_\mu(\hat{p})}\Theta_q(\hat{p})$. Thus, (B.8) implies

$$
\begin{aligned}
&\sum_{(s,b)\in\cup_{q\in Q_\mu(\hat{p})}\Theta_q(\hat{p})}\Pr(s,b)\sum_{q\in Q_\mu(s,b,\hat{p})}\mu(q)\cdot v(\hat{\alpha}\mid s,b)\cdot\mathbf{1}_{\{r(\hat{\alpha}|s,b)=(Y,Y)\}}\\
&>\sum_{(s,b)\in\cup_{q\in Q_\mu(\hat{p})}\Theta_q(\hat{p})}\Pr(s,b)\sum_{q\in Q_\mu(s,b,\hat{p})}\mu(q)\cdot v(\hat{p}\mid s,b)\cdot\mathbf{1}_{\{r(\hat{p}|s,b)=(Y,Y)\}}.
\end{aligned}
$$

Combined with (B.9), I have

$$
\begin{aligned}
&\sum_{q\in Q}\mu(q)V(q\circ\alpha,r)\\
&=\sum_{q\in Q_\mu(\hat p)}\mu(q)V(q\circ\alpha,r)+\sum_{q\notin Q_\mu(\hat p)}\mu(q)V(q,r)\\
&=\sum_{(s,b)\in\cup_{q\in Q_\mu(\hat p)}\Theta_q(\hat p)}\Pr(s,b)\Bigg[\sum_{q\in Q_\mu(s,b,\hat p)}\mu(q)\cdot v(\hat\alpha\mid s,b)\cdot\mathbf{1}_{\{r(\hat\alpha|s,b)=(Y,Y)\}}\\
&\qquad+\sum_{q\in Q_\mu(\hat p)\setminus Q_\mu(s,b,\hat p)}\mu(q)\cdot v(q(s,b)\mid s,b)\cdot\mathbf{1}_{\{r(q(s,b)|s,b)=(Y,Y)\}}\Bigg]\\
&\qquad+\sum_{(s,b)\notin\cup_{q\in q_\mu(\hat p)}\Theta_q(\hat p)}\Pr(s,b)\sum_{q\in Q_\mu(\hat p)}\mu(q)\cdot v(q(s,b)\mid s,b)\cdot\mathbf{1}_{\{r(q(s,b)|s,b)=(Y,Y)\}}\\
&\qquad+\sum_{q\notin Q_\mu(\hat p)}\mu(q)V(q,r)\\
&>\sum_{(s,b)\in\cup_{q\in Q_\mu(\hat p)}\Theta_q(\hat p)}\Pr(s,b)\Bigg[\sum_{q\in Q_\mu(s,b,\hat p)}\mu(q)\cdot v(\hat p\mid s,b)\cdot\mathbf{1}_{\{r(\hat p|s,b)=(Y,Y)\}}\\
&\qquad+\sum_{q\in Q_\mu(\hat p)\setminus Q_\mu(s,b,\hat p)}\mu(q)\cdot v(q(s,b)\mid s,b)\cdot\mathbf{1}_{\{r(q(s,b)|s,b)=(Y,Y)\}}\Bigg]\\
&\qquad+\sum_{(s,b)\notin\cup_{q\in Q_\mu(\hat p)}\Theta_q(\hat p)}\Pr(s,b)\sum_{q\in Q_\mu(\hat p)}\mu(q)\cdot v(q(s,b)\mid s,b)\cdot\mathbf{1}_{\{r(q(s,b)|s,b)=(Y,Y)\}}\\
&\qquad+\sum_{q\notin Q_\mu(\hat p)}\mu(q)V(q,r)\\
&=\sum_{q\in Q_\mu(\hat p)}\mu(q)V(q,r)+\sum_{q\notin Q_\mu(\hat p)}\mu(q)V(q,r)\\
&=\sum_{q\in Q}\mu(q)V(q,r).
\end{aligned}
$$

Hence, (2.1) does not hold.

Next, suppose that (B.7) is not satisfied for $\mu\in\Delta(Q\times r)$; that is, there exists some trader $i\in\{S,B\}$ of type $\hat\theta_i\in\Theta_i$ and his manipulation $(\hat\beta_i,\hat\gamma_i)\in\hat\Sigma_i$ such that

$$\sum_{q\in Q}\mu(q)U_i((q,r)\circ(\hat\beta_i,\hat\gamma_i)\mid\hat\theta_i)>\sum_{q\in Q}\mu(q)U_i(q,r\mid\hat\theta_i).$$

Consider an ex-ante manipulation $(\beta_i, \gamma_i) \in \Sigma_i$ of trader i such that

$$\beta_i(\theta_i) = \begin{cases} \hat{\beta}_i & \text{if } \theta_i = \hat{\theta}_i \\ \theta_i & \text{if } \theta_i \neq \hat{\theta}_i \end{cases}$$

and for any $r \in R_{\text{marg}}$ and $p \in \mathbb{R}_+$,

$$\gamma_i(\theta_i, r, p) = \begin{cases} \hat{\gamma}_i(r, p) & \text{if } \theta_i = \hat{\theta}_i \\ r_i(p \mid \theta_i, \theta_j) & \text{if } \theta_i \neq \hat{\theta}_i. \end{cases}$$

In other words, trader i disobeys as prescribed in $(\hat{\beta}_i, \hat{\gamma}_i)$ only when his type is $\hat{\theta}_i$. Then, I have

$$\begin{aligned}
&\sum_{q \in Q} \mu(q) U_i((q,r) \circ (\beta_i, \gamma_i)) \\
&= \Pr(\hat{\theta}_i) \sum_{q \in Q} \mu(q) U_i((q,r) \circ (\hat{\beta}_i, \hat{\gamma}_i) \mid \hat{\theta}_i) + \big(1 - \Pr(\hat{\theta}_i)\big) \sum_{q \in Q} \mu(q) U_i(q, r \mid \theta_i \neq \hat{\theta}_i) \\
&> \Pr(\hat{\theta}_i) \sum_{q \in Q} \mu(q) U_i(q, r \mid \hat{\theta}_i) + \big(1 - \Pr(\hat{\theta}_i)\big) \sum_{q \in Q} \mu(q) U_i(q, r \mid \theta_i \neq \hat{\theta}_i) \\
&= \sum_{q \in Q} \mu(q) U_i(q, r).
\end{aligned}$$

Hence, (2.2) does not hold. This comples the proof of the only-if-part. □

To prove the if-part, suppose that (2.1) is not satisfied for $\mu \in \Delta(Q \times r)$; that is, there exists some manipulation $\alpha \in \Sigma_I$ by the intermediary such that

$$\sum_{q \in Q} \mu(q) V(q \circ \alpha, r) > \sum_{q \in Q} \mu(q) V(q, r). \tag{B.10}$$

For any $\alpha \in \Sigma_I$, let P_α be the set of prices such that α prescribes disobeying; that is,

$$P_\alpha = \{p \in \mathbb{R}_+ : \alpha(p) \neq p\}.$$

Note that (B.10) can be written as

$$
\begin{aligned}
&\sum_{q\in Q}\mu(q)V(q\circ\alpha,r)\\
&=\sum_{q\notin\cup_{p\in P_\alpha}Q_\mu(p)}\mu(q)V(q,r)+\sum_{p\in P_\alpha}\sum_{q\in Q_\mu(p)}\mu(q)V(q\circ\alpha,r)\\
&=\sum_{q\notin\cup_{p\in P_\alpha}Q_\mu(p)}\mu(q)V(q,r)+\sum_{p\in P_\alpha}\sum_{q\in Q_\mu(p)}\mu(q)\sum_{(s,b)\in\Theta_q(p)}\Pr(s,b)\cdot v(\alpha(p)\mid s,b)\cdot\mathbf{1}_{\{r(\alpha(p)|s,b)=(Y,Y)\}}\\
&\quad+\sum_{q\in\cup_{p\in P_\alpha}Q_\mu(p)}\mu(q)\sum_{(s,b)\notin\cup_{p\in P_\alpha}\Theta_q(p)}\Pr(s,b)\cdot v(q(s,b)\mid s,b)\cdot\mathbf{1}_{\{r(q(s,b)|s,b)=(Y,Y)\}}\\
&>\sum_{q\notin\cup_{p\in P_\alpha}Q_\mu(p)}\mu(q)V(q,r)+\sum_{p\in P_\alpha}\sum_{q\in Q_\mu(p)}\mu(q)\sum_{(s,b)\in\Theta_q(p)}\Pr(s,b)\cdot v(p\mid s,b)\cdot\mathbf{1}_{\{r(p|s,b)=(Y,Y)\}}\\
&\quad+\sum_{q\in\cup_{p\in P_\alpha}Q_\mu(p)}\mu(q)\sum_{(s,b)\notin\cup_{p\in P_\alpha}\Theta_q(p)}\Pr(s,b)\cdot v(q(s,b)\mid s,b)\cdot\mathbf{1}_{\{r(q(s,b)|s,b)=(Y,Y)\}}\\
&=\sum_{q\notin\cup_{p\in P_\alpha}Q_\mu(p)}\mu(q)V(q,r)+\sum_{p\in P_\alpha}\sum_{q\in Q_\mu(p)}\mu(q)V(q,r)\\
&=\sum_{q\in Q}\mu(q)V(q,r).
\end{aligned}
$$

Then, there exists at least one recommendation $\hat{p} \in P_\alpha$ such that

$$
\begin{aligned}
&\sum_{q \in Q_\mu(\hat{p})} \mu(q) \sum_{(s,b) \in \Theta_q(\hat{p})} \Pr(s,b) \cdot v(\alpha(\hat{p}) \mid s,b) \cdot \mathbf{1}_{\{r(\alpha(\hat{p})|s,b)=(Y,Y)\}} \\
&\quad > \sum_{q \in Q_\mu(\hat{p})} \mu(q) \sum_{(s,b) \in \Theta_q(\hat{p})} \Pr(s,b) \cdot v(\hat{p} \mid s,b) \cdot \mathbf{1}_{\{r(\hat{p}|s,b)=(Y,Y)\}} \\
\Longleftrightarrow &\sum_{(s,b) \in \cup_{q \in Q_\mu(\hat{p})} \Theta_q(\hat{p})} \Pr(s,b) \sum_{q \in Q_\mu(s,b,\hat{p})} \mu(q) \cdot v(\alpha(\hat{p}) \mid s,b) \cdot \mathbf{1}_{\{r(\alpha(\hat{p})|s,b)=(Y,Y)\}} \\
&\quad > \sum_{(s,b) \in \cup_{q \in Q_\mu(\hat{p})} \Theta_q(\hat{p})} \Pr(s,b) \sum_{q \in Q_\mu(s,b,\hat{p})} \mu(q) \cdot v(\hat{p} \mid s,b) \cdot \mathbf{1}_{\{r(\hat{p}|s,b)=(Y,Y)\}} \\
\Longleftrightarrow &\sum_{(s,b) \in \cup_{q \in Q_\mu(\hat{p})} \Theta_q(\hat{p})} \underbrace{\frac{\Pr(s,b) \sum_{q \in Q_\mu(s,b,\hat{p})} \mu(q)}{\sum_{(\tilde{s},\tilde{b}) \in \Theta_S \times \Theta_B} \Pr(\tilde{s},\tilde{b}) \sum_{q \in Q_\mu(\tilde{s},\tilde{b},\hat{p})} \mu(q)}}_{=\,\xi(s,b|\hat{p})} \cdot v(\alpha(\hat{p}) \mid s,b) \cdot \mathbf{1}_{\{r(\alpha(\hat{p})|s,b)=(Y,Y)\}} \\
&\quad > \sum_{(s,b) \in \cup_{q \in Q_\mu(\hat{p})} \Theta_q(\hat{p})} \underbrace{\frac{\Pr(s,b) \sum_{q \in Q_\mu(s,b,\hat{p})} \mu(q)}{\sum_{(\tilde{s},\tilde{b}) \in \Theta_S \times \Theta_B} \Pr(\tilde{s},\tilde{b}) \sum_{q \in Q_\mu(\tilde{s},\tilde{b},\hat{p})} \mu(q)}}_{=\,\xi(s,b|\hat{p})} \cdot v(\hat{p} \mid s,b) \cdot \mathbf{1}_{\{r(\hat{p}|s,b)=(Y,Y)\}} \\
\Longleftrightarrow &\sum_{(s,b) \in \Theta_S \times \Theta_B} \xi(s,b \mid \hat{p}) \cdot v(\alpha(\hat{p}) \mid s,b) \cdot \mathbf{1}_{\{r(\alpha(\hat{p})|s,b)=(Y,Y)\}} \\
&\quad > \sum_{(s,b) \in \Theta_S \times \Theta_B} \xi(s,b \mid \hat{p}) \cdot v(\hat{p} \mid s,b) \cdot \mathbf{1}_{\{r(\hat{p}|s,b)=(Y,Y)\}} \\
\Longleftrightarrow &V(\mu \circ \alpha(\hat{p}) \mid \hat{p}) > V(\mu \mid \hat{p}).
\end{aligned}
$$

Hence, (B.6) does not hold.

Next, suppose that (2.2) is not satisfied for $\mu \in \Delta(Q \times r)$; that is, there exists some trader $i \in \{S,B\}$ and his manipulation $(\beta_i, \gamma_i) \in \Sigma_i$ such that

$$\sum_{q \in Q} \mu(q) U_i((q,r) \circ (\beta_i,\gamma_i)) > \sum_{q \in Q} \mu(q) U_i(q,r).$$

Note that, as I have seen in the proof of the only-if-part, trader i's ex-ante expected payoff is a weighted average of his interim expected payoffs. Thus, there exists at least one type $\hat{\theta}_i \in \Theta_i$ who gets a strictly higher interm expected payoff from this manipulation. Let $(\hat{\beta}_i, \hat{\gamma}_i) \in \hat{\Sigma}_i$ denote the interim manipulation by $\hat{\theta}_i$ obtained from (β_i, γ_i). I must have

$$\sum_{q \in Q} \mu(q) U_i((q,r) \circ (\hat{\beta}_i, \hat{\gamma}_i) \mid \hat{\theta}_i) > \sum_{q \in Q} \mu(q) U_i(q, r \mid \hat{\theta}_i).$$

Hence, (B.7) does not hold. This completes the proof.

C Alternative Assumptions on the Intermediary

C.1 Unbiased intermediary

C.1.1 Without commitment and enforcement power

If $\lambda = 1/2$, the intermediary's payoff no longer depends on price; he gets $(b-s)/2$ if trade occurs between (s,b) and 0 if no trade occurs. This gives the intermediary much less incentive to disobey, as he is now indifferent between prices that give the same expected trade probability. This effect is so strong that the refinement does not work at all; a pure FB-ASCE exists under the same condition as in Proposition 1. Proposition 4 summarizes this observation.

Proposition 4. *If the intermediary is unbiased ($\lambda = 1/2$), then a pure FB-ASCE exists if and only if*

$$\pi_S \pi_B b_H + (1-\pi_S) b_L \geq \pi_B s_H + (1-\pi_S)(1-\pi_B) s_L.$$

Proof. As in the proof of Proposition 1, I show that a pure FB-ASCE exists if and only if there exists some $p \in (b_L, s_H)$ such that $\pi_S \geq \frac{p-b_L}{b_H - s_H + p - b_L}$ and $\pi_B \leq \frac{b_L - s_L}{s_H - p + b_L - s_L}$, which is equivalent to $\pi_S \pi_B b_H + (1-\pi_S) b_L \geq \pi_B s_H + (1-\pi_S)(1-\pi_B) s_L$.

Let $q \in Q$ be such that $q(s_H, b_H) = s_H$, $q(s_H, b_L) \in (b_H, +\infty)$, $q(s_L, b_H) = p$ for some $p \in (b_L, s_H)$, and $q(s_L, b_L) = b_L$. I show that the FB mediation plan (q, r^A) is an ASCE if and only if $\pi_S \geq \frac{p-b_L}{b_H - s_H + p - b_L}$ and $\pi_B \leq \frac{b_L - s_L}{s_H - p + b_L - s_L}$, where $p \in (b_L, s_H)$. As shown in the proof of Proposition 1, the traders' interim IC constraints are satisfied if and only if $\pi_S \geq \frac{p-b_L}{b_H - s_H + p - b_L}$ and $\pi_B \leq \frac{b_L - s_L}{s_H - p + b_L - s_L}$.[14]

Next, consider the intermediary's incentive for manipulation. When he receives $q(s,b) \neq$

[14] Note that I have the same interim IC constaints in (q, r_q) and (q, r^A).

$q(s_H, b_L)$, his expected payoff is $(b-s)/2 > 0$ if he offers $p \in [s, b]$ and 0 otherwise. Hence, following the recommendation is optimal for him. When he receives $q(s_H, b_L)$, he believes that the traders are (s_H, b_L), who accept no offer. Hence, following the recommendation is optimal for him.

Thus, (q, r^A) is an ASCE if and only if $\pi_S \geq \frac{p-b_L}{b_H-s_H+p-b_L}$ and $\pi_B \leq \frac{b_L-s_L}{s_H-p+b_L-s_L}$. This completes the proof of the if-part.

To prove the only-if-part, suppose $\pi_S < \frac{p-b_L}{b_H-s_H+p-b_L}$ or $\pi_B > \frac{b_L-s_L}{s_H-p+b_L-s_L}$ for all $p \in (b_L, s_H)$. By Lemma 8, this implies $\pi_S < \frac{s_H-b_L}{b_H-b_L}$ and $\pi_B > \frac{b_L-s_L}{s_H-s_L}$. I show that no FB mediation plan is an ASCE. Note that a price-recommendation q of a FB mediation plan (q, r^A) must fall into one of the following three cases.

Case 1: Recommend two different prices for $(s, b) \neq (s_H, b_L)$; that is, $p_H = q(s_H, b_H) = q(s_L, b_H)$ and $p_L = q(s_L, b_L)$;

Case 2: Recommend two different prices for $(s, b) \neq (s_H, b_L)$; that is, $p_H = q(s_H, b_H)$ and $p_L = q(s_L, b_H) = q(s_L, b_L)$;

Case 3: Recommend three different prices for $(s, b) \neq (s_H, b_L)$; that is, $p_{HH} = q(s_H, b_H)$, $p_{LH} = q(s_L, b_H)$, and $p_{LL} = q(s_L, b_L)$.

As I focus on pure FB-ASCE, I restrict my attention to (q, r^A) in each case.

Case 1: Let $q \in Q$ be such that $q(s_H, b_H) = q(s_L, b_H) = p_H$ for some $p_H \in [s_H, b_H]$, $q(s_H, b_L) \in \mathbb{R}_+$, and $q(s_L, b_L) = p_L$ for some $p_L \in [s_L, b_L]$. As shown in the proof of Proposition 1, the traders' interim IC constraints are satisfied if and only if $\pi_S \geq \frac{p_H-p_L}{b_H-p_L}$.

Next, consider the intermediary's incentive for manipulation. When he receives p_H, if he offers p, then his expected payoff is

$$\begin{cases} \frac{\pi_S\pi_B}{\pi_S\pi_B+(1-\pi_S)\pi_B} \cdot \frac{b_H-s_H}{2} + \frac{(1-\pi_S)\pi_B}{\pi_S\pi_B+(1-\pi_S)\pi_B} \cdot \frac{b_H-s_L}{2} & \text{if } p \in [s_H, b_H] \\ \frac{(1-\pi_S)\pi_B}{\pi_S\pi_B+(1-\pi_S)\pi_B} \cdot \frac{b_H-s_L}{2} & \text{if } p \in [s_L, s_H) \\ 0 & \text{otherwise.} \end{cases}$$

Hence, following the recommendation is optimal for him. When he receives p_L, his expected payoff is $(b_L - s_L)/2$ if he offers $p \in [s_L, b_L]$ and 0 otherwise. Hence, following the recommendation is optimal for him. When he receives $q(s_H, b_L)$, he believes that the traders are (s_H, b_L), who accept no offer. Hence, following the recommendation is optimal for him.

Thus, (q, r^A) is an ASCE if and only if $\pi_S \geq \frac{p_H - p_L}{b_H - p_L}$, which is least stringent at $(p_H, p_L) = (s_H, b_L)$ and reduces to $\pi_S \geq \frac{s_H - b_L}{b_H - b_L}$. Since $\pi_S < \frac{s_H - b_L}{b_H - b_L}$, the mediation plan (q, r^A) cannot be an ASCE.

Case 2: Let $q \in Q$ be such that $q(s_H, b_H) = p_H$ for some $p_H \in [s_H, b_H]$, $q(s_H, b_L) \in \mathbb{R}_+$, and $q(s_L, b_H) = q(s_L, b_L) = p_L$ for some $p_L \in [s_L, b_L]$. As shown in the proof of Proposition 1, the traders' interim IC constraints are satisfied if and only if $\pi_B \leq \frac{p_L - s_L}{p_H - s_L}$.

Next, consider the intermediary's incentive for manipulation. When he receives p_H, his expected payoff is $(b_H - s_H)/2$ if he offers $p \in [s_H, b_H]$ and 0 otherwise. Hence, following the recommendation is optimal for him. When he receives p_L, if he offers p, then his expected payoff is

$$\begin{cases} \frac{(1-\pi_S)\pi_B}{(1-\pi_S)\pi_B + (1-\pi_S)(1-\pi_B)} \cdot \frac{b_H - s_L}{2} & \text{if } p \in (b_L, b_H] \\ \frac{(1-\pi_S)\pi_B}{(1-\pi_S)\pi_B + (1-\pi_S)(1-\pi_B)} \cdot \frac{b_H - s_L}{2} + \frac{(1-\pi_S)(1-\pi_B)}{(1-\pi_S)\pi_B + (1-\pi_S)(1-\pi_B)} \cdot \frac{b_L - s_L}{2} & \text{if } p \in [s_L, b_L] \\ 0 & \text{otherwise.} \end{cases}$$

Hence, following the recommendation is optimal for him. When he receives $q(s_H, b_L)$, by the same argument as in Case 1, following the recommendation is optimal for him.

Thus, (q, r^A) is an ASCE if and only if $\pi_B \leq \frac{p_L - s_L}{p_H - s_L}$, which is least stringent at $(p_H, p_L) = (s_H, b_L)$ and reduces to $\pi_B \leq \frac{b_L - s_L}{s_H - s_L}$. Since $\pi_B > \frac{b_L - s_L}{s_H - s_L}$, the mediation plan (q, r^A) cannot be an ASCE.

Case 3: Let $q \in Q$ be such that $q(s_H, b_H) = p_{HH}$ for some $p_{HH} \in [s_H, b_H]$, $q(s_H, b_L) \in \mathbb{R}_+$, $q(s_L, b_H) = p_{LH}$ for some $p_{LH} \in [s_L, b_H]$, and $q(s_L, b_L) = p_{LL}$ for some $p_{LL} \in [s_L, b_L]$, where $p_{HH} \neq p_{LH} \neq p_{LL}$. As shown in the proof of Proposition 1, the traders' interim IC constraints are satisfied if and only if $p_{HH} > p_{LH} > p_{LL}$, $\pi_S \geq \frac{p_{LH} - p_{LL}}{b_H - p_{HH} + p_{LH} - p_{LL}}$, and $\pi_B \leq \frac{p_{LL} - s_L}{p_{HH} - p_{LH} + p_{LL} - s_L}$.

Next, consider the intermediary's incentive for manipulation. When he receives $q(s,b) \neq q(s_H, b_L)$, his expected payoff is $(b-s)/2 > 0$ if he offers $p \in [s,b]$ and 0 otherwise. Hence, following the recommendation is optimal for him. When he receives $q(s_H, b_L)$, by the same argument as in Case 1 and 2, following the recommendation is optimal for him. Thus, (q, r^A) is an ASCE if and only if $p_{HH} > p_{LH} > p_{LL}$, $\pi_S \geq \frac{p_{LH}-p_{LL}}{b_H - p_{HH} + p_{LH} - p_{LL}}$, and $\pi_B \leq \frac{p_{LL}-s_L}{p_{HH}-p_{LH}+p_{LL}-s_L}$, which are least stringent at $(p_{HH}, p_{LL}) = (s_H, b_L)$ and reduce to $\pi_S \geq \frac{p_{LH}-b_L}{b_H-s_H+p_{LH}-b_L}$ and $\pi_B \leq \frac{b_L-s_L}{s_H-p_{LH}+b_L-s_L}$ for some $p_{LH} \in (b_L, s_H)$. Since $\pi_S < \frac{p-b_L}{b_H-s_H+p-b_L}$ or $\pi_B > \frac{b_L-s_L}{s_H-p+b_L-s_L}$ for all $p \in (b_L, s_H)$, the mediation plan (q, r^A) cannot be an ASCE.

In conclusion, if $\pi_S < \frac{p-b_L}{b_H-s_H+p-b_L}$ or $\pi_B > \frac{b_L-s_L}{s_H-p+b_L-s_L}$ for all $p \in (b_L, s_H)$, then, for each possible q above, a FB mediation plan (q, r^A) is not an ASCE. □

When I construct a FB-SCE, I can deter the intermediary from disobeying by letting the traders reject all off-path prices (namely, use r_q for each price-recommendation q). Intuitively, being unbiased has the same effect on the intermediary's incentive. Although the traders (s,b) will accept any price $p \in [s,b]$, the intermediary is indifferent between these prices.

C.1.2 With commitment and enforcement power

Matsuo (1989) studies the mechanism design problem for bilateral trade à la Myerson and Satterthwaite (1983) while putting the same assumptions on traders' valuations as in this paper. Namely, the traders' valuations are binary and independently distributed and satisfy $s_L < b_L < s_H < b_H$. The principal in his model can be viewed as an unbiased intermediary who can commit to and enforce the prices he offers. He shows that a Bayesithe incentive compatible, individually rational, and ex-post efficient mechanism exists if and only if $\pi_S \pi_B b_H + (1-\pi_S) b_L \geq \pi_B s_H + (1-\pi_S)(1-\pi_B) s_L$, the same condition as in Proposition 1 and 4. This implies that the lack of commitment and enforcement power has no bite on efficiency if the intermediary is unbiased.

C.2 Without intermediary

So far, I have studied the mediated bargaining game in which the intermediary makes an offer instead of the traders. To clarify the effect of intermediary, I consider another mediated bargaining game in which the seller makes an offer. Let $\hat{r}: \Theta_S \times \Theta_B \times \mathbb{R}_+ \to \{Y, N\}$ be a response-recommendation to the buyer and $\hat{R}$ be the set of all such recommendations. A mediation plan is a probability distribution over $Q \times \hat{R}$. I consider the mediated seller-offer bargaining game that proceeds as follows:

1. The mediator publicly commits to a mediation plan $\mu \in \Delta(Q \times \hat{R})$ that she will use;
2. The seller and the buyer privately observe the realizations of their types;
3. The traders confidentially report their types $s \in \Theta_S$ and $b \in \Theta_B$ to the mediator;
4. The mediator privately picks up a pure mediation plan $(q, \hat{r}) \in Q \times \hat{R}$ with probability $\mu(q, \hat{r})$. She confidentially recommends a price $q(s, b) \in \mathbb{R}_+$ to the seller and a response-rule $\hat{r}(\cdot \mid s, b)$ to the buyer;
5. The seller offers a price $p \in \mathbb{R}_+$;
6. The buyer responds to p, by either acceptance (Y) or rejection (N). If he accepts, trade occurs at the price p, and the payoffs are realized; otherwise, no trade occurs, and all players get 0.

It is straightforward to show that no acition is codominated in this game. Thus, any CE is an ASCE as in the mediated bargaining game. Let $\hat{r}^A \in \hat{R}$ be such that for all $(s, b) \in \Theta_S \times \Theta_B$,

$$\hat{r}^A(p \mid s, b) = \begin{cases} Y & \text{if } p \in [0, b] \\ N & \text{if } p \in (b, +\infty) \end{cases}.$$

A mediation plan $\mu \in \Delta(Q \times \hat{R})$ is an ASCE if it is an SCE and $\mu(Q \times \hat{r}^A) = 1$. With this definition of ASCE, I can show analogs of Lemma 3 and 4.

Lemma 11. *A FB mediation plan $\mu \in \Delta(Q \times \hat{r}^A)$ is an ASCE only if, for all $q \in \operatorname{supp}(\mu)$, $q(s_H, b_H) = b_H$, $q(s_L, b_H) = q(s_L, b_L) = b_L$, and $q(s_H, b_L) \notin [0, b_H)$.*

Proof. Consider an arbitrary mediation plan $\mu \in \Delta(Q \times \hat{r}^A)$. Suppose that there exist some $q \in \text{supp}(\mu)$ such that $q(s_H, b_H) \neq b_H$. If the high-type seller s_H is honest to the mediator and recommended $q(s_H, b_H)$, then his expected payoff from offering p is

$$\begin{cases} \xi(b_H \mid q(s_H, b_H))(p - s_H) & \text{if } p \in (b_L, b_H] \\ p - s_H & \text{if } p \in [0, b_L] \\ 0 & \text{otherwise} \end{cases},$$

where $\xi(b \mid p)$ is his posterior belief that the buyer is of type $b \in \Theta_B$ When he receives $p \in \mathbb{R}_+$. Since $p - s_H < 0$ for all $p \in [0, s_H)$ and $\xi(b_H \mid q(s_H, b_H)) > 0$, he has the incentive to disobey to offer b_H. Next, suppose that there exist some $q \in \text{supp}(\mu)$ and $b \in \Theta_B$ such that $q(s_L, b) \notin \{b_H, b_L\}$. If the seller of type s_L is honest to the mediator and recommended $q(s_L, b)$, then his expected payoff from offering p is

$$\begin{cases} \xi(b_H \mid q(s_L, b))(p - s_L) & \text{if } p \in (b_L, b_H] \\ p - s_L & \text{if } p \in [0, b_L] \\ 0 & \text{otherwise} \end{cases}.$$

Thus, he has the incentive to disobey to offer either b_H or b_L. This shows that $\mu \in \Delta(Q \times \hat{r}^A)$ is an ASCE only if, for all $q \in \text{supp}(\mu)$, $q(s_H, b_H) = b_H$ and $q(s_L, b) \in \{b_H, b_L\}$ for all $b \in \Theta_B$. In addition, any FB mediation plan $\mu \in \Delta(Q \times \hat{r}^A)$ must also satisfy $q(s, b) \in [s, b]$ for all $(s, b) \neq (s_H, b_L)$ and all $q \in \text{supp}(\mu)$, which implies $q(s_L, b_L) = b_L$.

If the high-type buyer b_H is honest and obedient, then he can get $b_H - b_L$ when the seller is of low-type and the price b_L is offered. If he misreports his type, then he can get $b_H - b_L$ when the seller is of low-type, and $b_H - p > 0$ when the seller is of high-type and the price $p \in [0, b_H)$

Table C.1: The price-recommendation $\hat{q}^*$.

	b_H	b_L
s_H	b_H	b_H
s_L	b_L	b_L

is offered. Thus, he has no profitable manipulation if

$$\begin{aligned}
&\sum_{q\in\{q'\in Q:\, q'(s_L,b_H)=b_L\}} \mu(q)(1-\pi_S)(b_H-b_L) \\
&\quad \geq \sum_{p\in[0,b_H)} \sum_{q\in\{q'\in Q:\, q'(s_H,b_L)=p\}} \mu(q)\pi_S(b_H-p)+(1-\pi_S)(b_H-b_L) \\
\iff &\sum_{p\in[0,b_H)} \sum_{q\in\{q'\in Q:\, q'(s_H,b_L)=p\}} \mu(q)\pi_S(b_H-p) \\
&\quad +\left(1-\sum_{q\in\{q'\in Q:\, q'(s_L,b_H)=b_L\}} \mu(q)\right)(1-\pi_S)(b_H-b_L)\leq 0.
\end{aligned}$$

This implies that μ is not an ASCE if there exists some $q\in \operatorname{supp}(\mu)$ such that $q(s_L,b_H)=b_H$ or $q(s_H,b_L)\in[0,b_H)$. This completes the proof. □

Lemma 12. *There exists a price-recommendation $\hat{q}^*\in Q$ such that if $(\hat{q}^*,\hat{r}^A)$ is not an ASCE, then any FB mediation plan $\mu\in\Delta(Q\times\hat{r}^A)$ is not either.*

Proof. Let $\hat{q}^*\in Q$ be such that $\hat{q}^*(s_H,b_H)=\hat{q}^*(s_H,b_L)=b_H$ and $\hat{q}^*(s_L,b_H)=\hat{q}^*(s_L,b_L)=b_L$ (see Table C.1). In other words, the price-recommendation only depends on the seller's report. As such, no manipulation by the buyer is profitable. Note that the seller's posterior belief about the buyer's type is the same as the prior regardless of the recommendation he receives. Thus, if the seller of type s offers p, then his expected payoff is

$$\begin{cases}
\pi_B(p-s) & \text{if } p\in(b_L,b_H] \\
p-s & \text{if } p\in[0,b_L] \\
0 & \text{otherwise.}
\end{cases}$$

Since $p-s_H<0$ for all $p\in[0,b_L]$, the high-type seller s_H has no profitable manipulation. The low-type seller s_L does not also have a profitable manipulation if

$$b_L - s_L \geq \pi_B(b_H - s_L) \iff \pi_B \leq \frac{b_L - s_L}{b_H - s_L}. \tag{C.1}$$

Consider an arbitrary FB mediation plan $\mu \in \Delta(Q \times \hat{r}^A)$ that satisfies the necessary conditions in Lemma 11 (otherwise, μ is not an ASCE and hence the statement of the lemma holds). Suppose that there exists some $q \in \text{supp}(\mu)$ such that $q(s_H, b_L) \neq b_H$. Then, the seller learns the buyer's type When he receives $q(s_H, b_L)$. Thus, if the low-type seller s_L misreports his type and receives $q(s_H, b_L)$, then he will offer b_L. When he misreports his type, he receives b_H with probability $\pi_B + (1-\pi_B)(1 - \sum_{q \in \{q' \in Q:\ q'(s_H, b_L) \neq b_H\}} \mu(q))$. In this case, he believes that the buyer is of high-type with probability $\pi_B / (\pi_B + (1-\pi_B)(1 - \sum_{q \in \{q' \in Q:\ q'(s_H, b_L) \neq b_H\}} \mu(q)))$. Therefore, he has no profitable manipulation if

$$b_L - s_L \geq \pi_B(b_H - s_L) + \sum_{q \in \{q' \in Q:\ q'(s_H, b_L) \neq b_H\}} \mu(q)(1-\pi_B)(b_L - s_L),$$

which is more stringent than (C.1). Note that even if such q exists, the interim IC constraints for the buyer and the high-type seller s_H are the same as those in $(\hat{q}^*, \hat{r}^A)$.

In conclusion, the players' IC constraints in μ are at least stringent as those in $(\hat{q}^*, \hat{r}^A)$. Therefore, if $(\hat{q}^*, \hat{r}^A)$ is not an ASCE, then μ is not either. □

Focusing on $(q^{**}, \hat{r}^A)$, I can establish the necessary and sufficient condition for the existence of FB-ASCE in the mediated seller-offer bargaining game.

Proposition 5. *In the mediated seller-offer bargaining game, a FB-ASCE exists if and only if* $\pi_B \leq \frac{b_L - s_L}{b_H - s_L}$.

Proof. As shown in the proof of Lemma 12, the mediation plan $(q^{**}, \hat{r}^A)$ is an ASCE if and only if $\pi_B \leq \frac{b_L - s_L}{b_H - s_L}$. By Lemma 12, no FB mediation plan $\mu \in \Delta(Q \times \hat{r}^A)$ is an ASCE if $\pi_B > \frac{b_L - s_L}{b_H - s_L}$. □

Proposition 5 implies that achieving the FB outcome in ASCE is possible with a seller-biased intermediary (if and) only if it is possible without an intermediary. Intuitively, a seller-biased intermediary has the same incentive as that the seller has in the mediated seller-offer bargaining game. Thus, a seller-biased intermediary is of no help in achieving the FB outcome, no matter how small the bias is.

D Implementation without Mediator

The FB- and SB-ASCE characterized in Section 3 and 4 are the equilibria of the game with the fictitious mediator. Therefore, the analysis so far tells nothing about under which communication structure the corresponding equilibrium outcomes can be achieved in the underlying game; that is, the game without the mediator. In this appendix, I come back to the underlying game to answer this question. I show that the noiseless direct communication between the traders and the intermediary is sufficient to achieve the FB outcome. However, to achieve the SB outcomes, I need to introduce artificial noise into the communication.

I consider the game between the seller, the buyer, and the intermediary. Their preferences are the same as those defined in Section 2. The timing of the game is as follows:

1. The seller and the buyer privately observe realizations of their types;
2. The traders confidentially report their types $s \in \Theta_S$ and $b \in \Theta_B$ to the intermediary;
3. The intermediary offers a price $p \in \mathbb{R}_+$;
4. The traders simultaneously respond to p, by either acceptance (Y) or rejection (N). If both accept, trade occurs at the price p, and the payoffs are realized; otherwise, no trade occurs, and all players get 0.

The communication between the traders and the intermediary at Time 2 is *noiseless* if the intermediary always receives what the traders have reported; otherwise, it is *noisy*.

D.1 Implementation of the FB outcome

The FB outcome induced by FB-ASCE (q^*, r^A) is summarized as follows:

- (s_H, b_H) trade at the price b_H;
- (s_H, b_L) do not trade;

Table D.1: The intermediary's offer in σ^* as a function of the reports

	b_H	b_L
s_H	b_H	b_H
s_L	b_L	b_L

- (s_L, b_H) trade at the price b_L;
- (s_L, b_L) trade at the price b_L.

Consider the following strategy profile and the intermediary's system of beliefs about the traders' types:

Strategy σ^*

- Both types of seller report truthfully;
- Both types of buyer report b_L;
- The intermediary offers b_H if he receives (s_H, b_H) or (s_H, b_L) and b_L if he receives (s_L, b_H) or (s_L, b_L) (see Table D.1);
- The seller (resp. the buyer) accepts an offer if and only if it is greater (resp. smaller) than or equal to his type.

The intermediary's system of beliefs ξ^*

- When he receives (s_H, b_H), he assigns probability 1 to (s_H, b_H);
- When he receives (s_H, b_L), he assigns probability π_B to (s_H, b_H) and $1 - \pi_B$ to (s_H, b_L);
- When he receives (s_L, b_H), he assigns probability 1 to (s_L, b_L);
- When he receives (s_L, b_L), he assigns probability π_B to (s_L, b_H) and $1 - \pi_B$ to (s_L, b_L).

Note that following σ^* results in the FB outcome induced by (q^*, r^A). Proposition 6 shows that (σ^*, ξ^*) is a perfect Bayesian equilibrium (PBE) of the game if the FB-ASCE exists.

Proposition 6. *Assume that the communication between the traders and the intermediary is noiseless. Then, (σ^*, ξ^*) is a PBE of the game if the FB-ASCE exists in the mediated barganing game.*

Proof. Note that the system of beliefs ξ^* are consistent with σ^*. The high-type seller s_H has no profitable deviation because misreporting his type leads to the offer b_L, which is unacceptable to him. The low-type seller has no profitable deviation if $b_L - s_L \geq \pi_B(b_H - s_L) \iff \pi_B \leq \frac{b_L - s_L}{b_H - s_L}$. Both types of buyer have no profitable deviation because the intermediary's offer does not depend on the buyer's report.

Given ξ^*, the intermediary's offers are clearly optimal except for when he receives (s_L, b_L). In this case, it is optimal for him to offer b_L if $b_L - s_L \geq \pi_B \lambda(b_H - s_L) \iff \pi_B \leq \frac{b_L - s_L}{\lambda(b_H - s_L)}$. Thus, no player has a profitable deviation and hence (σ^*, ξ^*) is a PBE if $\pi_B \leq \frac{b_L - s_L}{b_H - s_L}$; that is, when a FB-ASCE exists in the mediated bargaining game. □

This proposition implies that the noiseless direct communication is sufficient to achieve the FB outcome.

D.2 Implementation of the SB outcomes

As shown in Proposition 3, there are two possible SB-ASCE, one characterized by $x^{**1} = (1 - \varepsilon, 0, 0)$ and one characterized by $x^{**2} \equiv (1, 1, 1)$, where $\varepsilon = \frac{\pi_B(b_H - s_L) - (b_L - s_L)}{\pi_B(b_H - b_L)}$.

The SB outcome associated with x^{**1} is summarized as follows:

- (s_H, b_H) trade at the price b_H with probability $1 - \varepsilon$ and do not trade with probability ε;
- (s_H, b_L) do not trade;
- (s_L, b_H) trade at the price b_L;
- (s_L, b_L) trade at the price b_L.

Consider the following strategy profile, the intermediary's system of beliefs about the traders' types, and the noisy communication:

Strategy σ^{**1}

- Both types of seller report truthfully;
- Both types of buyer report b_H;

Table D.2: The intermediary's offer as a function of the reports in σ^{**1}

	b_H	b_L
s_H	b_H	$p \in (b_H, +, \infty)$
s_L	b_L	b_L

- The intermediary offers b_H if he receives (s_H, b_H), $p \in (b_H, +, \infty)$ if he receives (s_H, b_L), and b_L if he receives (s_L, b_H) or (s_L, b_L) (see Table D.2);
- The seller (resp. the buyer) accepts an offer if and only if it is greater (resp. smaller) than or equal to his type.

The intermediary's system of beliefs ξ^{**1}

- When he receives (s_H, b_H), he assigns probability π_B to (s_H, b_H) and $1-\pi_B$ to (s_H, b_L);
- When he receives (s_H, b_L), he assigns probability 1 to (s_H, b_L);
- When he receives (s_L, b_H), he assigns probability $\frac{\pi_S \pi_B \varepsilon}{\pi_S \varepsilon + (1-\pi_S)}$ to (s_H, b_H), $\frac{\pi_S(1-\pi_B)\varepsilon}{\pi_S \varepsilon + (1-\pi_S)}$ to (s_H, b_L), $\frac{(1-\pi_S)\pi_B}{\pi_S \varepsilon + (1-\pi_S)}$ to (s_L, b_H), and $\frac{(1-\pi_S)(1-\pi_B)}{\pi_S \varepsilon + (1-\pi_S)}$ to (s_L, b_L);
- When he receives (s_L, b_L), he assigns probability 1 to (s_L, b_L).

Noisy communication

- If the seller reports s_H, then the intermediary will receive s_H with probability $1-\varepsilon$ and s_L with probability ε;
- In the other cases, the intermediary always receives what the traders have reported.

Note that, combined with the above noisy communication, following σ^{**1} results in the SB outcome associated with x^{**1}. Proposition 7 shows that $(\sigma^{**1}, \xi^{**1})$ is a PBE of the game if the SB-ASCE is characterized by x^{**1}.

Proposition 7. *Assume that the communication between the traders and the intermediary is noisy as described above. Then, $(\sigma^{**1}, \xi^{**1})$ is a PBE of the game if the SB-ASCE is characterized by x^{**1}.*

Proof. Note that given the noisy communication described above, the system of beliefs ξ^{**1} are consistent with σ^{**1}. It is easy to see that the high-type seller s_H and both types of buyer have no profitable deviation. If the low-type seller s_L follows σ^{**1}, then his expected payoff is $b_L - s_L$. If he misreports his type, then his expected payoff is at most

$$\pi_B(1-\varepsilon)(b_H - s_L) + \pi_B\varepsilon(b_L - s_L) = \pi_B(b_H - s_L) - \pi_B\varepsilon(b_H - b_L) = b_L - s_L.$$

Thus, he has no profitable deviation.

Given ξ^{**1}, the intermediary's offers are clearly optimal except for when he receives (s_L, b_H). In this case, if he offers p, then his expected payoff is

$$\begin{cases} \xi^{**1}(s_H, b_H \mid s_L, b_H)v(p \mid s_H, b_H) + \xi^{**1}(s_L, b_H \mid s_L, b_H)v(p \mid s_L, b_H) & \text{if } p \in [s_H, b_H] \\ \xi^{**1}(s_L, b_H \mid s_L, b_H)v(p \mid s_L, b_H) & \text{if } p \in (b_L, s_H) \\ \xi^{**1}(s_L, b_H \mid s_L, b_H)v(p \mid s_L, b_H) + \xi^{**1}(s_L, b_L \mid s_L, b_H)v(p \mid s_L, b_L) & \text{if } p \in [s_L, b_L] \\ 0 & \text{otherwise,} \end{cases}$$

where $\xi^{**1}(\tilde{s}, \tilde{b} \mid s, b)$ is the intermediary's belief that the traders are $(\tilde{s}, \tilde{b}) \in \Theta_S \times \Theta_B$ when he receives $(s, b) \in \Theta_S \times \Theta_B$. Thus, it is optimal for him to offer b_L if

$$\begin{aligned} &\xi^{**1}(s_L, b_H \mid s_L, b_H)v(b_L \mid s_L, b_H) + \xi^{**1}(s_L, b_L \mid s_L, b_H)v(b_L \mid s_L, b_L) \\ &\geq \xi^{**1}(s_H, b_H \mid s_L, b_H)v(b_H \mid s_H, b_H) + \xi^{**1}(s_L, b_H \mid s_L, b_H)v(b_H \mid s_L, b_H) \\ \Longleftrightarrow\ &(1-\pi_S)\pi_B[\lambda(b_L - s_L) + (1-\lambda)(b_H - b_L)] + (1-\pi_S)(1-\pi_B)\lambda(b_L - s_L) \\ &\geq \pi_S\pi_B\varepsilon\lambda(b_H - s_H) + (1-\pi_S)\pi_B\lambda(b_H - s_L), \end{aligned}$$

which is equivalent to (4.2). Thus, no player has a profitable deviation and hence $(\sigma^{**1}, \xi^{**1})$ is a PBE if the SB-ASCE is characterized by x^{**1}. □

Next, the SB outcome associated with x^{**2} is summarized as follows:

- (s_H, b_H) trade at the price b_H;
- (s_H, b_L) do not trade;

Table D.3: The intermediary's offer as a function of the reports in σ^{**2}

	b_H	b_L
s_H	b_H	$p \in (b_H, +, \infty)$
s_L	b_H	b_H

- (s_L, b_H) trade at the price b_H;
- (s_L, b_L) do not trade.

Consider the following strategy profile and the intermediary's system of beliefs about the traders' types:

Strategy σ^{**2}

- Both types of seller report truthfully;
- Both types of buyer report b_H;
- The intermediary offers b_H if he receives (s_H, b_H), (s_L, b_H), or (s_L, b_L) and $p \in (b_H, +, \infty)$ if he receives (s_H, b_L) (see Table D.3);
- The seller (resp. the buyer) accepts an offer if and only if it is greater (resp. smaller) than or equal to his type.

The intermediary's system of beliefs ξ^{**2}

- When he receives (s_H, b_H), he assigns probability π_B to (s_H, b_H) and $1 - \pi_B$ to (s_H, b_L);
- When he receives (s_H, b_L), he assigns probability 1 to (s_H, b_L);
- When he receives (s_L, b_H), he assigns probability $\frac{\pi_S \pi_B \varepsilon}{\pi_S \varepsilon + (1-\pi_S)}$ to (s_H, b_H), $\frac{\pi_S(1-\pi_B)\varepsilon}{\pi_S \varepsilon + (1-\pi_S)}$ to (s_H, b_L), $\frac{(1-\pi_S)\pi_B}{\pi_S \varepsilon + (1-\pi_S)}$ to (s_L, b_H), and $\frac{(1-\pi_S)(1-\pi_B)}{\pi_S \varepsilon + (1-\pi_S)}$ to (s_L, b_L);
- When he receives (s_L, b_L), he assigns probability 1 to (s_L, b_H).

Two strategy profiles σ^{**1} and σ^{**2} differ only in the intermediary's offers when he receives (s_L, b_H) or (s_L, b_L). Also, two system of beliefs ξ^{**1} and ξ^{**2} differ only in the off-path belief when the intermediary receives (s_L, b_L). Note that, combined with the same noisy communication as before, following σ^{**2} results in the SB outcome associated with x^{**2}. Proposition 8 shows that $(\sigma^{**2}, \xi^{**2})$ is a PBE of the game if the SB-ASCE is characterized by x^{**2}.

Proposition 8. *Assume that the communication between the traders and the intermediary is noisy as described above. Then,* $(\sigma^{**2}, \xi^{**2})$ *is a PBE of the game if the SB-ASCE is characterized by* x^{**2}.

Proof. Note that given the noisy communication described above, the system of beliefs ξ^{**2} are consistent with σ^{**2}. It is easy to see that the traders have no profitable deviation.

By the proof of Proposition 7, the intermediary's offers are optimal given ξ^{**2} if

$$\begin{aligned}&(1-\pi_S)\pi_B[\lambda(b_L-s_L)+(1-\lambda)(b_H-b_L)]+(1-\pi_S)(1-\pi_B)\lambda(b_L-s_L)\\&\leq \pi_S\pi_B\varepsilon\lambda(b_H-s_H)+(1-\pi_S)\pi_B\lambda(b_H-s_L).\end{aligned}$$

Thus, no player has a profitable deviation and hence $(\sigma^{**2}, \xi^{**2})$ is a PBE if the SB-ASCE is characterized by x^{**2}. □

Proposition 7 and 8 imply that, in contrast to the implementation of the FB outcome, some artificial noise is necessary to implement the SB outcomes if I restrict my attention to the one round direct communication.

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