J. Zhou 02-dec-2014

Title: Derivation of Serdes Channel Models

References

[1] J. Zhou 07-Aug-2013

[2] J. Zhou 13-Sept-2013

[3] J. Zhou 23-Sept-2013

[4] J. Zhou 05-Nov-2014

Summary: [1] and [2] are single-ended only

[3] is differential without common-mode

[4] is differential with common mode however, it requires all references to be the same Zo, also, it doesn't have crosstalk

Objective: this derivation is based on all previous derivations to achieve following objectives

(1) differential and common-mode

(2) crosstalk

(3) no restrictions on reference/terminal impedances

(4) SSN

This is the most general case of Serdes channel modeling.

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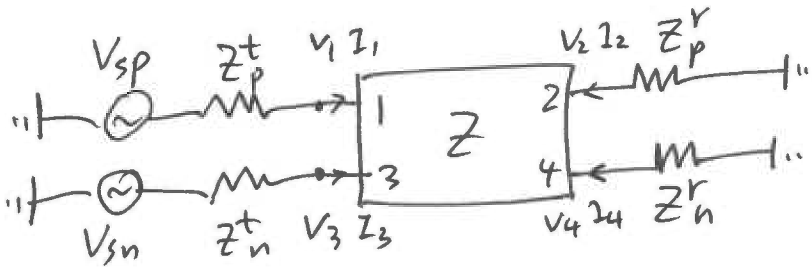
2 new

Section 1. Mixed-mode Solutions



system equations

$$V = [V_1\ V_2\ V_3\ V_4]^T \qquad (1a)$$

$$I = [I_1\ I_2\ I_3\ I_4]^T \qquad (1b)$$

$$V = Z I, \quad I = Y V \qquad (1c, 1d)$$

Terminal equations:

$$V_2 = -Z_p^r I_2 \qquad (2a)$$

$$V_4 = -Z_n^r I_4 \qquad (2b)$$

$$V_{sp} - V_1 = Z_p^t I_1 \qquad (3a)$$

$$V_{sn} - V_3 = Z_n^t I_3 \qquad (3b)$$

mixed-mode voltages and currents:

$$V_{d1} = V_1 - V_3, \quad V_{c1} = (V_1 + V_3)/2 \qquad (4a, 4b)$$

$$V_{d2} = V_2 - V_4, \quad V_{c2} = (V_2 + V_4)/2 \qquad (5a, 5b)$$

$$I_{d1} = (I_1 - I_3)/2, \quad I_{c1} = I_1 + I_3 \qquad (6a, 6b)$$

$$I_{d2} = (I_2 - I_4)/2, \quad I_{c2} = I_2 + I_4 \qquad (7a, 7b)$$

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The mapping from single-ended to differential:

$$\begin{bmatrix} V_{d1} \\ V_{d2} \\ V_{c1} \\ V_{c2} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}}_{M_V} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \qquad (mV)$$

$$\begin{bmatrix} I_{d1} \\ I_{d2} \\ I_{c1} \\ I_{c2} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}}_{M_I} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} \qquad (mI)$$

$$V_m = M_V V \qquad (mVb)$$

$$I_m = M_I I \qquad (mIb)$$

where V_m, I_m are mixed-mode voltage and current vectors as shown on the LHS of eqns (mV) and (mI)

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$$V_m = M_V Z I = M_V Z M_I^{-1} I_m \qquad (V_m)$$

The mixed-mode impedance matrix Z_m can be expressed as

$$Z_m = M_V Z M_I^{-1} \qquad (Zma)$$

$$= \begin{bmatrix} Z_{dd} & Z_{dc} \\ Z_{cd} & Z_{cc} \end{bmatrix} \qquad (Zmb)$$

$$\begin{pmatrix} V_{d1} \\ V_{d2} \\ V_{c1} \\ V_{c2} \end{pmatrix} = \begin{bmatrix} Z_{dd} & Z_{dc} \\ Z_{cd} & Z_{cc} \end{bmatrix} \begin{bmatrix} I_{d1} \\ I_{d2} \\ I_{c1} \\ I_{c2} \end{bmatrix} \qquad (V_m Z I_m)$$

in order to solve for V_m and I_m, from (3a) (3b) ~~subtract (3a) from (3b)~~

we obtain

$$V_1 - V_3 = (V_{sp} - V_{sn}) - Z_p^t I_1 + Z_n^t I_3 \qquad (10)$$

~~add~~ $$V_1 + V_3 = (V_{sp} + V_{sn}) - Z_p^t I_1 - Z_n^t I_3 \qquad (11)$$

define $$V_s^d = V_{sp} - V_{sn} \qquad (12a)$$

$$V_s^c = (V_{sp} + V_{sn})/2 \qquad (12b)$$

from eqn (4a, 4b)
we have

$$V_{d1} = V_s^d - Z_p^t I_1 + Z_n^t I_3 \qquad (13a)$$

$$V_{c1} = V_s^c - \frac{Z_p^t}{2} I_1 - \frac{Z_n^t}{2} I_3 \qquad (13b)$$

similarly from eqns (2a, 2b) we have

$$V_{d2} = V_2 - V_4 = -Z_p^r I_2 + Z_n^r I_4 \qquad (14a)$$

$$V_{c2} = \frac{V_2 + V_4}{2} = -\frac{Z_p^r}{2} I_2 - \frac{Z_n^r}{2} I_4 \qquad (14b)$$

eqns (13), (14) in matrix form:

$$\begin{bmatrix} V_{d1} \\ V_{d2} \\ V_{c1} \\ V_{c2} \end{bmatrix} = \underbrace{\begin{bmatrix} -Z_p^t & 0 & Z_n^t & 0 \\ 0 & -Z_p^r & 0 & Z_n^r \\ -\frac{Z_p^t}{2} & 0 & -\frac{Z_n^t}{2} & 0 \\ 0 & -\frac{Z_p^r}{2} & 0 & -\frac{Z_n^r}{2} \end{bmatrix}}_{Z^T} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} + \underbrace{\begin{bmatrix} V_s^d \\ 0 \\ V_s^c \\ 0 \end{bmatrix}}_{V_s} \quad (15)$$

from (1d) (and) (mvb) we have

$$I = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = [Y] \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = Y M_v^{-1} V_m \quad (16)$$

substitute (16) into (15)

$$V_m = Z^T Y M_v^{-1} V_m + V_s \quad (17)$$

where $Z^T = \begin{bmatrix} \text{see} \\ (15) \end{bmatrix}$, $V_s = \begin{bmatrix} \text{see} \\ (15) \end{bmatrix}$ (17b, 17c)

$$V_m = (U - Z^T Y M_v^{-1}) V_s \quad (18)$$

$$I_m = M_Z Y M_v^{-1} V_m \quad (19)$$

where U is the identity matrix

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The system transfer function $H_c(j\omega)$ is ~~given~~ defined as

$$V_m(j\omega) = H_c(j\omega)\, V_s(j\omega) \qquad (20)$$

$$H_c(j\omega) = (I - Z^T Y M_v^{-1})^{-1} \qquad (21)$$

$$V_s(j\omega) = FT(v_s(t)) \qquad (22a)$$

$$v_s(t) = IFT(V_s(j\omega))$$

$$v_m(t) = IFT(V_m(j\omega)) \qquad (23b)$$

$$V_m(j\omega) = FT(v_m(t)) \qquad (23a)$$

where FT is Fourier Transform

IFT is Inverse Fourier Transform

$$H_c(j\omega) = FT(h_c(t)) \qquad (24a)$$

$$h_c(t) = IFT(H_c(j\omega)) \qquad (24b)$$

~~to obtain $v_m(t)$ from~~

$$v_m(t) = h_c(t) * v_s(t) \qquad (25)$$

Conclusion:

the time-domain response $V_m(t)$ can be obtained in two ways

~~Option 1: obtain~~

Option 1:

(1.a) convert $V_s(t)$ to $V_s(j\omega)$ by FT

(1.b) obtain $V_m(j\omega)$ by eqn (18)

(1.c) obtain $V_m(t)$ by IFT

option 2:

(1.a) Obtain $H_c(j\omega)$ by (21)

(1.b) convert $H_c(j\omega)$ to $h_c(t)$ by IFT using (24b)

(1.c) convolve $h_c(t)$ with $V_s(t)$ to Obtain $V_m(t)$