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Derivation of Serdes Channel Simulation (Simplified)

References [1] 02-dec-2014 derivation

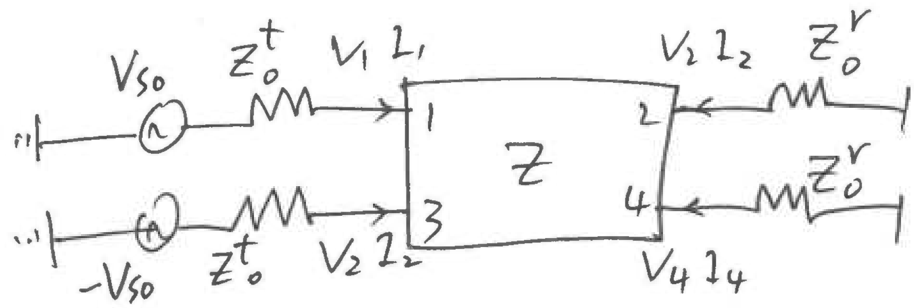
Objective: in 02-dec-2014 derivation, ~~all~~ there were no assumptions on excitations as well as impedance. The derivation is applicable to most general cases.

In this derivation, assumptions are made to simplify the process:

assumptions:

(1) there is no common-mode excitation

(2) p/n impedances are the same



① the exitations are purely differential

② the p/n impedances are equal

System equations:

$$V = [V_1\ V_2\ V_3\ V_4]^T \qquad (1a)$$

$$I = [I_1\ I_2\ I_3\ I_4]^T \qquad (1b)$$

$$V = Z\, I, \quad I = Y V \qquad (1c, 1d)$$

Terminal eqns:

$$V_2 = -Z_0^r I_2 \qquad 2a$$

$$V_4 = -Z_0^r I_4 \qquad 2b$$

$$V_{s0} - V_1 = Z_0^t I_1 \qquad 3a$$

$$-V_{s0} - V_3 = Z_0^t I_3 \qquad 3b$$

mixed mode voltages and currents

$$V_{d_1} = V_1 - V_3 \qquad V_{c_1} = (V_1 + V_3)/2 \qquad (4a, 4b)$$

$$V_{d_2} = V_2 - V_4 \qquad V_{c_2} = (V_2 + V_4)/2 \qquad (5a, 5b)$$

$$I_{d_1} = (I_1 - I_3)/2, \quad I_{c_1} = I_1 + I_3 \qquad (6a, 6b)$$

$$I_{d_2} = (I_2 - I_4)/2, \quad I_{c_2} = I_2 + I_4 \qquad (7a, 7b)$$

single-ended to mixed mode mapping

$$V_m = \begin{vmatrix} V_{d1} \\ V_{d2} \\ V_{c1} \\ V_{c2} \end{vmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 1/2 \end{pmatrix}}_{M_V} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$ eqn # (mV)

$$I_m = \begin{pmatrix} I_{d1} \\ I_{d2} \\ I_{c1} \\ I_{c2} \end{pmatrix} = \underbrace{\begin{pmatrix} 1/2 & 0 & -1/2 & 0 \\ 0 & 1/2 & 0 & -1/2 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}}_{M_I} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix}$$ eqn # (mI)

$$V_m = M_V V$$ eqn # (mVb)

$$I_m = M_I I$$ (mIb)

$$V_m = M_V Z M_I^{-1} I_m$$ eq # (Vm)

$$Z_m = M_V Z M_I^{-1}$$ # (Zma)

$$= \begin{bmatrix} Z_{dd} & Z_{dc} \\ Z_{cd} & Z_{cc} \end{bmatrix}$$ # (Zmb)

$$\begin{pmatrix} V_{d1} \\ V_{d2} \\ V_{c1} \\ V_{c2} \end{pmatrix} = \begin{bmatrix} Z_{dd} & Z_{dc} \\ Z_{cd} & Z_{cc} \end{bmatrix} \begin{pmatrix} I_{d1} \\ I_{d2} \\ I_{c1} \\ I_{c2} \end{pmatrix}$$ # (Vm Z Im)

to solve for V_m, I_m, from 3a, 3b.
we have

$$V_1 - V_3 = 2V_{s0} - Z_0^t (I_1 - I_3) \tag{10}$$

$$V_1 + V_3 = -Z_0^t (I_1 + I_3) \tag{11}$$

define $V_s^d = 2V_{s0}$ (12a)

$V_s^c = V_{s0} - V_{s0} = 0$ (12b)

from $\begin{pmatrix} 4a, & 4b \\ 6a, & 6b \end{pmatrix}$ we have

$$V_{d1} = V_s^d - 2Z_0^t I_{d1} \tag{13a}$$

$$V_{c1} = -\frac{Z_0^t}{2} I_{c1} \tag{13b}$$

similarly

$$V_{d2} = -Z_0^r (I_2 - I_4) \tag{14a}$$

$$= -2 Z_0^r I_{d2}$$

$$V_{c2} = -Z_0^r \frac{(I_2 + I_4)}{2} = -Z_0^r \frac{I_{c2}}{2} \tag{14b}$$

$$\begin{pmatrix} V_{d1} \\ V_{d2} \\ V_{c1} \\ V_{c2} \end{pmatrix} = \begin{pmatrix} -Z_0^t & 0 & Z_0^t & 0 \\ 0 & -Z_0^r & 0 & +Z_0^r \\ -\frac{Z_0^t}{2} & 0 & -\frac{Z_0^t}{2} & 0 \\ 0 & -\frac{Z_0^r}{2} & 0 & -\frac{Z_0^r}{2} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} + \begin{bmatrix} V_s^d \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

there is no common mode excitation

eqn #15

$$I = YV = YM_v^{-1}V_m \qquad (16)$$

$$V_m = Z^T Y M_v^{-1} V_m + V_s \qquad (17)$$

$$V_m = (U - Z^T Y M_v^{-1})^{-1} V_s \qquad (18)$$

$$I_m = M_I Y M_v^{-1} V_m \qquad (19)$$

$$V_m(j\omega) = H_c(j\omega) V_s(j\omega)$$

where $H_c(j\omega) = (U - Z^T Y M_v^{-1})^{-1}$

alternatively, from 12, 13 14

$$\begin{pmatrix} V_{d1} \\ V_{d2} \\ V_{c1} \\ V_{c2} \end{pmatrix} = \overbrace{\begin{bmatrix} -2Z_0^t & 0 & 0 & 0 \\ 0 & -2Z_0^r & 0 & 0 \\ 0 & 0 & -\frac{Z_0^t}{2} & 0 \\ 0 & 0 & 0 & -\frac{Z_0^r}{2} \end{bmatrix}}^{Z^{tm}} \begin{pmatrix} I_{d1} \\ I_{d2} \\ I_{c1} \\ I_{c2} \end{pmatrix} + \begin{pmatrix} V_s^d \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$V_m = Z^{tm} I_m + V_s$$

$$= Z^{tm} M_I Y M_V^{-1} V_m + V_s$$

$$V_m = (U - Z^{tm} M_I Y M_V^{-1})^{-1} V_s$$

this is equivalent to 18 provided that

$$Z^{tm} M_I = Z^T$$

at this point, it is obvious that by assuming

① the lack of common mode excitation

② ~~Zt~~ p/n impedances are equal

have not resulted in any real savings in the solution process, the main computation cost of this process in the inversion of eqn (18) of the 4x4 matrix. at all frequency points.

If one really needs to further reduce the cost of computation, ~~one must~~ more aggressive assumptions must be made:

in eqn #($V_m Z I_m$), let
on page 3

$Z_{dc} = Z_{cd} = Z_{cc} = 0$ we have

$$\begin{pmatrix} V_{d1} \\ V_{d2} \end{pmatrix} = Z_{dd} \begin{pmatrix} I_{d1} \\ I_{d2} \end{pmatrix} \quad \text{- - - -} \quad (101)$$

$$\begin{pmatrix} V_{d1} \\ V_{d2} \end{pmatrix} = \underbrace{\begin{pmatrix} -2Z_0^t & 0 \\ 0 & -2Z_0^r \end{pmatrix}} \begin{bmatrix} I_{d1} \\ I_{d2} \end{bmatrix} + \begin{bmatrix} V_s^d \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} V_{d1} \\ V_{d2} \end{pmatrix} = \left(\tilde{Z}^t \right] Z_{dd}^{-1} \begin{pmatrix} V_{d1} \\ V_{d2} \end{pmatrix} + \begin{pmatrix} V_s^d \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} V_{d1} \\ V_{d2} \end{pmatrix} = \left(u - \tilde{Z}^t Z_{dd}^{-1} \right)^{-1} \begin{pmatrix} V_s^d \\ 0 \end{pmatrix}$$

now, this inversion is 2x2 matrix which can be solved analytically.

I doubt the time saving from 4x4 inversion to 2x2 inversion is significant.