J Zhou 2015-01-22

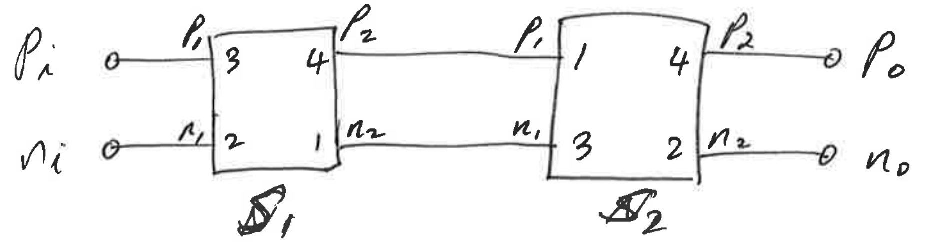

Fig. 1

Concatenation of 4 port S-parameters

J. Zhou

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input ports: p_i, n_i

output ports: p_o, n_o

each network S_1, S_2 has two ports on inputside (p_1, n_1) and 2 ports on output side (p_2, n_2)

the p ports are always connected to p ports
the n ports are always connected to n ports

The pindex matrix provides the port numbers corresponding to in/out and p/n:

$$\text{pindex}_{(S1)} = \begin{bmatrix} p_1 & p_2 \\ n_1 & n_2 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix} \quad (\text{for } S_1)$$

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$$\underset{(S2)}{pindex} = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

~~rearrange~~
resequence the S parameters so that

$$\begin{pmatrix} p_1 & p_2 \\ n_1 & n_2 \end{pmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

or

$p_1 \to 1$

$p_2 \to 3$

$n_1 \to 2$

$n_2 \to 4$

for example, for S1:

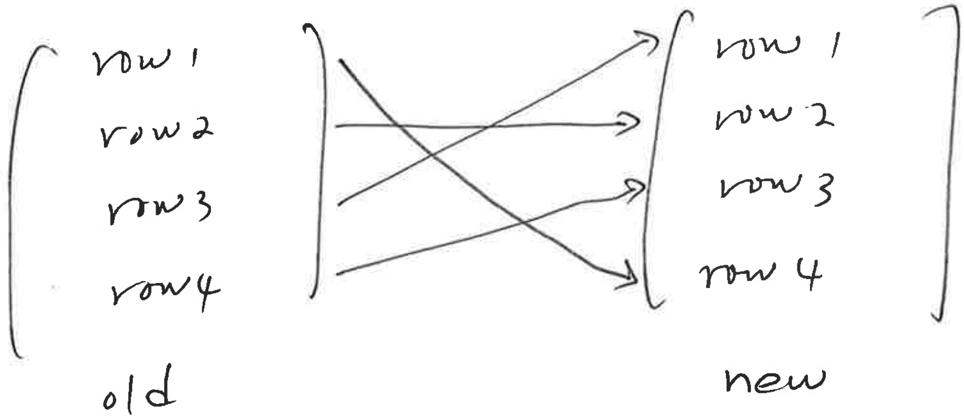
$p_1 = 3 \longrightarrow 1$

$p_2 = 4 \longrightarrow 3$

$n_1 = 2 \longrightarrow 2$

$n_2 = 1 \longrightarrow 4$

This is achieved by two operations of row and column swap row swap



do the column swap in similar way

do the same for S1 and S2

now we have

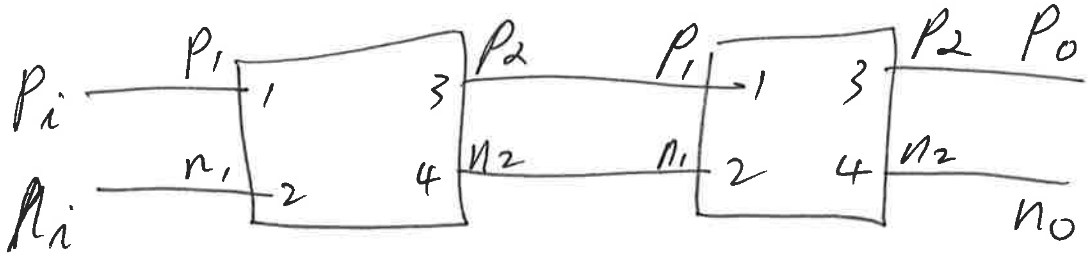


Fig. 2

The S parameter in block form:

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = \begin{bmatrix} S_{LL}^{(1)} & S_{LR}^{(1)} \\ S_{RL}^{(1)} & S_{RR}^{(1)} \end{bmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} \quad \text{for } S_1$$

where L = Left, (p_1, n_1) R = right (p_2, n_2)

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$$b_L^{(1)} = S_{LL}^{(1)} a_L^{(1)} + S_{LR}^{(1)} a_R^{(1)} \quad (1)$$

$$b_R^{(1)} = S_{RL}^{(1)} a_L^{(1)} + S_{RR}^{(1)} a_R^{(1)} \quad (2)$$

$\Big\}$ for S_1

from (2): $$a_L^{(1)} = \left(S_{RC}^{(1)}\right)^{-1}\left(b_R^{(1)} - S_{RR}^{(1)} a_R^{(1)}\right) \quad \text{-- (2.1)}$$

subst into (1):

$$b_L^{(1)} = S_{LL}^{(1)}\left(S_{RL}^{(1)}\right)^{-1} b_R^{(1)} - S_{LL}^{(1)}\left(S_{RL}^{(1)}\right)^{-1} S_{RR}^{(1)} a_R^{(1)} + S_{LR}^{(1)} a_R^{(1)}$$

$$= \left(S_{LR}^{(1)} - S_{LL}^{(1)}\left(S_{RL}^{(1)}\right)^{-1} S_{RR}^{(1)}\right) a_R^{(1)} + S_{LL}^{(1)}\left(S_{RL}^{(1)}\right)^{-1} b_R^{(1)} \quad \cdots (3)$$

$$b_L^{(1)} = T_{11}^{(1)} a_R^{(1)} + T_{12}^{(1)} b_R^{(1)} \quad \cdots (3.1)$$

where $T_{11}^{(1)} = S_{LR}^{(1)} - S_{LL}^{(1)}\left(S_{RL}^{(1)}\right)^{-1} S_{RR}^{(1)}$ -- (3.2)

$T_{12}^{(1)} = S_{LL}^{(1)}\left(S_{RL}^{(1)}\right)^{-1}$ -- (3.3)

from (2.1) let $T_{21}^{(1)} = -\left(S_{RL}^{(1)}\right)^{-1} S_{RR}^{(1)}$ -- (2.2)

$T_{22}^{(1)} = \left(S_{RL}^{(1)}\right)^{-1}$ -- (2.3)

we have $a_L^{(1)} = T_{21}^{(1)} a_R^{(1)} + T_{22}^{(1)} B_R^{(1)}$ --- (2.4) put (3.1) and (2.4) in matrix form

$$\begin{pmatrix} b_L^{(1)} \\ a_L^{(1)} \end{pmatrix} = \begin{pmatrix} T_{11}^{(1)} & T_{12}^{(1)} \\ T_{21}^{(1)} & T_{22}^{(1)} \end{pmatrix} \begin{pmatrix} a_R^{(1)} \\ b_R^{(1)} \end{pmatrix}$$

Similarly for S2

$$\begin{pmatrix} b_L^{(2)} \\ a_L^{(2)} \end{pmatrix} = \begin{pmatrix} T_{11}^{(2)} & T_{12}^{(2)} \\ T_{21}^{(2)} & T_{22}^{(2)} \end{pmatrix} \begin{pmatrix} a_R^{(2)} \\ b_R^{(2)} \end{pmatrix}$$

boundary condition at interface:

$$\begin{pmatrix} a_R^{(1)} \\ b_R^{(1)} \end{pmatrix} = \begin{pmatrix} b_L^{(2)} \\ a_L^{(2)} \end{pmatrix}$$

$$\begin{pmatrix} b_L^{(1)} \\ a_L^{(1)} \end{pmatrix} = (T_1)(T_2) \begin{pmatrix} a_R^{(2)} \\ b_R^{(2)} \end{pmatrix} \quad \ldots \quad (99)$$

$$\begin{pmatrix} b_L \\ a_L \end{pmatrix} = \begin{pmatrix} T_{LL} & T_{LR} \\ T_{RL} & T_{RR} \end{pmatrix} \begin{pmatrix} a_R \\ b_R \end{pmatrix} \quad \ldots \quad (100)$$

to get S parameter from (100)

$$b_L = T_{LL} a_R + T_{LR} b_R \quad \cdots (101)$$

$$a_L = T_{RL} a_R + T_{RR} b_R \quad (102)$$

from (102) $b_R = T_{RR}^{-1} a_L - T_{RR}^{-1} T_{RL} a_R$ (103)

sub (103) into (101):

$$b_L = T_{LL} a_R + T_{LR} T_{RR}^{-1} a_L - T_{LR} T_{RR}^{-1} T_{RL} a_R$$

$$b_L = \underbrace{T_{LR} T_{RR}^{-1}}_{S_{LL}} a_L + \underbrace{\left(T_{LL} - T_{LR} T_{RR}^{-1} T_{RL}\right)}_{S_{LR}} a_R$$

Let

$$S_{LL} = T_{LR} T_{RR}^{-1} \quad \cdots (110)$$

$$S_{LR} = T_{LL} - T_{LR} T_{RR}^{-1} T_{RL} \quad \cdot (111)$$

$$S_{RL} = T_{RR}^{-1}, \quad \cdots (112)$$

$$S_{RR} = -T_{RR}^{-1} T_{RL} \quad \cdots (113)$$

$$\begin{pmatrix} b_L \\ b_R \end{pmatrix} = \begin{pmatrix} S_{LL} & S_{LR} \\ S_{RL} & S_{RR} \end{pmatrix} \begin{pmatrix} a_L \\ a_R \end{pmatrix} \quad \cdots (200)$$

The end

Assignment

get two S4P

draw Fig. 1

write down pindex1, pindex2

row swap
column swap to get new S1, S2

draw Fig. 2

compute 3.2 3.3 ~~3~~2.2 2.3 to get $T^{①}$

and $T^{②}$

multiply $T^{①} T^{②}$ to get T eqn 99, 100

compute 110 111 112 113 to get S
in eqn 200