Channel Analysis Derivation
incident voltage Method

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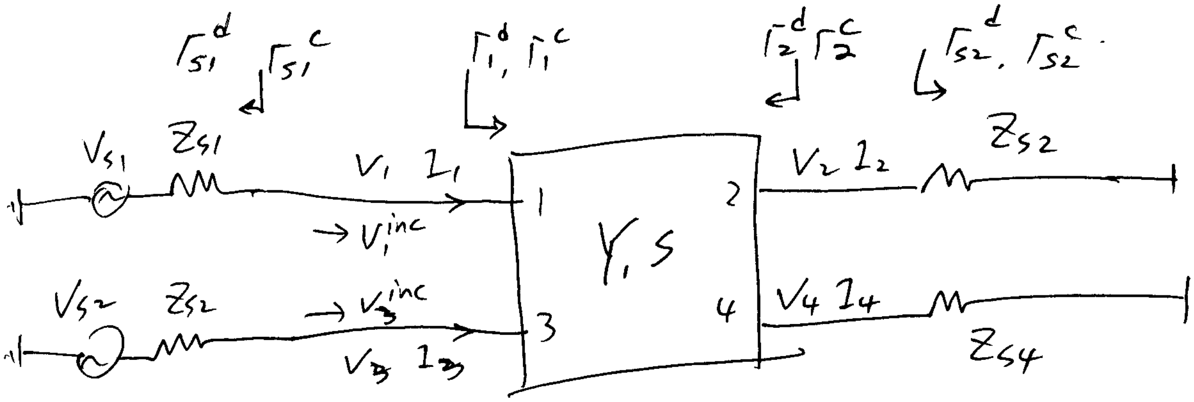

reference impedance of [S] is Z_0

source voltages V_{S1}, 0, V_{S3}, 0 at ports 1, 2, 3, 4

source impedances Z_{S1}, Z_{S2}, Z_{S3}, Z_{S4} at ports 1, 2, 3, 4

known incident voltages: $V_{d1}^{inc} = V_1^{inc} - V_3^{inc}$ ... ①

common mode incident $V_{C1}^{inc} = (V_1^{inc} + V_3^{inc})/2$ ②

differential source impedance Z_{d1}, ports 1, 3

Z_{d2}, ports 2, 4

common-mode source impedances Z_{C1}, ports 1, 3

Z_{C2}, ports 2, 4

This derivation uses incident voltages as the known excitation, rather than the source voltages.

The source impedances are known quantities:

Z_{S1}, Z_{S2}, Z_{S3}, Z_{S4}, Z_{d1}, Z_{d2}, Z_{c1}, Z_{c2}

The translation between single-ended and mixed-mode source impedances are derived separately.

Γ_1^d : differential reflection coefficient at port (1, 3)

Γ_1^c : common-mode reflection coefficient at port (1, 3)

Γ_2^d, Γ_2^c : at port (2, 4)

Γ_{S1}^d, Γ_{S1}^c : Mixed-mode reflection coefficient at sources Z_{S1}, Z_{S3}

Γ_{S2}^d, Γ_{S2}^c : at sources Z_{S2}, Z_{S4}

voltage at port (1,3)

$$V_{d_1} = V_{d_1}^{+} + V_{d_1}^{-}$$

$$= V_{d_1}^{inc}\left(1 + \Gamma_1^d \Gamma_{s1}^d + (\Gamma_1^d \Gamma_{s1}^d)^2 + \cdots\right)$$

$$+ V_{d_1}^{inc}\Gamma_1^d\left(1 + \Gamma_{s1}^d \Gamma_1^d + (\Gamma_{s1}^d \Gamma_1^d)^2 + \cdots\right) \quad (3)$$

Let $\Gamma_1^d \Gamma_{s1}^d = \xi_1^d$ (4a)

$$\Gamma_\sigma^d = \left(1 + \xi_1^d + \xi_1^{d^2} + \xi_1^{d^3} + \cdots\right) \quad (4b)$$

$$V_{d_1} = V_{d_1}^{inc}\,\Gamma_\sigma^d + V_{d_1}^{inc}\,\Gamma_1^d\,\Gamma_\sigma^d \quad (5)$$

$$V_{d_1} = V_{d_1}^{inc}\,\Gamma_\sigma^d(1 + \Gamma_1^d) \quad (5b)$$

similarly $V_{c_1} = V_{c_1}^{inc}\,\Gamma_\sigma^c(1+\Gamma_1^c)$ (6)

where $\Gamma_\sigma^c = \left(1 + \xi_1^c + \xi_1^{c^2} + \xi_1^{c^3} + \cdots\right)$ (6b)

$$\xi_1^c = \Gamma_1^c \Gamma_{s1}^c \quad (6c)$$

(5), (6): This is the solution at port (1,3)

$$\left[V_1^m\right] = \begin{pmatrix} V_{d_1} \\ V_{c_1} \end{pmatrix} = \begin{bmatrix} \Gamma_\sigma^d(1+\Gamma_1^d) & 0 \\ 0 & \Gamma_\sigma^c(1+\Gamma_1^c) \end{bmatrix} \begin{pmatrix} V_{d_1}^{inc} \\ V_{c_1}^{inc} \end{pmatrix} \quad (7)$$

$$V_1^m = \Gamma^m V^{inc} \quad \ldots \quad (7b)$$

where $V_1^m = \begin{bmatrix} V_{d_1} \\ V_{c_1} \end{bmatrix}$ (7c)

$$\Gamma^m = \begin{bmatrix} \Gamma_\sigma^d(1+\Gamma_1^d) & 0 \\ 0 & \Gamma_\sigma^c(1+\Gamma_1^c) \end{bmatrix} \quad (7d)$$

$$V^{inc} = \begin{bmatrix} V_{d_1}^{inc} \\ V_{c_1}^{inc} \end{bmatrix} \quad (7e)$$

now let's look at terminal conditions at port 2, 4.
by definition of Z_{d2} and Z_{c2}, we have:

$$V_{d2} = -Z_{d2} I_{d2} \quad (7a)$$

$$V_{c2} = -Z_{c2} I_{c2} \quad (7b)$$

$$V_2^m = \begin{bmatrix} V_{d2} \\ V_{c2} \end{bmatrix} = \underbrace{\begin{bmatrix} -Z_{d2} & 0 \\ 0 & -Z_{c2} \end{bmatrix}}_{\triangleq Z_2^m} \begin{bmatrix} I_{d2} \\ I_{c2} \end{bmatrix} \quad (8)$$

$$V_2^m = Z_2^m I_2^m \quad (8b)$$

The mixed-mode Y matrix

$$\begin{bmatrix} I_1^m \\ I_2^m \end{bmatrix} = \begin{bmatrix} Y_{11}^m & Y_{12}^m \\ Y_{21}^m & Y_{22}^m \end{bmatrix} \begin{bmatrix} V_1^m \\ V_2^m \end{bmatrix} \quad (9)$$

$$I_1^m = Y_{11}^m V_1^m + Y_{12}^m V_2^m \quad (9a)$$

$$I_2^m = Y_{21}^m V_1^m + Y_{22}^m V_2^m \quad (9b)$$

substitute (8b) into (9b) we have

$$I_2^m = Y_{21}^m V_1^m + Y_{22}^m Z_2^m I_2^m \quad (10)$$

$$I_2^m = (U - Y_{22}^m Z_2^m)^{-1} Y_{21}^m \cdot \Gamma^m V^{inc} \quad \cdots (10b)$$

where the last two terms are obtained from (7b)

substitute (10b) into (8b) to get V_2^m

$$V_2^m = Z_2^m (U - Y_{22}^m Z_2^m)^{-1} Y_{21}^m \Gamma^m V^{inc} \quad (11)$$

This is the solution for output voltage at port (2, 4)