Report of Model Predictive Control

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1 Introduction

This project is about model predictive control. One of the most important issue in model predictive control is the model of the plant. I will discuss the vehicle model in Section 2. Section 3 discusses time-step length and elapsed duration. Polynomial fitting and MPC preprocessing will be discussed in Section 4 followed by Model Predictive Control with Latency in Section 5.

2 The Model

There are two kinds of models can be utilized in model predictive control, they are *Kinematic Models* and *Dynamic Models*. Kinematic models are simplifications of dynamic models that ignore tire forces, gravity, and mass. On the other hand, Dynamic models aim to embody the actual vehicle dynamics as closely as possible.

I use four parameters to describe a vehicle in motion. They are: x, y, ψ, and v. The location of the vehicle can represented by x and y. ψ is the orientation of the vehicle. Finally, v is the velocity of the car. The above four variables are called the *states* of the model.

There are two control inputs of the model, they are *steering angle* (δ) and *throttle* (a). Control inputs are also called *actuators* of the control system.

Then, the kinematic model of a vehicle could be described by the following four equations:

$$
\begin{aligned}
x_{t+1} &= x_t + v_t cos(\psi_t) dt \\
y_{t+1} &= y_t + v_t sin(\psi_t) dt \\
\psi_{t+1} &= \psi_t + \frac{v_t}{L_f} \delta_t dt \\
v_{t+1} &= v_t + a_t d_t
\end{aligned}
\tag{1}
$$

where L_f measures the distance between the front of the vehicle and its center of gravity. The larger the vehicle, the slower the turn rate.

For model predictive control, there are two additional reference states should be considered, cte and $e\psi$. The two variables are used to measure the cost of the control problem. During the control process, the purpose is to minimize the cost such that the vehicle can reach desirable situations.

3 Timestep Length and Elapsed Duration

In this section, the chosen of N (timestep length) and dt (elapsed duration between timesteps) values will be discussed. Initially, I follow the settings of the course materials. That is, $N = 25$ and $dt = 0.05$. The car oscillated and almost ran off the road at N = 25. The cte vs. $time$ diagram is shown in Figure 1. Then, I tried to reduce N value. $N = 12$ is tried. The cte diagram showed the setting is worse than the initial one. Then, I tried to increase dt. This time, $N = 12$ and $dt = 0.1$ are used. The cte diagram showed this setting is good. Figure 3 and Figure 4 are two good settings.

4 Polynomial Fitting and MPC Preprocessing

A third order polynomial is fitted to waypoints. In the file `main.cpp`, line `134`, the following code

```
auto coeffs = polyfit(ptsx2, ptsy2, 3);
```

is used to find the coefficients of third order fitted polynomial.

In the `MPC.cpp`, line `127`, the fitted polynomial coefficients are used:

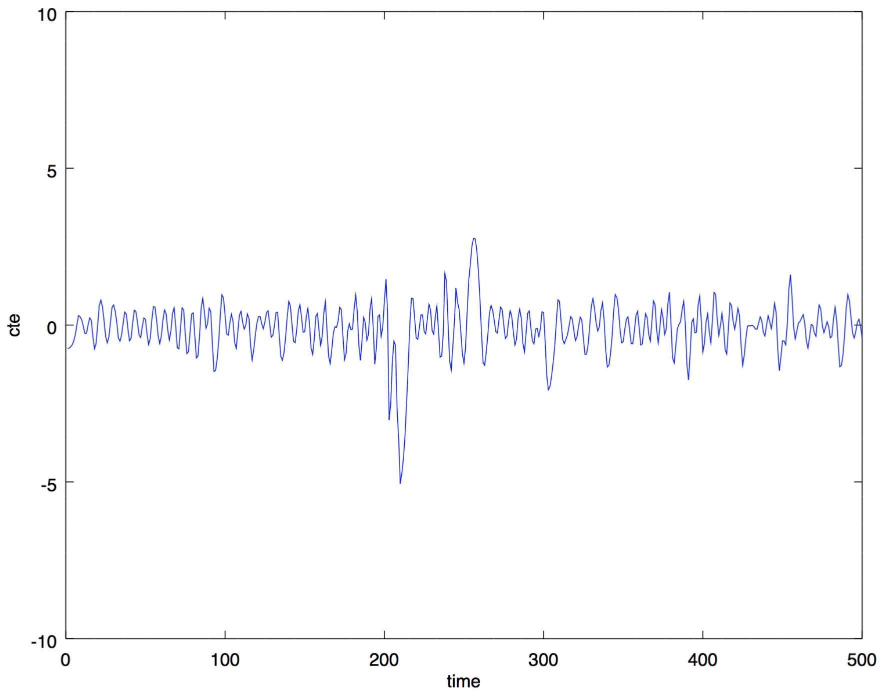

Figure 1: The cte vs. $time$: $N = 25$, $dt = 0.05$.

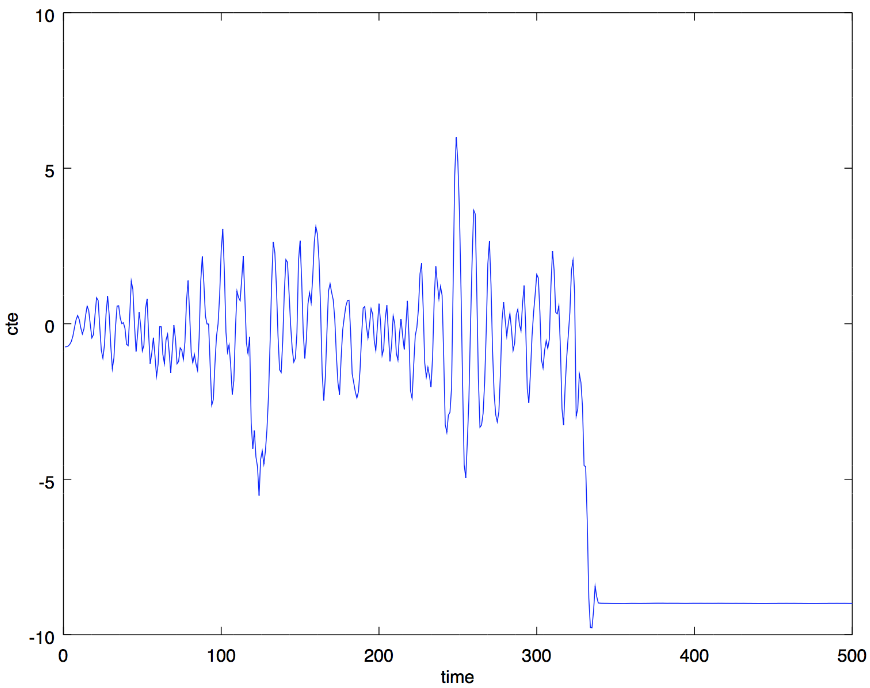

Figure 2: The cte vs. $time$: $N = 12$, $dt = 0.05$.

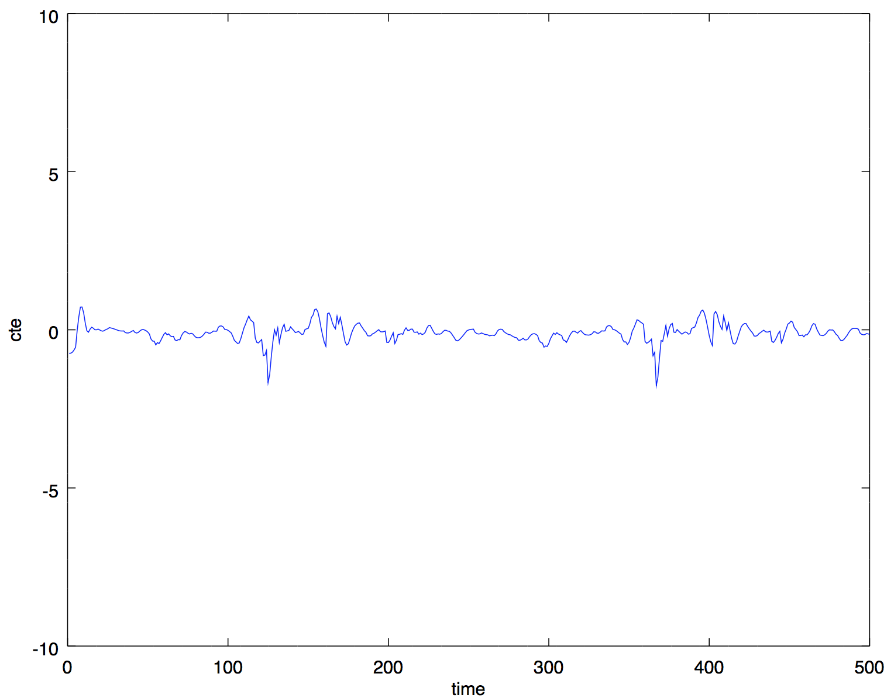

Figure 3: The cte vs. $time$: $N = 12$, $dt = 0.1$.

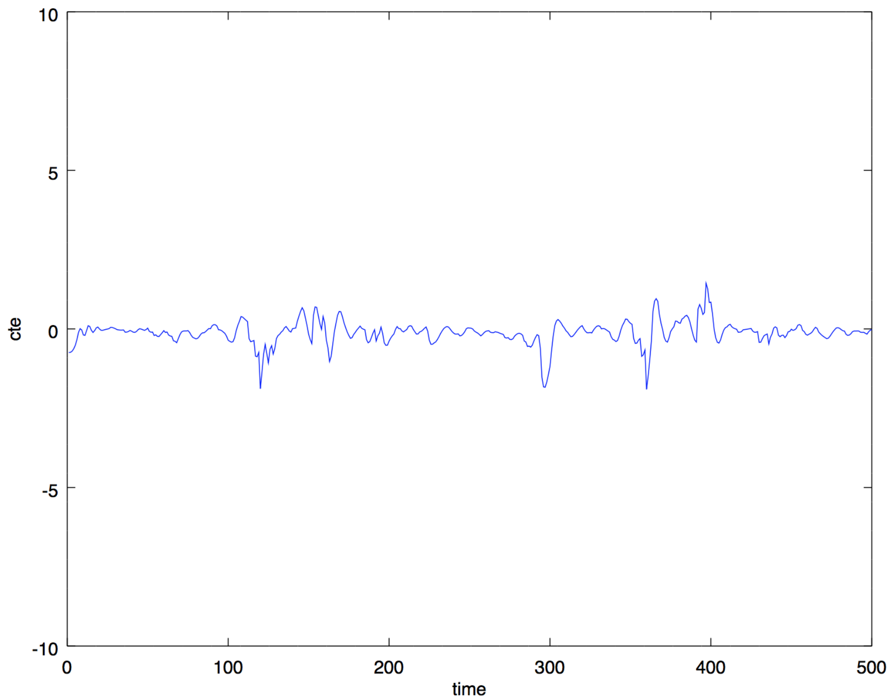

Figure 4: The *cte* vs. *time*: $N = 15$, $dt = 0.1$.

```
AD<double> f0 = coeffs[0] + coeffs[1]*x0 + coeffs[2]*x0*x0 + coeffs[3]*x0*x0*x0;
```

5 Model Predictive Control with Latency

Because there is a 100 millisecond latency of the simulator, I have to deal with the latency for stable control. My method is by adding latency corrections to all states. Line `128` to `133` in `main.cpp` show that.

```
//calculating coeffs, cte and epsi and adding latency corrections to all values
double latency = 0.1;
const double Lf = 2.67;
px = v*latency;
py = 0;
psi = -v*steer_value/Lf*latency;
v += throttle_value*latency;
```