ModSim Exercise 5

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1 Simple Conduction Model

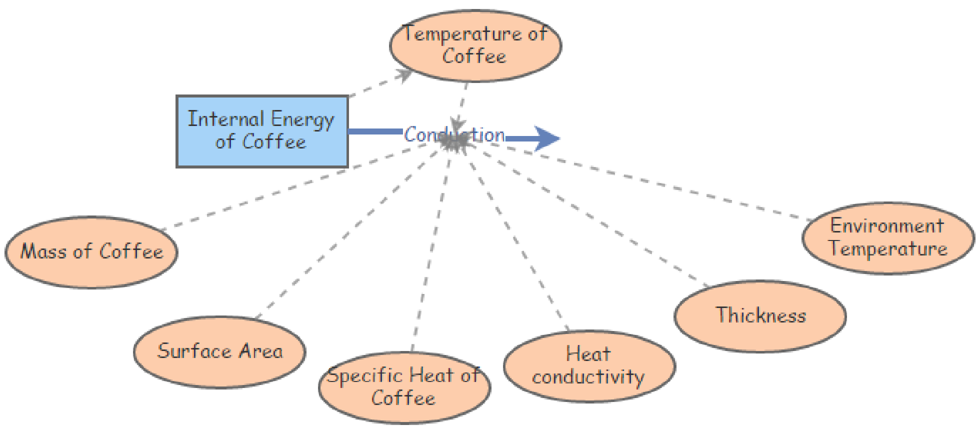

Figure 1: Rudimentary representation of the coffee-cup system

$$\begin{aligned}
\Delta U &= mc\Delta T \\
\Delta U &= Q - (W = 0) \\
Q &= -\frac{kA(T_{self} - T_{env})}{d} \\
mc\Delta T &= -\frac{kA(T_{self} - T_{env})}{d} \\
\therefore \Delta T &= -\frac{kA(T_{self} - T_{env})}{dmc}
\end{aligned}$$

In its MATLAB implementation, this flow was modeled as follows:

```
%Part I

r = 8/2 / 100; %m
h = 10 / 100; %m
A = pi*r^2 + 2*pi*r*h; %total surface area, m^2

d = 0.7 / 100;%m
c = 4186; %J/kg*K
rho = 1000; %kg/m^3
```

```
k = 1.5; %W/(m*K)

m = rho*pi*r^2*h; %kg

T_s = 370; %K
T_e = 290; %K

conduction = @(k,A,T_s,T_e,d,m,c) -k*A*(T_s-T_e)/(d*m*c);

range = 30;
vCoffee = zeros(1,range);
intv = 1:range;

for t = intv
    vCoffee(t) = T_s;
    T_s = T_s + conduction(k*60,A,T_s,T_e,d,m,c);%k to minutes
end
plot(intv-1,vCoffee);
title('The Time Series of the Temperature of Coffee')
xlabel('Time (minutes)');
ylabel('Temperature of Coffee(K)');
```

The resultant plot:

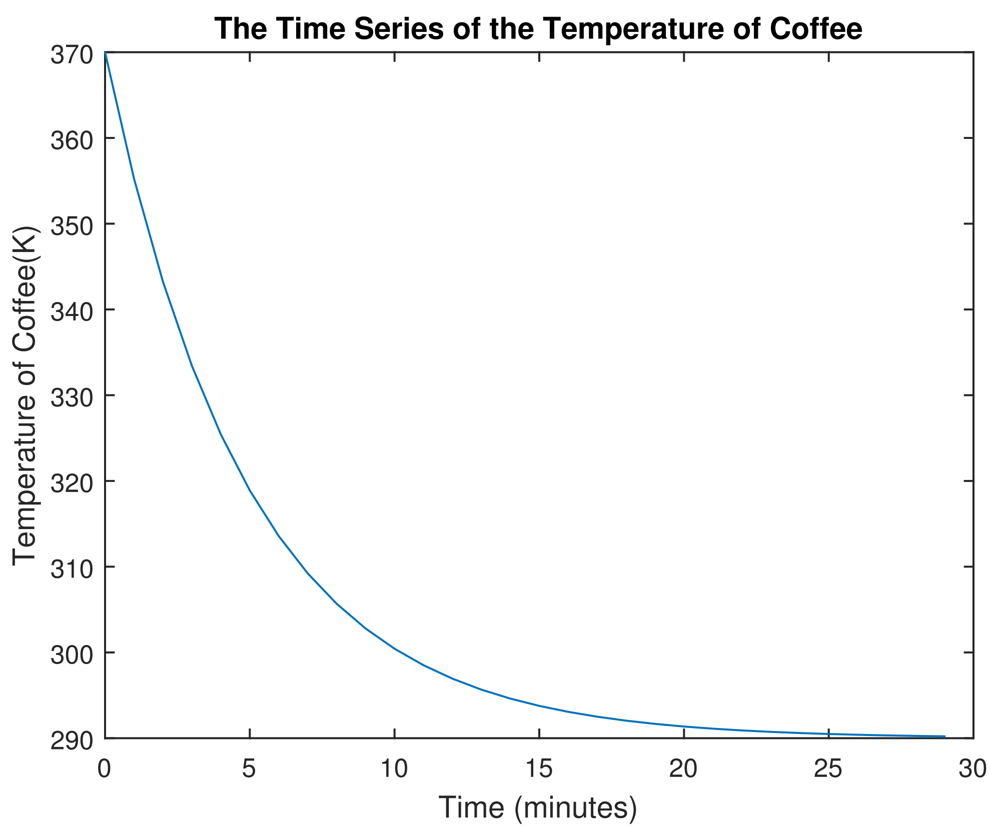

Figure 2: The cooling of 370K coffee over 30 minutes.

This graph is consistent to intuition; a hot coffee would not take more than 30 minutes to cool in room temperature; of course, with the addition of other factors, the coffee should cool down at a greater rate (unless given an external heat source).

2 Adding a Cream

In adding a cream, I would assume that the internal energy of the mixture after the addition is simply the sum of the internal energy of the two systems, and I would also set the internal energy to be proportional to the specific heat, mass, and the temperature of the respective substances. If I only knew the volume, I would further assume that the density of the cream similar to that of the coffee in order to deduce the mass. I will denote the coffee as system 1 and the cream as system 2 in the following qualitative analysis(as it is by no means numeric). Accordingly,

$$\begin{aligned}
E_{mix} &= E_1 + E_2 \\
E_1 &= k * c_1 * m_1 * T_1 \\
E_2 &= k * c_2 * m_2 * T_2 \\
E_{mix} &= k * c_1 * m_1 * T_1 + k * c_2 * m_2 * T_2 \\
&= k * c_1 * m_1 * T_{mix} + k * c_2 * m_2 * T_{mix} \\
T_{mix} &= \frac{k * c_1 * m_1 * T_1 + k * c_2 * m_2 * T_2}{k * c_1 * m_1 + k * c_2 * m_2} \\
T_{mix} &= \frac{c_1 * m_1 * T_1 + c_2 * m_2 * T_2}{c_1 * m_1 + c_2 * m_2}
\end{aligned}$$

Here, k is an arbitrary proportionality constant. Now, this is obviously a very rough sketch without knowing about how the physical system actually interacts. Upon this model, my next iteration would be to apply this to the simulation. The factors that impact the conduction flow are surface area, the temperature of the system and the environment, the thickness of the cup, the mass of the system, and the specific heat of the system. Of them, the surface area, the temperature of the system, the mass of the system, and the specific heat of the system undergoes a change when cream is added.

Making realistic assumptions, the surface area would change by $\frac{V_2}{\pi r^2} * 2\pi r$; the new temperature was determined above; the mass of the system is simply $m_1 + m_2$, and it is possible to deduce the new "specific heat" of the material under how I modeled the internal energy; since $E_{mix} = k * (c_1 * m_1 + c_2 * m_2) * T_{mix}$ and the internal energy is represented as $E = k * c * m * T$, it is reasonable to claim that the specific heat term would now correspond to $(c_1 * m_1 + c_2 * m_2)/(m_1 + m_2)$.

Therefore, in my next iteration, I will update the associated variables as described above at the moment I put in the cream. For the purposes of the model, I will assume that the temperature of the system would immediately settle down to the intermediate temperature, because it would be beyond the scope of this model to take into account the complex convection mechanism within the mixture itself. In determining the optimal moment at which to put the cream in the coffee, I would run the simulation 60 times, over an hour, and have the simulation return the moment at which the coffee temperature fell below the drinkable temperature. I would then take the resultant values and plot them against the moment at which the cream was put in. The punchline graph would roughly look like:

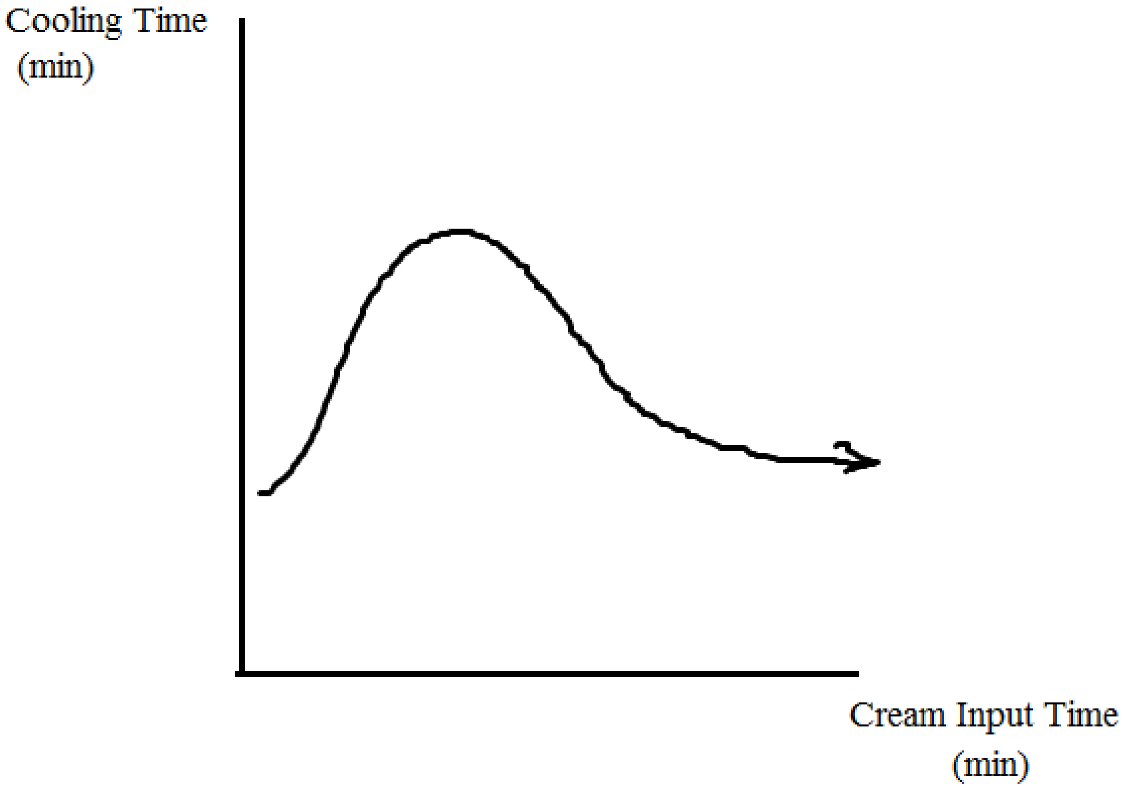

Figure 3: The graph of cream insertion time vs. time it took to cool down to a certain temperature (perhaps 60 degrees Celcius)