ISim Lab 5: Op Amps

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October 7 2015

1 Op Amp as Buffer

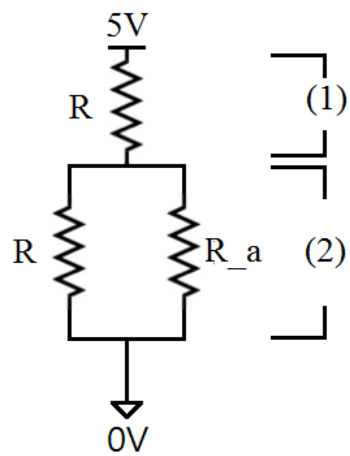

Figure 1: Simplified Circuit Diagram; Analog Discovery is represented as a resistor with resistance R_a.

Given:

$$\begin{aligned}
R_1 &= R \\
R_2 &= \frac{1}{\frac{1}{R} + \frac{1}{R_a}} \\
I_1 &= V_1/R_1 \\
I_2 &= V_2/R_2 \\
I_1 &= I_2
\end{aligned}$$

Yields

$$\begin{aligned}
R_2 = \frac{V_2 R}{V_1} &= \frac{1}{\frac{1}{R} + \frac{1}{R_a}} \\
\frac{V_2 R}{V_1} &= \frac{R_a * R}{R_a + R} \\
\frac{V_2}{V_1} &= \frac{R_a}{R_a + R} \\
R_a &= \frac{R * V_2}{V_1 - V_2}
\end{aligned}$$

Thus the impedence of the Analog Discovery is $\frac{1M\Omega * 1.73V}{3.27V - 1.73V} = 1.06M\Omega$.

Accordingly, the theoretical measured Voltage with $499K\Omega$ Resistors is:

$$\begin{aligned}\frac{R}{R_a} &= \frac{V_1 - V_2}{V_2} \\ &= \frac{5 - 2 * V_2}{V_2} \\ R * V_2 &= R_a(5 - 2 * V_2) \\ (R + 2 * R_a)V_2 &= 5R_a \\ V_2 &= \frac{5R_a}{R + 2R_a}\end{aligned}$$

So $V_2 = \frac{5V*1.06M\Omega}{.499M\Omega+2*1.06M\Omega} = 2.02V$.

Table 1: Voltage measurement through experiment

Resistance	Op Amp	Voltage
1 MΩ	X	1.73 V
499 KΩ	X	2.04 V
499 KΩ	O	2.51 V

As shown, the experimental value of 2.04V, when compared to the theoretical value of 2.02V, yields a % discrepancy of -0.99%. Meanwhile, the same circuit with the Op Amp demonstrated much closer proximity to the anticipated value (2.51 V), which evidences its capacity as a buffer. In the past labs, the value of R was small enough such that the value of R_2 approximated closely to R; in this experiment, we adopted a $1M\Omega$ resistor, which is on the same order of magnitude as the impedence of the Analog Discovery; hence, the voltage drop through the Analog Discovery was noticeably significant.

2 Inverting Amplifier

Assuming that no current goes through the Op Amp, the circuit is equivalent to the following diagram:

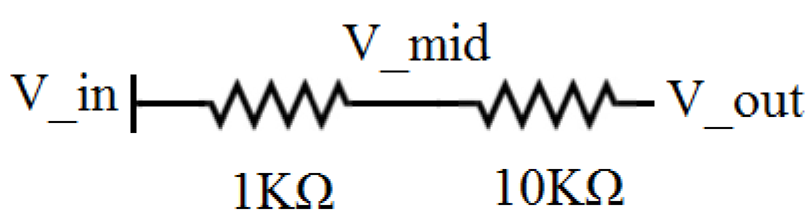

Figure 2: Simplified Circuit Diagram. When V_{out} is not at its extremes, V_{mid} is 2.5V when $V_{pos} = V_{neg}$; in reference to 2.5V, thus, it will be considered zero.

Since the circuit is congruent to that of a voltage-divider, it can be easily deduced that when V_{mid} is equivalent to $\frac{V_{mid}-V_{in}}{1K\Omega} = \frac{V_{out}-V_{mid}}{10K\Omega}$, and since $V_{mid} = 0$,

$$V_{out} = -10V_{in}$$

The output voltage will thus be amplified by 10. The below figure illustrates this relationship:

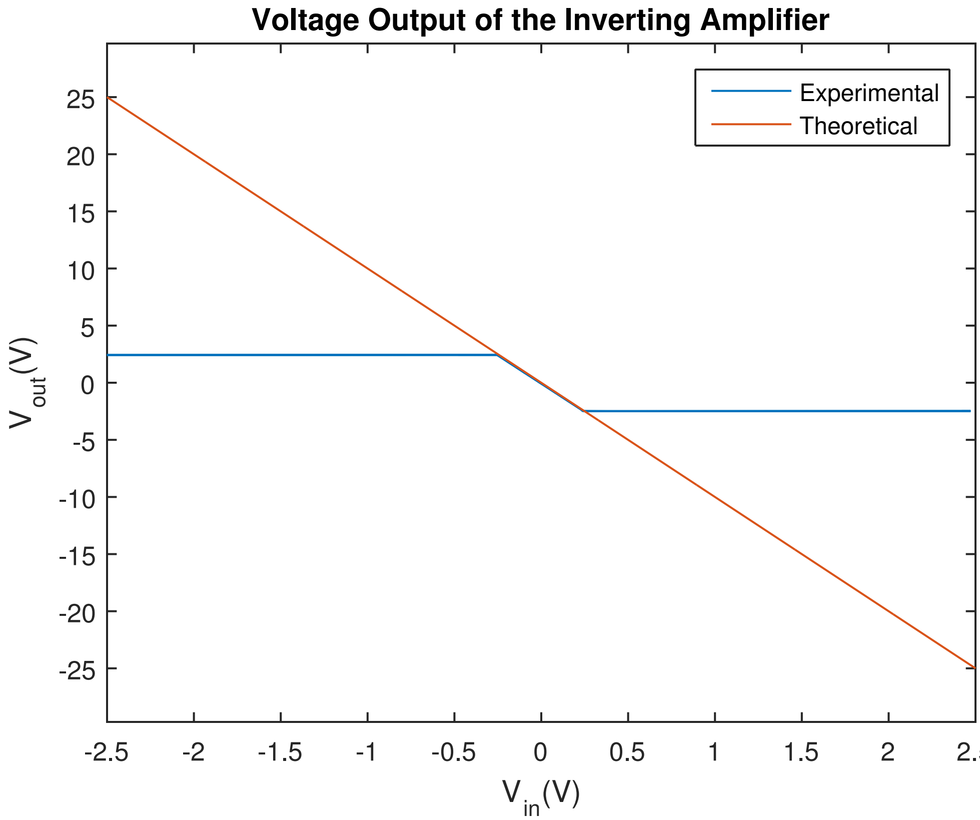

Figure 3: The relationship between input and output voltage through the inverting amplifier.

The theoretical line was generated simply by setting $V_{out} = -10V_{in}$; as seen, the slope of the line (where V_{out} is not limited by the rails) coheres closely to -10; in other places, the value is fixed at the rails.

3 Op Amp Filter

3.1 Approach with Resistance and Capacitance

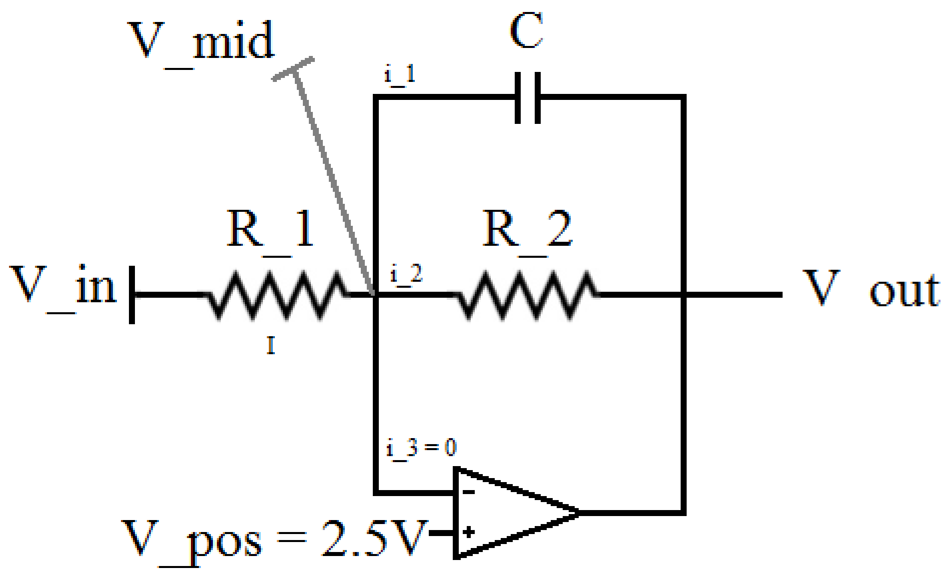

Figure 4: The circuit diagram of the Op-Amp Filter, which will be extensively referred to in the following section.

Since the voltage across the capacitor and the second resistor should be the same and the addition of the two currents, the relationship can be sumamrized as follows:

$$I = i_1 + i_2 \tag{1}$$

$$I = \frac{V_{mid} - V_{in}}{R_1} \tag{2}$$

$$i_1 = \frac{V_{out} - V_{mid}}{R_2} \tag{3}$$

$$i_2 = C\frac{d(V_{out} - V_{mid})}{dt} \tag{4}$$

Now, although V_{in} is actually $2.5 + 0.1sin(\omega t)$, taking the voltage in reference to V_{pos}, V_{in} can be represented as $0.1sin(\omega t)$. Accordingly, V_{mid} is also 0 if V_{out} $!= -2.5V$ and V_{out} $!= 2.5V$[1], in which case it is stuck at the 'rails' and the assumption that $V_{pos} == V_{neg}$ no longer holds true. Furthermore, V_{out} will be modeled

[1] The maximum/minimum values of the Op-Amp, not represented in the diagram for simplification.

as $A * 0.1 * sin(\omega t + \phi)$; the objective is to identify Amplitude(A) and Phase(ϕ).

$$\begin{aligned}
\frac{V_{mid} - V_{in}}{R_1} &= \frac{V_{out} - V_{mid}}{R_2} + C\frac{d(V_{out} - V_{mid})}{dt} \\
\frac{0 - 0.1sin(\omega t)}{R_1} &= \frac{0.1Asin(\omega t + \phi) - 0}{R_2} + C\frac{d(0.1Asin(\omega t + \phi) - 0)}{dt} \\
-\frac{0.1sin(\omega t)}{R_1} &= \frac{0.1Asin(\omega t + \phi)}{R_2} + C\frac{d(0.1Asin(\omega t + \phi))}{dt} \\
-\frac{sin(\omega t)}{R_1} &= \frac{Asin(\omega t + \phi)}{R_2} + C\frac{d(Asin(\omega t + \phi))}{dt} \\
-\frac{sin(\omega t)}{R_1} &= \frac{Asin(\omega t + \phi)}{R_2} + CA\omega cos(\omega t + \phi)
\end{aligned}$$

now, invoking trigonometric identities,

$$-\frac{sin(\omega t)}{R_1} = \frac{A}{R_2} * (sin(\omega t)cos(\phi) + cos(\omega t)sin(\phi)) + CA\omega(cos(\omega t)cos(\phi) - sin(\omega t)sin(\phi))$$

For this equation to be true at all times, $sin(\omega t)$ and $cos(\omega t)$ terms must balance separately:

$$0 = \frac{A}{R_2}cos(\omega t)sin(\phi) + CA\omega cos(\omega t)cos(\phi) \tag{5}$$

$$-\frac{sin(\omega t)}{R_1} = \frac{A}{R_2} * sin(\omega t)cos(\phi) - CA\omega sin(\omega t)sin(\phi) \tag{6}$$

in order to obtain ϕ, equation (5) simplifies to:

$$\begin{aligned}
0 &= \frac{A}{R_2}cos(\omega t)sin(\phi) + CA\omega cos(\omega t)cos(\phi) \\
0 &= \frac{sin(\phi)}{R_2} + C\omega cos(\phi) \\
-C\omega cos(\phi) &= \frac{sin(\phi)}{R_2} \\
-R_2C\omega &= \frac{sin(\phi)}{cos(\phi)}
\end{aligned}$$

thus,

$$\phi = atan(-R_2C\omega) \tag{7}$$

in order to obtain Amplitude, equation (6) simplifies to:

$$\begin{aligned}
-\frac{sin(\omega t)}{R_1} &= \frac{A}{R_2} * sin(\omega t)cos(\phi) - CA\omega sin(\omega t)sin(\phi) \\
-\frac{1}{R_1} &= \frac{A}{R_2} * cos(\phi) - CA\omega sin(\phi) \\
-\frac{1}{R_1} &= A * (\frac{1}{R_2} * cos(\phi) - C\omega sin(\phi))
\end{aligned}$$

thus,

$$A = \frac{1}{R_1 * (C\omega sin(\phi) - \frac{1}{R_2} * cos(\phi))} \tag{8}$$

3.2 Approach with Impedence

Alternatively, this circuit could be solved by using the concept of impedence, simply replacing resistors and capacitors with an equivalent "impedor":

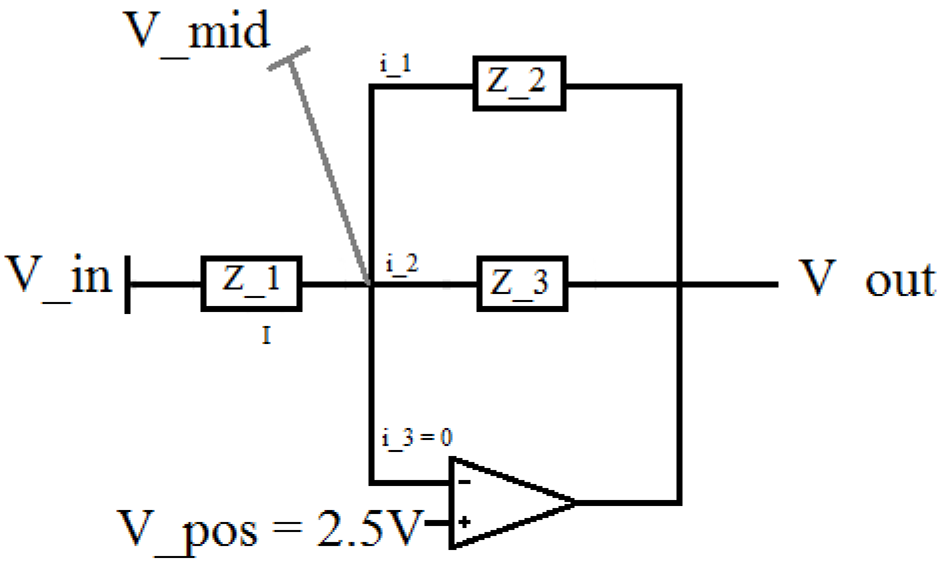

Figure 5: Caption

this circuit yields the following relationship:

$$\frac{V_{mid} - V_{in}}{Z_1} = \frac{V_{out} - V_{mid}}{\frac{1}{\frac{1}{Z_2}+\frac{1}{Z_3}}}$$
$$\frac{0 - V_{in}}{Z_1} = \frac{V_{out} - 0}{\frac{1}{\frac{1}{Z_2}+\frac{1}{Z_3}}}$$
$$\frac{V_{in}}{Z_1} = \frac{V_{out}}{\frac{1}{\frac{Z_2+Z_3}{Z_2 Z_3}}}$$
$$\frac{V_{in}}{Z_1} = V_{out}\frac{Z_2 + Z_3}{Z_2 Z_3}$$
$$\frac{V_{out}}{V_{in}} = \frac{Z_2 Z_3}{Z_1(Z_2 + Z_3)}$$

Since $Z_1 = 1\,\mathrm{k\Omega}$, $Z_2 = 1/j\omega C$, where C = 0.01 µF and $Z_3 = 1\,\mathrm{k\Omega}$,

$$\frac{V_{out}}{V_{in}} = \frac{1/(j\omega * 0.01\,\mu\mathrm{F}) * 1\,\mathrm{k\Omega}}{1\,\mathrm{k\Omega} * (1/(j\omega * 0.01\,\mu\mathrm{F}) + 1\,\mathrm{k\Omega})} \tag{9}$$

This yields a direct relationship of Amplitude and Phase to the frequency of V_{in}; the resultant Bode plot is shown below.

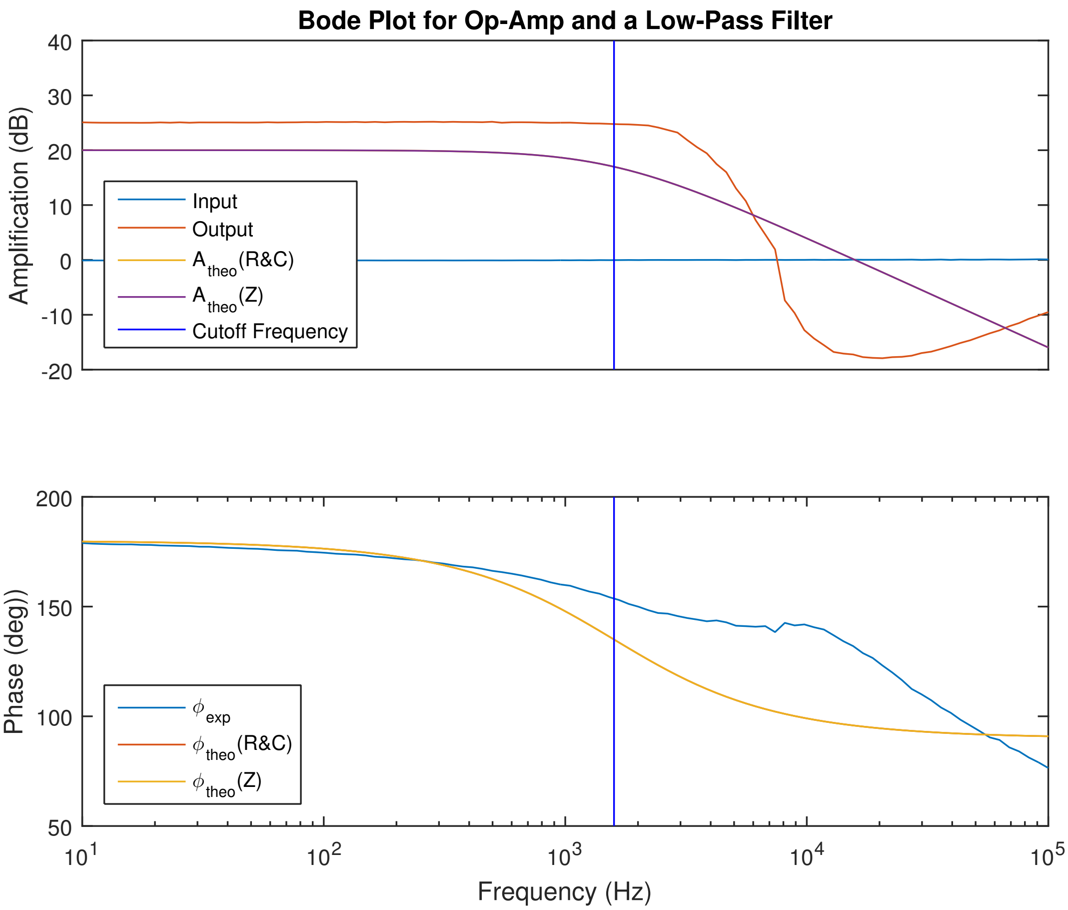

Figure 6: The Bode Plot of the circuit;paranthesized R & C indicates that the calculation was conducted by an analysis of resistance and capacitance; Z, on the other hand, indicates that the calculation was done with impedence. As seen, the two theoretical calculations exactly overlap.

while the results do not exactly match up, the two theoretical calculations are congruent; therefore, it is unlikely that the calculation themselves are both wrong. I would attribute the discrepancy between the two results (although they are indeed quite similar) to the limitations in the precision of devices, such as capacitors, as well as the ability of the analog discovery to pick up signals at high frequencies, as the discrepancy was more readily apparent in the higher frequency range.

4 Light Measurement

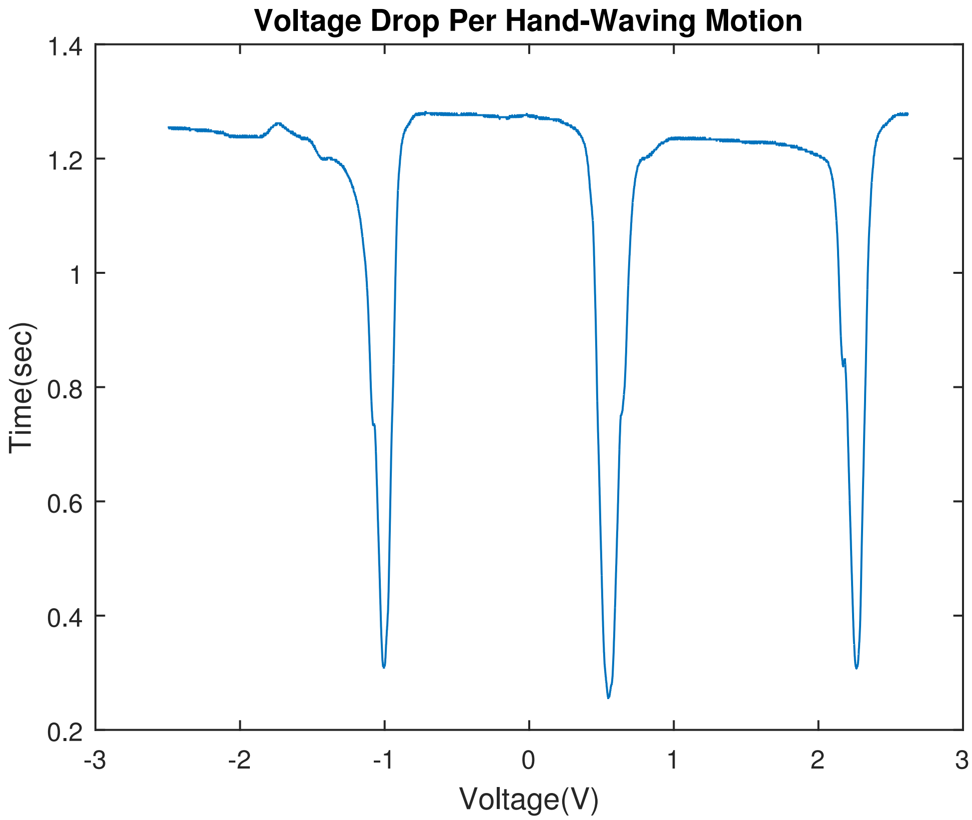

Figure 7: The troughs in the plot occurred when I waved my hand over the photodiode, which makes sense since it lost the supply of light, thereby reducing the electric potential. As soon as my hand was out of the way from the ambient light source, the voltage quickly recovered.

5 Pulse Measurement

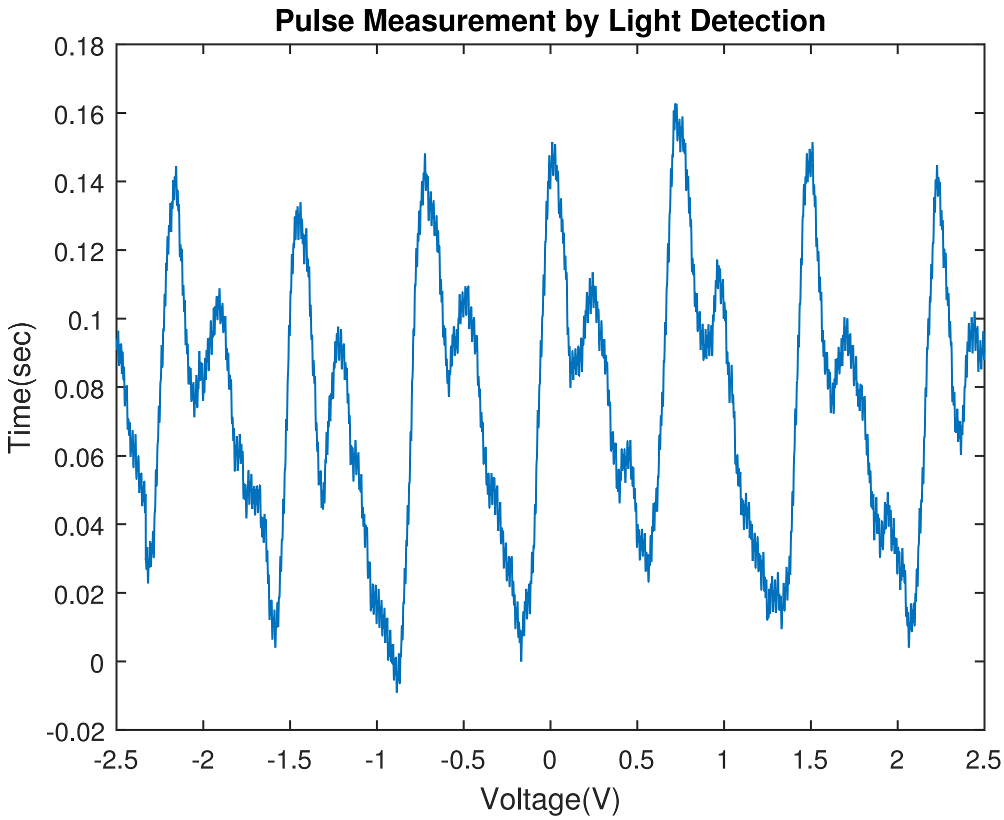

Figure 8: The Pulse, as measured by the intensity of light passing through the finger.

6 Photo

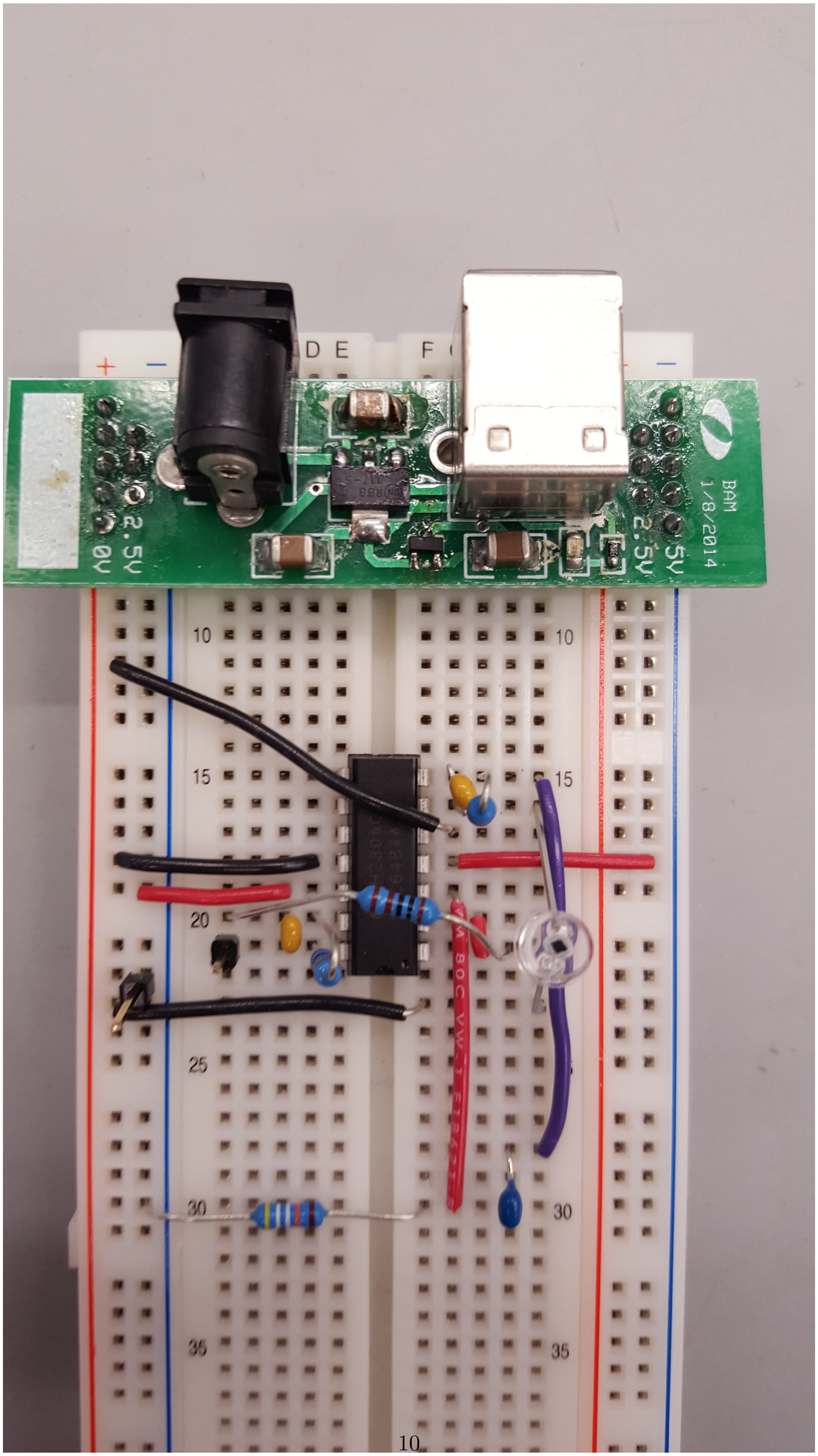

Figure 9: The picture of the pulse-measuring circuit.