ModSim Exercise 9

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November 9 2015

1 Model

The baseball is modelled as a particle, since the rotation is relatively trivial for the purposes of the investigation; modelling baseball as a rigid body involves more complex mechanics – namely aerodynamics, Magnus effects, etc. – which are beyond the scope of the question, as the primarily concern of the question is translational displacement.

2 Interactions

The forces that would act upon the ball are:

Gravity[1], $\vec{F_g} = -\frac{Gm1m2}{\vec{r}^2}\hat{\mathbf{r}}$

Drag[2], $\vec{F_d} = -1/2\rho C_d A\vec{v}^2\hat{\mathbf{v}}$

3 Free Body Diagram

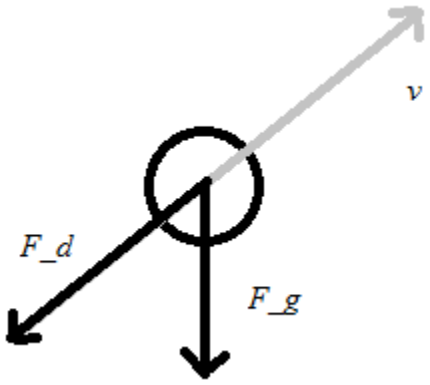

Figure 1: Free body diagram of the baseball.

In the diagram, F_d is Drag, F_g is Gravity, and v is the velocity(notated in grey since it does not contribute to net force).

4 Mathematical Abstraction

The ball was modeled in a 2D coordinate system, where sideways movement was ignored; in this manner, the x-axis was aligned parallel to the ground; likewise, the y-axis was normal to the ground. The mathematical

[1]Primary Factor

[2]Secondary Factor

relationships are as follows:

$$\frac{d\vec{r}}{dt} = \vec{v}$$
$$\frac{d\vec{v}}{dt} = \frac{\vec{F}}{m_b}$$
$$\vec{F} = \vec{F_g}(\vec{r}) + \vec{F_d}(\vec{v})$$
$$\vec{F_g} = -\frac{Gm_e m_b}{\vec{r}^2}\hat{\mathbf{r}}$$
$$\vec{F_d} = -1/2\rho C_d A\vec{v}^2\hat{\mathbf{v}}$$

As for initial conditions, at time $t = 0$, the ball would be some height above the origin (to accurately reflect baseball scenario), but its x-value would be at the reference(0); the velocity would depend on the occasion.

5 Explanation of Terms

Term	Description	Value	Unit
$\vec{r}$	Position of baseball	$< 0, 5 >$	m
$\vec{v}$	Velocity of baseball	$< 39, 12 >$	m/s
G	Gravitational Constant	$6.67408e-11$	$m^3kg^{-1}s^{-2}$
m_b	Mass of Baseball	.145	kg
m_e	Mass of Earth	$5.9742e24$	kg
A	Cross-sectional area of baseball	.004	m^2
$\vec{F_g}$	Gravity	$< 0, -1.42 >$	N
$\vec{F_d}$	Drag Force	$< -0.6, -0.09 >$	N
ρ	Density of Air	1.225	kg/m^3
C_d	Drag Coefficient of Baseball	0.3	-

Table 1: The terms in the differential equation; Vectors were notated element-wise. Constant values are taken verbatim; variable values are samples within reasonable range.