ModSim Exercise 6

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1 Implementing with ode45

The following is the same model as the previous exercise, now transcribed to an ode45-compatible code;

```
function simulate()



r = 8/2/100; %radius, m
h = 10/100; %height, m
A = pi*r^2+2*pi*r*h; %surface area, m^2
d = 0.7/100;%thickness, m
c = 4186; %specific heat, J/(kg*K)
rho = 1000; %density, kg/m^3
k = 1.5; %heat conductivity, W/(m*K) => (in seconds)
V = pi*r^2*h; %volume, m^3
m = rho*V; %mass, kg

T_s = 370;
T_e = 290;

TbyE = @(E,m,c) E/(m*c);
EbyT = @(T,m,c) T*m*c;

E_s_0 = EbyT(T_s,m,c);

[T,E_s] = ode45(@Q,[0 30],E_s_0);

plot(T,TbyE(E_s,m,c));


function cond = Q(t,E)
cond = -k*60*A*(TbyE(E,m,c)-T_e)/d;
end

end
```

And the resultant plot is shown below:

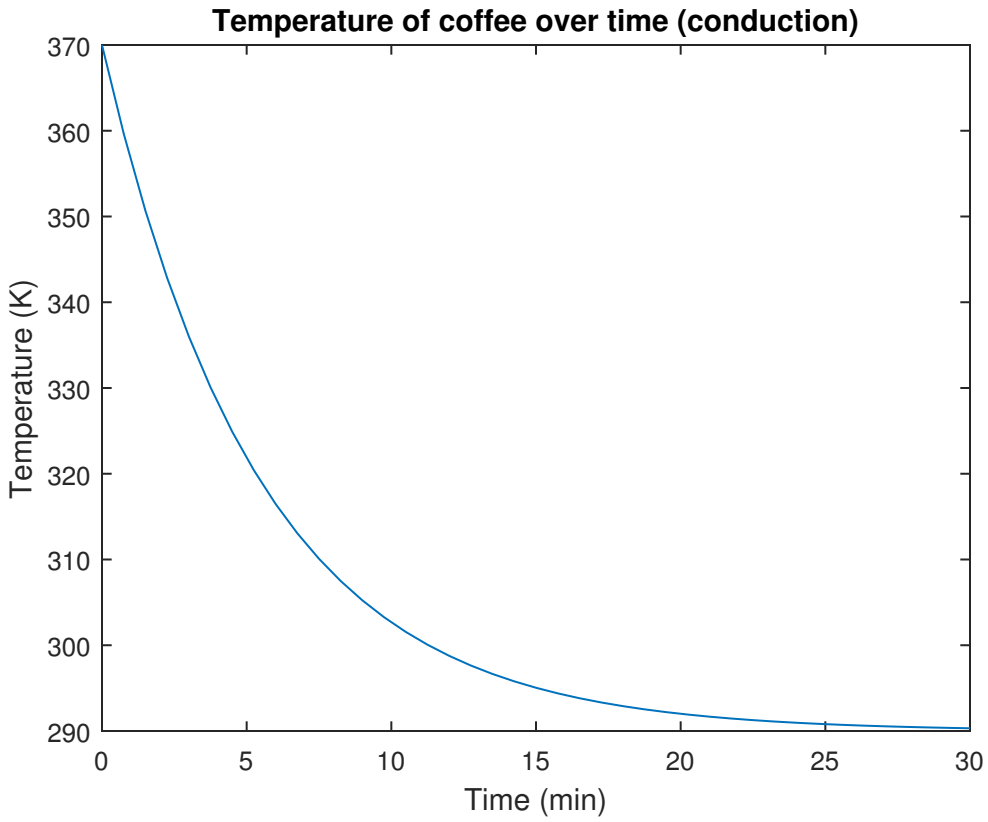

Figure 1: The temperature of coffee over time. In this elementary model, only conduction was taken into account in terms of the cooling of the coffee.

This graph is coherent the previous one, which did not utilize ode45.

2 Adding 100mL cream

The following MATLAB code is a function that takes the time to insert the cream as an argument to produce a plot accordingly.

```
function simulate(addCreamTime)
figure;

r = 8/2/100; %radius, m
h = 10/100; %height, m
A = pi*r^2+2*pi*r*h; %surface area, m^2
d = 0.7/100;%thickness, m
c = 4186; %specific heat, J/(kg*K)
rho = 1000; %density, kg/m^3
V = pi*r^2*h; %volume, m^3
m = rho*V; %mass, kg
k = 1.5; %heat conductivity, W/(m*K) => (in seconds)
T = 370;
T_e = 290;
E_0 = EbyT(T,m,c);

%Q = @(t,E) -k*60*A*(TbyE(E,m,c)-T_e)/d;
```

```
18
19 if(addCreamTime>0)
20     [t,E] = ode45(@conduction,[0 addCreamTime],E_0);
21     plot(t,TbyE(E,m,c));
22     [h,A,V,T,m,c,E_0] = addCream(h,r,V,m,c,E(end));
23 else
24     plot(0,TbyE(E_0,m,c),'o');
25     [h,A,V,T,m,c,E_0] = addCream(h,r,V,m,c,E_0);
26 end
27
28 hold on;
29
30 if(addCreamTime<30*60)
31     [t,E] = ode45(@conduction,[addCreamTime 30*60],E_0);
32     plot(t,TbyE(E,m,c));
33 else
34     plot(30*60,T,'o');
35 end
36 hold on;
37
38 line(xlim, [273+60 273+60]);
39
40     function Q = conduction(~,E)
41         Q = -k*A*(TbyE(E,m,c)-T_e)/d;
42     end
43
44     function [h_new,A_new,V_new,T_new,m_new,c_new,E_new] = addCream(h,r,V,m,c,
           E)
45         T = TbyE(E,m,c);
46
47         %CREAM STUFF
48         T_c = 275; %K
49         V_c = 100/1e6;% m^3
50         c_c = 4186; %specific heat, J/(kg*k)
51         rho_c = 1000; %density, kg/m^3
52         m_c = rho_c*V_c; %kg
53         E_c = EbyT(T_c,m_c,c_c); %J
54
55         %END OF CREAM STUFF
56         %update constants
57         h_new = h + V_c/(pi*r^2);
58         A_new = pi*r^2+2*pi*r*h;
59         V_new = V + V_c;
60         T_new = (E+E_c)/(m*c+m_c*c_c);
61         m_new = m + m_c;
62         c_new = (c*m+m_c*c_c)/(m_new);
63         E_new = E_c+E;
64     end
65 end
66
67
68 function res = TbyE(E,m,c)
69 res = E/(m*c);
70 end
```

```
71
72 function res = EbyT(T,m,c)
73 res = T*m*c;
74 end
```

In the case of adding the cream at the two endpoints (special cases), since ode45 needs to be executed for an interval of which the end must not be the same as the beginning, only one simulation was conducted by adjusting the control flow. In this case, the transition point was marked as a circle, as it couldn't be represented by a line. The resultant plot is shown below, for adding the cream at the beginning, middle, and the end, respectively:

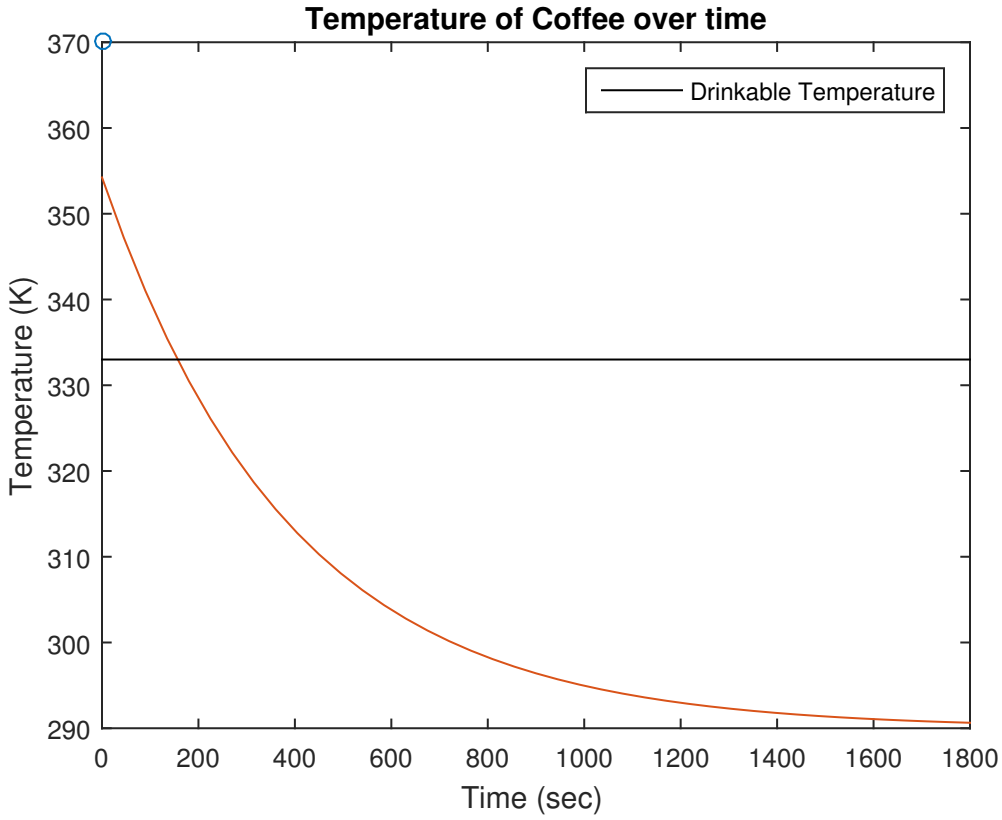

Figure 2: Cream added at t=0

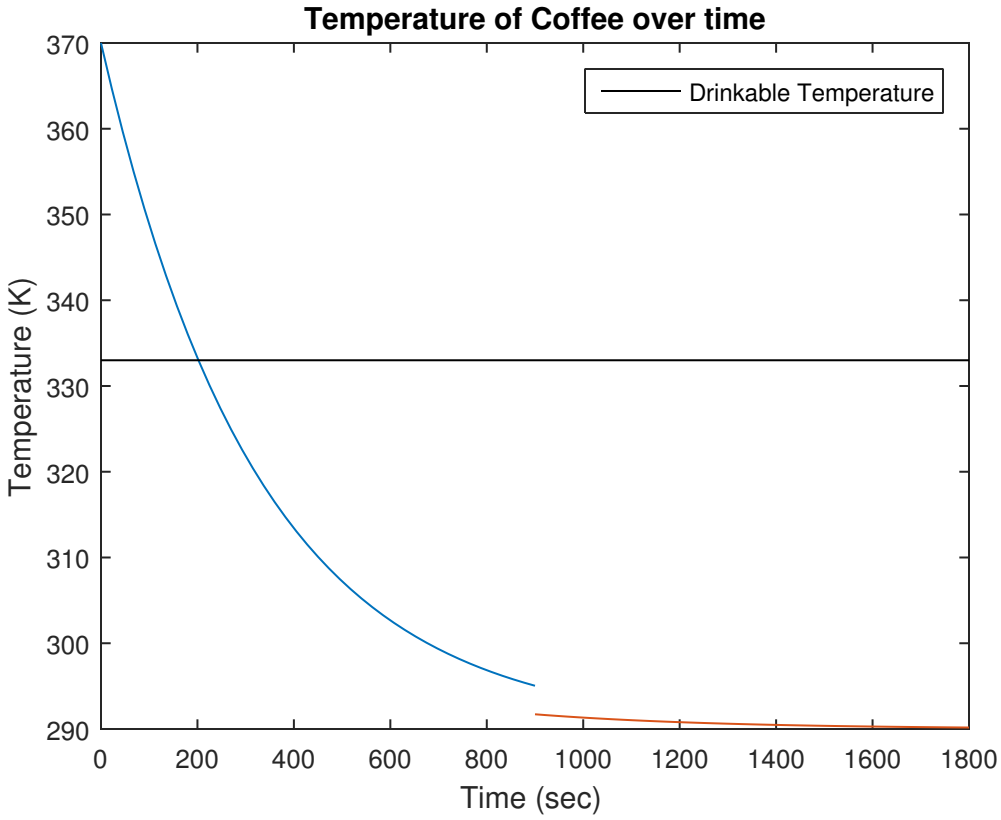

Figure 3: Cream added at t = 15 min

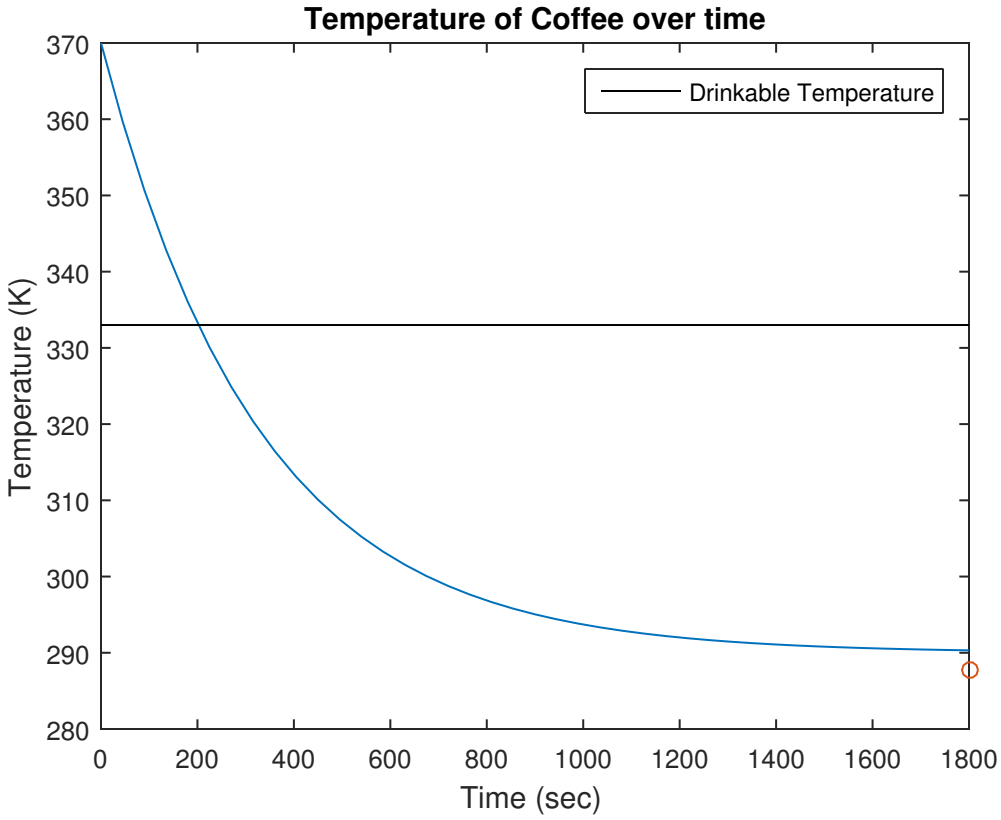

Figure 4: Cream added at t = 30 min

As seen, the falling slope is steepest at the beginning – which is the predicted behavior, as conduction is proportional to the differnce in temperature between the system and the environment. Therefore, it seems most advantageous to procrastinate the adding of the cream until the point at which adding the cream would immediately make the coffee drinkable.