# ISim Lab4: EKG

Yoonyoung Cho

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## 1 RC Filters

### 1.1 High-Pass Filter

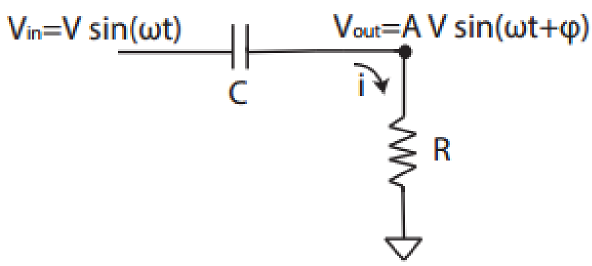


Figure 1: Circuit Diagram of a high-pass filter.

In a High-Pass Filter, the relationship between Frequency, Amplification, and Phase can be derived as follows:

$$V_{in} = V sin(\omega t) \tag{1}$$
$$V_{out} = AV sin(\omega t + \phi) \tag{2}$$
$$C\frac{d(V_{out} - V_{in})}{dt} = \frac{0 - V_{out}}{R} = I \tag{3}$$

This expands to:

$$C\frac{d(AV sin(\omega t + \phi) - V sin(\omega t))}{dt} = -\frac{AV sin(\omega t + \phi)}{R}$$
$$-RC(AV\omega cos(\omega t + \phi) - V\omega cos(\omega t)) = AV sin(\omega t + \phi)$$
$$-RC(A\omega cos(\omega t + \phi) - \omega cos(\omega t)) = Asin(\omega t + \phi)$$

Applying the trigonometric identities,

$$-RC(A\omega(cos(\omega t)cos(\phi) - sin(\omega t)sin(\phi)) - \omega cos(\omega t)) = A(sin(\omega t)cos(\phi) + cos(\omega t)sin(\phi)$$

As this equation must be true at all times, the terms with $sin(\omega t)$ and $cos(\omega t)$ must equate separately. Therefore,

$$RCA\omega(sin(\omega t)sin(\phi)) = Asin(\omega t)cos(\phi) \tag{4}$$
$$-RCA\omega(cos(\omega t)cos(\phi)) + RC\omega cos(\omega t)) = Acos(\omega t)sin(\phi) \tag{5}$$

equation (4) is now further simplified to yield the phase $\phi$:

$$RCA\omega(sin(\omega t)sin(\phi)) = Asin(\omega t)cos(\phi)$$
$$RC\omega sin(\phi) = cos(\phi)$$
$$cot(\phi) = RC\omega$$

$$\phi = acot(RC\omega) \tag{6}$$

and equation (5) yields Amplification in terms of $\omega$ and $\phi$.

$$-RCA\omega(cos(\omega t)cos(\phi)) + RC\omega cos(\omega t)) = Acos(\omega t)sin(\phi)$$
$$-RCA\omega cos(\phi) + RC\omega = Asin(\phi)$$
$$A(sin(\phi) + RC\omega cos(\phi)) = RC\omega$$

$$A = \frac{RC\omega}{sin(\phi) + RC\omega cos(\phi)} \tag{7}$$

As it is possible to sequentially solve for phase and amplification in MATLAB, there was no need to substitute $\phi$ and solve for the amplification expression. The resultant Bode plot for R = 1000 Ω, C = 0.1 µF is shown below:

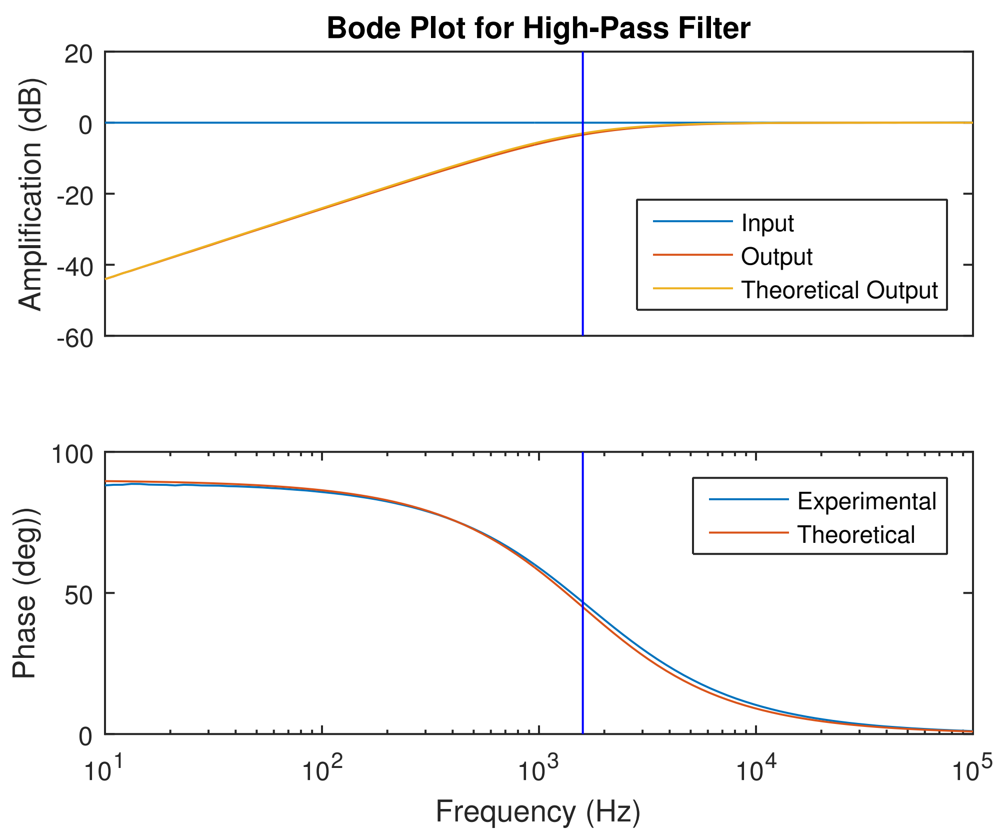


Figure 2: The Bode Plot for high-pass filter; the frequency ranges from 10Hz to 100kHz, and the blue line denotes the critical frequency, at 1592 Hz.

As shown, experimental and theoretical data are primarily consistent; the minor discrepancy in amplification and phase can be attributed to imperfections in the capacitor.

## 1.2 Low-Pass Filter

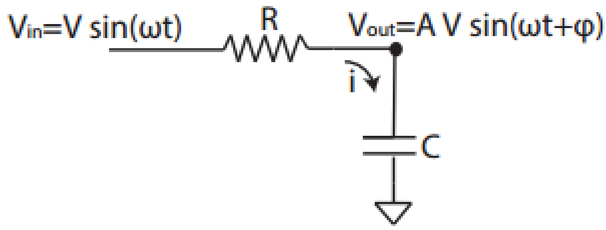


Figure 3: Circuit Diagram of a low-pass filter.

In a Low-Pass Filter, the relationship between Frequency, Amplification, and Phase can be derived as follows:

$$V_{in} = V sin(\omega t) \tag{8}$$
$$V_{out} = AV sin(\omega t + \phi) \tag{9}$$
$$C\frac{d(0 - V_{out})}{dt} = \frac{V_{out} - V_{in}}{R} = I \tag{10}$$

This expands to:

$$\begin{aligned}
-C\frac{dV_{out}}{dt} &= \frac{V_{out} - V_{in}}{R} \\
-RC\frac{d(AV sin(\omega t + \phi))}{dt} &= AV sin(\omega t + \phi) - V sin(\omega t) \\
-RCAV\omega cos(\omega t + \phi) &= AV sin(\omega t + \phi) - V sin(\omega t) \\
-RCA\omega cos(\omega t + \phi) &= A sin(\omega t + \phi) - sin(\omega t)
\end{aligned}$$

Applying the trigonometric identities,

$$-RCA\omega(cos(\omega t)cos(\phi) - sin(\omega t)sin(\phi)) = A(sin(\omega t)cos(\phi) + cos(\omega t)sin(\phi)) - sin(\omega t)$$

As this equation must be true at all times, the terms with $sin(\omega t)$ and $cos(\omega t)$ must equate separately. Therefore,

$$-RCA\omega(cos(\omega t)cos(\phi)) = Acos(\omega t)sin(\phi) \tag{11}$$
$$RCA\omega(sin(\omega t)sin(\phi)) = Asin(\omega t)cos(\phi) - sin(\omega t) \tag{12}$$

equation (11) is now further simplified to yield the phase $\phi$:

$$\begin{aligned}
-RCA\omega(cos(\omega t)cos(\phi)) &= Acos(\omega t)sin(\phi) \\
-RC\omega cos(\phi) &= sin(\phi) \\
-RC\omega &= \frac{sin(\phi)}{cos(\phi)} \\
tan(\phi) &= -RC\omega
\end{aligned}$$

$$\phi = atan(-RC\omega) \tag{13}$$

and equation (12) yields Amplification in terms of $\omega$ and $\phi$.

$$RCA\omega(sin(\omega t)sin(\phi)) = Asin(\omega t)cos(\phi) - sin(\omega t)$$
$$RCA\omega sin(\phi) = Acos(\phi) - 1$$
$$A(RC\omega sin(\phi) - cos(\phi)) = 1$$

$$A = 1/(RC\omega sin(\phi) - cos(\phi)) \tag{14}$$

The resultant Bode plot for R = 1000 Ω, C = 0.1 µF is shown below:

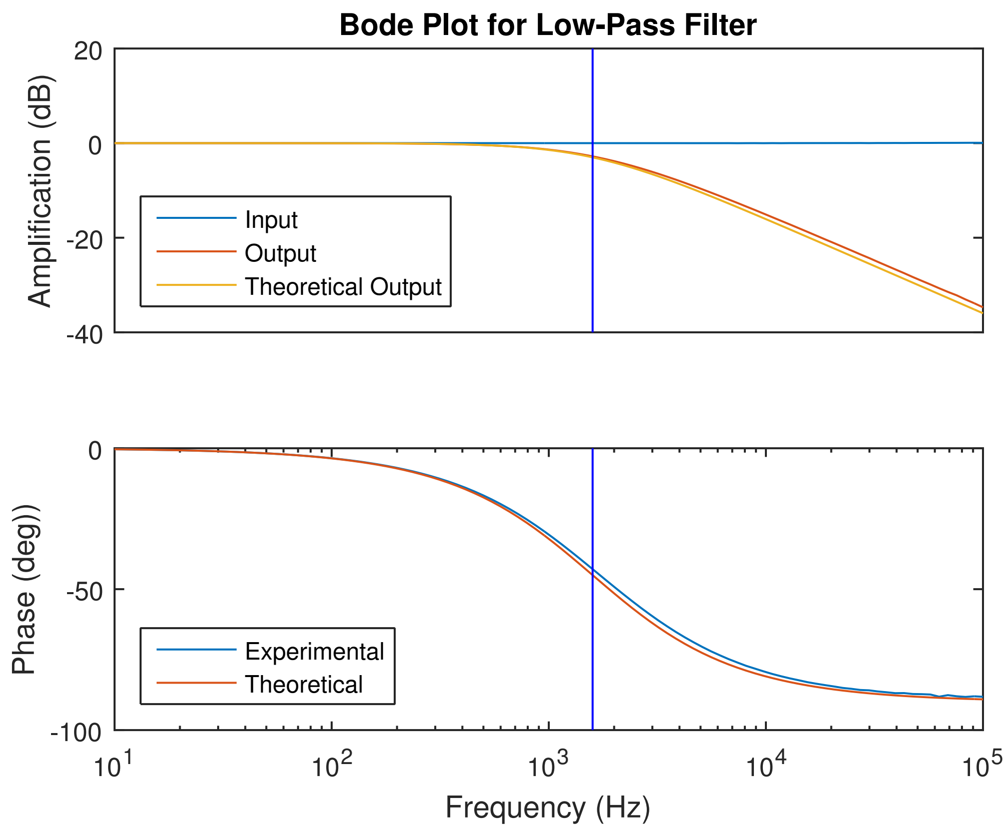


Figure 4: The Bode Plot for low-pass filter; the frequency ranges from 10Hz to 100kHz, and the blue line denotes the critical frequency, at 1592 Hz.

Such consistent coherence in the theoretical and experimental plots serve as sufficient verification of the theoretical model. The calculated cutoff frequency for both circuits was (since resistance and capacitance were held the same) $\frac{1}{1000\Omega * 0.1\,\mu F * 2\pi} = 1592Hz$, which is consistent to both of the plots.

## 1.3 Both Filters Applied

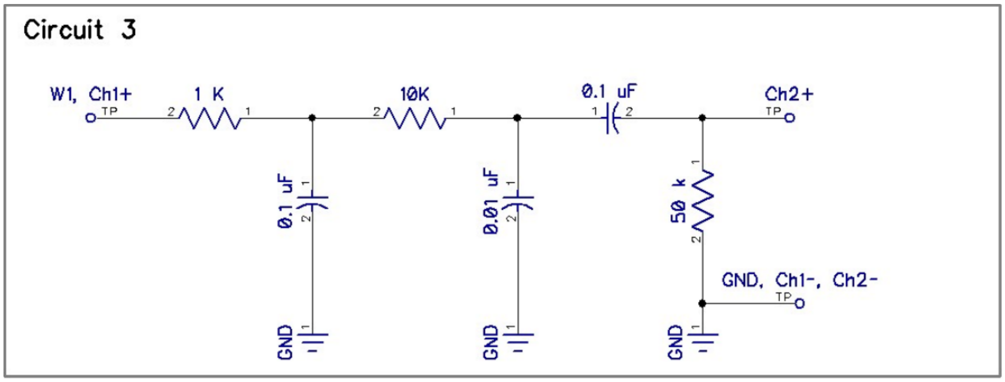


Figure 5: Circuit Diagram of two low-pass filters followed by a high-pass filter.

As for this scenario, in which multiple filters were applied to the source voltage consecutively, amplification was multiplied and phase was added per each filter, which is a natural assumption given the type of transformations they undergo in filtration. The resultant Bode plot which adheres to the above diagram is shown below:

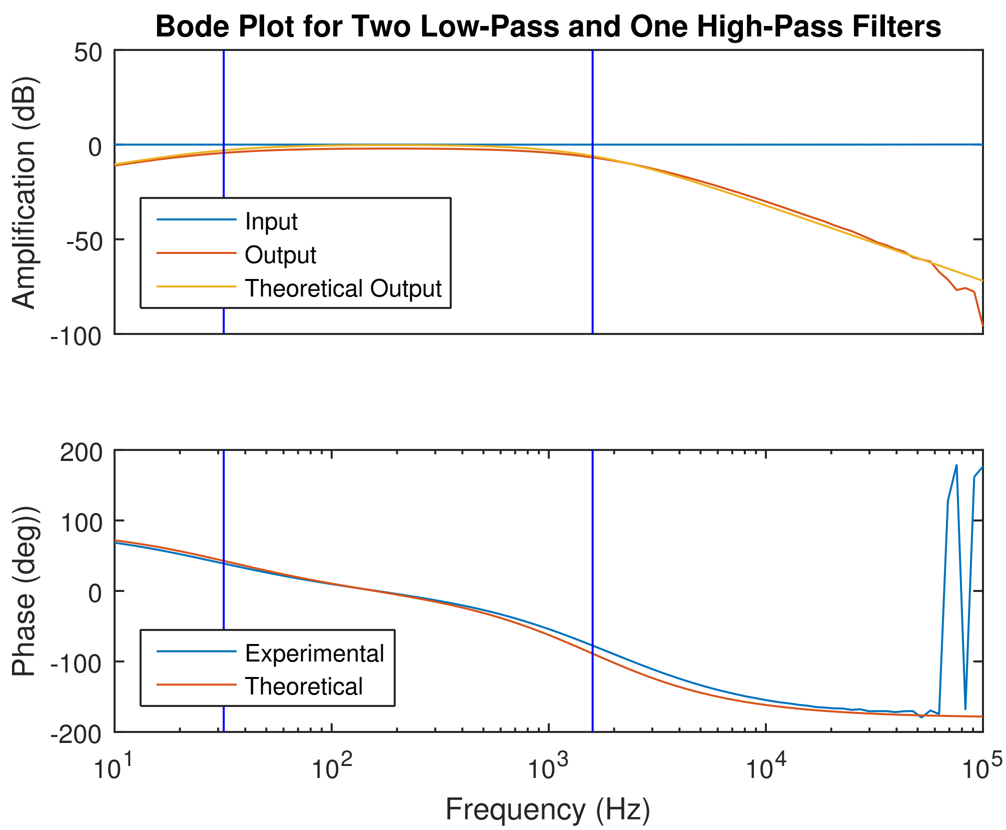


Figure 6: The Bode Plot for a combination of two low-pass filters and one high-pass filter, the frequency ranges from 10Hz to 100kHz. The blue line denotes the critical frequency, at 31.82Hz and 1592 Hz.

The irregularities shown an the end of the experimental plot are due to the confusion between 180° and -180°, as well as the inability for the analog discovery to measure in such fine resolutions.

# 2 The EKG

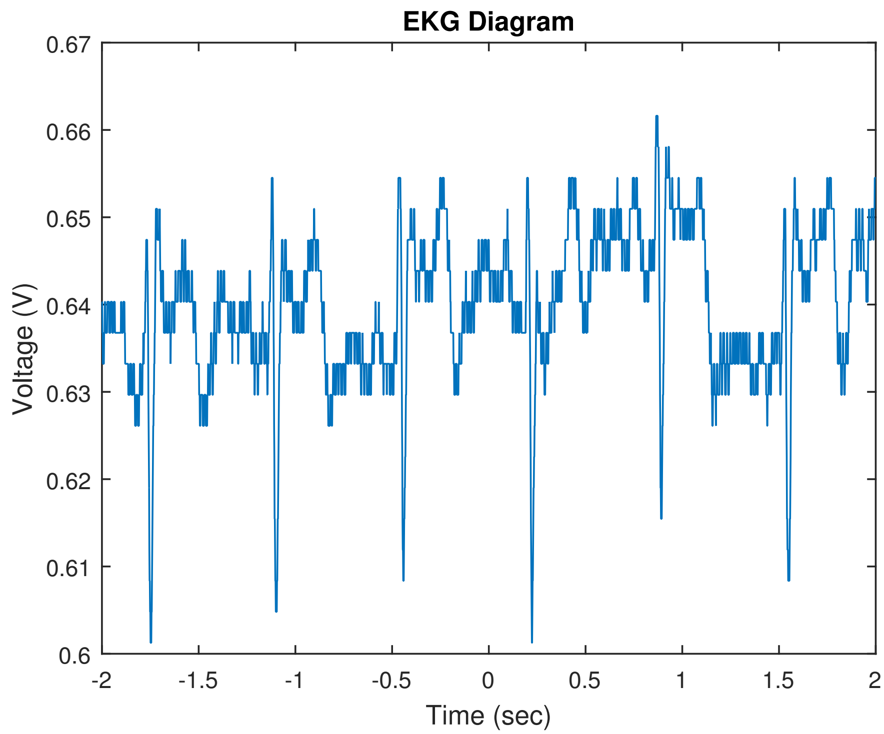


Figure 7: EKG Diagram

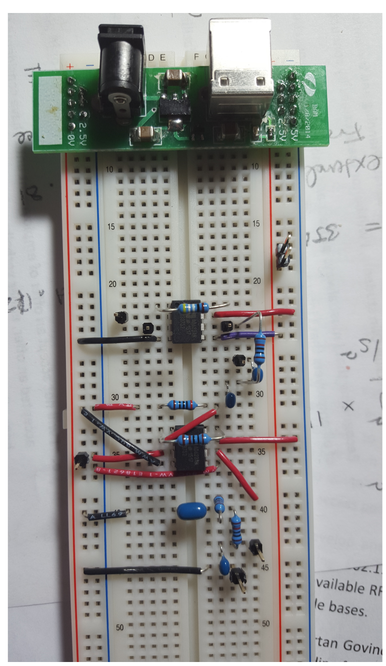

Figure 8: Circuit Picture