Analysis of the Gimbals Platform for the Three Degrees of Freedom using Differential Equations

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ANALYSIS OF THE GIMBALED PLATFORM FOR THE THREE DEGREES OF FREEDOM USING DIFFERENTIAL EQUATIONS

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Abstract- In this paper, Differential Equations of Motion (DEOMs) of three axis gimbaled platform are developed by using Lagrange's Approach. Gimbaled platform presented here consists of four interconnected bodies – stationary case, outer gimbal, inner gimbal and platform, each having one degree of freedom. The Lagrangian Approach is based on scalar quantities i.e. energy contained in the system unlike the other approaches which depend on vector quantities like forces and torques. Thus for a complicated system, one presented in this paper, Lagrange's Approach is the suitable method to develop DEOMs. DEOMs are then rearranged to get System State Equations.

Index Terms*- Differential equations of motion, Kinematics and Dynamics of Rigid Body, Lagrange's approach, System state equations.*

I. INTRODUCTION

Many applications from civil areas (like public security, fire protection, mobile surveillance and so on), from scientific explorations (like detection of rain or hail clouds, detection of turbulence and wind shear, air surveillance, in-flight collision avoidance, ground collision avoidance, surveillance of coastlines and areas of economic interest and many more), etc. requires camera, sensors, telescope, to position on vehicles ranging from submarines to spaceships [1]. For satisfactory operation of these instruments they cannot be mounted directly on vehicles as vehicles are mobile sites. Therefore an object should be developed which will maintain these instruments steady in a desired orientation even though vehicle is moving in different directions. Such an object is termed as stabilized platform.

In other words, stabilized platform is an object which is used to isolate payloads like sensors, cameras, telescopes, antennas etc. from the motion of vehicles, ships, aircrafts, spaceships etc. and even from some handheld and ground-mounted devices on which payload is mounted [2]. It can make the equipment which is mounted on the platform, to aim at and track object rapidly and exactly.

Although the requirements for stabilized platform vary widely depending on the application, they all have a common goal, which is to hold or control the line of sight of one object relative to another object irrespective of movements of vehicle on which payload is mounted. Also, in the stabilized platform systems, the basic requirements are to maintain stable operation even when there are changes in the system dynamics and to have very good disturbance rejection capability [3]. Three DOF gimbaled platform is one mechanism which can be used for stabilizing payload. Three-DOF gimbaled platform consists of four bodies namely – case, outer gimbal, inner gimbal and platform. To derive equations of motion of platform is main aim of this paper.

Differential equations of motion (DEOMs) for a particular system, gives response of that system over a period of time for various types of inputs. DEOMs are required to be derived in order to study motion of a system. Moreover, they are also very important in control system design. Various methods are documented in literature to derive DEOMs like – Newton's 2nd law of motion, D' Alembert's equation, Energy method, Virtual Work method, [4] etc.

The literature on DEOMs of three-DOF stabilized platform is limited to a few published articles. William Mendez et. al. [5] analyzed three-axis rotary platform for crystallographic study. Kinematics and dynamics of platform is developed by using Newton's 2nd law of rotational motion. Ravindra Singh et. al. [6] studied platform, having two DOF in azimuth and elevation, by considering platform rigid as well as flexible. Equations of motion are derived by using Newton's 2nd law of motion. F. Barnes [7] presented kinematics and dynamics of a three-DOF platform by applying Euler's simplified moment equations. S. Leghmizi and L. Sheng [8] developed kinematics model of three-DOF platform to study rates, acceleration of different components of a ship carried platform. Dynamics model of platform is proposed by S. Leghmizi et. al. [9] by using Euler's moment approach.

In the present study, a kinematic model of three-DOF gimbal platform is developed by assuming each member to be rigid for analyzing their rates and accelerations. Dynamics of platform is developed by using Lagrange's Equation which is energy approach.

System state equations are then obtained from DEOMs.

II. SYSTEM DESCRIPTION

As seen from Fig. 1, case is the outermost member which is fixed to the vehicle. Outer gimbal is mounted inside the case which can rotate only about one of its axes called as outer axis. Similarly, inner gimbal is positioned inside the outer gimbal in such a way that it can rotate only about inner axis. Moreover, platform that can spin about platform axis is located inside the inner gimbal. Thus, all members of gimbal arrangement have only one Degree of Freedom.

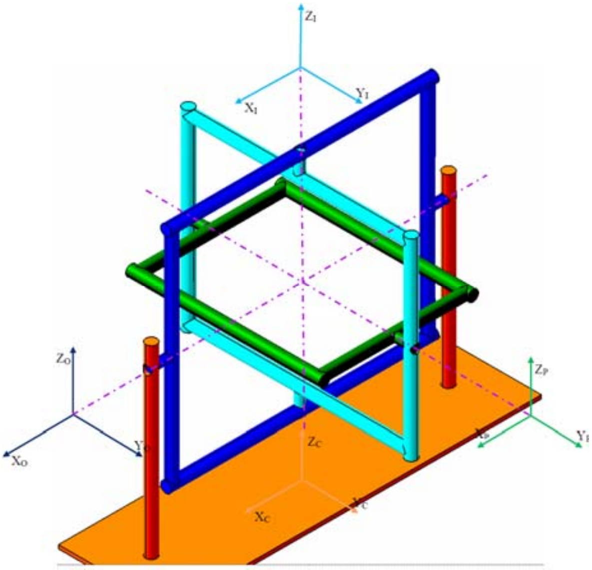

Fig. 1: Kinematic Model of gimbaled platform

An orthogonal right-handed co-ordinate system is defined, such that its origin coincides with the centre of mass, at each body of the gimbal arrangement. This co-ordinate system is called as body co-ordinate system. (X_P, Y_P, Z_P), (X_I, Y_I, Z_I), (X_O, Y_O, Z_O) and (X_C, Y_C, Z_C) are body co-ordinate systems at platform, inner gimbal, outer gimbal and case respectively.

Φ and $\dot{\Phi}$ are respectively the relative angle and relative angular rate between case and outer gimbal measured about X axis of outer gimbal (X_O). Similarly, ψ, $\dot{\Psi}$ are relative angle and relative angular rate between outer gimbal and inner gimbal measured about Z axis of inner gimbal (Z_I) and θ, $\dot{\theta}$ are relative angle and relative angular rate between inner gimbal and platform measured about Y axis of platform (Y_P).

A. Assumptions

To simplify analysis, assumptions are made which are as follows –

1) All members in the arrangement are rigid.
2) The body axes are chosen such that, they are the principal axes at the centre of mass of each member.
3) Initially, all the gimbals are stationary and are aligned orthogonal to each other.
4) C.G. of each gimbal is assumed to be present on axis of rotation thus change in potential energy is negligible. Therefore, potential energy is neglected.
5) System is directly fixed on the vehicle. Also, X–Z plane of system and that of vehicle are perfectly aligned. Thus, motion of vehicle is directly transferred to case of the system.

III. KINEMATICS OF PLATFORM

This INCLUDES developing the mathematical equations for angular rates and angular accelerations of different members of platform.

As assumed, system is directly fixed on the vehicle, the angular rate of vehicle and that of case are same. Hence,

$$\omega_c = \omega_{vehicle} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \qquad (1)$$

Where,

p, q, r are the roll, pitch and yaw rates of the vehicle. ω_c is the angular rate of case and $\omega_{vehicle}$ is the angular rate of vehicle.

Outer gimbal is rotating at relative angular rate of $\dot{\Phi}$ about X_O axis w.r.t case. So, the outer gimbal rate ω_o can be written as –

$$\omega_o^C = \omega_c^C + \omega_{o/c} \qquad (2)$$

Where,

ω_o^C is the absolute angular rate of outer gimbal in case co-ordinate system

ω_c^C is the angular rate of case in case co-ordinate system

$\omega_{o/c}$ is the relative angular rate between outer gimbal and case

Therefore,

$$\omega_o^C = \begin{bmatrix} p \\ q \\ r \end{bmatrix} + \begin{bmatrix} \dot{\Phi} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} p + \dot{\Phi} \\ q \\ r \end{bmatrix} \qquad (3)$$

To obtain the angular rate of the outer gimbal in its own co–ordinate system, ω_o^O, corresponds to the rotation of an angle □ w.r.t the case about X_O axis, the co–ordinate transformation is shown as below

$$\omega_o^O = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & \sin\varphi \\ 0 & -\sin\varphi & \cos\varphi \end{bmatrix} \times \begin{bmatrix} p + \dot{\Phi} \\ q \\ r \end{bmatrix}$$

$$\omega_o^O = \begin{bmatrix} p + \dot{\Phi} \\ q\cos\varphi + r\sin\varphi \\ -q\sin\varphi + r\cos\varphi \end{bmatrix} \qquad (4)$$

Following similar procedure rates for subsequent members can be found out as given below –

$\omega_I^I =$

$$\begin{bmatrix} (p+\dot{\Phi})\cos\varphi + (q\cos\varphi + r\sin\varphi)\sin\psi \\ -(p+\dot{\Phi})\sin\psi + (q\cos\varphi + r\sin\varphi)\cos\psi \\ \dot{\psi} - q\sin\varphi + r\cos\varphi \end{bmatrix} \quad (5)$$

$\omega_p^P =$

$$\begin{bmatrix} [(p+\dot{\phi})\cos\psi + (q\cos\varphi + r\sin\varphi)\sin\psi]\cos\theta - [\dot{\psi} - q\sin\varphi + r\cos\varphi]\sin\theta \\ \dot{\theta} - (p+\dot{\phi})\sin\psi + (q\cos\varphi + r\sin\varphi)\cos\psi \\ [(p+\dot{\phi})\cos\psi + (q\cos\varphi + r\sin\varphi)\sin\psi]\sin\theta + [\dot{\psi} - q\sin\varphi + r\cos\varphi]\cos\theta \end{bmatrix}$$

(6)

Angular acceleration of each member is obtained by evaluating the time derivative of the respective angular rates. It should be noted that p, q, r, □, □, Ψ are the functions time.

Hence, angular acceleration of outer gimbal is

$$\dot{\omega}_O^O = \frac{d(\omega_O^O)}{dt} = \begin{bmatrix} \dot{p} + \ddot{\phi} \\ (\dot{r} - q\dot{\varphi})\sin\varphi + (\dot{q} + r\dot{\varphi})\cos\varphi \\ (\dot{r} - q\dot{\phi})\cos\varphi - (\dot{q} + r\dot{\phi})\sin\varphi \end{bmatrix} \quad (7)$$

In similar manner, angular acceleration for inner gimbal and platform was calculated.

IV. DYNAMICS OF PLATFORM

DEOMs of platform were obtained by using Lagrange's Equation [10], given by (8).

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \quad (8)$$

Where,

L – Lagrangian function = T – V

T – Kinetic Energy of the system

V –Potential energy of the system

Q_i –Net external torque acting on the body.

q_i – eneralized co–ordinates, in this case □, □, Ψ.

As, change in potential energy is assumed to be negligible, Lagrangian function reduces to L = T. Therefore, Lagrange's Equation becomes –

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = Q_i \quad (9)$$

Kinetic energy of a body rotating about an axis is given by eq. 10

$$T = \frac{1}{2}\omega^T I \,\omega \quad (10)$$

Where,

I – Moment of inertial (3 x 3) matrix of the body.

ω – Angular rate vector of the body.

Thus, total kinetic energy of gimbaled platform is summation of kinetic energy of each member. Therefore, the total kinetic energy $T = T_P + T_I + T_O$

Where,

T_P, T_I, T_O – kinetic energy of platform, inner gimbal and outer gimbal respectively and calculated using eq. (10).

Applying Lagrange's equation given by (9) to each member of gimbal platform gives three-DEOMs.

As platform is having □ as a degree of freedom substituting generalized co–ordinates q_i as □ in Lagrange's Equation, which gives –

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} = Q_\theta \quad (11)$$

Calculating each term separately from eq. (11), grouping the terms and substituting values for angular rates and accelerations wherever necessary, as given by eq. (12) –

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = I_{Py}\left[\ddot{\theta} - (\dot{p}+\ddot{\Phi})\cos\psi\,\dot{\psi} - (\dot{p}+\ddot{\phi})\sin\psi - (q\cos\varphi + r\sin\varphi)\sin\psi\,\dot{\psi} + ((\dot{r} - q\dot{\varphi})\sin\varphi + (\dot{q} + r\dot{\varphi})\cos\varphi)\cos\psi\right]$$

$$\frac{\partial T}{\partial \theta} = -I_{Px}\omega_{px}\omega_{pz} + I_{Pz}\omega_{pz}\omega_{px} = (I_{Pz} - I_{Px})\omega_{px}\omega_{pz} \quad (12)$$

Thus, substituting values of $\frac{\partial T}{\partial \theta}$ and $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right)$ in the eq. (11), first DEOM is obtained as in (13) below

$Q_\theta =$

$$I_{Py}\left[\ddot{\theta} - (\dot{p}+\ddot{\Phi})\cos\psi\,\dot{\psi} - (\dot{p}+\ddot{\phi})\sin\psi - (q\cos\varphi + r\sin\varphi)\sin\psi\,\dot{\psi} + ((\dot{r} - q\dot{\varphi})\sin\varphi + (\dot{q} + r\dot{\varphi})\cos\varphi)\cos\psi\right] - (I_{Pz} - I_{Px})\omega_{px}\omega_{pz} \quad (13)$$

Rearranging the terms in above eq.

$$Q_\theta = I_{Py}(\ddot{\theta} - \ddot{\phi}\sin\Psi) + Q_{Py} \quad (14)$$

Where,

$Q_{Py} = (I_{Px} - I_{Pz})\omega_{Px}\omega_{Pz} - I_{Py}\beta$,

$\beta = (\dot{\phi}\sin\Psi + \omega_{Ix}\dot{\Psi} - \dot{\omega}_{Oy}\cos\Psi)$

By following similar procedure, other two-DEOMs are obtained as given by equations 15 and 16 below –

$$Q_\Psi = \ddot{\Psi}(I_{Iz} + I_{Pz}\cos^2\theta + I_{Px}\sin^2\theta) + \ddot{\phi}\sin\theta\cos\theta\cos\Psi(I_{Px} - I_{Pz}) + Q_{Iz} \quad (15)$$

$$Q_\varphi = \ddot{\phi}\begin{pmatrix} I_{Ox} + (I_{Ix} + I_{Px}\cos^2\theta + I_{Pz}\sin^2\theta)\cos\Psi^2 \\ + (I_{Iy} + I_{Py})\sin\Psi^2 \end{pmatrix} + \ddot{\Psi}\sin\theta\cos\theta\cos\Psi(I_{Pz} - I_{Px}) - \ddot{\theta}(I_{Py}\sin\Psi) + Q_{Ox} \quad (16)$$

Where,

$Q_{Iz} = I_{Iz}\dot{\omega}_{Oz} + (I_{Iy} - I_{Ix})\omega_{Iy}\omega_{Ix} + MPZ\cos\theta + MPX\sin\theta$

$MPZ = I_{Pz}(\alpha\sin\theta + \dot{\theta}\omega_{Px} + \dot{\omega}_{Oz}\cos\theta) + (I_{Py} - I_{Px})\omega_{Py}\omega_{Px}$

$MPX = I_{Px}(-\alpha\cos\theta + \dot{\theta}\omega_{Pz} + \dot{\omega}_{Oz}\sin\theta) + (I_{Py} - I_{Pz})\omega_{Py}\omega_{Pz}$

$\alpha = (\dot{\phi}\cos\Psi + \omega_{Iy}\dot{\Psi} + \dot{\omega}_{Oy}\sin\Psi)$

$Q_{Ox} = I_{Ox}\ddot{\phi} + (I_{Oz} - I_{Oy})\omega_{Oz}\omega_{Oy} + MIX\cos\Psi + MIY\sin\Psi$

$MIX = I_{Ix}\alpha + (I_{Iz} - I_{Iy})\omega_{Iz}\omega_{Iy} - MPX\cos\theta + MPZ\sin\theta$

$MIY = (I_{Iy} + I_{Py})\beta + (I_{Iz} - I_{Ix})\omega_{Iz}\omega_{Ix} + (I_{Pz} - I_{Px})\omega_{Pz}\omega_{Px}$

From the differential equations of motion (14, 15, 16) three simultaneous equations describing motion of gimbals are obtained as below –

$$A_1\ddot{\theta} + B_1\ddot{\varphi} = C_1$$
$$A_2\ddot{\Psi} + B_2\ddot{\varphi} = C_2 \qquad (17)$$
$$A_3\ddot{\varphi} + B_2\ddot{\Psi} + B_1\ddot{\theta} = C_3$$

Where,

$A_1 = I_{Py}$, $B_1 = -I_{Py}\sin\Psi$, $C_1 = (Q_\theta - Q_{Py})$,

$A_2 = (I_{Iz} + I_{Pz}\cos^2\theta + I_{Px}\sin^2\theta)$,

$B_2 = \sin\theta\cos\theta\cos\Psi\,(I_{Pz} - I_{Px})$, $C_2 = (Q_\Psi - Q_{Iz})$

$A_3 =$
$(I_{Ux} + (I_{Ix} + I_{Px}\cos^2\theta + I_{Pz}\sin^2\theta)\cos^2\Psi + (I_{Iy} + I_{Py})\sin^2\Psi)$

$C_3 = (Q_\varphi - Q_{Ox})$

The simultaneous eq. (17) are rearranged to obtain system state equations giving expression for $\ddot{\theta}, \ddot{\varphi}, and\ \ddot{\Psi}$ are obtained as below –

$$\ddot{\varphi} = \frac{A_1B_2C_2 + A_2B_1C_1 - A_1A_2C_3}{A_1B_2^2 - A_1A_2A_3 + A_2B_1^2}$$

$$\ddot{\theta} = \frac{C_1}{A_1} - \frac{B_1}{A_1} \times \left(\frac{A_1B_2C_2 + A_2B_1C_1 - A_1A_2C_3}{A_1B_2^2 - A_1A_2A_3 + A_2B_1^2}\right) \qquad (18)$$

$$\ddot{\Psi} = \frac{C_2}{A_2} - \frac{B_2}{A_2} \times \left(\frac{A_1B_2C_2 + A_2B_1C_1 - A_1A_2C_3}{A_1B_2^2 - A_1A_2A_3 + A_2B_1^2}\right)$$

System state equations given in eq. (18) can be solved to get values of angular displacement, angular rates and accelerations of different members in the gimballed platform.

V. DISCUSSIONS

Thus, the differential equations of motion for three-DOF platform are derived using Lagrange approach. System state equations are also obtained subsequently.

Here we observe that, developing DEOMs using Lagrange approach is much easier than that using Newton's 2nd law or by using Euler's approach. This is mainly because; Newton's 2nd Law and Euler's approach deal with vector quantities like forces, moments etc. On the other hand, Lagrange approach is based on scalar quantities like kinetic and potential energy. Scalar quantities are easier to handle than vector quantities.

Moreover, Lagrange approach automatically takes care of reaction terms, which are required to be calculated while using vector methods. Thus, calculation of reaction terms can be avoided by using Lagrange Approach to obtain differential equations of motion.

We also observe that, these equations are highly coupled as well as non–linear and thus needs to be simplified for their simulation by using some software like – MATLAB. Some appropriate assumptions are required to be employed to simplify these complicated, coupled and non–linear equations.

VI. CONCLUSIONS

In this paper, using Lagrange approach, a study of differential equations of motion for 3 DOF gimbaled stabilized platform is proposed. Kinematics model is developed to get expressions for angular displacement, rate and accelerations of different members of gimbaled platform. Differential equations of motion, obtained in dynamics model, are used to study behavior of system and to design control system as well.

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