Neutrino Oscillation

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Outline

1. **Recap of physics**
   - Neutrino oscillation
2. **Methodology and Results**
   - Data description
   - Preprocessing and models
   - Other models
3. **Conclusion and discussion**
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4. **Backup**
   - Supplementary materials

1.1 Neutrino Oscillation

- Mass Eigenstates:
  - The eigenstates to the free Hamiltonian are mass eigenstates.
- Flavor Eigenstates:
  - The eigenstates to the interaction Hamiltonian are flavor eigenstates.
- Superposition:
  - The standard model neutrinos are theorized to be a superposition of mass eigenstates.

$$|\nu_i\rangle = \sum_{\alpha} U^{*}_{\alpha i} \, |\nu_\alpha\rangle \, ,$$

$$|\nu_\alpha\rangle = \sum_{i} U_{\alpha i} \, |\nu_i\rangle \, ,$$

- Time evolution:

$$|\, \nu_j(t) \,\rangle = e^{-i \left(E_j t - \vec{p}_j \cdot \vec{x} \right)} \, |\, \nu_j(0) \,\rangle$$

A. Donini, M.B. Gavela, P. Hernández, S. Rigolin, Neutrino mixing and CP-violation, Nuclear Physics B. 574 (2000) 23–42.

- The **P**ontecorvo–**M**aki–**N**akagawa–**S**akata (PMNS) Matrix

$$
\begin{aligned}
U &= \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}
\end{aligned}
$$

- The **unitary transformation** between the flavor and mass eigenstates can be determined with 4 parameters: θ_{12}, θ_{13}, θ_{23}, and δ_{CP}.

- General form of the probability:

$$P_{\alpha\to\beta} = \left|\langle \nu_\beta \mid \nu_\alpha(L)\rangle\right|^2 = \left|\sum_j U^*_{\alpha j} U_{\beta j}\, e^{-i\frac{m_j^2 L}{2E}}\right|^2$$

$$P_{\alpha\to\beta} = \delta_{\alpha\beta} - 4\sum_{j>k} \mathcal{R}_e\left\{U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k}\right\} \sin^2\left(\frac{\Delta_{jk} m^2 L}{4E}\right)$$
$$+ 2\sum_{j>k} \mathcal{I}_m\left\{U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k}\right\} \sin\left(\frac{\Delta_{jk} m^2 L}{2E}\right)$$

- The main probability formulas we used:

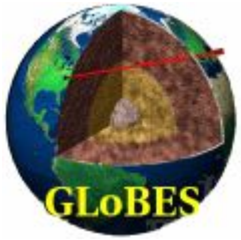
$$P_{\mu e} = \underbrace{4s_{13}^2 s_{23}^2 \frac{\sin^2(A-1)\Delta}{(A-1)^2}}_{\mathcal{O}_o} + \underbrace{\alpha^2 \cos^2\theta_{23} \sin^2 2\theta_{12} \frac{\sin^2 A\Delta}{A^2}}_{\mathcal{O}_2} + \underbrace{\alpha s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos(\Delta + \delta_{cp}) \frac{\sin(A-1)\Delta}{(A-1)} \frac{\sin A\Delta}{A}}_{\mathcal{O}_1}$$

$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2\Delta + \mathcal{O}(\alpha, s_{13})$$

N. Nath, M. Ghosh, and S. Goswami, *The Physics of Antineutrinos in DUNE and Determination of Octant and* δ_{CP}, Nuclear Physics B **913**, 381 (2016).

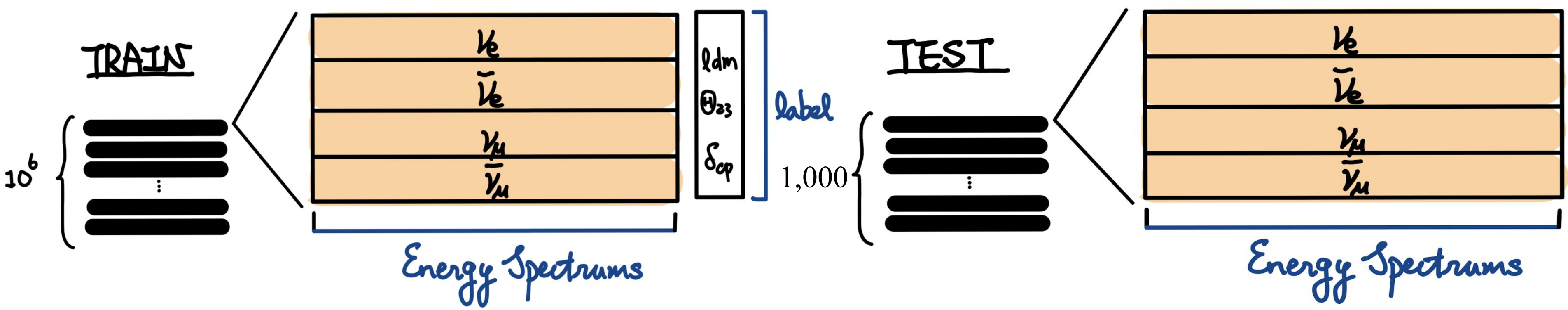
2.1 Data description

Data source: GLoBES simulation data.



Event rate
Neutrino flux
Cross section
Detector smearing
Oscillation probability

$$R(\vec{\mathbf{x}}) = \Phi(E_\nu) \times \sigma(E_\nu, \vec{\mathbf{x}}) \times \epsilon(\vec{\mathbf{x}}) \times P(\nu_A \rightarrow \nu_B)$$



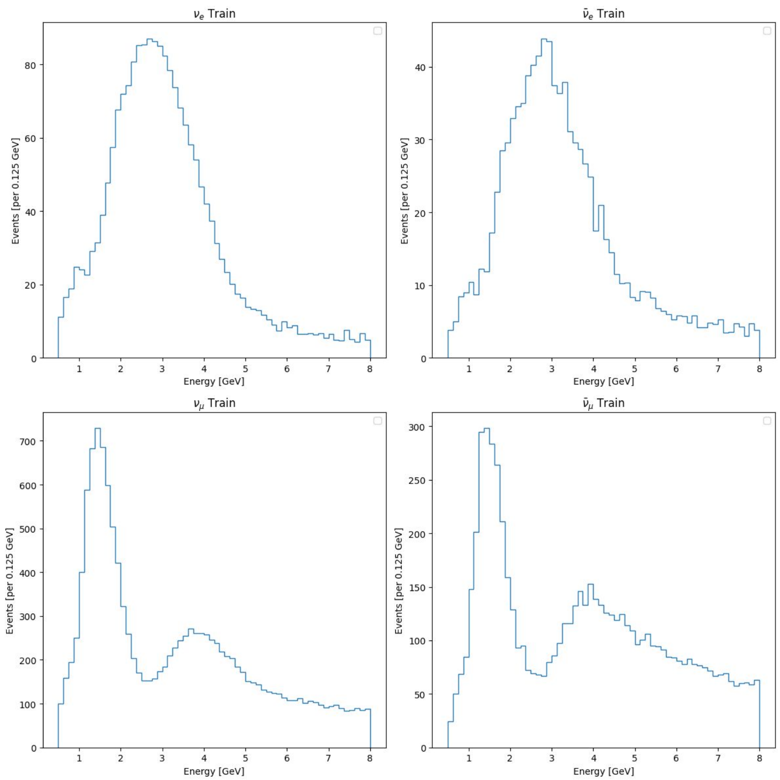
Callum Wilkinson, Neutrino oscillation physics and cross sections

In this project we have 4 channels (2 appearances and 2 disappearances), and 3 labels (ldm, θ_{23}, δ_{CP}).

- Four channels

$\nu_\mu \rightarrow \nu_e$
$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$

Appearance channels



$\nu_\mu \rightarrow \nu_\mu$
$\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$

Disappearance channels

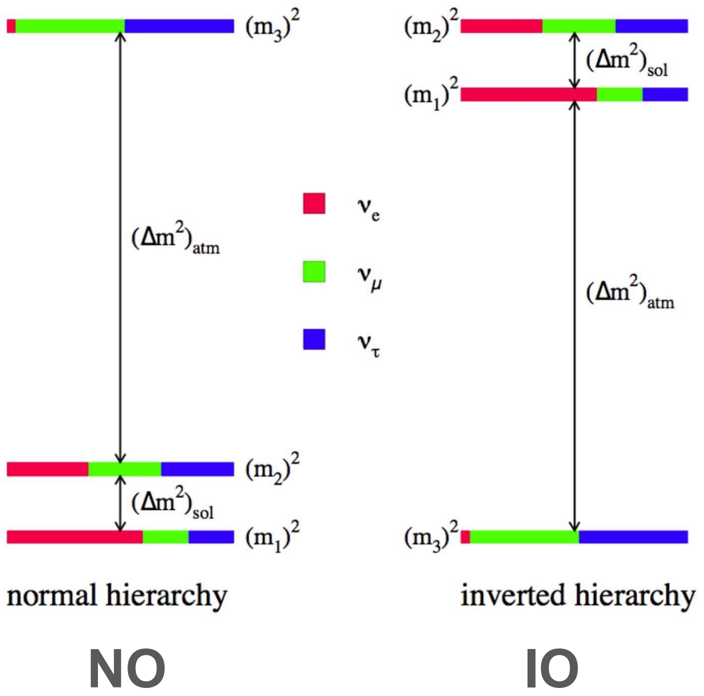
- Mixing angle θ_{23}, and CP phase δ_{CP}

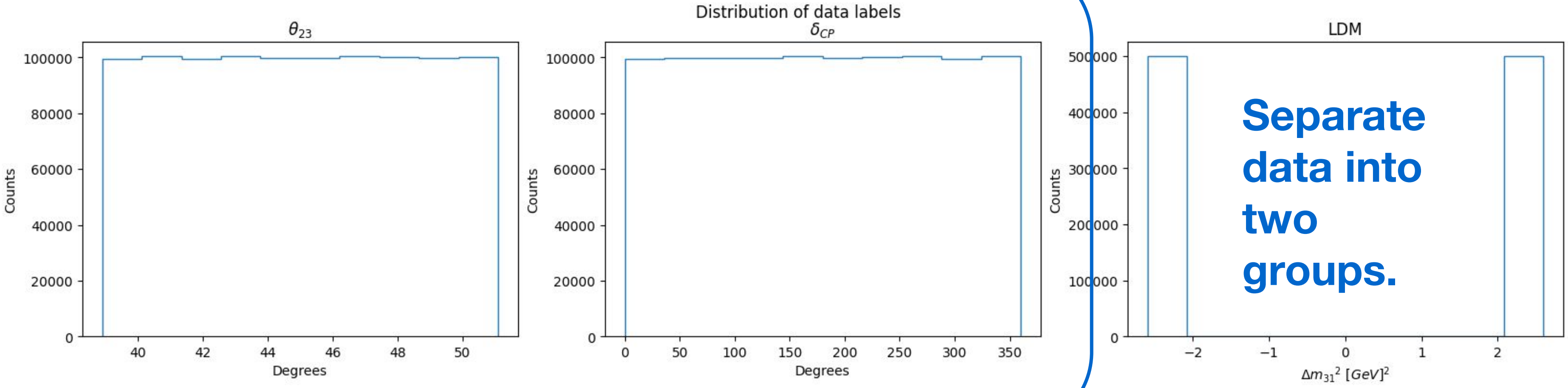
θ_{23} ranges from approximately 39 to 51 degrees, δ_{CP} ranges from 0 to 360 degrees.

- Mass hierarchy ldm

We are only able to know the difference and its orderings.

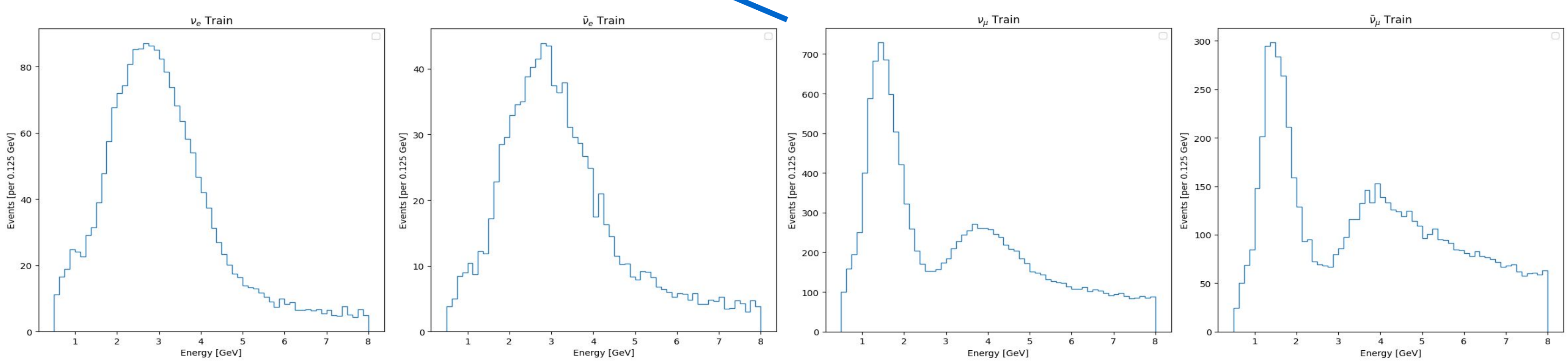
ldm only has two significant peaks, which tells us that we can split our train data into NO and IO two parts.





Separate data into two groups.

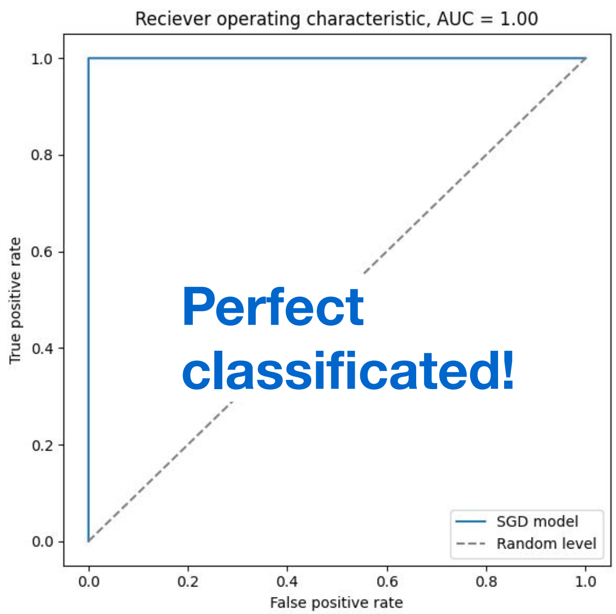
Tasked to find the correlation between **2 ANGLES** and **4 SPECTRUMS**.

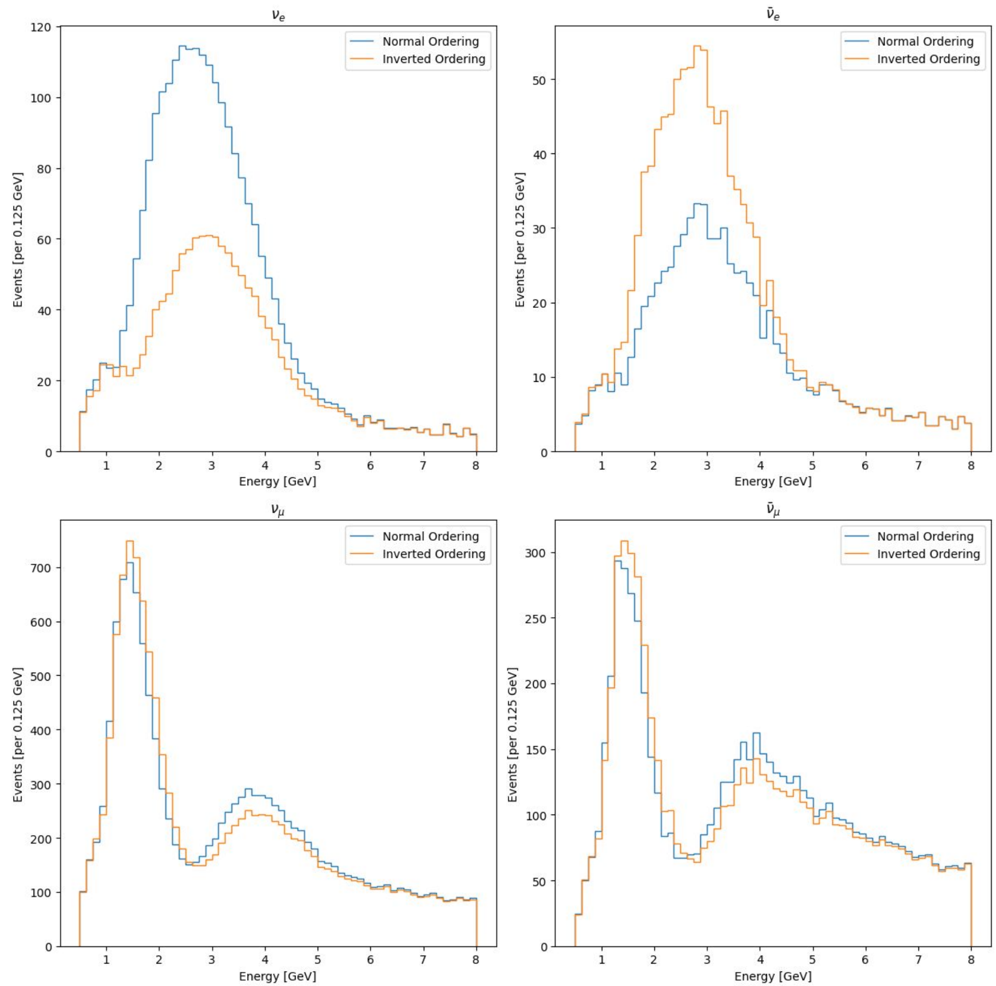


2.2 Preprocessing and models

I. LDM classification

- Distinct difference in these distributions.
- SVM with SGD
  → binary classification.





⬆️ averaged spectrum of the NO and IO

II. DCNN regression $\{\theta_{23}\}$

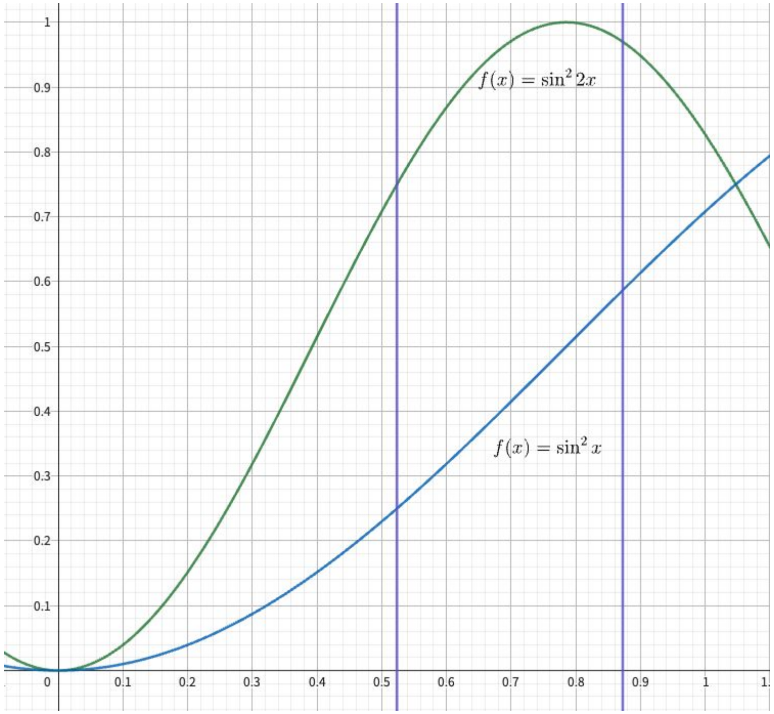
Theoretically, θ_{23} in (with non-trivial leading order):

- **appearance channel:** $\sim \sin^2 \theta_{23}$
- **disappearance channel:** $\sim \sin^2 2\theta_{23}$

$$P_{\mu e} = \underbrace{4s_{13}^2 s_{23}^2 \frac{\sin^2(A-1)\Delta}{(A-1)^2}}_{\mathcal{O}_o} + \underbrace{\alpha^2 \cos^2\theta_{23}\sin^2 2\theta_{12}\frac{\sin^2 A\Delta}{A^2}}_{\mathcal{O}_2}$$

$$+ \underbrace{\alpha s_{13}\sin 2\theta_{12}\sin 2\theta_{23}\cos(\Delta+\delta_{cp})\frac{\sin(A-1)\Delta}{(A-1)}\frac{\sin A\Delta}{A}}_{\mathcal{O}_1}$$

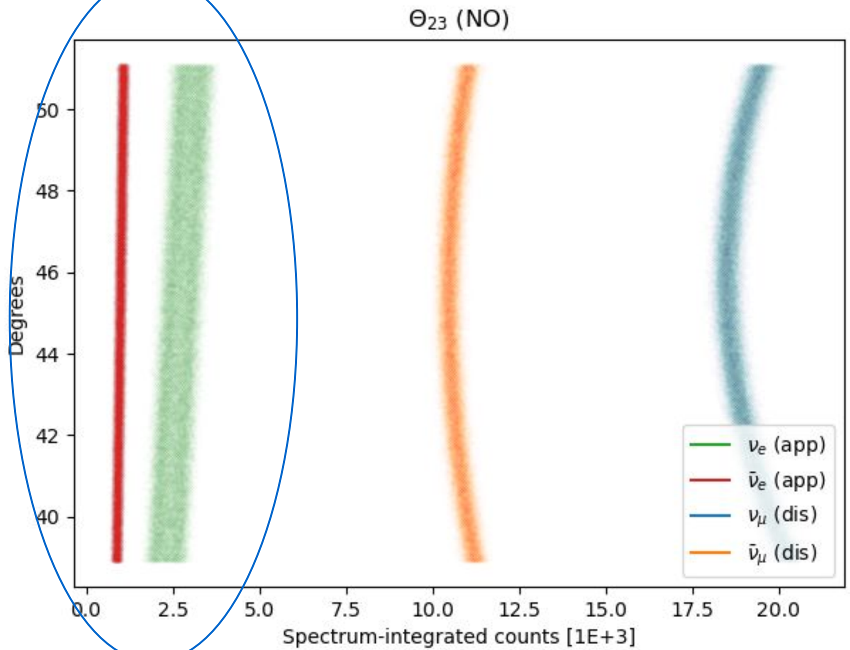
$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23}\sin^2\Delta + \mathcal{O}(\alpha, s_{13})$$

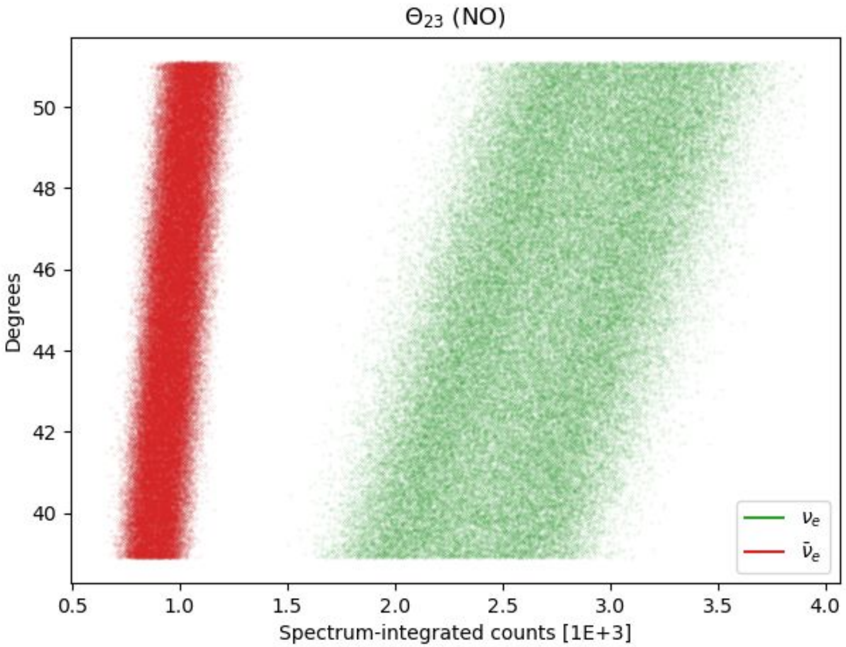


In this interval (30 ~ 50 degrees), the probability can be determined, due to the monotonicity of the function (for appearance channels).

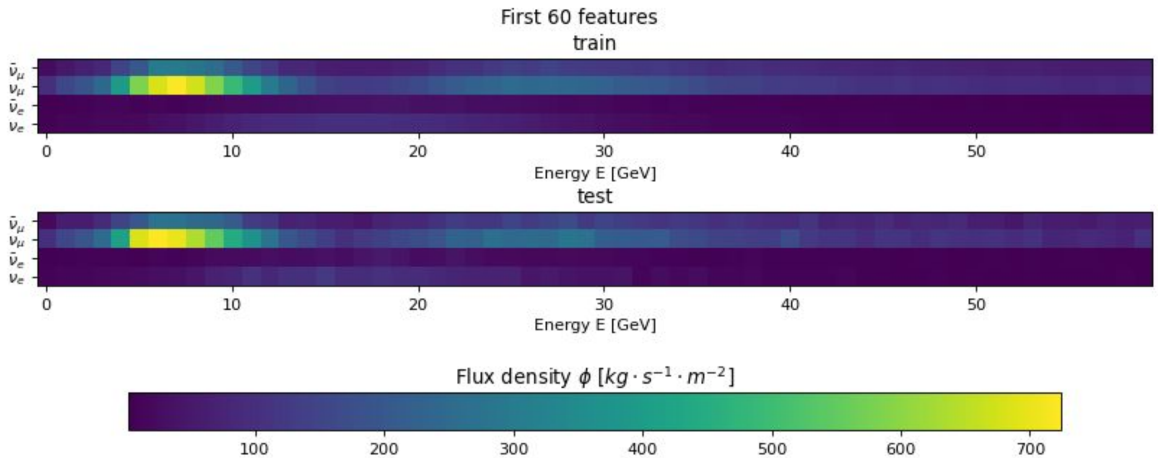
N. Nath, M. Ghosh, and S. Goswami, *The Physics of Antineutrinos in DUNE and Determination of Octant and* δ_{CP}, Nuclear Physics B **913**, 381 (2016).

Expect good predictability in θ_{23} !





No degeneracy in the appearance channels.



IN: MinMax Normalized CNN images +label

→ θ_{23}

OUT: MinMax Transpose

Loss: huber

DCNN

[Conv2D(64, 4) +BatchNorm +LeakyReLU(0.2)]*2 +AvgPooling2D {all 2D with same pad}

→ [Conv2D(32, 4) +BatchNorm +LeakyReLU(0.2)]*2 +AvgPooling2D {all 2D with same pad}

→ [Conv2D(16, 4) +BatchNorm +LeakyReLU(0.2)]*2 +AvgPooling2D {all 2D with same pad}

→ Flatten() +Dropout(0.2)

→ dense(64) +dense(16) +dense(8) +dense(1)

III-A. CNN regression $\{\delta_{CP}\}$

Theoretically, δ_{CP} relates to:

- appearance:

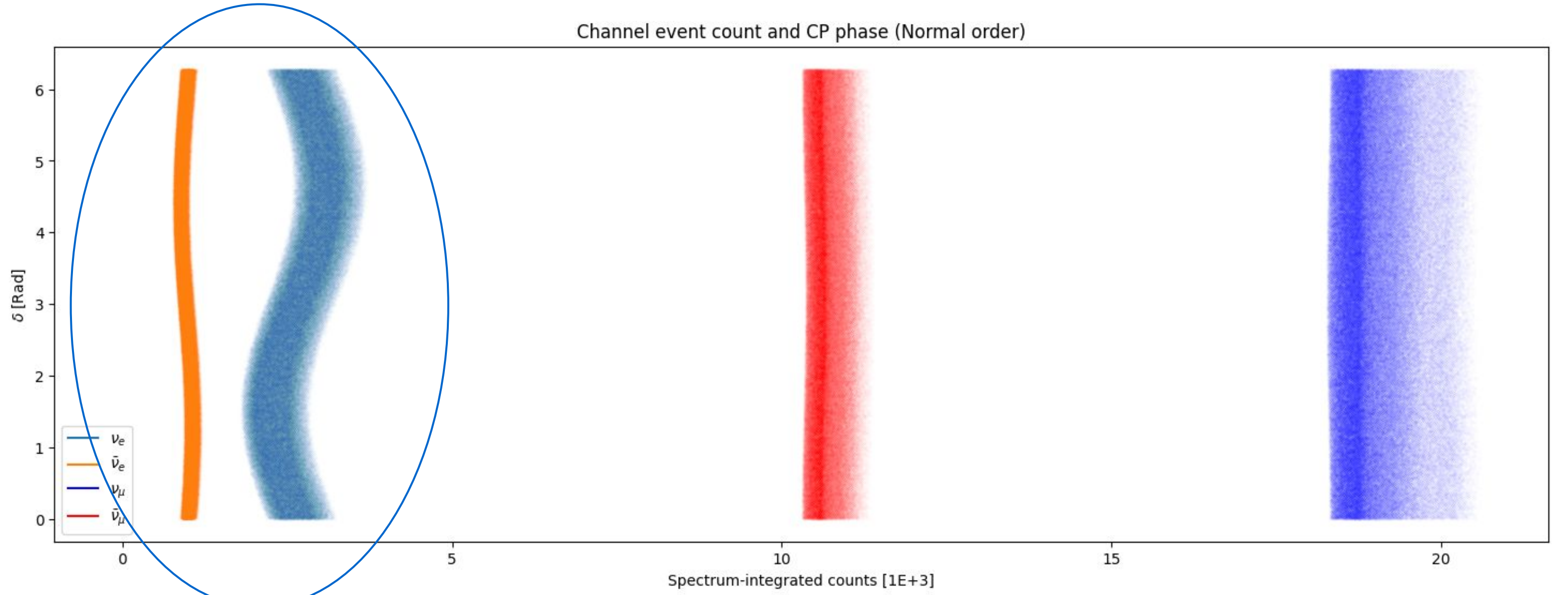
  $\sim cos(\Delta+\delta_{CP}) \sim \boldsymbol{sin\ \delta_{CP}}$

  In the vacuum oscillation maxima, Δ~90°

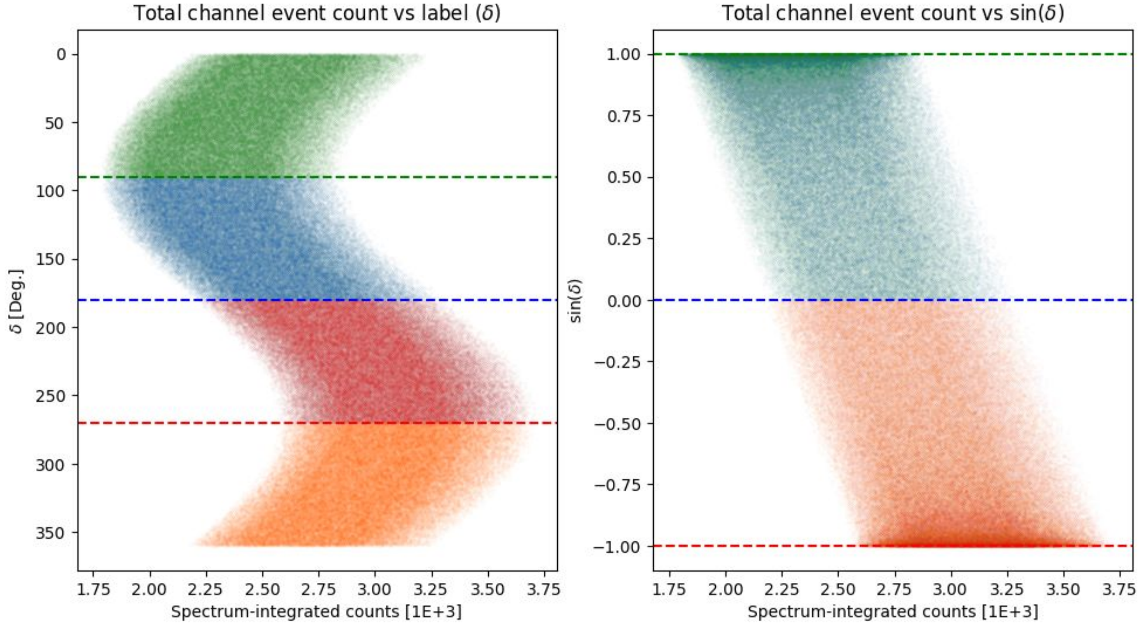
$$P_{\mu e} = \underbrace{4s_{13}^2 s_{23}^2 \frac{\sin^2(A-1)\Delta}{(A-1)^2}}_{\mathcal{O}_o} + \underbrace{\alpha^2 \cos^2\theta_{23}\sin^2 2\theta_{12}\frac{\sin^2 A\Delta}{A^2}}_{\mathcal{O}_2}$$
$$\underbrace{+\alpha s_{13}\sin 2\theta_{12}\sin 2\theta_{23}\cos(\Delta+\delta_{cp})\frac{\sin(A-1)\Delta}{(A-1)}\frac{\sin A\Delta}{A}}_{\mathcal{O}_1}$$

- disappearance: **X**

$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23}\sin^2\Delta + \mathcal{O}(\alpha, s_{13})$$



N. Nath, M. Ghosh, and S. Goswami, *The Physics of Antineutrinos in DUNE and Determination of Octant and* δ_{CP}, Nuclear Physics B **913**, 381 (2016).



IN: MinMax Normalized CNN images

$\rightarrow$ $\sin \delta_{CP}$

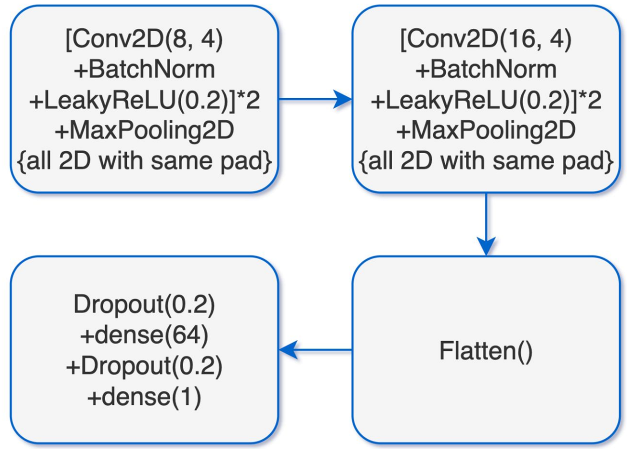
OUT: sine values

Reduce the degeneracy!

- Use sine and cosine with 2 transpose arc functions.
- Use sine and cosine with tangent and one transpose arc function.
- **Use one sine and one arc; then, do the classification.**

Reg

Loss: huber

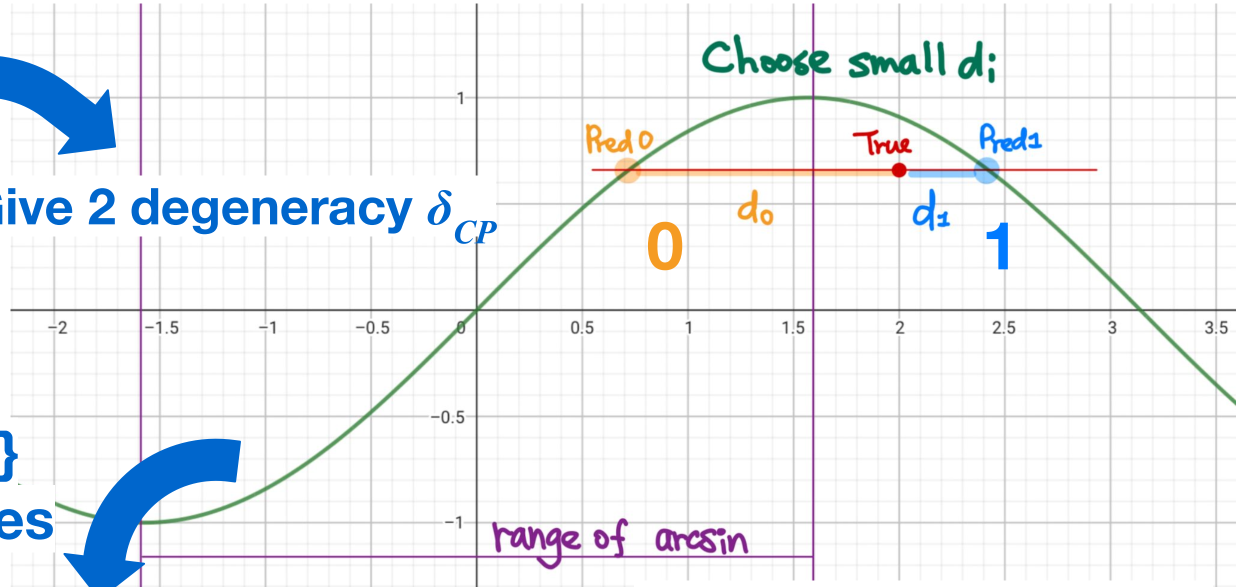


III-B. CNN classification $\{\delta_{CP}\}$

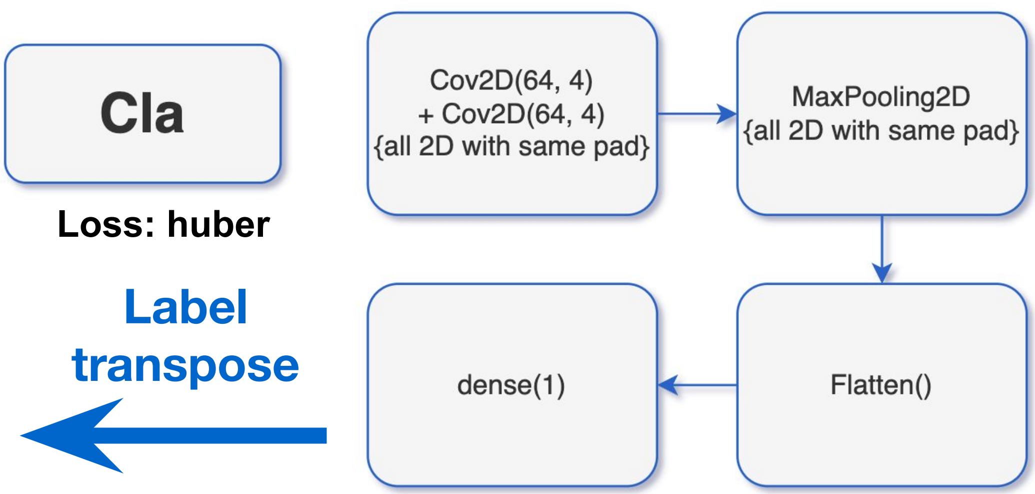
TRAIN procedure

$sin^{-1}(sin\ \delta_{cp})$

Give 2 degeneracy δ_{CP}

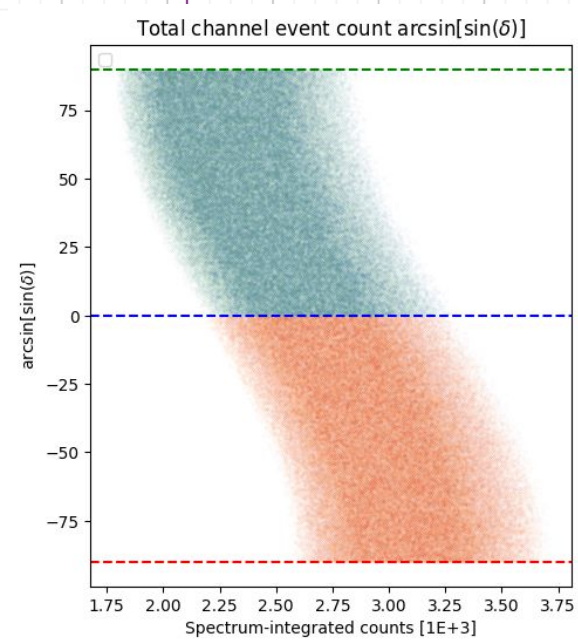


Label array with {0,1}
+MinMax CNN images



Label transpose

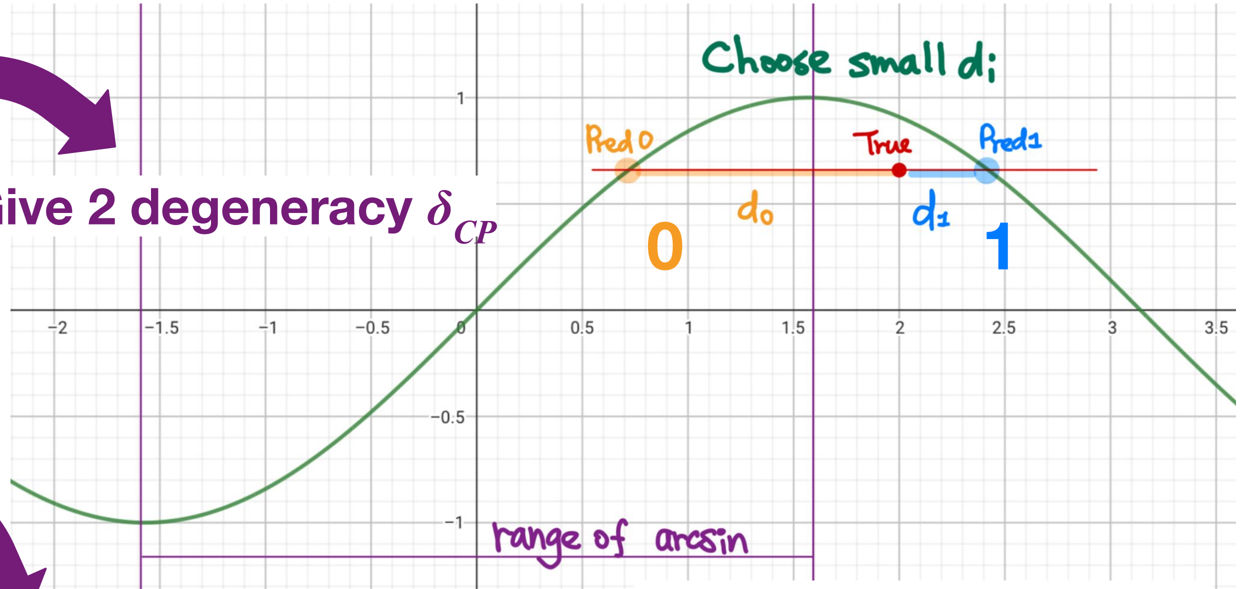
δ_{CP}



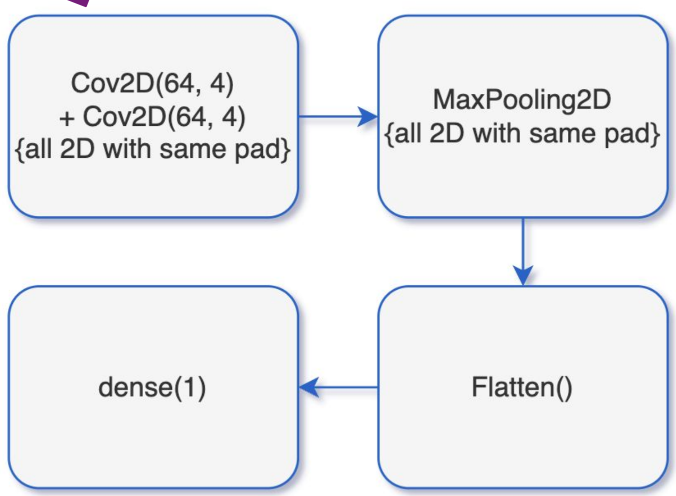
TEST procedure

$\sin^{-1}(\sin \delta_{cp})$

Give 2 degeneracy δ_{CP}



MinMax Normalized CNN images

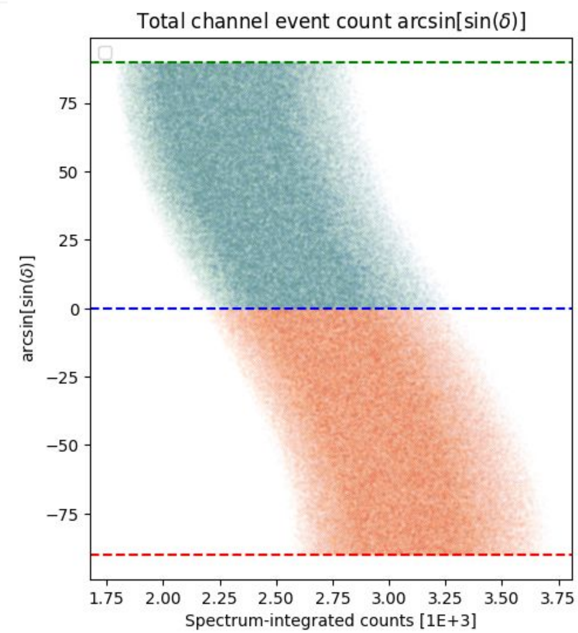


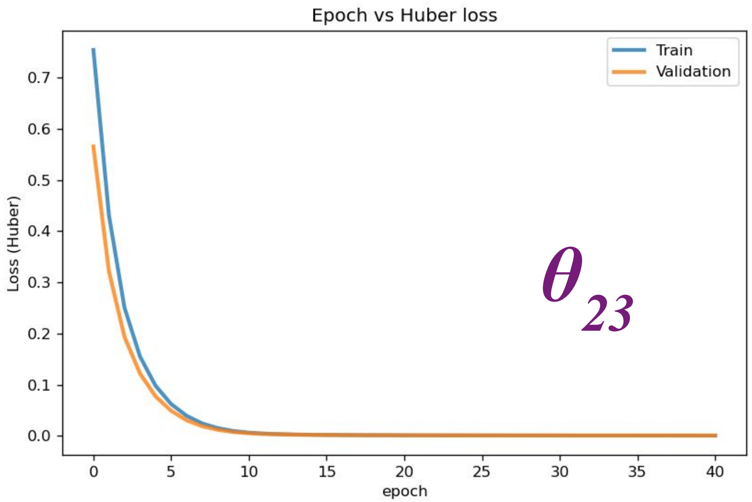
Cla

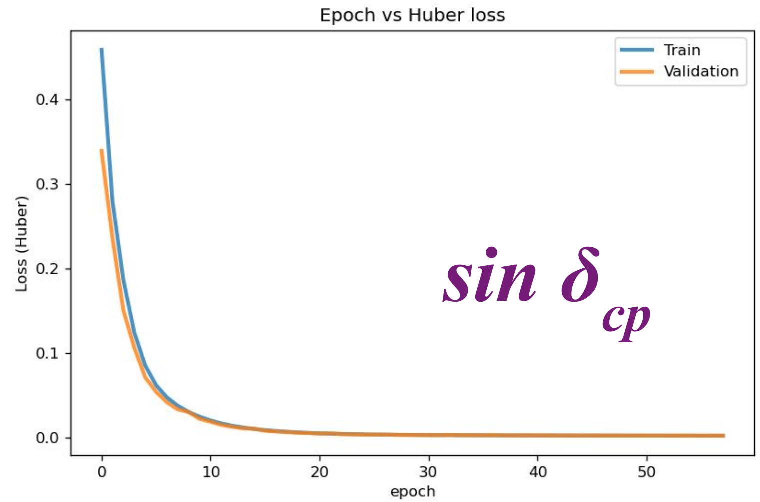
Loss: huber

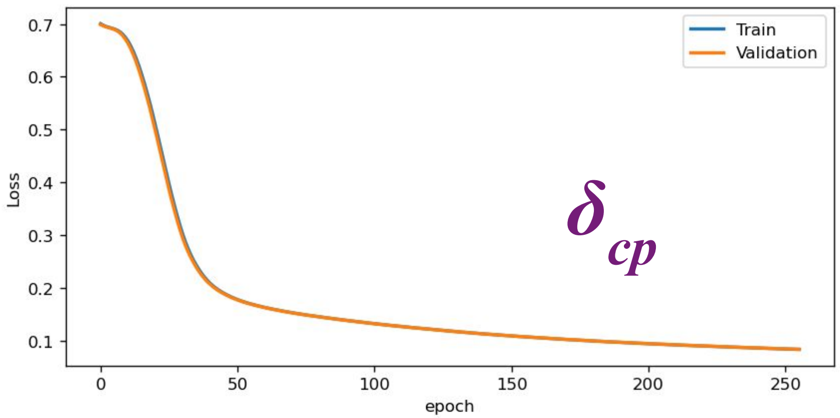
Label transpose

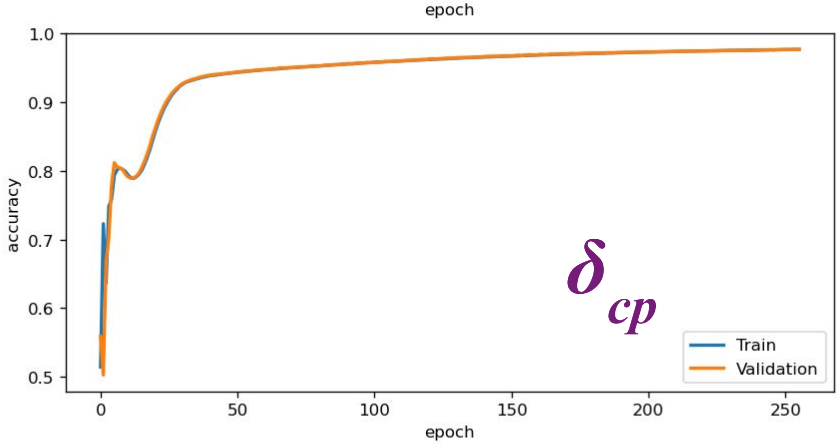
δ_{CP}

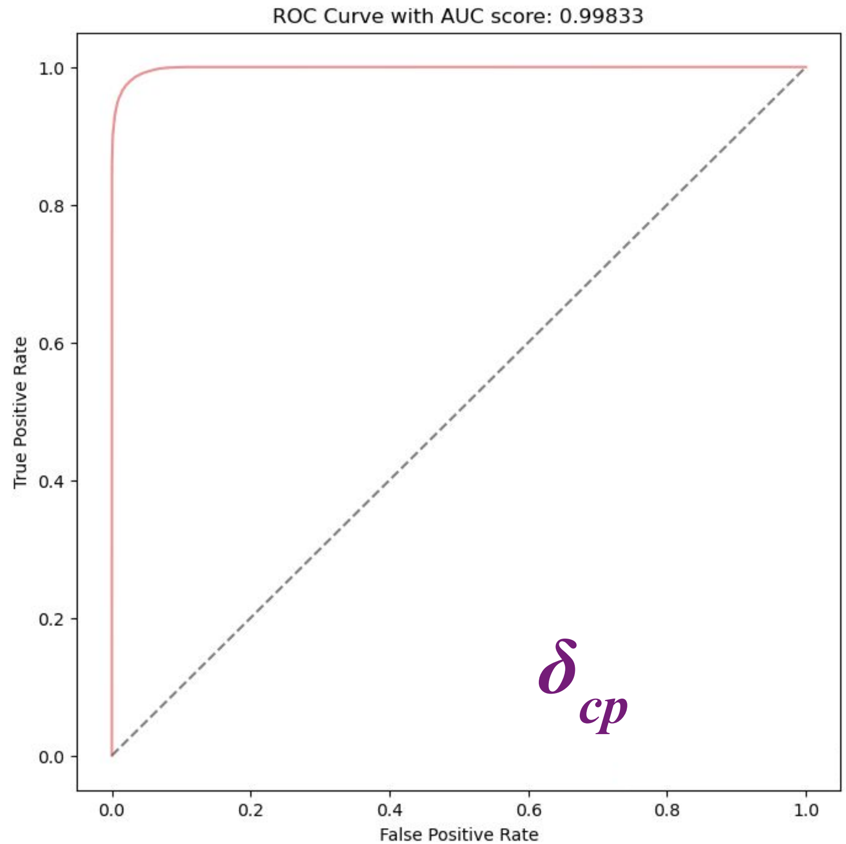


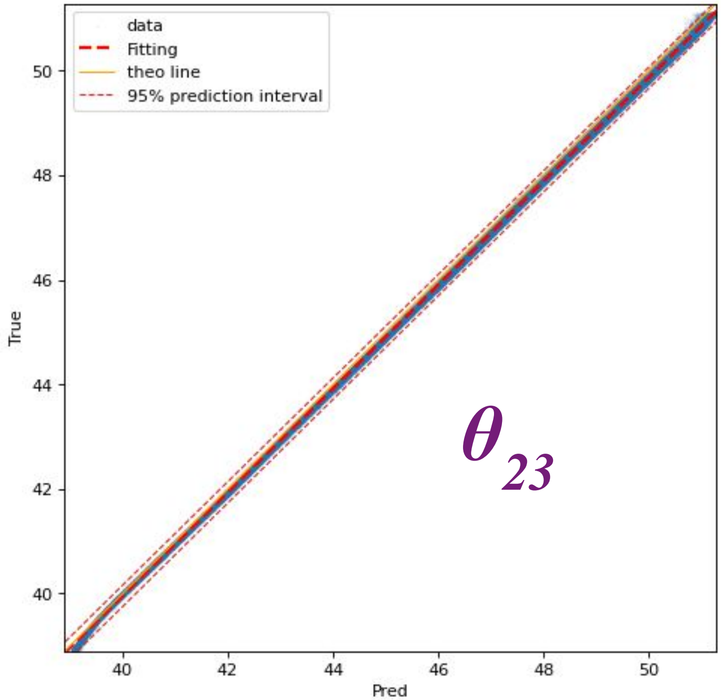

Epoch vs Huber loss
Train
Validation
Loss (Huber)
epoch
θ23
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
5
10
15
20
25
30
35
40

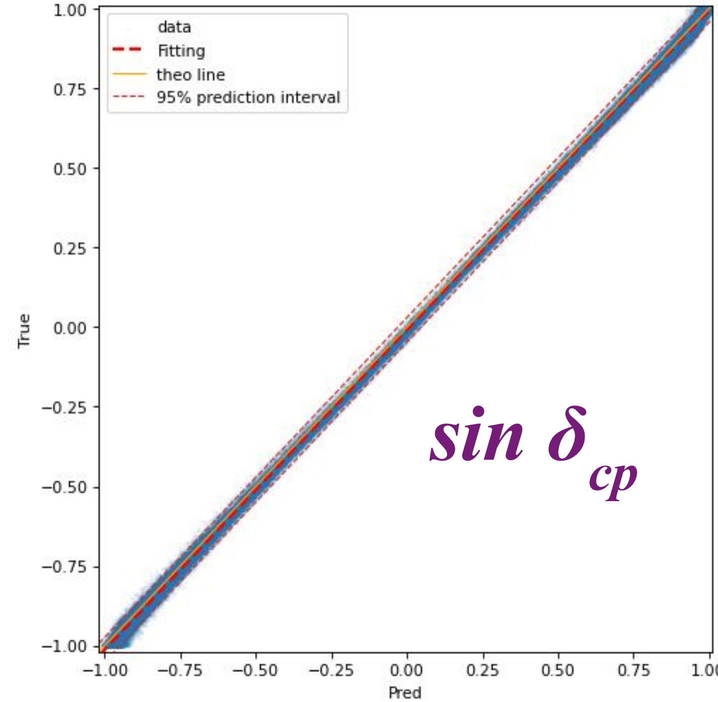

Epoch vs Huber loss
Train
Validation
Loss (Huber)
epoch
sin δcp
0.0
0.1
0.2
0.3
0.4
0
10
20
30
40
50

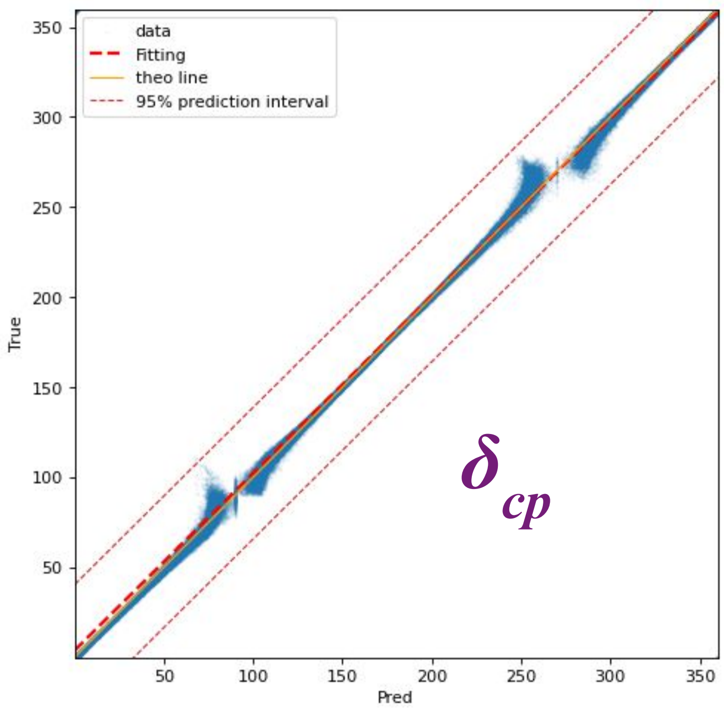

Train
Validation
Loss
epoch
δcp
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
50
100
150
200
250

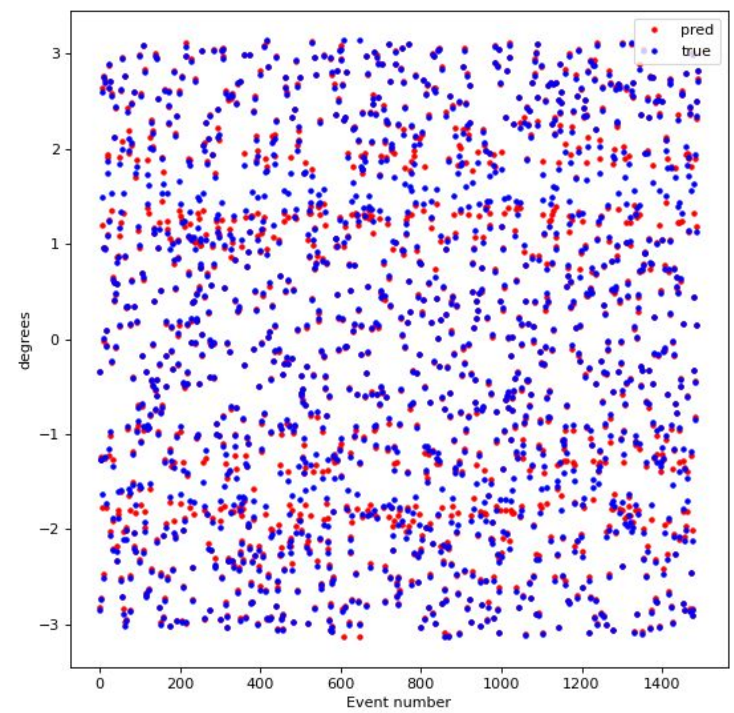

accuracy
epoch
δcp
Train
Validation
0.5
0.6
0.7
0.8
0.9
1.0
0
50
100
150
200
250


ROC Curve with AUC score: 0.99833
True Positive Rate
False Positive Rate
δcp
0.0
0.2
0.4
0.6
0.8
1.0


data
Fitting
theo line
95% prediction interval
True
Pred
40
42
44
46
48
50
θ_{23}


data
Fitting
theo line
95% prediction interval
True
Pred
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
$\sin \delta_{cp}$


data
Fitting
theo line
95% prediction interval
True
Pred
50
100
150
200
250
300
350
δ_{cp}


pred
true
degrees
Event number
0
200
400
600
800
1000
1200
1400
−3
−2
−1
0
1
2
3

δ_{cp}

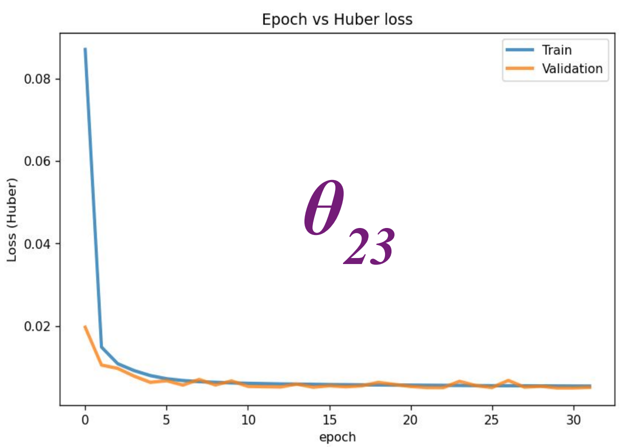
2.3 Other models

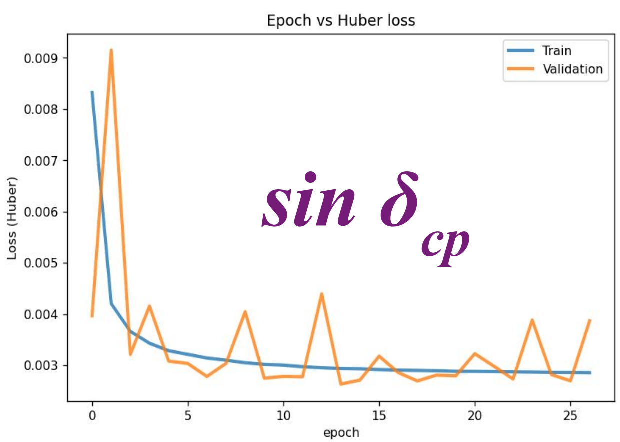
- Owing to the overfit with the test sample (base on public and private score), we add **noise** into the training datasets. Implementing similar models and methods.
  [improve a little bit]

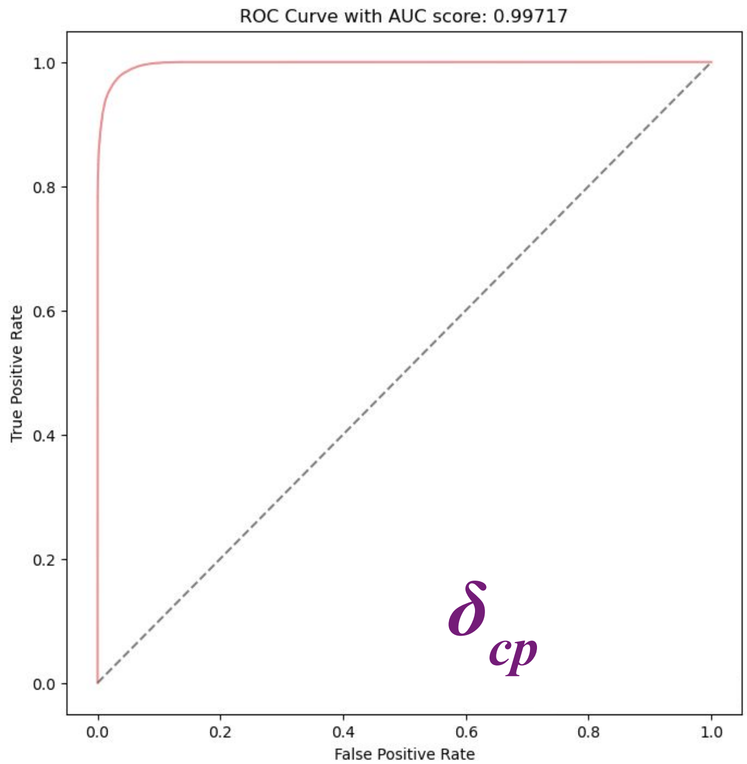
- Due to the arcsine classification distortion, we try to use the models reported in the last presentation.
  ⇒ Classificate δ_{cp} training data into 4 groups (each group is monotonic); then, do the regression. (**reversed procedure** of the main δ_{cp} model structure)
  [do not have obvious improvement]

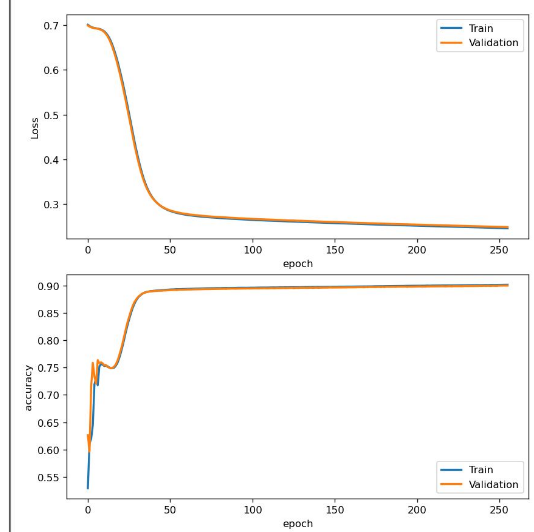
2.3.A Noisy samples

Owing to the overfit with the test sample (base on public and private score), we add noise into the training datasets. Implementing similar models and methods.

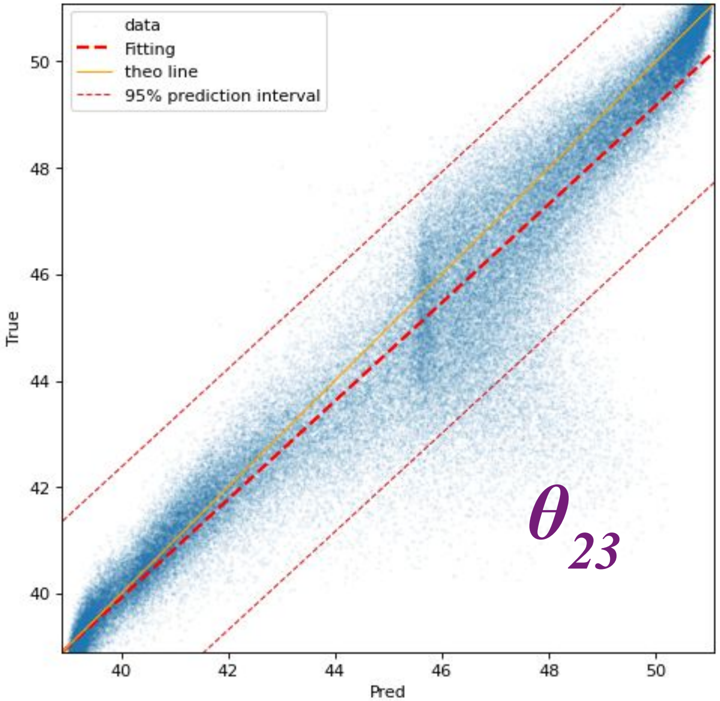


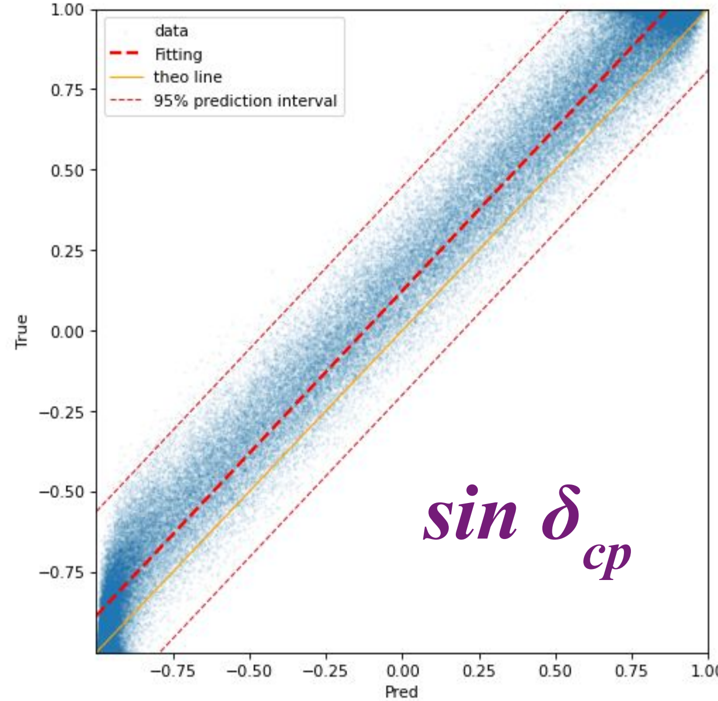


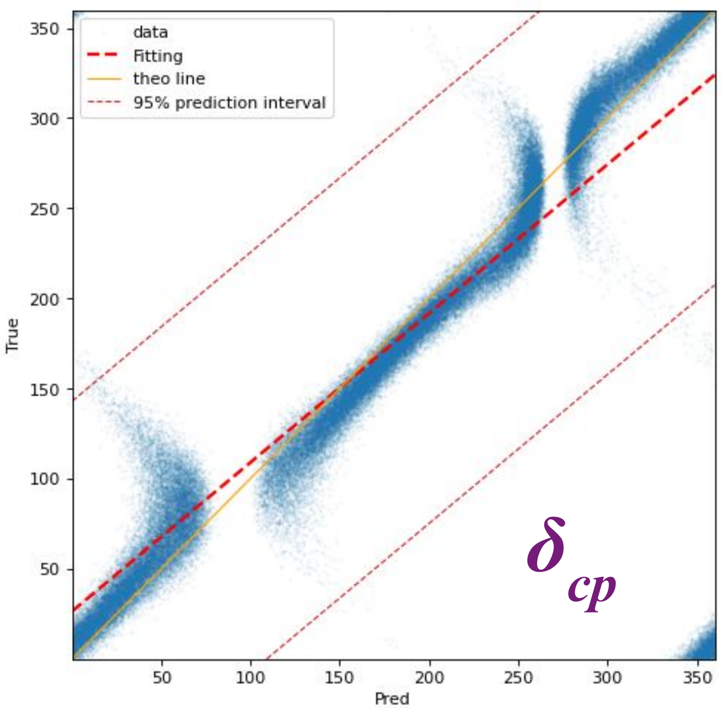


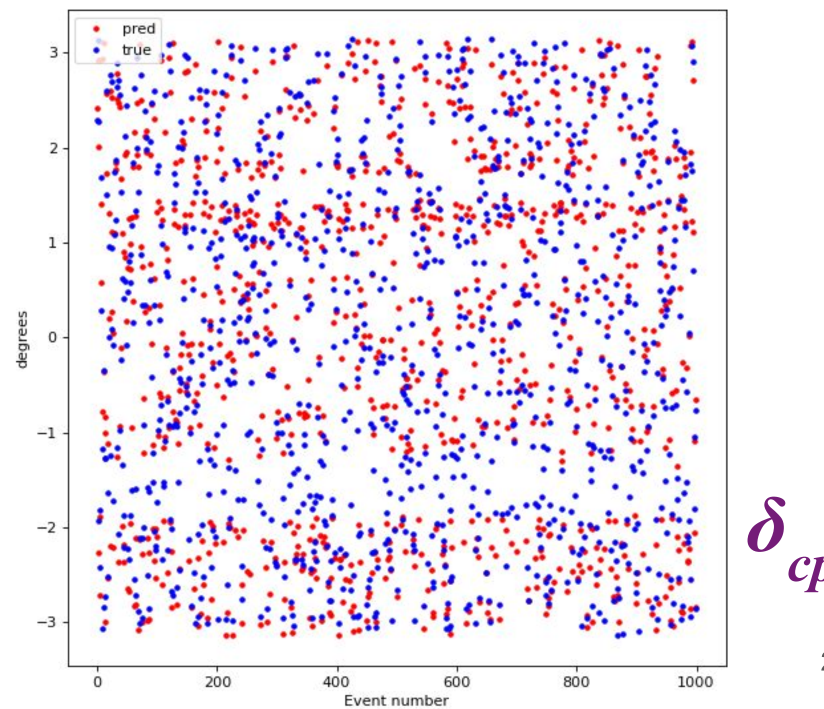


δ_{cp}


data
Fitting
theo line
95% prediction interval
True
Pred
θ_{23}


data
Fitting
theo line
95% prediction interval
True
Pred
$\sin \delta_{cp}$


data
Fitting
theo line
95% prediction interval
True
Pred
δ_{cp}


pred
true
degrees
Event number
δ_{cp}

2.3.B Reversed δ_{cp} procedure

Due to the arcsine classification distortion, we try to use the models reported in the last presentation.

⇒ Classificate δ_{cp} training data into 4 groups (each group is monocity); then, do the regression. (reversed procedure of the main model structure)

<LOSS CURVE>

時間太長 QQ 放結果就好 <RESULT>

3.1 Summary

- Physically, we recap some essential concepts of neutrino oscillation.
- For models, we first use the ldm binary model to split data into IO and NO two groups; after that, separating the regression of θ_{23} and δ_{cp} into several models. θ_{23}: regression; δ_{cp}: sine regression + arcsine classification.
- The results fit too well to tolerate the Poisson fluctuations in the test sample.
- Apply additional models to rule out the overfitting problems, such as the noisy model and the reversed δ_{cp} procedure model.
- Learn from the other group: implement AE to denoise; predict parameters at the same time.

BACK UP

Codes

General backup repo in GitHub:
https://github.com/yygarypeng/Neutrino-Oscillation-DUNE

Submission backup in Kaggle:
https://www.kaggle.com/competitions/phys591000-2023-final-project-i

The physics of antineutrinos in DUNE and determination of octant and δ_{CP}

cos(π/2 + δ) = -sin(δ), so this formula is also proportional to sin(δ) if in the vacuum ideal situations. (ref page 386, 387)

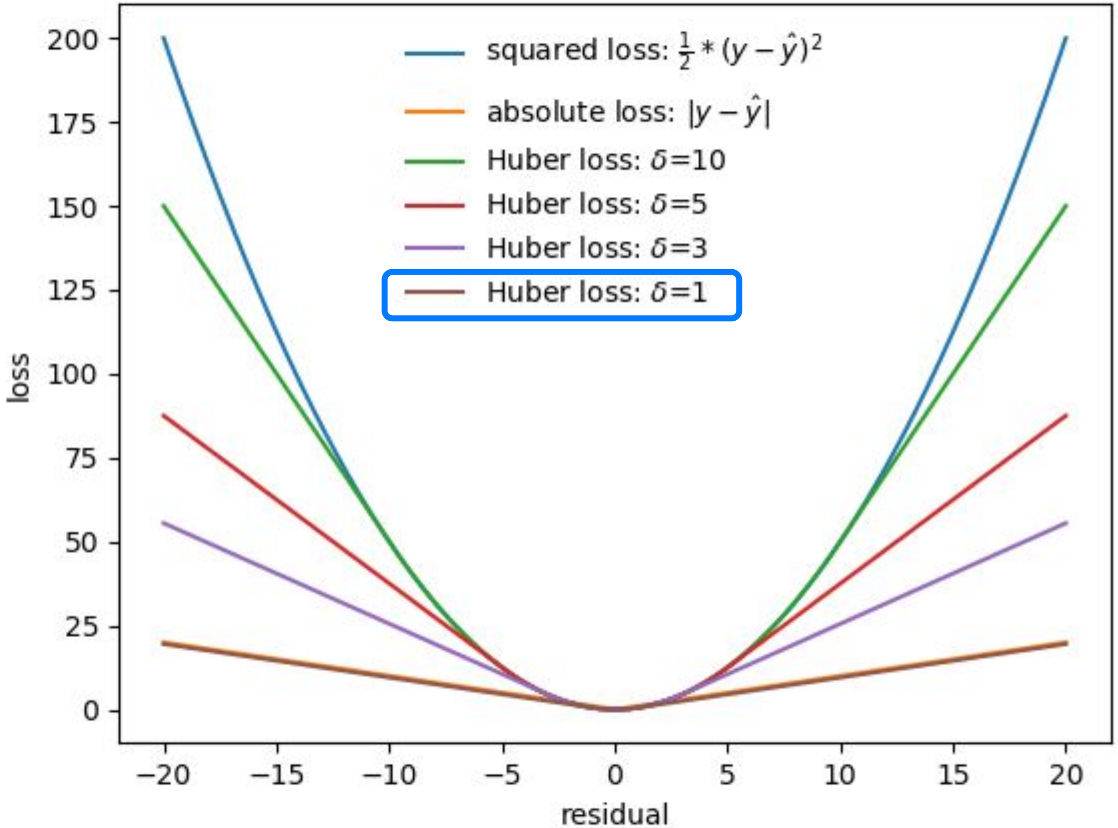
$$P_{\mu e} = \underbrace{4 s_{13}^2 s_{23}^2 \frac{\sin^2 (A-1)\Delta}{(A-1)^2}}_{\mathcal{O}_o} + \underbrace{\alpha^2 \cos^2\theta_{23} \sin^2 2\theta_{12} \frac{\sin^2 A\Delta}{A^2}}_{\mathcal{O}_2} + \underbrace{\alpha s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos(\Delta + \delta_{cp}) \frac{\sin(A-1)\Delta}{(A-1)} \frac{\sin A\Delta}{A}}_{\mathcal{O}_1}$$

$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2 \Delta + \mathcal{O}(\alpha, s_{13})$$

Note that for vacuum oscillation maxima, Δ corresponds to 90°. Thus in the appearance channel probability (cf. Eq. (2)), $\delta_{CP} = -90°(+90°)$ correspond to maximum (minimum) point in the probability for neutrinos. For antineutrinos it is the opposite. Thus, for these values of δ_{CP}, octant sensitivity is expected to be maximum if there is no degeneracy. Note that with the inclusion of matter effect, the appearance channel probability maxima does not coincide with the vacuum maxima and in that case the maximum and minimum points in the probability do not come exactly at $\pm 90°$ but gets slightly shifted. This can be seen from Fig. 1. However for illustration, we will take $\delta_{CP} = \pm 90°$ as the reference points to describe the physics of octant in DUNE.

Huber loss

$$Loss\left(y,\hat{y}\right)=\begin{cases}\frac{1}{2}\left(y-\hat{y}\right)^2, & \left|y-\hat{y}\right|\le\delta\\ \delta(\left|y-\hat{y}\right|-\frac{1}{2}\delta), & O.W.\end{cases}$$



Huber = MSE + MAE

MSE is more sensitive to outliers.
⇒ improve the **robustness** of the MSE to outliers using MAE.

However, MAE is not stable around the minimum.

[Huber loss]

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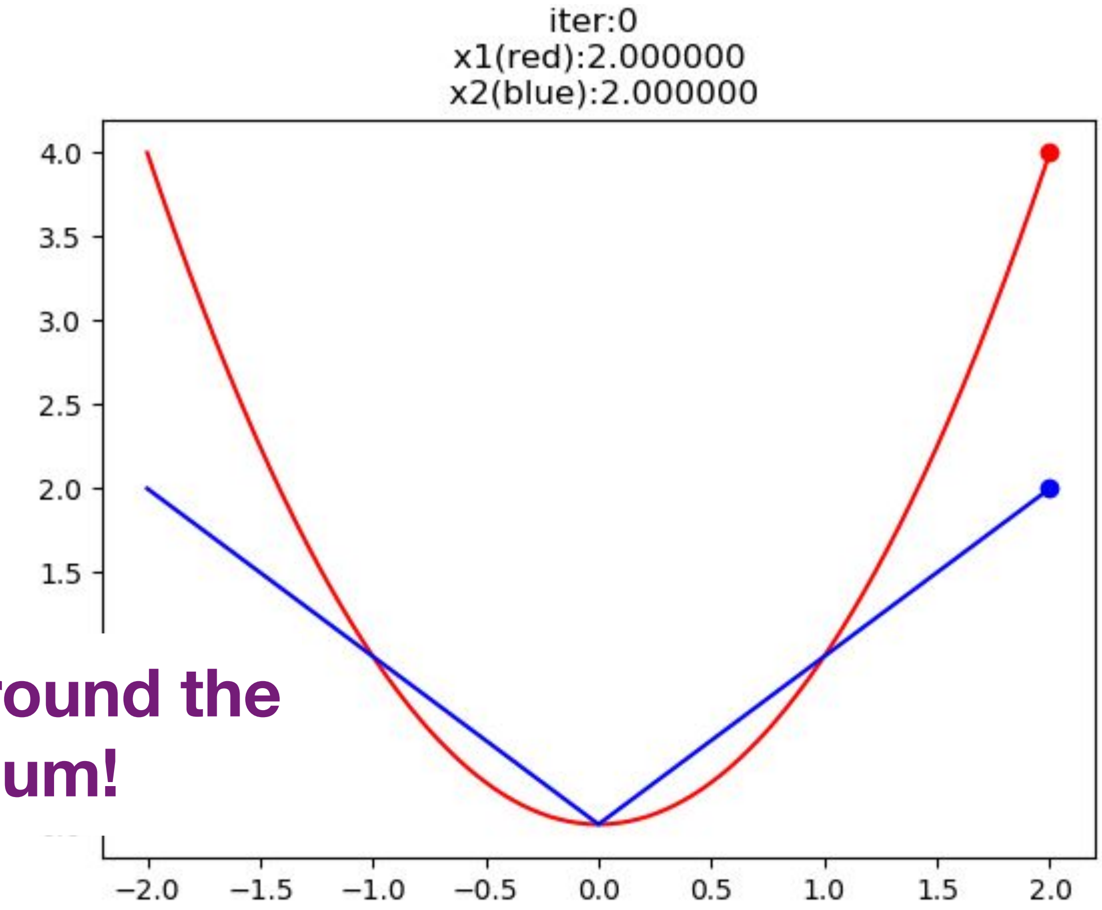
[Loss functions](#)

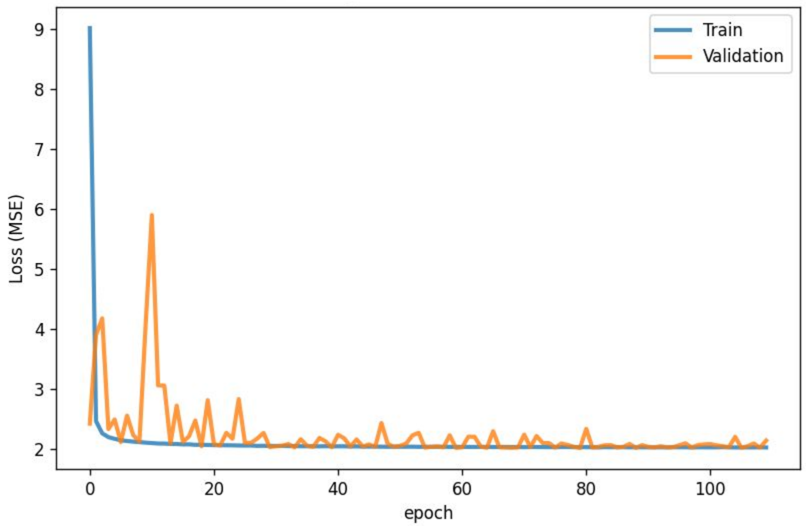
[MAE vs MSE](#)



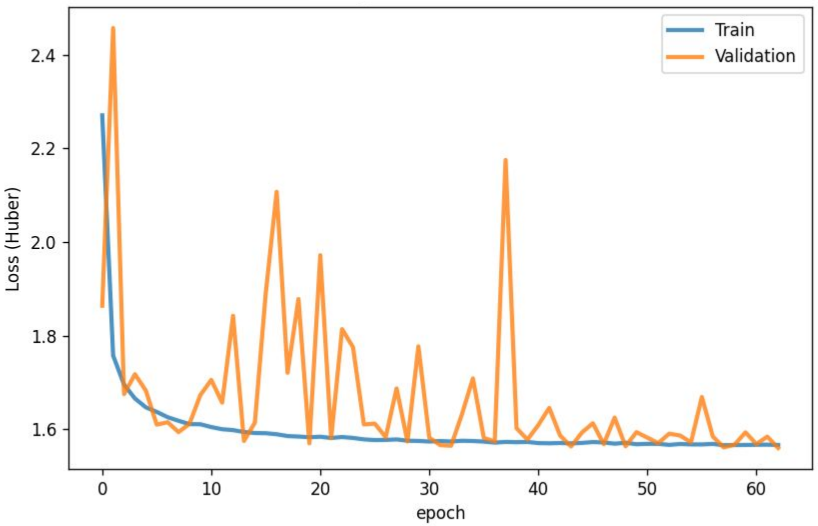
Unstable around the local minimum!

θ_{23} Loss curves (before normalization)

MSE

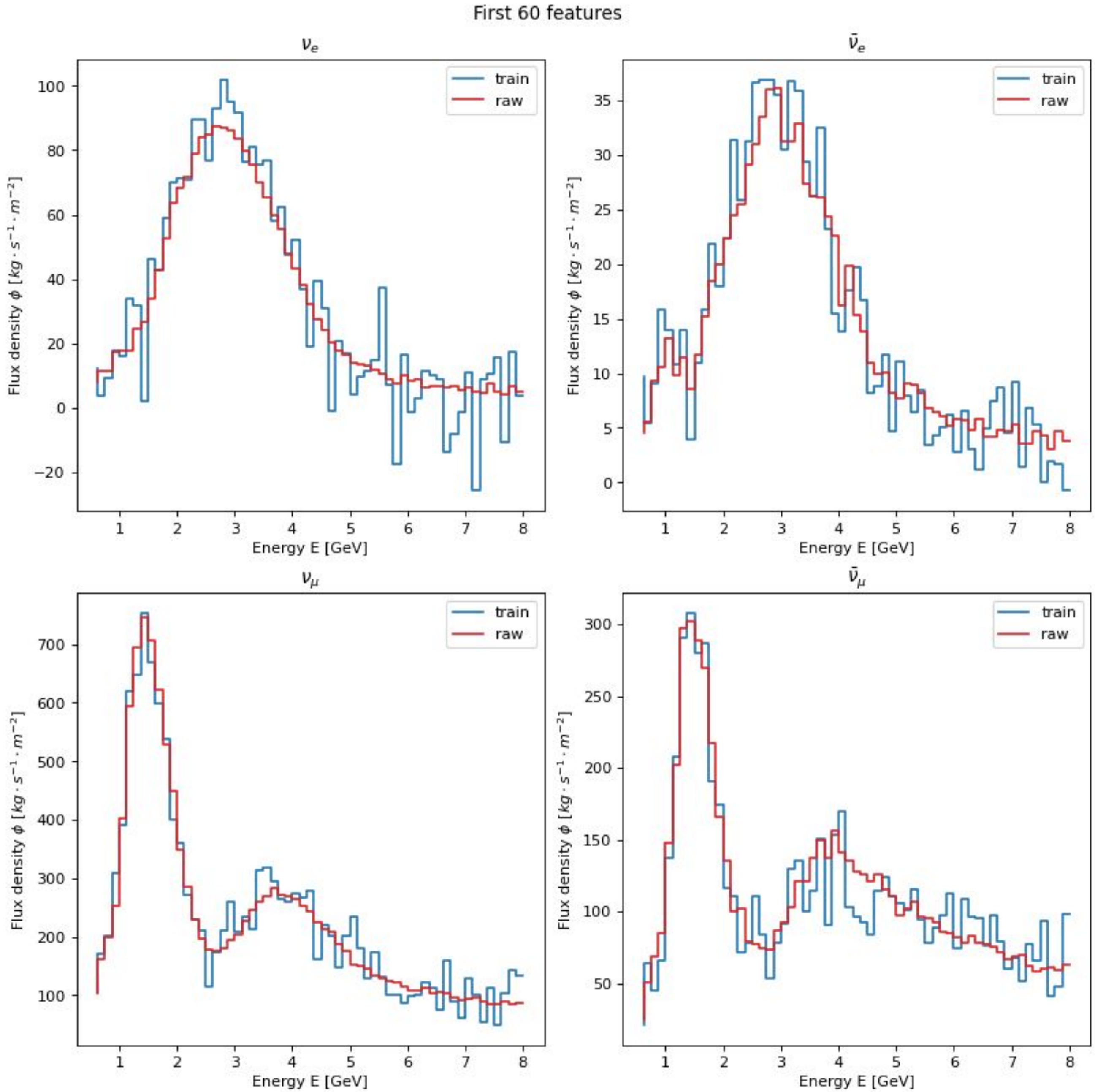


Huber

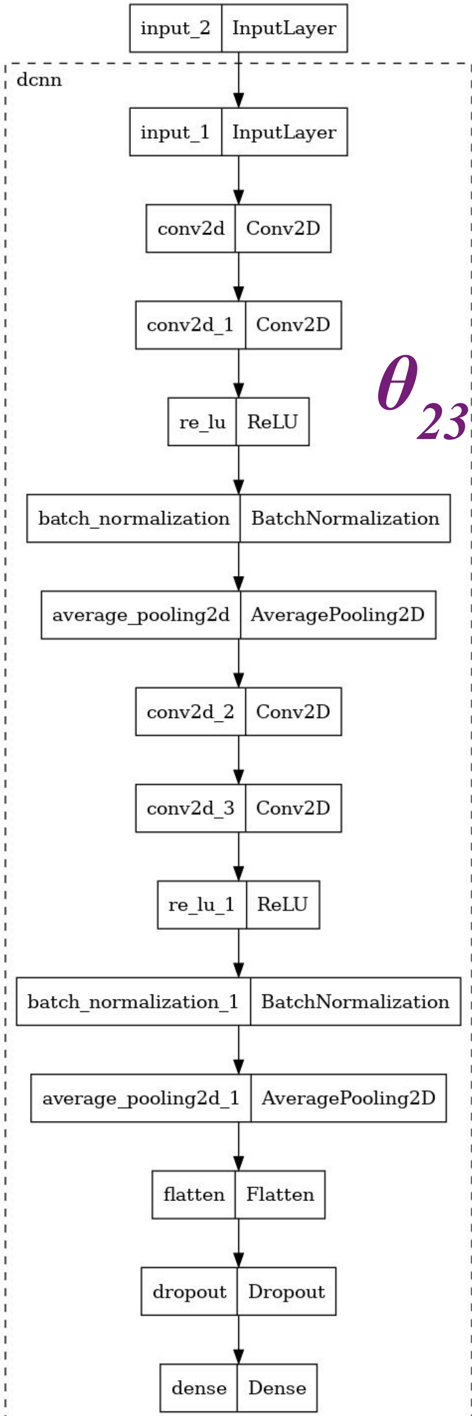


- Large fluctuations if no normalization!
- Huber loss can zoom out the large loss!

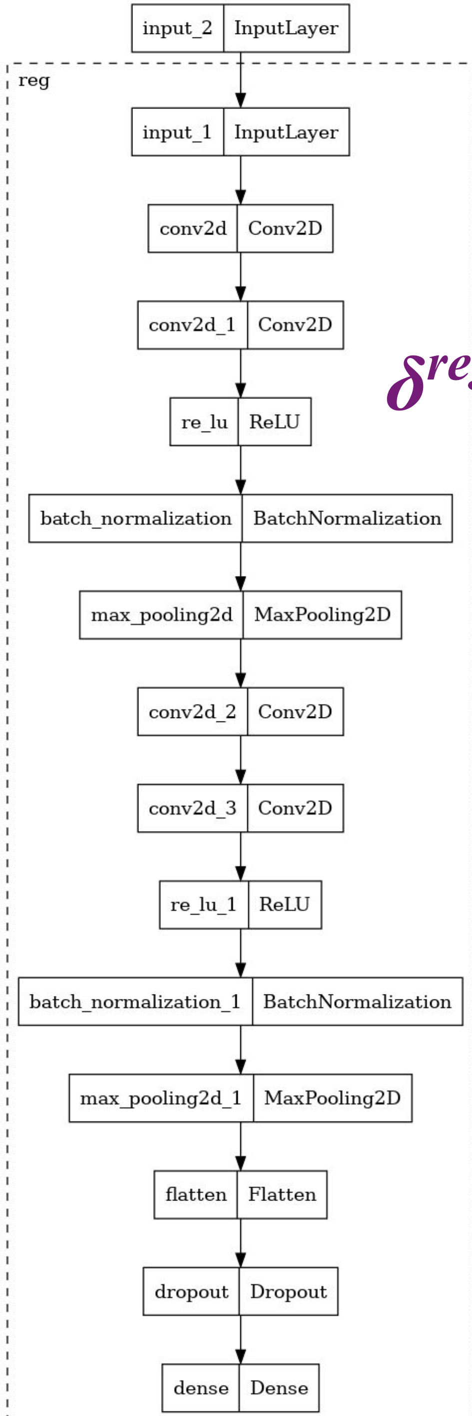
Noisy figures



Noisy model structures



θ_{23}



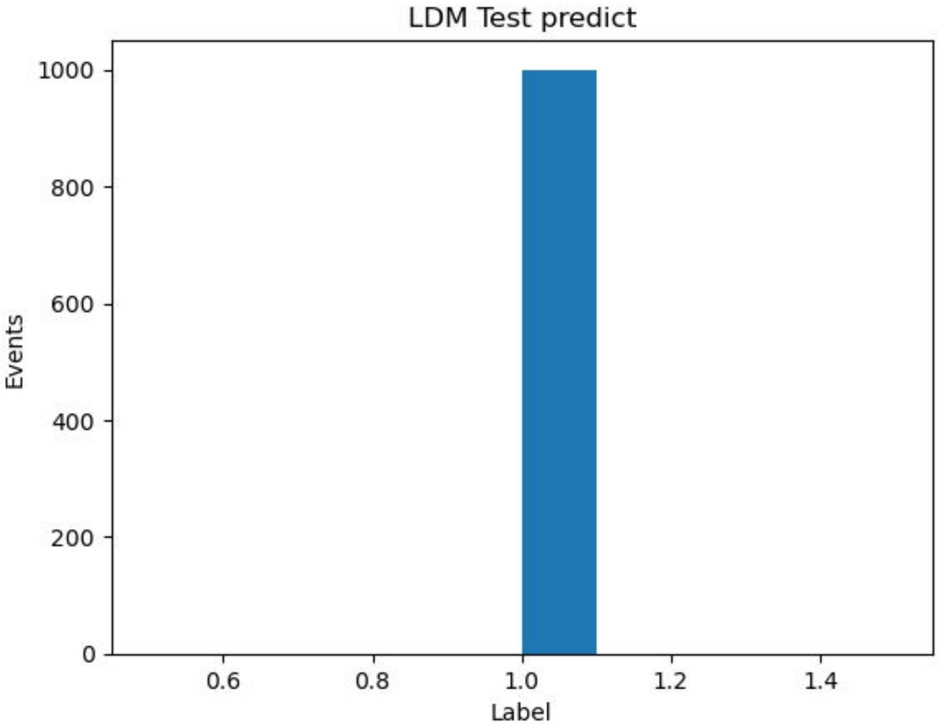
δ^{reg}_{cp}

```
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 input_1 (InputLayer)        [(None, 65, 4, 1)]        0

 conv2d_2 (Conv2D)           (None, 33, 2, 64)         320

 conv2d_3 (Conv2D)           (None, 17, 1, 64)         16448

 max_pooling2d_1 (MaxPooling  (None, 9, 1, 64)         0
 2D)

 flatten (Flatten)           (None, 576)               0

 dense (Dense)               (None, 1)                 577

=================================================================
Total params: 17,345
Trainable params: 17,345
Non-trainable params: 0
_________________________________________________________________
```

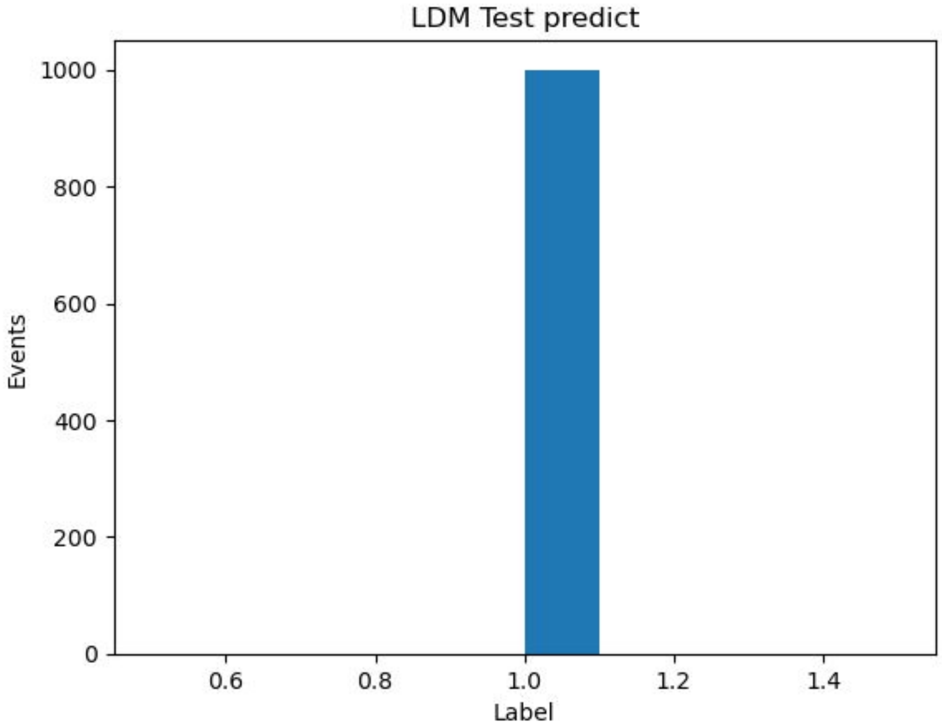
δ^{cla}_{cp}

Test prediction of LDM classification

With noise



Without noise



What is being measured?

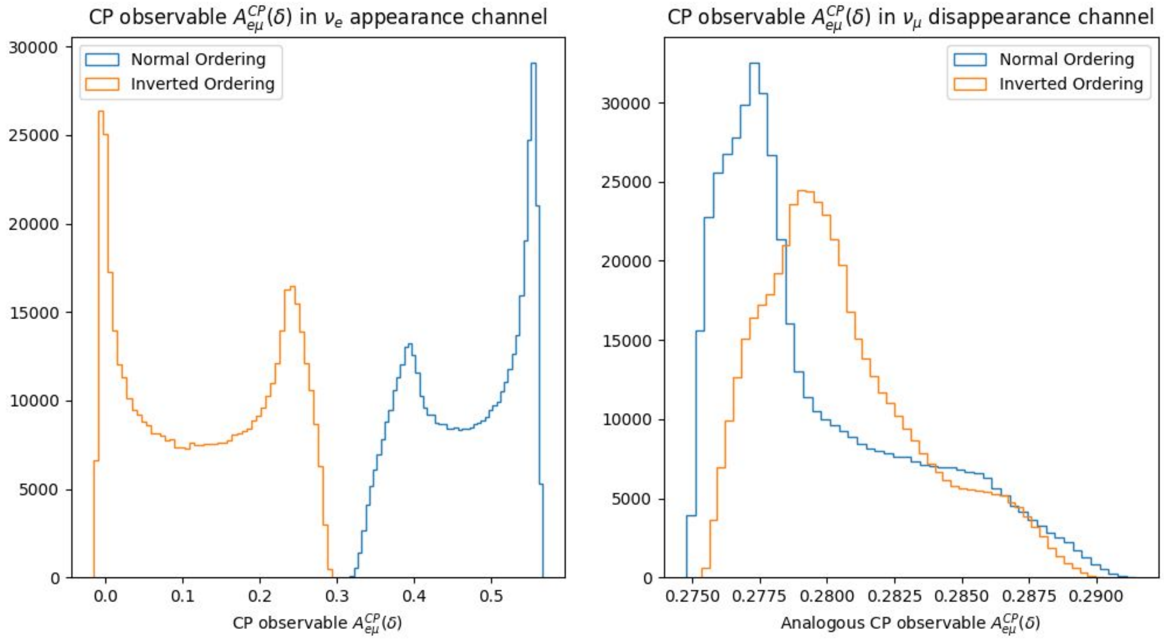
- The experiment aims to probe the effects of neutrino oscillations and CP violation measurements. Consider the CP asymmetry observable:

$$A_{ab}^{CP}(\delta) \equiv \frac{P(\nu_a \to \nu_b) - P(\bar{\nu}_a \to \bar{\nu}_b)}{P(\nu_a \to \nu_b) + P(\nu_a \to \nu_b)}$$

- In practice, this observable is constructed from the integrated spectrum of decaying leptons $N_{\ell\pm}$ and wrong-sign leptons $N_{wrong\ell\pm}$

$$A_{e\mu}^{\mathrm{CP}}(\delta) = \frac{N_{\mu^-}/N_{e^-}^{\mathrm{wrong}} - N_{\mu^+}/N_{e^+}^{\mathrm{wrong}}}{N_{\mu^-}/N_{e^-}^{\mathrm{wrong}} + N_{\mu^+}/N_{e^+}^{\mathrm{wrong}}}$$

The CP phase and asymmetry observable

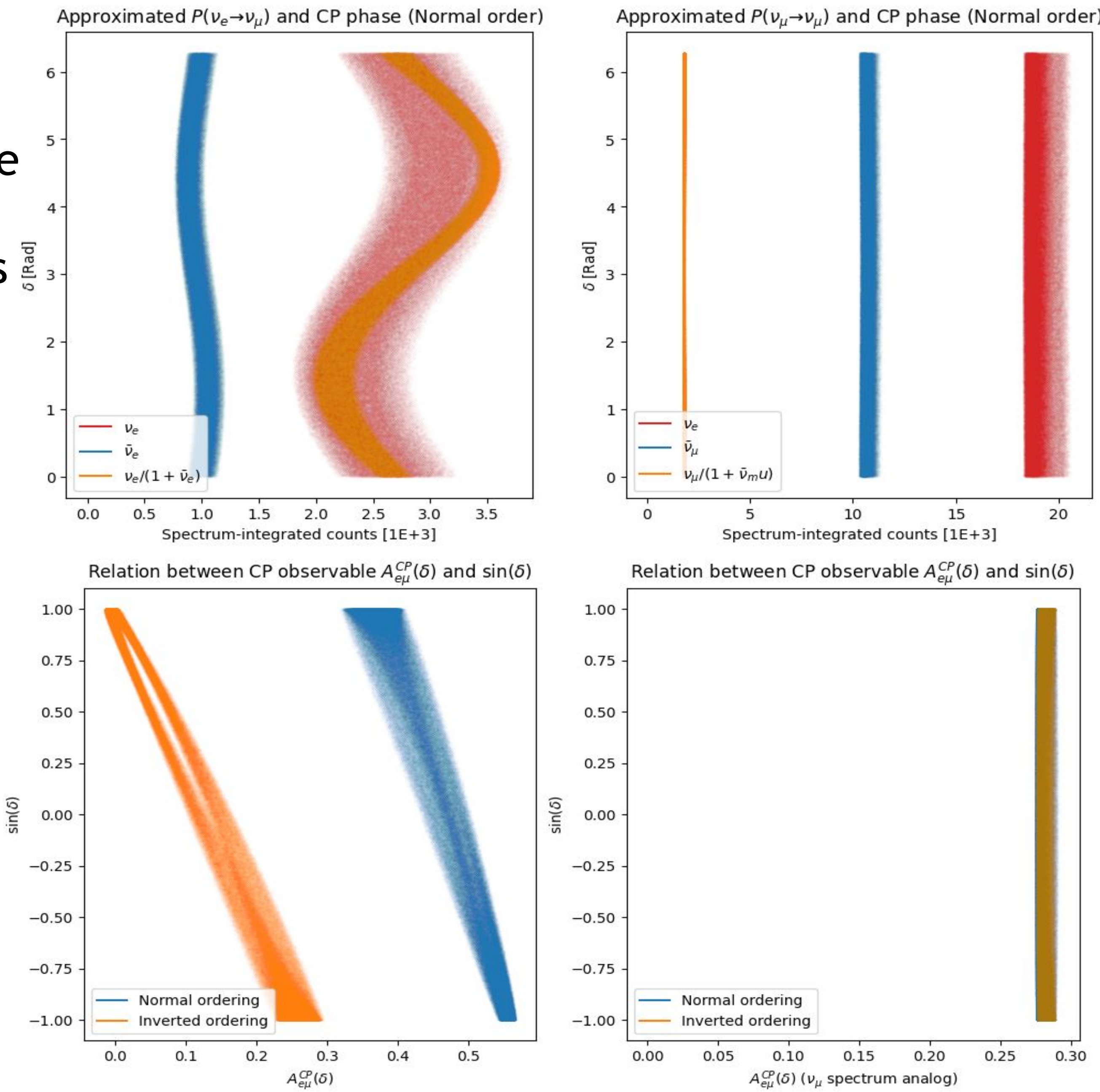


- As mentioned earlier, the CP phase can be quantified by the level of asymmetry.
- Here we present the distribution of CP phase and this observable in both the appearance channel and disappearance channel.

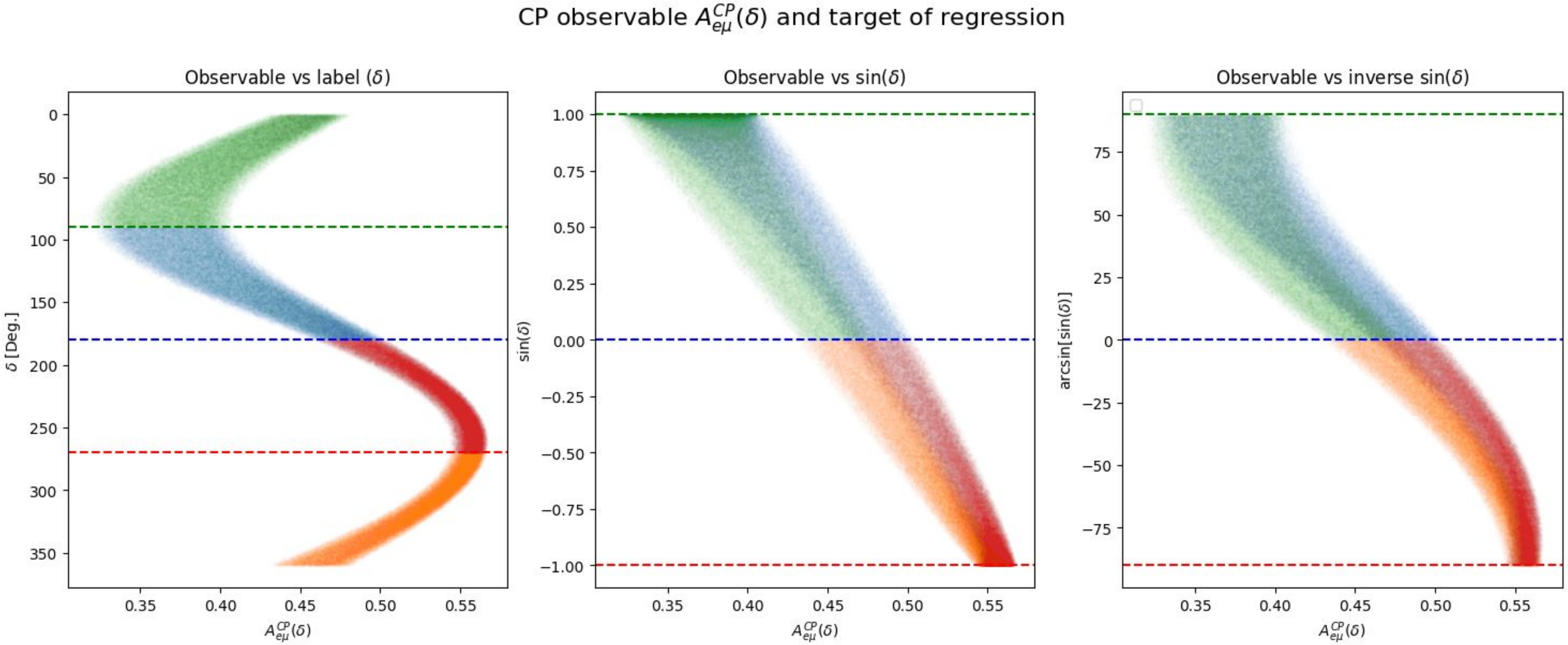
The upper and lower plots present the distribution of CP phase and its relationship with total event counts and the CP asymmetry observable respectively.

Plots of the ν_e and ν_μ spectrums are on the left and right side respectively.

Our observations show that the event counts in ν_μ spectrums are not correlated with the CP phase.

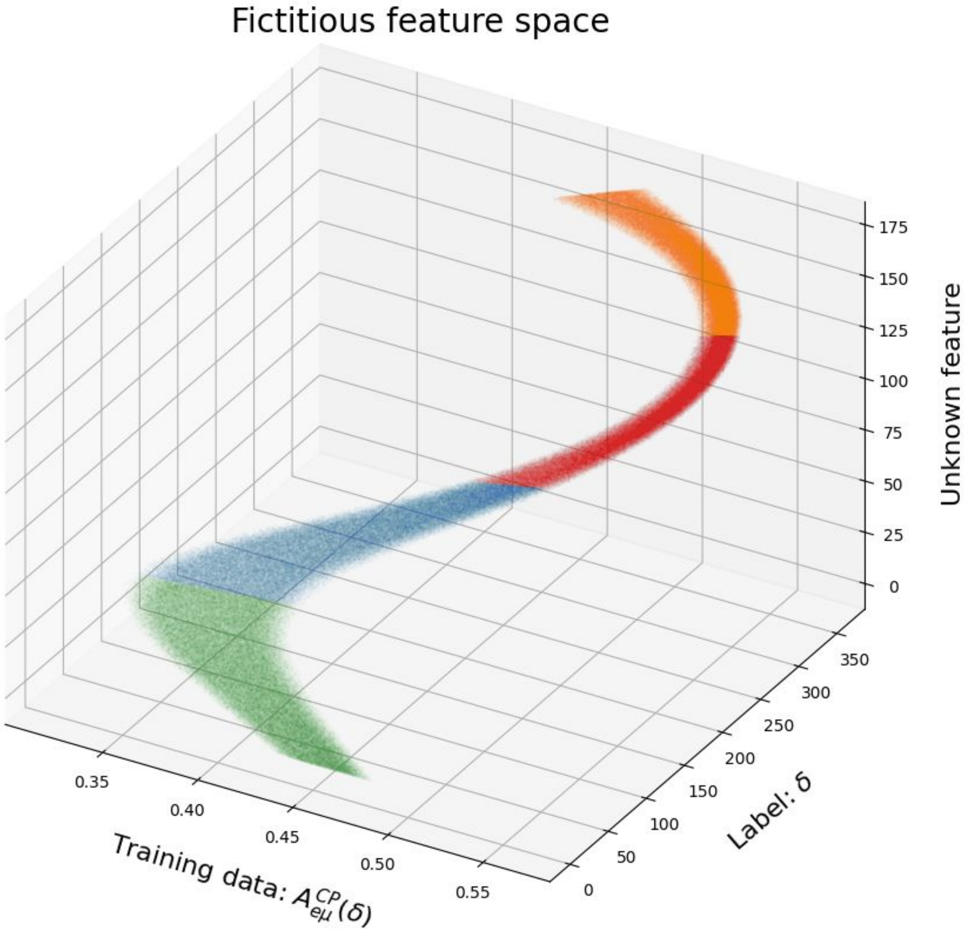


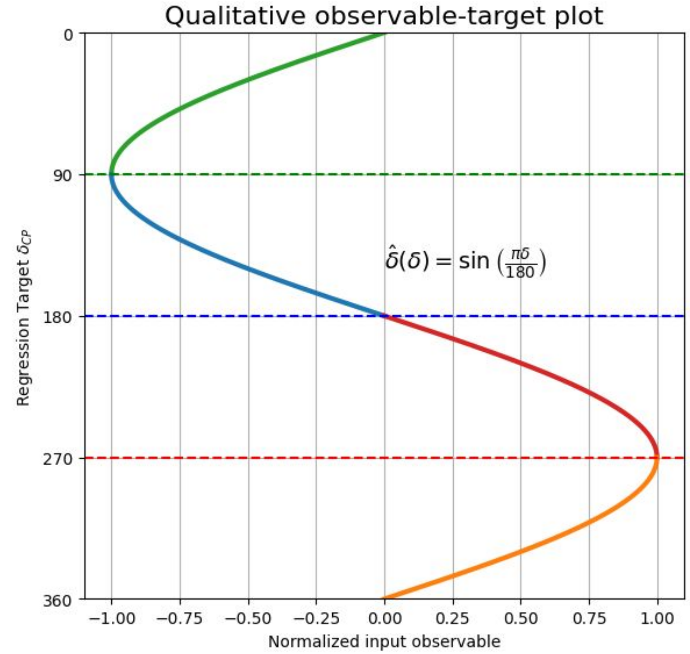
The CP phase Regression

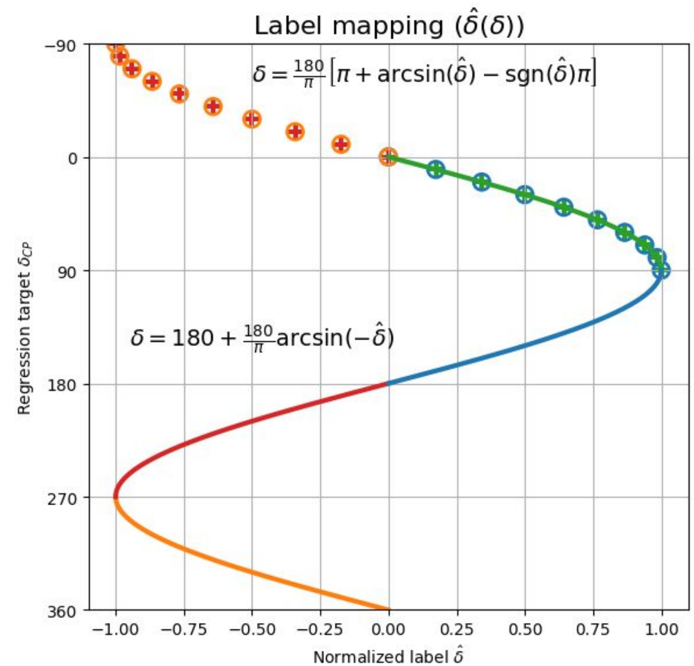


- By taking the sine value of the CP phase, we see a clear one-to-one relationship between the asymmetry observable and the sine value.
- However, transforming back using the arcsine function will only return values in the interval [-90,90].

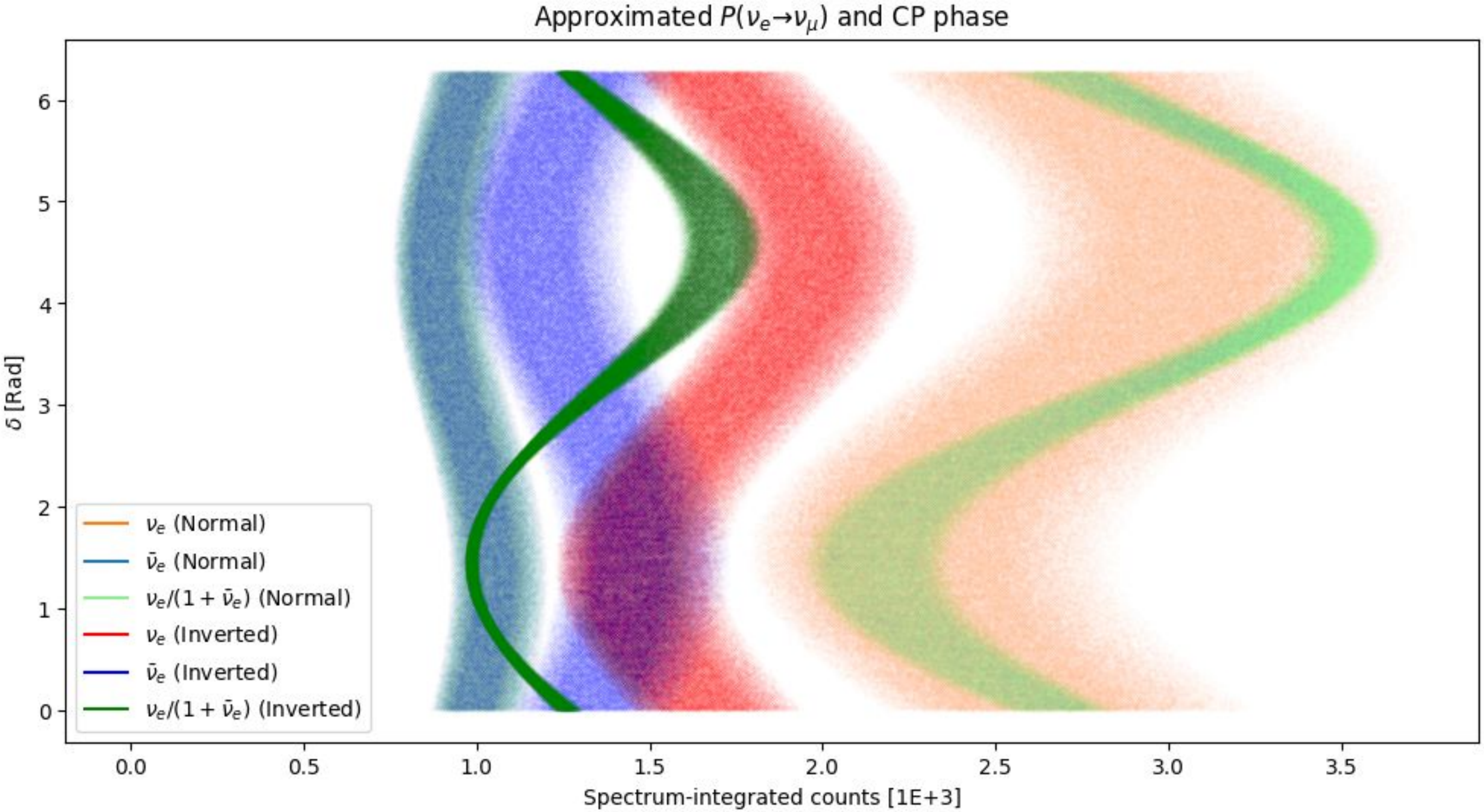
Idealized feature space







The CP phase Regression



ν_e

0 < δ < 50
38 < θ_{23} < 40
50 < δ < 100
38 < θ_{23} < 40
100 < δ < 150
38 < θ_{23} < 40
150 < δ < 200
38 < θ_{23} < 40
200 < δ < 250
38 < θ_{23} < 40
250 < δ < 300
38 < θ_{23} < 40
300 < δ < 350
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0 < δ < 50
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200 < δ < 250
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250 < δ < 300
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0 < δ < 50
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50 < δ < 100
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250 < δ < 300
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300 < δ < 350
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0 < δ < 50
44 < θ_{23} < 46
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150 < δ < 200
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$\bar{\nu}_e$


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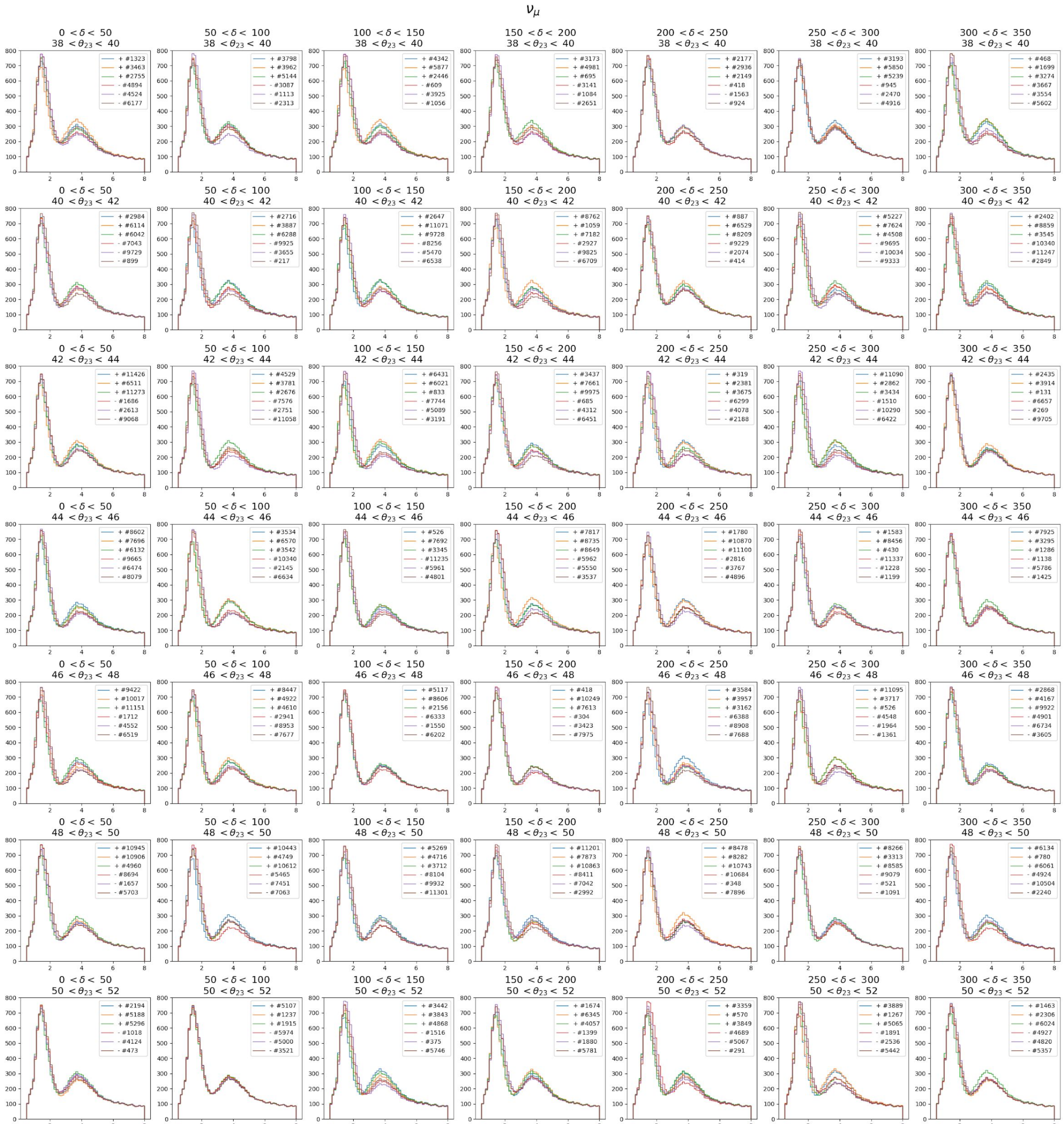
ν_μ
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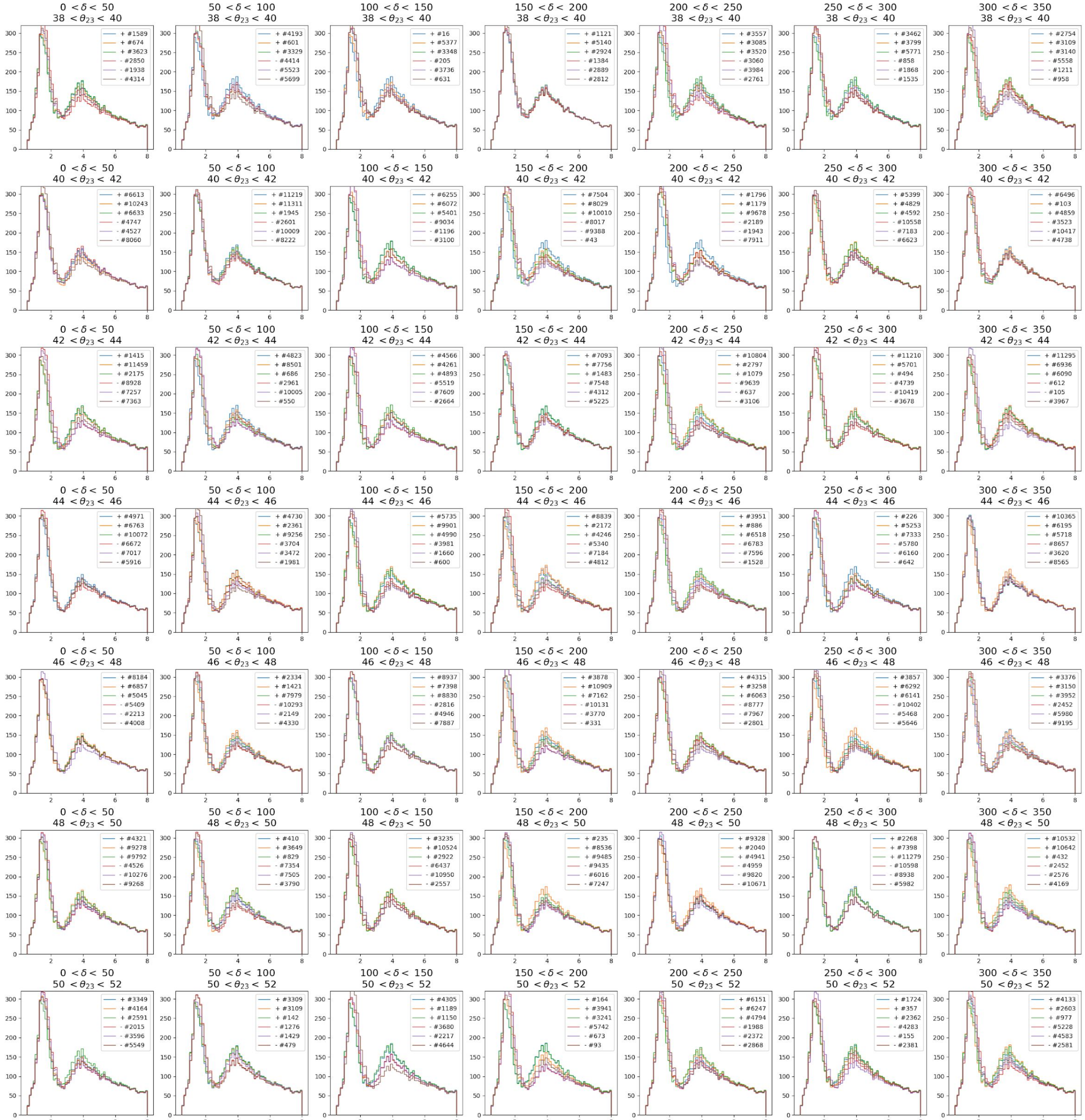
300 < δ < 350 50 < θ_{23} < 52

$\bar{\nu}_\mu$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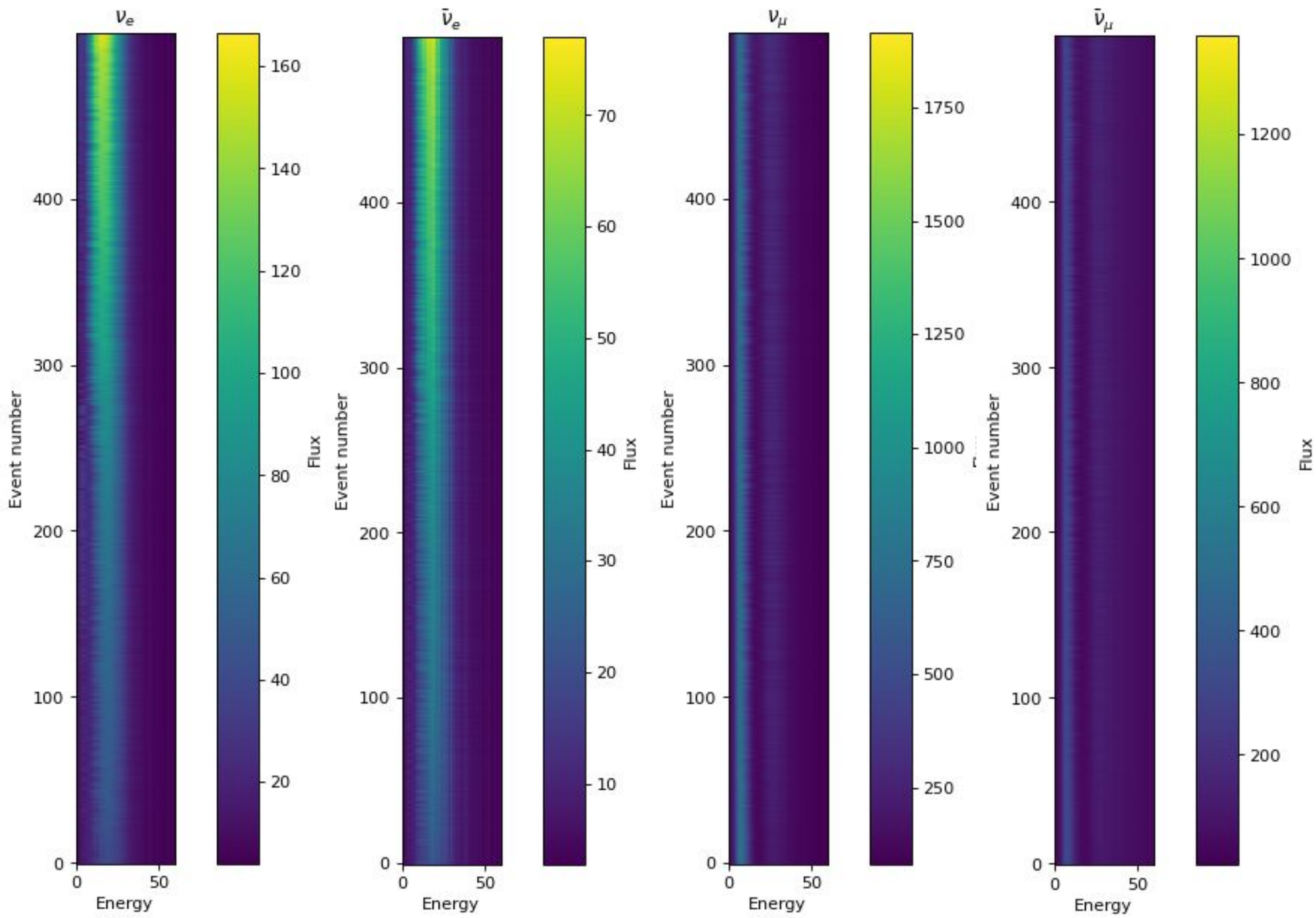
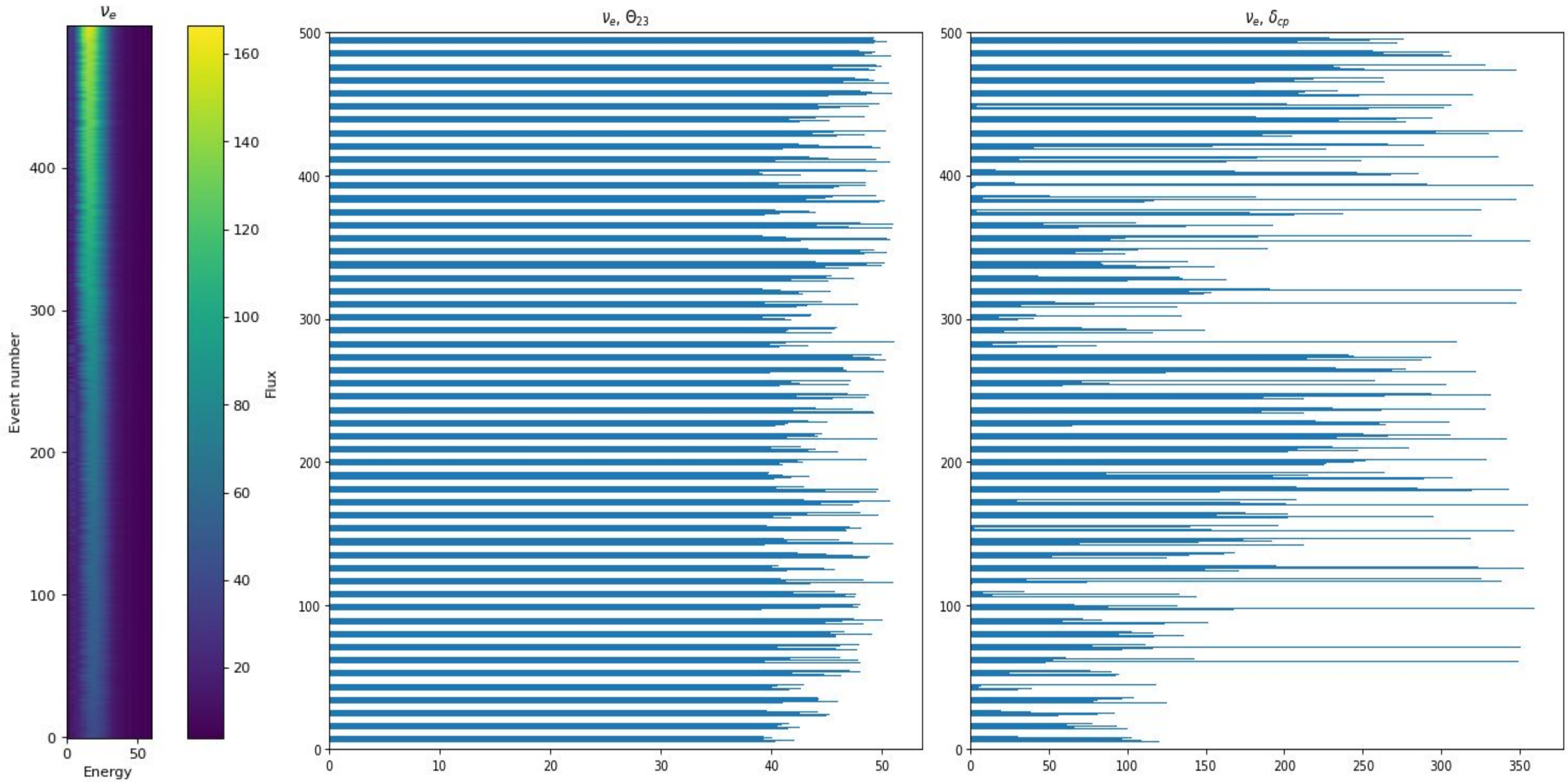
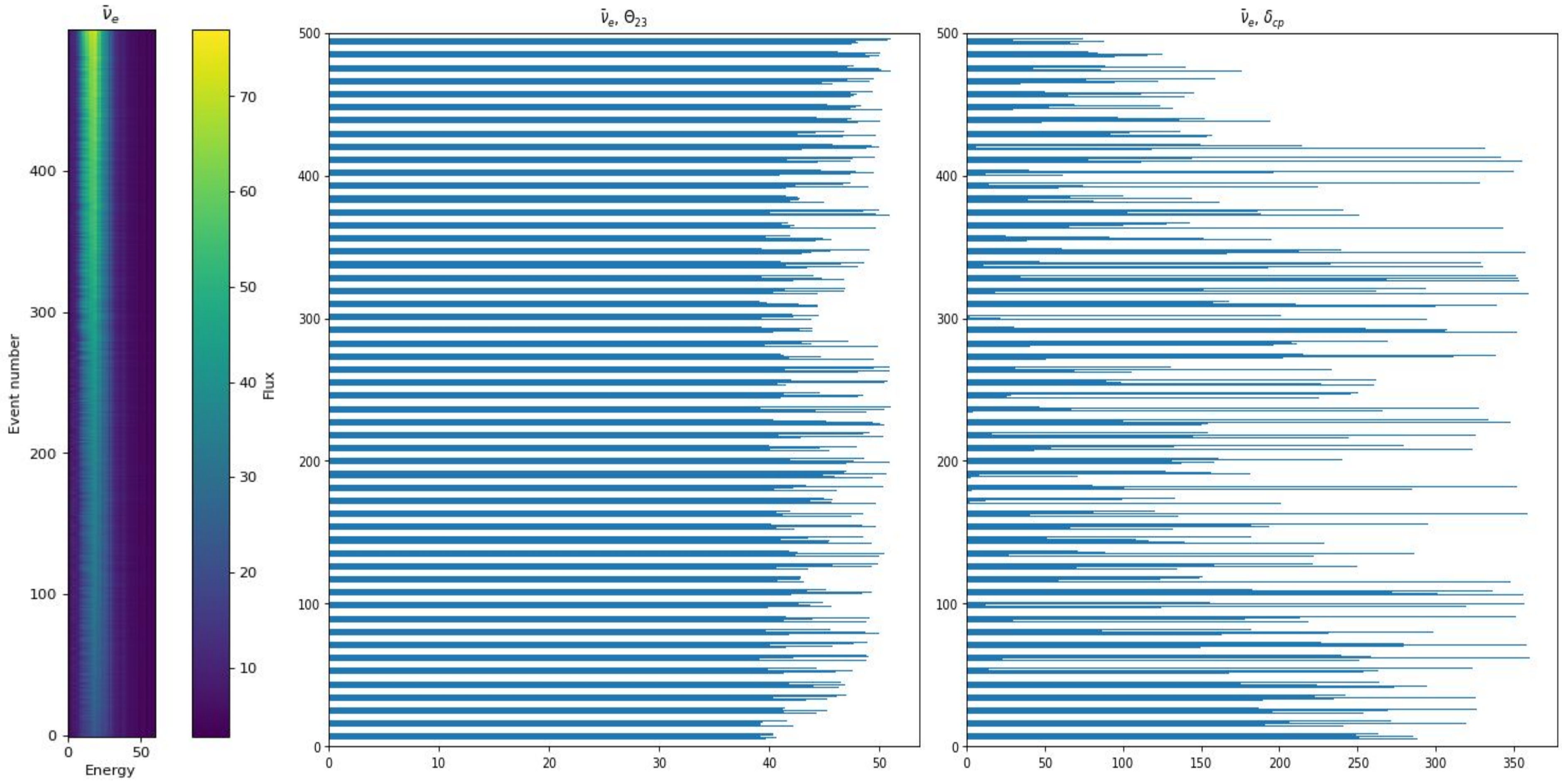
50 < θ23 < 52
300 < δ < 350
50 < θ23 < 52

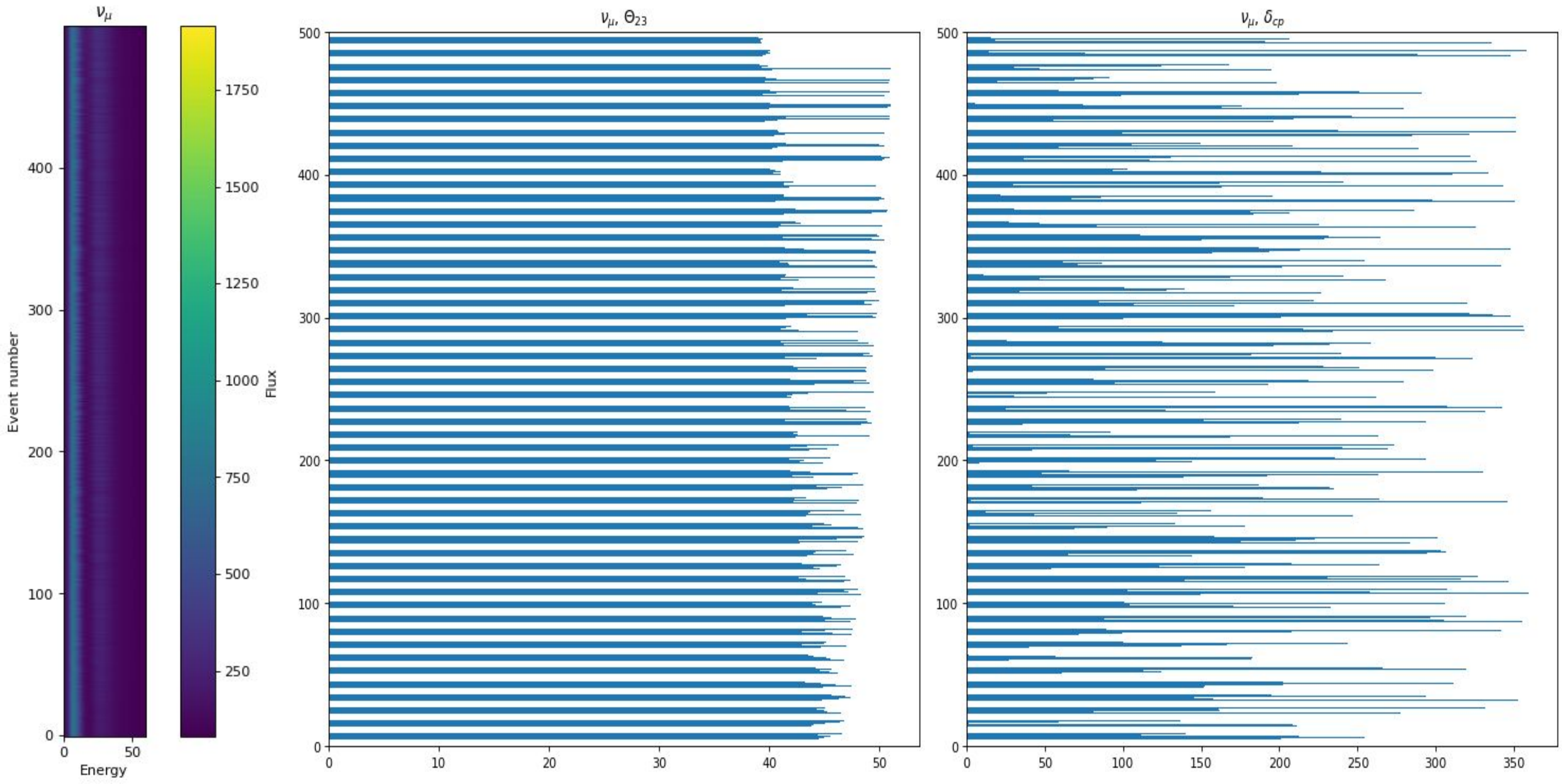
Sorted four channels with 500 events



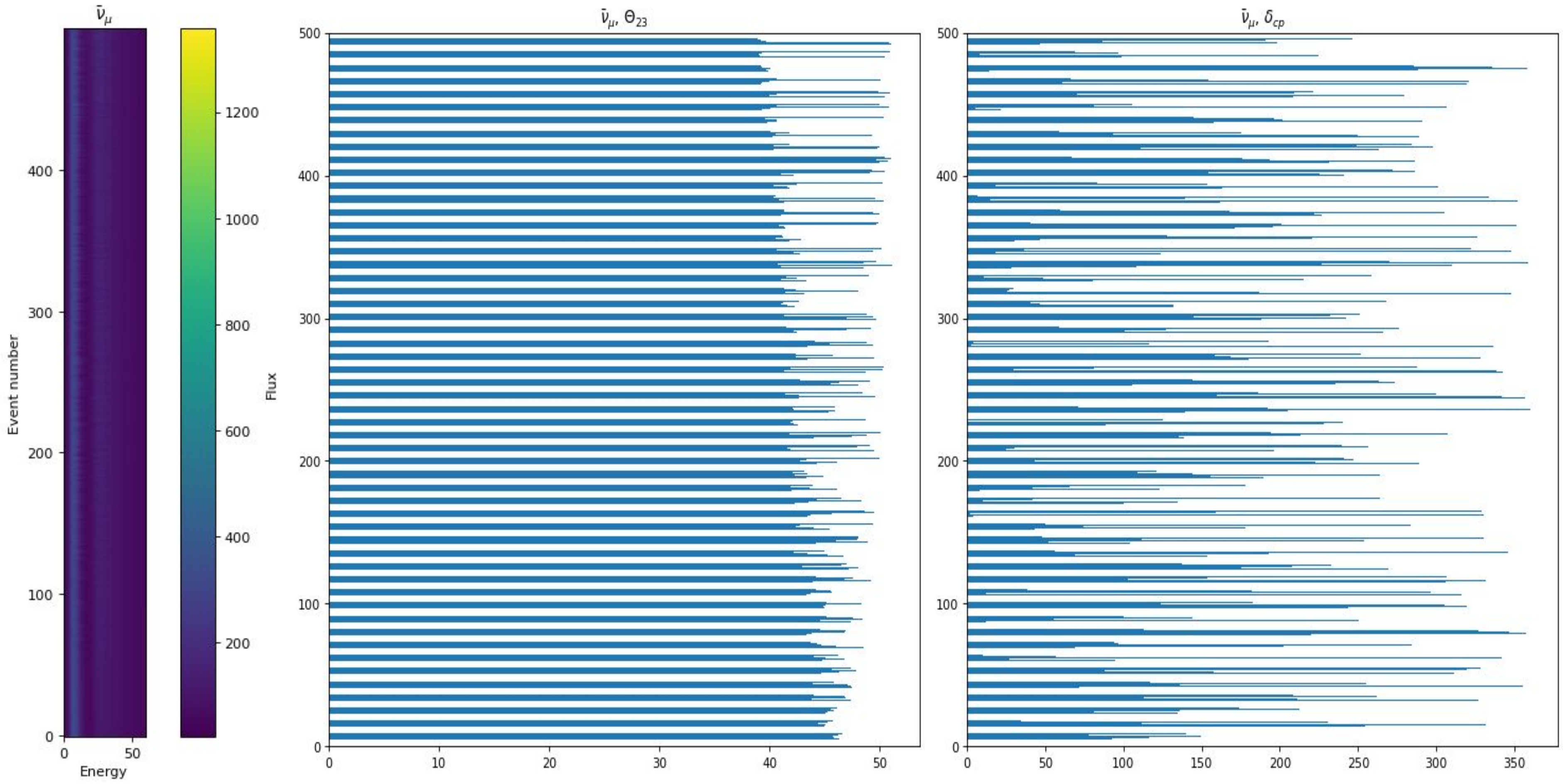
The results are sort by the sum of the specific channel.
(In this page, the y-axis is sorted by the sum of “event number ” by ve channel)




$\bar{\nu}_e$
$\bar{\nu}_e$, Θ_{23}
$\bar{\nu}_e$, δ_{cp}
Event number
Energy
Flux



ν_μ
Event number
Energy
Flux
ν_μ, Θ_{23}
ν_μ, δ_{cp}


$\bar{\nu}_\mu$
Event number
Energy
Flux
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100
200
300
400
0
50
200
400
600
800
1000
1200
$\bar{\nu}_\mu$, Θ_{23}
0
10
20
30
40
50
100
200
300
400
500
$\bar{\nu}_\mu$, δ_{cp}
0
50
100
150
200
250
300
350
100
200
300
400
500

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