Neutrino Physics

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- ★ Neutrino's history & lepton families
- ★ Dirac & Majorana neutrino masses
- ★ Lepton flavor mixing & CP violation
- ★ Neutrino oscillation phenomenology
- ★ Seesaw & leptogenesis mechanisms
- ★ Extreme corners in the neutrino sky

Lecture B

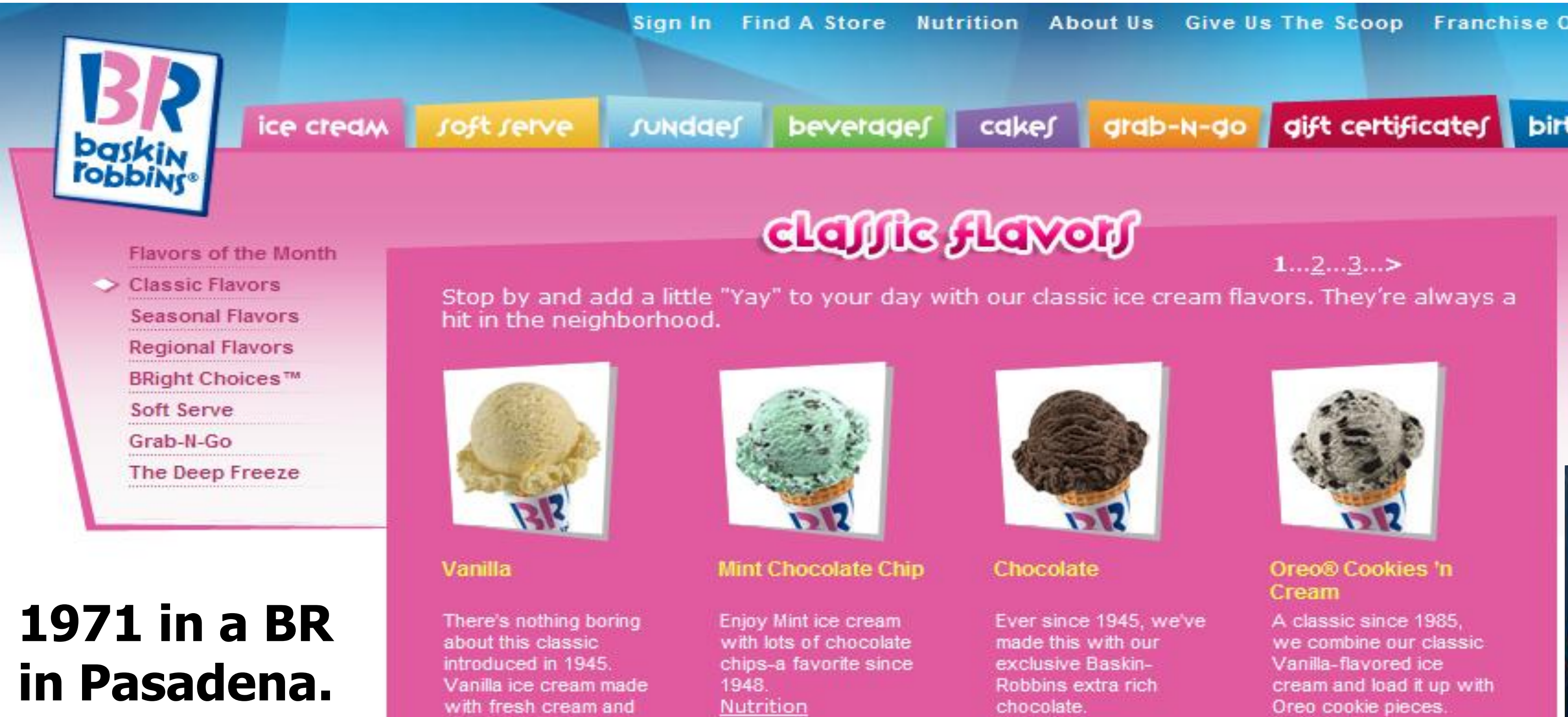
@ the 2nd Asia-Europe-Pacific School of HEP, 11/2014, Puri, India

12 known flavors

	Discoveries of lepton flavors, quark flavors and CP violation
1897	electron (Thomson, 1897)
1919	proton (up and down quarks) (Rutherford, 1919)
1932	neutron (up and down quarks) (Chadwick, 1932)
1933	positron (Anderson, 1933)
1937	muon (Neddermeyer and Anderson, 1937)
1947	Kaon (strange quark) (Rochester and Butler, 1947)
1956	electron antineutrino (Cowan *et al.*, 1956)
1962	muon neutrino (Danby *et al.*, 1962)
1964	CP violation in *s*-quark decays (Christenson *et al.*, 1964)
1974	charm quark (Aubert *et al.*, 1974; Abrams *et al.*, 1974)
1975	tau (Perl *et al.*, 1975)
1977	bottom quark (Herb *et al.*, 1977)
1995	top quark (Abe *et al.*, 1995; Abachi *et al.*, 1995)
2000	tau neutrino (Kodama *et al.*, 2000)
2001	CP violation in *b*-quark decays (Aubert *et al.*, 2001; Abe *et al.*, 2001)

USA

Harald Fritzsch and Murray Gell-Mann coined the “flavor” !



1971 in a BR in Pasadena.

Lecture B1

★ **The 3X3 Neutrino Mixing Matrix**

★ **Neutrino Oscillations in Vacuum**

★ **Neutrino Oscillations in Matter**

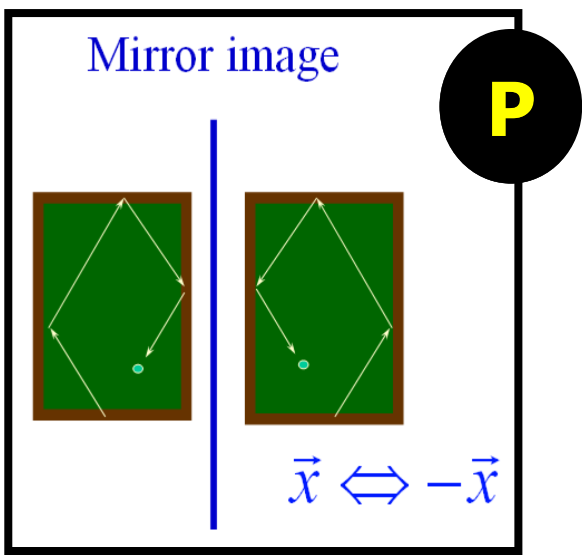
Flavor mixing

Flavor mixing: mismatch between **weak/flavor** eigenstates and **mass** eigenstates of fermions due to coexistence of **2** types of interactions.

Weak eigenstates: members of weak isospin doublets transforming into each other through the interaction with the *W* boson;

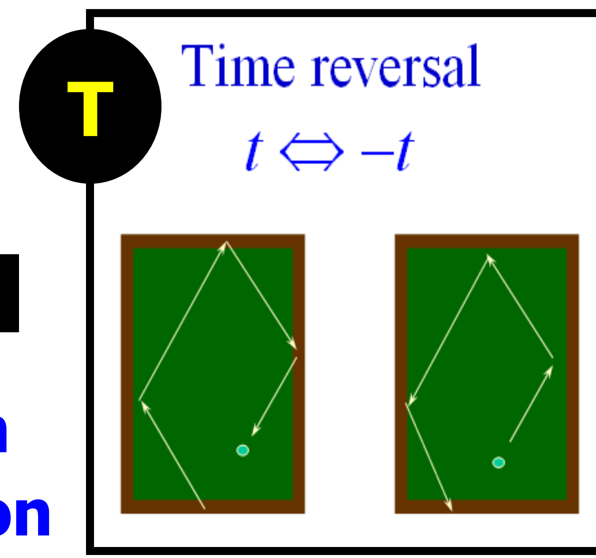
Mass eigenstates: states of definite masses that are created by the interaction with the Higgs boson (**Yukawa** interactions).

CP violation: **matter** and **antimatter**, or a reaction & its CP-conjugate process, are distinguishable --- coexistence of **2** types of interactions.



1957: **P violation**

1964: **CP violation**



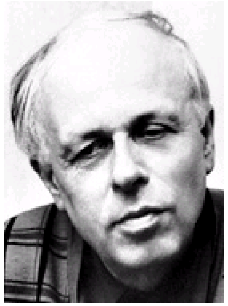
Towards the KM paper

1964: Discovery of CP violation in K decays (J.W. Cronin, Val L. Fitch)

NP 1980

1967: Sakharov conditions for cosmological matter-antimatter asymmetry (A. Sakharov)

NP 1975



1967: The standard model of electromagnetic and weak interactions without quarks (S. Weinberg)

0 citation for the first 4 yrs

NP 1979

1971: The first proof of the renormalizability of the standard model (G. 't Hooft)

NP 1999

KM in 1972

Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

In a framework of the renormalizable theory of weak interaction, problems of *CP*-violation are studied. It is concluded that no realistic models of *CP*-violation exist in the quartet scheme without introducing any other new fields. Some possible models of *CP*-violation are also discussed.

3 families allow for CP violation: Maskawa's bathtub idea!

"as I was getting out of the bathtub, an idea came to me"

Diagnosis of CP violation

In the minimal νSM (namely, SM+3 right-handed ν's) , the KM mechanism is responsible for CP violation.

$$\boxed{\mathcal{L}_{\nu\mathrm{SM}} = \mathcal{L}_{\mathrm{G}} + \mathcal{L}_{\mathrm{H}} + \mathcal{L}_{\mathrm{F}} + \mathcal{L}_{\mathrm{Y}}}$$

See the book by Xing + Zhou for a detailed proof

$$\mathcal{L}_{\mathrm{G}} = -\frac{1}{4}\left(W^{i\mu\nu}W^{i}_{\mu\nu} + B^{\mu\nu}B_{\mu\nu}\right)$$

$$\mathcal{L}_{\mathrm{H}} = \left(D^{\mu}H\right)^{\dagger}\left(D_{\mu}H\right) - \mu^{2}H^{\dagger}H - \lambda\left(H^{\dagger}H\right)^{2}$$

$$\mathcal{L}_{\mathrm{F}} = \overline{Q_{\mathrm{L}}}\,i\not{D}\,Q_{\mathrm{L}} + \overline{\ell_{\mathrm{L}}}\,i\not{D}\,\ell_{\mathrm{L}} + \overline{U_{\mathrm{R}}}\,i\not{\partial}'\,U_{\mathrm{R}} + \overline{D_{\mathrm{R}}}\,i\not{\partial}'\,D_{\mathrm{R}} + \overline{E_{\mathrm{R}}}\,i\not{\partial}'\,E_{\mathrm{R}} + \underline{\overline{N_{\mathrm{R}}}\,i\not{\partial}'\,N_{\mathrm{R}}}$$

$$\mathcal{L}_{\mathrm{Y}} = -\overline{Q_{\mathrm{L}}}Y_{\mathrm{u}}\tilde{H}U_{\mathrm{R}} - \overline{Q_{\mathrm{L}}}Y_{\mathrm{d}}HD_{\mathrm{R}} - \overline{\ell_{\mathrm{L}}}Y_{l}HE_{\mathrm{R}} - \underline{\overline{\ell_{\mathrm{L}}}Y_{\nu}\tilde{H}N_{\mathrm{R}}} + \mathrm{h.c.}$$

ν's Dirac mass

The strategy of diagnosis: **given proper CP transformations of gauge, Higgs and fermion fields, we may prove that the 1st, 2nd and 3rd terms are formally invariant, and hence the 4th term can be invariant only if provided the corresponding Yukawa coupling matrices are real. (Note that the SM spontaneous symmetry breaking itself doesn't affect CP.)**

CP transformations

Gauge fields:

$$\boxed{\left[B_\mu,\ W^1_\mu,\ W^2_\mu,\ W^3_\mu\right] \xrightarrow{\text{CP}} \left[-B^\mu,\ -W^{1\mu},\ +W^{2\mu},\ -W^{3\mu}\right]}$$

$$\boxed{\left[B_{\mu\nu},\ W^1_{\mu\nu},\ W^2_{\mu\nu},\ W^3_{\mu\nu}\right] \xrightarrow{\text{CP}} \left[-B^{\mu\nu},\ -W^{1\mu\nu},\ +W^{2\mu\nu},\ -W^{3\mu\nu}\right]}$$

Higgs fields:

$$\boxed{H(t,\mathbf{x}) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \xrightarrow{\text{CP}} H^*(t,-\mathbf{x}) = \begin{pmatrix} \phi^- \\ \phi^{0*} \end{pmatrix}}$$

Lepton or quark fields:

$$\boxed{\overline{\psi_1}\gamma_\mu\left(1\pm\gamma_5\right)\psi_2 \xrightarrow{\text{CP}} -\overline{\psi_2}\gamma^\mu\left(1\pm\gamma_5\right)\psi_1}$$

$$\boxed{\overline{\psi_1}\gamma_\mu\left(1\pm\gamma_5\right)\partial^\mu\psi_2 \xrightarrow{\text{CP}} \overline{\psi_2}\gamma^\mu\left(1\pm\gamma_5\right)\partial_\mu\psi_1}$$

Spinor bilinears:

	$\overline{\psi_1}\psi_2$	$i\overline{\psi_1}\gamma_5\psi_2$	$\overline{\psi_1}\gamma_\mu\psi_2$	$\overline{\psi_1}\gamma_\mu\gamma_5\psi_2$	$\overline{\psi_1}\sigma_{\mu\nu}\psi_2$
C	$\overline{\psi_2}\psi_1$	$i\overline{\psi_2}\gamma_5\psi_1$	$-\overline{\psi_2}\gamma_\mu\psi_1$	$\overline{\psi_2}\gamma_\mu\gamma_5\psi_1$	$-\overline{\psi_2}\sigma_{\mu\nu}\psi_1$
P	$\overline{\psi_1}\psi_2$	$-i\overline{\psi_1}\gamma_5\psi_2$	$\overline{\psi_1}\gamma^\mu\psi_2$	$-\overline{\psi_1}\gamma^\mu\gamma_5\psi_2$	$\overline{\psi_1}\sigma^{\mu\nu}\psi_2$
T	$\overline{\psi_1}\psi_2$	$-i\overline{\psi_1}\gamma_5\psi_2$	$\overline{\psi_1}\gamma^\mu\psi_2$	$\overline{\psi_1}\gamma^\mu\gamma_5\psi_2$	$-\overline{\psi_1}\sigma^{\mu\nu}\psi_2$
CP	$\overline{\psi_2}\psi_1$	$-i\overline{\psi_2}\gamma_5\psi_1$	$-\overline{\psi_2}\gamma^\mu\psi_1$	$-\overline{\psi_2}\gamma^\mu\gamma_5\psi_1$	$-\overline{\psi_2}\sigma^{\mu\nu}\psi_1$
CPT	$\overline{\psi_2}\psi_1$	$i\overline{\psi_2}\gamma_5\psi_1$	$-\overline{\psi_2}\gamma_\mu\psi_1$	$-\overline{\psi_2}\gamma_\mu\gamma_5\psi_1$	$\overline{\psi_2}\sigma_{\mu\nu}\psi_1$

$\boxed{\mathcal{L}_\mathrm{G}}$ $\boxed{\mathcal{L}_\mathrm{H}}$ $\boxed{\mathcal{L}_\mathrm{F}}$

formally invariant under CP

CP violation

The Yukawa interactions of fermions are formally invariant under CP if and only if

$$Y_{\rm u} = Y_{\rm u}^* \; , \qquad Y_{\rm d} = Y_{\rm d}^*$$
$$Y_l = Y_l^* \; , \qquad Y_\nu = Y_\nu^*$$

If the effective Majorana mass term is added into the SM, then the Yukawa interactions of leptons can be formally invariant under CP if

$$M_{\rm L} = M_{\rm L}^* \; , \qquad Y_l = Y_l^*$$

If the flavor states are transformed into the mass states, the source of flavor mixing and CP violation will show up in the *CC* interactions:

quarks

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(u \;\, c \;\, t)_{\rm L}} \; \gamma^\mu U \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\rm L} W^+_\mu + {\rm h.c.}$$

leptons

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e \;\, \mu \;\, \tau)_{\rm L}} \; \gamma^\mu V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

Comment A: CP violation exists since fermions interact with both the gauge bosons and the Higgs boson.

Comment B: both the CC and Yukawa interactions have been verified.

Comment C: the CKM matrix *U* is unitary, the MNSP matrix *V* is too?

Parameter counting

The 3×3 unitary matrix V can always be parametrized as a product of 3 unitary rotation matrices in the complex planes:

$$O_1(\theta_1, \alpha_1, \beta_1, \gamma_1) = \begin{pmatrix} c_1 e^{i\alpha_1} & s_1 e^{-i\beta_1} & 0 \\ -s_1 e^{i\beta_1} & c_1 e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{i\gamma_1} \end{pmatrix}$$

$$O_2(\theta_2, \alpha_2, \beta_2, \gamma_2) = \begin{pmatrix} e^{i\gamma_2} & 0 & 0 \\ 0 & c_2 e^{i\alpha_2} & s_2 e^{-i\beta_2} \\ 0 & -s_2 e^{i\beta_2} & c_2 e^{-i\alpha_2} \end{pmatrix}$$

$$O_3(\theta_3, \alpha_3, \beta_3, \gamma_3) = \begin{pmatrix} c_3 e^{i\alpha_3} & 0 & s_3 e^{-i\beta_3} \\ 0 & e^{i\gamma_3} & 0 \\ -s_3 e^{i\beta_3} & 0 & c_3 e^{-i\alpha_3} \end{pmatrix}$$

where $s_i \equiv \sin\theta_i$ and $c_i \equiv \cos\theta_i$ (for $i = 1, 2, 3$)

Category A: 3 possibilities

$$V = O_i O_j O_i \quad (i \neq j)$$

Category B: 6 possibilities

$$V = O_i O_j O_k \quad (i \neq j \neq k)$$

Phases

For instance, the standard parametrization is given below:

V

$$= \begin{pmatrix} e^{i\gamma_2} & 0 & 0 \\ 0 & c_2 e^{\alpha_2} & s_2 e^{-i\beta_2} \\ 0 & -s_2 e^{i\beta_2} & c_2 e^{-i\alpha_2} \end{pmatrix} \begin{pmatrix} c_3 e^{\alpha_3} & 0 & s_3 e^{-i\beta_3} \\ 0 & e^{i\gamma_3} & 0 \\ -s_3 e^{i\beta_3} & 0 & c_3 e^{-i\alpha_3} \end{pmatrix} \begin{pmatrix} c_1 e^{\alpha_1} & s_1 e^{-i\beta_1} & 0 \\ -s_1 e^{i\beta_1} & c_1 e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{i\gamma_1} \end{pmatrix}$$

$$= \begin{pmatrix} c_1 c_3 e^{i(\alpha_1+\gamma_2+\alpha_3)} & s_1 c_3 e^{i(-\beta_1+\gamma_2+\alpha_3)} & s_3 e^{i(\gamma_1+\gamma_2-\beta_3)} \\ -s_1 c_2 e^{i(\beta_1+\alpha_2+\gamma_3)} - c_1 s_2 s_3 e^{i(\alpha_1-\beta_2+\beta_3)} & c_1 c_2 e^{i(-\alpha_1+\alpha_2+\gamma_3)} - s_1 s_2 s_3 e^{i(-\beta_1-\beta_2+\beta_3)} & s_2 c_3 e^{i(\gamma_1-\beta_2-\alpha_3)} \\ s_1 s_2 e^{i(\beta_1+\beta_2+\gamma_3)} - c_1 c_2 s_3 e^{i(\alpha_1-\alpha_2+\beta_3)} & -c_1 s_2 e^{i(-\alpha_1+\beta_2+\gamma_3)} - s_1 c_2 s_3 e^{i(-\beta_1-\alpha_2+\beta_3)} & c_2 c_3 e^{i(\gamma_1-\alpha_2-\alpha_3)} \end{pmatrix}$$

$$= \begin{pmatrix} e^{ia} & 0 & 0 \\ 0 & e^{ib} & 0 \\ 0 & 0 & e^{ic} \end{pmatrix} \begin{pmatrix} c_1 c_3 & s_1 c_3 & s_3 e^{-i\delta} \\ -s_1 c_2 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 - s_1 s_2 s_3 e^{i\delta} & s_2 c_3 \\ s_1 s_2 - c_1 c_2 s_3 e^{i\delta} & -c_1 s_2 - s_1 c_2 s_3 e^{i\delta} & c_2 c_3 \end{pmatrix} \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix}$$

$$a = (\alpha_1 - \beta_1) - (\alpha_2 + \beta_2 - \gamma_2) - \gamma_3 \ , \quad b = -\beta_2 - \alpha_3 \ , \quad c = -\alpha_2 - \alpha_3 \ ;$$
$$x = \beta_1 + (\alpha_2 + \beta_2) + (\alpha_3 + \gamma_3) \ , \quad y = -\alpha_1 + (\alpha_2 + \beta_2) + (\alpha_3 + \gamma_3) \ , \quad z = \gamma_1 \ .$$

$$\delta = \beta_3 - \gamma_1 - \gamma_2$$

Physical phases

If neutrinos are Dirac particles, the phases *x*, *y* and *z* can be removed. Then the neutrino mixing matrix is

Dirac neutrino mixing matrix

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

If neutrinos are Majorana particles, left- and right-handed fields are correlated. Hence only a common phase of three left-handed fields can be redefined (e.g., $z = 0$). Then

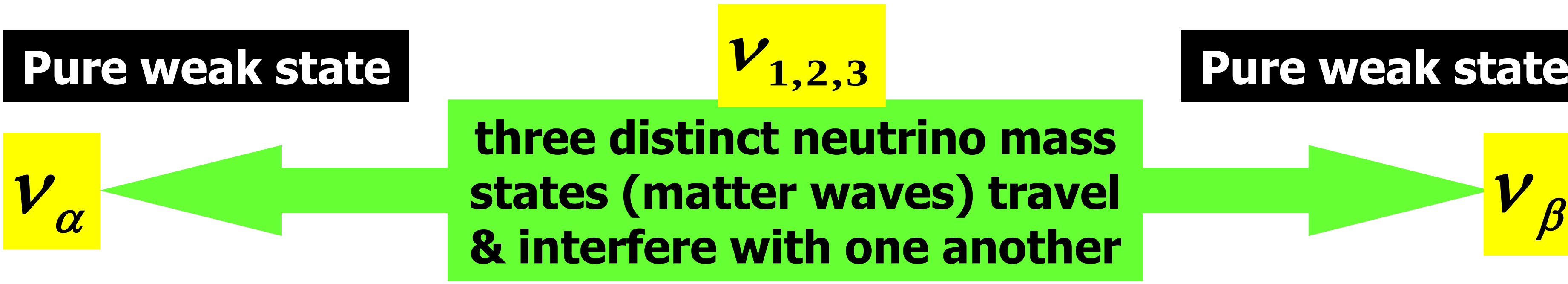
Majorana neutrino mixing matrix

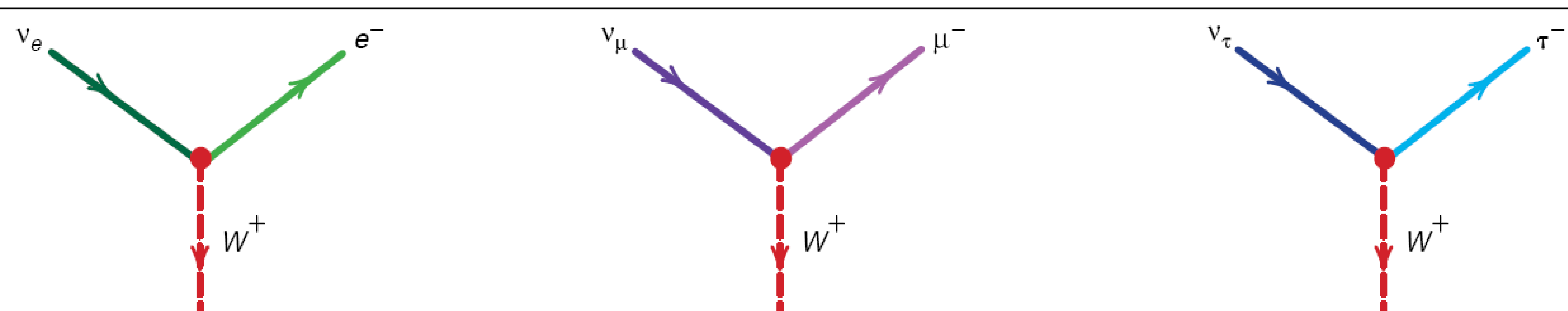
$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

What is oscillation?

Oscillation — a spontaneous periodic change from one neutrino flavor state to another, is a spectacular quantum phenomenon. It can occur as a natural consequence of neutrino mixing.

In a neutrino oscillation experiment, the neutrino beam is produced and detected via the weak charged-current interactions.





For example : $\overline{\nu_e}$ beam : β decay; ν_μ beam : π decay; ν_τ beam : D decay

How to calculate?

Boris Kayser (hep-ph/0506165): This change of neutrino flavor is a quintessentially quantum-mechanical effect. Indeed, it entails some quantum-mechanical subtleties that are still debated to this day. However, there is little debate about the "bottom line" ------- the expression for the flavor-change probability……

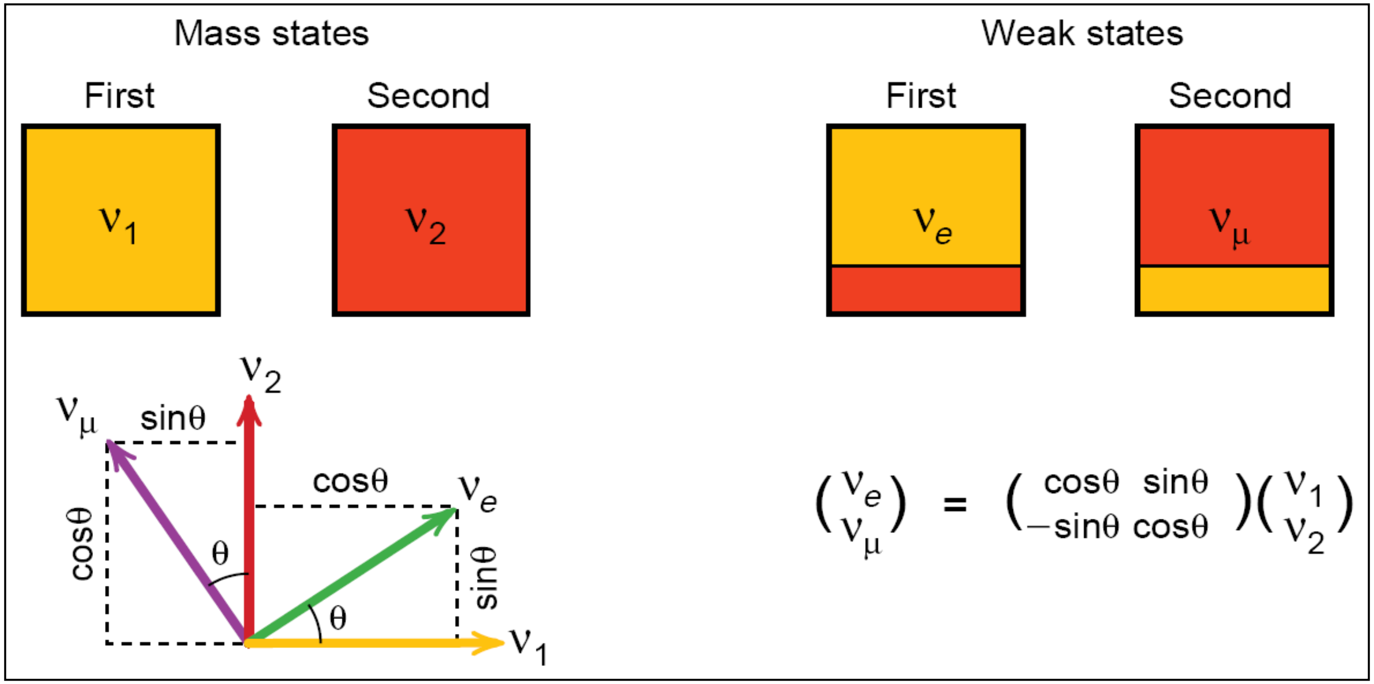
Some typical references:

- ♣ **Giunti, Kim, "Fundamentals of Neutrino Physics and Astrophysics" (2007)**
- ♣ **Cohen, Glashow, Ligeti: "Disentangling Neutrino Oscillations" (0810.4602)**
- ♣ **Akhmedov, Smirnov: "Paradoxes of Neutrino Oscillations" (0905.1903)**

Our strategy: follow the simplest way (which is conceptually ill) to derive the "bottom line" of neutrino oscillations: the leading-order formula of neutrino oscillations in phenomenology.

2-flavor oscillation (1)

For simplicity, we consider two-flavor neutrino mixing and oscillation:



Approximation:

a plane wave with a common momentum for each mass state

$$|\nu_\mu(0)\rangle = |\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$

$$|\nu_\mu(t)\rangle = -\sin\theta e^{-iE_1 t}|\nu_1\rangle + \cos\theta e^{-iE_2 t}|\nu_2\rangle$$

$$= e^{-iE_1 t}\left(-\sin\theta|\nu_1\rangle + \cos\theta e^{-i\Delta E t}|\nu_2\rangle\right)$$

$$\Delta E \equiv E_2 - E_1 = \sqrt{p^2 + m_2^2} - \sqrt{p^2 + m_1^2}$$

$$\approx \left(p + \frac{m_2^2}{2p}\right) - \left(p + \frac{m_1^2}{2p}\right) \approx \frac{\Delta m^2}{2E}$$

$$\Delta m^2 \equiv m_2^2 - m_1^2\,, \quad E \approx p \gg m_{1,2}\,(\text{relativistic neutrino beam})\,, \quad \hbar = c = 1\,(\text{natural units})$$

2-flavor oscillation (2)

The oscillation probability for appearance ν experiments:

$$\begin{aligned} P\left(\nu_\mu \to \nu_e\right) &= \left|\langle \nu_e | \nu_\mu(t)\rangle\right|^2 = \left|\left(\cos\theta\langle\nu_1| + \sin\theta\langle\nu_2|\right)\left(-\sin\theta|\nu_1\rangle + \cos\theta e^{-i\Delta E t}|\nu_2\rangle\right)\right|^2 \\ &= \left|\sin\theta\cos\theta\left(1 - e^{-i\Delta E t}\right)\right|^2 = 2\left(\sin\theta\cos\theta\right)^2\left(1 - \cos\frac{\Delta m^2 t}{2E}\right) \\ &= \sin^2 2\theta \sin^2\frac{\Delta m^2 L}{4E} \end{aligned}$$

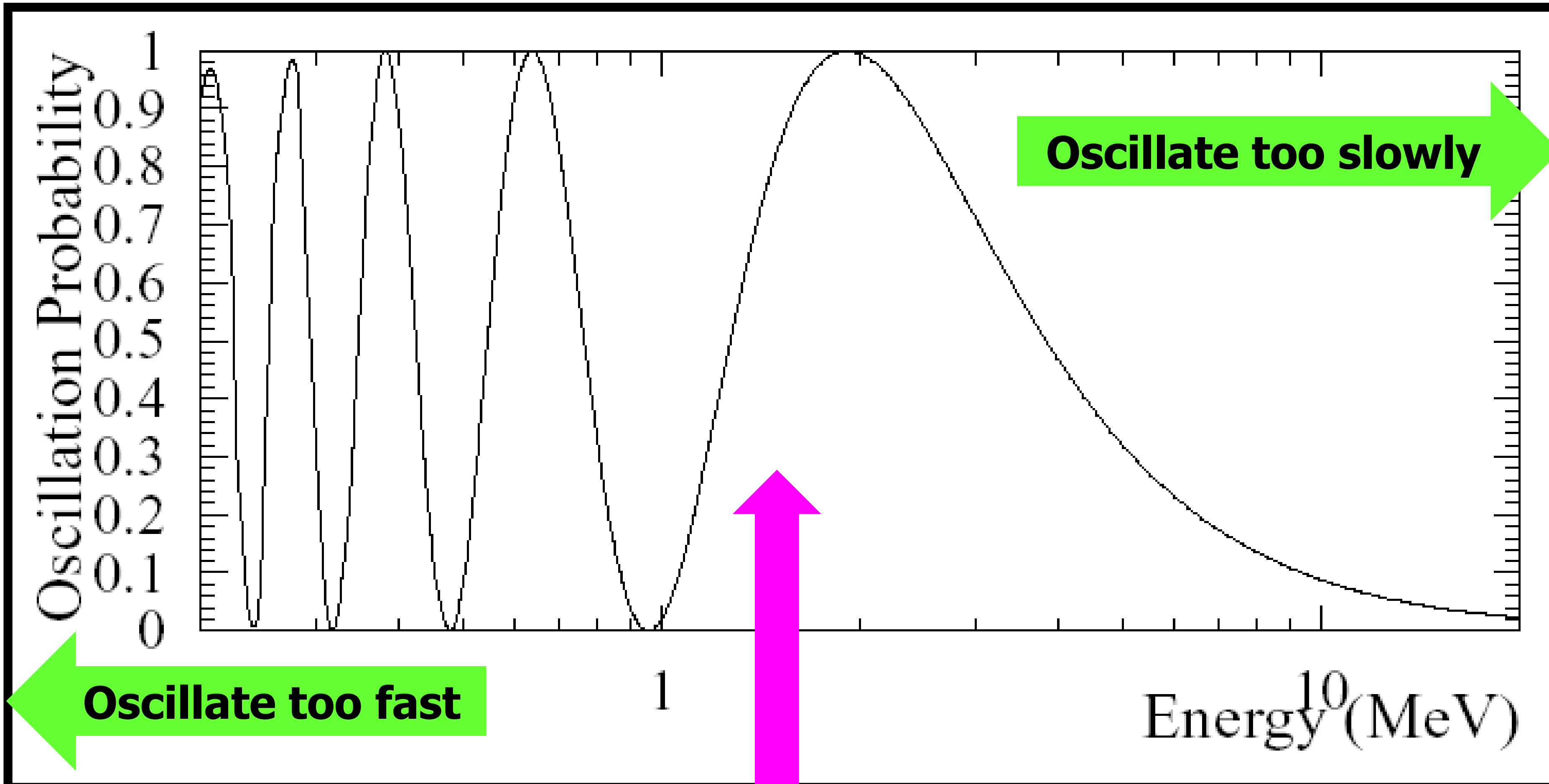
The conversion and survival probabilities in realistic units:

$$\begin{aligned} P\left(\nu_\mu \to \nu_e\right) &= \sin^2 2\theta \sin^2\frac{1.27\Delta m^2 L}{E} \\ P\left(\nu_\mu \to \nu_\mu\right) &= 1 - \sin^2 2\theta \sin^2\frac{1.27\Delta m^2 L}{E} \end{aligned}$$

Due to the smallness of (1,3) mixing, both solar & atmospheric neutrino oscillations are roughly the 2-flavor oscillation.

Δm^2 in unit of eV^2 , L in unit of km , E in unit of GeV

2-flavor oscillation (3)



$$P(\nu_e \to \nu_\mu) = |\langle \nu_\mu | \nu(t) \rangle|^2 = \sin^2 2\theta \sin^2 \left(\frac{1.27 \Delta m^2 L}{E} \right)$$

Exercise: why 1.27 ?

	Natural units	Realistic units
Phase factors	$\exp\left(-iE_{1,2}t\right)$	$\exp\left(-i\frac{E_{1,2}}{\hbar}t\right)$
Energies and momentum	$E_{1,2}=\sqrt{p^2+m_{1,2}^2}$	$E_{1,2}=\sqrt{p^2c^2+m_{1,2}^2c^4}$
Energy difference	$\Delta E=\frac{\Delta m^2}{2E}$	$\Delta E=\frac{\Delta m^2c^3}{2p}=\frac{\Delta m^2c^4}{2E}$
Time and distance	$t=L$	$t=\frac{L}{c}$
Oscillation argument	$\frac{1}{2}\Delta Et=\frac{\Delta m^2L}{4E}$	$\frac{1}{2}\frac{\Delta E}{\hbar}t=\frac{c^3}{\hbar}\cdot\frac{\Delta m^2L}{4E}$

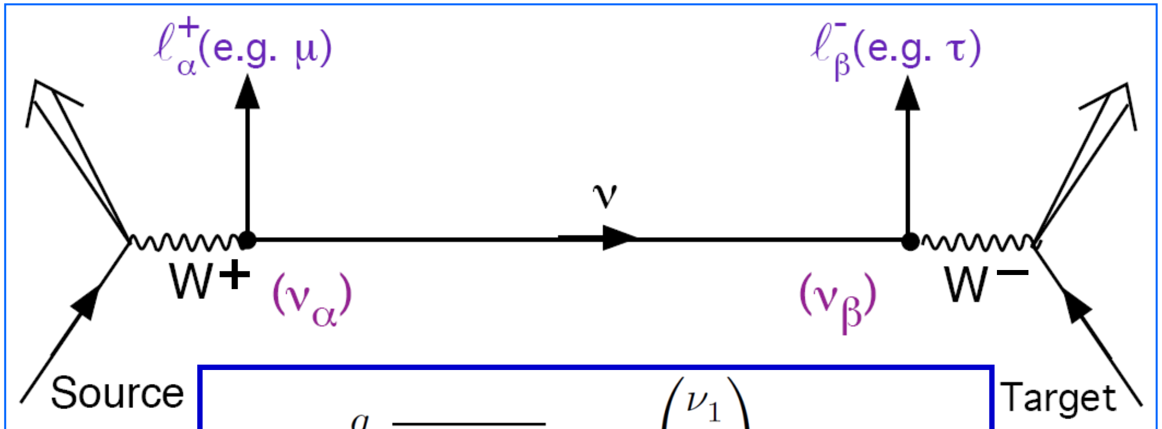
$$c = 2.998\times10^5\ \mathrm{km\,s^{-1}}$$
$$\hbar = 6.582\times10^{-25}\ \mathrm{GeV\,s}$$

$$\frac{c^3}{4\hbar}\ \Rightarrow\ \frac{1}{4\times0.1973}=1.267\approx1.27$$

$$c=1\quad\Rightarrow\quad \hbar=6.582\times10^{-25}\ \mathrm{GeV}\times2.998\times10^5\ \mathrm{km}$$
$$=1.973\times10^{-19}\ \mathrm{GeV\,km}=0.1973\mathrm{eV^2\,GeV^{-1}\,km}$$

3-flavor oscillation (1)

Production and detection of a neutrino beam by CC weak interactions:



$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e\ \mu\ \tau)_{\rm L}}\ \gamma^\mu V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W_\mu^- + {\rm h.c.}$$

$$|\nu_\alpha(0)\rangle = |\nu_\alpha\rangle = \sum_{i=1}^{3} \underline{V_{\alpha i}^*} |\nu_i\rangle$$

$$|\nu_\alpha(t)\rangle = \sum_{i=1}^{3} \underline{V_{\alpha i}^*} e^{-iE_i t} |\nu_i\rangle$$

$\alpha, \beta, \gamma = e, \mu, \tau$

$i, j, k = 1,2,3$

The amplitude and probability of neutrino oscillations:

$$A(\nu_\alpha \to \nu_\beta) = \langle \nu_\beta | \nu_\alpha(t) \rangle = \left(\sum_{j=1}^{3} V_{\beta j} \langle \nu_j | \right) \left(\sum_{i=1}^{3} V_{\alpha i}^* e^{-iE_i t} |\nu_i\rangle \right) = \sum_{i=1}^{3} V_{\alpha i}^* V_{\beta i} e^{-iE_i t}$$

$$P(\nu_\alpha \to \nu_\beta) = \left| \langle \nu_\beta | \nu_\alpha(t) \rangle \right|^2 = \left| \sum_{i=1}^{3} V_{\alpha i}^* V_{\beta i} e^{-iE_i t} \right|^2$$

$$= \sum_{i=1}^{3} \left| V_{\alpha i}^* V_{\beta i} \right|^2 + 2 \sum_{i<j}^{3} {\rm Re} \left[V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\beta j}^* e^{i(E_j - E_i)t} \right]$$

3-flavor oscillation (2)

The formula of three-flavor oscillation probability with CP/T violation:

$$
\begin{aligned}
P\left(\nu_\alpha \to \nu_\beta\right) &= \sum_{i=1}^{3} \left|V_{\alpha i}^* V_{\beta i}\right|^2 + 2\sum_{i<j}^{3} \mathrm{Re}\left(V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\beta j}^*\right) \cos\frac{\Delta m_{ji}^2 L}{2E} \\
&\quad - 2\sum_{i<j}^{3} \mathrm{Im}\left(V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\beta j}^*\right) \sin\frac{\Delta m_{ji}^2 L}{2E} \\
&= \sum_{i=1}^{3} \left|V_{\alpha i}^* V_{\beta i}\right|^2 + 2\sum_{i<j}^{3} \mathrm{Re}\left(V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\beta j}^*\right) \\
&\quad -4\sum_{i<j}^{3} \mathrm{Re}\left(V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\beta j}^*\right) \sin^2\frac{\Delta m_{ji}^2 L}{4E} - 2\sum_{i<j}^{3} \mathrm{Im}\left(V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\beta j}^*\right) \sin\frac{\Delta m_{ji}^2 L}{2E} \\
&= \left|\sum_{i=1}^{3} V_{\alpha i}^* V_{\beta i}\right|^2 - 4\sum_{i<j}^{3} \mathrm{Re}\left(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*\right) \sin^2\frac{\Delta m_{ji}^2 L}{4E} \\
&\quad + 2\sum_{i<j}^{3} \mathrm{Im}\left(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*\right) \sin\frac{\Delta m_{ji}^2 L}{2E}
\end{aligned}
$$

Jarlskog

$$\left|\sum_{i=1}^{3} V_{\alpha i}^* V_{\beta i}\right|^2 = \delta_{\alpha\beta}$$

$$\mathrm{Im}\left(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*\right) = \mathcal{J} \sum_{\gamma, k} \left(\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}\right)$$

3-flavor oscillation (3)

The final formula of 3-flavor oscillation probabilities with CP violation:

$$
\begin{aligned}
P\left(\nu_\alpha \to \nu_\beta\right) \quad &= \quad \delta_{\alpha\beta} - 4\sum_{i<j}^{3} \mathrm{Re}\left(V_{\alpha i}V_{\beta j}V_{\alpha j}^{*}V_{\beta i}^{*}\right)\sin^2\frac{\Delta m_{ji}^2 L}{4E} \\
&\quad + 8\mathcal{J}\sum_{\gamma}\epsilon_{\alpha\beta\gamma}\sin\frac{\Delta m_{21}^2 L}{4E}\sin\frac{\Delta m_{31}^2 L}{4E}\sin\frac{\Delta m_{32}^2 L}{4E}
\end{aligned}
$$

The 1st oscillating term: CP conserving; and the 2nd term: CP violating!

$$
\begin{aligned}
& 2\sum_{i<j}^{3}\mathrm{Im}\left(V_{\alpha i}V_{\beta j}V_{\alpha j}^{*}V_{\beta i}^{*}\right)\sin\frac{\Delta m_{ji}^2 L}{2E} \\
= \quad & +2\mathcal{J}\sum_{\gamma}\epsilon_{\alpha\beta\gamma}\left(\sin\frac{\Delta m_{21}^2 L}{2E} - \sin\frac{\Delta m_{31}^2 L}{2E} + \sin\frac{\Delta m_{32}^2 L}{2E}\right) \\
= \quad & -2\mathcal{J}\sum_{\gamma}\epsilon_{\alpha\beta\gamma}\left(\sin\frac{\Delta m_{12}^2 L}{2E} + \sin\frac{\Delta m_{23}^2 L}{2E} + \sin\frac{\Delta m_{31}^2 L}{2E}\right) \\
= \quad & +8\mathcal{J}\sum_{\gamma}\epsilon_{\alpha\beta\gamma}\sin\frac{\Delta m_{12}^2 L}{4E}\sin\frac{\Delta m_{23}^2 L}{4E}\sin\frac{\Delta m_{31}^2 L}{4E}
\end{aligned}
$$

NOTE: If you have seen a different sign in front of the CP-violating part in a lot of literature, it most likely means that a complex conjugation of *V* in the production point of neutrino beam was not properly taken into account.

Discrete symmetries

Basic expression

$$P\left(\nu_\alpha \to \nu_\beta\right) = \delta_{\alpha\beta} - 4\sum_{i<j}^{3} \mathrm{Re}\left(V_{\alpha i}V_{\beta j}V^*_{\alpha j}V^*_{\beta i}\right)\sin^2\frac{\Delta m^2_{ji}L}{4E} + 8\mathcal{J}\sum_{\gamma}\epsilon_{\alpha\beta\gamma}\sin\frac{\Delta m^2_{21}L}{4E}\sin\frac{\Delta m^2_{31}L}{4E}\sin\frac{\Delta m^2_{32}L}{4E}$$

CP transformation

$$V \to V^*$$

$$J \to -J$$

$$P\left(\overline{\nu}_\alpha \to \overline{\nu}_\beta\right) = \delta_{\alpha\beta} - 4\sum_{i<j}^{3} \mathrm{Re}\left(V_{\alpha i}V_{\beta j}V^*_{\alpha j}V^*_{\beta i}\right)\sin^2\frac{\Delta m^2_{ji}L}{4E} - 8\mathcal{J}\sum_{\gamma}\epsilon_{\alpha\beta\gamma}\sin\frac{\Delta m^2_{21}L}{4E}\sin\frac{\Delta m^2_{31}L}{4E}\sin\frac{\Delta m^2_{32}L}{4E}$$

T transformation

$$\alpha \leftrightarrow \beta$$

$$P\left(\nu_\beta \to \nu_\alpha\right) = \delta_{\alpha\beta} - 4\sum_{i<j}^{3} \mathrm{Re}\left(V_{\alpha i}V_{\beta j}V^*_{\alpha j}V^*_{\beta i}\right)\sin^2\frac{\Delta m^2_{ji}L}{4E} - 8\mathcal{J}\sum_{\gamma}\epsilon_{\alpha\beta\gamma}\sin\frac{\Delta m^2_{21}L}{4E}\sin\frac{\Delta m^2_{31}L}{4E}\sin\frac{\Delta m^2_{32}L}{4E}$$

CPT invariance

$$P\left(\overline{\nu}_\beta \to \overline{\nu}_\alpha\right) = P\left(\nu_\alpha \to \nu_\beta\right)$$

The 1st paper on CPV

Volume 72B, number 3 PHYSICS LETTERS 2 January 1978

TIME REVERSAL VIOLATION IN NEUTRINO OSCILLATION

Nicola CABIBBO*

*Laboratoire de Physique Théorique et Hautes Energies, Paris, France***

Received 11 October 1977

We discuss the possibility of CP or T violation in neutrino oscillation CP requires $\nu_\mu \leftrightarrow \nu_e$ and $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$ oscillations to be equal Time reversal invariance requires the oscillation probability to be an even function of time Both conditions can be violated, even drastically, if more than two neutrinos exist

Tri-maximal neutrino mixing + maximal CP violation:

$$\boxed{J = 1/6\sqrt{3}}$$

$$A = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^* \\ 1 & a^* & a \end{pmatrix}, \qquad a = \exp[2\pi i/3]$$

CP & T violation

Under CPT invariance, CP- and T-violating asymmetries are identical:

$$\begin{aligned} P\left(\nu_\alpha \to \nu_\beta\right) - P\left(\overline{\nu}_\alpha \to \overline{\nu}_\beta\right) &= P\left(\nu_\alpha \to \nu_\beta\right) - P\left(\nu_\beta \to \nu_\alpha\right) \\ &= 16\mathcal{J} \sum_\gamma \epsilon_{\alpha\beta\gamma} \sin\frac{\Delta m^2_{21} L}{4E} \sin\frac{\Delta m^2_{31} L}{4E} \sin\frac{\Delta m^2_{32} L}{4E} \end{aligned}$$

Intrinsic CPV × three oscillating terms

Comments:

- ★ **CP / T violation cannot show up in the disappearance neutrino oscillation experiments ($\alpha = \beta$);**
- ★ **CP / T violation is a small three-family flavor effect;**
- ★ **CP / T violation in normal lepton-number-conserving neutrino oscillations depends only upon the Dirac phase of *V*; hence such oscillation experiments cannot tell us whether neutrinos are Dirac or Majorana particles.**

$$J = \sin\theta_{12}\cos\theta_{12}\sin\theta_{23}\cos\theta_{23}\sin\theta_{13}\cos^2\theta_{13}\sin\delta \le 1/6\sqrt{3} \approx 9.6\%$$

Disappearance

Disappearance experiment: one flavor converts to the same one

Appearance experiment: one flavor oscillates into another one.

Most neutrino oscillation experiments are of the disappearance type

$$\begin{aligned} P(\nu_\alpha \to \nu_\alpha) &= 1 - 4\,|V_{\alpha 1}|^2\,|V_{\alpha 2}|^2 \sin^2\frac{\Delta m^2_{21} L}{4E} \\ &\quad -4\,|V_{\alpha 1}|^2\,|V_{\alpha 3}|^2 \sin^2\frac{\Delta m^2_{31} L}{4E} \\ &\quad -4\,|V_{\alpha 2}|^2\,|V_{\alpha 3}|^2 \sin^2\frac{\Delta m^2_{32} L}{4E} \end{aligned}$$

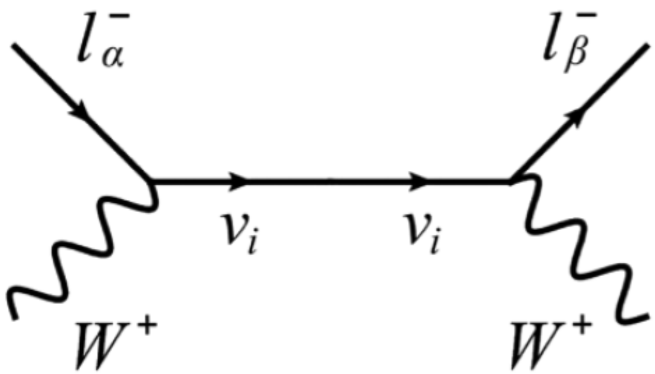
$$|\Delta m^2_{21}| = \Delta m^2_{\rm sun} \ll \Delta m^2_{\rm atm} = |\Delta m^2_{32}| \approx |\Delta m^2_{31}|$$

$\sim 7.6\times 10^{-5}\ \mathrm{eV}^2$ $\qquad$ $\sim 2.4\times 10^{-3}\ \mathrm{eV}^2$

This hierarchy & the small (1,3) mixing lead to the 2-flavor oscillation approximation for many experiments. A few upcoming experiments (long-baseline experiments) will probe the complete 3-flavor effects.

$\nu \Leftrightarrow \overline{\nu}$ oscillations

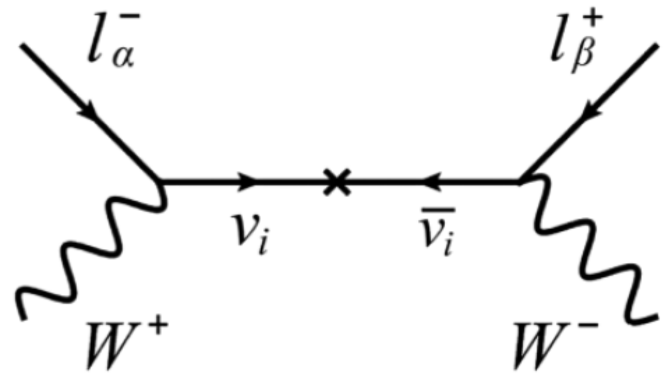
Comparison: neutrino-neutrino and neutrino-antineutrino oscillation experiments.



neutrino → neutrino

$$A = \sum_{k=1}^{3} V_{\alpha k}^{*} V_{\beta k} e^{-iE_k t}$$

Feasible and successful today!



neutrino → antineutrino

$$A = \frac{1}{E} \sum_{k=1}^{3} V_{\alpha k} V_{\beta k} m_k e^{-iE_k t}$$

Unfeasible, a hope tomorrow?

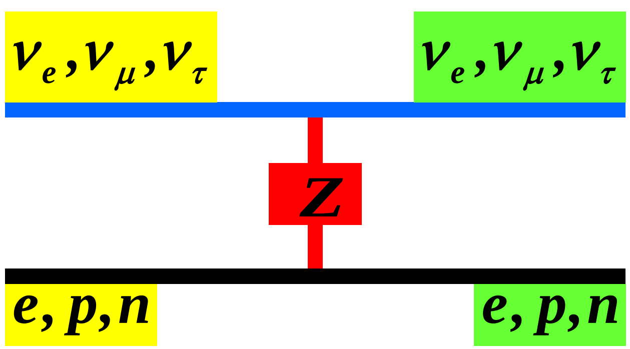
Sensitivity to CP-violating phase(s):

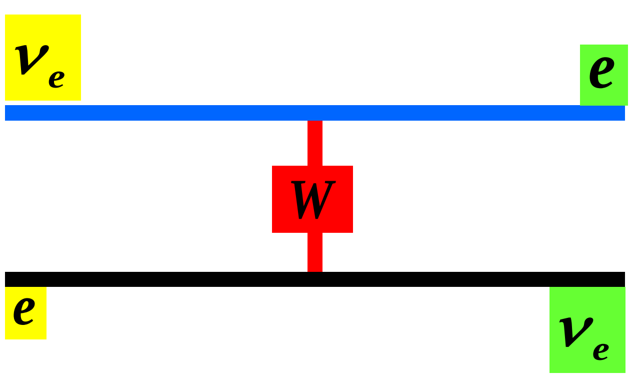
δ δ ρ σ

Matter effects

When light travels through a medium, it sees a refractive index due to coherent forward scattering from the constituents of the medium.

A similar phenomenon applies to neutrino flavor states as they travel through matter. All flavor states see a common refractive index from NC forward scattering, and the electron (anti) neutrino sees an extra refractive index due to CC forward scattering in matter.



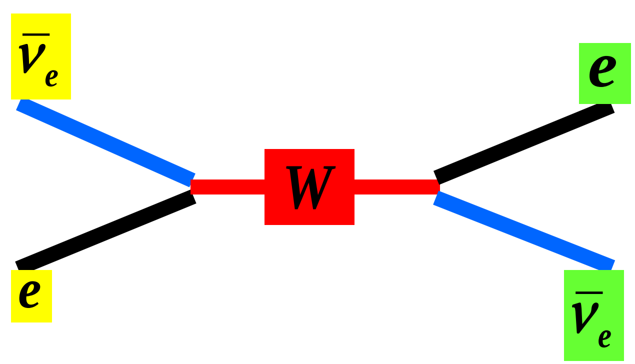


Refractive index

$$n_{\rm nc} = 1 + \frac{2\pi N_e}{p^2} f_{\rm nc}$$

$$n_{\rm cc} = \frac{2\pi N_e}{p^2} f_{\rm cc}$$

$$n_{\rm cc} = \frac{\sqrt{2} G_{\rm F} N_e}{p}$$

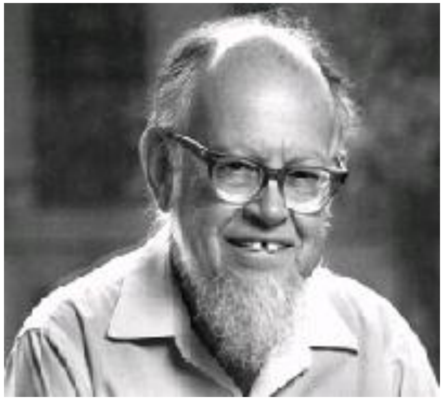


$-n_{cc}$

Matter may matter

In travelling a distance, each neutrino flavor state develops a "matter" phase due to the refractive index. The overall NC-induced phase is trivial, while the relative CC-induced phase may change the behaviors of neutrino oscillations: matter effects — L. Wolfenstein (1978)

$$\nu_e : \exp[ipx(n_{\rm nc} + n_{\rm cc} - 1)]$$
$$\nu_\mu : \exp[ipx(n_{\rm nc} - 1)]$$
$$\nu_\tau : \exp[ipx(n_{\rm nc} - 1)]$$



MSW

Matter effect inside the Sun can enhance the solar neutrino oscillation (S.P. Mikheyev and A.Yu. Smirnov 1985 — MSW effect); matter effect inside the Earth may cause a day-night effect. Note that matter effect in long-baseline experiments might result in fake CP-violating effects.

MSW resonance

Neutrino oscillation in matter (a 2-flavor treatment):

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}G_F N_e & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

$$P(\nu_e \to \nu_\mu)_{\mathrm{v}} = \sin^2 2\theta \sin^2\left(\frac{1.27\Delta m^2 L}{E}\right)$$

$$P(\nu_e \to \nu_\mu)_{\mathrm{m}} = \sin^2 2\tilde{\theta} \sin^2\left(\frac{1.27\Delta \tilde{m}^2 L}{E}\right)$$

The matter density changes for solar neutrinos to travel from the core to the surface

$$\begin{pmatrix}|\nu_e\rangle\\ |\nu_\mu\rangle\end{pmatrix} = \begin{pmatrix}\cos\tilde{\theta} & \sin\tilde{\theta}\\ -\sin\tilde{\theta} & \cos\tilde{\theta}\end{pmatrix}\begin{pmatrix}|\tilde{\nu}_1\rangle\\ |\tilde{\nu}_2\rangle\end{pmatrix}$$

$$\Delta\tilde{m}^2 = \sqrt{\left(\Delta m^2\cos 2\theta - 2\sqrt{2}\,G_{\mathrm{F}}N_e E\right)^2 + (\Delta m^2 \sin 2\theta)^2}$$

$$\tan 2\tilde{\theta} = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2\cos 2\theta - 2\sqrt{2}\,G_{\mathrm{F}}N_e E}$$

MSW

resonance →

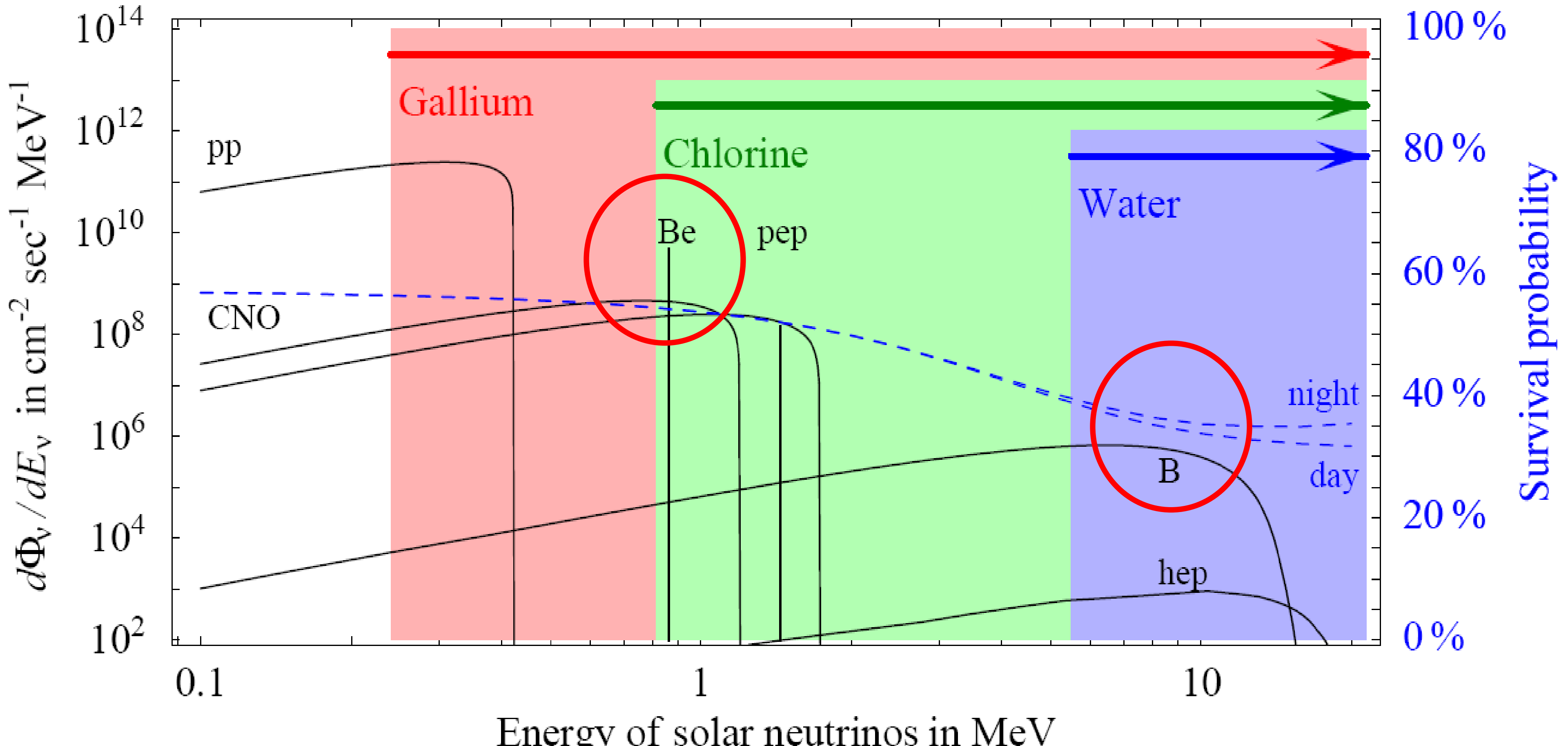
$$\tilde{\theta} = 45^\circ$$

Lecture B2

- ★ Evidence for Neutrino Oscillations
- ★ Lessons from Oscillation Data
- ★ Comparing Leptons with Quarks

Solar neutrinos

R. Davis observed a solar neutrino deficit, compared with J. Bahcall's prediction for the ν-flux, at the Homestake Mine in 1968.



Strumia & Vissani, hep-ph/0606054.

DATA

Examples: Boron (硼) ν's ~ 32%, Beryllium (铍) ν's ~ 56%

MSW solution

In the two-flavor approximation, solar neutrinos are governed by

$$N_e(0) \approx 6 \times 10^{25}\ \mathrm{cm}^{-3}$$

$$\mathcal{H}_{\mathrm{eff}} = \frac{\Delta m_{21}^2}{4E} \begin{bmatrix} -\cos 2\theta_{12} & \sin 2\theta_{12} \\ \sin 2\theta_{12} & \cos 2\theta_{12} \end{bmatrix} + \begin{bmatrix} \sqrt{2} G_{\mathrm{F}} N_e(r) & 0 \\ 0 & 0 \end{bmatrix}$$

$7.6 \times 10^{-5}\ \mathrm{eV}^2$

$0.75 \times 10^{-5}\ \mathrm{eV}^2 / \mathrm{MeV}$ (at $r = 0$)

Be-7 ν's: *E* ~ 0.862 MeV. The vacuum term is dominant. The survival probability on the earth is (for theta_12 ~ 34°):

$$\begin{aligned} P(\nu_e \to \nu_e) &\approx 1 - \frac{1}{2} \sin^2 2\theta_{12} \\ &\sim 0.56 \end{aligned}$$

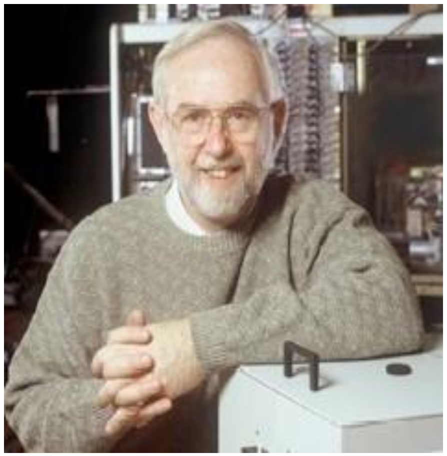
B-8 ν's: *E* ~ 6 to 7 MeV. The matter term is dominant. The produced ν is roughly ν_e ~ ν_2 (for *V* >0). The ν-propagation from the center to the outer edge of the Sun is approximately adiabatic. That is why it keeps to be ν_2 on the way to the surface (for theta_12 ~ 34°):

$$|\nu_2\rangle \approx \sin\theta_{12} |\nu_e\rangle + \cos\theta_{12} |\nu_\mu\rangle$$

$$P(\nu_e \to \nu_e) = |\langle \nu_e | \nu_2 \rangle|^2 = \sin^2\theta_{12} \approx 0.32$$

SNO in 2001

The **heavy water** Cherenkov detector at SNO confirmed the solar neutrino flavor conversion (A.B. McDonald 2001)



The Salient features:

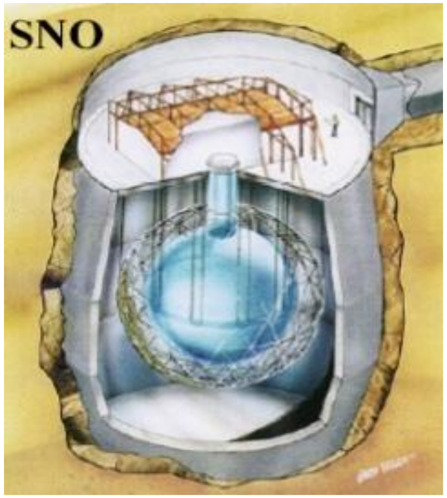
Boron-8 *e*-neutrinos

Flux and spectrum

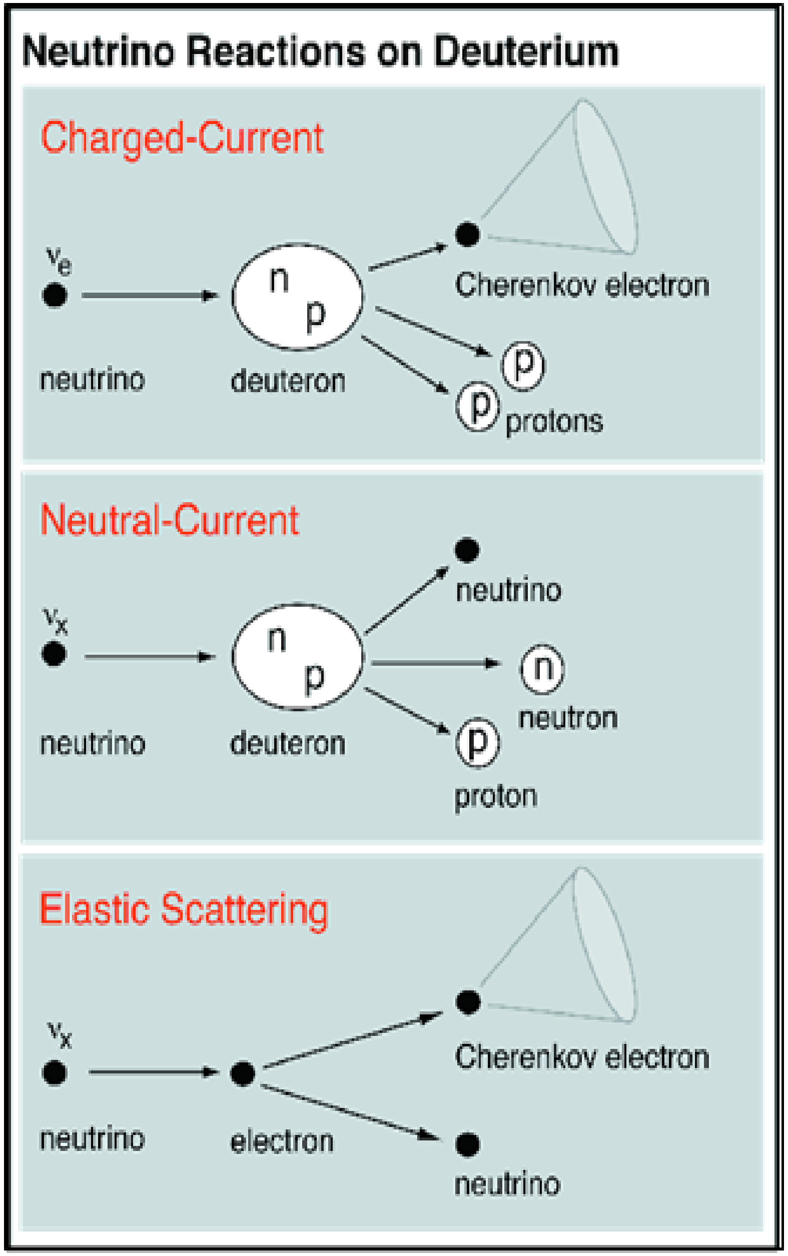
Deuteron as target

3 types of processes

Model-independent



At Super-Kamiokande only elastic scattering can happen between solar neutrinos & the ordinary water.



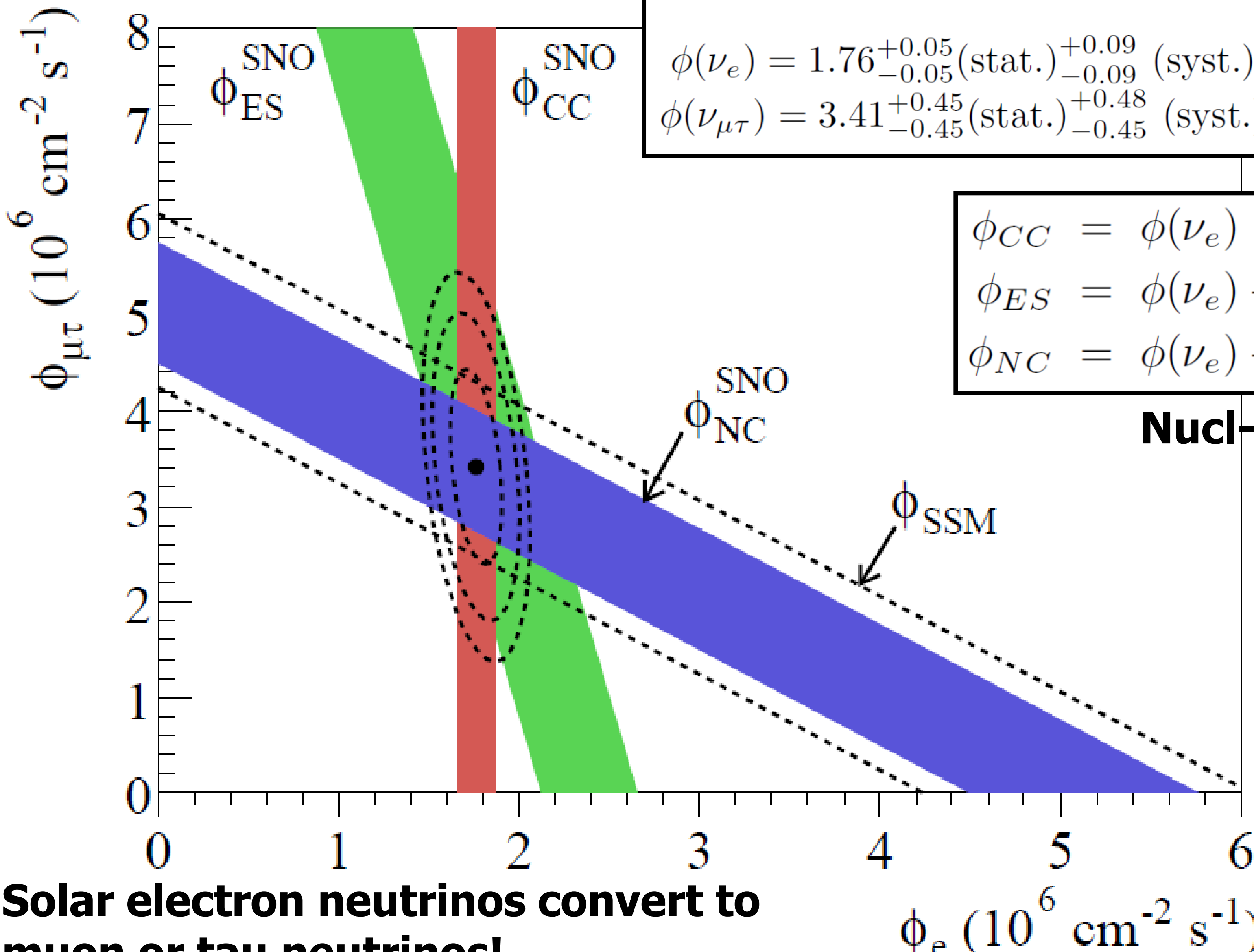
The SNO result

$$\phi_{CC} = 1.76^{+0.06}_{-0.05}(\text{stat.})^{+0.09}_{-0.09}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$
$$\phi_{ES} = 2.39^{+0.24}_{-0.23}(\text{stat.})^{+0.12}_{-0.12}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$
$$\phi_{NC} = 5.09^{+0.44}_{-0.43}(\text{stat.})^{+0.46}_{-0.43}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$

$$\phi(\nu_e) = 1.76^{+0.05}_{-0.05}(\text{stat.})^{+0.09}_{-0.09}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$
$$\phi(\nu_{\mu\tau}) = 3.41^{+0.45}_{-0.45}(\text{stat.})^{+0.48}_{-0.45}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$

$$\begin{aligned} \phi_{CC} &= \phi(\nu_e) \\ \phi_{ES} &= \phi(\nu_e) + 0.1559\phi(\nu_{\mu\tau}) \\ \phi_{NC} &= \phi(\nu_e) + \phi(\nu_{\mu\tau}) \end{aligned}$$

Nucl-ex/0610020

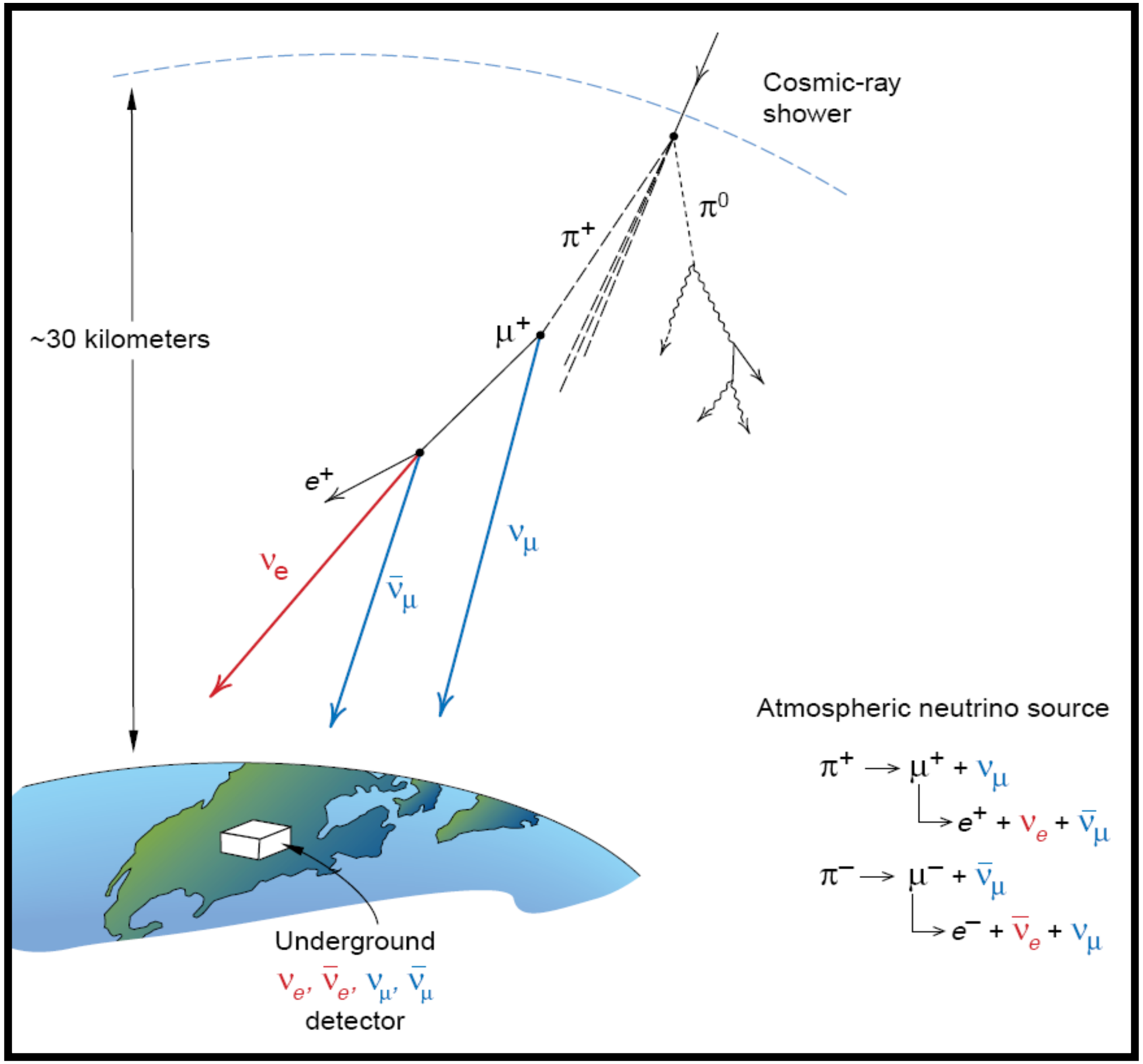


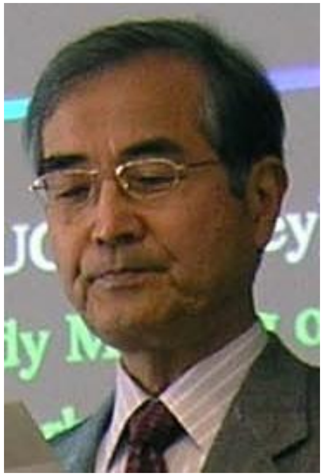
Solar electron neutrinos convert to muon or tau neutrinos!

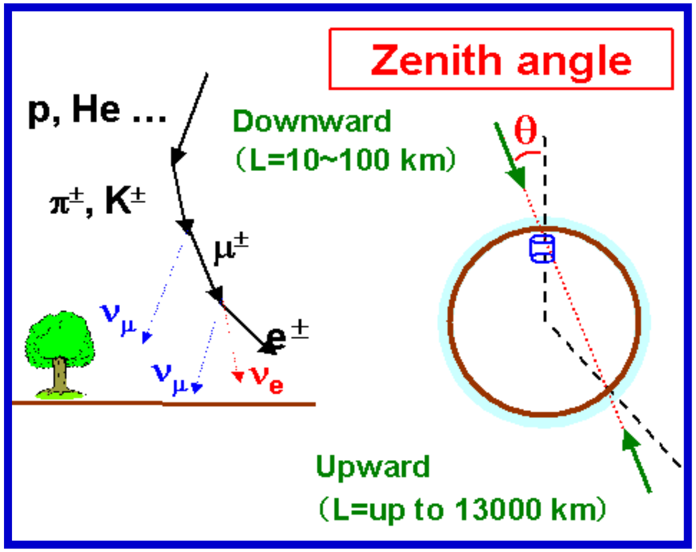
John Bahcall

Atmospheric neutrinos

Atmospheric muon neutrino deficit was firmly established at Super-Kamiokande (Y. Totsuka & T. Kajita 1998).







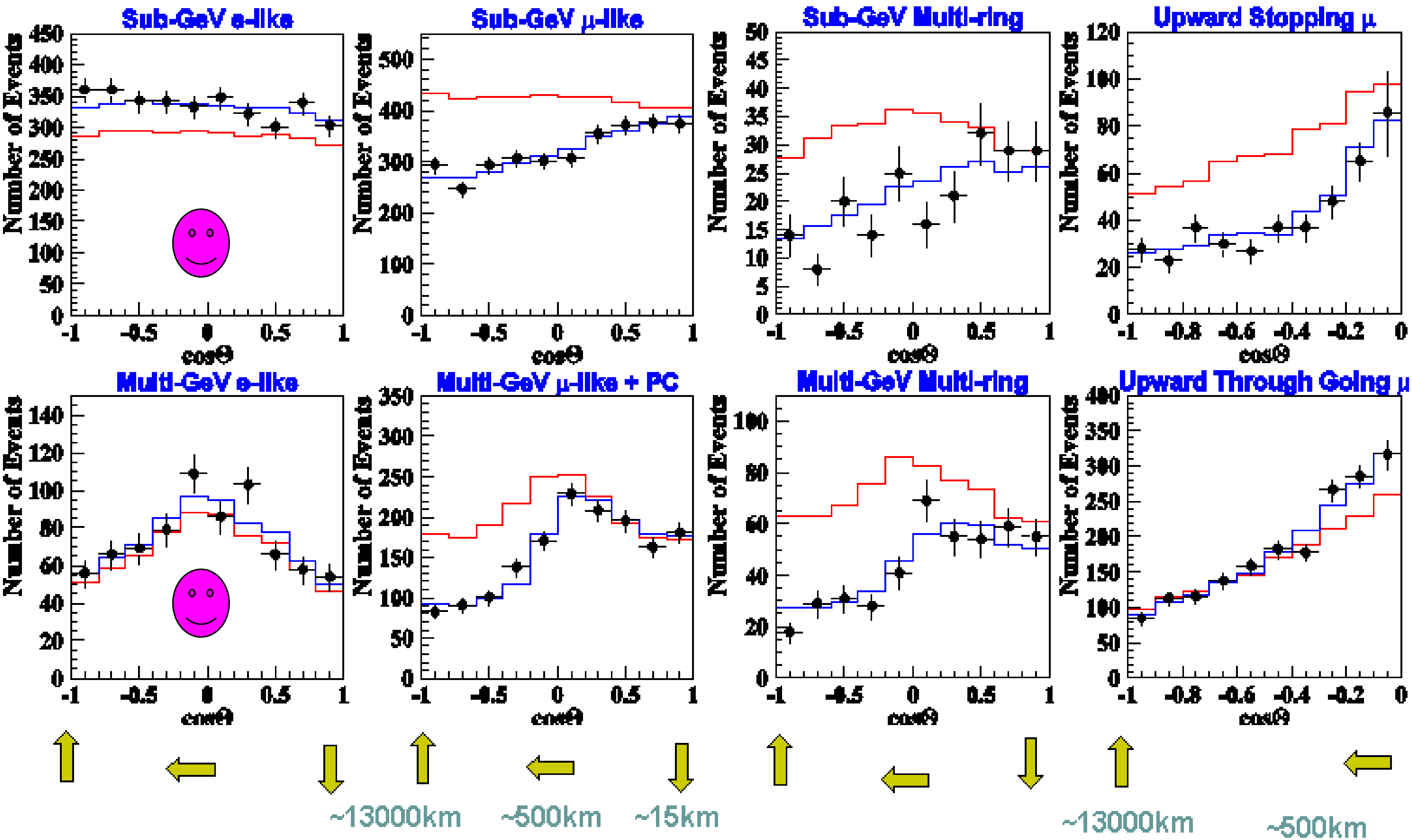
Zenith angle distributions

C. Sagi/ICHEP04

$\nu_\mu \leftrightarrow \nu_\tau$
2-flavor oscillations

Best fit
$\sin^2 2\theta=1.0$, $\Delta m^2=2.1\times10^{-3}$ eV^2

Null oscillation



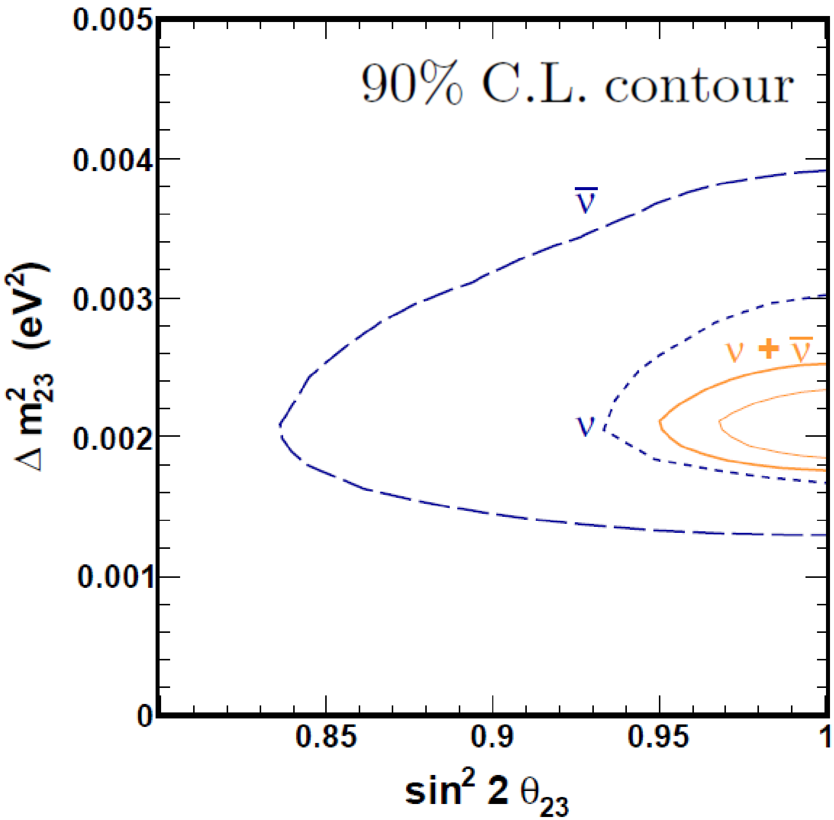
L/E Analysis: SK-I + SK-II

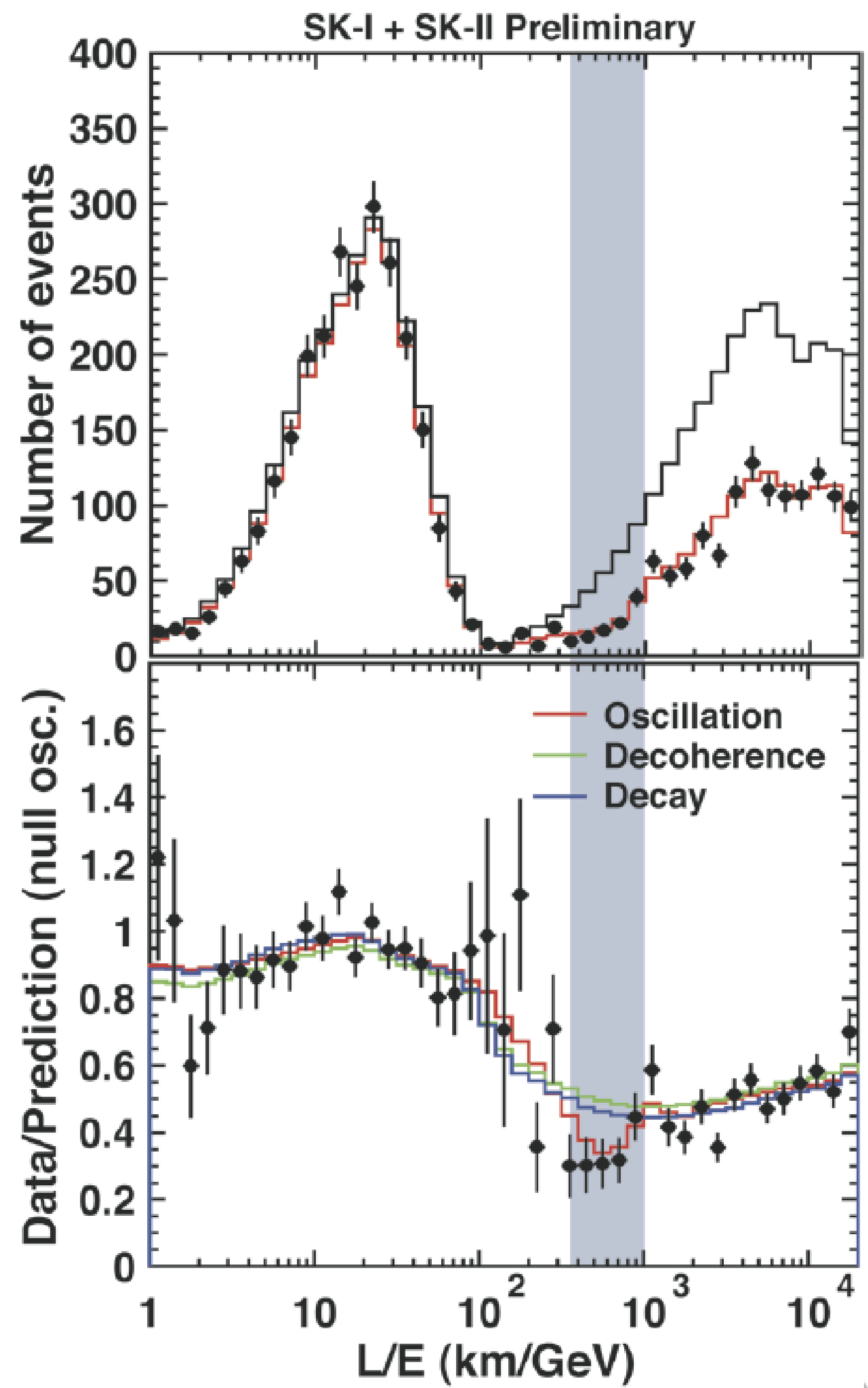
J. Raaf/Neutrino08

Datasets
SK-I FC/PC μ-like: 1489 days
SK-II FC/PC μ-like: 799 days

Phys. Rev. Lett. 107, 241801 (2011)

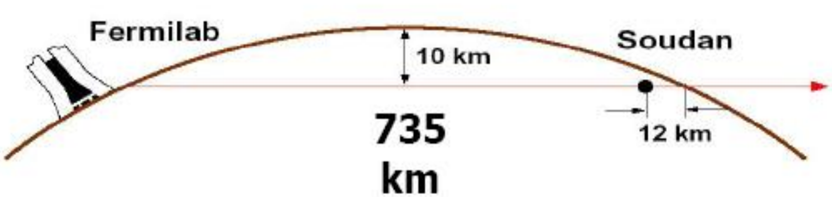
SK-I+II+III data set

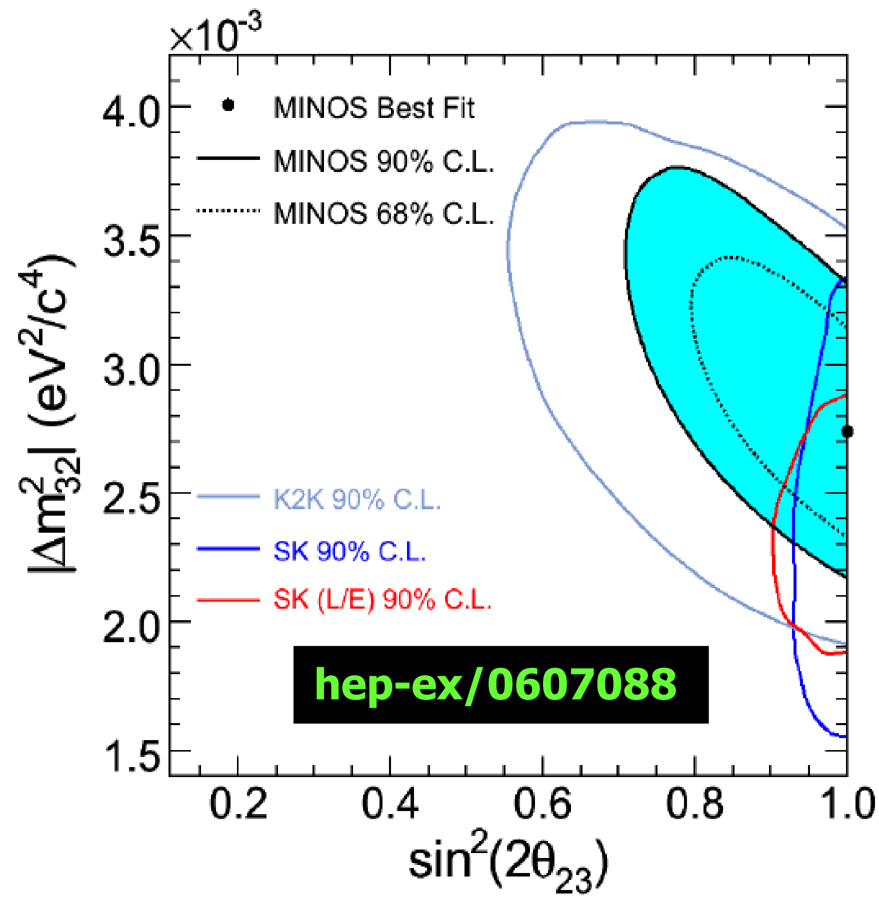




Accelerator neutrinos

The MINOS supports Super-K & K2K data





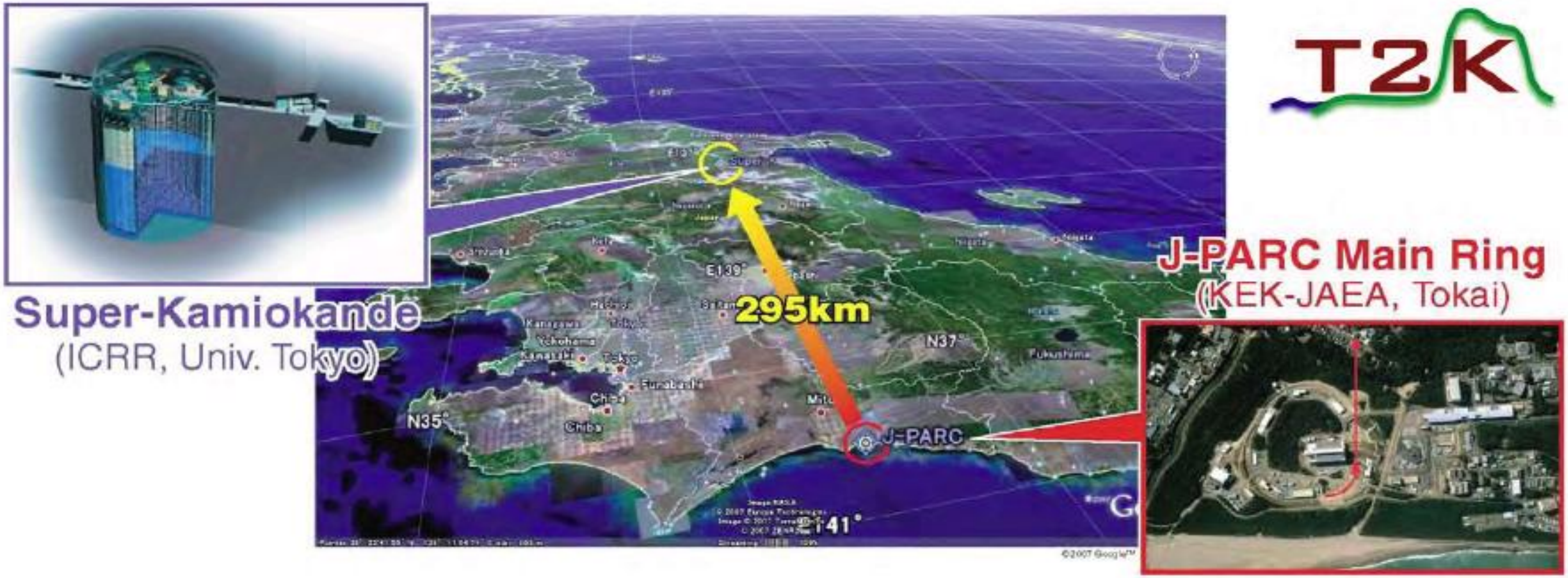
$$\left|\Delta m^2_{32}\right| = 2.74^{+0.44}_{-0.26}(\text{stat}+\text{syst})\times 10^{-3}\,\text{eV}^2$$

$$\sin^2 2\theta_{23} = 1.00_{-0.13}(\text{stat}+\text{syst})$$

Constrained to $\sin^2(2\theta_{23}) \le 1$

T2K in 2011

T2K (Tokai-to-Kamioka) experiment



arXiv:1106.2822 [hep-ex] 14 June 2011
Hint for unsuppressed theta(13) !

T2K Main Goals:

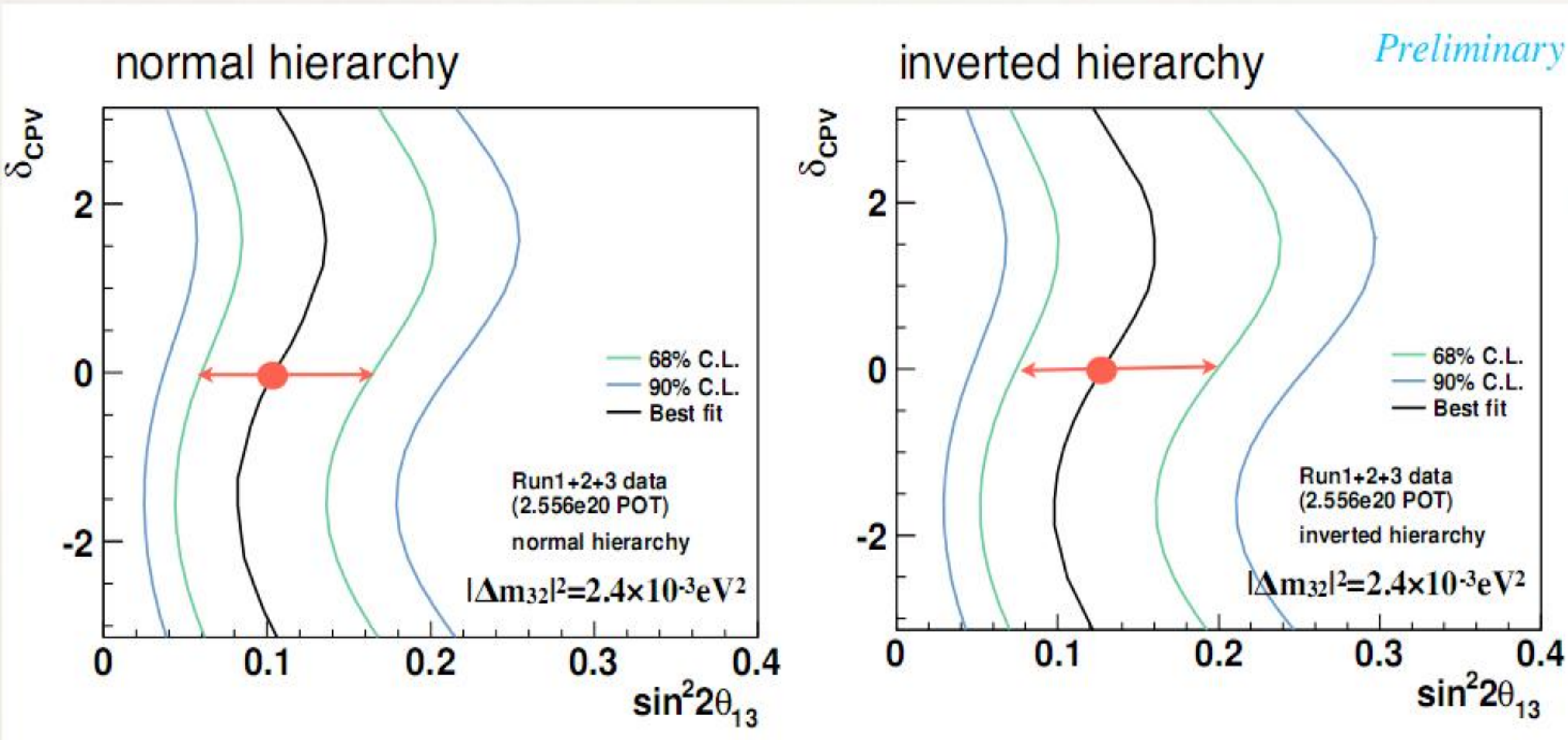
★ Discovery of $\nu_\mu \rightarrow \nu_e$ oscillation (ν_e appearance)

★ Precision measurement of ν_μ disappearance

T. Nakaya (Neutrino 2012)

Allowed Region (constant χ^2 method)

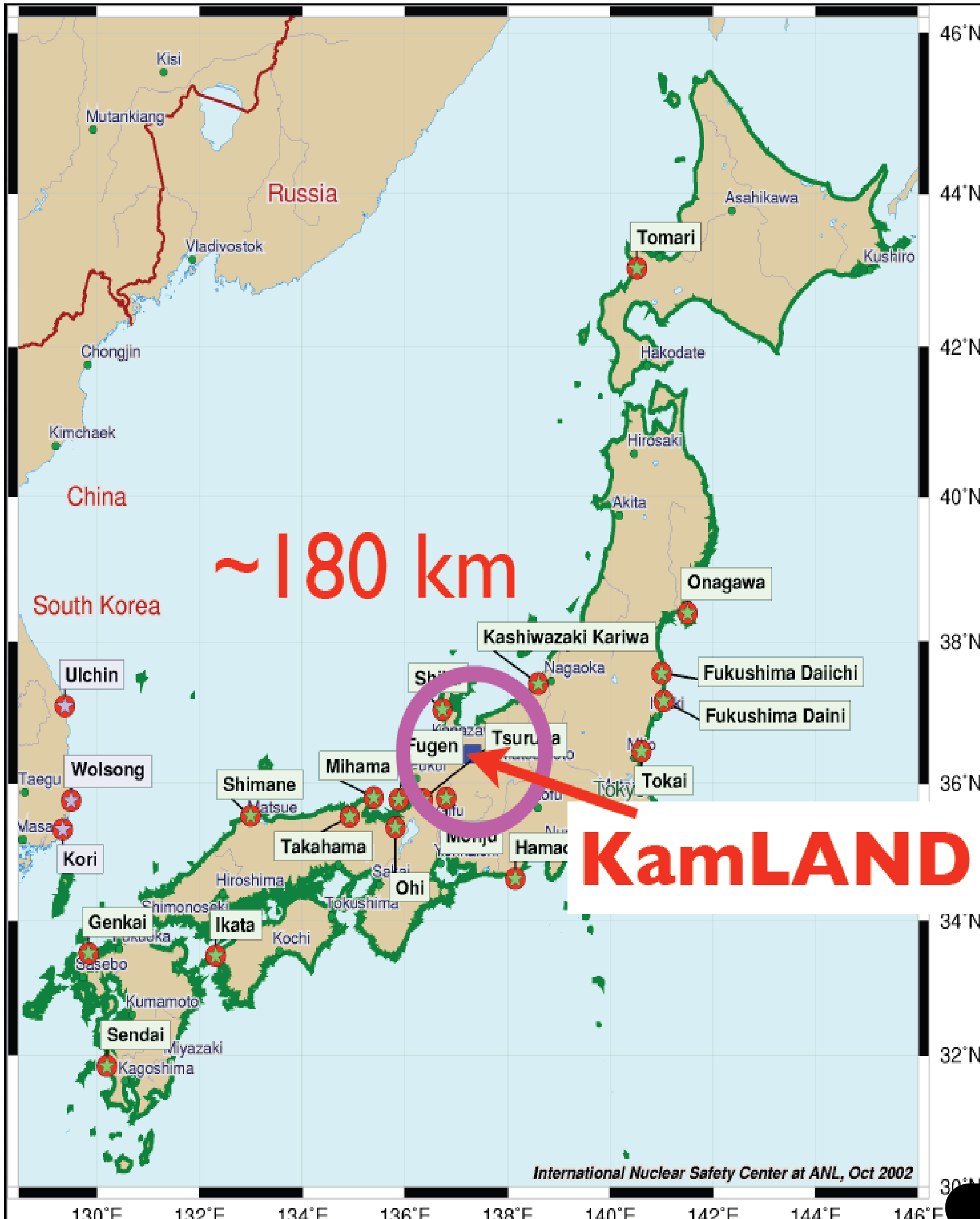
$P(\nu_\mu \rightarrow \nu_e) = \sin^2\theta_{23} \sin^2 2\theta_{13} \sin^2(1.27\Delta m_{32}^2 L/E) + CPV + \text{matter effect.} + \ldots$



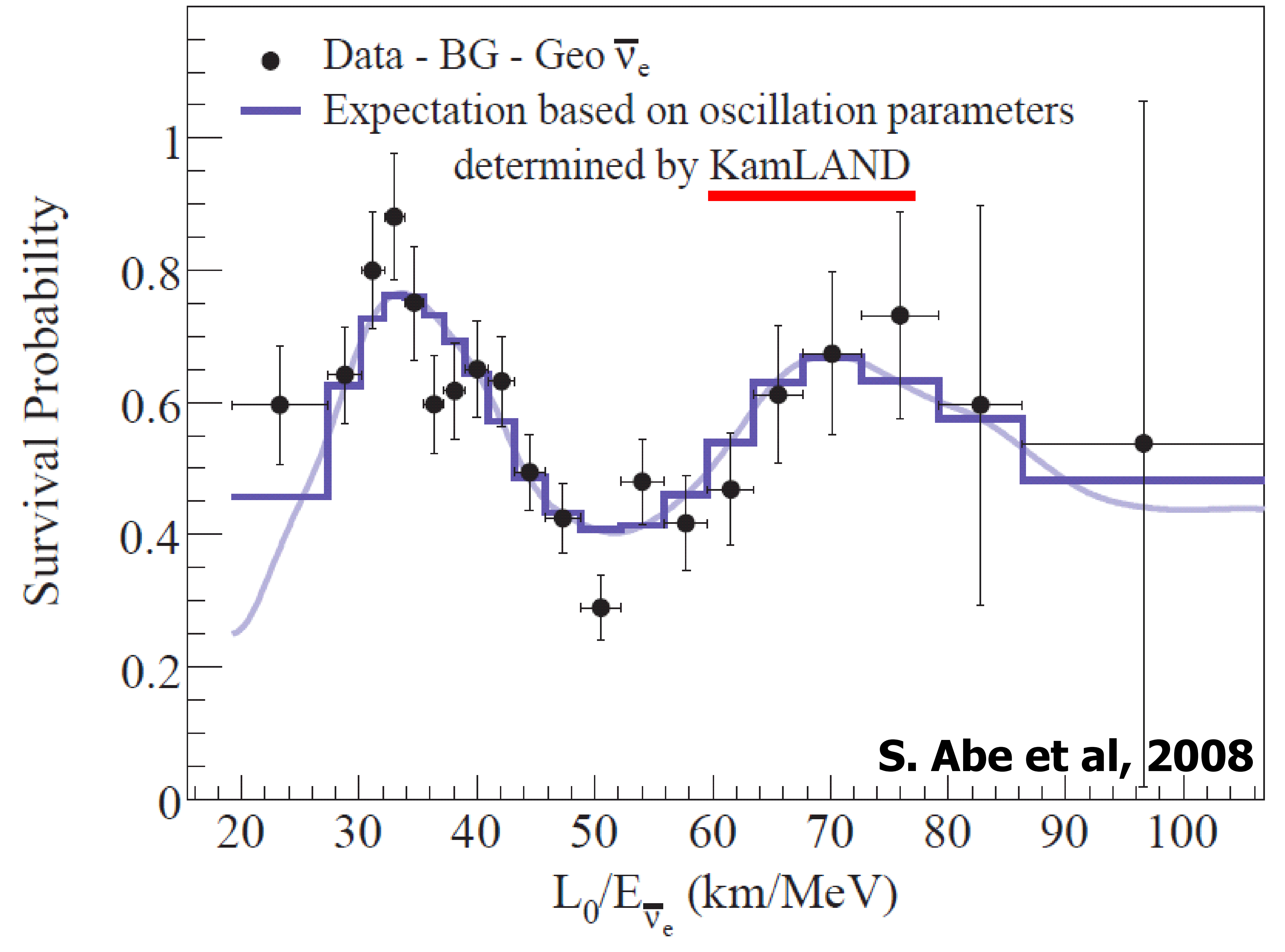
$\sin^2 2\theta_{13} = 0.104^{+0.060}_{-0.045}$ @ $\delta_{CP} = 0$

$\sin^2 2\theta_{13} = 0.128^{+0.070}_{-0.055}$ @ $\delta_{CP} = 0$

Reactor antineutrinos

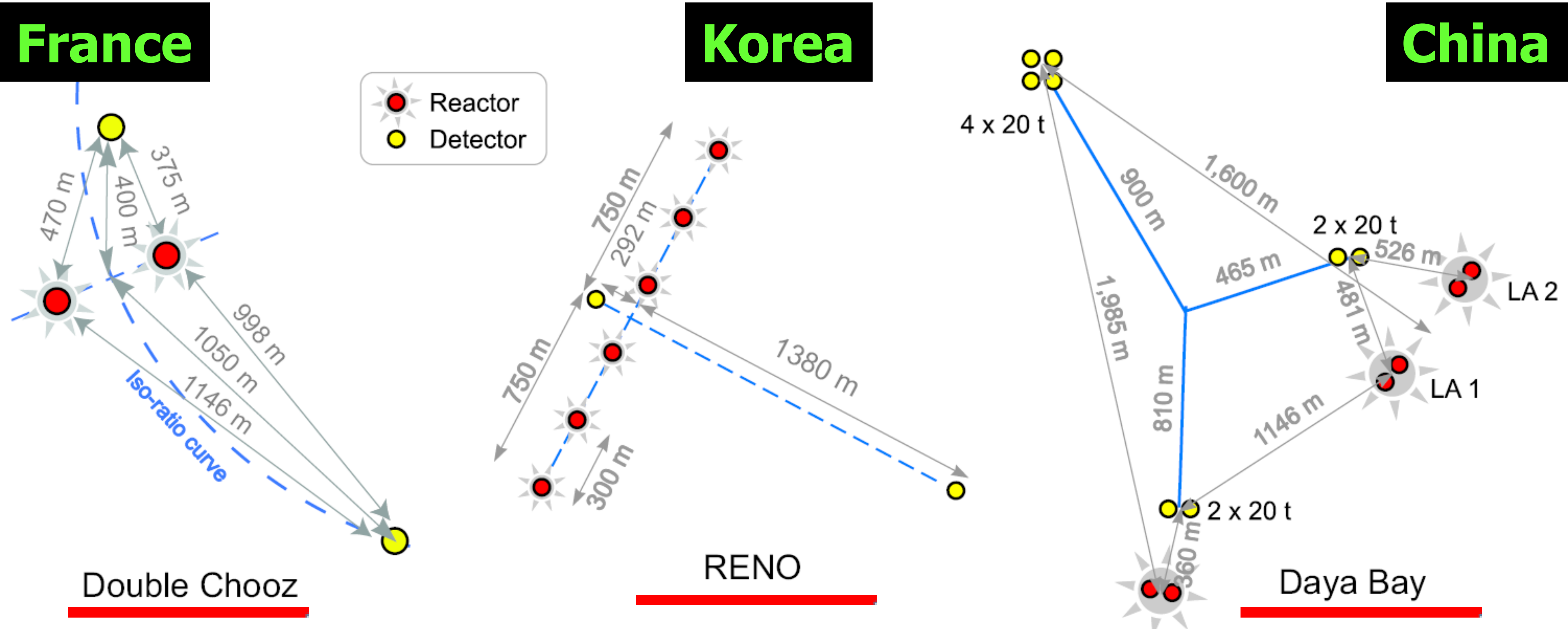


Verify the large angle MSW solution to the solar neutrino Problem



Data - BG - Geo $\overline{\nu}_e$
Expectation based on oscillation parameters
determined by KamLAND
Survival Probability
0
0.2
0.4
0.6
0.8
1
20
30
40
50
60
70
80
90
100
$L_0/E_{\overline{\nu}_e}$ (km/MeV)
S. Abe et al, 2008

Hunting for θ_{13}



Setup	Thermal power P_{Th} (GW)	Baseline L (m)	Detector mass m_{Det} (t)	Events/year	Backgrounds/day
Daya Bay [20]	17.4	1700	80	10×10^4	0.4
Double CHOOZ [21]	8.6	1050	8.3	1.5×10^4	3.6
RENO [22]	16.4	1400	15.4	3×10^4	2.6

Daya Bay in 2012

The Daya Bay Experiment

Adjacent mountains with horizontal access provide 860 (250) m.w.e cosmic shielding.

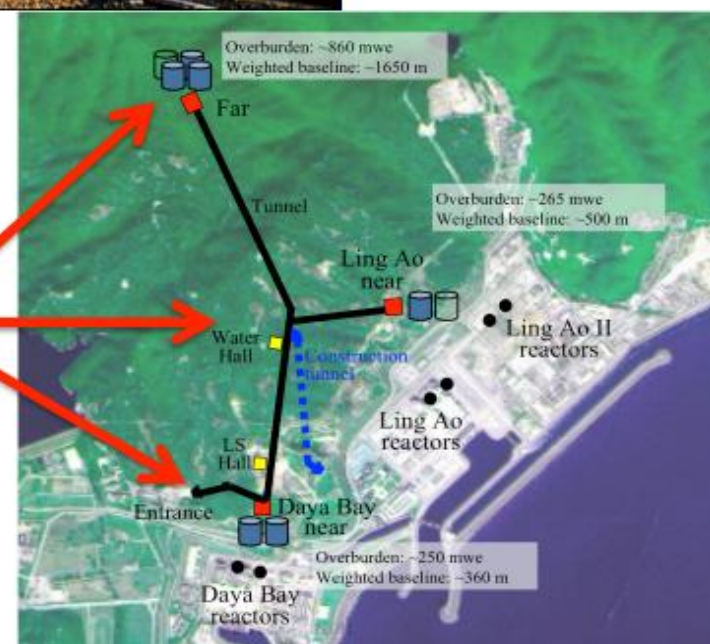
Daya Bay

Ling Ao I + II

6 commercial reactor cores with 17.4 GW_{th} total power.

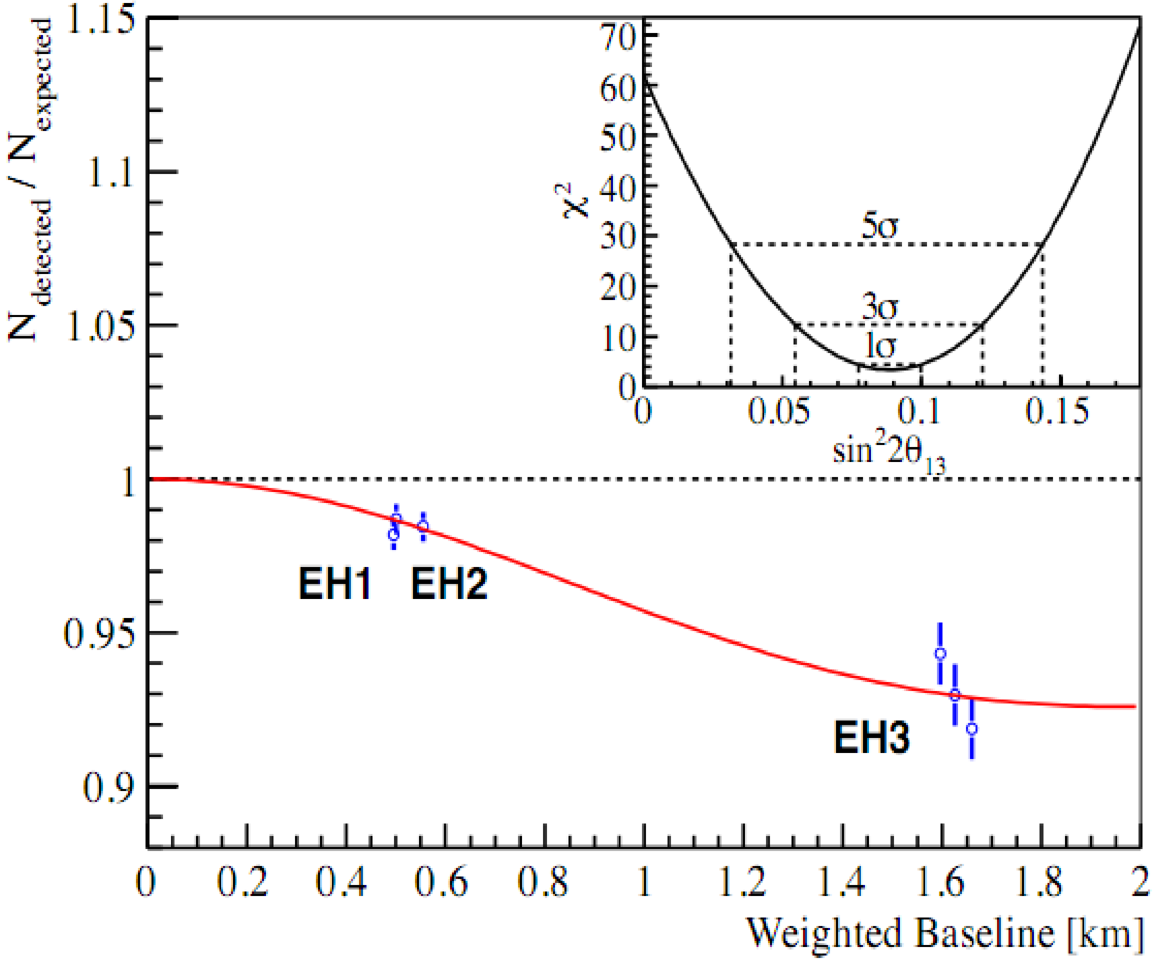
6 Antineutrino Detectors (ADs) give 120 tons total target mass.

Via GPS and modern theodolites, relative detector-core positions known to 3 cm.



Rate Analysis

Estimate θ_{13} using measured rates in each detector.



Uses standard χ^2 approach.

Far vs. near relative measurement. [Absolute rate is not constrained.]

Consistent results obtained by independent analyses, different reactor flux models.

Most precise measurement of $\sin^2 2\theta_{13}$ to date.

$$\sin^2 2\theta_{13} = 0.089 \pm 0.010\ (\text{stat}) \pm 0.005\ (\text{syst})$$

3-flavor global fit

M. Gonzalez-Garcia, M. Maltoni, T. Schwetz, e-Print: arXiv:1409.5439

	Normal Ordering ($\Delta\chi^2 = 0.97$)		Inverted Ordering (best fit)		Any Ordering
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.270 \to 0.344$	$0.304^{+0.013}_{-0.012}$	$0.270 \to 0.344$	$0.270 \to 0.344$
$\theta_{12}/^\circ$	$33.48^{+0.78}_{-0.75}$	$31.29 \to 35.91$	$33.48^{+0.78}_{-0.75}$	$31.29 \to 35.91$	$31.29 \to 35.91$
$\sin^2\theta_{23}$	$0.452^{+0.052}_{-0.028}$	$0.382 \to 0.643$	$0.579^{+0.025}_{-0.037}$	$0.389 \to 0.644$	$0.385 \to 0.644$
$\theta_{23}/^\circ$	$42.3^{+3.0}_{-1.6}$	$38.2 \to 53.3$	$49.5^{+1.5}_{-2.2}$	$38.6 \to 53.3$	$38.3 \to 53.3$
$\sin^2\theta_{13}$	$0.0218^{+0.0010}_{-0.0010}$	$0.0186 \to 0.0250$	$0.0219^{+0.0011}_{-0.0010}$	$0.0188 \to 0.0251$	$0.0188 \to 0.0251$
$\theta_{13}/^\circ$	$8.50^{+0.20}_{-0.21}$	$7.85 \to 9.10$	$8.51^{+0.20}_{-0.21}$	$7.87 \to 9.11$	$7.87 \to 9.11$
$\delta_{CP}/^\circ$	306^{+39}_{-70}	$0 \to 360$	254^{+63}_{-62}	$0 \to 360$	$0 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.50^{+0.19}_{-0.17}$	$7.02 \to 8.09$	$7.50^{+0.19}_{-0.17}$	$7.02 \to 8.09$	$7.02 \to 8.09$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.457^{+0.047}_{-0.047}$	$+2.317 \to +2.607$	$-2.449^{+0.048}_{-0.047}$	$-2.590 \to -2.307$	$\begin{bmatrix} +2.325 \to +2.599 \\ -2.590 \to -2.307 \end{bmatrix}$

Quark mixing: $\theta_{12} \simeq 13^\circ\ ,\ \theta_{23} \simeq 2^\circ\ ,\ \theta_{13} \simeq 0.2^\circ\ ,\ \delta \simeq 65^\circ$

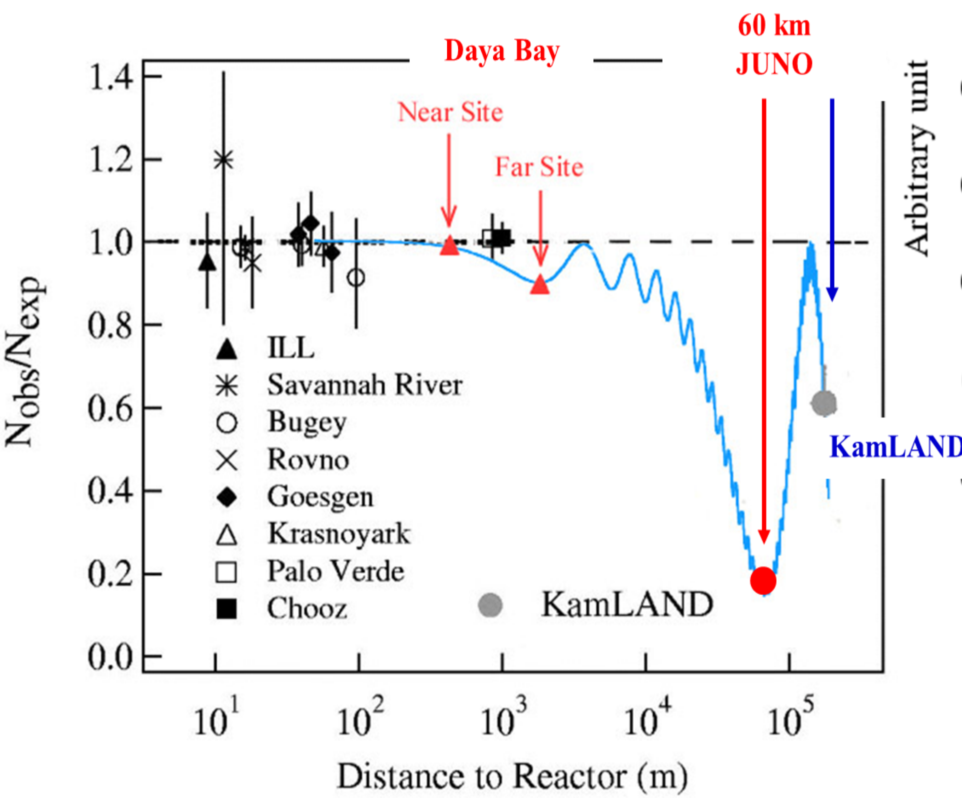
Lepton mixing: $\theta_{12} \simeq 33^\circ\ ,\ \theta_{23} \sim 45^\circ\ ,\ \theta_{13} \simeq 8.5^\circ\ ,\ \delta \sim 270^\circ$

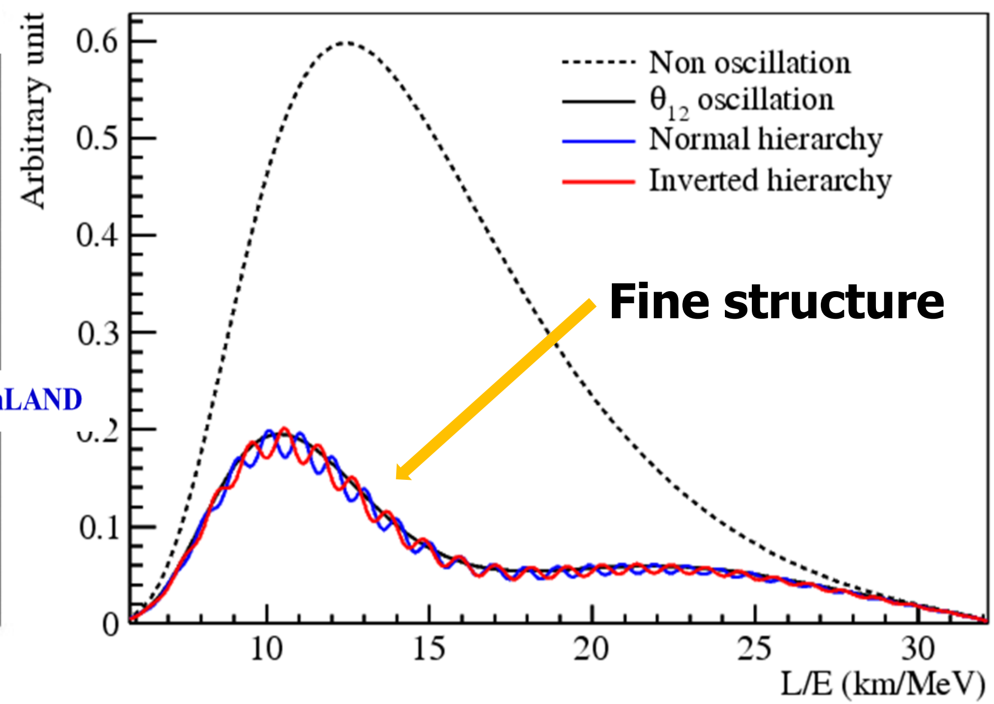
Mass ordering experiments

Accelerator (T2K) or atmospheric (INO/PINGU) experiments

$$\boxed{\Delta m^2_{31} + 2\sqrt{2}G_F N_e E}$$ **with the help of matter effects**

Reactor (JUNO): Optimum baseline at the minimum of Δm^2_{21} oscillations, corrected by fine structure of Δm^2_{31} oscillations.





Naïve understanding

Quark mixing

$$V_{\rm CKM} = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

Small quark mixing angles are due to large quark mass hierarchies?

$$m_u / m_c \sim m_c / m_t \sim \lambda^4$$

$$m_d / m_s \sim m_s / m_b \sim \lambda^2$$

$$\lambda \approx 0.22$$

3 CKM angles

$$\theta_{12} \sim \lambda \qquad \theta_{23} \sim \lambda^2 \qquad \theta_{13} \sim \lambda^4$$

A big CP-violating phase in the *CKM* matrix *V* is seen.

Lepton mixing

$$|U| = \begin{pmatrix} 0.801 \to 0.845 & 0.514 \to 0.580 & 0.137 \to 0.158 \\ 0.225 \to 0.517 & 0.441 \to 0.699 & 0.614 \to 0.793 \\ 0.246 \to 0.529 & 0.464 \to 0.713 & 0.590 \to 0.776 \end{pmatrix}$$

Large lepton mixing angles imply a small neutrino mass hierarchy?

$$m_e / m_\mu \sim \lambda^4 / 2 \qquad m_\mu / m_\tau \sim 4\lambda^2 / 3$$

$$\theta_{12} \sim \pi/6 \qquad \theta_{23} \sim \pi/4$$

$$m_1 \sim m_2 \sim m_3$$

CP violation?

What is behind?

Flavor Symmetry

Texture zeros | **Element correlations** | **GUT relations**

They reduce the number of free parameters, and thus lead to predictions for 3 flavor mixing angles in terms of either the mass ratios or constant numbers.

Example (Fritzsch ansatz)

$$M_{l,\nu} = \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$$

Dependent on mass ratios

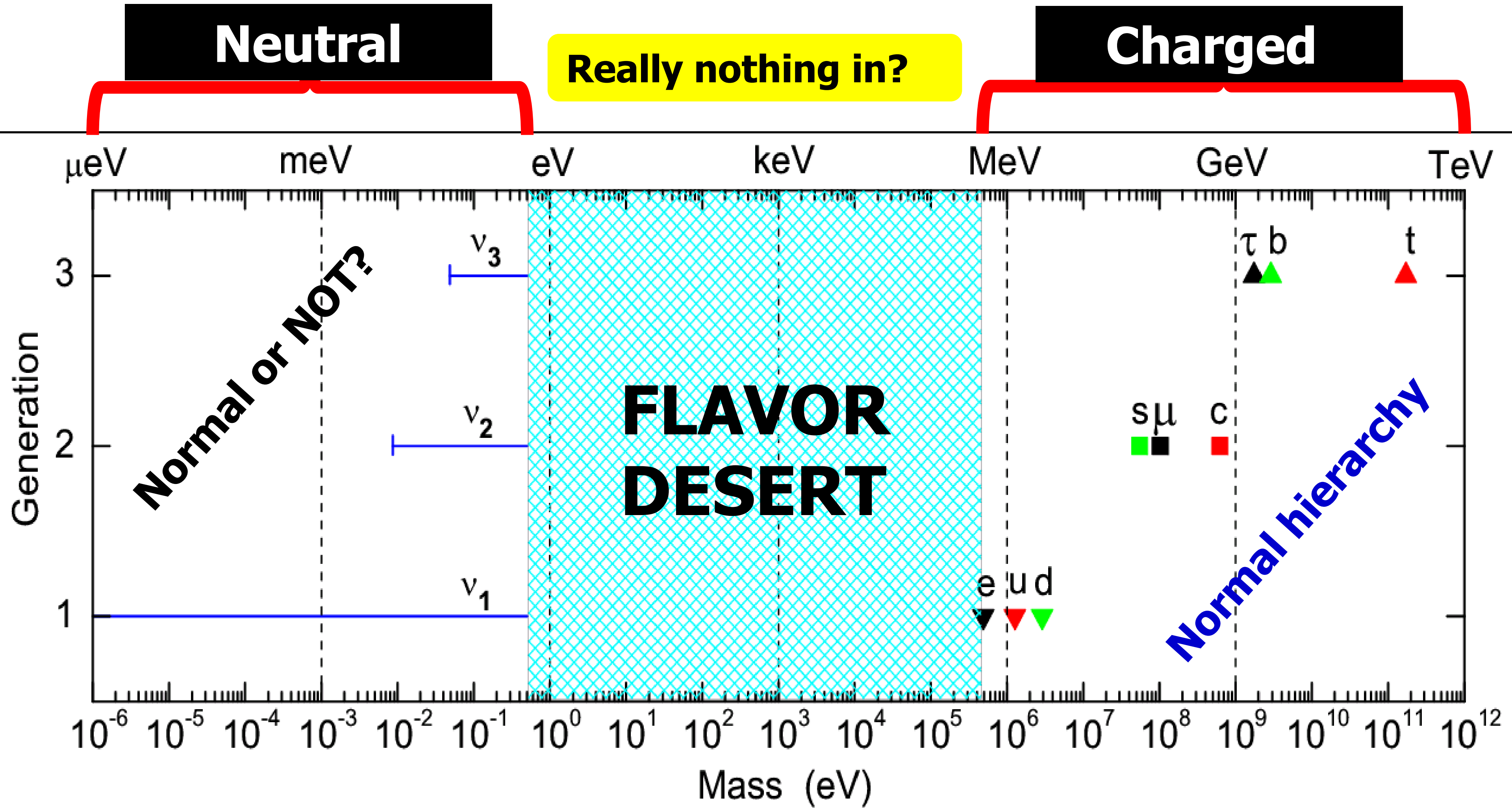
Example (Discrete symmetries)

$$M_{\nu} = \begin{pmatrix} b+c & -b & -c \\ -b & a+b & -a \\ -c & -a & a+c \end{pmatrix}$$

Dependent on simple numbers

PREDICTIONS

Summary (1)



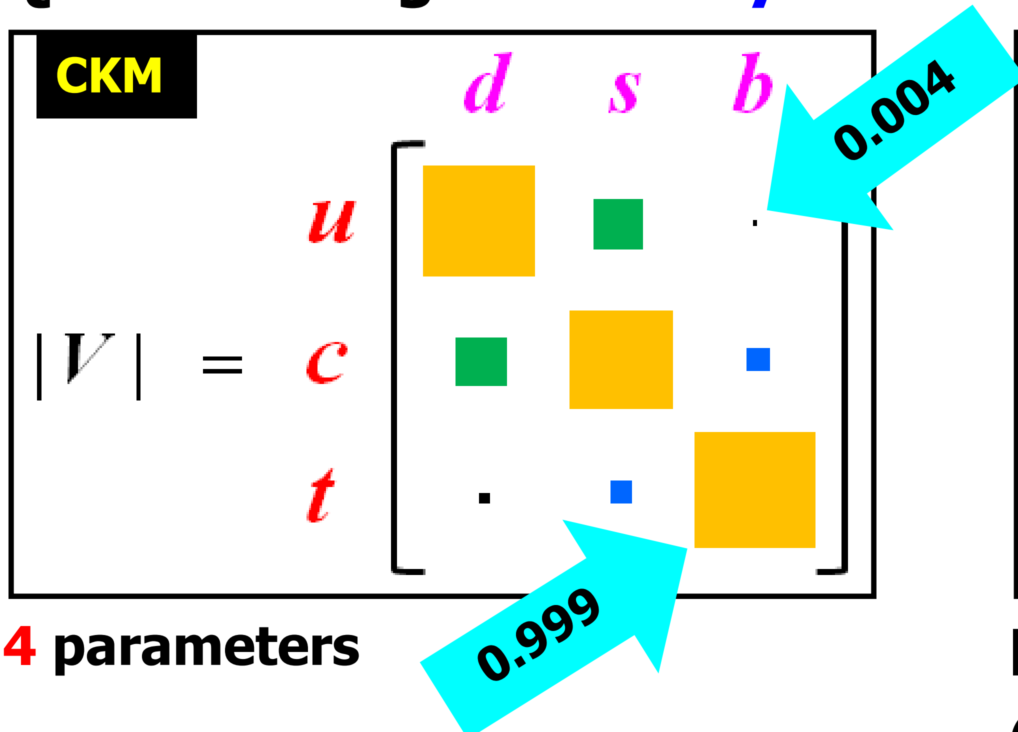
Flavor hierarchy + **Flavor desert** puzzles: **12** free (mass) parameters.

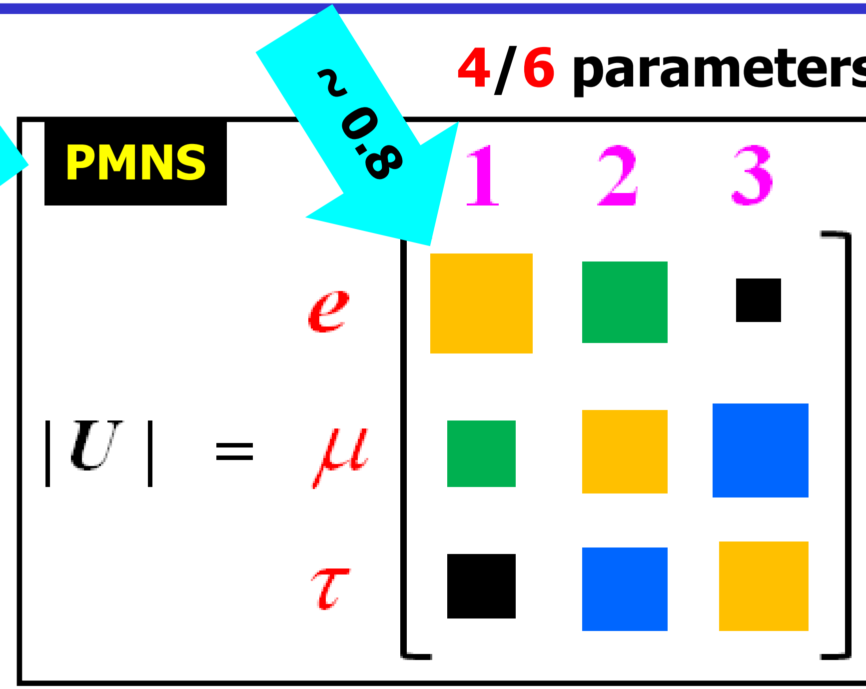
In the quark sector, why is the up quark lighter than the down quark?

Summary (2)

Quark mixing: hierarchy!

4/6 parameters

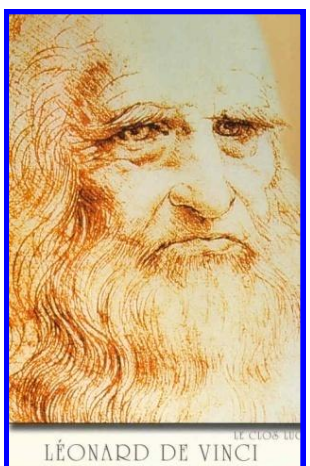




4 parameters

Lepton mixing: anarchy?

(Approximate μ-τ symmetry)



Our Philosophy

Although nature commences with reason and ends in experience, it is necessary for us to do the opposite, that is, to commence with experience and from this to proceed to investigate the reason