Collider Physics - Final Report

Atmospheric Neutrino Oscillation in Super-K

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I. Neutrino

Four types of neutrino sources can be classified as solar neutrino, atmospheric neutrino, accelerator neutrino, and reactor neutrino. Besides, these neutrino sources are investigated in different experimental facilities. As the aforementioned, the first affiliated neutrino observatory is SNO (Sudbury Neutrino Observatory) in Canada; the second representative neutrino observatory is Super-K (Super Kamiokande) in Japan; the third corresponding observatories are KEK-SuperK (Japan), T2K(Japan), etc; the last observatories representing are KamLAND, Daya bay (China), etc. In this final report, we are going to present and elaborate on atmospheric neutrino oscillation in Super-K.

i. Neutrino Oscillation

Neutrino oscillation is a phenomenon that describes the flavor of neutrino oscillating among three different flavors; further, this phenomenon can explain the existence of neutrino mass (i.e., Δm). In general, the "flavor eigenstates" are related to the "mass eigenstates" of the" with 3×3 unitary mixing matrix:

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle$$

The matrix U is the PMNS matrix (Pontecorvo-Maki-Nakagawa-Sakata matrix: mixing flavor eigenstates of leptons); ν_α is the mass eigenstate and ν_i is the flavor eigenstate. Thus, we need the PMNS matrix in order to find the relation between eigenspaces. The PMNS matrix is [1] [2]:

$$\begin{aligned} U_{\alpha i} &= \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \\ &\times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

where $\alpha = e,\ \mu,\ \tau$; $i = 1,\ 2,\ 3$; c_{ab} and s_{ab} means sin and cos with mixing angle θ_{ab}; δ means CP-violating phase (s.t. second term of the matrix is small). The first term of the matrix is corresponding to "solar mixing", the second term of the matrix is known to be small, and the third term of the matrix describes "atmospheric mixing".

The experiment of testing the neutrino oscillation needs a distance L to investigate. We, thence, need to consider the state propagating. For mass eigenstate

$$|\nu_j(t)\rangle = e^{-i(E_j t - \vec{p}_j \cdot \vec{x})}|\nu_j(0)\rangle$$

take ultrarelativistic limit $|\vec{p}_j| \gg m_j$ and also drop out the relative phase. When $t \approx L$: [2]

$$\begin{aligned} E_j &= \sqrt{\mathbf{p_j}^2 + m_j^2} = \mathbf{p_j}\sqrt{1 + \frac{m_j^2}{\mathbf{p_j^2}}} \\ &\approx \mathbf{p_j}\big(1 + \frac{m_j^2}{2\mathbf{p_j^2}}\big) = \mathbf{p_j} + \frac{m_j^2}{2\mathbf{p_j}} \end{aligned}$$

$$\Delta E_{ij} = \left(\mathbf{p_i} + \frac{m_i^2}{2\mathbf{p_i}}\right) - \left(\mathbf{p_j} + \frac{m_j^2}{2\mathbf{p_j}}\right) = \frac{\Delta m_{ij}^2}{2E}$$

Considering the difference of the state and taking the above approximation (here, we use ij to indicate the difference, but in the following equation

(1) we use jk to avoid garbling with imaginary), we can get:

$$|\nu_{ij}(L)\rangle = e^{-i(\frac{\Delta m_{ij}^2 L}{2E})}|\nu_{ij}(0)\rangle$$

In the wake of knowing the propagation of the mass eigenstate, we want to know the "interference" between the flavor eigenstates. Therefore, suppose the initial lepton with the flavor eigenstate α and what is the "probability" if we observe the flavor eigenstate β when $t \approx L$ is [1]:

$$\begin{aligned} P_{\alpha\to\beta} &= |\langle \nu_\beta|\nu_\alpha\rangle|^2 \\ &= \Big|\sum_j U_{\alpha j}^* U_{\beta j} e^{-i(\frac{m_j^2 L}{2E})}\Big|^2 \\ &\quad \text{drop out } \mathbb{Im} \\ &\Rightarrow \delta_{\alpha\beta} - 4\sum_{j>k} \mathbb{Re}\Big\{U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^*\Big\} \quad (1) \\ &\qquad \times \sin^2\left(\frac{\Delta_{jk} m^2 L}{4E}\right) \end{aligned}$$

ii. *One mass scale dominant approximation*

The one mass scale dominant approximation means there is one mass among the three masses which is manifest larger than the others in the mass square difference. (This approximation has been supported by the neutrino experiments [1]) Here, we suppose m_3 is the dominant mass :

$$|m_2^2 - m_1^2| \ll |m_3^2 - m_{1,2}^2|$$

Additionally, we can rewrite the phase inside the square of sin in equation (1) as:

$$\begin{aligned} \frac{\Delta_{jk}(mc)_2 L}{4\hbar c} &= \frac{GeV\ fm}{4\hbar c} \times \frac{\Delta_{jk} m^2}{eV^2}\frac{L}{km}\frac{GeV}{E} \\ &\approx 1.27 \times \frac{\Delta_{jk} m^2}{eV^2}\frac{L}{km}\frac{GeV}{E} \end{aligned}$$

According to this method, the propagating probabilities in vacuum for atmospheric neutrinos are [1] :

$$\begin{aligned} P(\nu_e \to \nu_\mu) &= P(\nu_\mu \to \nu_e) \\ &= \sin^2\theta_{23}\sin^2\theta_{13}\left(\frac{1.27\Delta m^2 L}{E}\right) \\ P(\nu_e \to \nu_e) &= 1 - \sin^2\theta_{13}\sin^2\left(\frac{1.27\Delta m^2 L}{E}\right) \\ P(\nu_\mu \to \nu_\mu) &= 1 - 4\cos^2\theta_{13}\sin^2\theta_{23} \\ &\quad \times \left(1 - \cos^2\theta_{13}\sin^2\theta_{23}\right) \\ &\quad \times \left(\frac{1.27\Delta m^2 L}{E}\right) \end{aligned}$$

II. Super Kamiokonde

III. Outlook

References

[1] J. Hosaka et al. Physical Review D (2006)

[2] Wikipedia Contributors, Neutrino oscillation, Wikipedia (2022).