**Computational Physics Lab**

# Midterm Proposal Report

Yuan-Yen Peng, Chih-Tian Ho
Advisor: Prof. Kuo-Chuan Pan
Dept. of Physics, NTHU
Hsinchu, Taiwan

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## 1 Motivation

We comprehend that the government is obligated to upgrade the whole prosperity level of society. Some stages, thereby, ought to be fulfilled; simultaneously, requiring some counterpoise with different hierarchies, inevitably. Some utilitarians may assert that it must quell the low class of people so as to haul up the entire echelon of humankind. We, however, cast doubt on this argument, reckoning that the wipe-out method is too arbitrary and transgressed as a viewpoint of humanity. Ergo, we want to use the numerical model to subvert utilitarianism's implementation.

## 2 Introduction

First, we build a city model and put people into this playground (here, we amend the method, which we presented on the slide sharing part. We recently set people wholly occupied the playground instead of scattering on random positions). Assigned playgrounders with the social rank randomly (instead of setting them according to where they were initially born), and we implement the exchange method (i.e., some rules) to simulate social mobility. After some period, we expect some stable stages, and we will observe the population distribution such as bell distribution, discussing some analogy "physical" quantities. Lastly, apply an "invisible hand" to interrupt the system; in other words, wipe out the low-class people, etc. Afterward, observe the analogical quantities, too.

## 3 Methodology

### 3.1 City

In reality, the source usage rate depends on distance, constructed a concentric structure, for example, Beijing (Figure 1), Chicago, etc. Here, our artificially built city will likewise develop around the center of sources; thus, we adopt the concentric zone model to discuss the hierarchy distribution.

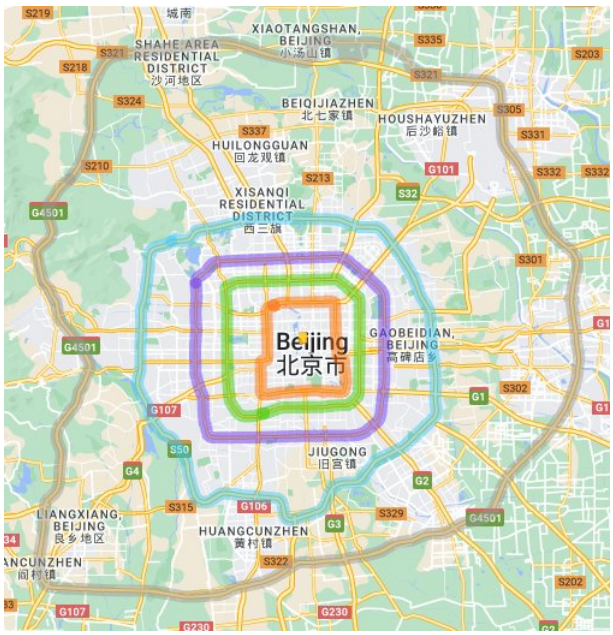


Figure 1: This is the example of ring city, Beijing, China.

In order to simplify the model, we use the concept of Alonso's bid rent curve model, taking the same radius ratio (i.e., $r1/r2 = r2/r3$ and so on) to classify different zones, see in Figure 3. After building the model, we will randomly put people into this playground. The "class" of people will be distinguished by the area where they initially locate because individuals cannot choose where to be born.

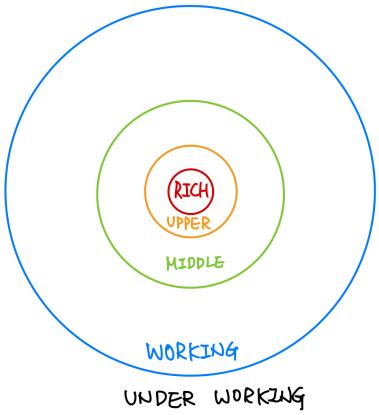


Figure 2: This is the depiction of our artificial city with a sort of geometrical radius. Different colors represent the different echelons of society.

## 3.2 Analogy

In this project, we want to use the Ising model-like analysis to discuss simulation outcomes. First and foremost, assigning the number to the classes one-on-one, we annotate them as "cross-like" spin (please, see the sec. Rule), and the temperature corresponds to how many steps it takes to change the "spin" (i.e. society mobility), time is the evolutionary time as well. The (internal) energy is $E = -J(\sigma_l\sigma_r\sigma_u\sigma_d) - h\sigma_i$, where $J$ is some energy constant, $h$ is the external field related to the background function, and $\sigma_i$, $i = left, right, upper, down$ are the spins of the surrounding sites (it will be discussed later in sec. Rule), and this physical quantity stands for the dynamical activity of

this artificial city; meanwhile, we also use magnetization $M$ as an analogy to the tendency of society; in other words, the more positive of M the more affluence people will gain, and on the contrary, the more negative M indicates the less copious individuals will suffer. There are many other metaphors of physics, we will likewise discuss during the process.

## 3.3 Hierarchy

In this project, we assign classes of people from +5 to -5 (see Figure 3)and set the population of +3 to -3 as 0.68 as initial conditions, which aims to fit the rule that the middle class is inside one standard deviation. We follow the real-life case that the Middle is the largest and two tails (rich and under-working) are the smallest.

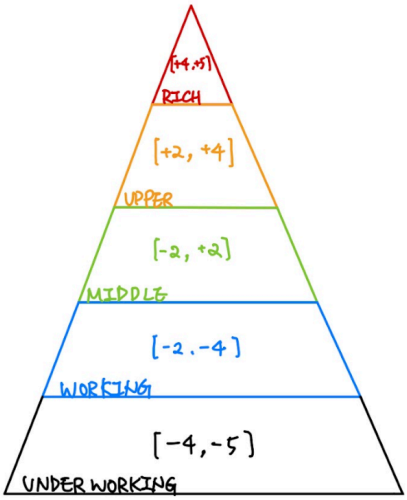


Figure 3: This is how we classify the hierarchy of people. Take Middle for example. MIDDLE means rank form -2 to +2 will be set to this rank.

## 3.4 Rule

In this project we use three functions to fit the real world's general phenomenon: $T(H)$, $A(H)$, $B$, $S(H)$ representing transporting, absorbing, background, and self functions respectively.

We define Transporting Function as:

$$T_{ab}(H) = -(H_a - H_b)$$

this represents the hierarchy difference between the two adjacent people. We use this function to describe that one will get a positive effect from the neighbor with a higher hierarchy, and vice versa.

We define Absorbing Function A(H) as:

$$A(H) = \frac{1}{\sqrt{2\pi a^2}} \times e^{-H^2/2a^2}, \quad a = 3$$

Here, the absorbing ability only depends on one's hierarchy, and here we assume the absorbing is a Gaussian distribution, with s.d. = 3, since we hope those people in the so-called middle class at H in $[-3, 3]$ is about 0.68. (within one standard deviation)

We define Background Function $B$ as:

$$B = exp\{-[\frac{(x - x_{mid})^2}{2\sigma_x^2} + \frac{(y - y_{mid})^2}{2\sigma_y^2}]\}, \ \sigma_x = \sigma_y = 3$$

This background function fits the city model because the resource is a two-dimensional Gaussian distribution.

We also define Self Function $S(H)$ as:

$$S(H) = \frac{b}{2} \times e^{H} + \frac{1}{2 \times \sqrt{2\pi a^2}} \times e^{-H^2/2a^2},\ a = 3, b = e^5$$

This Self function describes the level of consumption of environmental resources which we set as the combination of an exponential (higher hierarchy will have more chances to seize the fortune, so we assign the tendency as an exponential), and a Gaussian function (people in the middle hierarchy will be more aggressive to upgrade their social rank; therefrom we use the normal distribution to fit this appearance). In the Self function, both $a$ and $b$ are the normalization constants: a is the standard deviation we set the H of these people within $[-3, 3]$ and it is about 0.68; $b = e^5$ renders the maximum of an exponential term will be identical to one, and simultaneously, we divide both terms by two so as to make the weight of each are equal.

As a consequence, we can utilize the formula shown below to iterate everyone's hierarchy at the next time step:

$$H^{t+1}_{x,y} = H^t_{x,y} + A^t_{x,y} \times [T^t_{x,x,y,y+1} T^t_{x,x,y,y-1} T^t_{x,x-1,y,y} + T^t_{x,x+1,y,y}] + \eta \times S^t_{x,y} \times B^t_{x,y}$$

and

$$B^{t+1}_{x,y} = B^t_{x,y} - \eta \times S^t_{x,y} \times B^t_{x,y}$$

where $\eta$ is an influential constant related to the function of background and self functions.

We also set the maximum and minimum so that H must be in $[-5, 5]$. Hence, if $H > 5$, it will maintain "5" in N time steps, and vice versa.

Additionally, we set other interesting factors "p, q" (these correspond to the "probabilities" of altering social mobility):

$$\begin{cases} if\ one\ maintains\ +5\ in\ the\ N\ step,\ one\ will\ turn\ into\ -4\ by\ chance\ p; \\ if\ one\ maintains\ -5\ in\ the\ N\ step,\ one\ will\ turn\ into\ +4\ by\ chance\ q. \end{cases}$$

These two factors (p and q) aim to simulate the common phenomenon in the real world that rich people cannot hold for generations and genius may be born from poor families.

# 4 Outlook

## 4.1 Variables

We expect that outcomes of population distribution may look like the 2D bell shape, and this distribution may be distorted by other cities; that is to say that other background functions. (i.e., the main city will be affected by their satellite cities). Hence, if the first stage we completed successfully, we will add the extra cities to see how the population distribution of the single city change will be disturbed. Meanwhile, we will amend the initial conditions (i.e., the radius of zones, population, p, q, etc), seeing the difference, and discussing how the tendency of the population may prefer to be. On the other hand, we will also consider the invisible hands; that is to say, quell low classes, give more chances $(p, q)$ to let the different hierarchies alter, operate the different equality types (apply some model according to consequences), etc.

### 4.2 Analysis

In the analysis, we will see the capacity $dE/dT$ to see how adding heat (social mobility) can change energy. Besides, we will plot "M vs T, same t", "M vs t, same T", etc. Lastly, seeing class distribution (before and after the "invisible hands"), we expect to compare with some reality situations (rule of 2:8, etc). We will add the extra analysis if we find it during the process.

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