國立清華大學
NATIONAL TSING HUA UNIVERSITY

11110PHYS401300
Computational Physics Lab
計算物理實作

Release date: 2022.10.24
Due: 2022.11.07
(submit to google classroom)

Homework 2

Reading Assignments

1. Read the tutorials "The Git tutorial" (https://git-scm.com/docs/gittutorial and https://github.com/twtrubiks/Git-Tutorials)
2. Read the `scipy.integrate.solve_ivp` documentation (https://docs.scipy.org/doc/scipy/reference/generated/scipy.integrate.solve_ivp.html)

Written Assignments

1. Evaluate the general solutions of the below ODEs:
   (a) $m\ddot{x} + kx = 0$,
   (b) $m\ddot{x} + \lambda\dot{x} + kx = 0$,
   (c) $m\ddot{x} + \lambda\dot{x} + kx = F_0 \cos\omega_f t$,

Programming Assignments

1. Reuse your IVP solver `mysolver.py` and perform simulations of a damped oscillator (following the definition we used in the lecture). Make a plot for x and $\dot{x}$ versus t and a phase diagram (in polar coordinate) with the following initial conditions:
   Hint: In the diagram, we plot w versus u in polar coordinate, where $u = \omega_1 x = \sqrt{\omega_0^2 - \gamma^2} x$ and $w = \gamma x + \dot{x}$.
   (a) $A = 1$ cm, $\omega_0 = 1$ rad s^{-1}, $\gamma = 0.2$ s^{-1}, and $\phi = -\pi/2$ rad,
   (b) $A = 1$ cm, $\omega_0 = 1$ rad s^{-1}, $\gamma = 1.0$ s^{-1}, and $\phi = -\pi/2$ rad,
   (c) $A = 1$ cm, $\omega_0 = 1$ rad s^{-1}, $\gamma = 1.2$ s^{-1}, and $\phi = -\pi/2$ rad.
2. Following the programming problem 1(a), make two plots of the total energy and energy loss rate versus time for the damped oscillator.

3. Resonance: Now add a sinusoidal driving force ($F = F_0 \cos \omega_f t$) in your damped oscillator with $F_0 = 0.5$. Vary ω_f from 0.5 to 1.5 with an interval of 0.05. Rerun your simulation up to $t_{\max} = 50$. Measure the average amplitude of your oscillator (define $D = < |x(t)| >$ between $40 < t < 50$). Make plots of D versus ω_f with (a) $\lambda = 0.01$, (b) $\lambda = 0.1$, (c) $\lambda = 0.3$. Draw these three plots on the same figure. Do you find resonance? Are these resonance frequencies consistent with the analytical values?

4. A hanging mass-spring system can be analogized to RLC circuit systems. The gravitational force ($F = mg$) is analogous to the emf. The damping parameter has the electrical analog resistance (R). The displacement has the electrical analog charge (q), ... etc.

   Consider the series RLC circuit shown in the below Figure driven by an alternating emf of value $E_0 \sin \omega t$.

   (a) Show that the RLC system can be described by an ODE system,

   $$L\ddot{q} + R\dot{q} + \frac{q}{C} = E_0 \sin \omega t, \tag{1}$$

   Hint: Use the Kirchoff's equation to derive the governing equation.

   (b) Use the same IVP solver we developed in the class to numerically solve the system (Equation 1) with initial conditions: $L = C = E_0 = 1$, $R = 0.8$, and $\omega = 0.7$. Make plots of the current and the voltage V_L across the inductor as functions of time.

   (c) Redo the problem by varying ω from 0.3 to 1.5 with an interval 0.1. Do you see any special ω? What are the meaning of these frequencies?