

# Data Structures

CSCI 2270-202: REC 11

Sanskar Katiyar

# Logistics

## Office Hours (This week)

Today: 5 pm - 7 pm

Friday: 3 pm - 5 pm

## Assignment 7 Concerns

Interview grading opportunity

## No notes on Assignment 8, 9

Most stuff covered in recitation, Check Github

# Recitation Outline

1. Implementing Graph: Add/Remove, Display
2. BFS, DFS: Review
3. Dijkstra's Shortest Path Algorithm
4. Exercise

# BFS, DFS: Review

# DFS: Pseudocode (Recursive)

```
DFS(G, u) {  
    u.visited = true  
    for each v ∈ G.Adj[u]  
        if v.visited == false  
            DFS(G, v)  
}
```

```
for each u ∈ G  
    u.visited = false
```

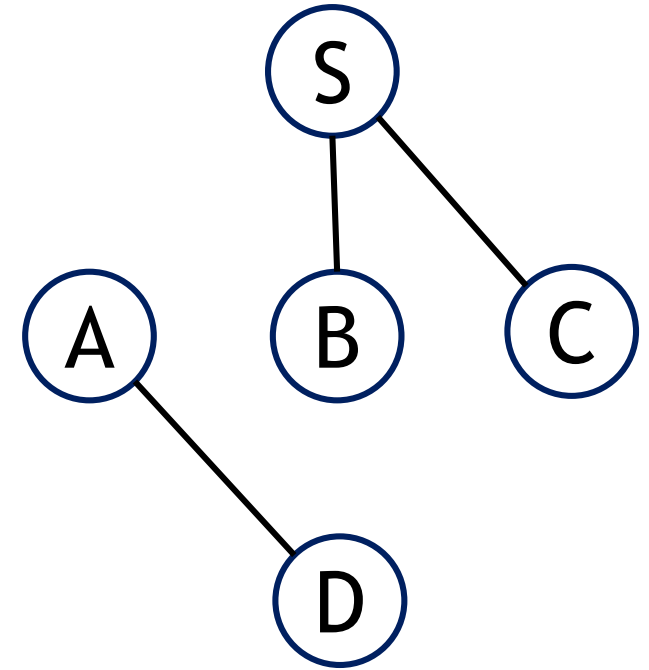
}

Initialize all nodes as unvisited

```
for each u ∈ G  
    if u.visited == false  
        DFS(G, u)
```

}

Loop: If there is more than one component



# BFS: Pseudocode (Iterative)

```
BFS(G, u) {  
    Q = Queue()  
    Q.enqueue(u)  
    u.visited = true  
    while !Q.isEmpty()  
        v = Q.peek(); Q.dequeue();  
  
        for each w ∈ G[v]  
            if w.visited == false  
                Q.enqueue(w)  
                w.visited = true  
}
```

# DFS: Finding Number of Components

```
DFS(G, u) {  
    u.visited = true  
    for each v ∈ G.Adj[u]  
        if v.visited == false  
            DFS(G, v)  
}
```

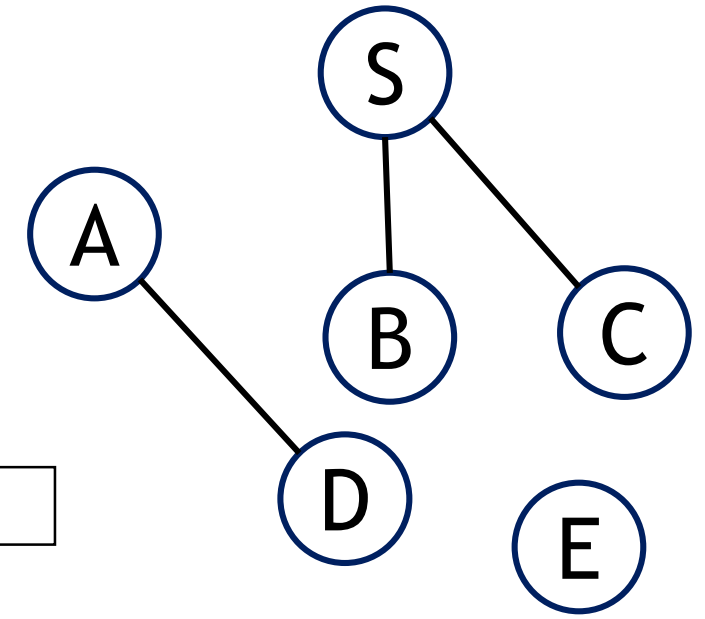
```
for each u ∈ G  
    u.visited = false
```

```
for each u ∈ G  
    if u.visited == false  
        DFS(G, u)
```

#components++;

} Initialize all nodes as unvisited

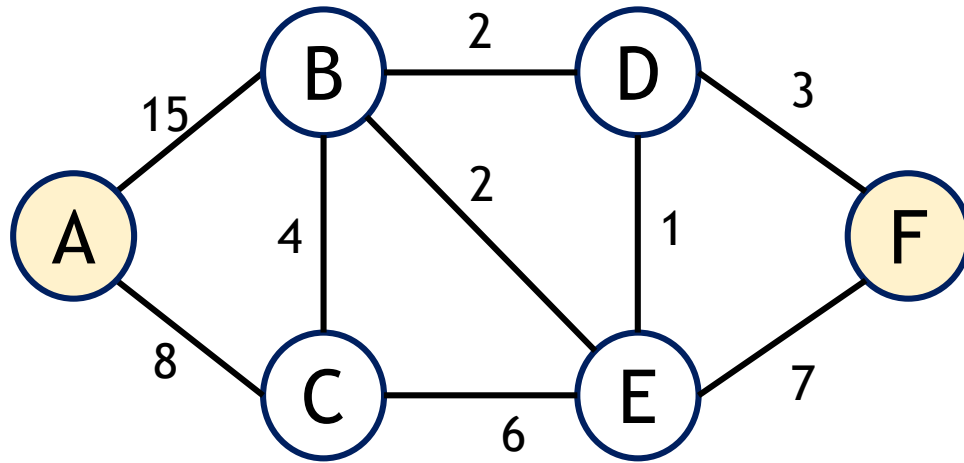
} Loop: If there is more than one component



# Dijkstra's Shortest Path



# Breadth First Search: Shortest Path



*Assume Edge weights are distances between two vertices*

**Shortest Path from A->F**

A -> B -> D -> F (20)

A -> C -> E -> F (21)

A -> B -> E -> F (24)

**A -> C -> B -> D -> F (17)**

## BFS: Shortest Path

Path with the smallest number of edges (agnostic to edge weight)

# Dijkstra's Shortest Path Algorithm

Edsger W. Dijkstra (in 1956)

Greedy Algorithm

Single-source shortest paths to all *reachable* vertices

Applicable to Weighted Graphs

***Applications:*** Network Routing Protocols, Path Planning (Potential Field Methods), AI

# Dijkstra's Shortest Path Algorithm

**Disclaimer:** The following implementation is different from what is covered in lectures and the textbook.

The idea is to demonstrate a different implementation approach for the same algorithm.

***Please ensure that you understand the implementation from the lecture before going forward with this.***

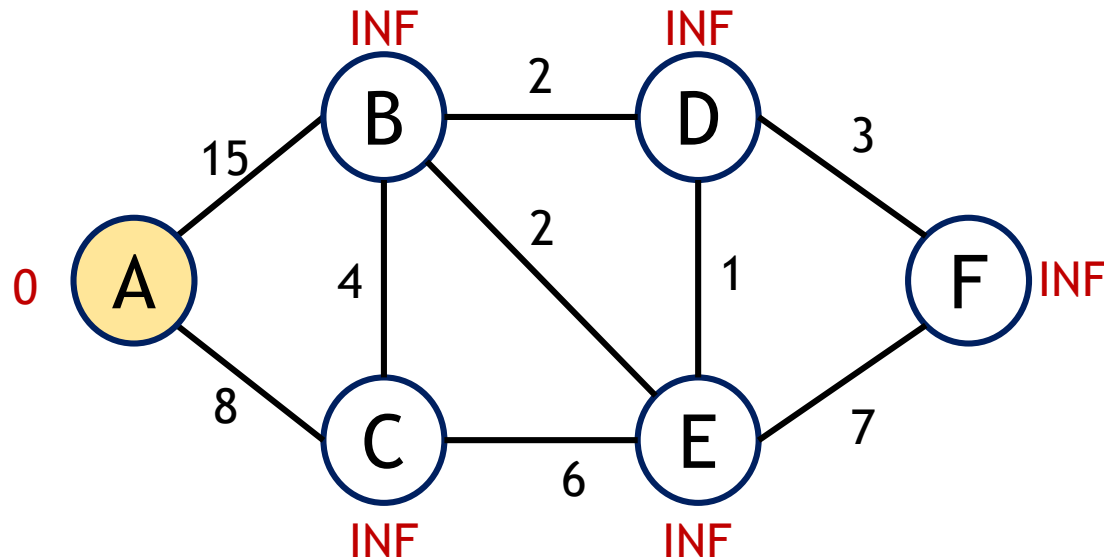
# Dijkstra's Shortest Path: Steps

1. From the source vertex, visit the vertex  $u$  with the least known distance
2. Once at the vertex  $u$ , check each of  $u$ 's neighbors
3. Calculate the distance for the neighbors by summing the cost of the edges leading from the source vertex
4. If the distance is less than a known distance, update the shortest distance for that vertex; mark the node from which the edge emanates as the predecessor

# Dijkstra's Shortest Path: Example

	A	B	C	D	E	F
DIST	0	INF	INF	INF	INF	INF
PREV	-	-	-	-	-	-

-	A	B	C	D	E	F
---	---	---	---	---	---	---



Initialize the distance of all nodes from A as INF; Populate Queue/Array with all vertices

Source's distance to itself is 0

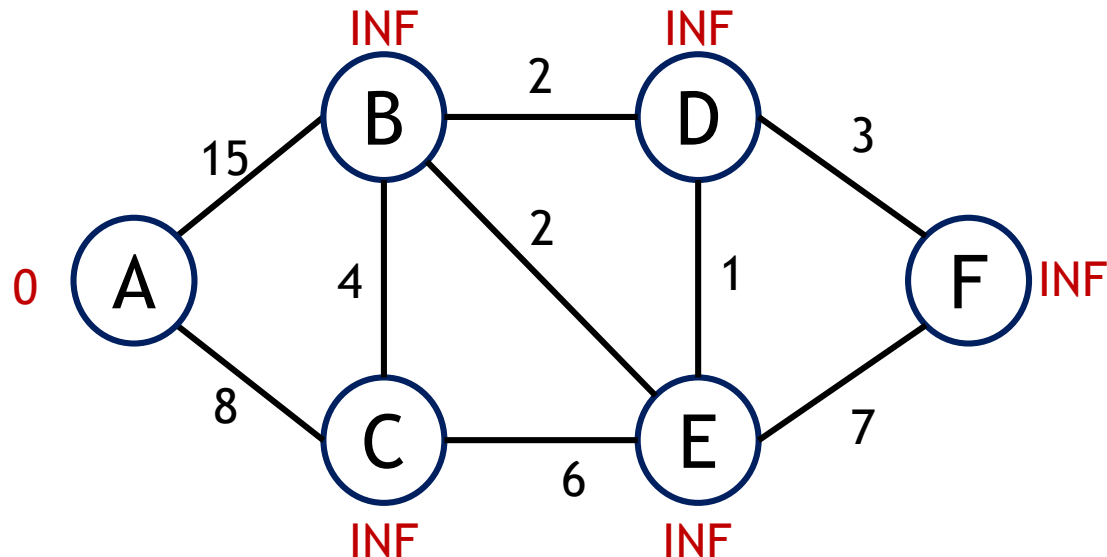
Why INF?

Think about non-reachable nodes, and comparison limits

# Dijkstra's Shortest Path: Example

	A	B	C	D	E	F
DIST	0	INF	INF	INF	INF	INF
PREV	-	-	-	-	-	-

-	A	B	C	D	E	F
---	---	---	---	---	---	---



```
while !Q.isEmpty()
    u = vertex in Q w. min(DIST)
    remove u from Q
    for each v in G[u]
        T = DIST[u] + DIST[u,v]
        if T < DIST[v]
            DIST[v] = T
            PREV[v] = u

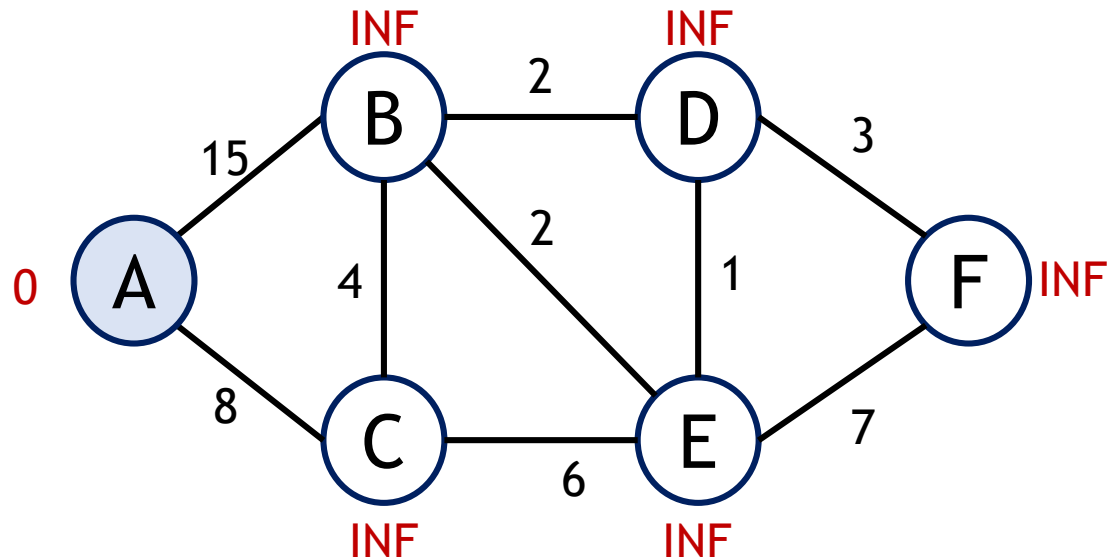
return DIST, PREV
```



# Dijkstra's Shortest Path: Example

	A	B	C	D	E	F
DIST	0	INF	INF	INF	INF	INF
PREV	-	-	-	-	-	-

-	A	B	C	D	E	F
---	---	---	---	---	---	---



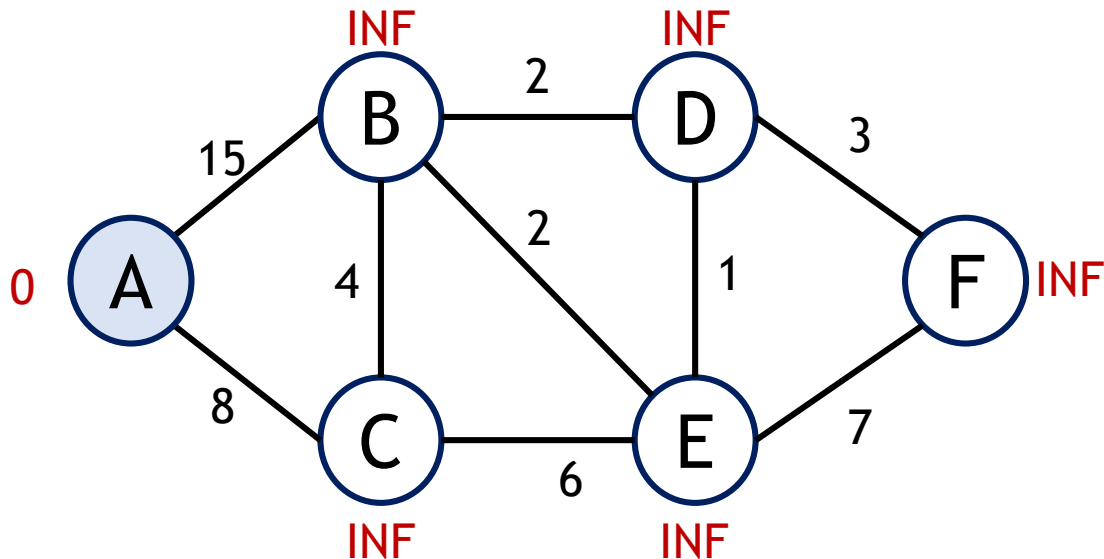
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return DIST, PREV
```

# Dijkstra's Shortest Path: Example

	A*	B	C	D	E	F
DIST	0	INF	INF	INF	INF	INF
PREV	-	-	-	-	-	-

A	B	C	D	E	F	
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return DIST, PREV
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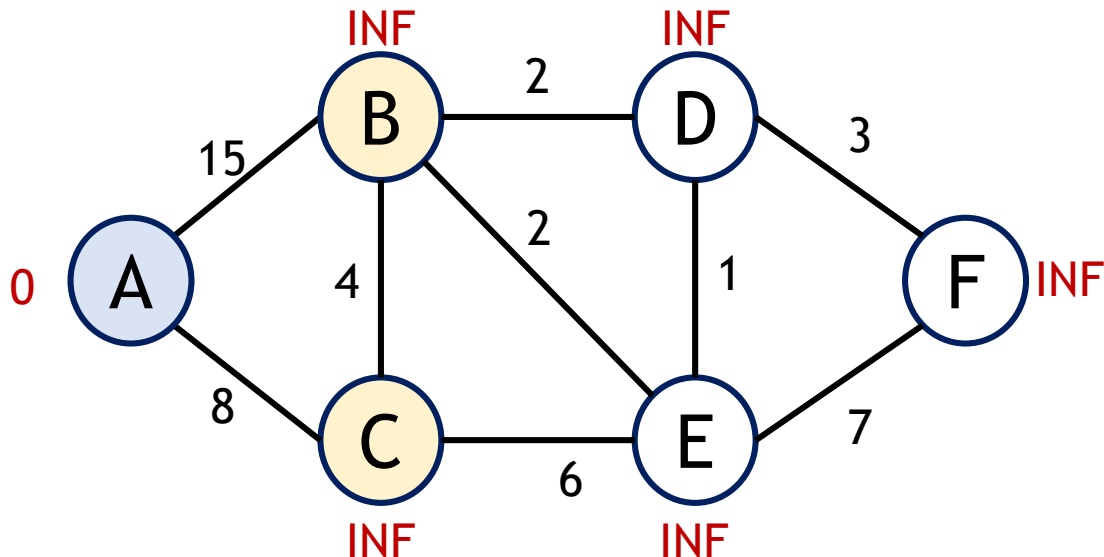




# Dijkstra's Shortest Path: Example

	A*	B	C	D	E	F
DIST	0	INF	INF	INF	INF	INF
PREV	-	-	-	-	-	-

A	B	C	D	E	F	



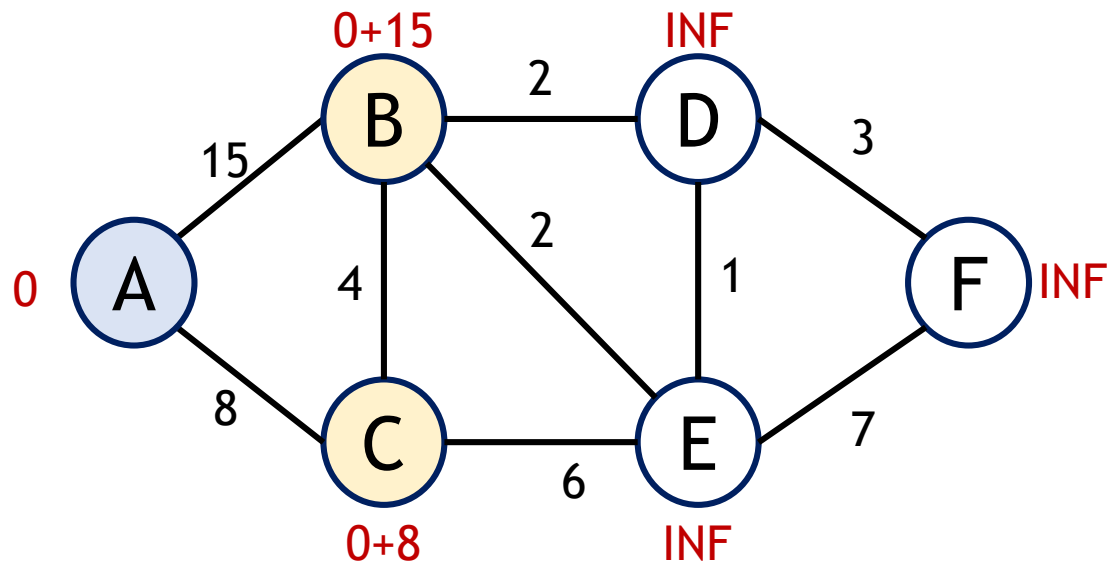
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DIST	0	INF	INF	INF	INF	INF
PREV	-	-	-	-	-	-

A	B	C	D	E	F	
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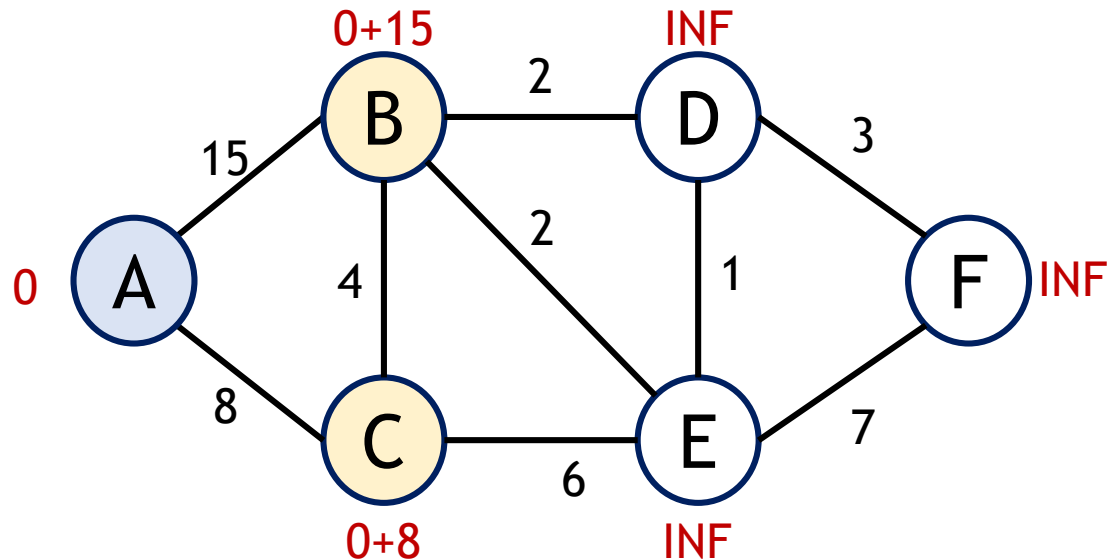
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        if T < DIST[v]
            DIST[v] = T
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    return DIST, PREV
    
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# Dijkstra's Shortest Path: Example

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DIST	0	INF	INF	INF	INF	INF
PREV	-	-	-	-	-	-

A	B	C	D	E	F	
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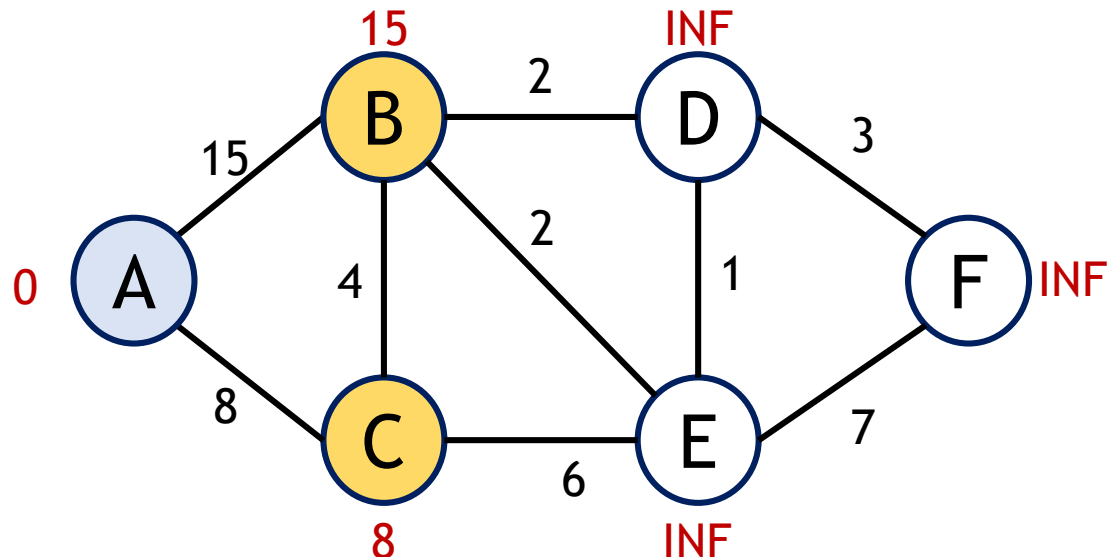
return DIST, PREV
    
```



# Dijkstra's Shortest Path: Example

	A*	B	C	D	E	F
DIST	0	15	8	INF	INF	INF
PREV	-	A	A	-	-	-

A	B	C	D	E	F	



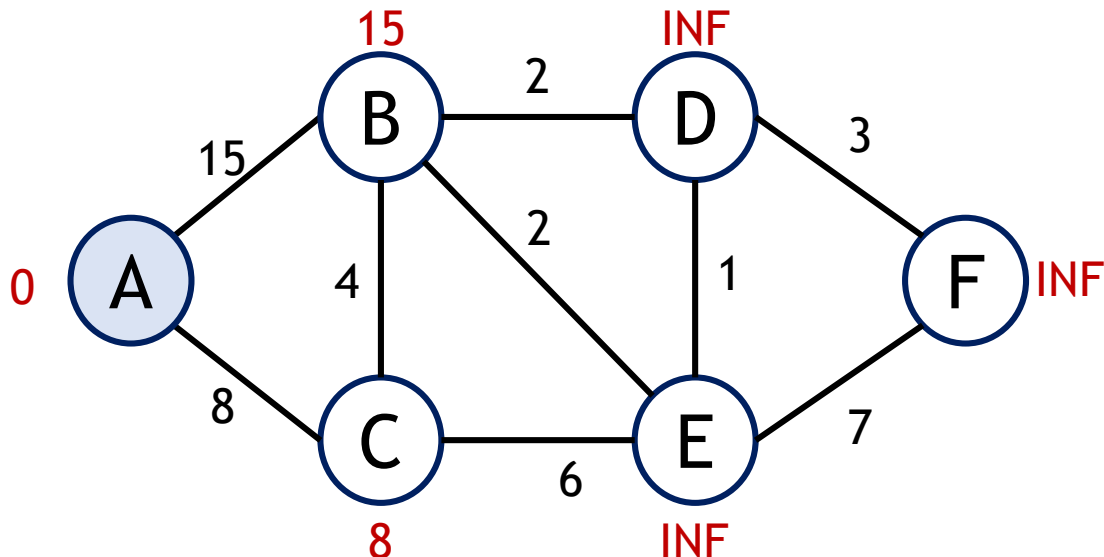
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        if T < DIST[v]
            DIST[v] = T
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return DIST, PREV
```

# Dijkstra's Shortest Path: Example

	A*	B	C	D	E	F
DIST	0	15	8	INF	INF	INF
PREV	-	A	A	-	-	-

A	B	C	D	E	F	
---	---	---	---	---	---	--



```
while !Q.isEmpty()
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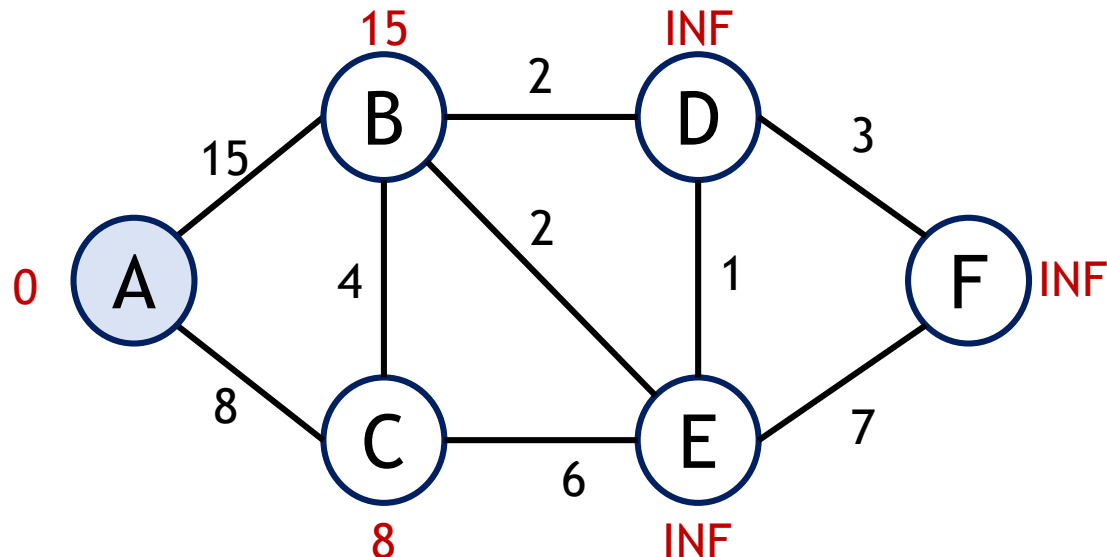
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# Dijkstra's Shortest Path: Example

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DIST	0	15	8	INF	INF	INF
PREV	-	A	A	-	-	-

A	B	C	D	E	F	



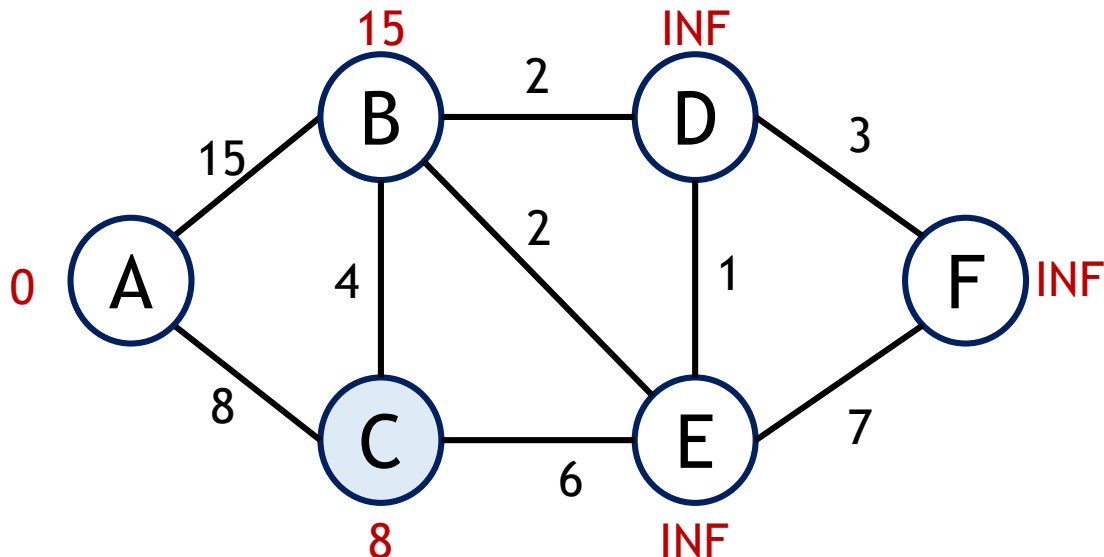
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# Dijkstra's Shortest Path: Example

	A*	B	C*	D	E	F
DIST	0	15	8	INF	INF	INF
PREV	-	A	A	-	-	-

C	B	D	E	F		
---	---	---	---	---	--	--



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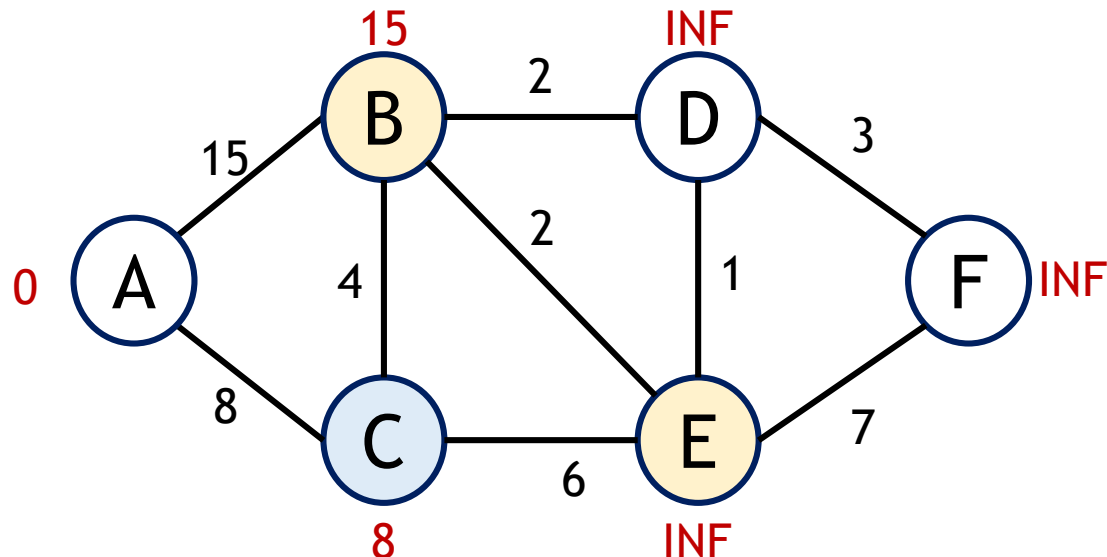
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# Dijkstra's Shortest Path: Example

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DIST	0	15	8	INF	INF	INF
PREV	-	A	A	-	-	-

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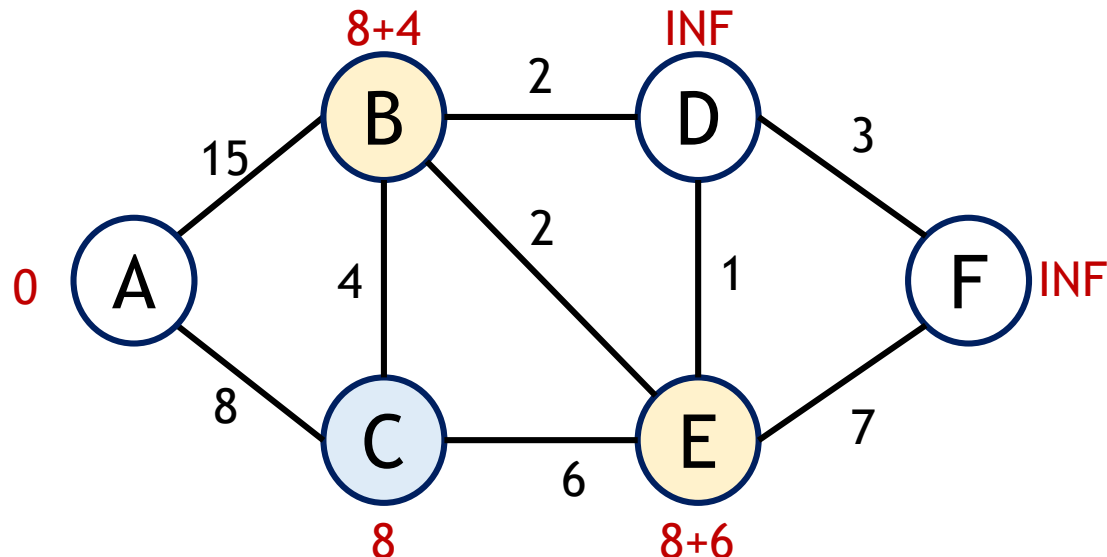
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DIST	0	15	8	INF	INF	INF
PREV	-	A	A	-	-	-

C	B	D	E	F		
---	---	---	---	---	--	--



```

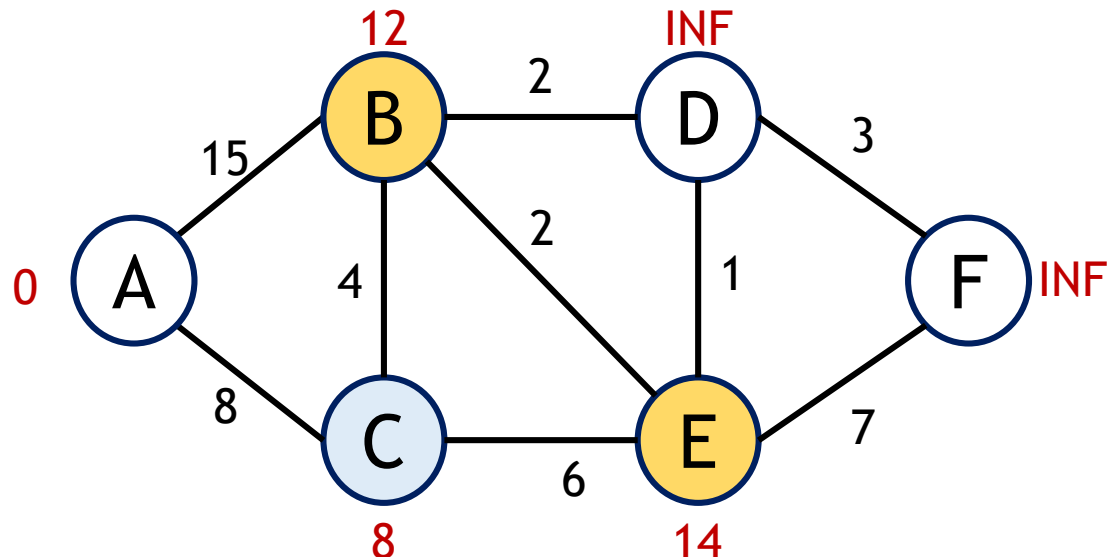
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```

# Dijkstra's Shortest Path: Example

	A*	B	C*	D	E	F
DIST	0	12	8	INF	14	INF
PREV	-	C	A	-	C	-

C	B	D	E	F		
---	---	---	---	---	--	--



```

while !Q.isEmpty()
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    remove u from Q
    for each v in G[u]
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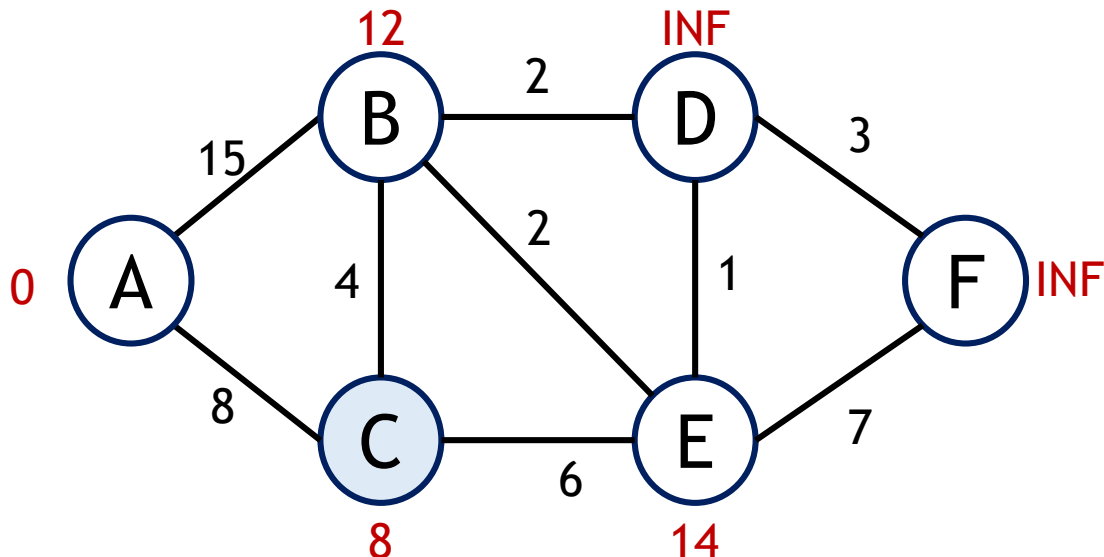
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# Dijkstra's Shortest Path: Example

	A*	B	C*	D	E	F
DIST	0	12	8	INF	14	INF
PREV	-	C	A	-	C	-

C	B	D	E	F		
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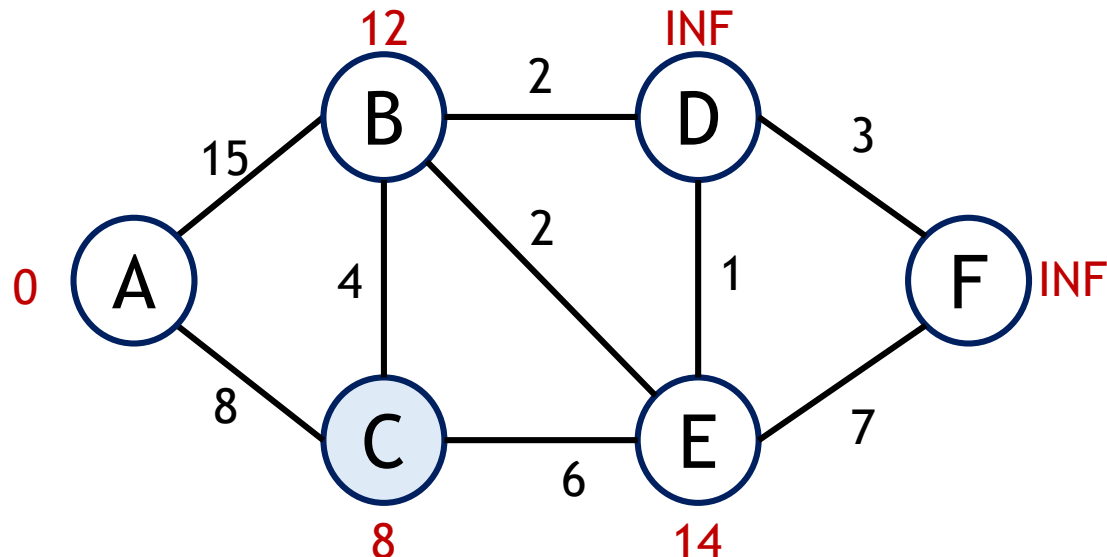
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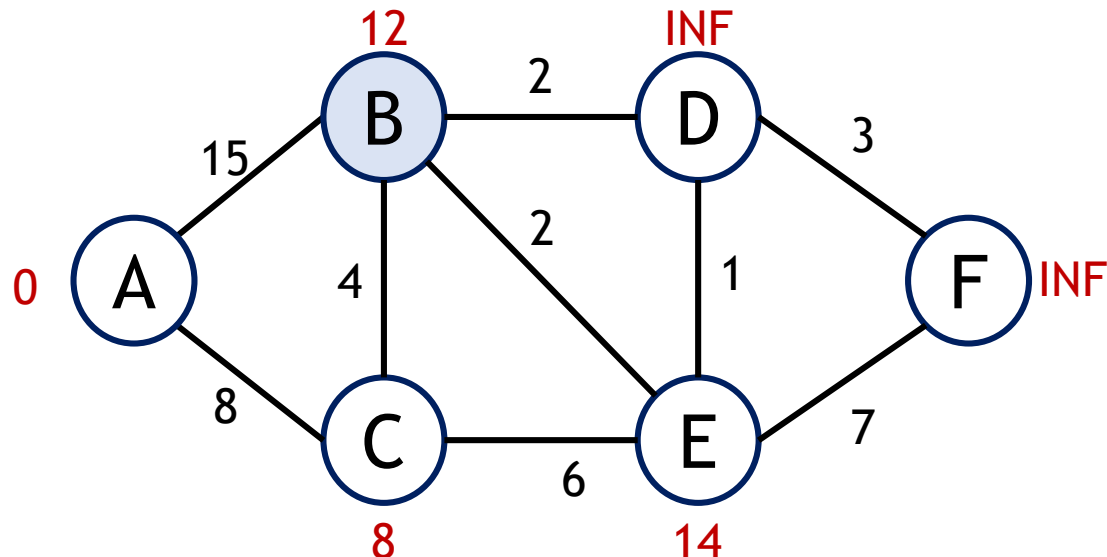
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DIST	0	12	8	INF	14	INF
PREV	-	C	A	-	C	-

B	D	E	F			
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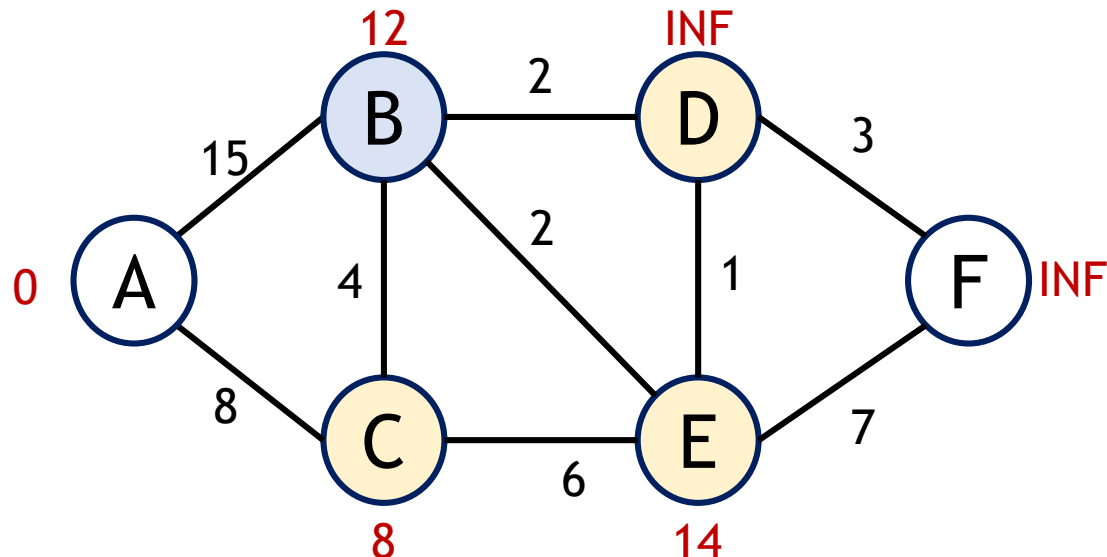
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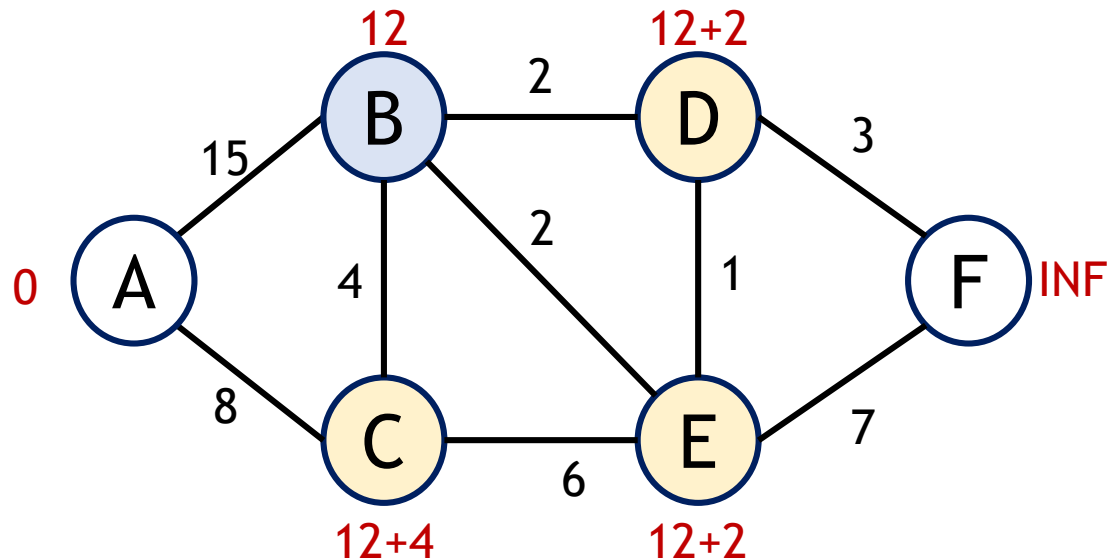


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DIST	0	12	8	INF	14	INF
PREV	-	C	A	-	C	-

B	D	E	F			
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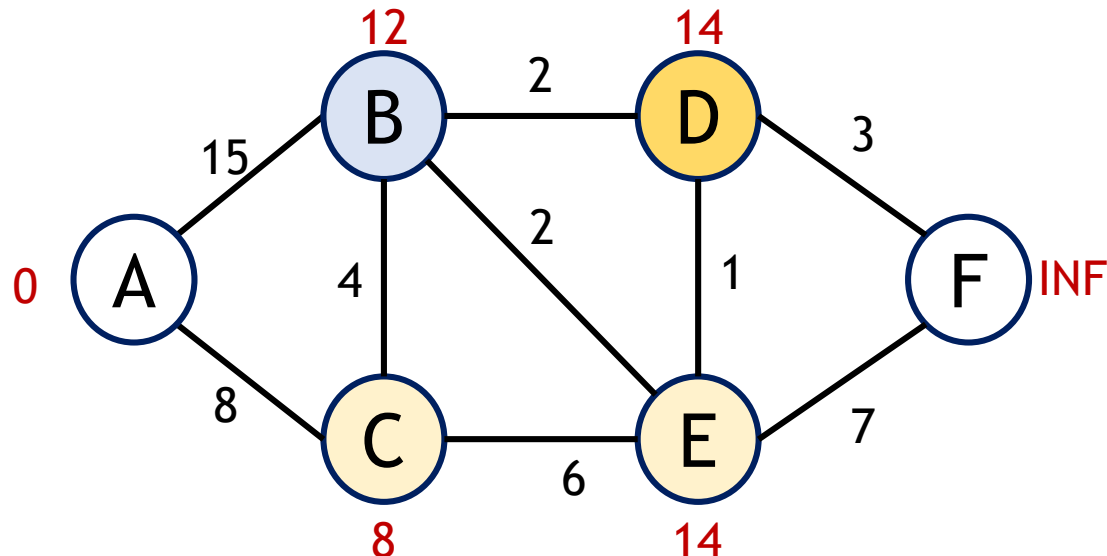
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PREV	-	C	A	B	C	-

B	D	E	F			
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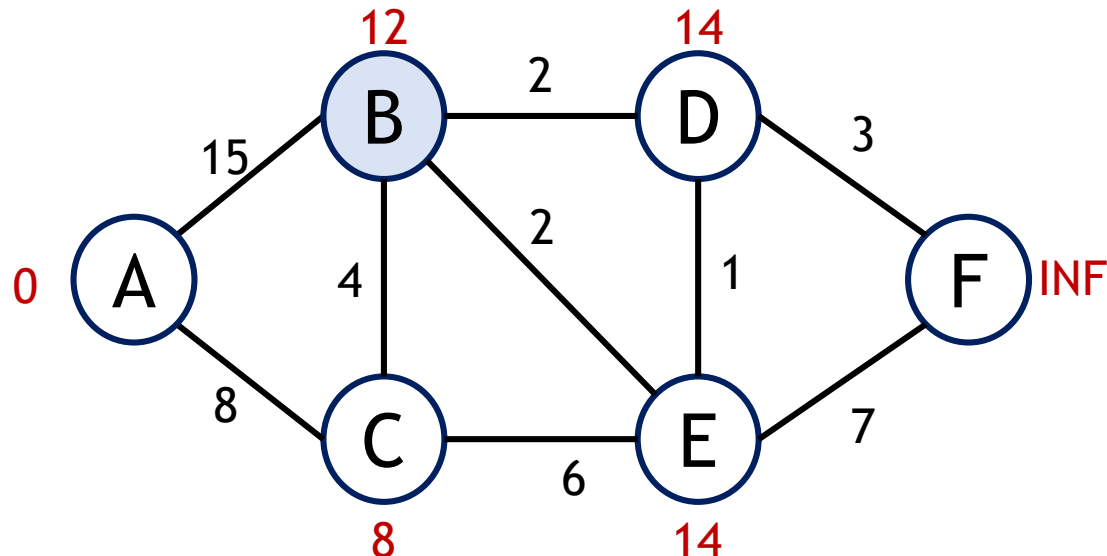




# Dijkstra's Shortest Path: Example

	A*	B*	C*	D	E	F
DIST	0	12	8	14	14	INF
PREV	-	C	A	B	C	-

B	D	E	F			
---	---	---	---	--	--	--



```

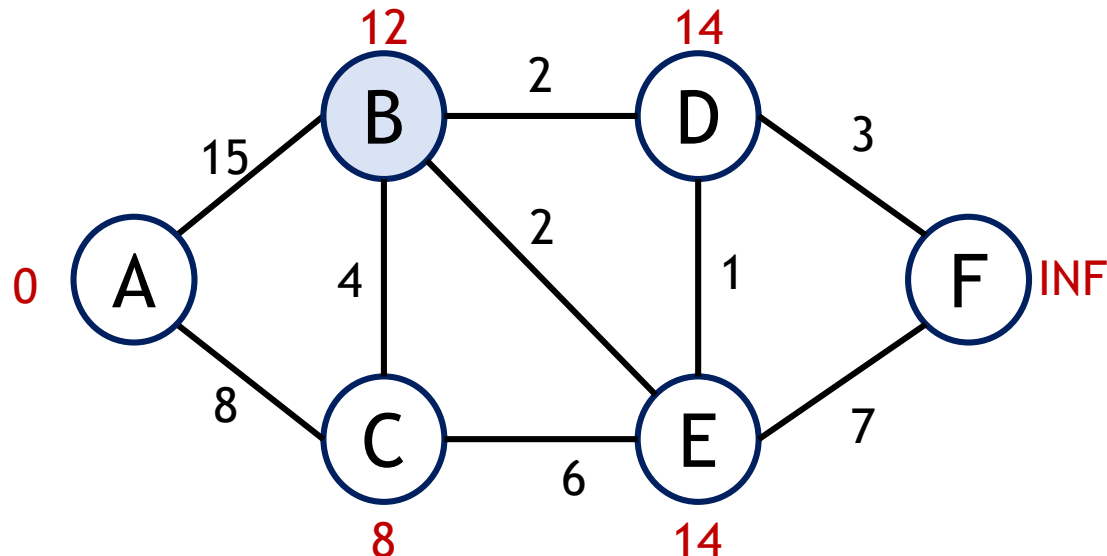
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DIST	0	12	8	14	14	INF
PREV	-	C	A	B	C	-

B	D	E	F			
---	---	---	---	--	--	--



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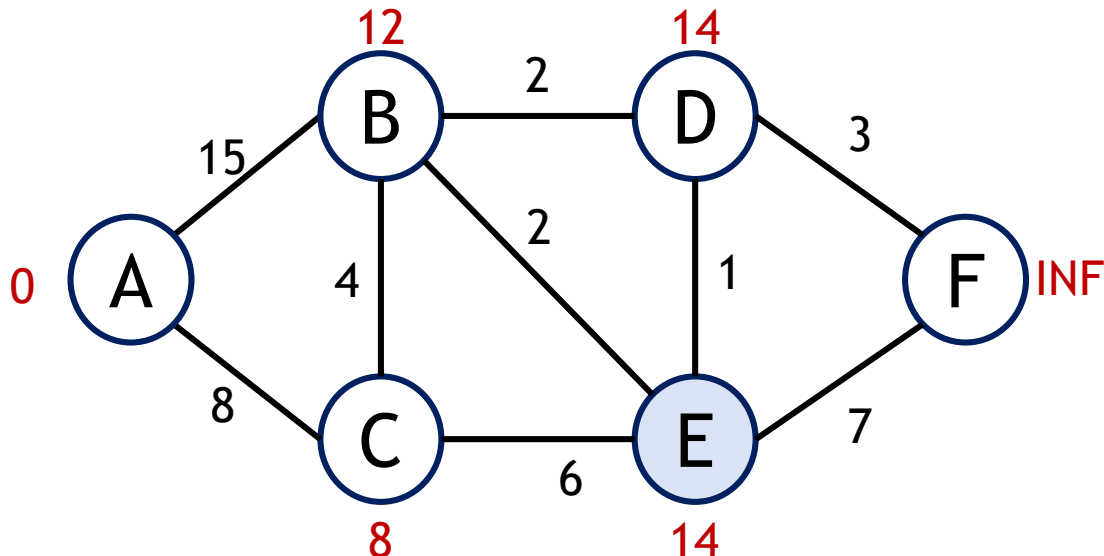
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	A*	B*	C*	D	E*	F
DIST	0	12	8	14	14	INF
PREV	-	C	A	B	C	-

E	D	F				
---	---	---	--	--	--	--



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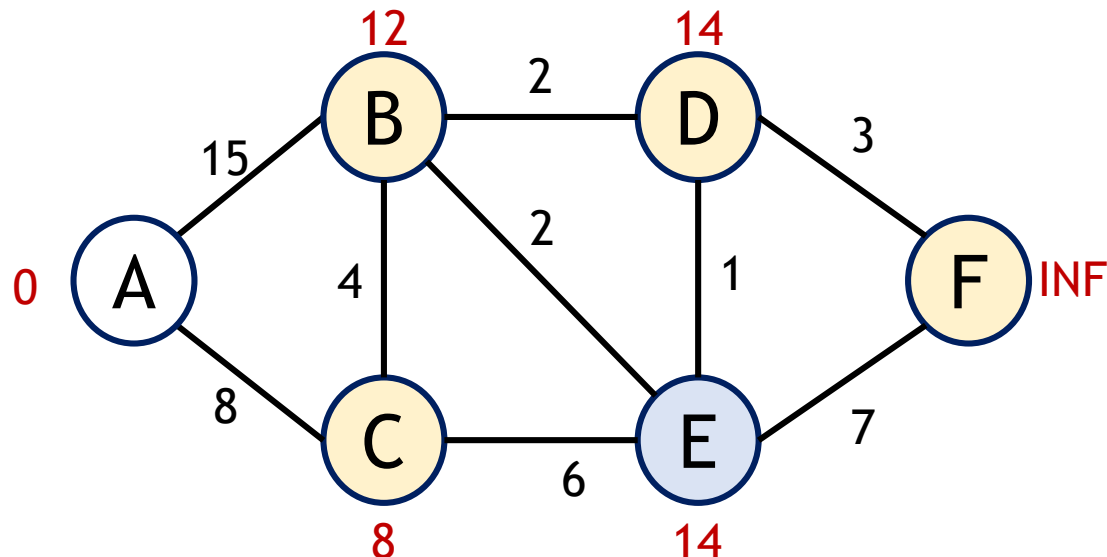
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            DIST[v] = T  
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```

```
return DIST, PREV
```

# Dijkstra's Shortest Path: Example

	A*	B*	C*	D	E*	F
DIST	0	12	8	14	14	INF
PREV	-	C	A	B	C	-

E	D	F				
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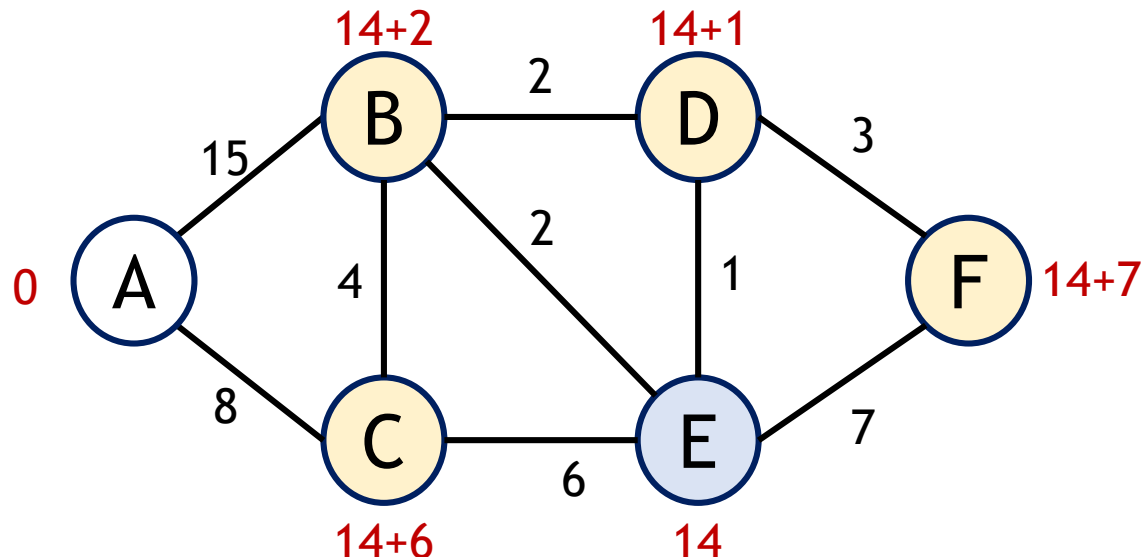


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	A*	B*	C*	D	E*	F
DIST	0	12	8	14	14	INF
PREV	-	C	A	B	C	-

E	D	F				
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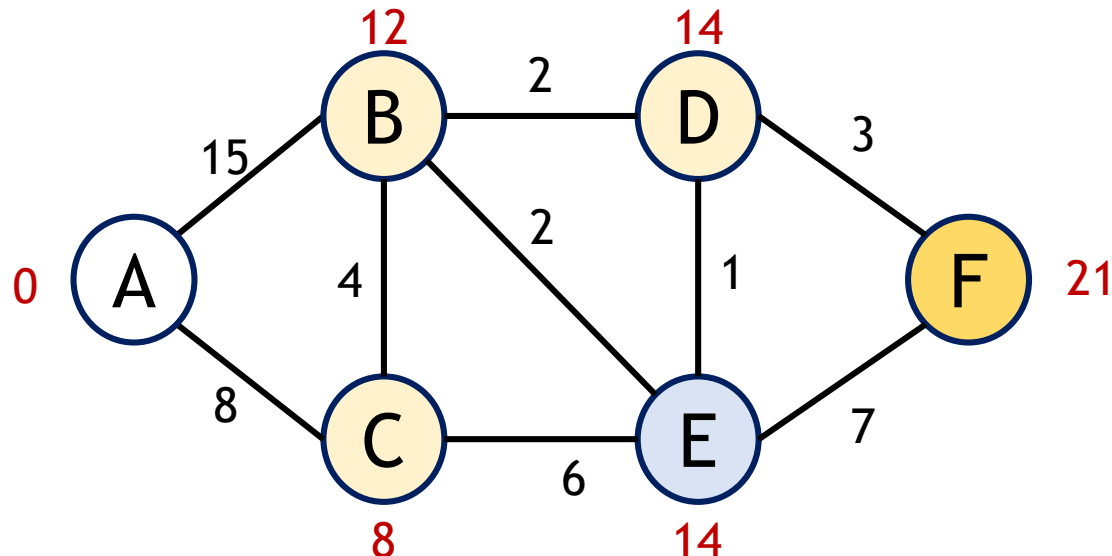


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return DIST, PREV
```

# Dijkstra's Shortest Path: Example

	A*	B*	C*	D	E*	F
DIST	0	12	8	14	14	21
PREV	-	C	A	B	C	E

E	D	F				
---	---	---	--	--	--	--



```

while !Q.isEmpty()
    u = vertex in Q w. min(DIST)
    remove u from Q
    for each v in G[u]
        T = DIST[u] + DIST[u,v]
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            DIST[v] = T
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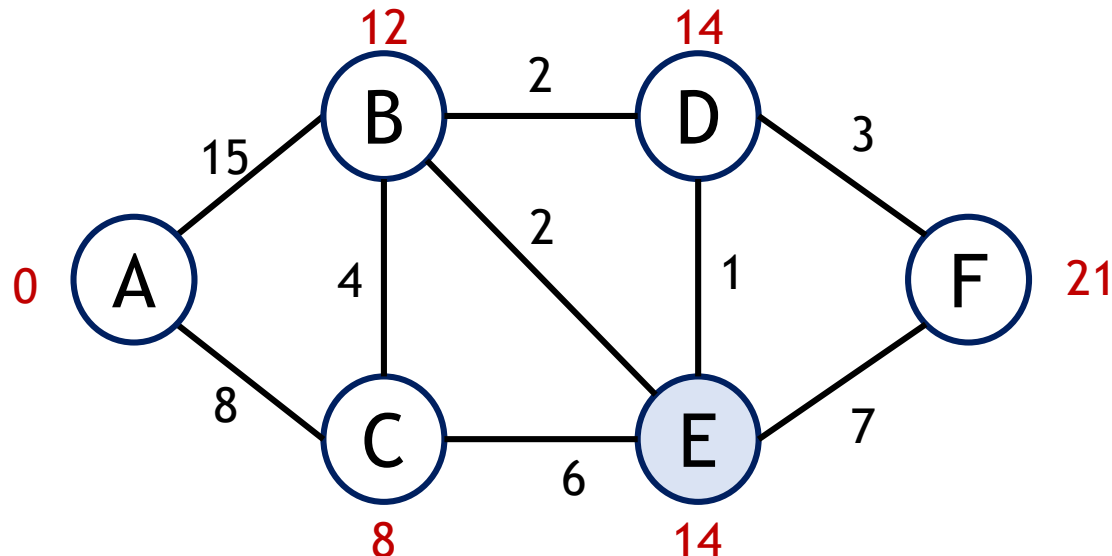
return DIST, PREV
    
```



# Dijkstra's Shortest Path: Example

	A*	B*	C*	D	E*	F
DIST	0	12	8	14	14	21
PREV	-	C	A	B	C	E

E	D	F				
---	---	---	--	--	--	--



```
while !Q.isEmpty()
    u = vertex in Q w. min(DIST)
    remove u from Q
    for each v in G[u]
        T = DIST[u] + DIST[u,v]
        if T < DIST[v]
            DIST[v] = T
            PREV[v] = u

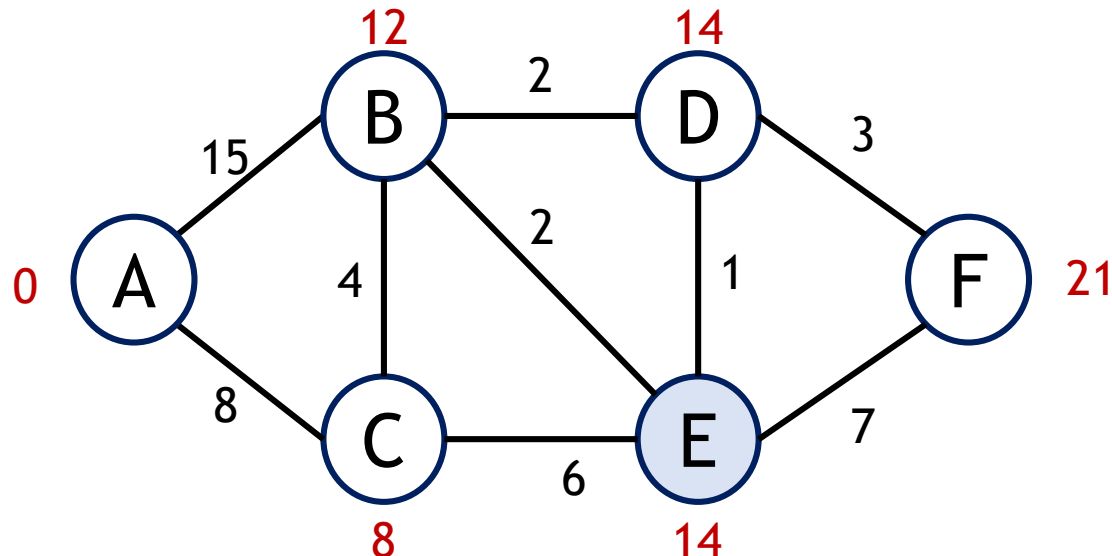
return DIST, PREV
```



# Dijkstra's Shortest Path: Example

	A*	B*	C*	D	E*	F
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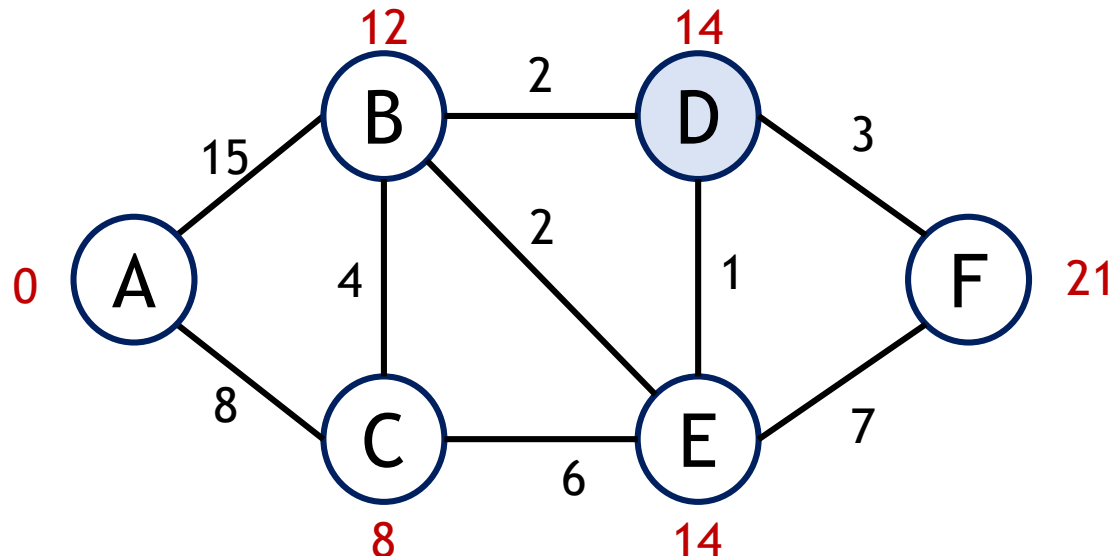
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```



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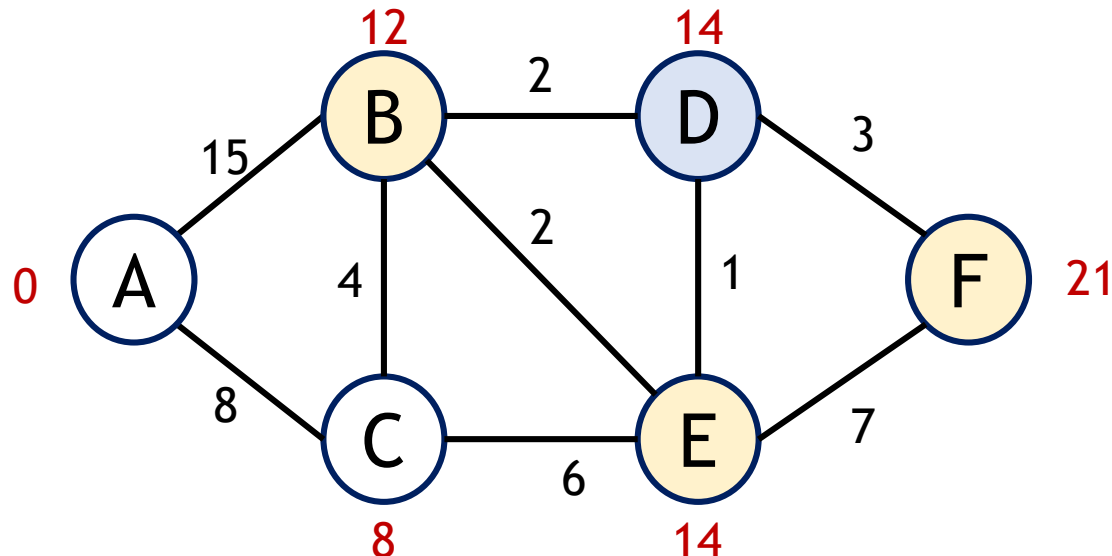
return DIST, PREV
```



# Dijkstra's Shortest Path: Example

	A*	B*	C*	D*	E*	F
DIST	0	12	8	14	14	21
PREV	-	C	A	B	C	E

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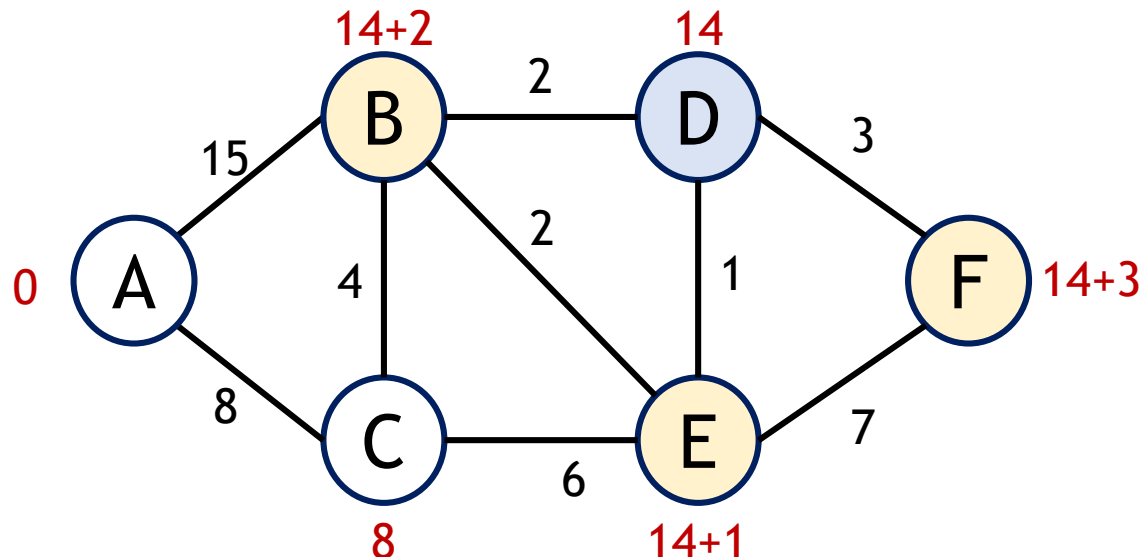
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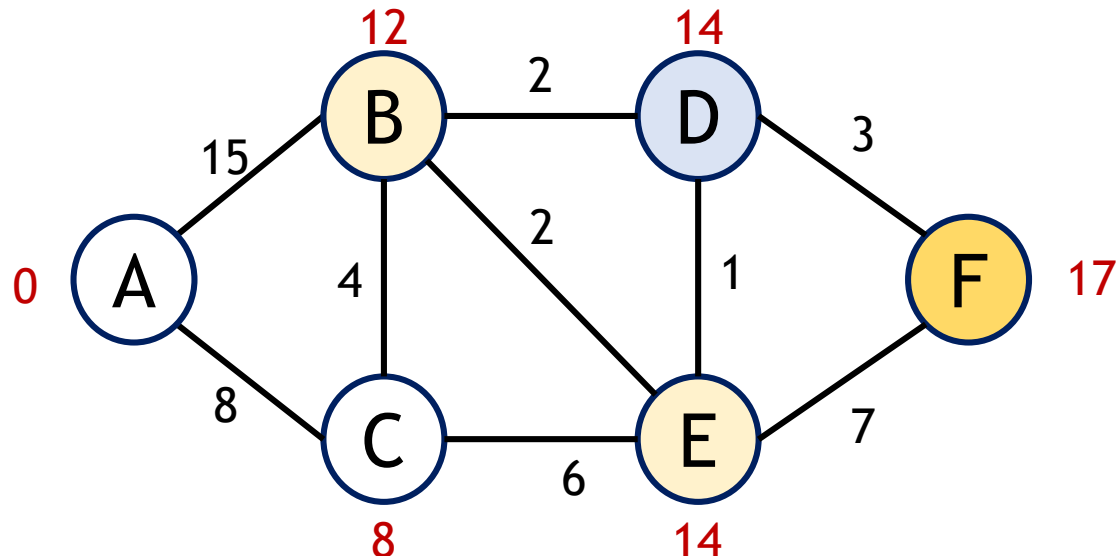
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```



# Dijkstra's Shortest Path: Example

	A*	B*	C*	D*	E*	F
DIST	0	12	8	14	14	17
PREV	-	C	A	B	C	D

D	F					
---	---	--	--	--	--	--



```
while !Q.isEmpty()
    u = vertex in Q w. min(DIST)
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    for each v in G[u]
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        if T < DIST[v]
            DIST[v] = T
            PREV[v] = u

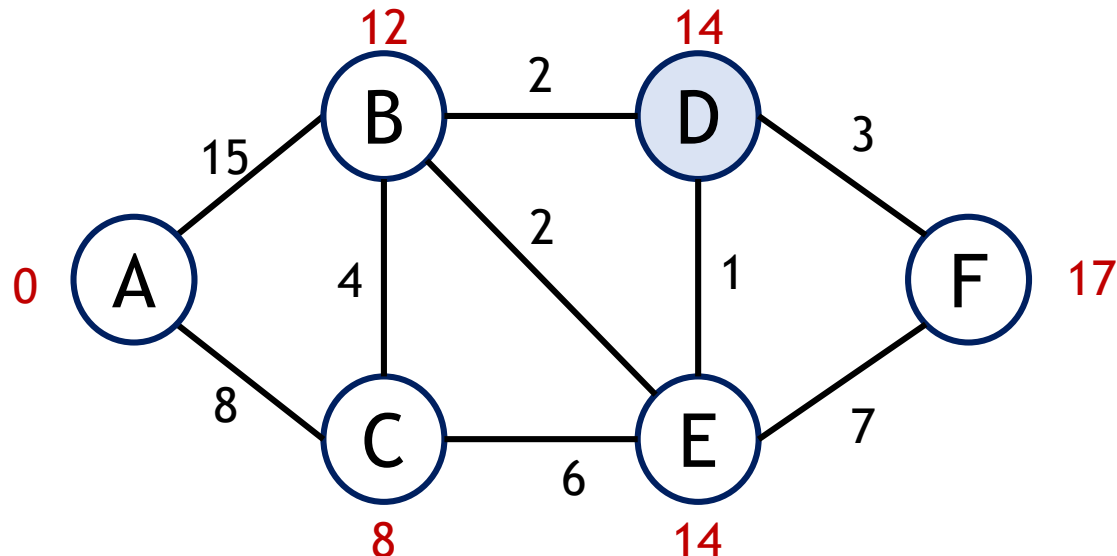
return DIST, PREV
```



# Dijkstra's Shortest Path: Example

	A*	B*	C*	D*	E*	F
DIST	0	12	8	14	14	17
PREV	-	C	A	B	C	D

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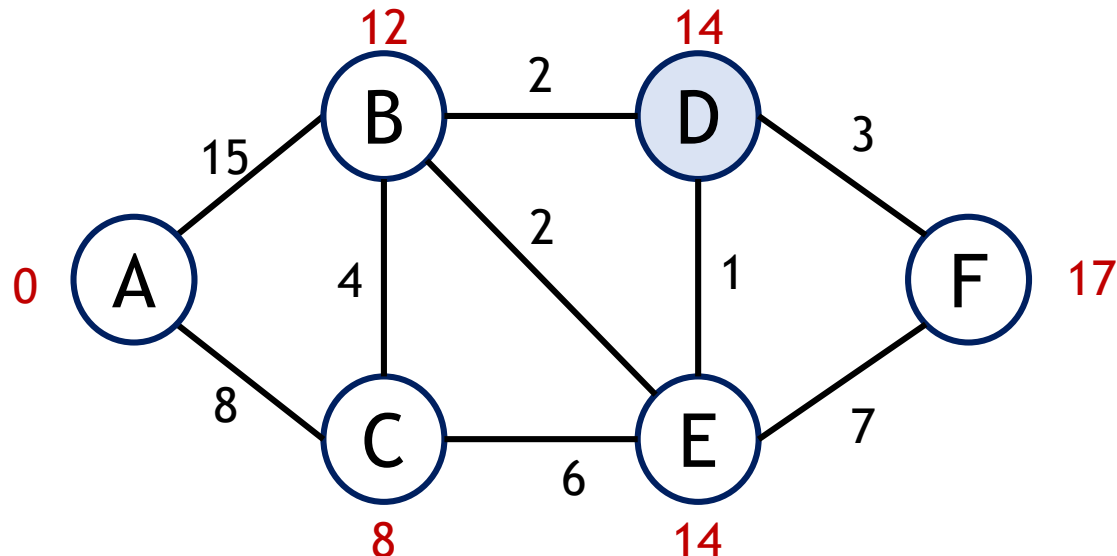
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# Dijkstra's Shortest Path: Example

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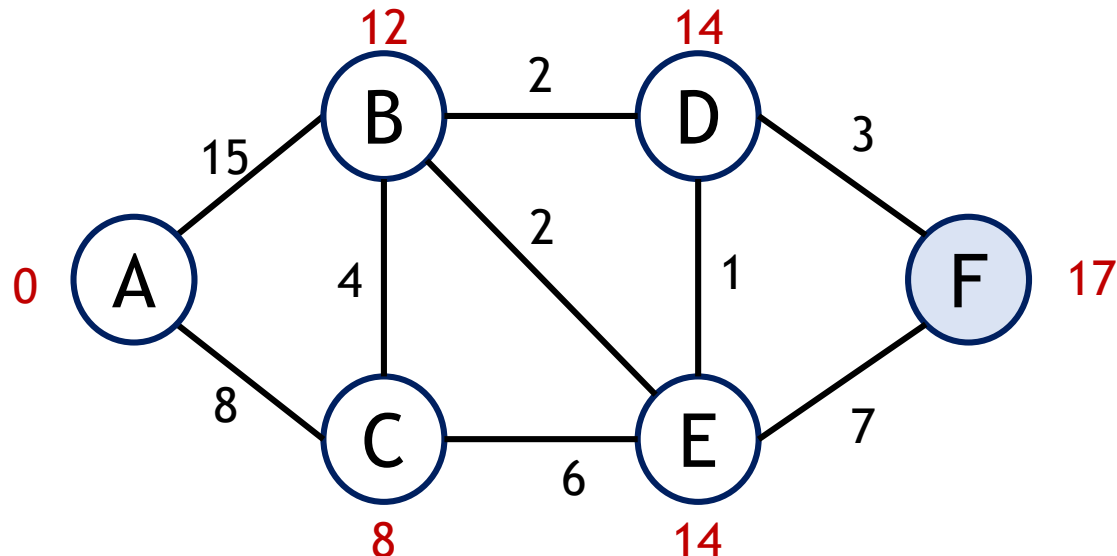
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---	--	--	--	--	--	--



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while !Q.isEmpty()  
    u = vertex in Q w. min(DIST)  
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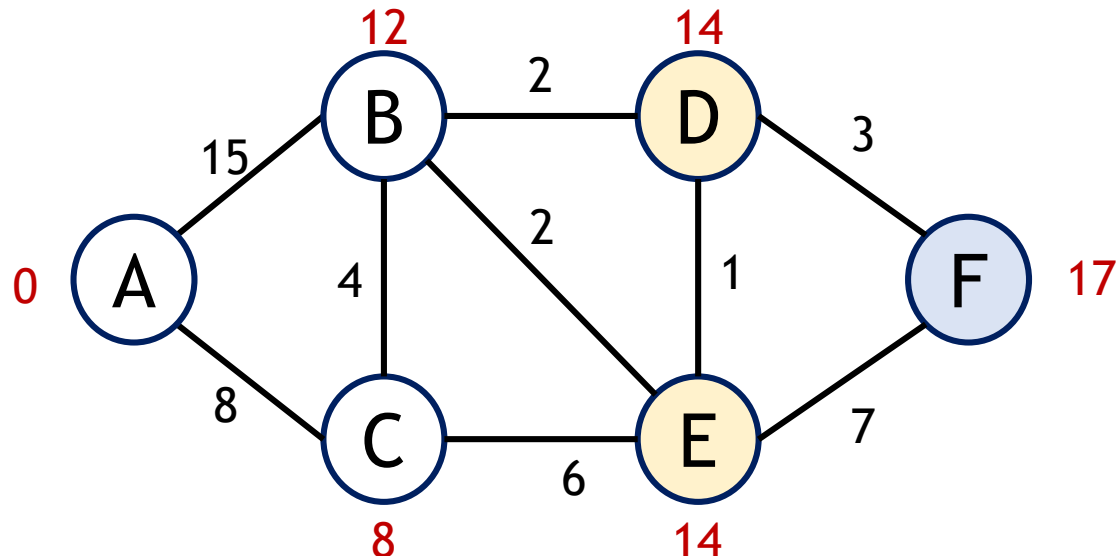
```
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        T = DIST[u] + DIST[u,v]  
        if T < DIST[v]  
            DIST[v] = T  
            PREV[v] = u
```

```
return DIST, PREV
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# Dijkstra's Shortest Path: Example

	A*	B*	C*	D*	E*	F*
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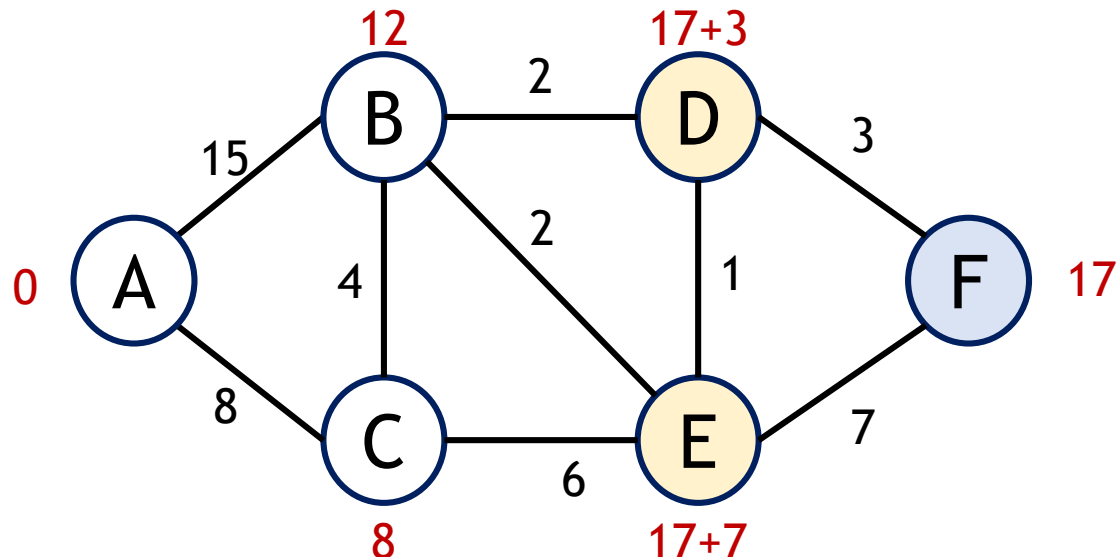
return DIST, PREV
```



# Dijkstra's Shortest Path: Example

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DIST	0	12	8	14	14	17
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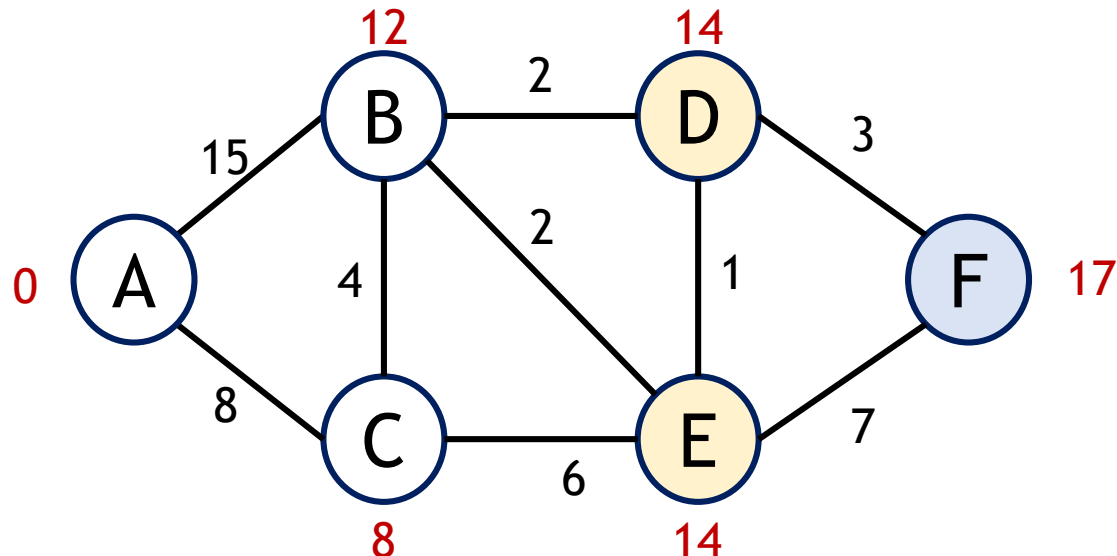
return DIST, PREV
    
```



# Dijkstra's Shortest Path: Example

	A*	B*	C*	D*	E*	F*
DIST	0	12	8	14	14	17
PREV	-	C	A	B	C	D

F						
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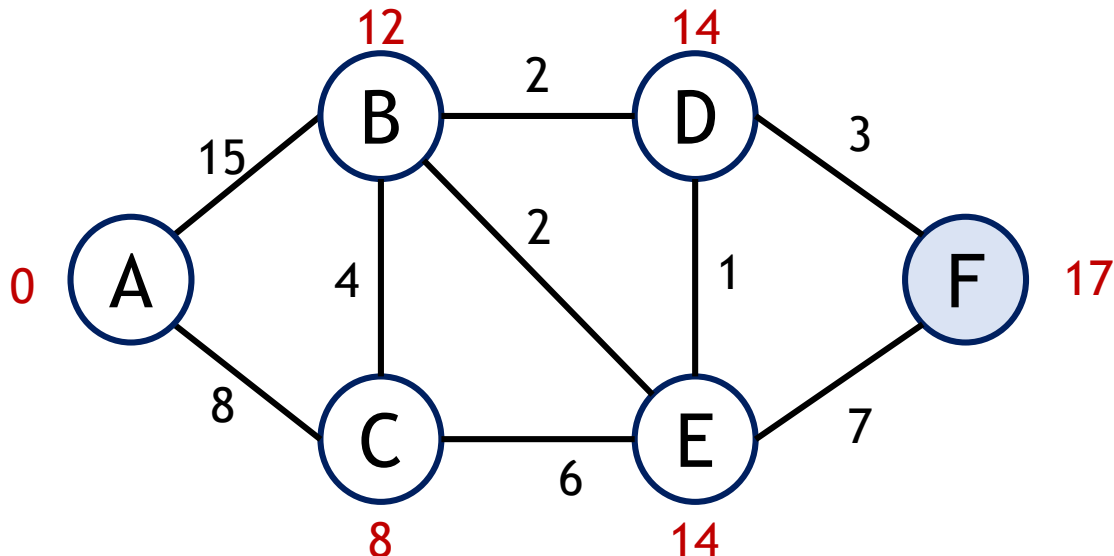
return DIST, PREV
    
```



# Dijkstra's Shortest Path: Example

	A*	B*	C*	D*	E*	F*
DIST	0	12	8	14	14	17
PREV	-	C	A	B	C	D

F						
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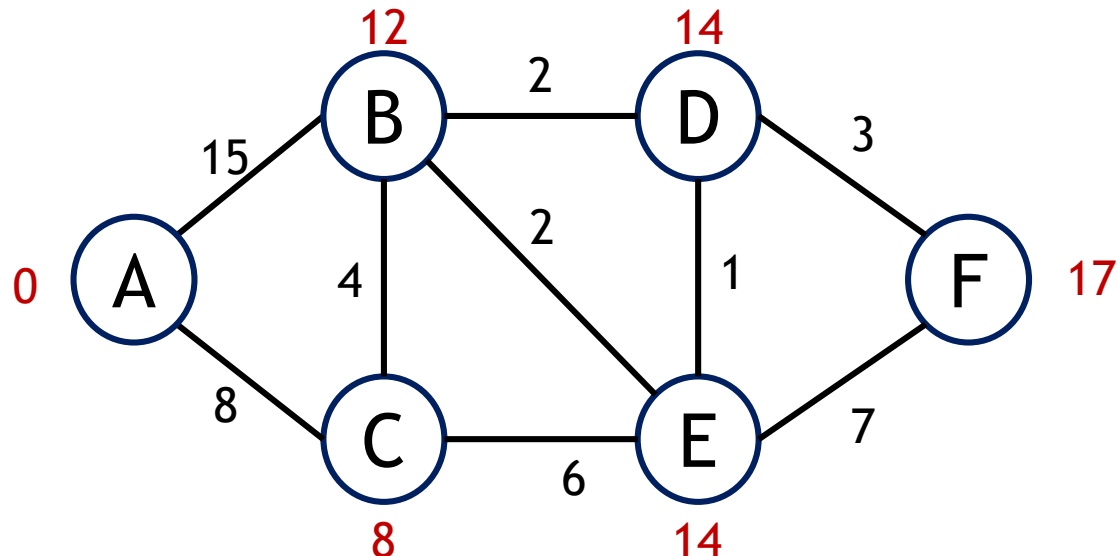
return DIST, PREV
```



# Dijkstra's Shortest Path: Example

	A*	B*	C*	D*	E*	F*
DIST	0	12	8	14	14	17
PREV	-	C	A	B	C	D

--	--	--	--	--	--	--



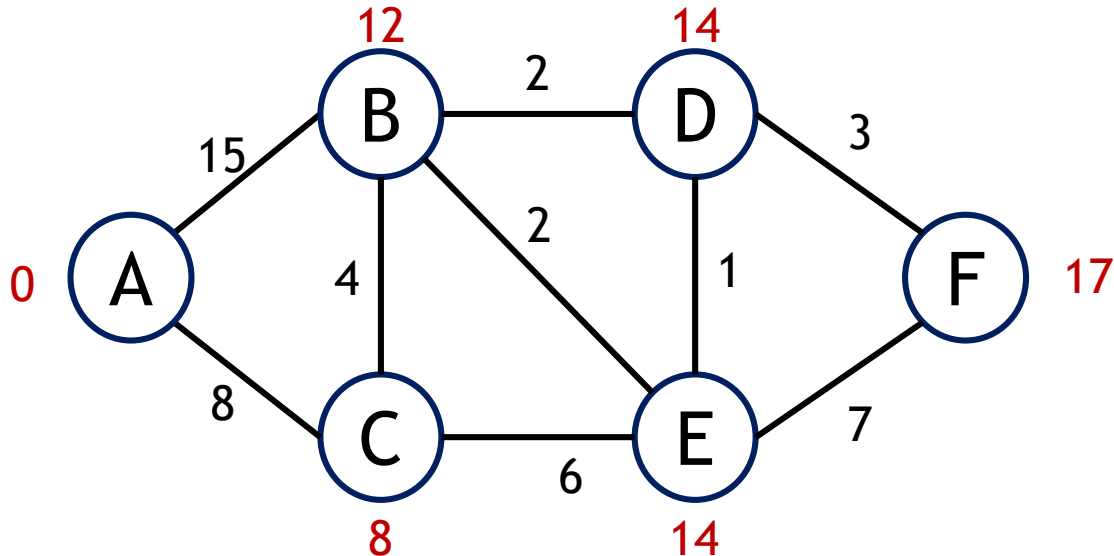
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    u = vertex in Q w. min(DIST)
    remove u from Q
    for each v in G[u]
        T = DIST[u] + DIST[u,v]
        if T < DIST[v]
            DIST[v] = T
            PREV[v] = u

return DIST, PREV
```



# Dijkstra's Shortest Path: Example

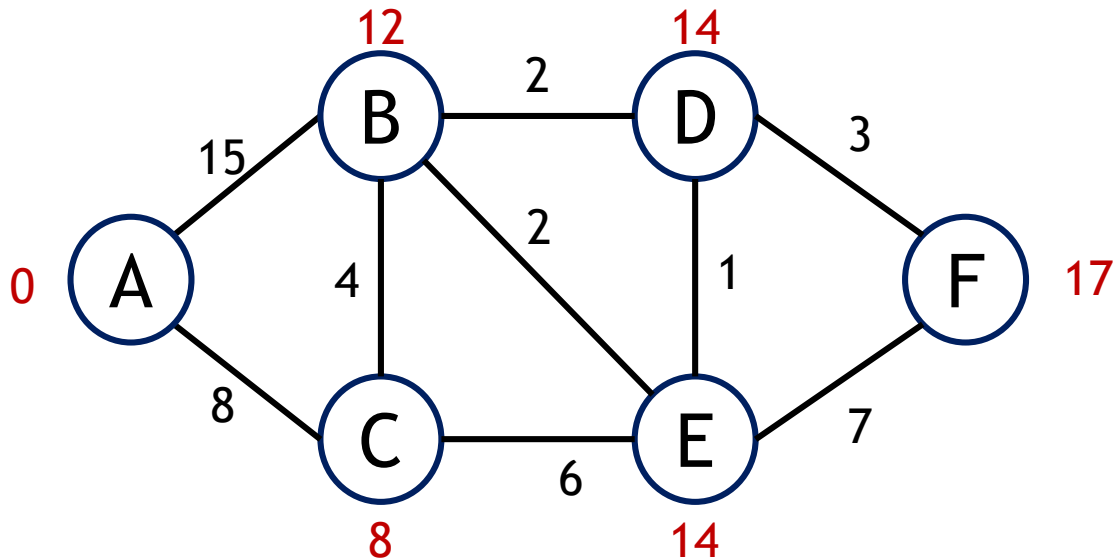
	A*	B*	C*	D*	E*	F*
DIST	0	12	8	14	14	17
PREV	-	C	A	B	C	D



To	Path	d
A	-	0
B	A -> C -> B	12
C	A -> C	8
D	A -> C -> B -> D	14
E	A -> C -> E	14
F	A -> C -> B -> D -> F	17

# Dijkstra's Shortest Path: Example

	A*	B*	C*	D*	E*	F*
DIST	0	12	8	14	14	17
PREV	-	C	A	B	C	D



Trace Path from A -> F

Start at F

$\text{PREV}(F) = D$

$\text{PREV}(D) = B$


$\text{PREV}(B) = C$

$\text{PREV}(C) = A$

**A, C, B, D, F**

# Dijkstra's Shortest Path: Implementation

```
while !Q.isEmpty()  
    u = vertex in Q w. min(DIST)  
    remove u from Q  
    for each v in G[u]  
        T = DIST[u] + DIST[u,v]  
        if T < DIST[v]  
            DIST[v] = T  
            PREV[v] = u
```



```
findmin(Q, DIST) {  
    ARG = 0; MIN = INF;  
    for(i=0; i<Q.size(); i++) {  
        if(DIST(Q[i]) < MIN) {  
            MIN = DIST(Q[i]);  
            ARG = i;  
        }  
    }  
    return Q[i];  
}
```

$O(|Q|) \approx O(|V|)$

```
return DIST, PREV
```

# Dijkstra's Shortest Path: Complexity

iter # : findmin(), neighbors

```

1: |V| reps, |V|-1 nbrs
2: |V| - 1 reps, |V|-1 nbrs
3: |V| - 2 reps, |V|-1 nbrs
.
.
i: |V| - i - 1 reps, |V|-1 nbrs
.
.
|V|: 1 reps, |V|-1 nbrs
    
```

$$O\left(\sum_{i=1}^{|V|} [|V| - i - 1] + [|V| - 1]\right)$$

$$O\left(\sum_{i=1}^{|V|} 2|V| - i - 2\right)$$

$$= O\left(2 \sum_{i=1}^{|V|} |V| - 2 \sum_{i=1}^{|V|} 1 - \sum_{i=1}^{|V|} i\right)$$

$$= O\left(2|V|^2 - 2|V| - \frac{|V|(|V|+1)}{2}\right)$$

$$= O(|V|^2)$$



# Dijkstra's Shortest Path: Implementation

```
while !Q.isEmpty()  
    u = vertex in Q w. min(DIST)  
    remove u from Q  
    for each v in G[u]  
        T = DIST[u] + DIST[u,v]  
        if T < DIST[v]  
            DIST[v] = T  
            PREV[v] = u
```

Utilize Priority Queue,  
which is implemented  
using a Binary Heap  
(Min-Heap)

$O(\log(|V|))$

```
return DIST, PREV
```

# References

<https://medium.com/basecs/finding-the-shortest-path-with-a-little-help-from-dijkstra-613149fbdc8e>

<https://www.hackerearth.com/practice/notes/dijkstras-algorithm/>

# Exercise: Identify if an edge is a bridge

## What is a bridge?

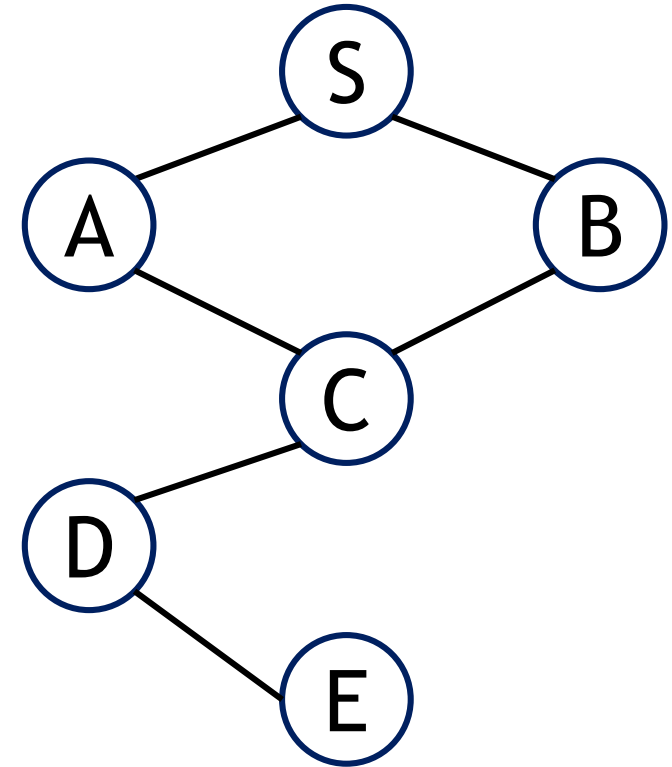
If removing an edge increases the number of components of the graph, then that edge is a bridge.

Implement following subroutines:

**removeEdge**

**DFTraversal**

**isBridge**



# Exercise: Identify if an edge is a bridge

1. Pick Edge  $e$  you wish to check
2. Perform DFS/BFS, count components  $ci$
3.  $tmp\_edge = e$ ;
4. Remove edge  $e$
5. Perform BFS/DFS, count components  $cf$
6. Restore Edge  $tmp\_edge$
7. if  $cf > ci$ : return True
8. return False

