# Data Structures

CSCI 2270-202: REC 12

**Sanskar Katiyar**

# Logistics

**Office Hours** (Zoom ID on Course Calendar)

Wednesday: 3 pm – 5 pm

Thursday: 5 pm – 6 pm

Friday: 3 pm – 5 pm

**Recitation Materials** *(Notes, Slides, Code, etc.)*

**sanskarkatiyar.github.io/CSCI2270**

# Recitation Outline

1. Hashing: Introduction
2. Hash Table, Hash Function
3. Collision Resolution
4. Exercise

# Hashing: Introduction

# **Hashing**: Introduction

A technique for *fast* insertion (storage) and search (retrieval)

Time Complexity for Search, Insertion:

- Average-case: **O(1)**, *under certain (reasonable) assumptions*
- Worst-case: **O(N)**

Allows maintaining data as (*key*, *value)* pairs over arbitrary keys

# Hashing: Introduction

## Key-Value pair

Use a key to identify/organize the data item

jodo0000

Identi(**key**)

*Identikey*: jodo0000
*Name*: John Doe
*Degree*: BS
*Major*: Computer Science
*Minor*: Psychology
. .
.

Student Record (**value**)

# **Hashing**: Example (1)

**Given a string S, find the first character that repeats (l-to-r).**

`In "abcdabcdabcd", 'a'` repeats first

**Approach #1**: Use an array

```
for(i = 0; i < S.size() - 1; i++)
    for(j= i + 1; j < S.size(); j++)
        if(S[i] == S[j]) return S[i];
```

Time: $\mathbf{O(N^2)}$

# Hashing: Example (1)

**Approach #2**: Use Hashing

```
Maintain an array A[256]
// 1 place for each ASCII character code

Initialize A[i] = 0, for all i

for(i = 0; i < S.size(); i++)
    A[(int) S[i]]++;
    if(A[(int) S[i]] > 1) return S[i];
```

Time: **O(N)**

# **Hashing**: Example (1)

**Approach #2**: Use Hashing

***Key***: character

***Value***: count of character

By maintaining a table and a map, we were able to reduce the retrieval to O(1) and thus the overall time complexity to O(N)

# **Hashing**: Example (2)

**LeetCode (TwoSum)**

Given an array of integers, return **indices** of the two numbers such that they add up to a specific target.

You may assume that each input would have ***exactly*** one solution, and you may not use the *same* element twice.

```
Given nums = [2, 7, 11, 15], target = 9,
Since nums[0] + nums[1] = 2 + 7 = 9,
return [0, 1].
```

# **Hashing**: Introduction

## Common Operations

`Search(key)`: Search for the *(key, value)* record identified by *key*

`Insert(key, value)`: Insert *(key, value)* record identified by *key*

`Remove(key)`: Delete *(key, value)* record identified by *key*

## Key Components

Hash Table (or Map)

Hash Function

Collision Resolution

# Hash Table, Hash Function

# **Hashing**: Hash Table

**Hash Table** (or Hash Map)

- Stores the (key, value) record
- Generalization of an array
- Uses a hash function to map the key to the associated value

Hash function ***h**: **X** → **Y**, where Y in [0, table_size) and Z*

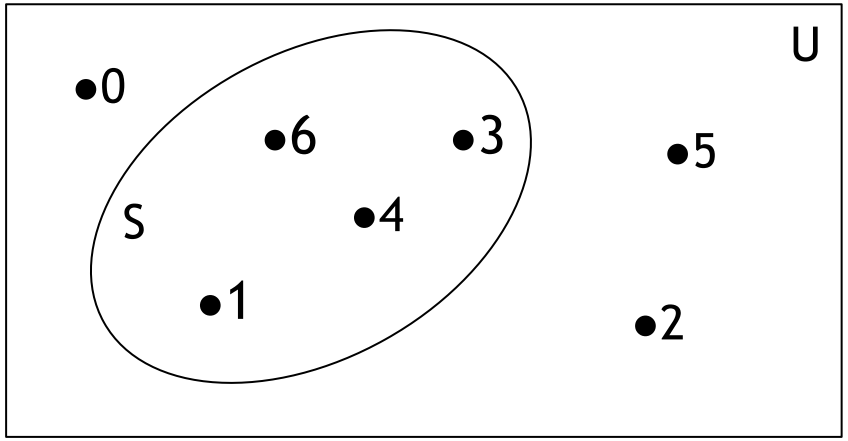
**Utilize a hash table when**

- *number of keys* << *number of possible keys*

# **Hashing**: Direct Addressing

$$h(x) = x$$

| | | | | | | |
|---|---|---|---|---|---|---|
| | 1 | | 3 | 4 | | 6 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |



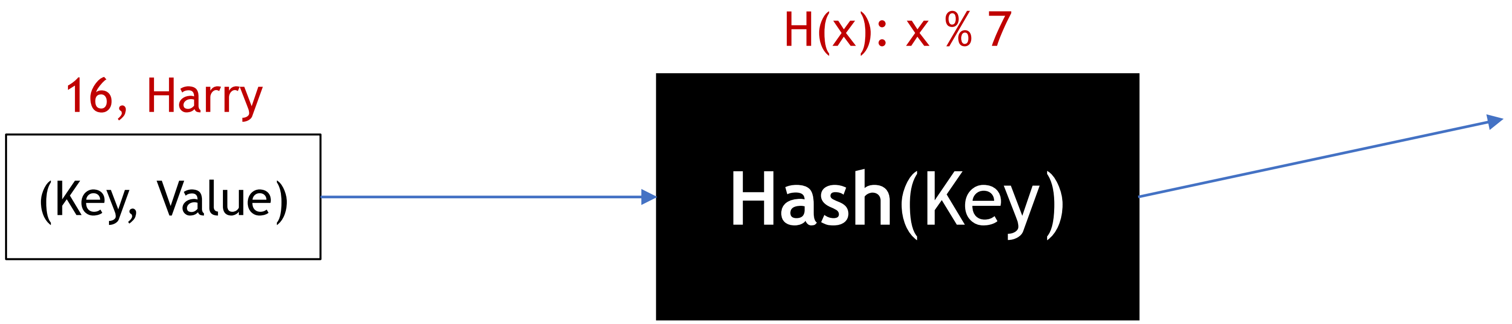

Direct Addressing is useful when **set of all *possible* keys is (finite) and *small***

Each **key identifies a unique record**

**Direct Address Tables**
*(Arrays: O(1) Access)*

# Hashing: Hash Function

A hash function maps the key to an appropriate integer index in the hash table.




| | | |
|---|---|---|
| 0 | | |
| 1 | | |
| 2 | 16 | Harry |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |

# **Hashing**: Hash Function

**Properties** of a good Hash function

- Quick Computation, O(1)
- Distribute key values uniformly across the table
- Have a high load factor for a given set of keys
- Minimize collision
- Allow efficient collision resolution

# **Hashing**: Hash Function

**Example:** *String*

**h(x) =** *(Sum of ASCII values of all characters)* **%** *113*

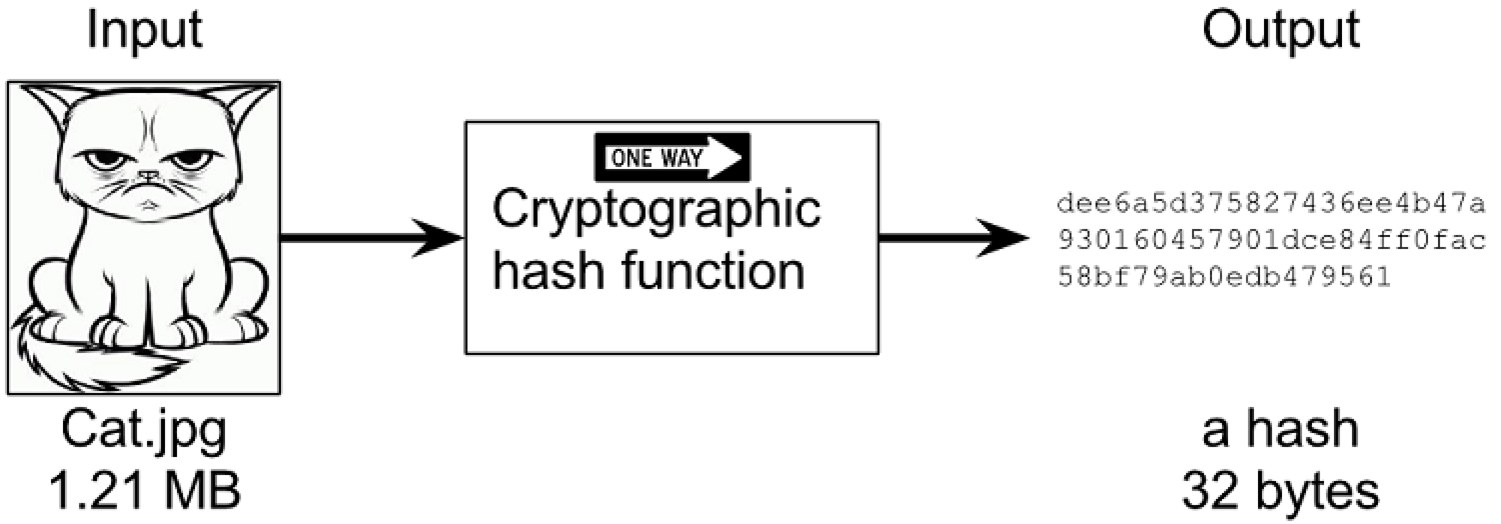
```
h(“Hashing”)  =>   706 % 113   => 28
h(“Table”)    =>   488 % 113   => 36
h(“2270”)     =>   203 % 113   => 90
```

# **Hashing**: Hash Function

Applications in **Cryptography**

**Message Digest for Authenticity**

**Password Storage and Verification**



# **Hashing**: Load Factor

$$Load\ Factor = \frac{Number\ of\ Elements\ in\ the\ Hash\ Table}{Size\ of\ Hash\ Table}$$

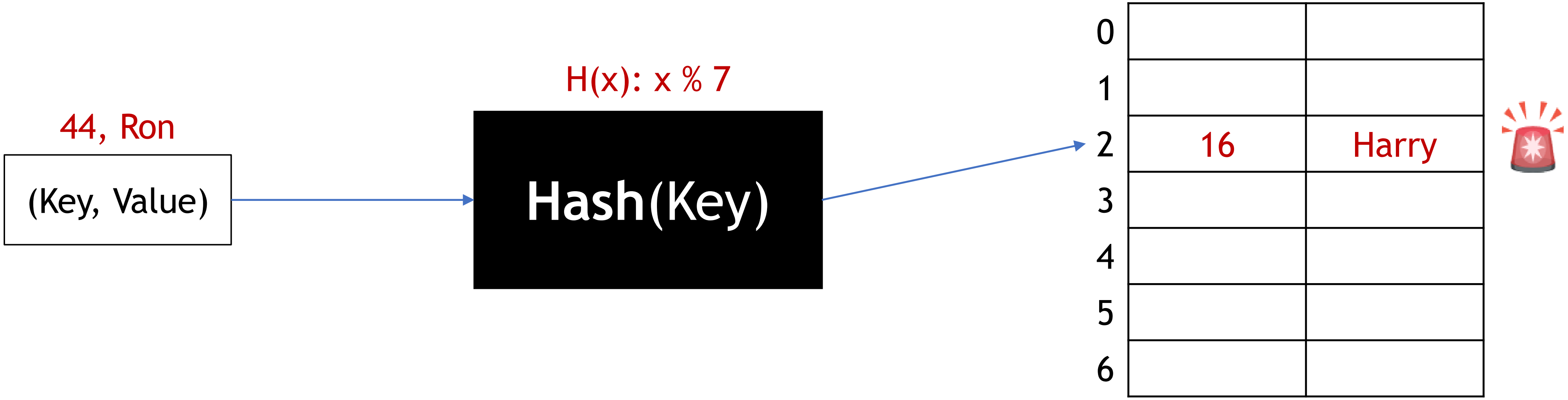
**Determines the efficiency of hash functions**

**Serves as a decision parameter**, whether to rehash or expand existing hash table entries

# Collision

# Hashing: Collision

**Collision**: When a hash function generates the same hash for two different key inputs



# **Hashing**: Collision Resolution

**Collision Resolution**: Finding an alternate location for some entry *x*, where *h(x)* causes a collision in the hash table

But then hashing is **no longer O(1)** search/insert/delete
Yes! **But still much better than O(N)**, on average

We will discuss 2 of N techniques here:

- **Linear Probing** (Open Addressing)
- **Separate Chaining** (Direct Chaining)

# Collision Resolution: Linear Probing

## Open Addressing

All elements are stored in the hash table

`table.size()` >= Number of keys

## Linear Probing

If index `[h(x) % S]` is full, then attempt insertion at `[(h(x) + 1) % S]`

If index `[(h(x) + 1) % S]` is full, then try `[(h(x) + 2) % S]...`

```
rehash(x) = [h(x) + (const) step_size] % table.size()
```

`h(x)` `S`

# Collision Resolution: Linear Probing

| Key | Key % 7 | Hash |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |

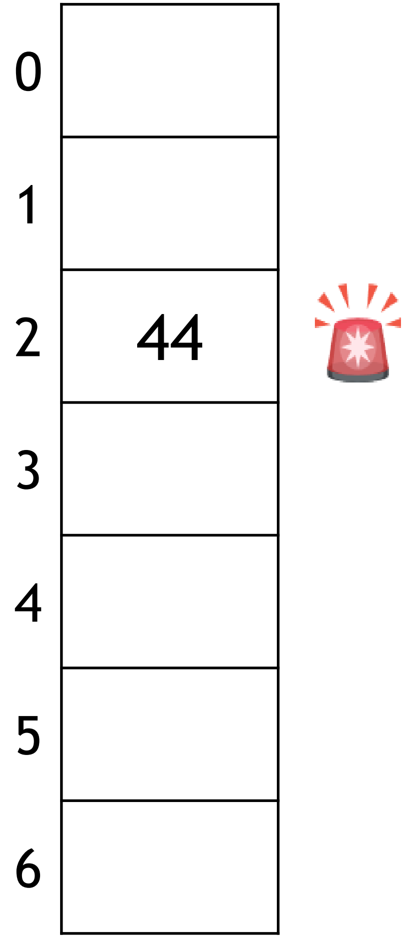
# Collision Resolution: Linear Probing

| Key | Key % 7 | Hash |
| --- | --- | --- |
| 44 | 44 % 7 | 2 |
| | | |
| | | |
| | | |
| | | |

| | |
| --- | --- |
| 0 | |
| 1 | |
| 2 | 44 |
| 3 | |
| 4 | |
| 5 | |
| 6 | |

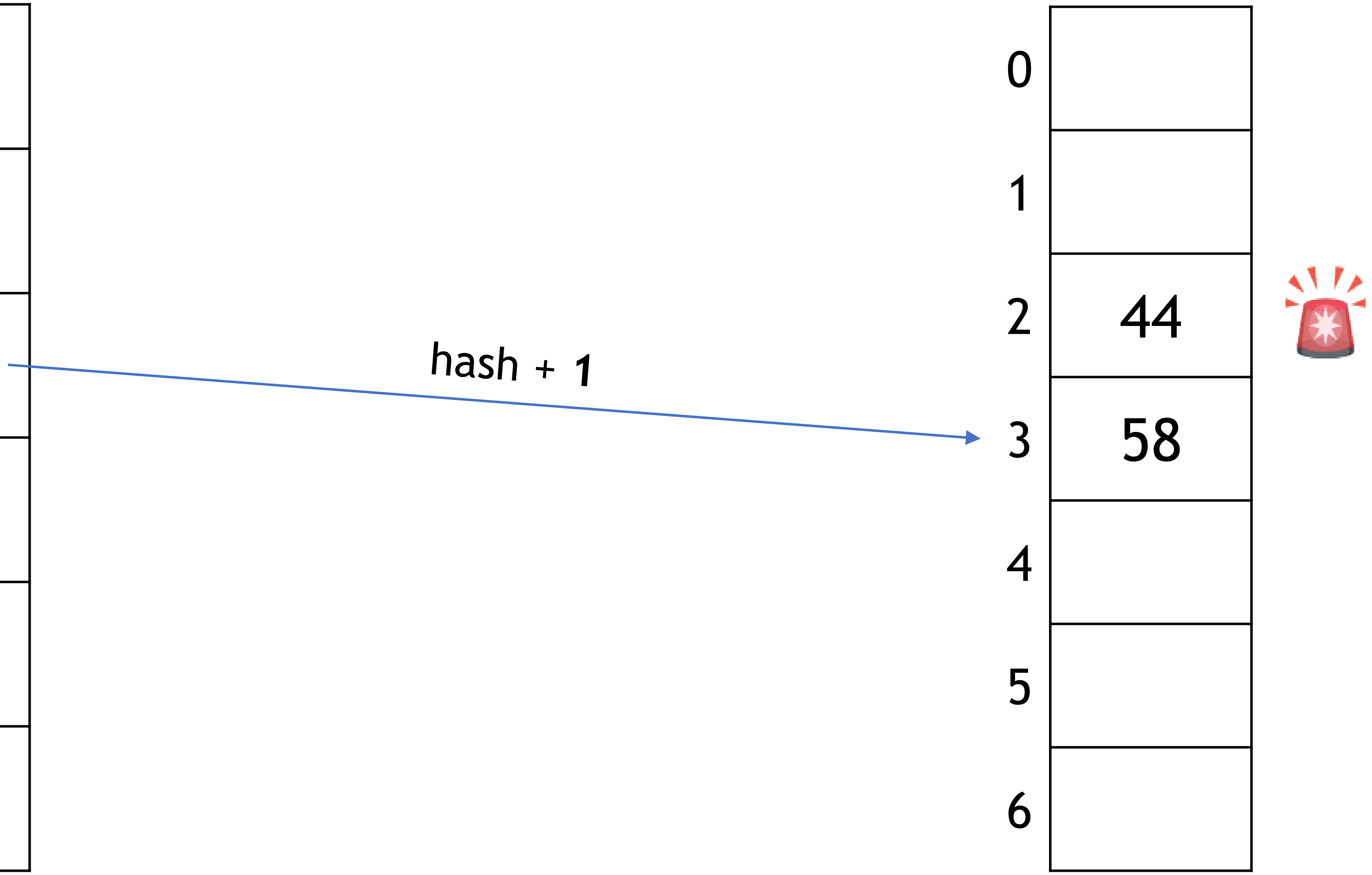
# Collision Resolution: Linear Probing

| Key | Key % 7 | Hash |
|---|---|---|
| 44 | 44 % 7 | 2 |
| 58 | 58 % 7 | 2 |
| | | |
| | | |
| | | |



# Collision Resolution: Linear Probing

| Key | Key % 7 | Hash |
|---|---|---|
| 44 | 44 % 7 | 2 |
| 58 | 58 % 7 | 2 |
| | | |
| | | |
| | | |



# Collision Resolution: Linear Probing

| Key | Key % 7 | Hash |
|---|---|---|
| 44 | 44 % 7 | 2 |
| 58 | 58 % 7 | 2 |
| 16 | 16 % 7 | 2 |
| | | |
| | | |

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | 44 |
| 3 | 58 |
| 4 | |
| 5 | |
| 6 | |

# Collision Resolution: Linear Probing

| Key | Key % 7 | Hash |
|---|---|---|
| 44 | 44 % 7 | 2 |
| 58 | 58 % 7 | 2 |
| 16 | 16 % 7 | 2 |
| | | |
| | | |

hash + 1

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | 44 |
| 3 | 58 |
| 4 | |
| 5 | |
| 6 | |

# Collision Resolution: Linear Probing

| Key | Key % 7 | Hash |
|---|---|---|
| 44 | 44 % 7 | 2 |
| 58 | 58 % 7 | 2 |
| 16 | 16 % 7 | 2 |
| | | |
| | | |

hash + 2

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | 44 |
| 3 | 58 |
| 4 | 16 |
| 5 | |
| 6 | |

# Collision Resolution: Linear Probing

| Key | Key % 7 | Hash |
|---|---|---|
| 44 | 44 % 7 | 2 |
| 58 | 58 % 7 | 2 |
| 16 | 16 % 7 | 2 |
| 7 | 7 % 7 | 0 |
| | | |

| | |
|---|---|
| 0 | 7 |
| 1 | |
| 2 | 44 |
| 3 | 58 |
| 4 | 16 |
| 5 | |
| 6 | |

`step_size = 1`

# Collision Resolution: Linear Probing

## Insertion

```
insert(x, T=table) {
for(i = 0; i < T.size(); i++) {
  if T[(h(x) + i) % T.size()].isEmpty()
  {
    T[(h(x) + i) % T.size()] = x;
    return SUCCESS_INSERTION;
  }
}

return ERROR_TABLE_FULL;
}
```

## Search (Item Exists)

```
search(x, T=table) {
for(i = 0; i < T.size(); i++) {
  if T[(h(x) + i) % T.size()] == x
  {
    return TRUE; // or x.data
  }
}

return FALSE;
}
```

# **Collision Resolution**: Linear Probing

## Clustering

Items tend to cluster together in the table (***retrieval time***)

If clusters merge, certain portions are densely occupied while others are relatively empty ***(wasted space)***

## Role of step_size

Cannot resolve clustering by simply increasing step_size

Choose step_size such that **step_size** and **table.size() are co-primes**

But **table.size() can't change**

# Collision Resolution: Quadratic Probing

## Linear Probing

`rehash(x) = [h(x) + (const) step_size] % table.size()`

## Quadratic Probing

If index `[h(x) % S]` is full, then try `[h(x) + 1²] % S`

If index `[(h(x) + 1²) % S]` is full, then try `[(h(x) + 2²) % S]...`

`rehash(x) = [h(x) + (const) step_size²] % table.size()`

`h(x)`

# Collision Resolution: Double Hashing

Use 2 separate hash functions: $\mathbf{h_1}$, $\mathbf{h_2}$

If index $[h_1(x)\ \%\ S]$ is full, then try $[h_1(x) + 1*h_2(x)]\ \%\ S$

If index $[(h_1(x) + k)\ \%\ S]$ is full, then try $[h_1(x) + (k+1) * h_2(x)]\ \%\ S$

...

**rehash(x) = [$h_1$(x) + step_size * $h_2$(x)] % table.size()**

# **Collision Resolution:** Direct Chaining

**Separate Chaining**

Each index of hash table stores a Linked List

The corresponding Linked List stores the items

**Direct Chaining**

Insert item *x* in the Linked List at index *h(x)* of the table

Insert item at head of Linked List ***(Why?)***

Search is O(N), *N*: length of the Linked List

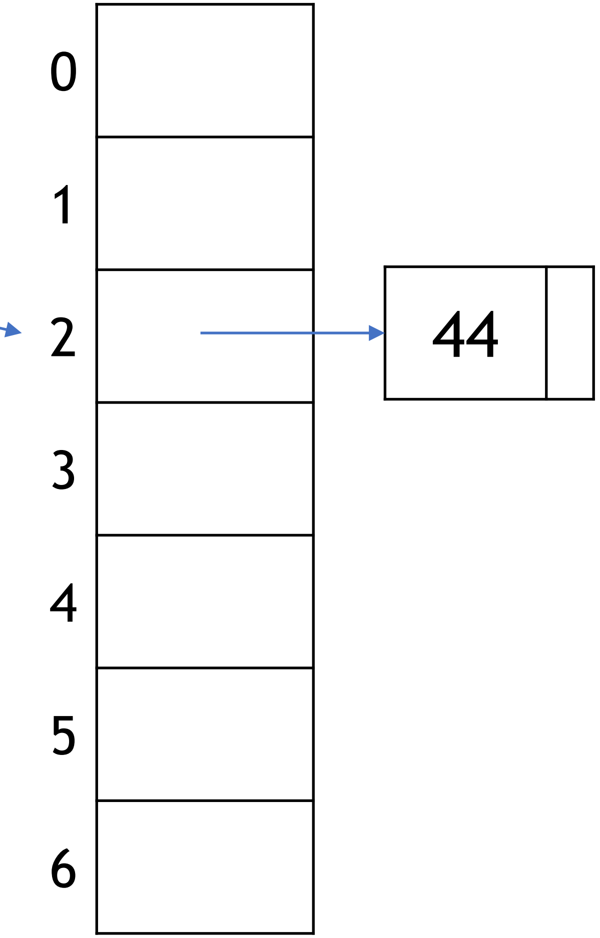
# Collision Resolution: Direct Chaining

| Key | Key % 7 | Hash |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |

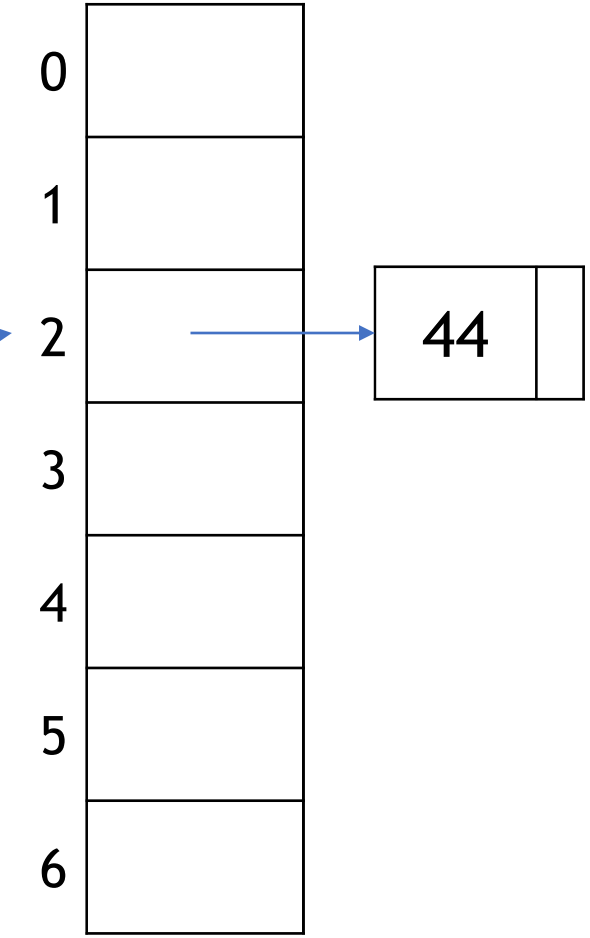
# Collision Resolution: Direct Chaining

| Key | Key % 7 | Hash |
|---|---|---|
| 44 | 44 % 7 | 2 |
| | | |
| | | |
| | | |
| | | |



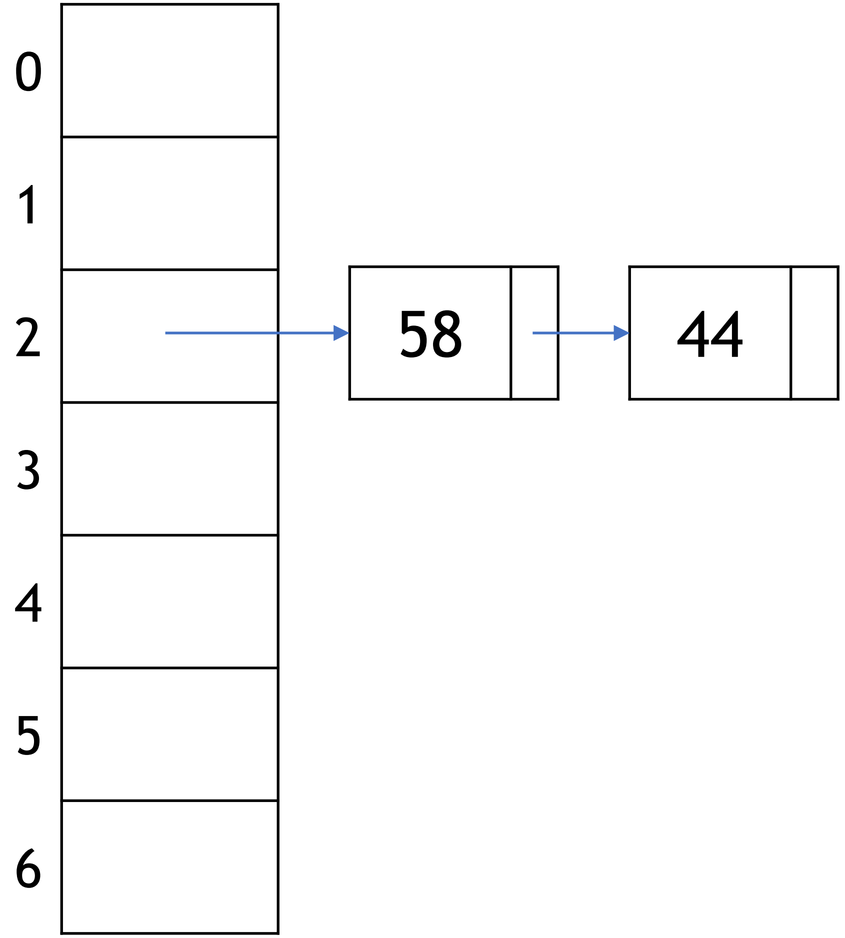
# Collision Resolution: Direct Chaining

| Key | Key % 7 | Hash |
|---|---|---|
| 44 | 44 % 7 | 2 |
| 58 | 58 % 7 | 2 |
| | | |
| | | |
| | | |



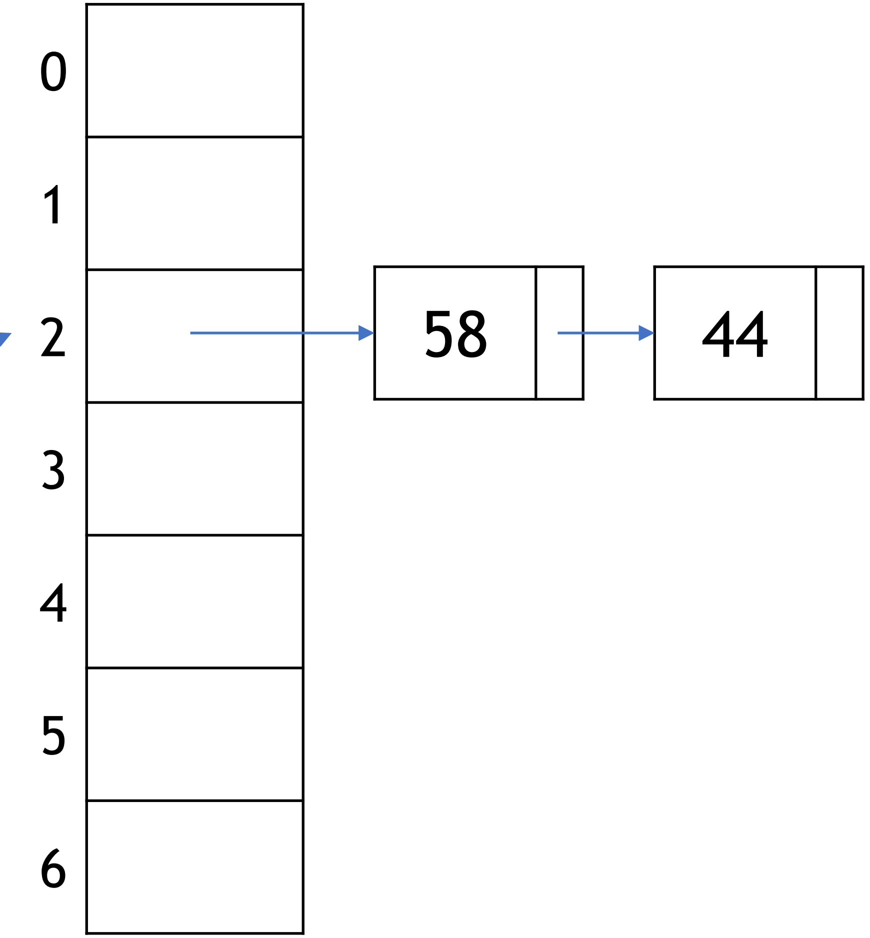
# Collision Resolution: Direct Chaining

| Key | Key % 7 | Hash |
|---|---|---|
| 44 | 44 % 7 | 2 |
| 58 | 58 % 7 | 2 |
| | | |
| | | |
| | | |



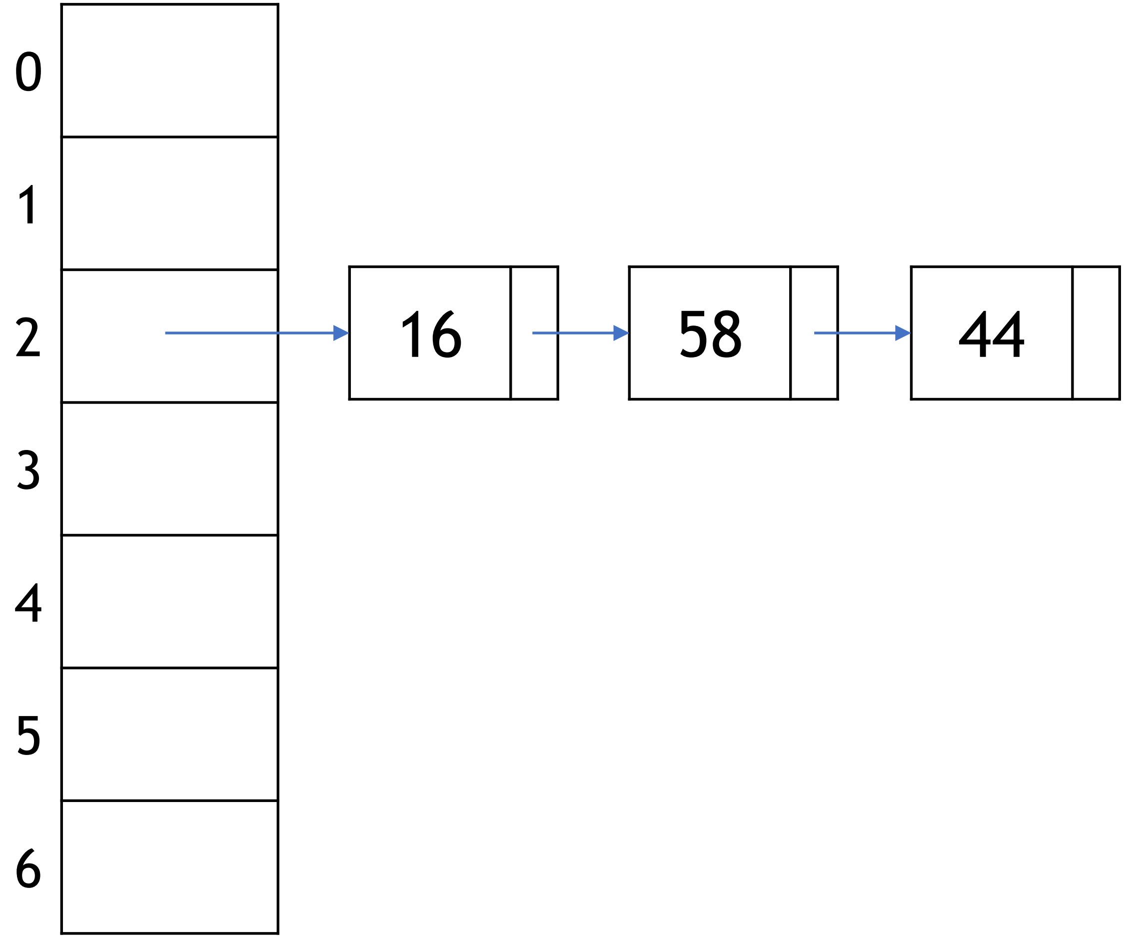
# Collision Resolution: Direct Chaining

| Key | Key % 7 | Hash |
|---|---|---|
| 44 | 44 % 7 | 2 |
| 58 | 58 % 7 | 2 |
| 16 | 16 % 7 | 2 |
| | | |
| | | |



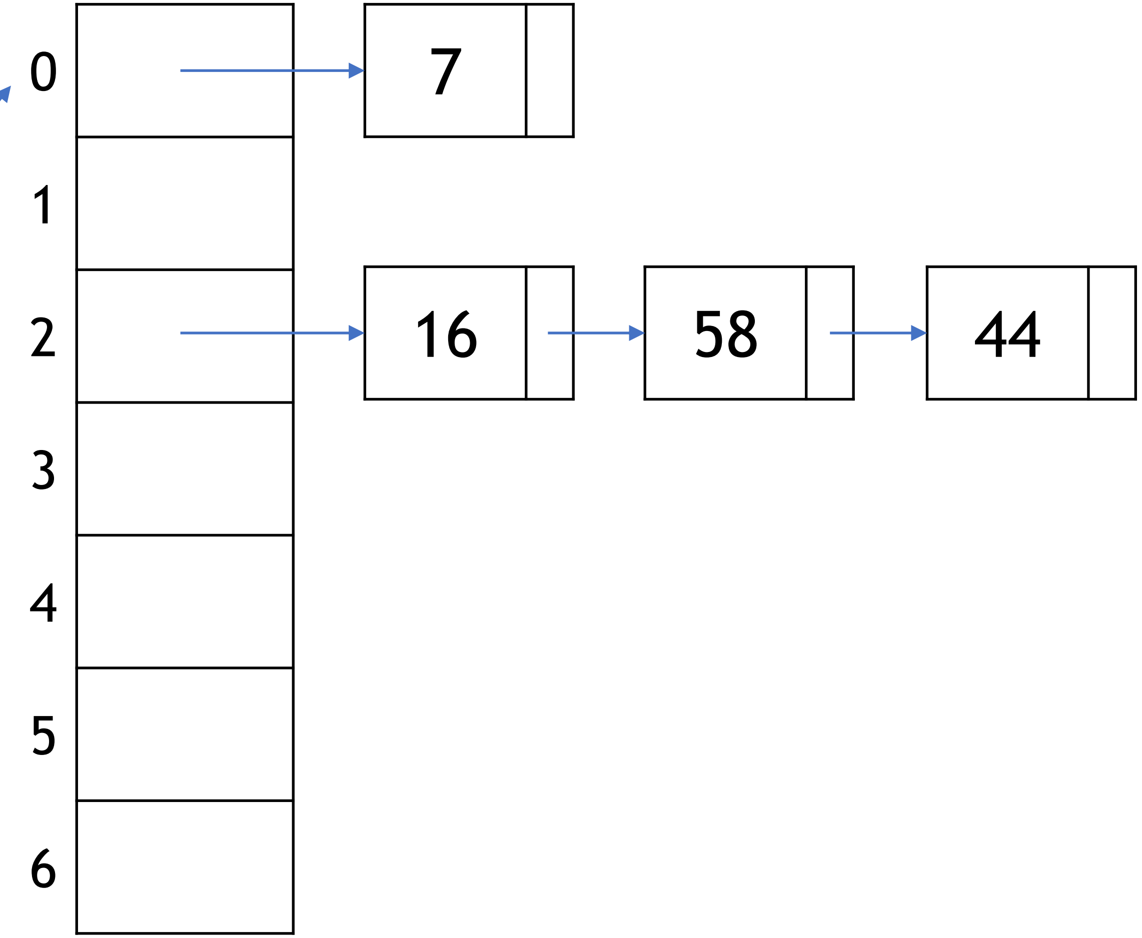
# Collision Resolution: Direct Chaining

| Key | Key % 7 | Hash |
|---|---|---|
| 44 | 44 % 7 | 2 |
| 58 | 58 % 7 | 2 |
| 16 | 16 % 7 | 2 |
| | | |
| | | |



# Collision Resolution: Direct Chaining

| Key | Key % 7 | Hash |
|---|---|---|
| 44 | 44 % 7 | 2 |
| 58 | 58 % 7 | 2 |
| 16 | 16 % 7 | 2 |
| 7 | 7 % 7 | 0 |
| | | |



# Collision Resolution: Direct Chaining

## Insertion

```
insert(x, T=table) {
  temp = createNode(x);
  temp->next = T[h(x)]->head;
  T[h(x)]->head = temp;

return true;
}
```

## Search

```
search(x, T=table) {
  temp = T[h(x)]->head;
  while(temp != NULL) {
    if(temp->data == x) {
      return temp;
    }
  }
return temp;
}
```

# **Collision Resolution:** Direct Chaining

**Chaining is Expensive**

- Storing Linked List links requires extra memory
- Cache locality is not available

**If Linked List becomes very large**

- Operation's time complexity is worsened

**Certain portions of the table may never be used**

# Open Addressing vs. Separate Chaining

| | Open Addressing | Separate Chaining |
|---|---|---|
| Hash Function | Requires extra care to avoid clustering | Less sensitive to hash function or load factor |
| Usability Scenario | If frequency of operations and number of keys are known | If number of keys, operation frequency are unknown |
| Cache Performance | Great. Everything is maintained in the table (contiguous) | Poor. Linked List allocations are spread all over memory |
| Memory Usage | Slots can be used even if hash function doesn't map to it | Certain portions of the table may go unused; Also requires extra space for LL links |

# References

https://www.hackerearth.com/practice/data-structures/hash-tables/basics-of-hash-tables/tutorial/

https://www.geeksforgeeks.org/hashing-set-3-open-addressing/