第六次上机作业 实验报告

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一、实验目的

通过快速傅里叶变换与快速傅里叶逆变换实现对给定函数的 Fourier 分析以及重建。

二、实验要求

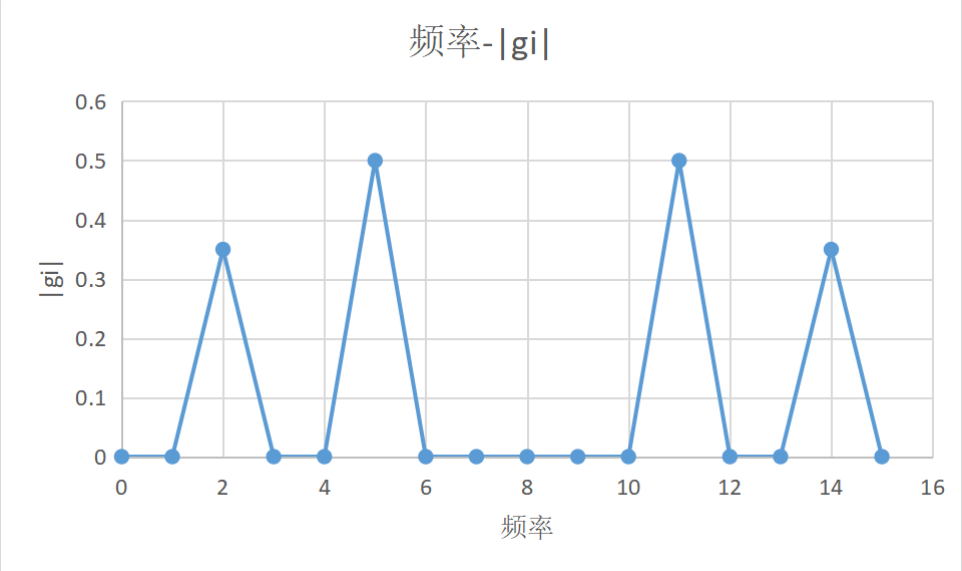
通过快速傅里叶变换与快速傅里叶逆变换实现对给定函数的 Fourier 分析，函数 f 以及划分数 n 如下：

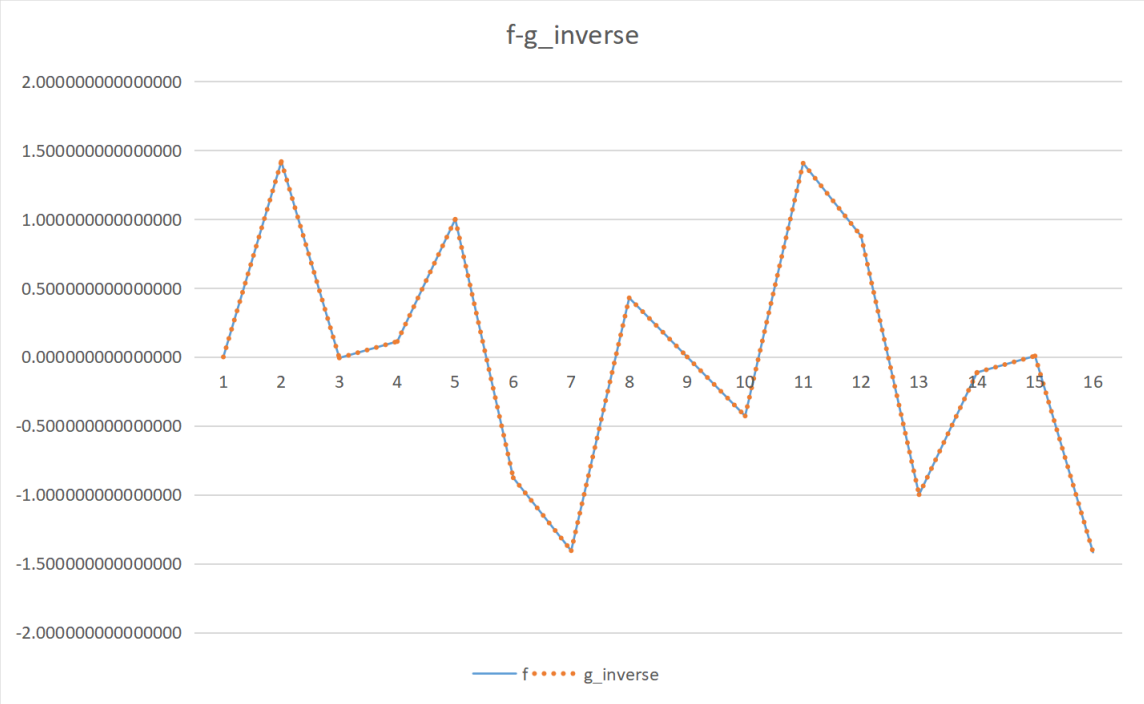
三、实验结果与分析

f_1:

$n = 2^4$:

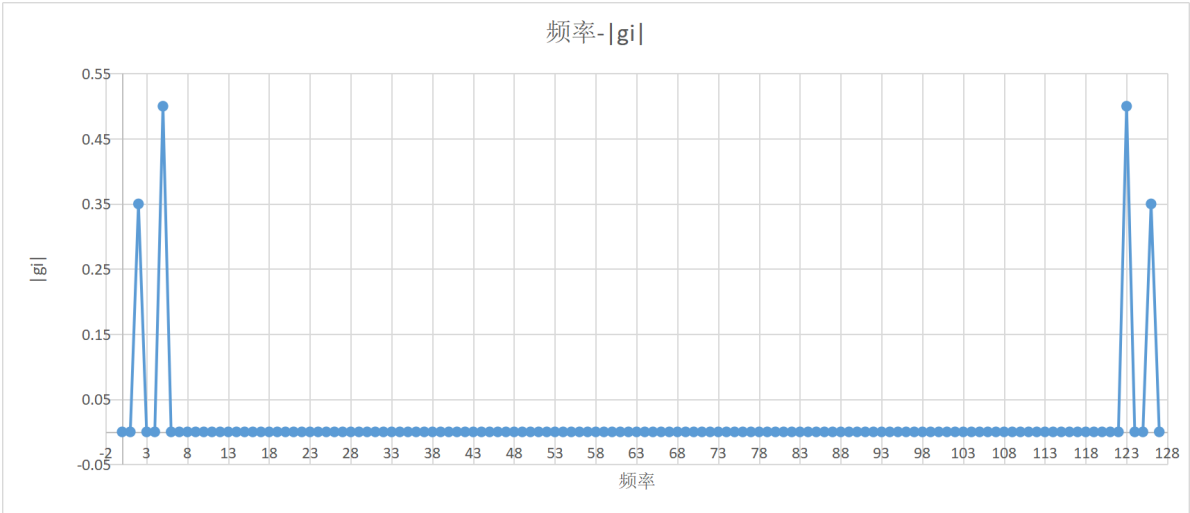
i	real	imag
0	0.000000000035917	0.000000000000000
1	0.000000000035917	0.000000000018121
2	-0.000000000011224	-0.349999999989650
3	0.000000000041529	0.000000000004723
4	0.000000000035917	0.000000000062214
5	-0.000000000154893	-0.499999999999687
6	0.000000000043774	-0.000000000061160
7	0.000000000041529	-0.000000000006493
8	0.000000000035917	0.000000000000000
9	0.000000000035917	0.000000000031266
10	0.000000000035917	0.000000000069017
11	-0.000000000227850	0.500000000002012
12	0.000000000035917	-0.000000000062214
13	0.000000000024693	-0.000000000018272
14	-0.000000000050509	0.349999999981793
15	0.000000000041529	-0.000000000031669

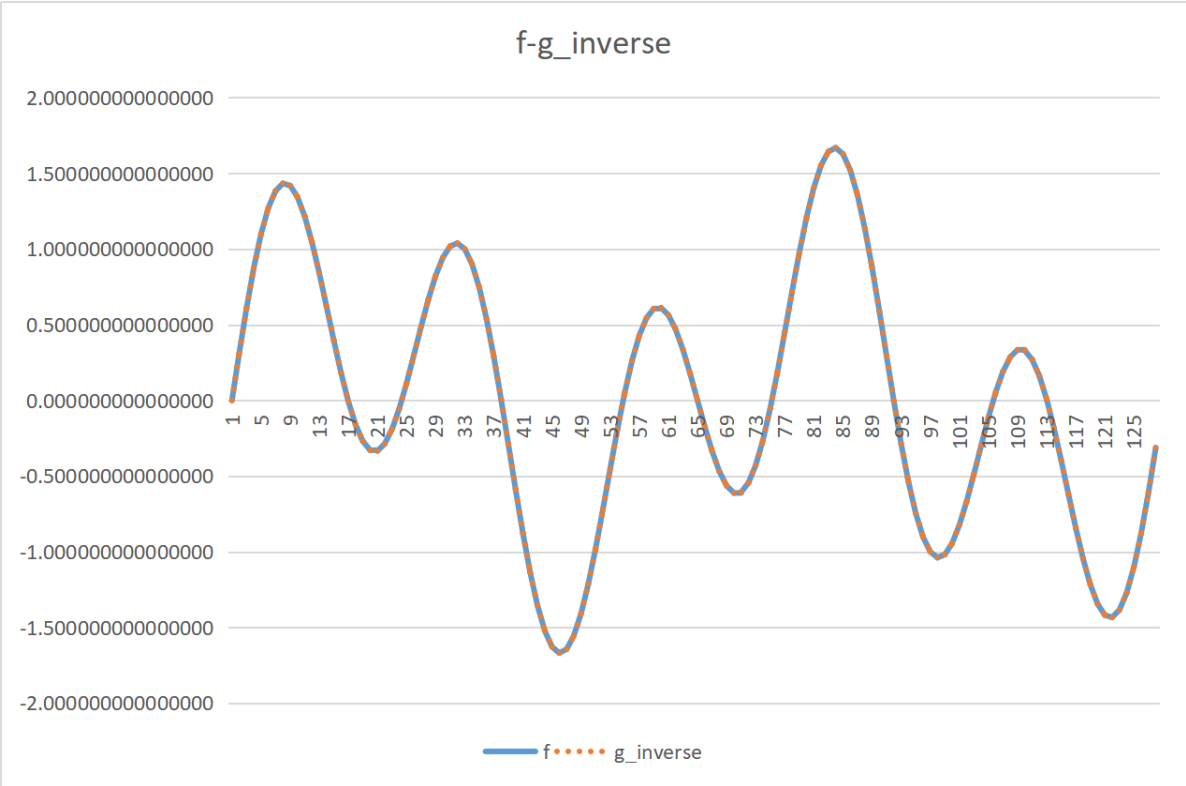




$n = 2^7$:

i	real	imag	i			i		
0	0.000000000004490	0.000000000000000	42	0.000000000004490	-0.000000000002671	84	0.000000000004490	0.000000000002438
1	0.000000000004490	0.000000000013404	43	0.000000000007997	-0.000000000003923	85	0.000000000003087	0.000000000002119
2	-0.000000000042652	-0.349999999996303	44	0.000000000004490	-0.000000000002438	86	0.000000000005472	0.000000000003459
3	0.000000000005191	-0.000000000009533	45	0.000000000003087	-0.000000000001240	87	0.000000000005191	0.000000000003590
4	0.000000000004490	0.000000000050325	46	0.000000000007436	-0.000000000003317	88	0.000000000004490	0.000000000003054
5	-0.000000000176500	-0.500000000017889	47	0.000000000005191	-0.000000000002126	89	0.000000000004490	0.000000000003351
6	0.000000000005472	-0.000000000087653	48	0.000000000004490	-0.000000000001887	90	0.000000000004490	0.000000000003469
7	0.000000000005191	-0.000000000043978	49	0.000000000004490	-0.000000000001657	91	0.000000000013609	0.000000000012834
8	0.000000000004490	-0.000000000034355	50	0.000000000004490	-0.000000000001578	92	0.000000000004490	0.000000000003762
9	0.000000000004490	-0.000000000024731	51	0.000000000005191	-0.000000000001547	93	0.000000000003087	0.000000000003173
10	0.000000000004490	-0.000000000022003	52	0.000000000004490	-0.000000000001380	94	0.000000000011364	0.000000000011125
11	0.000000000007997	-0.000000000026680	53	0.000000000003087	-0.000000000000585	95	0.000000000005191	0.000000000005399
12	0.000000000004490	-0.000000000017397	54	0.000000000005472	-0.000000000001284	96	0.000000000004490	0.000000000004603
13	0.000000000003087	-0.000000000007497	55	0.000000000005191	-0.000000000000995	97	0.000000000004490	0.000000000005026
14	0.000000000007436	-0.000000000020778	56	0.000000000004490	-0.000000000000905	98	0.000000000004490	0.000000000005200
15	0.000000000005191	-0.000000000013445	57	0.000000000004490	-0.000000000000706	99	0.000000000005191	0.000000000006616
16	0.000000000004490	-0.000000000011844	58	0.000000000004490	-0.000000000000625	100	0.000000000004490	0.000000000005643
17	0.000000000004490	-0.000000000010385	59	0.000000000024833	-0.000000000000473	101	0.000000000003086	0.000000000004778
18	0.000000000004490	-0.000000000009916	60	0.000000000004490	-0.000000000000448	102	0.000000000005472	0.000000000007877
19	0.000000000005191	-0.000000000010095	61	0.000000000003087	0.000000000000031	103	0.000000000005191	0.000000000008202
20	0.000000000004490	-0.000000000008894	62	0.000000000019221	-0.000000000000175	104	0.000000000004490	0.000000000006999
21	0.000000000003087	-0.000000000004589	63	0.000000000005191	0.000000000000044	105	0.000000000004490	0.000000000007723
22	0.000000000005472	-0.000000000009143	64	0.000000000004490	0.000000000000000	106	0.000000000004490	0.000000000008041
23	0.000000000005191	-0.000000000007950	65	0.000000000004490	0.000000000000198	107	0.000000000007997	0.000000000017192
24	0.000000000004490	-0.000000000006999	66	0.000000000004490	0.000000000000272	108	0.000000000004490	0.000000000008894
25	0.000000000004490	-0.000000000006352	67	0.000000000005191	0.000000000000562	109	0.000000000003087	0.000000000007856
26	0.000000000004490	-0.000000000006134	68	0.000000000004490	0.000000000000448	110	0.000000000007436	0.000000000017562
27	0.000000000013609	-0.000000000014832	69	0.000000000003087	0.000000000000649	111	0.000000000005191	0.000000000013840
28	0.000000000004490	-0.000000000005643	70	0.000000000005472	0.000000000000919	112	0.000000000004490	0.000000000011844
29	0.000000000003087	-0.000000000003052	71	0.000000000005191	0.000000000001091	113	0.000000000004490	0.000000000013621
30	0.000000000011364	-0.000000000011860	72	0.000000000004490	0.000000000000903	114	0.000000000004490	0.000000000014532
31	0.000000000005191	-0.000000000005231	73	0.000000000004490	0.000000000001112	115	0.000000000005191	0.000000000020121
32	0.000000000004490	-0.000000000004603	74	0.000000000004490	0.000000000001191	116	0.000000000004490	0.000000000017397
33	0.000000000004490	-0.000000000004210	75	0.000000000007997	0.000000000002805	117	0.000000000003087	0.000000000017861
34	0.000000000004490	-0.000000000004074	76	0.000000000004490	0.000000000001380	118	0.000000000005472	0.000000000028641
35	0.000000000005191	-0.000000000004274	77	0.000000000003087	0.000000000001321	119	0.000000000005191	0.000000000037016
36	0.000000000004490	-0.000000000003762	78	0.000000000007436	0.000000000002904	120	0.000000000004490	0.000000000034355
37	0.000000000003087	-0.000000000002031	79	0.000000000005191	0.000000000002231	121	0.000000000004490	0.000000000053632
38	0.000000000005472	-0.000000000003976	80	0.000000000004490	0.000000000001887	122	0.000000000004490	0.000000000087750
39	0.000000000005191	-0.000000000003466	81	0.000000000004490	0.000000000002124	123	-0.000000000311891	0.500000000015088
40	0.000000000004490	-0.000000000003054	82	0.000000000004490	0.000000000002215	124	0.000000000004490	-0.000000000050325
						125	0.000000000003087	-0.000000000018854
						126	-0.000000000122203	0.349999999986332
						127	0.000000000005191	-0.000000000032378

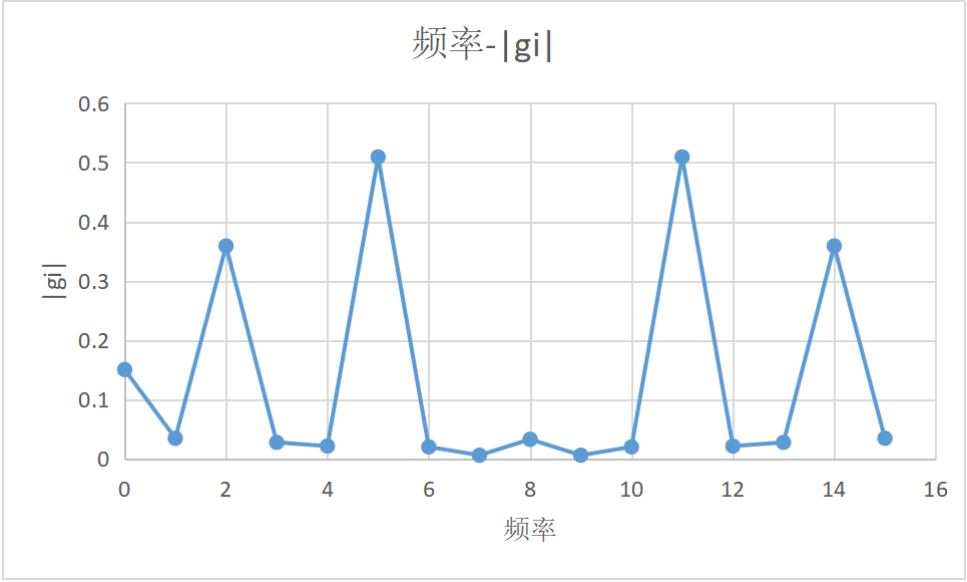


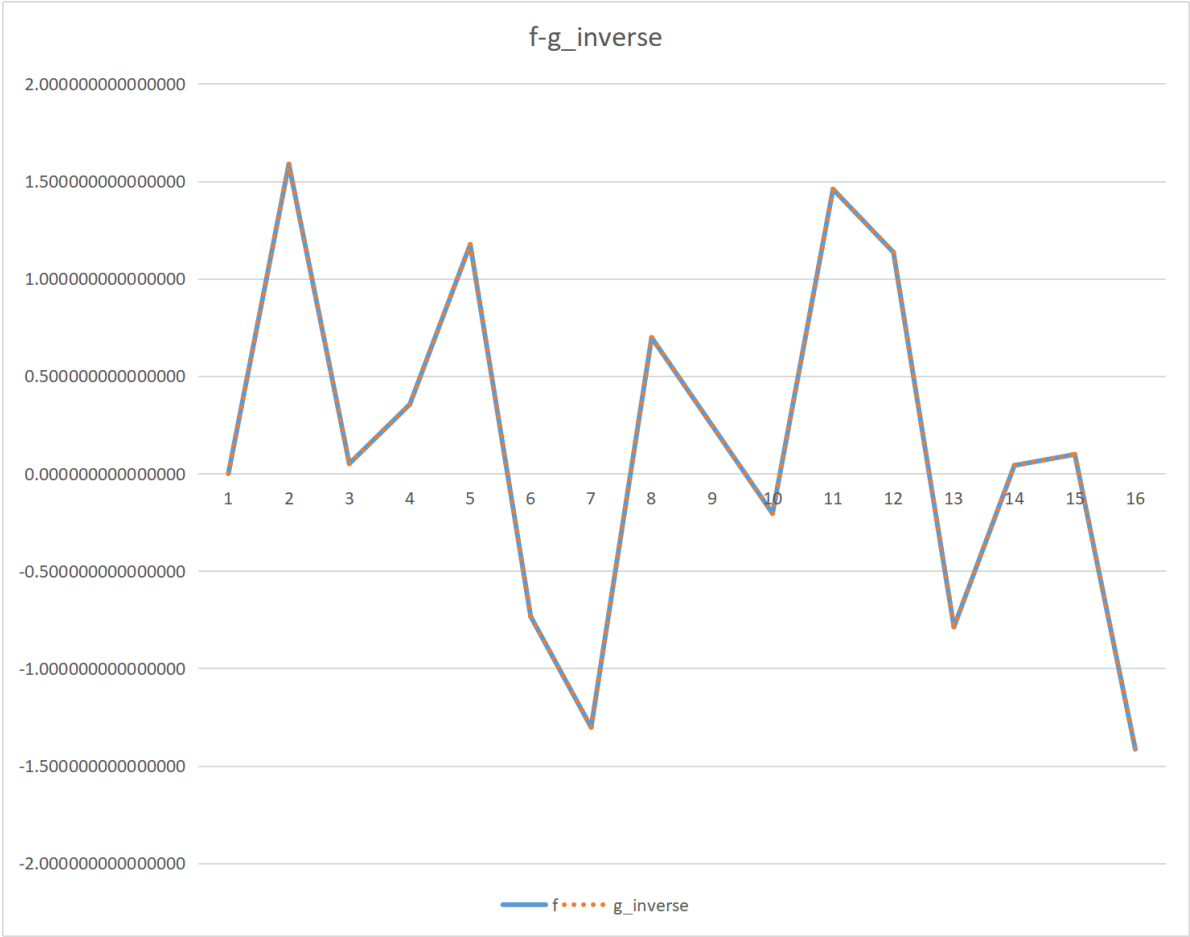


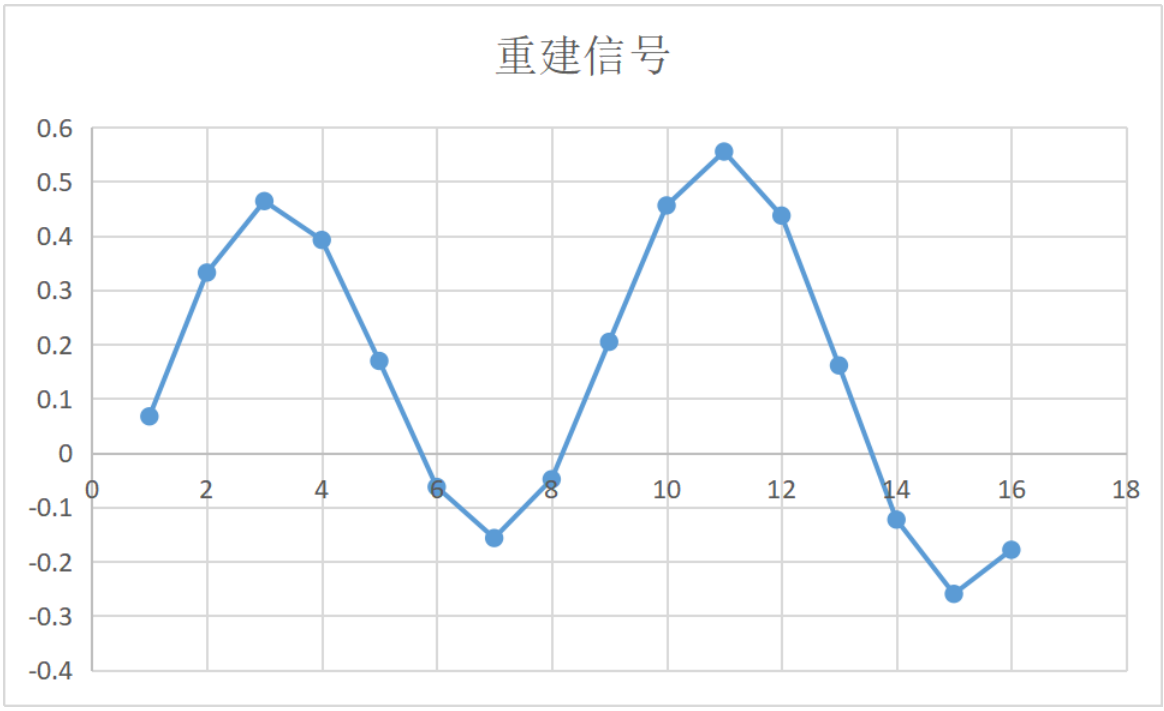
f_2:

$n = 2^4$:

i	real	imag
0	0.150436592137968	0.000000000000000
1	-0.034313084335389	-0.002070579680914
2	-0.014671698404089	-0.358853829260604
3	-0.022392781208580	-0.015913118434736
4	0.020585060155315	0.005156707825886
5	-0.007697271518806	-0.509470722861954
6	-0.003005990159815	-0.019611275619753
7	0.002785857979382	-0.005039805219438
8	-0.032641410791719	0.000000000000000
9	0.002785857972368	0.005039805248540
10	-0.003005990170333	0.019611275629034
11	-0.007697271595353	0.509470722866135
12	0.020585060155778	-0.005156707825886
13	-0.022392781229695	0.015913118419506
14	-0.014671698445070	0.358853829251323
15	-0.034313084331022	0.002070579662861




f-g_inverse
2.000000000000000
1.500000000000000
1.000000000000000
0.500000000000000
0.000000000000000
-0.500000000000000
-1.000000000000000
-1.500000000000000
-2.000000000000000
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
f
g_inverse


重建信号
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0 2 4 6 8 10 12 14 16 18

$n = 2^7$ ：

	real	imag
0	0.150333380703647	0.000000000000000
1	-0.003247209451043	0.002059557099264
2	0.000745900460923	-0.343059889227170
3	0.011558305677864	0.000233731639280
4	-0.003060403114695	-0.009886117109227
5	0.002376247234203	-0.501766860762195
6	-0.006434845284046	-0.005963422724461
7	0.003253790959701	0.003071695142975
8	-0.007995746687560	0.000470161824377
9	-0.003095307956621	0.010133116894996
10	0.003436854703482	-0.003391517861204
11	-0.007005436663333	0.008004366012200
12	-0.003998324170654	0.008856862978253
13	0.004638507446824	-0.007605753700059
14	0.002695846646918	0.000224660997949
15	0.003118908121680	-0.002021052219370
16	-0.012521644462553	-0.002022634726977
17	-0.001809519760954	0.008501132670776
18	-0.001447576398414	0.009781394530303
19	-0.005690088624724	-0.004857272942821
20	0.005997742351071	-0.003078886538540
21	-0.003604427876821	0.002167871712266
22	-0.002566113991687	-0.007676522017601
23	-0.010868270861069	0.002571616028336
24	0.001463904961315	0.003677459095909
25	0.000809820846388	-0.005852867614875
26	-0.008650487792806	0.000004499253702
27	-0.004671023036165	0.004357747611291
28	-0.007489050388369	0.002385093004337
29	-0.003808748003032	0.003508256223313
30	-0.003373150698467	-0.006500683811780
31	0.011486016824524	0.002211618433105
32	0.010235524182155	-0.002550601963831
33	0.009176817266736	0.002182556439855
34	-0.006832542913077	0.000273191174152
35	0.003621933920452	0.006778267637864
36	-0.003068220059780	0.001790346538854
37	-0.006379031286566	-0.000884981341278
38	0.002570548917478	-0.008949905109211
39	0.000958823604089	0.002177861088390
40	-0.003154574738178	-0.004462919045563
41	-0.005638559657609	0.001275143476131
42	-0.004418458784438	-0.008676995874612
43	0.003656541744786	0.005556976651185
44	-0.003633604234989	0.002318063212253
45	0.007297042279577	-0.004970716903811
46	0.001582697240161	-0.006823104649136
47	0.002546854798694	-0.000536437347168
48	-0.001205114485249	0.003897393970223
49	-0.009064689183365	0.000968507220261
50	-0.008802290605991	-0.004783137651853
51	0.006822695511152	-0.006183364841054
52	-0.001523248869468	-0.005969692046493
53	0.006113755856032	0.009967357605311
54	-0.007745535690360	-0.001462588985637
55	-0.002885842946663	0.000674236655214
56	0.000481933450897	-0.003987793648423
57	0.000434454725928	0.004946739618899
58	0.002303286317497	-0.004302500403577
59	0.000491793020490	-0.003556519359315
60	-0.009879897885803	-0.003623641791944
61	-0.001164726625647	-0.000924825540674
62	0.001962723403451	-0.004257341878539
63	-0.009994766051675	0.002137724504390
64	-0.000179600711148	0.000000000000000
65	-0.009994766051004	-0.002137724507435
66	0.001962723388390	0.004257341880743
67	-0.001164726623161	0.000924825540100
68	-0.009879897886897	0.003623641790947
69	0.000491792998145	0.003556519359027
70	0.002303286316314	0.004302500406094
71	0.000434454728430	-0.004946739618924
72	0.000481933450201	0.003987793649215
73	-0.002885842946071	-0.000674236655923
74	-0.007745535691937	0.001462588985187
75	0.006113755864152	-0.009967357600933
76	-0.001523248869563	0.005969692046676
77	0.006822695506841	0.006183364842341
78	-0.008802290604817	0.004783137651935
79	-0.009064689182518	-0.000968507222394
80	-0.001205114485111	-0.003897393969503
81	0.002546854799353	0.000536437347731
82	0.001582697236562	0.006823104649542
83	0.007297042279983	0.004970716907479
84	-0.003633604234863	-0.002318063213046
85	0.003656541740761	-0.005556976652972
86	-0.004418458785872	0.008676995874919
87	-0.005638559656604	-0.001275143476487
88	-0.003154574738941	0.004462919045238
89	0.000958823603972	-0.002177861088529
90	0.002570548914368	0.008949905110144
91	-0.006379031276056	0.000884981350400
92	-0.003068220058835	-0.001790346538465
93	0.003621933920617	-0.006778267638338
94	-0.006832542906851	-0.000273191168970
95	0.009176817268572	-0.002182556436410
96	0.010235524181926	0.002550601963831
97	0.011486016823886	-0.002211618430015
98	-0.003373150706540	0.006500683804636
99	-0.003808748000270	-0.003508256220983
100	-0.007489050387638	-0.002385093005319
101	-0.004671023046551	-0.004357747622130
102	-0.008650487792012	-0.000004499254403
103	0.000809820845049	0.005852867617053
104	0.001463904961434	-0.003677459095475
105	-0.010868270861221	-0.002571616030880
106	-0.002566113994201	0.007676522015791
107	-0.003604427870899	-0.002167871700234
108	0.005997742350996	0.003078886540464
109	-0.005690088627977	0.004857272938274
110	-0.001447576392739	-0.009781394523246
111	-0.001809519757459	-0.008501132667932
112	-0.012521644462946	0.002022634726257
113	0.003118908120913	0.002021052221216
114	0.002695846644472	-0.002246610027796
115	0.004638507446742	0.007605753713355
116	-0.003998324169289	-0.008856862978497
117	-0.007005436665430	-0.008004366023506
118	0.003436854703506	0.003391517868946
119	-0.003095307952706	-0.010133116882827
120	-0.007995746687703	-0.000470161825279
121	0.003253790960681	-0.003071695131404
122	-0.006434845286519	0.005963422722746
123	0.002376247098750	0.501766860760926
124	-0.003060403116773	0.009886117109747
125	0.011558305675784	-0.002233731664131
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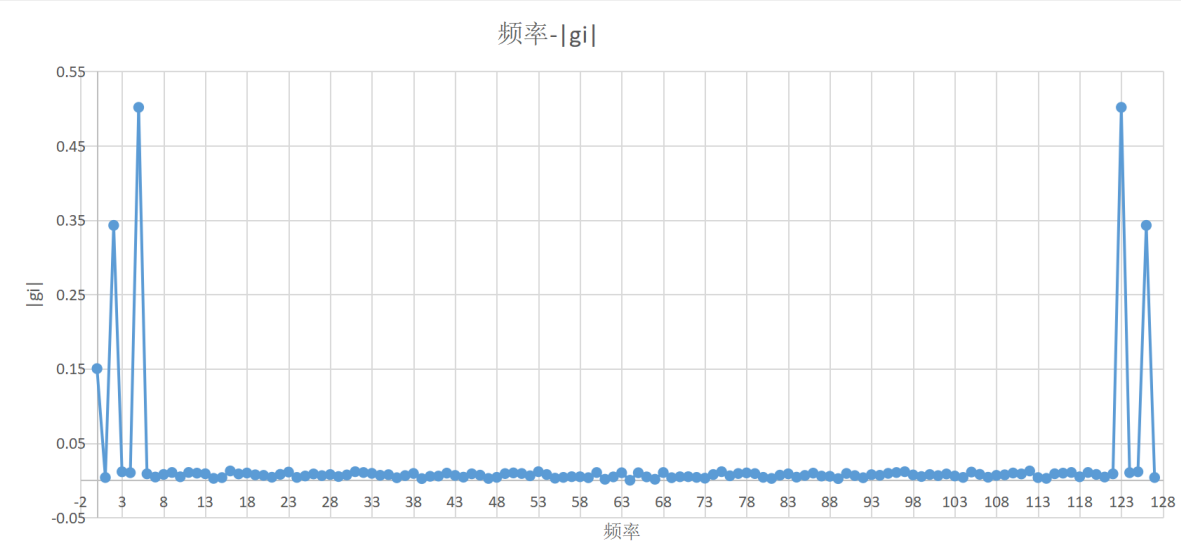
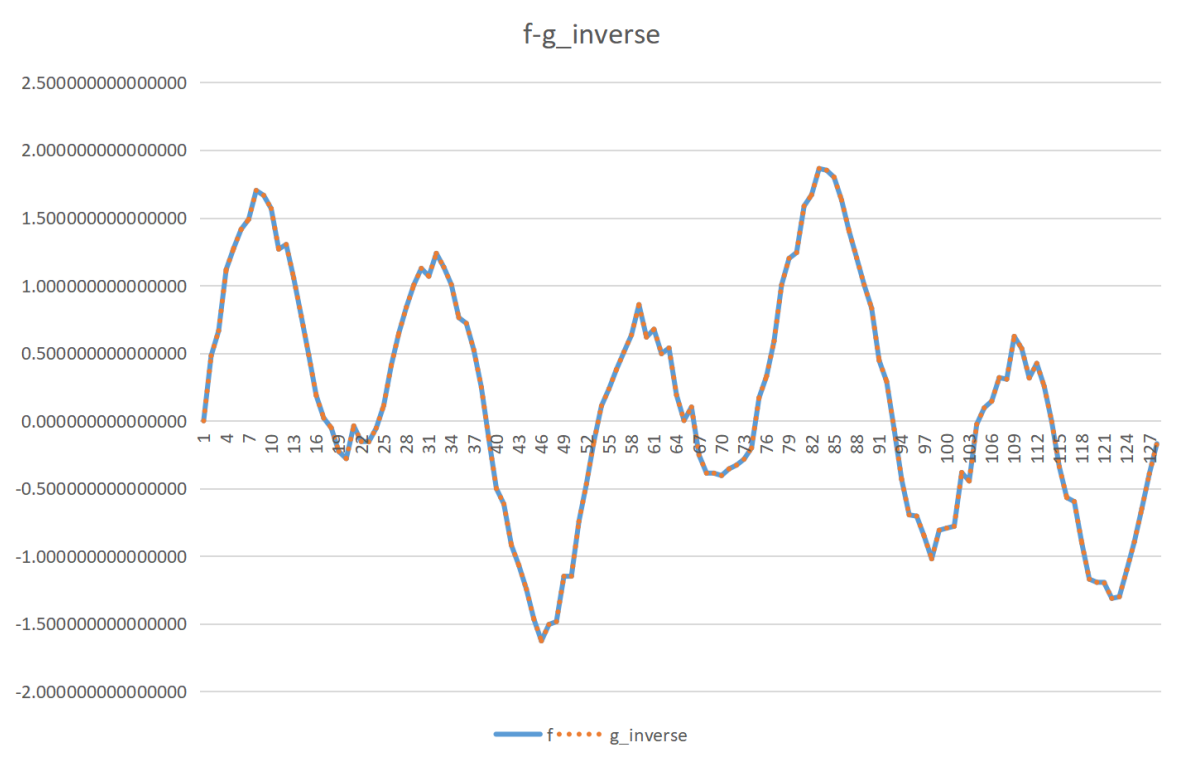
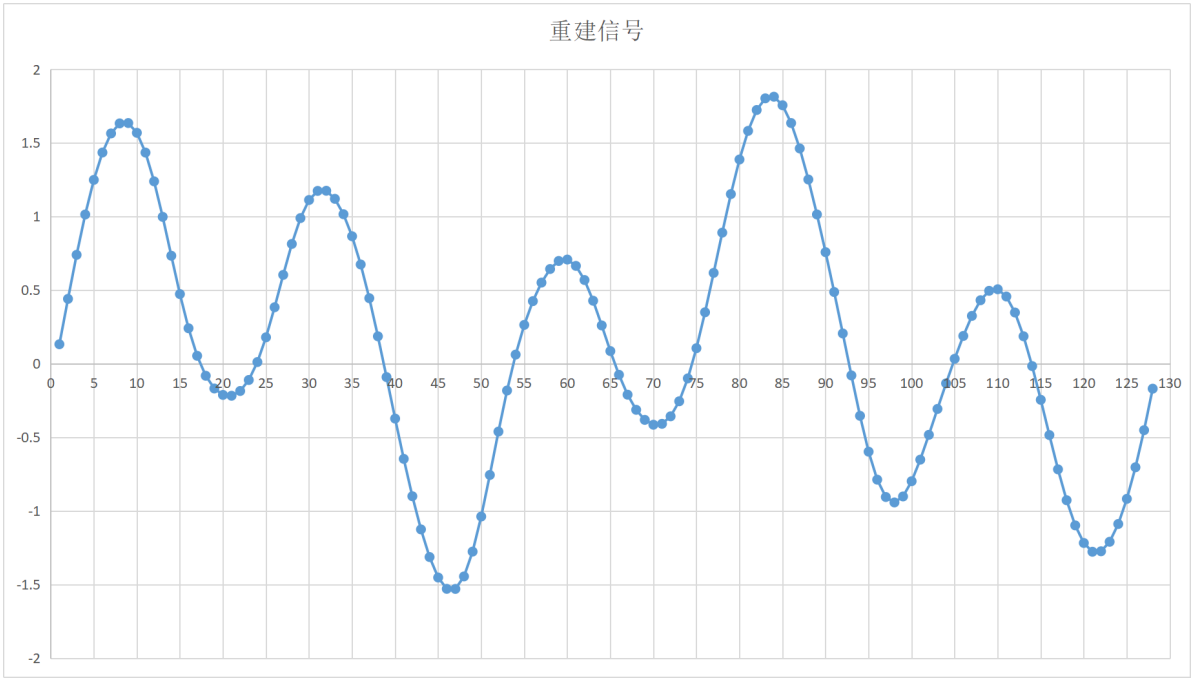
127	-0.003247209449699	-0.002059557118198







分析：

1. 采样数目n越大，结果越精确，重建的图像越光滑，但计算效率会下降。在保证足够精度的情况下，取适中的n值即可。
2. 去掉高频系数重建会使重建的信号比原来的信号光滑。