c) $\lambda_{MAP} = \underset{\lambda \in [0,+\infty)}{\operatorname{argmax}} \; p(\lambda|D) = \underset{\lambda \in [0,+\infty)}{\operatorname{argmax}} \; p(D|\lambda)\, p(\lambda)$

$\theta = \frac{1}{2}$

$$p(\lambda|D) = \frac{\lambda^{\sum_{i=1}^{n} x_i} e^{-n\lambda}}{\prod_{i=1}^{n} x_i!} \cdot \frac{1}{2} e^{-\frac{1}{2}\lambda}$$

$$ll(D|\lambda) = \ln p(\lambda|D) = \ln\lambda \sum_{i=1}^{n} x_i - n\lambda - \sum_{i=1}^{n} \ln(x_i!) + \ln(\tfrac{1}{2}) - \tfrac{1}{2}\lambda$$

$$\frac{\partial ll(D|\lambda)}{\partial \lambda} = \frac{1}{\lambda} \sum_{i=1}^{n} x_i - n - \frac{1}{2} = 0$$

$$\lambda_{MAP} = \frac{1}{n+\frac{1}{2}} \sum_{i=1}^{n} x_i = \frac{158}{19}$$

d) For MLE, we now know that $\lambda_{MLE} = \frac{79}{7}$, the number of accidents follow the Poisson Distribution with mean $\lambda_{MLE} = 79/7$.

For MAP, it will follow the Poisson Distribution with mean $\lambda_{MAP} = 158/19$

They can predict probability of each number of accidents that may happen tomorrow. $p(X=1)$, $p(X=2)$ ....

e) The prior is used to incorporate prior knowledge and helps to model the estimate distribution with mean $\lambda_{MAP}$ since $p(\lambda|D) \propto p(D|\lambda)\, p(\lambda)$

f) $\lambda_{MAP} = \frac{1}{n+\theta} \sum_{i=1}^{n} x_i$. $\theta$ should increase to reflect this new belief. It can also be seen from plots

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