Q5:

a) $P(y_i = 1 \mid x_i, \lambda) = f(x_i, \lambda)$

Parameter : $Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$, $y_i \in \{0, 1\}$

n is number of samples

$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$, $x_i \in \{0, 1\}$

$\lambda \in \mathbb{R}$

$y_i = \begin{cases} 0 & \text{table not free} \\ 1 & \text{table free} \end{cases}$ $\quad x_i = \begin{cases} 0 & \text{not sunny} \\ 1 & \text{sunny} \end{cases}$

or distribution

f is model function with parameters λ, X and output y, in this case may be Bernoulli Distribution with binary input X and parameter λ.

Determine the maximum likelihood estimate of λ

b) Now we have λ_{MLE}, if it is sunny today,

$P(y_i = 1 \mid 1, \lambda_{MLE}) = f(1, \lambda_{MLE})$

c) We could expand X to $\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ \vdots & \vdots \\ x_{n1} & x_{n2} \end{bmatrix}$

$x_{i1} \in \{0, 1\} = \begin{cases} 0 & \text{not sunny} \\ 1 & \text{sunny} \end{cases}$

$x_{i2} \in \{0, 1, 2\} = \begin{cases} 0 & \text{morning} \\ 1 & \text{afternoon} \\ 2 & \text{evening} \end{cases}$

and expand λ to $\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$

f is then a model function or distribution with new parameters λ and X and output y

vector

$P\left(y_i = 1 \mid [x_{i1}, x_{i2}], \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}\right) = f(x_i, \lambda)$

Determine the maximum likelihood estimate of vector λ

Hilroy