Q3 :

a) dim = 1

σ \ sample #	10	100	1000
1	-0.151	-0.004	-0.074
10	-3.518	-1.378	0.131

We can find that sample mean when $\sigma = 1$ is closer to 0 than sample mean when $\sigma = 10$

b) Since the covariance matrix is an identity matrix, X, Y, Z are independent

c) samples lie on the plane $X = Z$

Q4 :

a) $\lambda = 0$ when there is no data

$p(\lambda)$ decreases monotonically as λ increase, $\lambda \in [0, +\infty)$

$p(0) \Rightarrow \lambda = 0$ has maximum probability

b) $p(x|\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$

$\lambda_{MLE} = \underset{\lambda \in [0,+\infty)}{\operatorname{argmax}} \; p(D|\lambda)$

$p(D|\lambda) = p(\{x_i\}_{i=1}^{n} | \lambda) = \frac{\lambda^{\sum_{i=1}^{n} x_i} \cdot e^{-n\lambda}}{\prod_{i=1}^{n} x_i!}$

$ll(D|\lambda) = \ln p(D|\lambda) = \ln\lambda \sum_{i=1}^{n} x_i - n\lambda - \sum_{i=1}^{n} \ln(x_i!)$

$\frac{\partial ll(D|\lambda)}{\partial \lambda} = \frac{1}{\lambda} \sum_{i=1}^{n} x_i - n = 0$

$\lambda_{MLE} = \frac{1}{n} \sum_{i=1}^{n} x_i = \frac{79}{9}$