CMPUT 466 A1
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Q1 :

a) $E[f(x)] = \sum_{x \in X} f(x)\, p(x)$
$= 10 \cdot 0.1 + 5 \cdot 0.2 + \frac{10}{7} \cdot 0.7 = 3$

b) $E[1/p(x)] = \sum_{x \in X} 1/p(x) \cdot p(x) = \sum_{x \in X} 1 = 3$

c) For an arbitrary pmf p. $E[1/p(x)]$ = the number of elements in the outcome space

Q2 :

a) $E[x] = E[a_1 x_1 + a_2 x_2 + \cdots a_m x_m]$
$= a_1 E[x_1] + a_2 E[x_2] + \cdots a_m E[x_m]$
$= a_1 u_1 + a_2 u_2 + \cdots a_m u_m$
$= \sum_{i=1}^{m} a_i u_i$

the dimension of $E[x]$ is d

b) $Cov[X] = Cov(\sum_{i=1}^{m} a_i x_i, \sum_{j=1}^{m} a_j x_j) = \sum_{i=1}^{m} \sum_{j=1}^{m} a_i a_j Cov(x_i, x_j)$
Since $x_1, x_2 \cdots x_m$ are independent,
$Cov(x_i, x_j) = \begin{cases} 0, & i \neq j \\ \Sigma_i, & i = j \end{cases}$
$Cov[X] = a_1^2 \Sigma_1 + a_2^2 \Sigma_2 + \cdots a_m^2 \Sigma_m$
$= \sum_{i=1}^{m} a_i^2 \Sigma_i$ ~~with dimension $d \times d$~~
with dimension d

if X_1, X_2 are dependent, $Cov[X_1, X_2] \neq 0$
$Cov[X_1, X_2] = E[(X_1 - \mu_1)(X_2 - \mu_2)]$
since for X_i, $Cov[X_i] = \Sigma_i \in R^{d \times d}$
$Cov[X_1, X_2] = \Lambda$ for $\Lambda \in R^{d \times d}$

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