

Bonus.

d)	<del>d</del>	avg. distance
	d	
	1	0.792
	2	1.247
	4	1.873
	8	2.742
	16	3.918
	32	5.618
	64	7.951
	128	11.279

As  $d$  increases, average distance increases.  
We can infer that for  $k$ -means clustering, it won't work or it has bad performance for a high dimensional space since the volume of the space increases at a great rate relative to increase of dimensions. Average distance to means (centroid) could be too large.

e) the ratio is  $(1-\epsilon)^d$ , ( $0 < \epsilon < 1$ )

As  $d$  gets large, the ratio becomes small

and  $(1-\epsilon)^d \xrightarrow{d \rightarrow \infty} 0$ , the volume of

$d$ -dimensional hypercube with side length  $1-\epsilon$  has nearly 0 volume.

For high  $d$ , if side length, here distance to the origin is less than 1, samples will be clustered in a very small volume. While the distance is larger than 1, samples will be distributed in a large volume. For random generated multivariate Gaussian samples in such large volume, average distance will increase as  $d$  increases since  $P(-0.5 < x < 0.5)$  for a Gaussian Normal is only 38.3%