# Fertility · Family · Nation · World

*A Survey on Promoting Fertility Rate*

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Summary Sheet

Population is an essential issue since ancient times. Whether it overloads or lacks all results in some immeasurable trouble to individuals, groups, societies, nations, even the world. Under specific circumstance, we need to find out different measures related to various factors applicable to the citizen who suits to fertility.

As for Question 1, not only do we consider economic development, higher education the two factors associated with birth rate, but also we link gender ratio over years with fertility. We first assume both are linear correlation and then we adopt functional fitting tools in Matlab to approximately represent its intrinsic relationship. Ultimately, we combine all the three factors and construct mathematics models to analyze what concrete influence occur to birth rate when they function jointly.

In Question 2, we collect necessary date charts from Internet, so as to uncover which measure can powerful maintain the positive birth rate. As a result, we choose the number of workforce and the average asset of families within babies as our main focus. With the mathematics models we build, we find out the rules in it and explain why they shape like that through examples in reality.

Last but not the least, to solve Question 3 perfectly, we summarize the rules and the experience in Question 1 and 2. According to current situation in China, we suggest that: government and society are supposed to unite every industry and every field, ①improving the gender ratio ②increasing the number of employment, ③stimulating the welfare of fertility, ④Increase the proportion of middle-income population.

Key Word
mathematics models birth rate fertility policy measures

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# 1 Introduction of Problem

## 1.1 Background

In traditional Chinese culture, it is generally believed that “the more children, the more blessings”, which means having more children will bring more happiness and blessings. However, after the relaxation of the policy, the fertility level and the number of births still did not increase significantly. According to the China Statistical Yearbook 2024 compiled and published by the National Bureau of Statistics recently, at the end of 2023, the population of the country was 140967 million, a decrease of 2.08 million from the end of the last year, and the natural population growth rate is -1.48‰. Among them, 23 provinces have a negative natural population growth rate, 3 more than those in 2022. This is also a continuation of negative population growth in two years in the country since the negative growth occurred in 2022.

So what factors are at play in the essential decline of the fertility level? As far as we’re concerned, most of those factors more or less could be owed to the giant population base of China, approximately 1.4 billion. Considering the general environment, accordingly, housing shortage pushes more and more adults to shoulder a huge debt of cars, houses and so on, while the phenomenon of academic qualification’s depreciation leads to less chance for undergraduates to hunt a job for living, which prompts them to extend their study career to gain high education such as bachelor or doctor. Correspondingly, the more time committed to study means the less time they have to finish some vital events such as dating and marriage, so that they potentially lose eagerness for fertility. Besides, more and more common unemployment or midlife crisis in Chinese society make employers take more and more burden about dismissal. Meanwhile, competitions among colleagues in the same industry are heated, all of which contribute to the severe fertility situation.

As an old saying goes: ”Fertility in the present, benefits on generations.” The problem of population’s developing benignly is an eternal issue, not just for China, but also for whole human, which is associated with the destiny of world. To solve it is of great urgency right now. Furthermore, we hope our survey also could be applied to other nations and religions as a good example to handle the population problem there.

## 1.2 Clarification

### 1.2.1 Question 1

Utilizing the data on the Internet, we’re supposed to construct mathematics models analyzing the association between the decline of birth rate in China, economic development and high education.

### 1.2.2 Question 2

What more effective measures should be adopted to achieve the increase of birth rate? We’re supposed to construct mathematics models and analyze their concrete implementation effect.

### 1.2.3 Question 3

We’re expected to write a letter to relevant department, comprehensively elaborating our plan in increase birth rate.

# 2 Analysis of Problem

## 2.1 Question 1

We hope when we construct a mathematics model to represent the association with two, three or more factors, we could just focus on these points and ignore other irrelevant variables because of their minimal influence on our model. In view of the low effect since the relevant policy to facilitate birth rate in China has been adopted, we selectively neglect the disturbance factor of policy within it. As well, we have collected relevant data from 2004 to 2023 (Table 1:Data output). Naturally, we set our sample range from 2004 to 2023.

Synthetically, we choose high education proportion (the ratio of student enrollment and graduates to the population from 18 to 25), per capita disposable income(reflecting economic development), the ratio of male to female and birth rate as research objectives.

Among them, fitting the point set is the key to our model constructing. After a series of previous tests, we decide cubic polynomial equations as our function form. The derivation process of fitting function is below:

### Step 1: Set the form of the polynomial function

Let the polynomial function for fitting be $y = a_0 + a_1x + a_2x^2 + a_3x^3$, where $a_0, a_1, a_2, a_3$ are coefficients to be determined.

### Step 2: Establish the system of equations

Suppose there are $n$ dot - matrix data $(x_i, y_i)$, $i = 1, 2, \cdots, n$. Substituting each data point into the polynomial function, we can get:

$$
\begin{aligned}
y_1 &= a_0 + a_1x_1 + a_2x_1^2 + a_3x_1^3 \\
y_2 &= a_0 + a_1x_2 + a_2x_2^2 + a_3x_2^3 \\
&\cdots \\
y_n &= a_0 + a_1x_n + a_2x_n^2 + a_3x_n^3
\end{aligned}
$$

It can be written in matrix form $Y = XA$, where

$$
Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, X = \begin{bmatrix} 1 & x_1 & x_1^2 & x_1^3 \\ 1 & x_2 & x_2^2 & x_2^3 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_n & x_n^2 & x_n^3 \end{bmatrix}, A = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}
$$

### Step 3: Solve the coefficient matrix

When $n \geq 4$, the least - squares method is generally used to solve the coefficient $A$. According to the least - squares method, the objective is to minimize $\sum_{i=1}^{n}(y_i - (a_0 + a_1x_i + a_2x_i^2 + a_3x_i^3))^2$. Let $J = \sum_{i=1}^{n}(y_i - (a_0 + a_1x_i + a_2x_i^2 + a_3x_i^3))^2$. Take the partial derivatives of $J$ with respect to $a_0, a_1, a_2, a_3$ respectively and set the partial derivatives to 0, and the following system of equations is obtained:

$$
\begin{aligned}
\frac{\partial J}{\partial a_0} &= -2\sum_{i=1}^{n}(y_i - (a_0 + a_1x_i + a_2x_i^2 + a_3x_i^3)) = 0 \\
\frac{\partial J}{\partial a_1} &= -2\sum_{i=1}^{n}x_i(y_i - (a_0 + a_1x_i + a_2x_i^2 + a_3x_i^3)) = 0 \\
\frac{\partial J}{\partial a_2} &= -2\sum_{i=1}^{n}x_i^2(y_i - (a_0 + a_1x_i + a_2x_i^2 + a_3x_i^3)) = 0 \\
\frac{\partial J}{\partial a_3} &= -2\sum_{i=1}^{n}x_i^3(y_i - (a_0 + a_1x_i + a_2x_i^2 + a_3x_i^3)) = 0
\end{aligned}
$$

After sorting, we can get $(X^TX)A = X^TY$. Solving this system of equations can obtain the coefficients $a_0, a_1, a_2, a_3$, so as to determine the fitted polynomial function.

Now that we have gotten four functions of high education proportion, per capita disposable, the ratio of male to female and birth rate. In order to uncover the association with birth rate and the three factors, let's assume they have linear relationship,

$$f(x) = ag(x) + bh(x) + cl(x) + d \tag{1}$$

Suppose we want to fit the function $f(x)$, and its expression is:

$$f(x) = ag(x) + bh(x) + cl(x) + d \tag{2}$$

where $a$, $b$, $c$ and $d$ are the coefficients to be determined, and $g(x)$, $h(x)$, $l(x)$ are known functions.

In order to determine these coefficients, we usually use the least squares method. Suppose we have a set of observed data points $\{(x_i, y_i)\}$, where $y_i$ is the observed value of $f(x)$ at $x = x_i$, for $i = 1, 2, \cdots, n$.

According to the principle of the least squares method, we want to minimize the sum of squared errors ($SSE$), which is defined as:

$$SSE = \sum_{i=1}^{n} [y_i - (ag(x_i) + bh(x_i) + cl(x_i) + d)]^2 \tag{3}$$

In order to find the values of $a$, $b$, $c$ and $d$ that minimize $SSE$, we take the partial derivatives of $SSE$ with respect to $a$, $b$, $c$ and $d$ respectively, and set these partial derivatives equal to zero.

First, take the partial derivative with respect to $a$:

$$\frac{\partial SSE}{\partial a} = \frac{\partial}{\partial a} \sum_{i=1}^{n} [y_i - (ag(x_i) + bh(x_i) + cl(x_i) + d)]^2 \tag{4}$$

$$= -2 \sum_{i=1}^{n} g(x_i)[y_i - (ag(x_i) + bh(x_i) + cl(x_i) + d] \tag{5}$$

Set $\frac{\partial SSE}{\partial a} = 0$, we get:

$$\sum_{i=1}^{n} g(x_i)[y_i - (ag(x_i) + bh(x_i) + cl(x_i) + d] = 0 \tag{6}$$

Then, take the partial derivative with respect to $b$:

$$\begin{aligned}\frac{\partial SSE}{\partial b} &= \frac{\partial}{\partial b}\sum_{i=1}^{n}[y_i - (ag(x_i) + bh(x_i) + cl(x_i) + d)]^2 &(7)\\ &= -2\sum_{i=1}^{n} h(x_i)[y_i - (ag(x_i) + bh(x_i) + cl(x_i) + d] &(8)\end{aligned}$$

Set $\frac{\partial SSE}{\partial b} = 0$, we get:

$$\sum_{i=1}^{n} h(x_i)[y_i - (ag(x_i) + bh(x_i) + cl(x_i) + d] = 0 \tag{9}$$

Next, take the partial derivative with respect to $c$:

$$\begin{aligned}\frac{\partial SSE}{\partial c} &= \frac{\partial}{\partial c}\sum_{i=1}^{n}[y_i - (ag(x_i) + bh(x_i) + cl(x_i) + d)]^2 &(10)\\ &= -2\sum_{i=1}^{n} l(x_i)[y_i - (ag(x_i) + bh(x_i) + cl(x_i) + d] &(11)\end{aligned}$$

Set $\frac{\partial SSE}{\partial c} = 0$, we get:

$$\sum_{i=1}^{n} l(x_i)[y_i - (ag(x_i) + bh(x_i) + cl(x_i) + d] = 0 \tag{12}$$

Finally, take the partial derivative with respect to $d$:

$$\begin{aligned}\frac{\partial SSE}{\partial d} &= \frac{\partial}{\partial d}\sum_{i=1}^{n}[y_i - (ag(x_i) + bh(x_i) + cl(x_i) + d)]^2 &(13)\\ &= -2\sum_{i=1}^{n}[y_i - (ag(x_i) + bh(x_i) + cl(x_i) + d] &(14)\end{aligned}$$

Set $\frac{\partial SSE}{\partial d} = 0$, we get:

$$\sum_{i=1}^{n}[y_i - (ag(x_i) + bh(x_i) + cl(x_i) + d] = 0 \tag{15}$$

We can write the above system of equations in matrix form. Let:

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} g(x_1) \\ g(x_2) \\ \vdots \\ g(x_n) \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} h(x_1) \\ h(x_2) \\ \vdots \\ h(x_n) \end{bmatrix}, \quad \mathbf{L} = \begin{bmatrix} l(x_1) \\ l(x_2) \\ \vdots \\ l(x_n) \end{bmatrix},$$

$$\mathbf{X} = \begin{bmatrix} g(x_1) & h(x_1) & l(x_1) & 1 \\ g(x_2) & h(x_2) & l(x_2) & 1 \\ \vdots & \vdots & \vdots & \vdots \\ g(x_n) & h(x_n) & l(x_n) & 1 \end{bmatrix}, \quad \beta = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

The system of equations can be expressed as:

$$\mathbf{X}^T \mathbf{X} \beta = \mathbf{X}^T \mathbf{y} \tag{16}$$

Solving the above matrix equation gives the least squares estimates of the coefficients $\beta = (a, b, c, d)^T$.

| Year | Education | PCDI | Sex Rate | Birth Rate |
|---|---|---|---|---|
| 2004 | 0.093967634 | 05661 | 104.6 | 0.0126 |
| 2005 | 0.107824675 | 06385 | 104.6 | 0.0127 |
| 2006 | 0.118409243 | 07229 | 104.6 | 0.0128 |
| 2007 | 0.127319781 | 08584 | 104.7 | 0.0130 |
| 2008 | 0.135296502 | 09957 | 104.7 | 0.0133 |
| 2009 | 0.141159965 | 10977 | 104.7 | 0.0134 |
| 2010 | 0.148236332 | 12520 | 104.8 | 0.0133 |
| 2011 | 0.156071179 | 14551 | 104.8 | 0.0131 |
| 2012 | 0.165486774 | 16510 | 104.8 | 0.0141 |
| 2013 | 0.177003071 | 18311 | 104.8 | 0.0133 |
| 2014 | 0.190884291 | 20167 | 104.8 | 0.0135 |
| 2015 | 0.206663578 | 21966 | 104.8 | 0.0125 |
| 2016 | 0.225604786 | 23821 | 104.7 | 0.0131 |
| 2017 | 0.243530653 | 25974 | 104.7 | 0.0129 |
| 2018 | 0.257292027 | 28228 | 104.6 | 0.0108 |
| 2019 | 0.278134803 | 30733 | 104.5 | 0.0103 |
| 2020 | 0.306029721 | 32189 | 104.4 | 0.0830 |
| 2021 | 0.330059514 | 35128 | 104.3 | 0.0730 |
| 2022 | 0.358348689 | 36883 | 104.2 | 0.0660 |
| 2023 | 0.376610209 | 39218 | 104.0 | 0.0630 |

表 1: Data output

(Data sources: National Bureau of Statistics of China, World Population Prospects)

## 2.2 Question 2

When it comes to what more effective measures to facilitate birth rate, we Chinese have seen the two-child or three-child policies could not reverse the situation currently. Accordingly, purely encouraging citizen to have more children is not a applicable option. We’re expected to come up with another practicable choices and analyze them deeply and quantitatively. After looking through numerous articles and papers, we have a better understanding of what leads to a positive fertility environment, and choose these elements below as our analysis objects:

1.The population of workforce in China yearly According to China National Bureau of Statistics, the population of workforce in China has declined continually since 2015, roughly coordinating the trend of the decrease of birth rate, which inspire us to link the two as consideration. Then we apply the concept of fitting in statistics into our analysis.

After several times of tests, we decide quadratic polynomial as our goal. $y = a_0+a_1x+a_2x^2$ The basic principle of solution is the same as above(cubic polynomial equations).

Actually, the result of the analysis proves our assumption: the decline of the population of workforce has a certain association with the birth rate. In other words, the two have a strong correlation. The establishment of the conjecture also explains why citizens resist to birth under the circumstance where the population of workforce has been declining. Human beings have a tendency to make plans for the uncertain future, which suggest that when the population of workforce has been declining they potentially lose source of income and have difficulty to make living when they' re in unemployment. However, Fertility is a time-consume and high-cost event, which also need parents' patience and perseverance. To guarantee their normal lifestyle not to be disturbed, they have to give up having one or more children.

| Year | Workforce |
|---|---|
| 2004 | 75290 |
| 2005 | 76120 |
| 2006 | 76310 |
| 2007 | 76531 |
| 2008 | 77046 |
| 2009 | 77510 |
| 2010 | 78388 |
| 2011 | 78579 |
| 2012 | 78894 |
| 2013 | 79300 |
| 2014 | 79690 |
| 2015 | 80091 |
| 2016 | 79282 |
| 2017 | 79042 |
| 2018 | 78653 |
| 2019 | 78985 |
| 2020 | 78392 |
| 2021 | 78024 |
| 2022 | 76863 |
| 2023 | 77216 |

表 2: Workforce over Years

(Data sources: National Bureau of Statistics of China)

2.The asset status of citizens Under the premise where we have collected efficient materials about the data of the number of children in families with different income in America, we’re supposed to take it as a reference to let us survey posses more levels and perspectives.

Similarly, we decide quartic polynomial as our fitting function after tests.

$y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5$

Completing the construction of the mathematics model, we surprisingly find its trend is a U-shaped curve, which suggests there is the a point where birth rate can achieve the highest value as families with certain income.

As the curve shows, if American government attempts to maintain or enhance the ratio of household income within a certain interval, the birth rate may increase meanwhile, so do Chinese: under usual condition, this measure could be applied domestically.

| Income | Children |
|---|---|
| 10000-14999 | 52.31 |
| 15000-24999 | 56.41 |
| 25000-34999 | 58.82 |
| 35000-49999 | 52.69 |
| 50000-74999 | 53.04 |
| 75000-99999 | 52.69 |
| 100000-149999 | 49.86 |
| 150000-199999 | 46.46 |

表 3: Income with Children

(Data sources:www. statista. com)

3.Other measures In addition, we also find some interesting and useful information related to birth rate in other countries, such as building fertility-friendly hospitals based on the theory of population balance, improving the treatment in Andrology and Gynecology [2] , and granting or increasing childcare allowance, extending maternity and paternity leave... [1] The policies and their effects are shown in the following figure:

**表 1　各国的政策措施及实施效果比较**

| 措施及效果 | 韩国 | 日本 | 新加坡 | 俄罗斯 | 德国 | 法国 | 瑞典 |
|---|---|---|---|---|---|---|---|
| 物质层面 | | | | | | | |
| 生育奖励 | √ | | √ | √ | | | |
| 育儿津贴 | √ | √ | √ | √ | √ | √ | √ |
| 提供育儿服务 | √ | √ | √ | | √ | | √ |
| 教育费用补助 | √ | √ | | √ | | √ | √ |
| 辅助生殖资助 | √ | | | √ | √ | | |
| 医疗援助 | √ | √ | √ | | | | |
| 住房保障和补助 | √ | √ | √ | √ | √ | | √ |
| 大家庭交通费减免 | | | | | | √ | |
| 低收入或多子女家庭补贴 | | | | √ | √ | √ | |
| 减免税收 | | | √ | | √ | √ | |
| 精神层面 | | | | | | | |
| 孕产假及育儿假 | | | | | | | |
| 母亲产假、哺乳假 | √ | √ | √ | √ | √ | √ | √ |
| 男士育儿假 | √ | √ | √ | | | √ | √ |
| 育儿病事假 | | | √ | | | √ | √ |
| 建立社交网站 | | | √ | | | | |
| 多孩家长延长退休年龄 | √ | | | | | | |
| 多孩家庭子女就业优惠政策 | √ | | | | | | |
| 兴建育儿机构 | | √ | | | √ | √ | √ |
| 改善公共教育服务 | √ | √ | √ | | | | |
| 提供协调工作和生活培训 | | | √ | | | | |
| 实施年份 | 2005 | 1994 | 1986 | 2006 | 1986 | 1939 | 20 世纪 20 年代 |
| 政策起效用时间/年 | 2 | 12 | 23 | 6 | 20 | 60 | — |
| 实施初期 TFR[a] | 1.08 | 1.49 | 1.43 | 1.36 | 1.41 | — | — |
| 政策实施后 TFR 最低值[a] | 0.88 | 1.27 | 0.95 | 1.36 | 1.25 | 1.66 | 1.50 |
| 政策实施后 TFR 最高值[a] | 1.26 | 1.45 | 1.79 | 1.80 | 1.59 | 2.99 | 2.46 |
| 2023 年 TFR[1] | 0.88 | 1.31 | 1.04 | 1.52 | 1.53 | 1.79 | 1.67 |

注：TFR：总和生育率；[a]联合国人口司《世界人口展望 2022》估计数据（https://population.un.org/dataportal/home）

(a) Related Policies

Considering the time limitation and the lack of necessary knowledge, we only scratch the surface of what factors matter in our survey. As for quantitative analysis, we leave it to readers to solve for the moment in the future.

## 2.3 Question 3

Refer to Question 1 and 2, we can basically make sure the measures to boost fertility as below:

①Optimize the gender ratio. From the fitting effects of Question 1, we see there is a positive correlation between the gender ratio and birth rate from the fitting function

$$\text{Final fitted function:} \quad y = -0.0019566j(x) + 0.00089974h(x) + 0.0014802g(x) + 0.011660.000314 \tag{17}$$

, which indicates we should take optimizing the gender ratio as consideration.

②Improve employment rate. From some relevant analysis in Question 2, we find there is a strong correlation between the decline of the population of workforce and birth rate. In conclusion, it’s vital to improve employment rate, guaranteeing the live of citizen and living up with their expectation to life forward.

③Increase the proportion of middle-income population. According to the report mentioned above in www. statista. com, we understand that it’s a function model that monotonically increase firstly and then monotonically decrease. Furthermore, we utilize Matlab tools to figure out the extreme point, it represents which level of families within household income has the most potential of the eagerness to birth. Similarly, it could also be seen as a effective strategy applied in China.

④Stimulate the welfare of fertility. Besides, through the chart above provided by TANGLong-mei, XU Jia-lin, ZHANG Jun,HONG Ping,ZHANG Wei-hong, we had better consider how to improve those infrastructure related and stimulate the welfare of fertility, easing the burden of families within children, which will motivate family to make plan for a baby or more for sure.

Generally speaking, The State Family Planning Commission of the People’s Republic of China is the administration managing China’s fertility policy. So the letter sent to it will be like this as below:

Dear officials in the State Family Planning Commission :

In order to fully and effectively implement China’s population strategy and coordinate population development with economic prosperity, we sincerely offer you a few suggestion as below:

①Optimize the gender ratio. In view of the common concept of preference for sons over daughters in some parts of regions, we are supposed to correct the unreasonable concept and advocate the equality of men and women in rights.

②Improve employment rate. The decline of the population of workforce is not good for fertility. So we’re concerned that government should make policies to enhance the development of small and medium-sized enterprises, increasing the number of jobs and promoting economy and business to boom dynamically.

③Increase the proportion of middle-income population. Gorgeously accomplishing the great task of comprehensively poverty alleviation, we are expected to continue our mission of serving people, letting them enjoy the reform dividend and raise their happiness.

④Stimulate the welfare of fertility. It’s believed that society and government should cover a part of costs of baby-parenting, such as granting or increasing childcare allowance, extending maternity and paternity leave and so on, which contributes to an alleviation of families’ worry.

# 3 Description of symbols and definitions of terms

## 3.1 Question 1

$x_i(i = 1, 2, 3...)$ means the decentralized independent variable of each fitting function corresponding to each year ;

$f(x_i)$ means the birth rate in specific year;

$h(x_i)$ means the per capita income in specific year;

$g(x_i)$ means the gender ratio in specific year;

$j(x_i)$ means the higher education ratio in specific year;

## 3.2 Question 2

$m(x_i)$ means the number of workforce in specific year.

$n(x_i)$ means the birth rate in families within certain range of assets.

# 4 Assumptions of Models

(1)Ignore the effects of changes of marriage rate, divorce rate, female family status and other variables on the models.

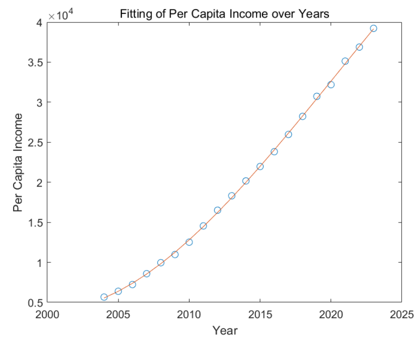
(2)Assume that the policies of two-child and three-child haven’t make quite a obvious contribution to the current situation of the decline of birth rate so far.

(3)Agree that our mathematics models meet the conditions needed in the 21st centuries( from 2003 to 2023).

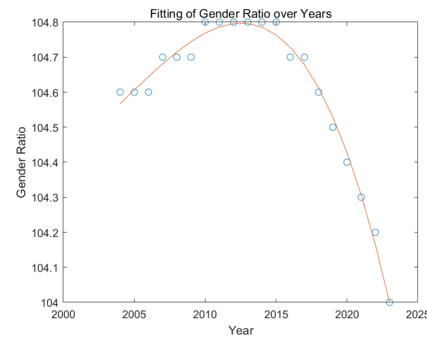
# 5 Constructions of Mathematics Models

## 5.1 Question 1

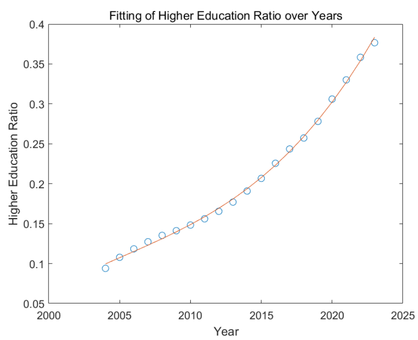
Combined the derivation process mentioned as above, we generate five figures in Matlab(the codes are in Appendix 9.1).



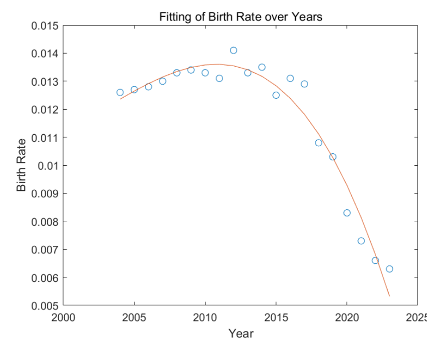

(a) Fitting of Per Capita Income over Years



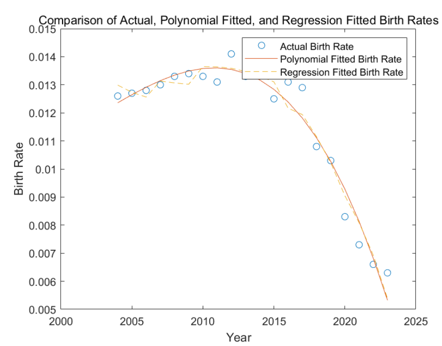

(b) Fitting of Gender Ratio over Years




(c) Fitting of Higher Education Ratio over Years




(d) Fitting of Birth Rate over Years




(e) Comparison of Actual, Polynomial Fitted, and Regression Fitted Birth Rates

$$
\begin{aligned}
&\text{per capita income:} \quad h(x) = 19022.251515 + 11399.777291x + 1291.945774x^2 - 362.397261x^3 \text{(figure a)} \\
&\text{gender ratio:} \quad g(x) = 104.792311 - 0.060941x - 0.197169x^2 - 0.044891x^3 \text{(figure b)} \\
&\text{higher education ratio:} \quad j(x) = 0.187141 + 0.075535x + 0.021111x^2 + 0.004997x^3 \text{(figure c)} \\
&\text{birth rate:} \quad f(x) = 0.013304 - 0.001382x - 0.001730x^2 - 0.000314x^3 \text{(figure d)} \\
&\text{Final fitted function:} \quad y = -0.0019566j(x) + 0.00089974h(x) + 0.0014802g(x) + 0.011660.000314 \text{(figure e)}
\end{aligned}
\tag{18}
$$

## 5.2 Question 2

We fit the two factors and get the two figures as below.
(The MatLab code is in Appendix 8.2 and 8.3)

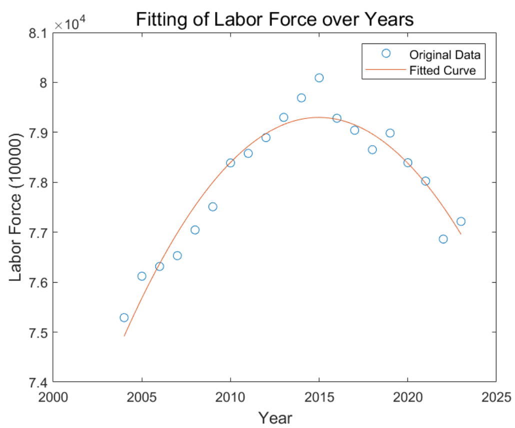



图 3: Fitting of WorkForce over Years

$$
\text{WorkForce:} \quad m(x) = -36.364x^2 + 146580.866x - 147635790.547 \tag{19}
$$

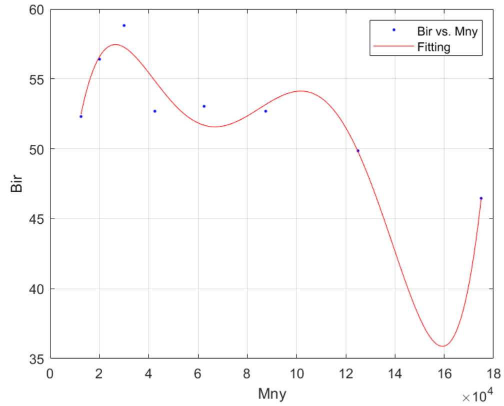



图 4: The asset status of citizens

$$
\begin{aligned}
n(x) = {} & 10.0272\left(\frac{x-69380}{56790}\right)^5 - 16.9764\left(\frac{x-69380}{56790}\right)^4 - 13.1050\left(\frac{x-69380}{56790}\right)^3 \\
& + 16.3041\left(\frac{x-69380}{56790}\right)^2 + 1.4708\left(\frac{x-69380}{56790}\right) + 51.6067
\end{aligned} \tag{20}
$$

Obviously, there is an extreme point lying between 2 and 3. Naturally, If we can work out it, the mathematics model will surely meaningful for our survey.

The function reaches its maximum value of 57.45 at $x = 2.6481 \times 10^4$ in the interval $(10000, 160000)$.

# 6 Strengths and Weakness

## 6.1 Strengths

① We fully consider the factors associated with birth rate, such as higher education ratio, per capita disposable income, gender ratio and so on, which makes our survey applicable and integrate.

② Each factor is fitted as polynomial form, which can help us analyze and predict efficiently.

③ Foreign excellent policies to facilitate fertility are introduced and analyzed in our survey, which makes our solution variable.

### 6.2 Weakness

① In view of the lack of relevant samples and necessary means, we can not conduct analysis of the stability of our mathematics models.

② Some policies and its actual effect can not be quantitative analyzed and worked out, which causes difficulties for us to make a exact explanation.

## 7 Conclusion

So to speak, a period of situation of fertility reflects human being’s tomorrow. And now, a low birth rate clearly is not good for sustainable and beneficial development, whether in terms of ecosystem, economy or politics.

Aiming to reverse the bad trend and maintain a normal birth rate year by year, we have discussed several measures as above, including improving the gender ratio, increasing the number of employment, stimulating the welfare of fertility, and so on. Among them, most of the opinions have been analyzed and proved mathematically and seriously.

As far as we’re concerned, the ups and downs , and valleys and peaks of population is linked with our country’s development orient and path. As one of the member, we should keep alert and take initiative to shoulder parts of responsibilities and obligation in response to the advocates, contributing hopeful and harmonious community, society, nation, even the world.

## 8 Appendix

### 8.1 Quetion1 Related MCode

Listing 1: Related MATLAB Code

```
data = readtable('data_output2.xlsx');
Year = data.Year;
HigherEducationRatio = data.Education;
PerCapitaIncome = data.InputMoney;
GenderRatio = data.SexRate;
```

```
BirthRate = data.Birth_1000000_;
Year_centered = (Year - mean(Year)) / std(Year);
coefficients_HE = polyfit(Year_centered, HigherEducationRatio, 3);
fitted_HE = polyval(coefficients_HE, Year_centered);
coefficients_Income = polyfit(Year_centered, PerCapitaIncome, 3);
fitted_Income = polyval(coefficients_Income, Year_centered);
coefficients_Gender = polyfit(Year_centered, GenderRatio, 3);
fitted_Gender = polyval(coefficients_Gender, Year_centered);
coefficients_BirthRate = polyfit(Year_centered, BirthRate, 3);
fitted_BirthRate = polyval(coefficients_BirthRate, Year_centered);
X = [ones(length(Year),1), HigherEducationRatio, PerCapitaIncome, GenderRatio];
X_centered = [ones(length(Year),1), (HigherEducationRatio - mean(HigherEducationRatio)) / std(HigherEducationRatio),...
(PerCapitaIncome - mean(PerCapitaIncome)) / std(PerCapitaIncome),...
(GenderRatio - mean(GenderRatio)) / std(GenderRatio)];
if license('test','Statistics_Toolbox')
[b, bint, r, rint, stats] = regress(BirthRate, X_centered);
disp(['Intercept b0: ', num2str(b(1))]);
disp(['Coefficient b1 (HigherEducationRatio): ', num2str(b(2))]);
disp(['Coefficient b2 (PerCapitaIncome): ', num2str(b(3))]);
disp(['Coefficient b3 (GenderRatio): ', num2str(b(4))]);
BirthRate_Fitted = X_centered * b;
else
disp('Please install Statistics and Machine Learning Toolbox first.');
end
fprintf('Fitting function of higher education ratio: y = %f + %f*x + %f*x^2\n', coefficients_HE(3), coefficients_HE(2), coefficients_HE(1));
fprintf('Fitting function of per capita income: y = %f + %f*x + %f*x^2\n', coefficients_Income(3), coefficients_Income(2), coefficients_Income(1));
fprintf('Fitting function of gender ratio: y = %f + %f*x + %f*x^2\n', coefficients_Gender(3), coefficients_Gender(2), coefficients_Gender(1));
fprintf('Fitting function of birth rate: y = %f + %f*x + %f*x^2 + %f*x^3\n', coefficients_BirthRate(4), coefficients_BirthRate(3), coefficients_BirthRate(2), coefficients_BirthRate(1));
figure;
plot(Year, HigherEducationRatio, 'o', Year, fitted_HE, '-');
title('Fitting of Higher Education Ratio over Years');
xlabel('Year');
```

```
ylabel('Higher Education Ratio');
figure;
plot(Year, PerCapitaIncome, 'o', Year, fitted_Income, '-');
title('Fitting of Per Capita Income over Years');
xlabel('Year');
ylabel('Per Capita Income');
figure;
plot(Year, GenderRatio, 'o', Year, fitted_Gender, '-');
title('Fitting of Gender Ratio over Years');
xlabel('Year');
ylabel('Gender Ratio');
figure;
plot(Year, BirthRate, 'o', Year, fitted_BirthRate, '-');
title('Fitting of Birth Rate over Years');
xlabel('Year');
ylabel('Birth Rate');
if license('test','Statistics_Toolbox')
disp(['Regression coefficients: ', num2str(b')]);
disp(['R - squared value: ', num2str(stats(1))]);
figure;
plot(Year, r, 'o');
title('Residual Plot');
xlabel('Year');
ylabel('Residual');

figure;
plot(Year, BirthRate, 'o', 'DisplayName', 'Actual Birth Rate');
hold on;
plot(Year, fitted_BirthRate, '-', 'DisplayName', 'Polynomial Fitted Birth Rate');
plot(Year, BirthRate_Fitted, '--', 'DisplayName', 'Regression Fitted Birth Rate');
title('Comparison of Actual, Polynomial Fitted, and Regression Fitted Birth Rates');
xlabel('Year');
ylabel('Birth Rate');
legend show;
hold off;
end
```

### 8.2 Quetion2 Related MCode1

Listing 2: Related MATLAB Code

```matlab
% Read data from Excel file
data = readtable('birth_rate_data.xlsx');

% Extract relevant columns
Mny = data.AnnualHouseholdIncomeInU_S_Dollars;
Bir = data.NumberOfBirthsPer1_000Women;

% Convert Mny from cell array to numeric array
Mny = str2double(Mny);

% Perform polynomial fitting
coefficients = polyfit(Mny, Bir, 2);

% Print the fitted function
fprintf('The fitted function is: y = %.4fx^2 + %.4fx + %.4f\n', coefficients(1), coefficients(2), coefficients(3));

% Calculate the extreme point
a = coefficients(1);
b = coefficients(2);
if a ~= 0
extreme_point_x = -b / (2 * a);
extreme_point_y = polyval(coefficients, extreme_point_x);
fprintf('The x - coordinate of the extreme point: %.4f\n', extreme_point_x);
fprintf('The y - coordinate of the extreme point: %.4f\n', extreme_point_y);
else
fprintf('The fitted function is linear and has no extreme point.\n');
end

% Generate data for plotting the fitted curve
Mny_fit = linspace(min(Mny), max(Mny), 100);
Bir_fit = polyval(coefficients, Mny_fit);

% Plot the original data points and the fitted curve
```

```
figure;
plot(Mny, Bir, 'o', 'DisplayName', 'Original␣Data');
hold on;
plot(Mny_fit, Bir_fit, '-', 'DisplayName', 'Fitted␣Curve');
if a ~= 0
plot(extreme_point_x, extreme_point_y, 'ro', 'DisplayName', 'Extreme␣Point');
end
xlabel('Annual␣Household␣Income');
ylabel('Number␣of␣Births␣per␣1,000␣Women');
title('Fitting␣of␣Birth␣Rate␣vs.␣Household␣Income');
legend show;
hold off;
```

### 8.3 Quetion2 Related Mcode2

Listing 3: Related MATLAB Code

```
function [fitresult, gof] = createFit(Mny, Bir)
%CREATEFIT(MNY,BIR)

[xData, yData] = prepareCurveData( Mny, Bir );


ft = fittype( 'poly5' );


[fitresult, gof] = fit( xData, yData, ft );


figure( 'Name', 'Fitting' );
h = plot( fitresult, xData, yData );
legend( h, 'Bir␣vs.␣Mny', 'Fitting', 'Location', 'NorthEast', 'Interpreter', 'none' );

xlabel( 'Mny', 'Interpreter', 'none' );
ylabel( 'Bir', 'Interpreter', 'none' );
grid on
```

## 8.4 Quetion2 Related Mcode3

Listing 4: Related MATLAB Code

```matlab
% Read data from Excel file
data = readtable('YearData.xls');

% Extract relevant columns
Year = data.Year
Lab = data.Lab



% Perform polynomial fitting
coefficients = polyfit(Year, Lab, 2);

% Print the fitted function
fprintf('The fitted function is: y = %.4fx^2 + %.4fx + %.4f\n',
    coefficients(1), coefficients(2), coefficients(3));



% Generate data for plotting the fitted curve
Mny_fit = linspace(min(Year), max(Year), 100);
Bir_fit = polyval(coefficients, Mny_fit);

% Plot the original data points and the fitted curve
figure;
plot(Year, Lab, 'o', 'DisplayName', 'Original Data');
hold on;
plot(Mny_fit, Bir_fit, '-', 'DisplayName', 'Fitted Curve');

xlabel('Year', 'FontSize', 12);
ylabel('Labor Force (10000)', 'FontSize', 12);
title('Fitting of Labor Force over Years', 'FontSize', 14);

legend show;
hold off;
```

# 9 Reference

## References

[1] 唐龙妹, 许佳琳, 张军, 洪苹, and 张维宏. 鼓励生育政策的国际比较. 生殖医学杂志, 32(11):1753–1759, 11 2023.

[2] 孔晓明 and 孔星星. 基于人口均衡理论构建生育友好型医院研究. 江苏卫生事业管理, 35(9):1233–1234, 1245, 9 2024.