Finance 514 | Spring 2018
Financial Engineering II | Martin Widdicks

Valuation Project 3

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1. Product for valuation

i. Accelerated Return Securities Based on the Value of the Worst Performing of the S&P 500® Index and the Russell 2000® Index (Two stock indices and averaging feature)

ii. Product details

Maturity date:	11-Sep-23
Underlying indices:	S&P 500® Index (the "SPX Index") and Russell 2000® Index (the "RTY Index")
Principal amount:	$1,000 per security
Payment at maturity:	If the **final average index value** of **each** underlying index **is greater than or equal to** 121% of its respective initial index value: $1,000 + $1,000 × [(index performance factor of the worst performing underlying index – 121%) × 334%] + $415
	If the **final average index value** of **either** underlying index **is less than** 121% of its respective initial index value but the **final average index value** of **neither** underlying index **is less than** its respective initial index value: $1,000 + $1,000 × [(index performance factor of the worst performing underlying index – 100%) × 150%] + $100
	If the **final average index value** of **either** underlying index **is less than** its respective initial index value but the **final average index value** of **neither** underlying index **is less than** 95% of its respective initial index value: $1,000 + $1,000 × [(index performance factor of the worst performing underlying index – 95%) × 200%]
	If the **final average index value** of **either** underlying index **is less than** 95% of its initial index value: ($1,000 × index performance factor of the worst performing underlying index) + $50
Index performance factor:	Final average index value / initial index value
Initial index value:	SPX: 2,614.45 / RTY: 1,512.155
Final average index value:	The arithmetic average on each of the averaging dates
Averaging dates:	Business day during 3-month from and including June 6, 2023 to and including September 6, 2023
Maximum payment	$2,116.40
Pricing date:	4-Apr-18
Original issue date:	9-Apr-18

2. Data collection and parameter selection

All of the data for estimating the product chosen was sourced from Bloomberg and Investing.com. For more specification, please refer to the excel file attached.

i. Volatility: Implied volatilities (20.702% for SPX and 23.259% for RTY) with different moneynesses were used for the valuation.

ii. LIBOR rate: Interpolated LIBOR continuously compounded rate from 04/04/2018 to 09/11/2023, which is 2.74275 % was used for performing the Monte-Carlo simulation and interpolated LIBOR from 04/09/2018 to 09/11/2013, 2.74252%, was used for adjusting the value of product from pricing date to issue date.

iii. Dividend: 2.0420% for SPX and 1.1228% for RTY as continuous dividend rates.

iv. Correlation: 0.849. Historical price data of both SPX and RTY from 04/04/2016 to 04/04/2018 (2 years) were used to estimate the correlation between the two index via calculating continuously compounded rate of returns.

3. Valuation

The valuation of the Accelerated Return Securities Based on the Value of the Worst Performing of the S&P 500® Index and the Russell 2000® Index was constructed under the Monte Carlo method.

i. Given that the chosen product has averaging feature, calculate the business days for the period of averaging to determine the necessary steps for each simulation.

- M = 66 business days
- For each Monte Carlo simulation, there should be M paths to determine the prices of each index at each path.

```
M = np.busday_count(date(2023, 6, 6), date(2023, 9,6))
```

ii. Generate identically, independently distributed standard normal values(ϕ) for calculating each stock path under risk neutral measure.

- We need total $M \times N \times 2$ ϕs for valuing the product.

```
for num in range(num_simulation):
    for i in range(M-1):  # index paths
        phi1 = np.random.standard_normal()
        phi2 = np.random.standard_normal()
        SS[i+1] = SS[i]*np.exp((r - Sdiv - 0.5 * Ssig**2)*dt + Ssig * np.sqrt(dt) * phi1)
        RS[i+1] = RS[i]*np.exp((r - Rdiv - 0.5 * Rsig**2)*dt + Rsig * corr * np.sqrt(dt)
        * phi1 + Rsig * np.sqrt(1-corr**2) * np.sqrt(dt) * phi2)
```

iii. After getting M prices of each index, calculating arithmetic average of them to get the final average index value.

```
avgs[num] = np.average(SS) # final average index values
avgr[num] = np.average(RS)
```

iv. Determine the worst performing underlying index based on the final average index value to finalize payoffs at maturity date.

```
pfs = avgs[num] / SS0
pfr = avgr[num] / RS0
if pfs > pfr:
    wpf = pfr
else:
    wpf = pfs
```

v. Determine the payoffs at maturity in accordance with the given rule.

```
# payoffs
if avgs[num] >= SS0 * 1.21 and avgr[num] >= RS0 * 1.21:
    VT[num] = principal + principal * ((wpf-1.21) * 3.34) + 415
elif SS0 <= avgs[num] < SS0 * 1.21 or RS0 <= avgr[num] < RS0 * 1.21:
    VT[num] = principal + principal * ((wpf-1) * 1.50) + 100
elif SS0*0.95 <= avgs[num] < SS0 or RS0*0.95 <= avgr[num] < RS0:
    VT[num] = principal + principal * ((wpf-0.95) * 2)
elif avgs[num] < SS0*0.95 or avgr[num] < RS0*0.95:
    VT[num] = principal * wpf + 50
```

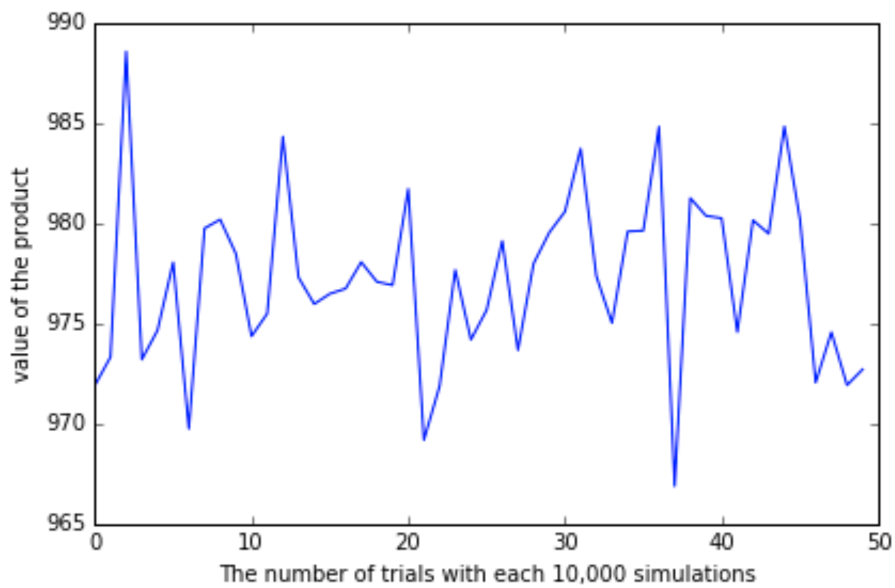
vi. Iterate the steps from ii. to v. for given number of simulation, sum all of the payoffs at maturity, divide it by the number of simulation, discount it using the interpolated LIBOR rate, and, finally adjusting the discounted value from pricing date to issue date.

```
# Final value of the product (with adjustment btwn Issue date and Pricing date)
V0 = np.exp(ro * ((Idate-Pdate).days / 365)) * np.exp(-r * T) * 1/num_simulation * np.sum(VT)
```

4. Discussion

i. Errors

It is hard to determine the error terms in Monte Carlo method since it is a random for each path, which depends on the random draw of ϕ. We can easily see as below that the estimated values of the product of fluctuate randomly (the number of simulation = 10,000, total trials = 50).



ii. Volatility Variation

To see the sensitivity from different volatilities, the number of simulation = 10,000 was used for each simulation. It was expected that there would be no patterns in the value of the product with increasing or decreasing implied volatility since it is based on random values, ϕs. The results confirmed this view as below.

Moneyness	90%	95%	100%	105%	110%
Implied vol (SPX)	21.264	20.691	20.137	19.601	18.963
Implied vol (RTY)	23.825	23.249	22.654	22.089	21.562
Value	987.8492	982.5730	979.4501	984.2386	975.2980

5. Conclusion

The final value is $986.6143 given the implied volatility of 20.702%(SPX) and 23.259%(RTY) with the 10,000 simulations. For the $986.6143, it is quite close to but above $971.30, the estimated value from the issuer, Morgan Stanley. Moreover, it seems that the value of the product ranges from 966.8800 to 988.5752 considering with different implied volatilities by different moneynesses for each underlying asset, SPX and RTY, and a number of 50 trials of the simulations. Though the estimated value of the structured product was close to and below the principal amount, which is a desired result, there was a difficult issue in regards with conducting the Monte Carlo simulation. The convergence of the methods is too slow to determine the exact value of the product with demanding considerable amount of computing power even with taking into account of the effort to use antithetic sampling to improve the convergence to match the distribution of the path.

```
# Antithetic sampling
phia1 = np.random.standard_normal(int(M/2))
phiao1 = -phia1
phi1 = np.concatenate((phia1, phiao1))

phia2 = np.random.standard_normal(int(M/2))
phiao2 = -phia2
phi2 = np.concatenate((phia2, phiao2))
```