



Multiple fleet aircraft schedule recovery following hub closures

Benjamin G. Thengvall^a, Gang Yu^b, Jonathan F. Bard^{c,*}

^a *Department of Mechanical Engineering, Graduate Program in Operations Research, University of Texas, Austin, Texas 78712-1063, USA*

^b *Department of Management Science and Information Systems, Graduate School of Business, University of Texas, Austin, Texas 78712-1175, USA*

^c *Department of Mechanical Engineering, Graduate Program in Operations Research, University of Texas, Austin, Texas 78712-1063, USA*

Received 26 February 1999; received in revised form 3 November 1999; accepted 6 November 1999

Abstract

This paper presents three multi-commodity network-type models for determining a recovery schedule for all aircraft operated by a large carrier following a hub closure. The first is a pure network with side constraints, the second is a generalized network, and the third is a pure network with side constraints in which the time horizon is discretized. Each model allows for cancellations, delays, ferry flights, and substitution between fleets and subfleets. In the first two cases, the objective is to maximize a “profit” function which includes an incentive to maintain as much of the original aircraft routings as possible. In the third case, the objective is to minimize the sum of cancellation and delay costs.

After comparing solution quality and computation times for each of the three models, the first was seen to outperform the others and was singled out for further analysis. Results for a comprehensive set of scenarios are presented along with ideas for continuing work. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Irregular operations; Multi-commodity networks; Multiple fleets; Cancellations; Delays; Flight disruptions

1. Introduction

When faced with a lack of resources, airlines are often unable to fly their published schedule. This is frequently the result of equipment failure, air-traffic-control restrictions, inclement

* Corresponding author. Tel.: +1-512-471-3076; fax: 1-512-232-1489.

E-mail addresses: benji@mail.utexas.edu (B.G. Thengvall), yu@uts.cc.utexas.edu (G. Yu), jbard@mail.utexas.edu (J.F. Bard).

weather, or crew shortages. When problems like these arise, corrective action must be taken in real time in a manner that salvages revenue, placates customers and returns the airline to the original schedule in a timely fashion. This general situation is called the airline irregular operations problem and involves not only aircraft routing, but also crew scheduling, gate assignment and passenger recovery. The more specific problem of rerouting aircraft is termed the aircraft schedule recovery problem.

In this paper we address a subset of the more general problem, namely, determining the best response when a hub is closed. This situation represents the most extreme of foreseeable disruptions and is most often the result of major weather phenomena such as snowstorms, thunderstorms or hurricanes. When a station through which a large number of flights travel is closed for any length of time, major disruptions to an airline's published flight schedule are unavoidable. Intelligently rescheduling aircraft in such situations can save airlines thousands of dollars and minimize the adverse impact on passengers.

The purpose of this paper is to present three multi-commodity network-type models for determining a recovery schedule for all aircraft operated by a large carrier following a hub closure. Each model allows for cancellations, delays, ferry flights, and substitution between fleets and subfleets. After comparing solution quality and computation times for each of the three models, the first is singled out for further analysis. Results for a comprehensive set of scenarios are presented along with ideas for continuing work. AMPL was used to code the model and all solutions were obtained with CPLEX.

The outline of this paper is as follows. Section 2 contains a brief literature review. Section 3 defines the problem and introduces the three approaches. Section 4 explains the scenarios and provides preliminary test results for the three models. Further computational results for the best model are contained in Section 5 along with a discussion of the model's strengths. We conclude with a discussion of the limitations of the proposed approach and several suggestions for future work.

2. Related literature

Many aspects of airline operations have long been the focus of operations research practitioners. Flight and crew scheduling, though, have received most of the attention in the literature. While increased efforts in recent years have been made to solve the irregular operations problem, the body of research in this area is still relatively small.

Teodorovic and Guberinic (1984) were one of the first to model several aspects of the problem. Using a graph construct they developed a solution procedure based on branch and bound to minimize total customer delay. Their work was extended by Teodorovic and Stojkovic (1990) to include cancellations and station curfews.

Jarrah et al. (1993) present a thorough overview of the problem along with two models based on a space-time network: one for delays and the other for cancellations. While delays and cancellations are not addressed simultaneously, the approach was considered practical enough to be implemented by United Airlines (Rakshit et al. 1996).

Yan and Yang (1996) were the first to incorporate flight cancellations, delays and ferry flights in a single model. Using a basic space-time network representation, they present a small case study

using real flight data from China Airlines. The framework introduced was extended by Yan and Lin (1997) to handle station closures and by Yan and Tu (1997) to handle multiple fleets.

Argüello et al. (1997a) present a greedy randomized adaptive search procedure (GRASP) to reconstruct aircraft routings. Argüello et al. (1997b) provide a time-band optimization model to solve the same problem. The formulation is an integral minimum-cost network flow problem with side constraints.

Cao and Kanafani (1997a,b) present a 0–1 quadratic programming (QP) model based on the delay model of Jarrah et al. (1993). They extended the model to include both delays and cancellations simultaneously, as well as the entire network of stations.

Thengvall et al. (2000) extended the work of Yan and Lin (1997), Yan and Tu (1997), Yan and Yang (1996), Yan and Young (1996) to build recovery schedules for aircraft by including an incentive in the formulation to minimize deviation from the original aircraft routings. They present extensive test results for a single fleet. In addition, a rounding heuristic is described that can be used to obtain feasible integer solutions from the LP relaxation of the mixed-integer programming (MIP) formulation.

To date, the work of Argüello et al. (1997a,b), Yan and Lin (1997), Yan and Tu (1997), Yan and Yang (1996), Yan and Young (1996) and Thengvall et al. (2000) represents the most comprehensive and practical approaches to the irregular operations problem. Only Yan and Tu (1997) have addressed the multi-fleet problem in the context of irregular operations. Their case study is relatively small and considers only one plane out of service at a time. The current paper presents three models for solving the multi-fleet aircraft schedule recovery problem during hub closures. The model studied in greatest detail includes an incentive to minimize deviation from the original aircraft routings and is tested using real-world data. The largest problem instances solved in terms of stations, flights and unique aircraft types considered are significantly larger than those found in previous papers.

3. Problem definition and formulations

The problem addressed in this paper is that of rescheduling aircraft over a specified recovery period in response to the closing of a hub. From the time the station closes until it reopens no transient activity is permitted. Thus, given the position of planes at the closure time, the original flight schedule, the time of station closure and reopening, and a time set for recovery, we are interested in finding the “best” assignment of flights to all available aircraft such that once operations resume, all flights can be flown as originally scheduled.

Many factors serve to complicate the problem of finding an optimal or best aircraft rerouting. One is the variety of aircraft operated by most major carriers. Another is the goal of recovering the schedule in a finite time period. Recovery by time t is defined as having the appropriate aircraft in place to fly all flights as scheduled from time t onward. Therefore, a recovery schedule must be built that ensures all aircraft will be at correct stations by certain times to achieve this goal. This constraint is called aircraft balance at the end of the recovery period.

For carriers with multiple equipment types, each fleet and subfleet has different characteristics that limit their use in various situations. For example, certain aircraft may not be allowed into certain stations or may not be approved for flight over water. Different types of aircraft also have

Table 1
Subfleet hierarchies

Fleet	Subfleet	Hierarchy #
B727-200	72L149	90
B727-200	723153	80
B727-200	722153	60
B727-200	72W149	30
B727-200	72S149	20
B727-200	72F000	10

Table 2
Allowable fleet substitutions

Fleet	DC9-30	MD-80	B737-100	B737-200	B737-300	B737-500	B737-700
DC9-30	–	No	Yes	Yes	No	No	No
MD-80	Yes	–	Yes	Yes	No	No	No
B737-100	Yes	No	–	Yes	No	No	No
B737-200	Yes	No	Yes	–	No	No	No
B737-300	Yes	No	Yes	Yes	–	Yes	Yes
B737-500	Yes	No	Yes	Yes	Yes	–	Yes
B737-700	Yes	No	Yes	Yes	Yes	Yes	–

different passenger carrying capacities. The general problem does not decompose into separate problems for each fleet and subfleet because substitution is allowed between some fleets and within fleets according to subfleet hierarchies.

The following general rules were derived from interviews with Continental Airlines personnel. Substitutions within subfleets are allowed according to hierarchy numbers. An aircraft of a given subfleet can substitute for any aircraft in its fleet having a hierarchy number less than or equal to its own. Table 1 shows hierarchy numbers for the B727-200 fleet where a member of subfleet 72L149 can substitute for a member of subfleet 72W149, but not vice versa. Fleet substitutions are described in Table 2 where a ‘yes’ indicates that the row fleet can substitute for the column fleet. A member of fleet MD-80 can substitute for a member of fleet DC9-30, but not vice versa. Fleets not listed in Table 2 are not allowed to substitute for other fleets nor are other fleets allowed to be substituted for them.

The aircraft schedule recovery problem was investigated with three different models. Each is based on a multi-commodity network representation, provides for flight delays and cancellations, and ensures aircraft balance at the end of the recovery period. Balance is maintained at the fleet level. This implies that aircraft of the correct fleet types are located at the correct stations in time to resume all normally scheduled flights following the recovery period. Each model is an MIP. Their unique properties are described below.

3.1. Preference model

This model is derived from the work of Yan and his co-authors (Yan and Yang, 1996; Yan and Young, 1996; Yan and Lin, 1997; Yan and Tu, 1997), and solves the aircraft recovery problem for

multiple fleets with the objective of maximizing a modified “profit” function over the flight schedule during the recovery period. The proposed objective function accounts for passenger revenues, flight delays and cancellations. In addition, it includes an incentive to minimize deviation from the original aircraft routings. The benefits of a particular flight along with an incentive to maintain it as a part of the flight path of a particular aircraft are incorporated in a series of coefficients. The result is an integer multi-commodity network model with side constraints, which is NP-hard due to the addition of side constraints to the otherwise pure network structure (Garey and Johnson, 1979).

The proposed model can be thought of as a collection of simple space-time networks that are connected with a set of flight cover constraints and demand nodes at the fleet level. Each aircraft equipment type (subfleet) has a single space-time network, as illustrated in Fig. 1. Flows on the network are aircraft. The **vertical arcs** represent aircraft on the ground that are waiting for a flight or that have no more flights during the recovery period. In the diagram, all flow is from top to bottom. Arrows are omitted on the ground arcs to avoid clutter. The **sloping diagonal arcs** represent flights from one station to another. These arcs begin at the flight’s scheduled departure time and end at the scheduled arrival time plus the turnaround time for the specific aircraft type. The **turnaround time** is the time it takes to prepare the plane for another flight. **Diagonal flight arcs** are restricted to binary values while vertical ground arcs, which are only required to be nonnegative, will contain integer flow by construction. This can be seen from Fig. 1. Beginning with integer values on the vertical arcs, because only binary flows can enter or leave each station via flight arcs, integrality will always be maintained on the vertical arcs. In some instances, not all flights will be covered. Flight arcs with no flow in the final solution are canceled.

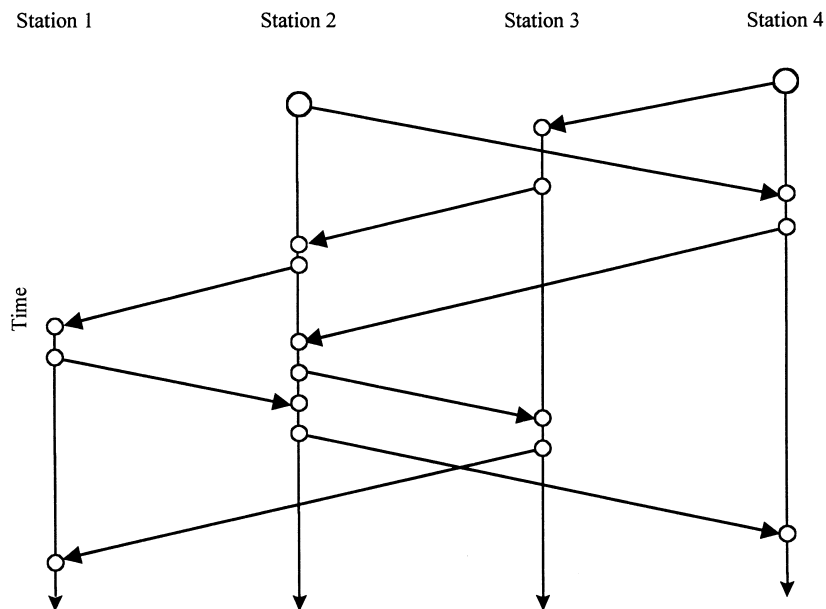


Fig. 1. Space-time network representation.

The two larger nodes in Fig. 1 at stations 2 and 4 are **supply points**. Not all stations need have a supply. The model presented can accommodate recovery periods of arbitrary length. Aircraft are supplied to the model at various times depending on when they become available. If the recovery period begins in the middle of the day, as it often does, supply nodes will contain all aircraft on the ground at that time. Planes in the air can be added to the model when they reach their current destination through intermediate supply nodes. The smaller nodes at each arc endpoint are balance of flow (intermediate) nodes and are present at each flight departure or arrival. Ground arcs allow aircraft to remain at a station indefinitely depending on whether they are waiting to take a flight or are unassigned for the remainder of the recovery period. In the diagram, the vertical axis represents time and the horizontal axis space.

3.1.1. Incorporating delays

To account for delays on a particular flight leg, a series of flight arcs are created to represent the available options for taking the flight at a later time. In Fig. 2, two delay options are shown for each of the four flight legs representing, say, 20 and 60 min delays. To ensure that each leg is flown at most once a cover constraint is added. This side constraint requires that the sum of flows on all arcs (variables) representing the same flight be less than or equal to one. The revenue for delayed flights is adjusted to include a cost per minute delayed. Note that supply and intermediate nodes have been omitted in this and the following figures to reduce the amount of unnecessary detail.

3.1.2. Extra arcs for discouraging deviation in aircraft routings

A common complaint of managers using optimization-based decision support systems is that small changes in input often translate into drastically different solutions (Brown et al.,

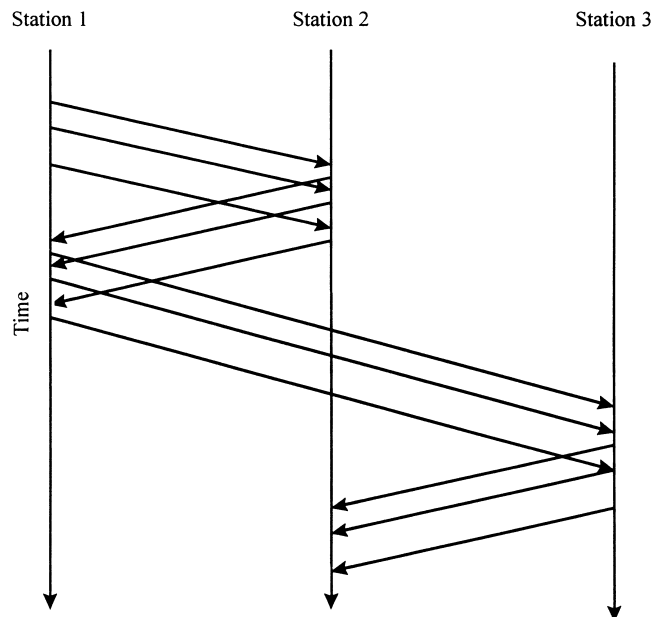


Fig. 2. Incorporating delays.

1997). In the airline industry, many issues are taken into account in the construction of the original flight schedule. Changes to aircraft routings affect crew schedules, gate assignments, scheduled maintenance and passenger connections. Consequently, dealing with irregular operations in a way that encourages minimal deviation to the original aircraft routings is crucial.

To “protect” original flight paths additional flight arcs are incorporated in the general model. This new construct, represented by the circular arc in Fig. 3 will be called a *protection arc*. Protection arcs span a set of contiguous flight legs on an original flight path. The cover constraint introduced for delayed flights must now be expanded to include the extra flight option shown here. The new arc in Fig. 3 corresponds to one plane flying both flight legs, rather than a single new flight. By providing an incentive on the new arc a single plane is encouraged to fly both legs, thereby reducing the attractiveness of flying the two legs with separate aircraft. Incentive values are determined through experimentation and can be adjusted to obtain favorable solution characteristics in terms of the number of cancellations, number of delays, and the amount of deviation to the original aircraft routings.

The use of a protection arc to discourage schedule deviation can be extended to a flight path containing any number of legs. The incentive on an arc that spans three flights should be greater than the incentive on an arc that spans two flight legs together. This is accomplished by adding a constant bonus value for each flight leg covered. Of course, each cover constraint, ensuring that each flight is assigned at most once, must be expanded to include all flight and protection arcs that represent that flight. Additional protection arcs can be added that span any subset of contiguous flight legs originally assigned to an aircraft.

3.1.3. Through-flight considerations

Through flights are mechanisms to serve long-haul markets with one or more intermediate stops, e.g., Boston–Newark–Los Angeles represents a through flight from Boston to Los Angeles. Passengers traveling from Boston to Los Angeles are able to stay on the same plane throughout the trip and often adjust their itinerary to do so. The revenues associated with flight legs that make up through flights are not independent. If one of the legs is canceled, the through flight passengers will not make it to their destination as scheduled. To address this issue, an additional arc is added to the model that goes from the through flight’s origin to its destination. It does not represent a new flight option, but rather one aircraft assigned to all of the flight legs in the through flight. Again, this additional arc must be included in the cover constraint to ensure flights are not duplicated in the final solution. This construct can be applied to through flights with any number of intermediate stops.

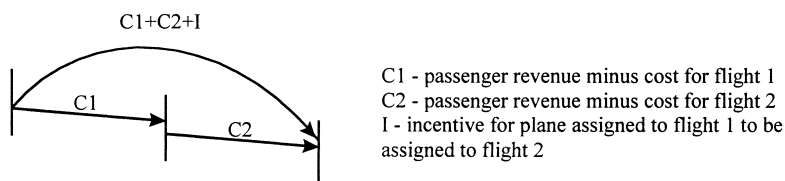


Fig. 3. Protection arcs.

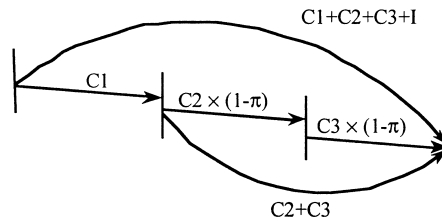


Fig. 4. Inclusion of through-flight arcs.

Assume that the last two of the three flight legs shown in Fig. 4 represent a through flight. Let $0 < \pi < 1$ be the proportion of passengers on the first leg of the through flight that are taking both legs of the through flight. **If either leg is flown individually or if the legs are flown by different aircraft, the revenue gained will be discounted by the factor $(1 - \pi)$.** If the new arc, shown on the bottom, is taken the full revenue will be received for the through flight. Fig. 4 includes the protection arcs from the previous section to demonstrate how the two constructs are used together and how the arcs are assigned revenue. Although many airline information systems track individual passenger connections in real time, including this data in an irregular operations model would make it too unwieldy to solve. However, when a large percentage of passengers are traveling the same set of flight legs a through flight can be designated.

3.1.4. Ferry arcs

It is not always possible to recover the original schedule by a given time using only flights contained in an airline's original schedule. In such cases, ferry flights are necessary to achieve aircraft balance by the end of the recovery period. The model includes three types of ferry flights, as illustrated in Fig. 5. Type I ferries connect each supply point with each demand point for a particular equipment type. Type II ferries go from each supply point to each hub and type III ferries travel from each hub to each demand point. An estimation of airspeed coupled with the distance between stations is used to determine the ending time for a Type II ferry and the starting time for a Type III ferry. Type III ferries arrive "just in time" to meet the expected demand.

The inclusion of Type I ferry arcs ensures feasible flows for the network model under all circumstances. While not desirable, it is always feasible to assign all available aircraft to Type I ferry arcs traveling directly from supply points to demand points.

3.1.5. Considerations for multiple fleets

The network for each equipment type (subfleet) will contain all flights originally scheduled to be flown by that equipment type as well as delayed flights, protection arcs for flight paths, through flights and ferry flights as described above. In addition, the network for each equipment type will also contain all flights, delayed flights and through flights for which that equipment type can substitute according to the fleet substitution and subfleet hierarchy rules described earlier in this section (Tables 1 and 2). Moreover, the cover constraints must be expanded to include these

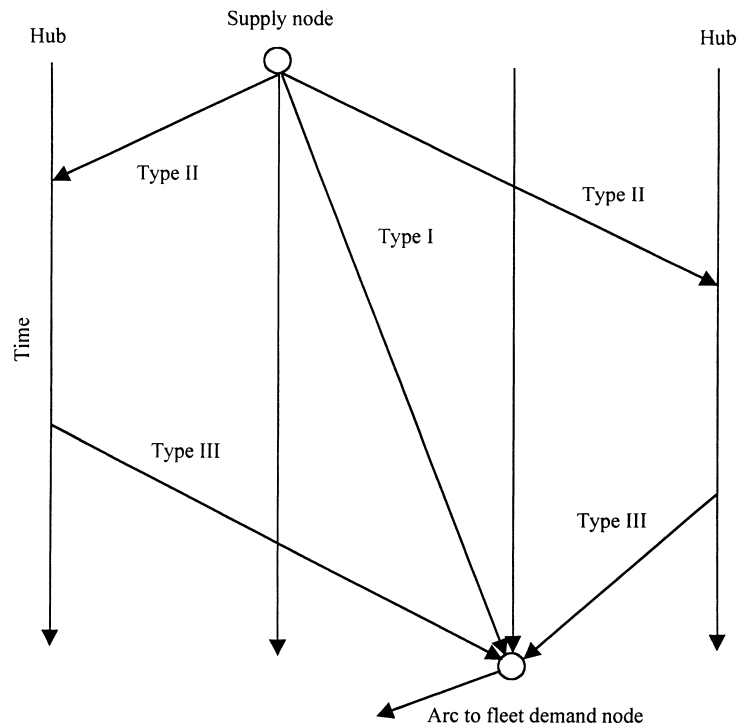


Fig. 5. Types I, II and III ferries.

substitution arcs. Protection and ferry arcs appear only in the simple network associated with their original equipment types and are not involved in any substitutions.

If balance were required at the subfleet level, demand points would appear on each individual network just as the supply points do (Fig. 5). However, end-of-period aircraft balance is only required at the fleet level in our model. Therefore, fleet demand nodes are established to collect flow from the subfleet networks. For each demand point, a sink arc emanates at the demand time and station from each subfleet network that belongs to the fleet represented by that demand (Fig. 5). These arcs terminate at a fleet demand node.

A primary strength of this modeling approach is its ability to generate solutions that reflect changing user preferences. By adjusting the number of delay options, the cost per minute of delaying flights and the bonuses awarded for protecting flights, schedules with different average solution properties can be obtained. (Empirical results demonstrating this property for the case of a single fleet and minor disruptions are given by Thengvall et al. (2000).) Costs for interfleet and intrafleet substitution, as well as a cost for subfleet imbalance, can also be introduced. An airline that is the lone provider in a particular market may prefer lengthy delays to a flight cancellation, while an airline in a competitive market may cancel a flight rather than delay it more than a short time. With simple parameter adjustments, preferences like these can be easily addressed. This adds to the versatility and power of the model. In real-time control, it is essential to provide the decision-maker with a set of diverse, high quality solutions from which he or she can choose the most preferred, depending on the current situation.

*Mathematical model**Sets*

D	fleet demand nodes
E	equipment types (subfleets)
F	flight arcs (including delayed flights)
G	ground arcs
I	intermediate nodes (for flow balance and introduction of supply)
P	protection and through-flight arcs
R	ferry arcs
S	sink arcs
N	unique flight number representing each flight leg
$O(i,e)$	arcs originating at node i for equipment type e
$T(i,e)$	arcs terminating at node i for equipment type e
$F(\eta)$	arcs covering flight η ; $F(\eta) \subset F \cup P$

Parameters

C_{fe}	passenger revenue minus flight cost minus delay cost minus substitution penalty for flight arc f flown by equipment type e
C_p	for protection arc p – passenger revenues minus flight costs minus delay costs of all flights covered plus the appropriate incentive value; for through-flight arc p – passenger revenues minus flight costs minus delay costs of all flights covered
C_r	cost for ferry flight r
C_{se}	cost of subfleet imbalance on arc s for equipment type e
B_{ie}	balance of aircraft at intermediate node i for equipment type e (positive integer if aircraft supplied at node i , 0 otherwise)
B_d	fleet demand for aircraft at demand node d
U_{ge}	upper bound on ground arc g for equipment type e

Variables

u_r	binary flow on ferry arc r
v_{se}	integer flow on sink arc s for equipment type e
x_{fe}	binary flow on flight arc f for equipment type e
y_p	binary flow on protection or through-flight arc p
z_{ge}	flow on ground arc g for equipment type e

Maximize

$$\sum_{\substack{f \in F \\ e \in E}} C_{fe} x_{fe} + \sum_{p \in P} C_p y_p - \sum_{r \in R} C_r u_r - \sum_{\substack{s \in S \\ e \in E}} C_{se} v_{se} \quad (1a)$$

subject to

$$\begin{aligned} & \sum_{\substack{g \in G \cap O(i,e) \\ e \in E}} z_{ge} - \sum_{\substack{g \in G \cap T(i,e) \\ e \in E}} z_{ge} + \sum_{\substack{f \in F \cap O(i,e) \\ e \in E}} x_{fe} - \sum_{\substack{f \in F \cap T(i,e) \\ e \in E}} x_{fe} + \sum_{p \in P \cap O(i,e)} y_p - \sum_{p \in P \cap T(i,e)} y_p + \sum_{r \in R \cap O(i,e)} u_r \\ & - \sum_{r \in R \cap T(i,e)} u_r + \sum_{s \in S \cap O(i,e)} v_{se} = B_{ie} \quad \forall i \in I \quad \forall e \in E, \end{aligned} \quad (1b)$$

$$\sum_{\substack{s \in S \cap T(d,e) \\ e \in E}} v_{se} = B_d \quad \forall d \in D, \quad (1c)$$

$$\sum_{\substack{f \in F \cap F(\eta) \\ e \in E}} x_{fe} + \sum_{p \in P \cap F(\eta)} y_p \leq 1 \quad \forall \eta \in N, \quad (1d)$$

$$u_r \in \{0, 1\} \quad \forall r \in R, \quad (1e)$$

$$v_{se} \geq 0 \text{ and integer} \quad \forall s \in S \quad \forall e \in E, \quad (1f)$$

$$x_{fe} \in \{0, 1\} \quad \forall f \in F \quad \forall e \in E, \quad (1g)$$

$$y_p \in \{0, 1\} \quad \forall p \in P, \quad (1h)$$

$$0 \leq z_{ge} \leq U_{ge} \quad \forall g \in G \quad \forall e \in E. \quad (1i)$$

The objective function (1a) maximizes the total profit associated with the recovery period schedule. The first term captures revenue, delay cost and substitution cost on individual flight arcs. The second term duplicates these figures for protection and through-flight arcs. Recall that the protection arcs include an additional incentive value. The third and fourth terms account for ferry and subfleet imbalance costs. At the end of the period, subfleet imbalance may occur because we only require fleet balance. A sufficiently large penalty value C_{se} is assigned to ensure an acceptable amount of subfleet imbalance.

Flow balance at intermediate nodes on each subfleet network is maintained by constraint (1b). The aircraft balance constraint (1c) accounts for the flow to fleet demand nodes at the end of the recovery period. Eq. (1d), the flight cover constraint, assures that at most one arc associated with a particular flight receives flow. All ferry arcs (1e), flight arcs (1g) and protection arcs (1h) are binary variables. Sink arcs (1f) are integer variables. Ground arcs (1i) are required only to be nonnegative but will be integral in any feasible solution as previously explained. Note that if a flight is canceled, no benefit is received or cost incurred. To account explicitly for cancellations all that is necessary is to introduce a slack variable, s_η , in Eq. (1d), a corresponding cost coefficient, C_η , and an additional term in the objective function of the form $-\Sigma_\eta C_\eta s_\eta$.

The addition of incentive values and delay costs makes the objective function artificial. Including these factors, though, allows us to weight the arcs so that the accompanying solution has favorable properties in terms of the number of cancellations, number of delays, and the amount of deviation to the original aircraft routings. Through parametric analysis, we are then able to

generate a wide range of solutions as a function of the relative importance of these criteria, given the current operating conditions.

3.2. Generalized network preference model

The second preference model was motivated by the idea that a generalized network model has unique structure that can be exploited using specialized network algorithms to solve more quickly than a general linear program. When solving mixed-integer programs, many relaxed LP subproblems need to be solved. If each of these subproblems was solved more quickly this could result in a significant time reduction in the solution process. Our second model represents the same situation as the first model, but introduces new network modeling constructs that take the place of the cover constraints (1d) in model (1).

In this formulation, every original flight arc and delayed flight arc is split into three components, as illustrated in Fig. 6. The first component is the same as the original flight arc with the exception that it ends at a new intermediate node instead of the flight's destination station. Two new arcs originate at this new intermediate node, each having an upper bound of 0.5 and a gain (or multiplier) of 2. One arc terminates at the flight's destination station and the other terminates at a new flight sink node. Thus the flow on the original flight arc is split into two parts. The first part continues to flow through the network allowing the aircraft to take additional flights, while the second goes directly to the flight sink node to ensure flights are not repeated. As at most one unit of flow is allowed to enter the flight sink node, each flight in the original schedule can only be flown once, thus accomplishing what the side constraint did in the original formulation.

Protection and through-flight arcs are handled in a similar fashion. The original arc is split into the number of legs covered by the original arc plus two new arcs. The multipliers on each arc flowing out of the new intermediate node are set to the number of legs covered by the original arc

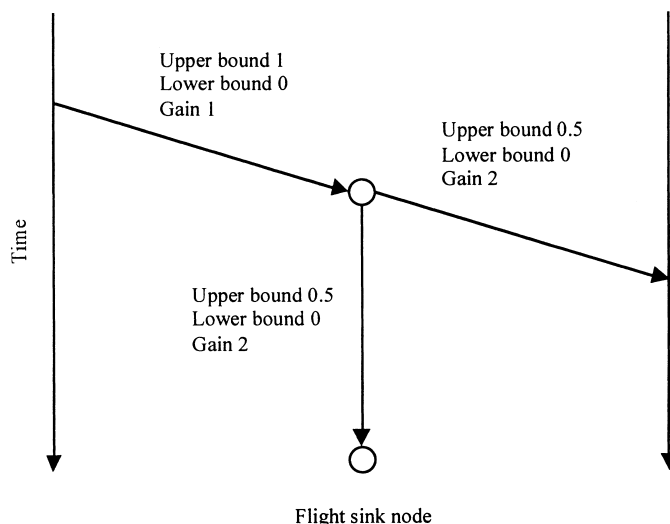


Fig. 6. Generalized network arcs.

and each arc has an upper bound equal to the reciprocal of the multiplier assigned. An arc goes from the new node to the flight sink node for each flight covered by the original arc. For each flight that an arc represents, whether a flight arc, protection arc or through-flight arc, there is a split arc that goes to the flight sink node for that flight. This approach allows us to drop the side constraints from the original preference model, leaving a generalized network model.

After formulating this new model, however, an easy to implement generalized network code could not be located. In addition, initial testing showed that the LP relaxation of this formulation provided very poor upper bounds which, in turn, led to very long branch and bound runtimes (some scenarios were stopped after 10 h). If a generalized network code had been available, it would have needed to be much faster than the standard dual or barrier algorithms to have made this formulation attractive. To achieve better bounds, and in so doing speed solution time, cuts were added to the model. These cuts are side constraints enforcing equal flow on all arcs that originate at the new intermediate nodes. For example, for a flight arc, the new arc going to the flight sink node must be equal to the new arc going to the flight's destination station. For a protection arc, the new arc going to the final protected flight's destination must be equal to each of the new arcs that split off to the appropriate flight sink nodes. This set of cuts dramatically improved the upper bound achieved from the LP relaxation and in turn the overall branch and bound solution times. All results in the comparative portion of Section 4 concerning the generalized network preference model correspond to this adaptation.

3.3. *Time band model*

The third formulation is a version of the model developed by Argüello et al. (1997b) extended to include multiple fleets. In the time band formulation, all station activity is aggregated into discrete time bands. For example, with a 30-min time band setting, all arrivals or departures between 9:00 and 9:29 at a particular station depart from or arrive at a single node. Time bands of 30 min were tested. Unlike the first two models, there are no ground arcs, protection arcs or through-flight arcs in this formulation. Flight arcs carry flow between stations and termination arcs are used to fulfill demands at sink nodes.

The original time band model was adapted for multiple fleets in a manner similar to that described for the preference model. Substitution was allowed between and within fleets and ferry arcs were added for increased flexibility.

4. Testing and preliminary results

Data for testing was provided by Continental Airlines which operates three hubs: IAH (Houston), EWR (Newark) and CLE (Cleveland). For the given schedule, 49% of the flights have IAH as their origin or destination, 35% have EWR as their origin or destination and 14% have CLE as their origin or destination. The data spans 2 1/2 days and includes 332 active aircraft from 12 different fleets. Each fleet has from one to six subfleets for a total of 28 different types of aircraft. The schedule includes 2921 flights between 149 domestic and international locations.

A test set consisting of 9 scenarios was developed to compare the three formulations. Only output for the Houston and Newark hubs is presented. The 9 scenarios are defined by the length

of the closure and a recovery length (see below). All closures begin at 10 a.m. The first instance, for example, represents a 2-h down time and a recovery length of 4 h. No flights can enter or leave between 10 a.m. and 12 p.m., and all operations must be back on schedule by 2 p.m.

Closure length (h)	Recovery options (h)
2	4, 6
6	8, 11, 14
10	12, 16, 20, 24

These scenarios provided 18 instances, 9 Houston closings and 9 Newark closings, on which to compare the performance of the proposed models. All problems were solved using CPLEX's MIP solver beginning with the barrier algorithm and switching to the dual simplex algorithm during

Table 3
Solution times (s) and problem sizes for Models 1–3

IAH (Houston)					
Scenario	AMPL	CPLEX	Rows	Columns	Nonzeros
(a) For Model 1					
120–240	20.5	20.9	4581	10,823	24,749
120–360	33.7	61.8	7934	18,997	44,752
360–480	59.1	267.7	13,980	30,508	74,235
360–660	111.2	480.1	21,759	46,226	115,233
360–840	116.7	513.7	22,671	47,932	119,663
600–720	137.8	400.0	19,273	40,854	100,817
600–960	124.7	474.6	19,589	41,419	102,476
600–1200	146.5	519.6	20,731	43,372	107,949
600–1440	348.6	1489.8	30,243	61,843	157,269
(b) For Model 2					
120–240	21.0	26.1	4581	10,823	24,749
120–360	27.6	82.9	7931	18,867	44,238
360–480	61.1	370.5	13,959	30,040	72,297
360–660	88.4	669.9	21,726	45,091	110,285
360–840	89.0	808.7	22,634	46,716	114,336
600–720	90.5	641.7	19,167	39,786	961,58
600–960	99.2	573.9	19,476	40,289	975,34
600–1200	157.6	681.3	20,620	42,059	102,025
600–1440	452.8	2331.9	30,007	59,455	145,632
(c) For Model 3					
120–240	9.39	2.73	826	6003	13,295
120–360	12.17	6.54	1628	11,052	26,552
360–480	17.90	20.14	2488	17,874	47,212
360–660	28.19	163.79	5336	41,024	114,045
360–840	38.61	390.77	7624	56,486	159,553
600–720	30.89	346.87	5423	45,159	125,651
600–960	46.26	823.69	9530	71,385	203,226
600–1200	69.27	977.05	12,380	85,819	246,390
600–1440	112.29	11,802.85	18,291	141,601	415,205

Table 4
Percent canceled and delayed for each model

IAH (Houston)						
Scenario	Model 1		Model 2		Model 3	
	Canceled	Delayed	Canceled	Delayed	Canceled	Delayed
120–240	19	3	18	2	31	8
120–360	18	2	17	3	26	8
360–480	26	10	27	9	32	16
360–660	21	10	22	9	26	12
360–840	20	10	22	8	18	11
600–720	36	9	35	9	36	11
600–960	35	9	35	8	30	12
600–1200	33	8	34	8	30	12
600–1440	27	5	27	4	26	10

branch and bound. The stopping criterion for the search tree was set to 1% of optimality. A Pentium 200 with 128 Mb of RAM was used for computations.

Solution times and problem sizes for the three models are reported in Table 3 (a, b and c) for the instances in which Houston was closed. Problem sizes are reported following AMPL and CPLEX preprocessing and are thus smaller than originally formulated. This preprocessing step explains the similar problem sizes displayed for models 1 (preference model) and 2 (generalized network preference model). Over the nine problem instances, on average 22% of the variables in model 1 are binary, while all of the variables in model 2 are continuous, and 85% of the variables in model 3 are binary. In each model, there is a side constraint for each flight in the recovery period. The number of flights for each instance is reported in Table 6. In addition, model 2 has a cut added for each flight arc and each protection arc. All other constraints are node balance constraints. Solution times are divided into two parts, construction time (s) in AMPL and solution time (s) using CPLEX. Solution times and problem sizes for the Newark hub are similar and may be obtained from the authors.

Models 1 and 2 have very similar characteristics in terms of model size and the percentage of flights canceled and delayed. However, solution times for model 1 are shorter in every instance than the solution times for model 2. Model 3 (time band model) solves very quickly for the shorter closure times, but is much slower for the instances with the largest closure length. In addition, the percentage of delays generated by model 3 is greater than the percentage generated by the other two models in all cases. This last point can be seen in Table 4 where the percentage of flights canceled and delayed are listed for each model and each scenario. Following this testing, a decision was made to pursue the preference model.

5. Additional results for model 1

Table 5 lists the statistics collected in the further analysis of model 1. Tables 6–9 show results from closing Continental's Houston hub. Similar results were obtained for Newark and Cleveland. Because the latter hubs have less traffic, the accompanying solutions exhibited less disruption to the original schedule.

Table 5
Key to statistics

Flights	Number of flights in the model
Must alter	Flights that cannot be flown as scheduled due to the station closure, i.e., flights scheduled to arrive or depart during the period the station is closed
Canceled	Number of flights canceled in solution
Model delays	Number of flights delayed in the model solution
True delays	Number of delays in post-processed solution
Percent of canceled to or from closed station	Percent of flights canceled whose origin or destination is the closed station
Number of must alter canceled	Number of flights canceled that cannot be flown as scheduled due to the station closure
Percent of must alter canceled	Percent of flights canceled that cannot be flown as scheduled due to the station closure
Percent of delayed to or from closed station	Percent of flights delayed whose origin or destination is the closed station
Number of must alter delayed	Number of flights delayed that cannot be flown as scheduled due to the station closure
Percent of must alter delayed	Percent of flights delayed that cannot be flown as scheduled due to the station closure
Interfleet	Number of substitutions between fleets
Intrafleet	Number of substitutions within fleets
Imbalance	Number of cases of subfleet imbalance at a demand point
Type I ferries	Number of type I ferries in the solution
Types II and III	Number of types II and III ferries in the solution
Swaps	Number of flight legs assigned to aircraft other than the one to which it was originally scheduled routes
Percent correct	Percent of aircraft that get to their original destination on time
Percent unaltered	Percent of aircraft that fly their originally scheduled routes
Through flights	Number of through flights in the model
Broken throughs	Number of through flights that are split and flown by different aircraft or are partially flown

Table 6
General statistics

Scenario	Flights	Must alter	Canceled	Model delays	True delays
120–240	343	57	58	32	25
120–360	534	57	47	42	35
360–480	729	248	252	60	42
360–660	945	248	267	64	35
360–840	983	248	265	71	41
600–720	976	404	411	79	51
600–960	997	404	415	77	52
600–1200	1056	404	420	74	50
600–1440	1434	404	446	66	46

The model solutions do not alter many more flights than are required by the closure. This can be seen by comparing ‘must alter’ with the sum of ‘canceled’ and ‘true delays’ (Table 6). On average the model solution alters 1.25 times the number of flights it must alter when IAH is closed.

Table 7
Detailed cancellation and delay statistics

Scenario	Percent of canceled to or from closed station	Number/percent of must alter canceled	Percent of delayed to or from closed station	Number/percent of must alter delayed
120–240	91	30/53	97	27/47
120–360	91	26/46	98	31/54
360–480	96	200/81	88	48/19
360–660	96	207/83	86	41/17
360–840	95	204/82	86	44/18
600–720	94	353/87	87	51/13
600–960	94	354/88	87	50/12
600–1200	94	358/89	86	46/11
600–1440	96	365/90	91	39/10

Table 8
Miscellaneous statistics

Scenario	Interfleet	Intrafleet	Imbalance	Type I ferries	Types II and III
120–240	14	4	9	3	8
120–360	13	4	4	0	1
360–480	36	10	22	0	6
360–660	42	12	8	0	3
360–840	46	9	11	0	1
600–720	39	7	12	5	14
600–960	45	8	19	4	11
600–1200	43	7	16	4	10
600–1440	70	11	13	1	4

Table 9
Miscellaneous statistics

Scenario	Swaps	Percent correct	Percent unaltered	Through flights	Broken throughs
120–240	28	81	80	24	7
120–360	45	88	84	48	8
360–480	116	65	55	93	14
360–660	188	64	58	146	18
360–840	212	62	58	157	17
600–720	154	53	46	156	26
600–960	152	54	47	158	26
600–1200	174	52	45	160	24
600–1440	341	56	46	220	35

For example, in the first scenario 57 flights had to be canceled or delayed due to the hub closure. In total 58 were canceled and 25 were delayed in the recovery schedule. The model solution altered 1.46 times the number of flights it had to alter. In addition, the majority of the flights altered are flights that originate or terminate at the closed station. When IAH is closed, 94% of cancellations and 90% of delays are flights with IAH as an endpoint (Table 7).

Notice that there are two types of delays: ‘model delays’ and ‘true delays’ (Table 6). This results from the way delays are structured in the preference model. Recall that only finite delay lengths are allowed. Consequently, the times associated with ‘model delays’ are nearly always overstated. For example, a flight may only need to be delayed 12 min, but if the only options in the model are for 10, 30 and 60 min delays, a 30 min delay will be chosen. This results in an 18 min overstatement of delay. Through post-processing, in which flights are assigned to each available aircraft, an accurate picture of the delays in the recovery schedule can be obtained. The actual delays found in the post-processed solution are referred to as ‘true delays’.

Also illustrated in these tables is one of this model’s most compelling features. Namely, the protection of original aircraft routings when possible. This can be seen by looking at ‘percent unaltered’ (Table 9) which counts the number of aircraft that fly their originally scheduled routes. A slightly more relaxed measure is ‘percent correct’ (Table 9), which records the number of aircraft that get to their originally scheduled destination on time. In the shorter hub closure scenarios at least four out of five planes fly their original routes, and even for the longest closure length nearly half of the aircraft fly their originally scheduled routes. As changes to aircraft routings affect crew schedules, gate assignments, scheduled maintenance and passenger connections, minimizing deviation from the original schedule is an important consideration during major disruptions.

The quality of solutions for a given closure length vary only slightly with the different recovery length scenarios. This was true with few exceptions at all three hubs, and can be seen for the Houston hub by comparing ‘canceled’, ‘model delays’, ‘percent correct’ and ‘percent unaltered’ (Tables 6 and 9) for the same closure length across the different recovery lengths. However, the number of ferry flights flown tends to decrease with increased recovery lengths (Table 8).

The ability to substitute aircraft between fleets and within fleets is also seen to be a valuable contribution of the model (Table 8). In every scenario, many flights (columns ‘interfleet’ and ‘intrafleet’) are flown by fleets and subfleets different from those originally scheduled. As tested, no costs were included for interfleet or intrafleet substitutions. If less substitution is desirable, though, a penalty cost could be added.

When examining the above results one must realize that the solution characteristics found in these tables are parameter dependent. This is especially true in scenarios with longer recovery periods. For example, when the cost per minute for delays is reduced, solutions show an increase in the number of delays and a decrease in the number of cancellations. In Thengvall et al. (2000), the ability to adjust solutions to incorporate user preferences through parameter manipulation is explored in detail for the single fleet case. Similar results demonstrating how the solution characteristics change for the preference model developed here when bonus and delay costs are adjusted can be obtained from the authors.

6. Summary and conclusions

To solve the aircraft schedule recovery problem three multi-commodity network formulations were proposed. After an initial comparison of their solution times and quality, the preference model was chosen for further study. This model provided high-quality solutions in a reasonable amount of time. In addition to the model, an algorithm was developed to assign flights to all

available aircraft. Implementation details can be found in Thengvall (1999). Combining the model and the algorithm, a user has the tools to generate a complete aircraft recovery schedule when faced with a hub closure.

Maintenance issues have not been taken into account in the assignment of aircraft, but they can be addressed in one of the following ways. Aircraft with imminent maintenance requirements could be treated as another type of equipment. Each special case would add another commodity to the model. A simpler approach when feasible is to fix the route of any plane requiring maintenance, i.e., do not allow the model to alter that route.

An interesting extension of this work would be to shrink the preference model through preprocessing in a way that maintained the solution characteristics of the full-scale model. As evidenced in Table 7, almost all disrupted flights have their origin or termination at the closed station. Knowing this, many flights or perhaps entire flight paths could be placed in the solution without including them in the model. By studying the properties of full-scale solutions, many decisions would be obvious and could be made without inclusion in the model. Preprocessing in this manner, based on the characteristics of these solutions, could shorten computation times even further.

Acknowledgements

The authors would like to express their appreciation to CALEB Technologies Corp. for their sponsorship of this research. We would also like to thank Continental Airlines for providing the data. It should also be noted that valuable comments and suggestions were provided by the referees and the associate editor.

References

- Argüello, M.F., Bard, J.F., Yu, G., 1997a. A GRASP for aircraft routing in response to groundings and delays. *Journal on Combinatorial Optimization* 5, 211–228.
- Argüello, M.F., Bard, J.F., Yu, G., 1997b. Models and methods for managing airline irregular operations aircraft routing. In: Yu, G. (Ed.), *Operations Research in the Airline Industry*. Kluwer Academic Publishers, Boston, pp. 1–45.
- Brown, G.G., Dell, R.F., Wood, R.K., 1997. Optimization and persistence. *Interfaces* 27, 15–37.
- Cao, J., Kanafani, A., 1997a. Real-time decision support for integration of airline flight cancellations and delays, part I: mathematical formulations. *Transportation Planning and Technology* 20, 183–199.
- Cao, J., Kanafani, A., 1997b. Real-time decision support for integration of airline flight cancellations and delays, part II: algorithms and computational experiments. *Transportation Planning and Technology* 20, 201–217.
- Garey, M.R., Johnson, D.S., 1979. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. Freeman, New York.
- Jarrah, A.I.Z., Yu, G., Krishnamurthy, N., Rakshit, A., 1993. A decision support framework for airline flight cancellations and delays. *Transportation Science* 27, 266–280.
- Rakshit, A., Krishnamurthy, N., Yu, G., 1996. Systems operations advisor: a real-time decision support system for managing airline operations at United Airlines. *Interfaces* 26, 50–58.
- Teodorovic, D., Guberinic, S., 1984. Optimal dispatching strategy on an airline network after a schedule perturbation. *European Journal of Operational Research* 15, 178–182.

- Teodorovic, D., Stojkovic, G., 1990. Model for operational daily airline scheduling. *Transportation Planning and Technology* 14, 273–285.
- Thengvall, B.G., 1999. Models and Solution Techniques for the Aircraft Schedule Recovery Problem. Doctoral Dissertation, Graduate Program in Operations Research and Industrial Engineering, University of Texas, Austin.
- Thengvall, B.G., Bard, J.F., Yu, G., 2000. Balancing user preferences for aircraft schedule recovery during irregular operations. *IIE Transactions on Operations Engineering* 32 (3).
- Yan, S., Lin, C., 1997. Airline scheduling for the temporary closure of airports. *Transportation Science* 31, 72–82.
- Yan, S., Tu, Y., 1997. Multifleet routing and multistop flight scheduling for schedule perturbation. *European Journal of Operational Research* 103, 155–169.
- Yan, S., Yang, D., 1996. A decision support framework for handling schedule perturbations. *Transportation Research. Part B: Methodology* 30, 405–419.
- Yan, S., Young, H., 1996. A decision support framework for multi-fleet routing and multi-stop flight scheduling. *Transportation Research. Part A: Policy and Planning* 30, 379–398.