HW 4

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1. (a) Since they didn't regularize their data at first.

They got different answer.

I assume the difference is caused by the value of Joe's GDP is relative much higher then other features. Therefore, it contributes most part of the data's characteristics which leads to the result that Joe's first eigenvector explained 99.9%.

(b) First, regularize their data.

$+\beta \quad +\beta \cdot \beta$
$\cdot \beta$
$-\alpha \quad \cdot \alpha \cdot \alpha$
$\alpha^2 \quad \alpha^2$
$-\alpha$
$\alpha^2 \quad \alpha^2$

2. (a) $\bar{x}_1 = 0, \quad \bar{x}_2 = 0 \qquad X = \begin{bmatrix} \alpha & -\alpha & -\beta & \beta \\ \alpha & -\alpha & \beta & -\beta \end{bmatrix}$

$$\operatorname{cov}(X) = \frac{1}{4-1}\left([x_1]^T[x_1] + [x_2]^T[x_2] + \cdots + [x_4]^T[x_4]\right)$$

$$= \frac{1}{3}\begin{bmatrix} \alpha^2+\alpha^2+\beta^2+\beta^2 & \alpha^2+\alpha^2-\beta^2-\beta^2 \\ \alpha^2+\alpha^2-\beta^2-\beta^2 & \alpha^2+\alpha^2+\beta^2+\beta^2 \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \alpha^2+\beta^2 & \alpha^2-\beta^2 \\ \alpha^2-\beta^2 & \alpha^2+\beta^2 \end{bmatrix} \#$$

(b) let $\operatorname{cov}(X) = A$, $\Rightarrow \det(A-\lambda I) = 0$

$$A - \lambda I = \begin{bmatrix} \frac{2\alpha^2+2\beta^2}{3} - \lambda & \frac{2\alpha^2-2\beta^2}{3} \\ \frac{2\alpha^2-2\beta^2}{3} & \frac{2\alpha^2+2\beta^2}{3} - \lambda \end{bmatrix}, \quad \det(A-\lambda I) = \left(\frac{2\alpha^2+2\beta^2}{3}\right)^2 - 2\left(\frac{2\alpha^2+2\beta^2}{3}\right)\cdot\lambda + \lambda^2 - \left(\frac{2\alpha^2-2\beta^2}{3}\right)^2$$

$$= \lambda^2 - \frac{4}{3}(\alpha^2+\beta^2)\lambda + \frac{4}{9}(\alpha^2+\beta^2)^2$$

$$= \lambda^2 - \frac{4}{3}(\alpha^2+\beta^2)\lambda + \frac{16}{9}(\alpha^2\beta^2)$$

$$\det(A) = \left(\lambda - \frac{4}{3}\alpha^2\right)\left(\lambda - \frac{4}{3}\beta^2\right) = 0$$

$\lambda = \frac{4}{3}\alpha^2$ or $\frac{4}{3}\beta^2 \#$ $\Rightarrow$ eigenvalue rise accordingly with α and β

- eigenvector e

$$Ae = \lambda e \Rightarrow (A - \lambda I)e = 0$$

if $\lambda = \frac{4}{3}\alpha^2$:

$$\begin{bmatrix} \frac{2\alpha^2+2\beta^2}{3} - \frac{4}{3}\alpha^2 & \frac{2\alpha^2-2\beta^2}{3} \\ \frac{2\alpha^2-2\beta^2}{3} & \frac{2\alpha^2+2\beta^2}{3} - \frac{4}{3}\alpha^2 \end{bmatrix} = \begin{bmatrix} \frac{-2\alpha^2+2\beta^2}{3} & \frac{2\alpha^2-2\beta^2}{3} \\ \frac{2\alpha^2-2\beta^2}{3} & \frac{-2\alpha^2+2\beta^2}{3} \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\frac{2}{3}(-\alpha^2+\beta^2)e_1 + \frac{2}{3}(\alpha^2-\beta^2)e_2 = 0$$

$$\frac{2}{3}(-\alpha^2+\beta^2)e_1 = \frac{2}{3}(-\alpha^2+\beta^2)e_2, \quad e_1 = e_2 = 1 \quad e = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \#$$

if $\lambda = \frac{2}{3}\beta^2$:

$$\begin{bmatrix} \frac{2\alpha^2+2\beta^2}{3} - \frac{4}{3}\beta^2 & \frac{2\alpha^2-2\beta^2}{3} \\ \frac{2\alpha^2-2\beta^2}{3} & \frac{2\alpha^2+2\beta^2}{3} - \frac{4}{3}\beta^2 \end{bmatrix} = \begin{bmatrix} \frac{2\alpha^2-2\beta^2}{3} & \frac{2\alpha^2-2\beta^2}{3} \\ \frac{2\alpha^2-2\beta^2}{3} & \frac{2\alpha^2-2\beta^2}{3} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

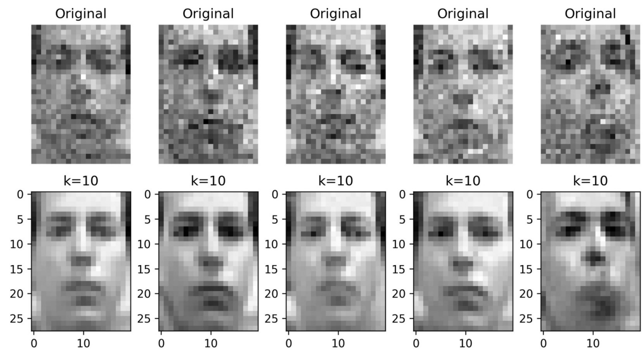
$$\frac{2}{3}(\alpha^2-\beta^2)e_1 + \frac{2}{3}(\alpha^2-\beta^2)e_2 = 0$$

$$\frac{2}{3}(\alpha^2-\beta^2)e_1 = -\frac{2}{3}(\alpha^2-\beta^2)e_2 \;,\; e_1 = -e_2 = 1 \;,\; e = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$ #

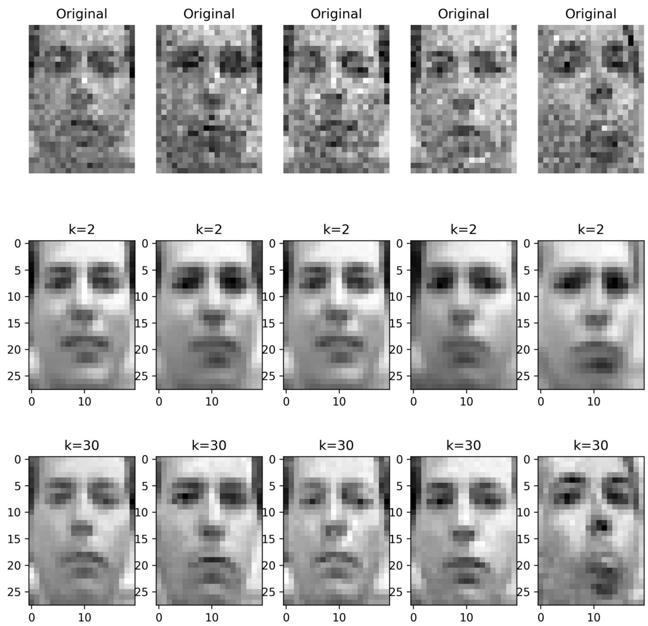
$\exists$ eigenvectors do not be affected by α or β.

3.

(a)

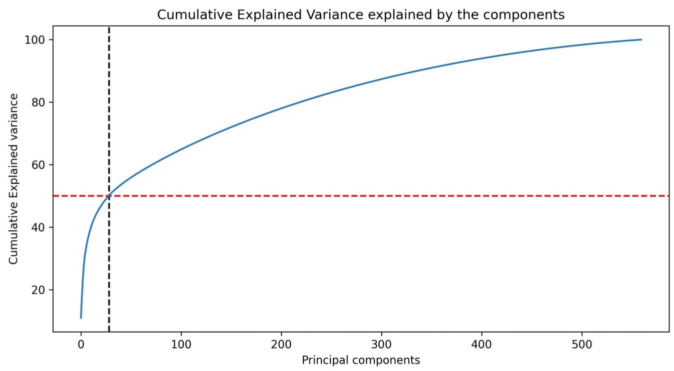


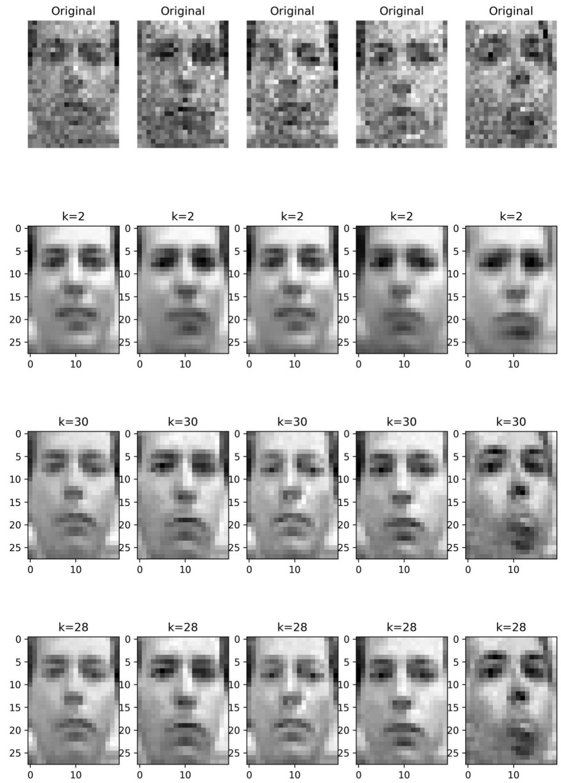
(b)



(c)

I choose 50%





28 components