Simulation and Modeling Coursework 1 Report

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Question 1

When *k* cores are either using or queued on, the lock, corresponding arrival rate is: $\lambda_k = \frac{n-k}{t_x}$, for$(0 \leq k \leq n-1)$

As described in the coursework specification, average cache-update time is proportional to the half of the total number of cores queued on the lock. Then we have the completion rate (output rate): $r_k = \frac{1}{t_c + \frac{k-1}{2} t_u}$, for$(1 \leq k \leq n)$.

Question 2

Transition from state *k* is equal to the transition into the state *k* when the system is at the equilibrium. Then the general relationship (closed-form) between state *k* and state *k+1* is: $p_k \lambda_k = p_{k+1} r_{k+1}$. Therefore:

$$p_k = p_0 \prod_{k=1}^{n} \frac{\lambda_{k-1}}{r_k} = p_0 \left(\frac{n!}{(n-k)!\, t_x^k} \prod_{k=1}^{n} (t_c + \frac{k-1}{2} t_u)^k \right)$$

which gives the general solution for state *k* in terms of probability of state 0. Since the total probability of all states is 1, p_0 can be calculated: $p_0 = \frac{1}{1+\sum_{i=1}^{n} \prod_{k=1}^{i} \frac{\lambda_{k-1}}{r_k}} = \frac{1}{1+\sum_{i=1}^{n} \frac{n!}{(n-k)!\, t_x^k} \prod_{k=1}^{n} (t_c + \frac{k-1}{2} t_u)^k}$

Total population *N* (the mean number of cores either using or waiting for the lock) is: $N = 0 \times p_0 + \sum_{k=1}^{n} k\, p_k$

By definition, throughput is the product of completion rate multiplied by the probability of each state, which is also equal to the product of the arrival rate multiplied by corresponding probabilities: $\tau = \sum_{k=0}^{n-1} p_k \lambda_k = \sum_{k=1}^{n} p_k r_k$

According to the Little's Law: $W = \frac{N}{\tau}$. Hence, since t_c is service time, queuing time W_Q should be: $W_Q = W - t_c$

Question 3

From the coursework specification, "speed-up" can be expressed by: $\text{Speedup} = n - N$

Figure 1.1 and Figure 1.2 show the queuing time and speed-up against number of cores with different t_x.

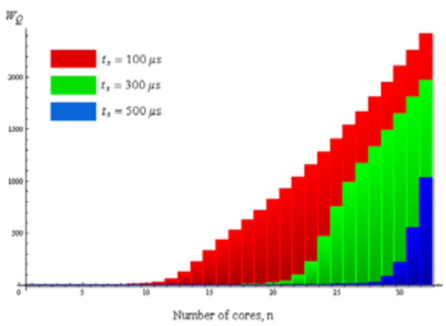

Figure 1.1, W_Q

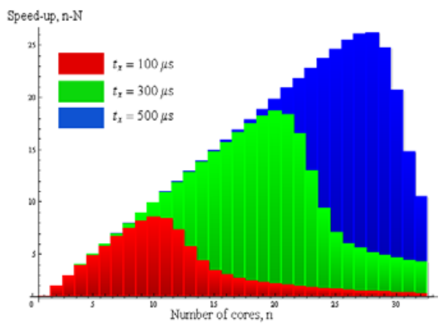

Figure 1.2, Speed-Up

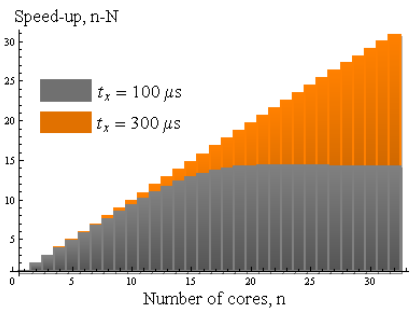

Figure 1.3, Speed-Up (ideal lock)

Question 4

For an ideal ticket spinlock, completion rate becomes: $r_k = \frac{1}{t_c + t_u}$, for $(1 \leq k \leq n)$.

Corresponding speed-up with different number of cores are shown in Figure 1.3 (using the same equation for "speedup" and completion rate $r_k = \frac{1}{t_c + t_u}$). The population is illustrated in Figure 2.2.

Question 5

Collapse in the speed-up was observed in Figures 1.2 due to the average number of either executing or waiting cores (N) has been increasing faster than number of cores (n) itself. The main reason is the cache consistency which is assumed to be the bottle-neck of the spinlock tickets model. This is due to the fact that time to update every locally-cached copy of