Received: 8 March 2016 | Revised: 7 April 2017 | Accepted: 14 June 2017
DOI: 10.1002/qre.2191

RESEARCH ARTICLE

WILEY

Methods for interpreting the out-of-control signal of multivariate control charts: A comparison study

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Abstract

Multivariate control charts have proved to be a useful tool for identifying an out-of-control process. However, one of the main drawbacks of these charts is that they do not indicate which measured variables have been shifted. To overcome this issue, several alternative approaches that aim to diagnose faults the responsible variable(s) for the out-of-control signal and help identify aberrant variables may be found in the literature. This paper reviews several techniques that are used to diagnose the responsible variable(s) for the out-of-control signal and attempt to make a comparative study among them. In particular, we evaluate the performance of each method under different simulation scenarios in terms of successful identification of the out-of-control variables. Special attention has also been given to the computational approaches and especially in the ability of artificial neural networks to identify out-of-control signals. The obtained results show that: there is no particular method that can be considered as panacea; the artificial neural networks' performance depends heavily on the training data set.

KEYWORDS

artificial neural networks (ANNs), control charts, interpretation of an out-of-control signal, multivariate statistical process monitoring (MSPM)

1 | INTRODUCTION

Multivariate statistical process monitoring (MSPM) consists of a set of advanced techniques for the monitoring and control of the operating performance of batch and continuous processes.[1] In general, the multivariate statistical quality control problem considers a continuous process where each item is characterized by p quality characteristics (correlated random variables), $X_1, X_2, \dots, X_p$.[2] These characteristics have to be monitored to ensure that the process works stably and satisfactorily.

Jackson[3] stated that any multivariate process control procedure should fulfill 4 conditions: (1) an answer to the question: "Is the process in control?" must be available; (2) an overall probability for the event "Procedure diagnoses an out-of-control state erroneously" must be specified; (3) the relationships among the variables-attributes should be taken into account; (4) an answer to the question: "If the process is out-of-control, what is the problem?" should be available.

In our work, we focus our attention in the fourth condition of the Jakson statement. More specifically, we investigate the existing methods that can be applied not only to identify that the process is out of control but they may be used also to identify which of the measured variables have been shifted.

Multivariate control charts have been proven to be a useful tool in the field of MSPM to identify out-of-control processes.[4] However, interpreting an out-of-control process is a hard task to be achieved in a multivariate out-of-control process.[4]

To overcome this issue, several different approaches that aim to diagnose faults in out-of-control conditions and help identify aberrant variables may be found in the literature. These approaches can be classified in 2 broad categories:

- the analytical approaches[5-9] that are based on heuristic statistical algorithms and do not require usually any training.
- the computational approaches[10-12,14,15] that aim to detect out-of-control signals faster than analytical ones and require training.

Bersimis et al[16] categorized the analytical methods in 5 main categories. Specifically, the identification methods are categorized in methods that are

- univariate control charts with standard or appropriately adjusted control limits (usually Bonferroni adjustment is applied),[6,17-20]
- decomposition of the T^2 statistic,[5,7,21]
- cause-selecting control charts and regression-adjusted variables (usually these methods pre-assume an in-depth knowledge of the process itself),[8,22,23]
- projection methods (usually using principal components analysis), and[3,24-26]
- methods that are based on graphical techniques (usually descriptive methods).[4,27-29]

However, although there is a diversity of approaches, an overall evaluation of the performance of these methods is missing from the literature. Das and Prakash[30] presented a comparative study among the methods of Mason et al,[5] Doganaksoy et al,[6] Murphy,[7] and Hawkins.[8] However, the number of samples that they used in their simulation studies is very small (ie, only 1000 samples) and details concerning the simulation setup are missing. Moreover, the number of the methods reviewed is restricted.

In this paper, our contribution is twofold:

- to present a comparative study among the most promising techniques that are used to diagnose faults in out-of-control conditions, confirming and extending the work of Das and Prakash.[30]
- to evaluate the performance of artificial neural networks (ANNs) in identifying out-of-control signals, confirming and extending the works of Aparisi,[10] Chen and Wang,[31] and Hwarng.[32]

The rest of the paper is organized as follows: Section 2 overviews multivariate control charts and several out-of control interpretation methods that may found in the literature. Section 3 presents the simulation setup parameters, while Section 4 presents the obtained results. Finally, Section 5 concludes the paper and presents future work.

2 | PRELIMINARIES-MSPM INTERPRETATION METHODS

2.1 | Multivariate control charts

Multivariate statistical process monitoring techniques are usually distinct in 2 phases:

- Phase 1: A retrospective analysis is applied to assess if the process is in control since the first sample was collected. Once this is accomplished, the control chart is used to define what is meant by statistical in-control.[1]
- Phase 2: The control charts are used to verify if the process remain in control in the future.

In case of multivariate processes, several extensions of univariate control charts may be applied, such as the multivariate Shewhart type control charts, multivariate cumulative sum control charts, and multivariate exponentially weighted moving average control charts, to identify an out-of-control process. Since the most widely used multivariate control chart is the Hotelling T^2 control chart, in our work, we use the Hotelling T^2 control chart.

More specifically, we focus on phase 2, and we assume that if the under examination in our paper vector $\boldsymbol{X}$ follows a p-dimensional normal distribution, denoted as $\sim N_p(\mu, \Sigma)$, then the T^2 can be given by

$$T^2 = n(\boldsymbol{X} - \boldsymbol{\mu})\Sigma^{-1}(\boldsymbol{X} - \boldsymbol{\mu})', \tag{1}$$

where μ and Σ are the known vector of means and known variance-covariance matrix, respectively, and n the number of observations. It is proven that T^2 follows a χ^2 distribution with p degrees of freedom, and thus in the case that the parameters are known, the upper control limit (UCL) is given by $UCL = \chi^2(\alpha, p)$ where α denotes the upper percentile of the χ^2 distribution.

It should be noted that in the following of the paper, the value of variable n is set to 1, ie, we work with individual observations.

2.2 | The Doganaksoy, Faltin, and Tucker out-of-control variable selection algorithm

The main idea of the Doganaksoy, Faltin, and Tucker (DFT) algorithm[6] is to apply the univariate t-ranking procedure along with the use of Bonferroni-type limits.[17] More specifically, the following univariate statistic X is used, in the case that the vector of means and the variance-covariance matrix are known:

$$S_{DFT} = \frac{X_i - \mu_i}{\sigma_{ii}}, \tag{2}$$

where X_i and μ_i denote the new sample vector and the reference mean of the ith variable, respectively. σ_{ii} is the (i, i)th element of the covariance matrix Σ. For the Bonferroni control limits, the UCL is given by

$$UCL(i) = \mu_i + Z_{1-\frac{\alpha}{2p}} \frac{\sigma_i}{\sqrt{(n)}} \tag{3}$$

and the lower control limit (LCL) can be expressed as

$$LCL(i) = \mu_i - Z_{1-\frac{\alpha}{2p}} \frac{\sigma_i}{\sqrt{(n)}}. \tag{4}$$

The steps of the DFT algorithm are the following:

Step 1: Calculate the T^2 statistic. If the statistic is greater than the UCL, then go to step 2.

Step 2: For each variable, calculate the smallest significance level α_{ind} that would yield an individual confidence interval for $(X_i - \mu_i), i = 1, \ldots, p$ that contains zero.

Step 3: Plot α_{ind} for each variable on a $(0-1)$ scale. Note that variables with larger α_{ind} values are the ones with relatively smaller univariate x statistic values that require closer investigation as possible being among those components that have undergone a change. If indications of highly suspect variables are desired, then continue.

Step 4: Compute the confidence interval α_{Bonf} that yields the desired nominal confidence interval α_{sim} of the Bonferroni-type simultaneous confidence intervals for $(X_i - \mu_i)$. Here, $\alpha_{Bonf} = (p + \alpha_{sim} - 1)/p$.

Step 5: Components having $\alpha_{ind} > \alpha_{Bonf}$ are classified as being those that are most likely to have changed.

2.3 | The Mason, Young, Tracy decomposition method

The basic idea of the Mason, Young, Tracy (MYT) decomposition method[5] is that the Hotelling's T^2 chart can be decomposed into orthogonal components, each of which reflects the contribution of an individual variable.

For example, for the case of p variables, the general decomposition of Hotelling T^2 can be given by

$$T^2 = T_1^2 + T_{2.1}^2 + \cdots + T_{p.1,2,\ldots,p-1}^2 = T_1^2 + \sum_{j=2}^{p} T_{j.1,2,\ldots,j-1}^2. \quad (5)$$

The first term of Equation 5 denotes the Hotelling T^2 statistic for the first variable of the observation vector $\boldsymbol{X}$ and is given as

$$T_1^2 = \left(\frac{X_1 - \mu_1}{\sigma_1}\right)^2, \quad (6)$$

where μ_1 and σ_1^2 denote the mean and the variance of the first variable of the observation vector $\boldsymbol{X}$, respectively.

All the T_j^2 values are computed using the general formula:

$$T_{j.1,2,\ldots,j-1}^2 = (X_j - \mu_j)' \Sigma_{jj}^{-1} (X_j - \mu_j), \quad (7)$$

where X_j represents the appropriate subvector, and Σ_{jj} denotes the corresponding covariance submatrix obtained from the overall Σ matrix by deleting the jth variable.

The steps of the MYT algorithm are the following:

Step 1: Calculate the T^2 statistic. If the statistic is greater than the UCL, then go to step 2.

Step 2: Compute the individual T_i^2 statistic for every component of the $\boldsymbol{X}$ vector. Remove variables whose observations produce a significant T_i^2. The observations on these variables are out of individual control, and it is not necessary to check how they relate to the other observed variables. With significant variables removed, we have a reduced set of variables. Check the subvector of the remaining k variables of a signal. If there is still an out-of-control signal, go to the next step; otherwise, we have located the source of the problem.

Step 3: Optional: Examine the correlation structure of the reduced set of variables. Remove any variable having a very weak correlation (0.3 or less) with all the other variables. The contribution of a variable that falls in this category is measured by the T_i^2 component.

Step 4: If a signal remains in the subvector of k variables not deleted compute all $T_{i.j}^2$ terms. Remove from study all pairs of variables, (X_i, X_j), that have a significant $T_{i.j}^2$ term. This indicates that something is wrong with the bivariate relationship, and whenever this occurs, the set of variables under consideration should be further reduced. Examine all the removed variables for the cause of the signal. Compute the T^2 terms for the remaining subvector. If no signal is present, the source of the problem is with the bivariate relationships and those variables that were out of individual control.

Step 5: If the subvector of the remaining variables still contains a signal, compute all $T_{i.j,k}^2$ terms. Remove any triple, (X_i, X_j, X_k), of variables that show significant results and check the remaining subvector for a signal.

Step 6: Continue computing the higher order terms in this way until there are no variables left in the reduced set. The worst-case situation is the one where all the unique terms have to be computed.

2.4 | The Murphy out-of-control algorithm

The Murphy (MUR) method[7] identifies the out-of-control variables based on discriminant analysis. The algorithm is triggered by an out-of-control signal from a Hotelling T^2 control chart.

The steps of this algorithm are the following:

Step 1: Calculate the T^2 statistic. If an out-of-control condition is signaled, then continue with step 2.

Step 2: Calculate the individual chart $T_i^2(X_i)$ for $i = 1, \ldots, p$, and calculate the p differences $D_1(i) = [T^2 - T_1^2(X_i)]$. Choose the min $(D_1(i)) = D_1(r)$, and test this minimum difference. If $D_1(r)$ is not significant, eg, $D_1(r) > \chi^2(\alpha, 2)$, then the rth variable only requires attention. If $D_1(r)$ is significant, then continue with step 3.

Step 3: Calculate the $p - 1$ differences $D_2(r,j) = [T^2 - T_2^2(X_r, X_j)], 1 \leqslant j \neq r \leqslant p$. Choose the min $(D_2(r,j)) = D_2(r,s)$, and test this minimum difference. If $D_2(r,s)$ is not significant, eg, $D_2(r) \leqslant \chi^2(\alpha, 3)$, then the rth and the sth variables only require attention. If $D_2(r,s)$ is significant, then continue with step 4.

Step: Continue with a similar manner.

Step p: If the final $D_{p-(p-1)}$ test is significant, then all p variables will require attention.

2.5 | The Hawkins out-of-control algorithm

The Hawkins (HAW) method[8] proposed this method for the case where the shift occurs only in one of the component.[30] The basic idea is to rescale the vector $\boldsymbol{X}$ to $\boldsymbol{Z}$, as

$$\boldsymbol{Z} = \boldsymbol{A}(\boldsymbol{X} - \boldsymbol{\mu}), \tag{8}$$

where the transformation matrix is given by

$$\boldsymbol{A} = [diag(\Sigma^{-1})]^{-\frac{1}{2}}\Sigma^{-1}. \tag{9}$$

It follows from definition that $\boldsymbol{Z} \sim N(0, \boldsymbol{B})$ where

$$\boldsymbol{B} = [diag(\Sigma^{-1})]^{-\frac{1}{2}}\Sigma^{-1}[diag(\Sigma^{-1})]^{-\frac{1}{2}}. \tag{10}$$

Since the HAW algorithm is based on cumulative sums, it is not taken into account in our performance evaluation comparison to compare methods that follow the same approach. Das and Prakash[30] includes the HAW method in their study; however, details on the parameters used in the implementation of the algorithm, as well as, for the simulation setup are not given.

2.6 | The Maravelakis and Bersimis out-of-control algorithm

The Maravelakis and Bersimis (MAB) algorithm[4] uses the well known Andrews curves[33] for solving the problem of interpreting out-of-control variables. More specifically, the main idea of the algorithm is that the variables responsible for the out-of-control signal will give one or more specific subdomains of $(-\pi, \pi)$ outside the control limits.

The steps of the MAB algorithm are the following:

Step 1: Calculate the T^2 statistic. If the statistic is greater than the UCL, then go to step 2.

Step 2: For the multivariate observation $\boldsymbol{x} = (x_1, x_2, \dots, x_p)$, construct the corresponding Andrews curve as $f_x(t) = \boldsymbol{a'(t)x}$ where $-\pi \leqslant t \leqslant \pi$ and $a(t) = (\frac{1}{\sqrt{2}}, sin(t), cos(t), sin(2t), cost(2t), \dots)$. Compute the UCL and the LCL limits for the f_x curve from the following formulas: $UCL = \boldsymbol{\mu a'(t)} + \sqrt{\frac{1}{n}c_\alpha \boldsymbol{a(t)}\Sigma\boldsymbol{a'(t)}}$ and $LCL = \boldsymbol{\mu a'(t)} - \sqrt{\frac{1}{n}c_\alpha \boldsymbol{a(t)}\Sigma\boldsymbol{a'(t)}}$ where c_α denotes the upper percentage point of the p-variate χ^2 distribution, and μ denotes the known vector of means. Plot these limits along with $f_x(t_r)$ for every $t_r = -\pi + \frac{\pi r}{180}, r = 0, 1, \dots, 360$.

Step 3: For each one of the t_r points that lie outside the control limits, calculate the value of the $f_x(t_r)$. From all those $a_i(t_r), i = 1, \dots, p$ with the same sign as $f_x(t_r)$, the one with the largest contribution in $f_x(t_r)$ pinpoints the out-of-control variable.

Step 4: From the set of the t_r variables, the one with the largest frequency is the out-of-control variable.

Step 5: Calculate the statistic used in step 1 for the incoming observation after removing the out-of-control variable indicated in step 4. If there is still an out-of-control signal, go to step 2 using the incoming vector without the out-of-control variable.

2.7 | The Maravelakis, Bersimis, Panaretos, and Psarakis method–rPCA method

The basic idea of the algorithm is that in case an out-of control signal occurs, we may investigate the impact of each variable by using its contribution to the total principal component analysis (PCA) score. More specifically, assuming a p variable vector, the typical form of a PCA model is the following: $Z_k = u_{1k}X_1 + u_{2k}X_2 + \cdots + u_{pk}X_p$ where Z_k is the kth PC, and $u_{1k}, u_{2k}, \dots, u_{pk}$ is the corresponding k-dimensional eigenvector and $X_1, \dots, X_p$ are the process variables.

The steps of rPCA algorithm in case that an out-of control signal is occurred are the following:

Step 1 Calculate the T^2 statistic. If the statistic is greater than the critical value, then go to step 2.

Step 2 Calculate the ratios for each variable based on the covariance matrix used. If a covariance matrix with positive correlations is considered, then the ratio is given by

$$r_{ki} = \frac{u_{1k}X_1 + u_{2k}X_2 + \cdots + u_{pk}X_p}{Y_{1i} + Y_{2i} + \cdots + Y_{di}}, \tag{11}$$

where Y_{ij} is the score of the jth PC, and d denotes the number of significant principal components.

In the case that a covariance matrix with negative correlations is considered, then the ratio is given by

$$r_{ki} = \frac{u_{1k}X_1 + u_{2k}X_2 + \cdots + u_{pk}X_p}{\overline{Y}_{1i} + \overline{Y}_{2i} + \cdots + \overline{Y}_{di}}, \tag{12}$$

where $\overline{Y}_{ij}$ is the score of the jth PC using in place of each X_k, their in control values.

Step 3 Plot the ratios for each variable in a control chart. Compute the α and the $1 - \alpha$ percentage points of the ratio of the two normal distributions with suitable parameters, and use them as LCL and UCL, respectively.

Step 4 Observe which variable, or variables, issue an out-of-control signal.

Step 5 Fix the problem, calculate the χ^2 statistic for the incoming observation, and go to step 1 in case of an out-of-control signal.

2.8 | Artificial neural networks

The notion of the ANNs was inspired by the computations similar the human brain routinely performs. More specifically, in the human brain, a typical neuron collects signals from others through a host of fine structures called dendrites. The neuron sends out spikes of electrical activity through a long, thin stand known as an axon, which splits into thousands of branches. At the end of each branch, a structure called a "synapse" converts the activity from the axon into electrical effects that inhibit or excite activity in the connected neurons.

In this context, in neural computing an ANN is composed by a number of neurons that are connected together. Each neuron within the network is usually a simple-processing unit that takes one or more inputs and produces an output. At each neuron, every input has an associated weight that modifies the strength of each input. The neuron simply adds together all the inputs and calculates an output to be passed on.

The basic components of the neural network are the following:

- Input layer - The activity of the input units represents the raw information that is fed into the network.
- Hidden layer - The activity of each hidden unit is determined by the activities of the input units and the weights on the connections between the input and the hidden units.
- Output layer - The behavior of the output units depends on the activity of the hidden units and the weights between the hidden and output units.

In an ANN, 2 modes of operation exist:

- the training mode, where the network is trained to get the required output (this mode coincides with the phase 1 in SPM literature) and
- the using mode, to investigate the accuracy of the network (this mode coincides with the phase 2 in SPM literature).

Many researchers have investigated the application of ANN to MSPM. Aparisi et al[10] proposed a back-propagation network–based model that classifies the variables into shifted and nonshifted variables. Chen and Wang[31] proposed a back-propagation network–based model that can both identify the characteristic or group of characteristics that cause the signal and classify the magnitude of the shifts. Hwarng[32] proposed an NN-based method to detect out-of-control signals and identify which variable caused the out-of-control signal for the bivariate case. A survey on the applicability of NNs in SPM charts may be found in Psarakis.[13]

Although different ANN configurations are applied, the authors argue that for the detection of mean shifts, the following parameters should be considered: (1) the size of the input data; (2) the number of layers; (3) the number of hidden nodes; (4) the applied activation function; (5) the size of training file; (6) the length of the training, etc. The significance of these parameters in the interpretation of out-of-control signals is indicated in Psarakis.[13]

The basic methodology that is applied to detect out-of-control signals is comprised of the following steps:

Step 0: Generate the required data set to train the network.

Step 1: Calculate the T^2 statistic. If the statistic is greater than the critical value, then go to step 2.

Step 2: Start the using mode operation and check the output to detect an out-of-control signal.

3 | SIMULATION SETUP

To evaluate the performance of the algorithms presented in the previous section in terms of successful identification of the out-of-control variables, different simulation scenarios were considered.

More specifically, we consider 3 different scenarios based on the variance-covariance matrix that is applied, with 3 different correlation values, ie, $\rho = -0.45$, $\rho = 0.2$, and $\rho = 0.8$ for the case of 3 variables and $\rho = -0.2$, $\rho = 0.5$, and $\rho = 0.8$ for the case of 5 variables (since the intraclass correlation cannot assume values less than or equal to $\frac{-1}{p-1}$,[30] ie, for the case of 3 variables $\rho \leqslant -0.5$ and for the case of 5 variables $\rho \leqslant -0.25$).

3.1 | Simulation setup 1

In the first scenario, the following compound symmetry matrices are used for the cases of the 3 and 5 variables, respectively:

$$\Sigma = \begin{bmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{bmatrix}, \tag{13}$$

$$\Sigma = \begin{bmatrix} 1 & \rho & \rho & \rho & \rho \\ \rho & 1 & \rho & \rho & \rho \\ \rho & \rho & 1 & \rho & \rho \\ \rho & \rho & \rho & 1 & \rho \\ \rho & \rho & \rho & \rho & 1 \end{bmatrix}, \tag{14}$$

where ρ denotes the correlation among the variables.

3.2 | Simulation setup 2

In the second scenario, the following spatial symmetry variance-covariance matrices are used for the cases of the 3 and 5 variables, respectively:

$$\Sigma = \begin{bmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & \rho \\ \rho^2 & \rho & 1 \end{bmatrix}, \quad (15)$$

$$\Sigma = \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 & \rho^4 \\ \rho & 1 & \rho & \rho^2 & \rho^3 \\ \rho^2 & \rho & 1 & \rho & \rho^2 \\ \rho^3 & \rho^2 & \rho & 1 & \rho \\ \rho^4 & \rho^3 & \rho^2 & \rho & 1 \end{bmatrix}. \quad (16)$$

3.3 | Simulation setup 3

In the third scenario, the following unstructured variance-covariance matrices are used for the cases of the 3 and 5 variables, respectively:

$$\Sigma = \begin{bmatrix} 1 & -0.7 & -0.8 \\ -0.7 & 1 & -0.9 \\ -0.8 & 0.9 & 1 \end{bmatrix}, \quad (17)$$

$$\Sigma = \begin{bmatrix} 1 & -0.7 & 0.8 & 0.85 & 0.95 \\ 0.7 & 1 & 0.9 & -0.8 & 0.7 \\ 0.8 & 0.9 & 1 & -0.9 & -0.8 \\ -0.85 & -0.8 & -0.9 & 1 & -0.75 \\ 0.95 & 0.7 & 0.8 & -0.75 & 1 \end{bmatrix}. \quad (18)$$

In all scenarios, we have generated data from a multivariate normal distribution $\boldsymbol{X} \sim N_p(\mu, \Sigma)$. Under different scenarios, the first and/or the second component of the in-control mean $\mu = [0\,0\,0]$ is shifted in various values from -3 to 3 representing in this way various out-of-control cases, ie, if we denote as K the set that contains the shift values, then $K = \{-3, -2, -1.5, -1, -0.5, 0.5, 1, 1.5, 2, 2.5, 3\}$, as in Das and Prakash.[30]

For each case, a file containing 10 000 samples was generated. All the analytical approaches were implemented in Matlab. The significance level α was set to 0.05. For the DFT method, the α_{nom} and the α_{sim} values are set to 0.80 and 0.95, respectively.

For the neural network the fast artificial neural network library (http://leenissen.dk/fann/wp/) was used and for the training mode the back-propagation training algorithm was applied.

The neurons in the input layer represent the values of the variables of the scenario (ie, 3 or 5), the Hotelling T^2 statistic, the correlations among variables, and the sample size, respectively. For the training phase, we have used data generated from the multivariate normal distribution $\boldsymbol{X} \sim N_p(\mu, \Sigma)$ described above.

The outputs of the network are 3- or 5-variable vectors where $X_i = 0$ or 1 where 1 indicates that the specific variable is out of control; otherwise, the value is set to 0. It should be noted that in cases that the output values are close to 0 or 1, we round them down or up to get output values exactly 0 or 1.

TABLE 1 Simulation parameters used for the artificial neural network

Parameter	Value
Number of layers	3
Number of inputs	8 or 12
Number of outputs	3 or 5
Number of neurons in the hidden layer	12
Desired error	0.01
Activation function (hidden and output)	Sigmoid

The performance metric for our evaluation is the successful identification of the out control variables, ie, the algorithm should detect at least one of the out-of-control variables. It should be noted that in our comparison, the results of HAW method have been omitted, since the method implies an in-depth knowledge of the process itself and thus it cannot be comparable with the other methods.

As it was mentioned in the previous subsection, different ANN configurations will lead to different degree of accuracy of the mean shift. Therefore, to achieve the "optimal" ANN configuration, the parameters that affect the performance of the ANN detection capability have been examined through simulations. Table 1 depicts the parameters set on the ANN software for the training and the using modes.

4 | PERFORMANCE EVALUATION RESULTS

In this section, we evaluate the performance of the methods in different simulation cases. More specifically, in the first subsection, we examine the performance of the methods when only the first component of the in-control mean μ is shifted. In the second subsection, the performance of the methods, when both the first and the second components of the in-control mean μ are shifted, is investigated. For the tables 3, 5, 7, 9 and 12 that summarize the overall performance of the methods with bold text we highlight the successful identification percentage that exceeds the 50%, while for the rest of the tables with bold text we highlight the largest successful identification percentage for each scenario.

4.1 | One-shift case

4.1.1 | First scenario–compound symmetry variance-covariance matrix

For the case of 3 variables (Table 2):

For medium negative correlation, ie, $\rho = -0.45$, all the analytical methods besides the MUR method perform equally well for large shifts and below average for small shifts. The performance of the MUR method remains ineffective for large shifts and its performance is improved for small shifts, as also indicated in Das and Prakash.[30] The ANN method has the best

TABLE 2 One-shift successful identification percentage (%) for the first scenario with 3 variables

Shift			$\rho = -0.45$							$\rho = 0.2$							$\rho = 0.8$						
K1	K2	K3	δ	MYT	MUR	DFT	MAB	ANN	rPCA	δ	MYT	MUR	DFT	MAB	ANN	rPCA	δ	MYT	MUR	DFT	MAB	ANN	rPCA
−3	0	0	5.84	77.1	0.4	79.0	81.2	**91.4**	78.8	3.11	83.1	86.1	79.7	58.6	**98.2**	1.5	5.58	78.6	0.6	81.7	86.0	**99.8**	5.0
−2	0	0	3.90	47.7	8.5	49.1	63.2	**86.9**	49.4	2.07	74	80.6	62.0	43.5	**96.2**	1.7	3.72	51.5	10.9	52.6	64.5	**98.6**	4.2
−1.5	0	0	2.92	36.3	16.9	33.8	52.4	**83.8**	38.2	1.55	66.3	74.1	49.4	36.4	**93.3**	1.5	2.79	37.6	20.6	34.1	46.8	**97.4**	2.8
−1	0	0	1.95	27.5	24.5	21.8	39.4	**79.1**	30.1	1.04	50.6	59.8	33.8	29.1	**86.8**	1.5	1.86	28.8	27.8	20.5	36.3	**95.4**	2.4
−0.5	0	0	0.97	17.1	28.0	14.2	21.6	**73.3**	20.9	0.52	30.9	39.8	20.1	20.5	**76.9**	1.1	0.93	16.9	27.6	11.0	24.4	**87.0**	0.8
0.5	0	0	0.97	17.4	26.0	13.9	14.1	**45.0**	19.8	0.52	32.9	**42.3**	21.2	12.9	40.8	0.6	0.93	17.4	26.4	11.0	20.8	**34.9**	1.5
1	0	0	1.95	27.2	24.5	21.4	26.2	**36.8**	29.8	1.04	50.2	**59.6**	33.3	22.6	28.5	1.4	1.86	27.9	28.6	20.2	**29.7**	27.9	2.5
1.5	0	0	2.92	35.1	17.2	32.9	**40.3**	29.1	36.9	1.55	65.4	**74.5**	49.3	29.3	18.8	1.1	2.79	38.7	21.5	35.2	**43.5**	17.6	3.3
2	0	0	3.90	48.4	7.9	50.3	**53.8**	21.2	50.1	2.07	73.2	**80.9**	60.6	36.2	15.3	1.8	3.72	51.0	11.5	52.7	**58.6**	9.7	3.8
2.5	0	0	4.87	64.3	2.0	**67.5**	66.5	14.0	65.8	2.59	79.6	**83.4**	71.9	46.5	9.9	1.5	4.65	65.1	3.8	69.3	**74.2**	4.0	5.3
3	0	0	5.84	76.1	0.3	78.0	76.3	7.8	**78.2**	3.11	83.1	**86.0**	79.8	54.8	6.8	1.3	5.58	78.2	0.8	81.1	**84.0**	0.9	5.5

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE 3 Overall one-shift succesful identification percentage (%) for the first scenario with 3 variables

ρ		MUR	DFT	MAB	ANN	rPCA
−0.45	43.1	14.2	42.0	48.6	**51.7**	45.3
0.2	**62.7**	**69.7**	**51.0**	35.5	**52.0**	1.4
0.8	44.7	16.4	42.7	**51.7**	**52.1**	3.4

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

performance for values $K1 \leqslant 1$, due to the fact that our training set contains more data generated from these cases. This is actually the reason for the asymmetric results along with the magnitude of shift. The MAB method is the second one for negative values and the first one for shifts $K1 > 1$.

For small positive correlation, ie, $\rho = 0.2$, all the analytical methods besides the rPCA perform equally well for large shifts and below average for small shifts. The ANN method has the best performance for values $K1 \leqslant 1$, due to the fact that in our training set, we used more data generated from these cases. The DFT method has the second best performance from the analytical methods after the MUR method.

For large positive correlation, ie, $\rho = 0.8$, all the analytical methods besides the MUR and the rPCA methods perform equally well for large shifts and below average for small shifts. The performance of the MUR method decreases for large shifts, and its performance is improved for small shifts.The MAB method has the best performance from the analytical methods.

As Table 3 presents, the identification capability of all the methods besides the MAB and the rPCA methods increases when the correlation is medium or large. In all cases, the percentage of successful identification of the ANN method is above average.

It should be noted that although in our simulation results, we have included the rPCA method since both the means of the numerator and the denominator of every ratio are equal to zero and the correlations among them is positive the method cannot be applied correctly. As it can also be seen from the results of the following cases the performance of the method is very poor.

For case of 5 variables (Table 4), whatever the correlation is, the performance of ANN method is lower than the one in the 3-variable case, since in our training set we used contains even more data generated from the 2-variables mean shift case.

For all the different correlation values, both the MYT and DFT methods perform equally well for large shifts and below average for small shifts.The performance of the MAB method remains ineffective for large shifts and small shifts.

As also stated in Maravelakis and Bersimis,[4] whenever the number of the variables increases, the performance of the

TABLE 4 One-shift successful identification percentage (%) for the first scenario with 5 variables

Shift					$\rho = -0.2$							$\rho = 0.5$							$\rho = 0.8$						
K1	K2	K3	K4	K5	δ	MYT	MUR	DFT	MAB	ANN	rPCA	δ	MYT	MUR	DFT	MAB	ANN	rPCA	δ	MYT	MUR	DFT	MAB	ANN	rPCA
−3	0	0	0	0	3.87	74.8	44.8	**76.7**	29.5	8.8	7.1	3.87	74.0	45.2	**79.1**	30.1	7.6	7.9	6.04	69.3	0.6	**77.6**	57.1	41.7	16.2
−2	0	0	0	0	2.58	59.4	54.2	**60.3**	15.7	7.2	6.6	2.58	57.6	53.9	**60.7**	13.9	6.9	5.7	4.0	44.7	10.2	**47.1**	22.9	6.9	10.6
−1.5	0	0	0	0	1.94	45.9	**49.7**	46.7	9.9	6.7	1.0	1.94	47.5	**52.5**	50.3	9.9	6.8	3.5	3.02	31.3	19.6	**33.3**	12.3	6.8	7.4
−1	0	0	0	0	1.29	32.9	**40.7**	34.2	7.3	6.6	1.0	1.29	30.4	**43.9**	34.3	6.3	6.4	2.6	2.01	20.9	**24.0**	22.7	6.0	6.4	5.2
−0.5	0	0	0	0	0.65	18.5	**27.3**	19.7	4.2	5.8	2.3	0.65	18.1	**26.8**	19.2	3.6	8.0	1.2	1.01	10.8	**21.7**	11.5	3.4	8.0	2.7
0.5	0	0	0	0	0.65	19.5	**29.8**	21.0	1.6	6.2	1.0	0.65	18.1	**28.5**	18.3	2.4	9.0	1.4	1.01	9.4	**21.6**	12.6	1.4	9.0	2.0
1	0	0	0	0	1.29	33.2	**42.1**	34.1	3.4	5.1	0.6	1.29	32.8	**43.7**	34.2	4.3	9.0	2.6	2.01	22.0	**25.6**	24.7	4.8	9.0	5.0
1.5	0	0	0	0	1.94	46.3	**48.6**	47.4	6.3	4.1	1.7	1.94	45.7	**50.8**	49.3	7.3	8.5	4.1	3.02	32.2	20.1	**32.9**	8.4	8.5	7.9
2	0	0	0	0	2.58	57.8	53.1	**58.6**	10.1	3.7	5.5	2.58	58.7	55.1	**62.5**	12.3	7.7	6.2	4.02	44.9	10.4	**46.8**	18.7	7.7	10.4
2.5	0	0	0	0	3.23	67.1	51.4	**67.9**	15.9	3.5	6.1	3.23	67.9	53.2	**72.2**	18.6	6.7	6.6	5.03	57.9	3.1	**63.2**	34.6	6.7	14.0
3	0	0	0	0	3.87	74.6	45.8	**76.5**	24.6	3.9	10.5	3.87	73.4	45.7	**78.4**	27.5	6.1	8.2	6.04	69.0	0.4	**77.9**	53.9	6.1	16.5

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE 5 Overall one-shift successful identification percentage (%) for the first scenario with 5 variables

ρ		MUR	DFT	MAB	ANN	rPCA
−0.2	48.18	44.32	49.37	11.68	5.60	3.95
0.5	47.65	45.39	**50.77**	12.38	7.52	4.55
0.8	37.49	14.30	40.94	20.32	10.62	8.90

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

MAB method decreases. This is caused because whenever the dimensionality of the problem increases, the ability of the curves to address such a complicated problem diminishes. Additionally, for medium positive correlation, ie, $\rho = 0.5$, the DFT method performs inadequate.

As Table 5 presents, the overall performance of all the methods is poor. When the number of variables increases, the identification of the only one out-of-control variable is a hard task to be achieved.

4.1.2 | Second scenario–spatial power variance-covariance matrix

For the case of 3 variables (Table 6), for medium negative correlation, ie, $\rho = -0.45$, the MYT, DFT, MAB, and the MUR methods perform equally well for large shifts and below average for small shifts. The DFT method has the best performance, followed by the MUR method.

For small positive correlation, ie, $\rho = 0.2$, all the methods besides the rPCA method perform equally well for large shifts and below average for small shifts.The MUR method has the best performance, and the DFT method has the second best performance after MUR method.

For large positive correlation, ie, $\rho = 0.8$, all the analytical methods perform besides the MUR and rPCA methods equally well for large shifts and below average for small shifts. The MUR method remains ineffective for large shifts, and its performance is improved for small shifts. The ANN method has the best performance for shifts $K1 > 1$ followed by the MAB method, while the rPCA method for the reason described in the previous scenario performs poorly.

As it can be seen from Table 7, whenever the spatial power variance-covariance matrix is used, the performance of the methods besides the rPCA method is improved, especially for large positive correlation, ie, $\rho = 0.8$, compared with the results of the first scenario (Table 3).

For the case of 5 variables (Table 8):

The accuracy on out-of-control signals interpretation of the DFT and the ANN methods decrease when the spatial power variance-covariance matrix is used, while for all the other methods, the accuracy is increased.

The MUR method performs poor for large positive correlation, ie, $\rho = 0.8$, while for all the other cases perform equally well for large shifts and below average for small shifts.

TABLE 6 One-shift succesful identification percentage (%) for the second scenario with 3 variables

Shift			$\rho = -0.45$							$\rho = 0.2$							$\rho = 0.8$						
K1	K2	K3	δ	MYT	MUR	DFT	MAB	ANN	rPCA	δ	MYT	MUR	DFT	MAB	ANN	rPCA	δ	MYT	MUR	DFT	MAB	ANN	rPCA
−3	0	0	3.36	83.2	68.6	**84.4**	67.3	58.7	3.0	3.06	83.2	**88.3**	83.4	58.7	63.8	3.3	5.00	78.2	3.8	81.2	**84.9**	84.2	3.7
−2	0	0	2.24	71.5	71.3	**73.5**	48.2	48.9	3.1	2.04	74.5	**82.3**	74.8	43.4	54.8	1.7	3.33	54.2	20.1	59.4	66.4	**79.7**	2.9
−1.5	0	0	1.68	62.2	**67.7**	65.0	40.3	45.4	2.7	1.53	66.9	**75.5**	66.9	38.9	47.7	1.3	2.50	41.7	29.4	47.3	53.2	**74.1**	2.5
−1	0	0	1.12	48.0	**55.9**	50.6	28.5	33.7	2.2	1.02	52.3	**63.0**	53.0	30.8	37.5	0.7	1.67	32.7	34.9	37.7	40.6	**65.7**	1.9
−0.5	0	0	0.56	28.4	**39.4**	32.1	20.7	24.0	0.9	0.51	34.3	**43.7**	35.5	18.6	27.7	0.6	0.83	19.4	29.9	23.1	27.6	**52.0**	1.8
0.5	0	0	0.56	31.8	**40.2**	33.3	14.2	17.7	1.2	0.51	30.7	**38.3**	31.4	11.7	19.3	1.3	0.83	20.2	32.1	23.3	25.1	**47.6**	0.6
1	0	0	1.12	50.2	**58.7**	53.3	28.5	27.3	1.7	1.02	50.3	**60.2**	51.6	21.1	29.6	1.8	1.67	32.3	33.4	38.1	38.0	**59.6**	1.8
1.5	0	0	1.68	63.2	**67.9**	65.6	33.7	36.3	1.7	1.53	65.4	**74.5**	66.1	30.3	37.2	2.4	2.50	43.3	30.3	48.2	49.4	**75.0**	2.9
2	0	0	2.24	71.8	70.5	**73.4**	42.8	43.1	2.0	2.04	73.9	**82.7**	74.1	38.2	45.7	1.8	3.33	53.5	20.4	59.1	63.2	**74.8**	3.1
2.5	0	0	2.80	78.2	72.1	**79.8**	53.9	46.8	2.8	2.55	79.8	**87.0**	80.2	47.0	52.0	2.1	4.17	67.0	10.6	71.6	75.2	**80.2**	3.5
3	0	0	3.36	83.1	69.1	**83.9**	66.0	53.7	2.6	3.06	83.3	**88.6**	83.1	57.2	56.3	1.3	5.00	78.7	3.7	81.5	84.1	**83.9**	3.5

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE 7 Overall one-shift successful identification percentage (%) for the second scenario with 3 variables

ρ	MYT	MUR	DFT	MAB	ANN	rPCA
−0.45	**61.1**	**61.9**	**63.2**	40.4	39.6	2.2
0.2	**63.1**	**71.3**	**63.6**	36.0	42.9	1.7
0.8	47.4	22.6	**51.9**	**55.2**	**70.6**	2.6

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

Whatever the correlation is, the performance of ANN method, as Table 9 presents, is lower than the one in the 3-variable case, since our training set contains even more data generated from the 2-variables mean shift case.

4.1.3 | Third scenario–unstructured variance-covariance matrix

All the analytical methods besides the MUR and the rPCA methods perform better for the 3-variable case (Table 10) than for the 5-variable case (Table 11). Specifically, for the 3-variables case (Table 10), they perform equally well for large shifts and below average for small shifts. The MYT method has the best performance uniformly, while the ANN method follows. For the case of 5 variables (Table 11), all the methods besides the MUR method perform generally below average, while the MUR method perform equally well for large shifts and below average for small shifts. As Table 12 depicts only the DFT, the MAB and the ANN methods perform above average for the case of 5 variables.

Figures 1-4, as well as Figures B1-B4, present the performance of the 4 basic analytical methods: the MYT, DFT, MAB, and the MUR methods for the cases of 3 and 5 variables, respectively. As it can be seen from the figures, the performance of the methods for the 1-shift identification decreases as the number of variables increases, especially when either the MUR method or the MAB methods are applied. Also, the correlation among the variables plays an important role in the successful identification ratio of each method.

4.2 | Two-shift case

4.2.1 | First scenario–compound symmetry variance-covariance matrix

For the case of 3 variables:

As Tables A1-A4 present, the performance of all the methods besides the rPCA is above average. All the methods besides the rPCA method succeed in identifying at least one out-of-control variable. The MAB has the second lowest performance due to the fact that there are cases where there are no t_r points outside the control limits (step 3 of the method), and thus, we cannot determine which variable is out of control.

TABLE 8 One-shift successful identification percentage (%) for the second scenario with 5 variables

Shift					$\rho = -0.2$							$\rho = 0.5$							$\rho = 0.8$						
K1	K2	K3	K4	K5	δ	MYT	MUR	DFT	MAB	ANN	rPCA	δ	MYT	MUR	DFT	MAB	ANN	rPCA	δ	MYT	MUR	DFT	MAB	ANN	rPCA
−3	0	0	0	0	3.06	75.4	**88.2**	79.7	24.0	5.5	1.5	3.46	75.0	67.3	**79.6**	28.6	4.9	1.9	5.00	71.0	7.5	**76.7**	44.4	2.1	7.9
−2	0	0	0	0	2.04	62.0	**76.6**	66.5	15.3	4.9	1.0	2.31	60.3	**66.5**	63.9	15.6	5.6	1.9	3.33	50.6	26.8	**53.4**	19.7	2.0	5.3
−1.5	0	0	0	0	1.53	49.3	**64.4**	53.1	10.7	4.9	1.7	1.73	51.5	**60.4**	54.6	10.9	4.8	1.3	2.50	37.8	32.8	**41.1**	11.9	3.1	3.6
−1	0	0	0	0	1.02	33.1	**45.7**	36.3	7.8	4.8	1.4	1.15	35.1	**46.8**	39.0	8.0	5.3	1.1	1.67	24.9	**32.8**	28.7	5.9	4.3	2.2
−0.5	0	0	0	0	0.51	21.9	**30.2**	23.9	5.9	5.1	1.5	0.58	18.5	**29.0**	20.6	5.3	5.1	0.5	0.83	14.4	**24.8**	15.7	3.5	5.1	0.7
0.5	0	0	0	0	0.51	20.9	**32.3**	23.2	2.6	6.8	0.7	0.58	18.1	**30.3**	20.7	0.9	5.3	0.5	0.83	13.8	**25.9**	17.4	2.7	5.5	0.8
1	0	0	0	0	1.02	35.8	**48.8**	38.0	4.9	8.6	0.2	1.15	35.7	**47.4**	38.7	4.8	5.8	0.7	1.67	27.3	**35.0**	31.3	5.1	4.7	2.0
1.5	0	0	0	0	1.53	49.5	**65.1**	53.2	7.6	9.4	1.9	1.73	49.1	**57.4**	52.5	8.3	6.1	0.6	2.50	37.7	32.7	**39.8**	8.3	3.5	3.7
2	0	0	0	0	2.04	60.1	**76.5**	64.1	11.3	10.5	0.0	2.31	61.2	**66.3**	65.2	11.8	6.8	0.0	3.33	49.4	26.5	**51.9**	15.8	4.4	4.8
2.5	0	0	0	0	2.55	69.7	**83.7**	73.2	16.2	11.0	2.7	2.89	70.4	68.9	**73.6**	19.1	8.8	0.0	4.17	61.3	17.0	**65.3**	27.8	7.7	6.5
3	0	0	0	0	3.06	75.2	**88.0**	78.7	21.6	11.4	2.1	3.46	74.1	67.7	**78.3**	26.0	10.8	1.7	5.00	70.3	7.5	**76.8**	42.0	10.9	7.7

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE 9 Overall one-shift successful identification percentage (%) for the second scenario with 5 variables

ρ	MYT	MUR	DFT	MAB	ANN	rPCA
−0.2	**50.3**	**63.6**	**53.6**	11.6	7.5	1.3
0.5	49.9	**55.3**	**53.3**	12.7	6.3	0.9
0.8	41.7	24.5	45.3	17.0	4.8	4.1

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE 10 One-shift successful identification percentage (%) for the third scenario with 3 variables

K1	K2	K3	δ	MYT	MUR	DFT	MAB	ANN	rPCA
−3	0	0	5.01	79.7	3.9	83.3	**88.2**	85.7	0.9
−2	0	0	3.34	54.6	19.8	59.9	**77.8**	76.7	0.8
−1.5	0	0	2.51	43.6	29.2	49.1	69.0	**72.2**	0.8
−1	0	0	1.67	32.3	33.7	37.4	56.7	**65.7**	0.4
−0.5	0	0	0.84	22.9	33.3	25.3	37.9	**52.0**	0.3
0.5	0	0	0.84	23.6	34.0	26.6	33.3	**48.6**	0.3
1	0	0	1.67	33.4	35.1	37.8	46.3	**58.6**	0.4
1.5	0	0	2.51	43.8	29.4	49.1	60.7	**73.0**	0.7
2	0	0	3.34	54.2	21.0	59.2	71.2	**76.0**	0.9
2.5	0	0	4.18	67.6	10.6	72.4	**79.6**	76.2	1.0
3	0	0	5.01	79.6	3.7	82.7	**86.0**	84.8	0.9

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE 11 One-shift successful identification percentage (%) for the third scenario with 5 variables

$K1$	$K2$	K3	K4	K5	δ	MYT	MUR	DFT	MAB	ANN	rPCA
−3	0	0	0	0	16	24.9	**78.2**	0.0	38.2	6.1	10.7
−2	0	0	0	0	10.67	16.4	**43.8**	0.0	10.60	11.0	6.4
−1.5	0	0	0	0	8	7.3	**24.2**	0.0	3.1	5.1	4.3
−1	0	0	0	0	5.33	1.7	10.7	0.1	0.8	**13.1**	2.5
−0.5	0	0	0	0	2.67	0.2	4.6	5.1	0.0	**51.0**	0.8
0.5	0	0	0	0	2.67	0.1	4.0	4.4	0.1	**40.4**	0.7
1	0	0	0	0	5.33	1.6	**10.6**	0.2	0.5	10.1	2.5
1.5	0	0	0	0	8	6.5	**24.4**	0.0	3.0	5.2	4.2
2	0	0	0	0	10.67	16.2	**42.8**	0.0	9.6	5.0	6.7
2.5	0	0	0	0	13.33	24.1	**62.6**	0.0	21.1	4.5	8.9
3	0	0	0	0	16	24.5	**77.9**	0.0	36.8	3.4	10.4

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE 12 Overall one-shift successful identification percentage (%) for the third scenario with 5 variables

Shifts		MUR	DFT	MAB	ANN	rPCA
1st shift	48.7	23.1	**53.0**	**64.2**	**70.0**	**0.7**
2nd shift	11.2	34.9	0.9	11.3	14.1	5.3

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

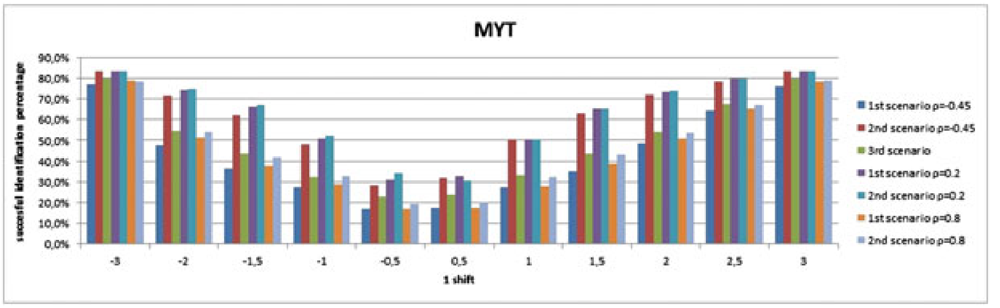

FIGURE 1 One-shift Mason, Young, Tracy's (MYT) successful identification ratio for the 3-variable case [Colour figure can be viewed at wileyonlinelibrary.com]

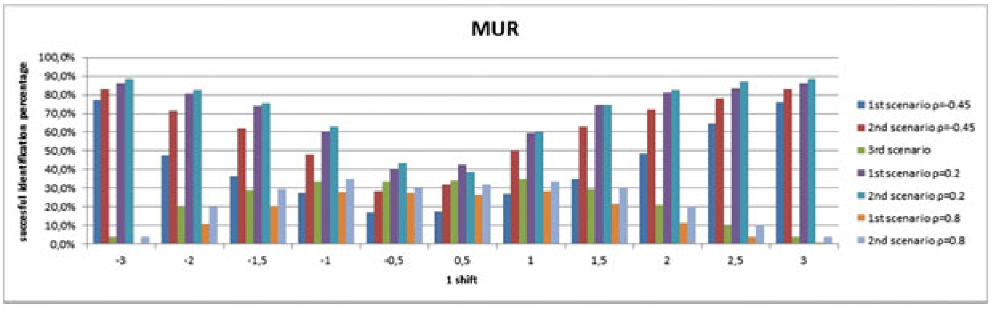

FIGURE 2 One-shift Murphy's (MUR) successful identification ratio for the 3-variable case [Colour figure can be viewed at wileyonlinelibrary.com]

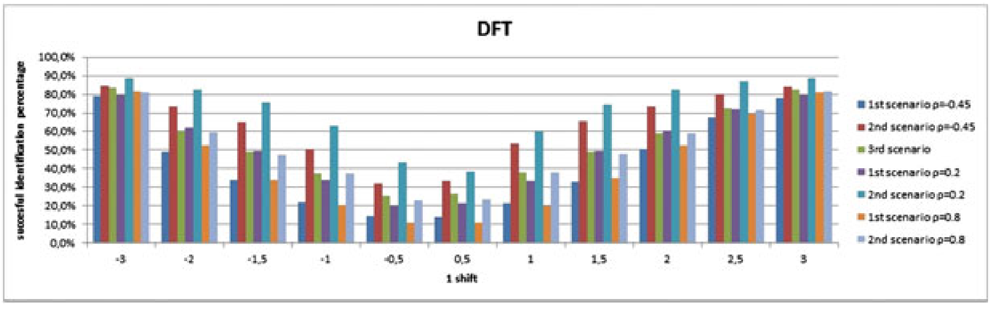

FIGURE 3 One-shift Doganaksoy, Faltin, Tucker's (DFT) successful identification ratio for the 3-variable case [Colour figure can be viewed at wileyonlinelibrary.com]

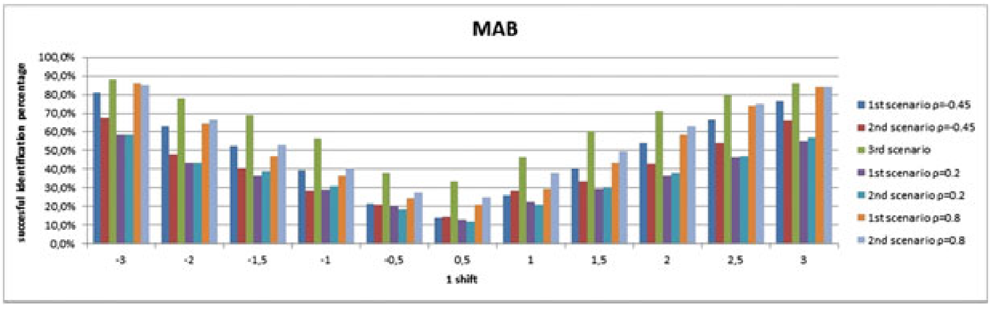

FIGURE 4 One-shift Maravelakis and Bersimis' (MAB) successful identification ratio for the 3-variable case [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 13 Two-shift successful identification percentage (%) for the third scenario

*K*1	*K*2	*K*3	δ	MYT	MUR	DFT	MAB	ANN	rPCA
−3	1	0	5.68	97.8	**100.0**	88.4	99.9	91.0	46.8
−3	1.5	0	6.30	**100.0**	**100.0**	98.2	99.9	91.0	77.7
−3	2	0	7.06	97.6	**100.0**	88.1	99.9	96.0	94.8
−3	2.5	0	7.91	97.7	**100.0**	91.0	99.7	92.0	99.1
−3	3	0	8.84	97.8	**100.0**	93.8	98.9	94.0	99.6
−2	−3	0	7.44	**100.0**	**100.0**	99.6	98.9	97.0	73.3
−2	−2	0	5.48	**100.0**	**100.0**	93.6	99.1	95.0	57.6
−2	−1.5	0	4.62	**100.0**	**100.0**	85.3	99.0	97.0	49.9
−2	−1	0	3.91	99.9	**100.0**	77.6	97.4	93.0	42.7
−2	1	0	4.20	97.7	**99.9**	60.3	96.2	90.0	73.6
−2	1.5	0	4.99	**100.0**	**100.0**	85.3	99.0	**100.0**	90.5
−2	2	0	5.89	97.5	**100.0**	69.4	98.5	**100.0**	95.9
−2	2.5	0	6.87	97.4	**100.0**	78.8	97.8	95.0	97.0
−2	3	0	7.90	97.6	**100.0**	89.2	96.8	94.0	98.0
−1.5	1	0	3.53	97.1	**99.5**	46.5	86.0	97.0	80.1
−1.5	1.5	0	4.42	**100.0**	99.9	74.5	96.5	94.0	90.2
−1.5	2	0	5.41	97.7	**100.0**	62.1	93.7	96.0	93.2
−1.5	2.5	0	6.45	97.8	**100.0**	76.7	95.9	97.0	94.9
−1.5	3	0	7.52	97.5	**100.0**	88.6	96.0	**100.0**	96.4
−1	−3	0	6.98	99.7	**100.0**	95.7	98.2	98.0	82.6
−1	−2	0	4.77	99.8	**100.0**	74.4	94.6	94.0	69.6
−1	−1.5	0	3.72	99.8	**99.9**	61.5	88.5	99.0	61.7
−1	−1	0	2.74	**99.8**	99.4	57.5	84.9	92.0	55.8
−0.5	−3	0	6.89	99.3	**100.0**	92.6	97.1	94.0	86.1
0.5	3	0	6.89	99.3	**100.0**	92.7	95.7	95.0	86.6
1	−3	0	7.23	97.8	**100.0**	88.5	91.2	99.0	94.5
1	−2	0	5.01	97.7	**100.0**	59.4	75.0	95.0	89.3
1	−1.5	0	3.95	97.5	99.9	43.7	59.8	**100.0**	86.2
1	−1	0	2.95	97.0	**98.9**	38.2	53.0	94.0	81.6
1	1	0	2.74	99.8	99.6	56.6	82.5	**100.0**	55.2
1	1.5	0	3.72	97.5	**99.9**	43.7	59.8	93.0	62.5
1	2	0	4.77	99.8	**100.0**	74.8	92.3	93.0	70.8
1	2.5	0	5.87	99.7	**100.0**	88.2	95.7	99.0	76.5
1	3	0	6.98	99.8	**100.0**	96.0	96.9	97.0	82.1
1.5	−3	0	7.52	**100.0**	**100.0**	98.4	91.6	92.0	96.4
1.5	−2	0	5.41	**100.0**	**100.0**	84.4	79.7	91.0	93.1
1.5	−1.5	0	4.42	**100.0**	**100.0**	74.5	71.4	97.0	90.1
1.5	−1	0	3.53	**99.9**	99.6	66.6	69.3	92.0	80.9
2	−3	0	7.90	97.4	**100.0**	89.6	92.3	90.0	97.9
2	−2	0	5.89	97.7	**100.0**	69.2	83.9	91.0	96.0
2	−1.5	0	4.99	97.2	**100.0**	62.2	80.6	95.0	90.4
2	−1	0	4.20	97.6	**99.8**	60.6	80.0	99.0	73.4
3	−3	0	8.84	97.7	**100.0**	94.0	93.7	95.0	99.7
3	−2	0	7.06	97.7	**100.0**	89.3	91.3	95.0	94.8
3	−1.5	0	6.30	97.5	**100.0**	88.4	90.3	93.0	77.5
3	−1	0	5.68	97.6	**100.0**	87.9	90.2	94.0	47.3
3	−0.5	0	5.23	98.1	**100.0**	88.3	90.1	96.0	25.9
3	0.5	0	5.06	99.7	**100.0**	92.6	96.9	93.0	24.9
3	1	0	5.36	**100.0**	**100.0**	96.1	99.4	90.0	31.9

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE 14 Two-shift successful identification percentage (%) for the third scenario with 5 variables

$K1$	$K2$	$K3$	$K4$	$K5$	δ	MYT	MUR	DFT	MAB	ANN	rPCA
−3	1	0	0	0	3.47	**99.7**	99.4	99.0	82.7	91.0	15.3
−3	1.5	0	0	0	3.74	**99.8**	99.6	99.0	89.6	94.0	20.2
−3	2	0	0	0	4.05	**99.9**	**99.9**	99.4	95.2	90.0	23.6
−3	2.5	0	0	0	4.40	**99.9**	**99.9**	**99.5**	98.1	99.0	25.0
−3	3	0	0	0	4.78	**100.0**	**99.9**	99.8	99.4	99.0	25.8
−2	−3	0	0	0	3.35	**99.5**	99.3	99.4	94.6	99.0	0.2
−2	−2	0	0	0	2.61	**98.5**	97.9	98.1	87.7	98.0	0.4
−2	−1.5	0	0	0	2.33	**97.9**	97.1	98.0	80.5	98.0	1.1
−2	−1	0	0	0	2.13	97.6	96.5	97.6	71.7	**98.0**	1.3
−2	1	0	0	0	2.49	98.4	97.4	95.9	77.2	99.0	15.1
−2	1.5	0	0	0	2.82	99.0	98.3	96.8	85.9	98.0	19.6
−2	2	0	0	0	3.19	99.3	99.1	97.6	92.4	97.0	20.6
−2	2.5	0	0	0	3.60	99.6	99.4	98.5	96.0	**100.0**	18.4
−2	3	0	0	0	4.03	**99.8**	99.7	99.3	97.8	94.0	15.8
−1.5	1	0	0	0	2.03	96.9	95.7	94.3	77.1	92.0	15.5
−1.5	1.5	0	0	0	2.39	97.7	96.8	94.9	84.3	**99.0**	16.9
−1.5	2	0	0	0	2.80	**98.6**	98.0	96.1	90.0	97.0	15.9
−1.5	2.5	0	0	0	3.24	**99.4**	99.2	98.0	94.1	96.0	13.9
−1.5	3	0	0	0	3.70	**99.6**	**99.6**	98.9	96.3	95.0	10.1
−1	−3	0	0	0	3.03	**99.2**	99.0	99.0	86.9	98.0	0.6
−1	−2	0	0	0	2.10	**97.4**	96.5	97.0	80.7	97.0	1.1
−1	−1.5	0	0	0	1.67	95.6	94.3	95.5	77.1	**100.0**	1.6
−1	−1	0	0	0	1.31	**92.0**	89.7	92.4	73.5	90.0	2.9
−1	1	0	0	0	1.59	**94.3**	92.2	90.5	75.3	94.0	13.7
−1	1.5	0	0	0	2.01	**96.6**	95.3	93.2	82.5	95.0	14.0
−1	2	0	0	0	2.47	**97.9**	97.1	95.4	86.6	96.0	12.5
−1	2.5	0	0	0	2.94	**98.9**	98.6	97.2	90.7	93.0	10.2
−1	3	0	0	0	3.42	**99.5**	**99.5**	98.8	94.1	95.0	7.1
−0.5	−3	0	0	0	3.00	99.0	98.9	99.0	84.9	**100.0**	1.5
0.5	3	0	0	0	3.00	99.2	99.1	99.1	84.7	92.0	1.5
1	−3	0	0	0	3.42	99.6	99.6	99.0	92.2	90.0	8.0
1	−2	0	0	0	2.47	**97.9**	97.4	95.7	83.5	95.0	12.0
1	−1.5	0	0	0	2.01	**96.7**	95.2	93.6	75.3	95.0	14.1
1	−1	0	0	0	1.59	93.6	91.9	90.5	64.1	93.0	14.2
1.5	−3	0	0	0	3.70	99.7	99.7	98.8	95.1	94.0	11.5
1.5	−2	0	0	0	2.80	98.6	98.1	96.3	85.8	**99.0**	15.8
1.5	−1.5	0	0	0	2.39	**97.9**	97.0	94.7	76.7	91.0	17.3
1.5	−1	0	0	0	2.03	**96.0**	95.0	93.2	66.0	93.0	15.7
2	−3	0	0	0	4.03	**99.8**	**99.8**	99.2	96.7	94.0	15.8
2	−2	0	0	0	3.19	99.2	99.1	97.4	88.0	**100.0**	20.4
2	−1.5	0	0	0	2.82	**99.0**	98.3	96.6	80.2	98.0	18.7
2	−1	0	0	0	2.49	**97.9**	97.0	96.1	68.6	90.0	16.4
3	−3	0	0	0	4.78	**100.0**	**100.0**	**99.9**	98.6	97.0	26.8
3	−2	0	0	0	4.05	**99.9**	99.8	99.5	92.6	96.0	24.2
3	−1.5	0	0	0	3.74	**99.8**	99.6	99.1	86.9	94.0	20.5
3	−1	0	0	0	3.47	**99.5**	99.3	99.0	78.5	91.0	15.1
3	−0.5	0	0	0	3.26	**99.5**	99.1	99.3	68.9	98.0	10.4
3	0.5	0	0	0	3.06	99.3	98.9	**99.3**	64.2	94.0	3.1
3	1	0	0	0	3.09	**99.5**	99.1	99.6	69.5	**100.0**	1.4

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE 15 Overall two-shift successful identification percentage (%) for the third scenario with 5 variables

Shifts		MUR	DFT	MAB	ANN	rPCA
1st shift	98.6	**99.6**	74.7	86.9	94.9	68.6
2nd shifts	94.1	92.2	93.0	74.4	**94.4**	9.3

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

For the case of 5 variables:

Tables A5-A8 summarize the obtained results. As expected, the increased number of variables deteriorates the performance of the methods. This is more obvious for the MAB and rPCA methods.

4.2.2 | Second scenario–spatial power variance-covariance matrix

When the spatial power variance-covariance matrix is applied, the obtained results, presented in Tables A9-A16, are quite similar with ones obtained from the first scenario. Therefore, we may conclude whenever the performance criterion is the identification of at least one out-of-control variable the degree of correlation among the variables does not affect significantly the performance of the methods.

All the methods besides the rPCA succeed in identifying at least an out-of-control variable, as Table A16 presents.

4.2.3 | Third scenario–unstructured variance-covariance matrix

When the unstructured variance-covariance matrix is applied all the methods besides the rPCA method succeed in interpreting out-of-control signals. The performance of the MAB method is increased when the unstructured variance-covariance matrix is applied, as Tables 13-15 present.

5 | CONCLUSIONS

This paper aims to evaluate the performance of several techniques that are used to diagnose faults in out-of-control conditions in multivariate environments and that work in Shewhart manner, ie, they do not have the property of memory. From the obtained results, the ANN and the MYT methods seem to be the most promising ones. However, their performance depends on the number of shifted variables, the correlation among the variables, the shifted value, etc. The ANN method could achieve good performance on identifying out-of-control signals; however, special need should be given on several parameters, ie, the number of parameters to be monitored, the correlation degree among the variables, etc. Otherwise, its performance could be very poor.

Our future work will focus on investigating the applicability of the interpretation methods on big data.

ACKNOWLEDGMENTS

The authors wish to thank the editor and the referees for their thoughtful and detailed suggestions that improved the final version of the paper.

This work is part of the "Advanced Stochastic Models of Multivariate Statistical Process Control for Non-Industrial Processes" – B.829 research project, implemented within the framework of the "Aristeia" and cofinanced by National and Community Funds (from the Greek Ministry of Development-General Secretariat of Research and Technology and from EU—European Social Fund).

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How to cite this article: Bersimis S, Sgora A, Psarakis S. Methods for interpreting the out-of-control signal of multivariate control charts: A comparison study. *Qual Reliab Engng Int*. 2017;33:2295–2326. https://doi.org/10.1002/qre.2191

APPENDIX A

TABLE A1 Two-shift successful identification percentage (%) for the first scenario $\rho = -0.45$ with 3 variables

*K*1	*K*2	*K*3	δ	MYT	MUR	DFT	MAB	ANN	rPCA
−3	1	0	3.43	99.6	99.4	98.8	80.1	**100.0**	10.0
−3	1.5	0	3.70	99.8	99.7	99.2	88.1	**100.0**	13.7
−3	2	0	4.01	99.8	99.7	99.4	93.4	**100.0**	18.4
−3	2.5	0	4.36	99.9	99.9	99.6	96.1	**100.0**	20.6
−3	3	0	4.74	**100.0**	100.0	99.8	96.6	**100.0**	21.9
−2	−3	0	3.43	99.4	99.4	99.1	92.9	**100.0**	0.3
−2	−2	0	2.67	98.4	98.2	97.7	86.6	**100.0**	0.9
−2	−1.5	0	2.37	97.5	97.4	96.5	80.9	**100.0**	0.9
−2	−1	0	2.15	97.2	96.1	96.1	71.6	**100.0**	1.4
−2	1	0	2.46	98.2	97.1	96.6	76.0	**100.0**	13.2
−2	1.5	0	2.79	98.6	98.1	96.8	84.6	**100.0**	14.9
−2	2	0	3.16	99.5	99.1	98.1	90.9	**100.0**	16.6
−2	2.5	0	3.57	99.6	99.4	98.5	94.8	**100.0**	18.5
−2	3	0	4.01	99.9	99.8	99.5	96.7	**100.0**	16.2
−1.5	1	0	2.00	96.9	95.3	94.0	74.3	**100.0**	11.6
−1.5	1.5	0	2.37	97.7	96.9	95.8	84.2	**100.0**	14.6
−1.5	2	0	2.79	98.7	98.1	96.6	89.9	**100.0**	14.9
−1.5	2.5	0	3.23	99.5	99.2	98.2	93.3	**100.0**	16.2
−1.5	3	0	3.70	99.7	99.7	99.2	95.5	**100.0**	13.9
−1	−3	0	3.11	99.1	99.0	99.0	86.4	**100.0**	1.4
−1	−2	0	2.15	96.6	96.0	96.1	80.4	**100.0**	2.1
−1	−1.5	0	1.72	94.3	93.1	93.0	75.1	**100.0**	2.1
−1	−1	0	1.34	92.2	89.4	90.2	72.9	**100.0**	2.4
−1	1	0	1.58	94.6	91.7	91.6	74.6	**100.0**	12.3
−1	1.5	0	2.00	96.8	95.9	94.7	83.0	**100.0**	14.8
−1	2	0	2.46	98.4	97.5	96.8	86.1	**100.0**	13.0
−1	2.5	0	2.94	99.2	98.8	97.8	90.1	**100.0**	11.7
−1	3	0	3.43	99.6	99.3	99.0	93.7	**100.0**	10.3
−0.5	−3	0	3.06	99.1	99.0	98.8	85.6	**100.0**	2.7
0.5	3	0	3.06	99.3	99.1	98.9	85.5	**100.0**	2.4
1	−3	0	3.43	99.6	99.5	99.1	93.1	**100.0**	10.6
1	−2	0	2.46	98.0	97.4	96.1	82.6	**100.0**	12.4
1	−1.5	0	2.00	96.4	95.0	93.2	74.3	**100.0**	12.7
1	−1	0	1.58	94.1	91.9	91.1	63.5	**100.0**	12.3
1.5	−3	0	3.70	99.8	99.7	99.2	94.8	**100.0**	14.4
1.5	−2	0	2.79	98.9	98.1	97.0	85.6	**100.0**	15.8
1.5	−1.5	0	2.37	97.7	96.6	95.4	76.7	**100.0**	14.4
1.5	−1	0	2.00	96.5	94.7	93.6	65.9	**100.0**	12.5

Continues

TABLE A1 Continued

*K*1	*K*2	*K*3	δ	MYT	MUR	DFT	MAB	ANN	rPCA
2	−3	0	4.01	**100.0**	99.9	99.5	96.6	**100.0**	17.2
2	−2	0	3.16	99.4	99.3	98.1	88.6	**100.0**	16.9
2	−1.5	0	2.79	99.0	98.4	97.3	79.2	**100.0**	15.3
2	−1	0	2.46	98.1	97.4	96.2	66.7	**100.0**	11.8
3	−3	0	4.74	**100.0**	100.0	99.8	98.3	**100.0**	22.7
3	−2	0	4.01	99.9	99.9	99.5	92.1	**100.0**	17.6
3	−1.5	0	3.70	99.7	99.6	99.0	85.1	**100.0**	14.3
3	−1	0	3.43	99.4	99.3	98.7	77.8	**100.0**	10.2
3	−0.5	0	3.23	99.4	99.3	98.8	68.7	**100.0**	7.5
3	0.5	0	3.06	99.1	99.0	98.9	64.0	**100.0**	2.5
3	1	0	3.11	99.1	99.0	98.9	70.2	**100.0**	1.2

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE A2 Two-shift successful identification percentage (%) for the first scenario $\rho = 0.2$ with 3 variables

*K*1	*K*2	*K*3	δ	MYT	MUR	DFT	MAB	ANN	rPCA
−3	1	0	3.43	99.6	99.4	98.8	80.1	**100.0**	10.0
−3	1.5	0	3.70	99.8	99.7	99.2	88.1	**100.0**	13.7
−3	2	0	4.01	99.8	99.7	99.4	93.4	**100.0**	18.4
−3	2.5	0	4.36	99.9	99.9	99.6	96.1	**100.0**	20.6
−3	3	0	4.74	**100.0**	100.0	99.8	96.6	**100.0**	21.9
−2	−3	0	3.43	99.4	99.4	99.1	92.9	**100.0**	0.3
−2	−2	0	2.67	98.4	98.2	97.7	86.6	**100.0**	0.9
−2	−1.5	0	2.37	97.5	97.4	96.5	80.9	**100.0**	0.9
−2	−1	0	2.15	97.2	96.1	96.1	71.6	**100.0**	1.4
−2	1	0	2.46	98.2	97.1	96.6	76.0	**100.0**	13.2
−2	1.5	0	2.79	98.6	98.1	96.8	84.6	**100.0**	14.9
−2	2	0	3.16	99.5	99.1	98.1	90.9	**100.0**	16.6

Continues

TABLE A2 Continued

*K*1	*K*2	*K*3	δ	MYT	MUR	DFT	MAB	ANN	rPCA
−2	2.5	0	3.57	99.6	99.4	98.5	94.8	**100.0**	18.5
−2	3	0	4.01	99.9	99.8	99.5	96.7	**100.0**	16.2
−1.5	1	0	2.00	96.9	95.3	94.0	74.3	**100.0**	11.6
−1.5	1.5	0	2.37	97.7	96.9	95.8	84.2	**100.0**	14.6
−1.5	2	0	2.79	98.7	98.1	96.6	89.9	**100.0**	14.9
−1.5	2.5	0	3.23	99.5	99.2	98.2	93.3	**100.0**	16.2
−1.5	3	0	3.70	99.7	99.7	99.2	95.5	**100.0**	13.9
−1	−3	0	3.11	99.1	99.0	99.0	86.4	**100.0**	1.4
−1	−2	0	2.15	96.6	96.0	96.1	80.4	**100.0**	2.1
−1	−1.5	0	1.72	94.3	93.1	93.0	75.1	**100.0**	2.1
−1	−1	0	1.34	92.2	89.4	90.2	72.9	**100.0**	2.4
−1	1	0	1.58	94.6	91.7	91.6	74.6	**100.0**	12.3
−1	1.5	0	2.00	96.8	95.9	94.7	83.0	**100.0**	14.8
−1	2	0	2.46	98.4	97.5	96.8	86.1	**100.0**	13.0
−1	2.5	0	2.94	99.2	98.8	97.8	90.1	**100.0**	11.7
−1	3	0	3.43	99.6	99.3	99.0	93.7	**100.0**	10.3
−0.5	−3	0	3.06	99.1	99.0	98.8	85.6	**100.0**	2.7
0.5	3	0	3.06	99.3	99.1	98.9	85.5	**100.0**	2.4
1	−3	0	3.43	99.6	99.5	99.1	93.1	**100.0**	10.6
1	−2	0	2.46	98.0	97.4	96.1	82.6	**100.0**	12.4
1	−1.5	0	2.00	96.4	95.0	93.2	74.3	**100.0**	12.7
1	−1	0	1.58	94.1	91.9	91.1	63.5	**100.0**	12.3
1.5	−3	0	3.70	99.8	99.7	99.2	94.8	**100.0**	14.4
1.5	−2	0	2.79	98.9	98.1	97.0	85.6	**100.0**	15.8
1.5	−1.5	0	2.37	97.7	96.6	95.4	76.7	**100.0**	14.4
1.5	−1	0	2.00	96.5	94.7	93.6	65.9	**100.0**	12.5
2	−3	0	4.01	**100.0**	99.9	99.5	96.6	**100.0**	17.2
2	−2	0	3.16	99.4	99.3	98.1	88.6	**100.0**	16.9
2	−1.5	0	2.79	99.0	98.4	97.3	79.2	**100.0**	15.3
2	−1	0	2.46	98.1	97.4	96.2	66.7	**100.0**	11.8
3	−3	0	4.74	**100.0**	100.0	99.8	98.3	**100.0**	22.7
3	−2	0	4.01	99.9	99.9	99.5	92.1	**100.0**	17.6
3	−1.5	0	3.70	99.7	99.6	99.0	85.1	**100.0**	14.3
3	−1	0	3.43	99.4	99.3	98.7	77.8	**100.0**	10.2
3	−0.5	0	3.23	99.4	99.3	98.8	68.7	**100.0**	7.5
3	0.5	0	3.06	99.1	99.0	98.9	64.0	**100.0**	2.5
3	1	0	3.11	99.1	99.0	98.9	70.2	**100.0**	1.2

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE A3 Two-shift successful identification percentage (%) for the first scenario $\rho = 0.8$ with 3 variables

*K*1	*K*2	*K*3	δ	MYT	MUR	DFT	MAB	ANN	rPCA
−3	1	0	6.62	**100.0**	100.00	97.2	99.3	**100.0**	22.0
−3	1.5	0	7.27	**100.0**	100.00	99.0	98.8	**100.0**	27.3
−3	2	0	7.97	**100.0**	100.00	99.8	98.0	**100.0**	30.2
−3	2.5	0	8.71	**100.0**	100.00	**100.0**	97.9	**100.0**	34.9
−3	3	0	9.49	**100.0**	100.00	**100.0**	97.4	**100.0**	38.2
−2	−3	0	5.15	98.2	99.99	88.6	84.4	**100.0**	3.6
−2	−2	0	3.92	97.7	99.82	70.9	84.7	**100.0**	4.0
−2	−1.5	0	3.52	97.4	99.48	67.1	86.4	**100.0**	5.2
−2	−1	0	3.34	97.4	99.45	65.8	84.6	**100.0**	7.0
−2	1	0	4.84	**100.0**	99.98	75.8	96.2	**100.0**	19.9
−2	1.5	0	5.56	**100.0**	100.00	86.3	96.0	**100.0**	23.1
−2	2	0	6.32	**100.0**	100.00	95.0	93.1	**100.0**	26.5
−2	2.5	0	7.13	**100.0**	100.00	99.0	90.3	**100.0**	29.2
−2	3	0	7.97	**100.0**	100.00	99.8	88.8	**100.0**	30.6
−1.5	1	0	3.98	99.9	99.87	59.9	91.6	**100.0**	18.1
−1.5	1.5	0	4.74	**100.0**	99.98	73.1	96.4	**100.0**	19.9
−1.5	2	0	5.56	**100.0**	100.00	85.8	94.6	**100.0**	24.0
−1.5	2.5	0	6.40	**100.0**	100.00	95.4	91.4	**100.0**	26.3
−1.5	3	0	7.27	**100.0**	100.00	99.1	89.4	**100.0**	26.4
−1	−3	0	5.03	99.4	99.97	88.4	87.5	**100.0**	7.9
−1	−2	0	3.34	97.7	99.28	66.1	69.7	**100.0**	7.1
−1	−1.5	0	2.57	96.7	98.35	58.2	65.7	**100.0**	6.2
−1	−1	0	1.96	95.3	96.72	53.4	64.7	**100.0**	5.9
−1	1	0	3.16	99.8	99.44	51.5	87.4	**100.0**	14.6
−1	1.5	0	3.98	**100.0**	99.89	61.1	95.2	**100.0**	17.2
−1	2	0	4.84	**100.0**	99.99	76.0	96.1	**100.0**	20.3
−1	2.5	0	5.73	**100.0**	100.00	89.6	95.6	**100.0**	22.3
−1	3	0	6.62	**100.0**	100.00	97.5	94.0	**100.0**	22.8
−0.5	−3	0	5.23	99.8	99.98	88.8	92.9	**100.0**	10.6
0.5	3	0	5.23	99.9	100.00	89.7	97.1	**100.0**	10.3
1	−3	0	6.62	**100.0**	100.00	97.2	97.7	**100.0**	22.7
1	−2	0	4.84	**100.0**	99.99	76.2	96.2	**100.0**	19.1
1	−1.5	0	3.98	**100.0**	99.91	60.4	93.5	**100.0**	17.7
1	−1	0	3.16	99.9	99.48	50.5	86.2	**100.0**	14.7
1.5	−3	0	7.27	**100.0**	100.00	99.1	98.7	**100.0**	27.9
1.5	−2	0	5.56	**100.0**	100.00	86.2	98.1	**100.0**	23.4
1.5	−1.5	0	4.74	**100.0**	99.98	72.6	97.3	**100.0**	20.6
1.5	−1	0	3.98	99.9	99.83	59.2	90.3	**100.0**	17.6
2	−3	0	7.97	**100.0**	100.00	99.9	99.5	**100.0**	31.4
2	−2	0	6.32	**100.0**	100.00	95.4	99.3	**100.0**	25.9
2	−1.5	0	5.56	**100.0**	100.00	87.2	98.9	**100.0**	23.4
2	−1	0	4.84	**100.0**	99.99	75.4	95.5	**100.0**	19.6
3	−3	0	9.49	**100.0**	100.00	**100.0**	**100.0**	**100.0**	38.5
3	−2	0	7.97	**100.0**	100.00	99.9	**100.0**	**100.0**	31.2
3	−1.5	0	7.26	**100.0**	100.00	98.9	99.7	**100.0**	27.5
3	−1	0	6.62	**100.0**	100.00	97.1	98.8	**100.0**	22.7
3	−0.5	0	6.05	**100.0**	100.00	93.8	96.4	**100.0**	17.6
3	0.5	0	5.23	99.8	99.98	88.8	89.3	**100.0**	10.8
3	1	0	5.04	99.5	99.99	88.1	89.0	**100.0**	8.1

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE A4 Overall two-shift successful identification percentage (%) for the first scenario with 3 variables

ρ		MUR	DFT	MAB	ANN	rPCA
−0.45	98.3	98.5	81.5	81.1	**99.9**	77.9
0.2	97.1	96.2	96.0	77.9	**100.0**	7.3
0.8	98.8	99.2	76.3	85.8	**100.0**	13.9

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE A5 Two-shift successful identification percentage (%) for the first scenario $\rho = -0.2$ with 5 variables

$K1$	$K2$	$K3$	K4	K5	δ	MYT	MUR	DFT	MAB	rPCA	ANN
−3	1	0	0	0	3.42	**100.0**	95.5	98.0	33.7	**100.0**	0.8
−3	1.5	0	0	0	3.35	**100.0**	96.8	98.0	40.8	**100.0**	2.4
−3	2	0	0	0	3.42	**100.0**	98.0	98.5	49.8	**100.0**	3.0
−3	2.5	0	0	0	3.59	**100.0**	99.2	98.9	57.6	**100.0**	18.2
−3	3	0	0	0	3.87	**100.0**	99.5	99.4	64.8	**100.0**	5.1
−2	−3	0	0	0	5.63	**100.0**	92.9	**100.0**	74.6	**100.0**	4.4
−2	−2	0	0	0	4.47	**100.0**	77.7	99.4	51.2	**100.0**	11.5
−2	−1.5	0	0	0	3.93	**100.0**	71.3	98.4	40.4	**100.0**	6.9
−2	−1	0	0	0	3.42	**100.0**	69.3	96.4	31.4	**100.0**	9.6
−2	1	0	0	0	2.24	**100.0**	88.6	91.3	27.5	**100.0**	5.2
−2	1.5	0	0	0	2.33	**100.0**	93.2	93.8	36.5	**100.0**	17.9
−2	2	0	0	0	2.58	**100.0**	95.5	94.7	43.0	**100.0**	17.1
−2	2.5	0	0	0	2.96	**100.0**	97.1	97.1	46.7	**100.0**	14.1
−2	3	0	0	0	3.42	**100.0**	97.9	98.4	50.7	**100.0**	2.4
−1.5	1	0	0	0	1.71	**100.0**	85.8	86.3	29.0	**100.0**	12.0
−1.5	1.5	0	0	0	1.94	**100.0**	89.9	88.5	35.6	**100.0**	14.6
−1.5	2	0	0	0	2.33	**100.0**	93.1	93.6	36.4	**100.0**	8.7
−1.5	2.5	0	0	0	2.81	**100.0**	94.8	95.8	39.1	**100.0**	11.9
−1.5	3	0	0	0	3.35	**100.0**	96.3	98.0	43.3	**100.0**	18.9
−1	−3	0	0	0	4.65	**100.0**	87.8	99.7	47.7	**100.0**	12.8
−1	−2	0	0	0	3.42	**100.0**	69.4	96.7	27.8	**100.0**	13.6
−1	−1.5	0	0	0	2.81	**100.0**	60.0	92.6	22.0	**100.0**	10.6
−1	−1	0	0	0	2.24	**100.0**	54.4	86.7	19.6	**100.0**	12.4
−1	1	0	0	0	1.29	**100.0**	79.9	77.9	26.8	**100.0**	3.7
−1	1.5	0	0	0	1.71	**100.0**	84.5	85.7	28.2	**100.0**	2.7
−1	2	0	0	0	2.24	**100.0**	87.8	91.3	28.6	**100.0**	13.2
−1	2.5	0	0	0	2.81	**100.0**	91.7	95.5	29.3	**100.0**	1.2
−1	3	0	0	0	3.42	**100.0**	93.8	98.0	34.4	**100.0**	1.0
−0.5	−3	0	0	0	4.23	**100.0**	88.0	99.1	37.1	**100.0**	17.5
0.5	3	0	0	0	4.23	**100.0**	88.0	99.2	33.8	**100.0**	12.6
1	−3	0	0	0	3.42	**100.0**	94.1	97.9	33.4	**100.0**	12.6
1	−2	0	0	0	2.24	**100.0**	88.2	92.1	24.6	**100.0**	4.6
1	−1.5	0	0	0	1.71	**100.0**	83.9	85.5	22.4	**100.0**	3.7
1	−1	0	0	0	1.29	**100.0**	78.3	78.0	15.7	**100.0**	3.7
1.5	−3	0	0	0	3.35	**100.0**	96.4	98.1	39.8	**100.0**	3.8
1.5	−2	0	0	0	2.33	**100.0**	92.6	92.7	31.3	**100.0**	3.2
1.5	−1.5	0	0	0	1.94	**100.0**	90.9	88.6	25.3	**100.0**	5.5
1.5	−1	0	0	0	1.71	**100.0**	85.1	86.3	16.8	**100.0**	12.6

Continues

TABLE A5 Continued

*K*1	*K*2	*K*3	K4	K5	δ	MYT	MUR	DFT	MAB	rPCA	ANN
2	−3	0	0	0	3.42	**100.0**	98.0	98.4	48.5	**100.0**	2.8
2	−2	0	0	0	2.58	**100.0**	96.1	95.2	36.3	**100.0**	14.3
2	−1.5	0	0	0	2.33	**100.0**	93.3	93.4	28.2	**100.0**	3.4
2	−1	0	0	0	2.24	**100.0**	88.9	91.8	19.6	**100.0**	9.2
3	−3	0	0	0	3.87	**100.0**	99.6	99.5	63.2	**100.0**	13.6
3	−2	0	0	0	3.42	**100.0**	98.3	98.4	44.7	**100.0**	0.1
3	−1.5	0	0	0	3.35	**100.0**	97.1	98.4	34.6	**100.0**	13.4
3	−1	0	0	0	3.42	**100.0**	95.0	98.0	29.2	**100.0**	17.3
3	−0.5	0	0	0	3.59	**100.0**	92.0	98.1	24.8	**100.0**	4.4
3	0.5	0	0	0	4.23	**100.0**	88.2	99.2	34.8	**100.0**	10.1
3	1	0	0	0	4.65	**100.0**	88.6	99.6	45.2	**100.0**	13.8

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE A6 Two-shift successful identification percentage (%) for the first scenario $\rho = 0.5$ with 5 variables

*K*1	*K*2	*K*3	K4	K5	δ	MYT	MUR	DFT	MAB	rPCA	ANN
−3	1	0	0	0	4.32	98.8	92.1	**99.5**	54.0	98.4	23.1
−3	1.5	0	0	0	4.66	97.4	94.5	**99.7**	67.3	98.5	24.2
−3	2	0	0	0	5.07	94.9	96.3	**99.9**	79.8	98.4	27.2
−3	2.5	0	0	0	5.52	94.0	98.0	**100.0**	89.0	99.0	29.5
−3	3	0	0	0	6.00	95.8	98.7	**100.0**	94.4	98.5	32.7
−2	−3	0	0	0	4.20	99.3	89.6	**99.4**	80.4	98.8	8.0
−2	−2	0	0	0	3.27	97.5	78.4	97.2	65.0	**98.4**	9.1
−2	−1.5	0	0	0	2.90	97.0	75.8	95.8	54.1	**99.1**	10.4
−2	−1	0	0	0	2.65	97.0	75.2	93.8	39.7	**99.4**	11.2
−2	1	0	0	0	3.11	97.9	78.0	97.0	34.2	**99.9**	19.5
−2	1.5	0	0	0	3.52	97.2	82.2	**98.5**	43.7	99.5	21.7
−2	2	0	0	0	4.00	96.9	87.5	**99.2**	56.7	98.5	23.1
−2	2.5	0	0	0	4.52	96.8	92.4	99.7	68.8	**99.6**	25.6
−2	3	0	0	0	5.07	97.9	96.4	**99.9**	80.2	99.0	26.9
−1.5	1	0	0	0	2.53	97.7	70.5	93.9	28.8	**99.3**	17.4
−1.5	1.5	0	0	0	3.00	97.4	76.2	96.6	36.1	**99.8**	19.0
−1.5	2	0	0	0	3.52	97.4	82.8	98.1	46.3	**98.6**	20.5
−1.5	2.5	0	0	0	4.08	98.2	88.9	**99.4**	56.3	99.3	22.2
−1.5	3	0	0	0	4.66	99.0	93.8	**99.8**	69.5	98.8	25.4
−1	−3	0	0	0	3.83	**99.9**	90.0	98.5	63.8	98.3	10.7
−1	−2	0	0	0	2.65	99.2	75.2	94.3	45.6	**99.5**	10.8
−1	−1.5	0	0	0	2.10	98.6	66.5	88.4	40.5	**98.9**	9.2
−1	−1	0	0	0	1.63	97.9	62.4	84.3	33.9	**99.4**	8.5
−1	1	0	0	0	2.00	97.3	62.8	88.8	23.0	**98.9**	14.5
−1	1.5	0	0	0	2.53	98.0	69.9	94.0	29.5	**99.6**	17.8
−1	2	0	0	0	3.11	98.2	77.8	96.9	35.2	**98.3**	18.9
−1	2.5	0	0	0	3.71	**98.8**	85.1	98.7	45.6	98.6	20.8
−1	3	0	0	0	4.32	99.6	91.5	99.7	57.7	**99.8**	22.4

Continues

TABLE A6 Continued

*K*1	*K*2	*K*3	K4	K5	δ	MYT	MUR	DFT	MAB	rPCA	ANN
−0.5	−3	0	0	0	3.80	**100.0**	89.8	98.8	52.9	98.0	12.8
0.5	3	0	0	0	3.80	**99.9**	90.0	98.7	51.5	99.0	12.9
1	−3	0	0	0	4.32	99.6	92.1	**99.7**	55.1	99.6	21.8
1	−2	0	0	0	3.11	98.1	77.8	97.1	31.0	**99.5**	19.2
1	−1.5	0	0	0	2.53	97.7	70.7	93.5	23.0	**98.4**	16.8
1	−1	0	0	0	2.00	97.7	62.4	89.0	15.9	**99.2**	15.0
1.5	−3	0	0	0	4.66	98.7	93.8	99.8	65.9	**99.0**	24.5
1.5	−2	0	0	0	3.52	97.1	82.2	**98.5**	40.7	98.3	21.1
1.5	−1.5	0	0	0	3.00	96.9	74.9	97.0	30.6	**98.3**	19.5
1.5	−1	0	0	0	2.53	97.3	70.0	94.6	22.7	**99.9**	17.2
2	−3	0	0	0	5.07	98.0	96.2	**99.9**	77.8	98.9	27.6
2	−2	0	0	0	4.00	96.9	87.5	**99.4**	53.3	99.3	22.8
2	−1.5	0	0	0	3.52	97.3	82.0	98.4	40.2	**98.0**	19.6
2	−1	0	0	0	3.11	**98.0**	78.4	96.9	28.2	98.6	18.5
3	−3	0	0	0	6.00	95.4	99.0	**100.0**	94.3	98.8	32.1
3	−2	0	0	0	5.07	94.9	96.3	**99.9**	76.9	99.5	26.8
3	−1.5	0	0	0	4.66	97.0	94.6	99.8	63.5	**98.5**	24.4
3	−1	0	0	0	4.32	**98.9**	91.7	99.5	48.6	98.3	22.7
3	−0.5	0	0	0	4.05	99.4	90.8	99.2	37.6	99.7	20.3
3	0.5	0	0	0	3.80	**99.1**	90.2	98.6	34.7	99.0	15.6
3	1	0	0	0	3.83	98.3	90.1	98.7	45.5	**99.7**	13.1

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE A7 Two-shift successful identification percentage (%) for the first scenario $\rho = 0.8$ with 5 variables

*K*1	*K*2	*K*3	K4	K5	δ	MYT	MUR	DFT	MAB	rPCA	ANN
−3	1	0	0	0	6.80	91.2	90.9	**100.0**	84.5	99.5	35.9
−3	1.5	0	0	0	7.36	72.1	95.4	**100.0**	93.5	99.9	39.3
−3	2	0	0	0	8.00	68.8	97.4	**100.0**	97.2	99.1	41.4
−3	2.5	0	0	0	8.72	80.2	98.7	**100.0**	96.9	98.1	43.7
−3	3	0	0	0	9.49	91.5	98.7	**100.0**	95.0	99.9	46.5
−2	−3	0	0	0	6.42	99.8	83.1	**100.0**	98.5	98.8	15.9
−2	−2	0	0	0	4.98	95.6	59.7	99.8	90.2	99.2	15.3
−2	−1.5	0	0	0	4.43	93.3	53.2	**99.5**	78.3	98.7	15.3
−2	−1	0	0	0	4.05	95.3	52.3	**98.7**	59.8	98.1	18.4
−2	1	0	0	0	4.90	96.0	62.0	**99.9**	60.7	99.0	26.7
−2	1.5	0	0	0	5.57	88.1	72.7	**100.0**	80.1	99.4	30.3
−2	2	0	0	0	6.32	83.5	85.8	**100.0**	92.7	98.3	33.5
−2	2.5	0	0	0	7.14	89.1	94.3	**100.0**	96.6	98.9	37.1
−2	3	0	0	0	8.00	95.0	97.6	**100.0**	97.9	99.6	41.8
−1.5	1	0	0	0	4.00	97.2	45.9	99.6	42.3	**98.7**	22.9
−1.5	1.5	0	0	0	4.74	95.0	57.1	**99.9**	63.8	99.5	25.8
−1.5	2	0	0	0	5.57	93.8	73.5	**100.0**	80.2	98.3	29.7
−1.5	2.5	0	0	0	6.45	95.2	87.2	**100.0**	91.2	99.2	34.1
−1.5	3	0	0	0	7.36	97.5	95.3	**100.0**	95.8	99.6	38.7

Continues

TABLE A7 Continued

*K*1	*K*2	*K*3	K4	K5	δ	MYT	MUR	DFT	MAB	rPCA	ANN
−1	−3	0	0	0	5.90	**100.0**	83.0	99.9	93.2	98.6	18.4
−1	−2	0	0	0	4.05	**99.9**	53.1	98.9	69.9	98.4	14.9
−1	−1.5	0	0	0	3.21	99.3	40.5	97.1	56.1	**100.0**	13.2
−1	−1	0	0	0	2.49	97.6	34.8	92.8	41.3	**98.1**	12.5
−1	1	0	0	0	3.16	97.2	37.1	98.4	26.9	**99.7**	19.7
−1	1.5	0	0	0	4.00	98.0	45.6	**99.6**	42.2	98.5	23.5
−1	2	0	0	0	4.90	98.5	60.7	**99.9**	62.4	98.3	28.2
−1	2.5	0	0	0	5.84	98.8	79.3	**100.0**	79.0	98.6	31.4
−1	3	0	0	0	6.80	99.2	91.2	**100.0**	89.1	99.2	35.9
−0.5	−3	0	0	0	5.88	**100.0**	82.7	99.9	86.6	99.2	22.8
0.5	3	0	0	0	5.88	**100.0**	83.1	99.9	82.1	99.9	21.7
1	−3	0	0	0	6.80	99.2	90.9	**100.0**	87.2	99.5	36.1
1	−2	0	0	0	4.90	98.4	61.9	**100.0**	56.6	98.8	27.2
1	−1.5	0	0	0	4.00	97.8	46.0	**99.5**	38.7	98.4	24.0
1	−1	0	0	0	3.16	97.3	38.2	98.3	22.9	**98.9**	20.0
1.5	−3	0	0	0	7.36	97.5	95.1	**100.0**	94.1	99.7	38.7
1.5	−2	0	0	0	5.57	93.2	72.9	**100.0**	78.2	99.2	30.2
1.5	−1.5	0	0	0	4.74	94.7	57.0	99.9	60.4	99.0	26.3
1.5	−1	0	0	0	4.00	97.5	46.5	**99.5**	38.9	98.7	23.1
2	−3	0	0	0	8.00	95.0	97.7	**100.0**	97.7	99.9	40.8
2	−2	0	0	0	6.32	83.3	85.9	**100.0**	91.6	98.3	33.6
2	−1.5	0	0	0	5.57	88.0	73.6	**100.0**	80.0	99.1	30.2
2	−1	0	0	0	4.90	96.2	62.0	**99.9**	58.5	99.8	26.7
3	−3	0	0	0	9.49	91.5	98.6	**100.0**	99.8	99.2	46.8
3	−2	0	0	0	8.00	68.3	97.7	**100.0**	98.0	**100.0**	41.2
3	−1.5	0	0	0	7.36	73.5	95.4	**100.0**	93.4	99.7	38.7
3	−1	0	0	0	6.80	90.3	91.1	**100.0**	83.3	98.4	36.4
3	−0.5	0	0	0	6.35	98.7	85.8	**100.0**	68.0	99.8	32.4
3	0.5	0	0	0	5.88	98.1	82.8	**99.9**	64.1	99.5	26.6
3	1	0	0	0	5.90	91.3	83.1	**99.9**	74.0	99.2	23.6

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE A8 Overall two-shift successful identification percentage (%) for the first scenario with 5 variables

ρ	MYT	MUR	DFT	MAB	ANN	rPCA
−0.2	100.0%	82.0%	92.9%	33.9%	**100.0%**	9.5%
0.5	98.0%	79.3%	94.3%	45.0%	**98.9%**	15.1%
0.8	94.7%	65.9%	98.3%	63.7%	**99.0%**	23.3%

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE A9 Two-shift successful identification percentage (%) for the second scenario $\rho = -0.45$ with 3 variables

*K*1	*K*2	*K*3	δ	MYT	MUR	DFT	MAB	ANN	rPCA
−3	1	0	3.38	99.2	**98.9**	98.5	76.6	96.0	3.3
−3	1.5	0	3.36	99.2	**99.2**	98.9	83.3	99.0	2.1
−3	2	0	3.44	99.3	**99.4**	98.7	90.7	96.0	0.8
−3	2.5	0	3.60	99.5	**99.6**	98.9	95.3	96.0	0.4
−3	3	0	3.83	**99.5**	99.7	99.0	96.8	92.0	0.2
−2	−3	0	4.91	**100.0**	**100.0**	98.4	99.7	94.0	24.4
−2	−2	0	3.94	99.7	99.8	93.2	97.3	**100.0**	24.2
−2	−1.5	0	3.50	**99.6**	99.4	90.3	92.7	90.0	22.0
−2	−1	0	3.09	**99.0**	98.5	88.9	85.9	93.0	18.7
−2	1	0	2.24	96.6	96.1	95.2	73.5	**100.0**	4.2
−2	1.5	0	2.34	94.7	97.3	95.3	83.3	**100.0**	2.1
−2	2	0	2.56	95.8	**97.5**	95.1	87.3	92.0	1.5
−2	2.5	0	2.87	96.8	**98.1**	95.9	88.5	92.0	1.0
−2	3	0	3.25	97.6	**98.8**	97.6	90.3	96.0	0.6
−1.5	1	0	1.72	**98.8**	93.3	91.5	72.9	93.0	3.7
−1.5	1.5	0	1.92	**99.6**	95.6	92.4	78.8	92.0	2.3
−1.5	2	0	2.24	98.6	96.9	93.0	79.8	**99.0**	2.0
−1.5	2.5	0	2.65	97.8	**98.6**	94.6	82.1	91.0	2.2
−1.5	3	0	3.10	95.8	**99.3**	97.0	86.0	90.0	1.5
−1	−3	0	4.02	99.6	**99.8**	95.5	97.8	91.0	15.1
−1	−2	0	2.96	98.6	**99.0**	86.3	91.9	96.0	16.6
−1	−1.5	0	2.45	**97.8**	97.6	81.9	87.0	91.0	17.0
−1	−1	0	1.97	95.8	94.0	77.4	80.0	**100.0**	16.3
−1	1	0	1.28	**92.1**	89.4	86.3	68.5	93.0	3.6
−1	1.5	0	1.62	93.5	92.8	87.1	72.9	**98.0**	3.3
−1	2	0	2.07	96.6	**96.9**	91.2	72.8	94.0	3.1
−1	2.5	0	2.55	97.6	98.5	93.8	78.2	**99.0**	2.8
−1	3	0	3.07	98.6	**99.4**	96.1	83.8	95.0	2.3
−0.5	−3	0	3.65	99.5	**99.8**	95.0	95.3	94.0	10.0
0.5	3	0	3.65	99.4	**99.8**	94.9	92.8	97.0	10.6
1	−3	0	3.07	98.5	**99.3**	96.0	83.1	92.0	2.7
1	−2	0	2.07	96.4	**96.6**	90.8	70.7	92.0	3.0
1	−1.5	0	1.62	92.4	94.7	88.9	67.3	**95.0**	3.6
1	−1	0	1.28	78.2	89.0	85.6	61.3	**93.0**	3.7
1.5	−3	0	3.10	98.5	**99.6**	96.7	85.1	97.0	1.4
1.5	−2	0	2.24	97.3	**97.7**	93.7	77.1	93.0	2.3
1.5	−1.5	0	1.92	**96.0**	95.4	91.8	71.1	93.0	2.0
1.5	−1	0	1.72	95.1	93.3	91.9	62.3	**95.0**	3.2
2	−3	0	3.25	99.0	**99.5**	97.9	88.7	94.0	0.6
2	−2	0	2.56	97.8	**98.1**	95.5	82.5	96.0	1.2
2	−1.5	0	2.34	**97.5**	96.8	95.5	75.0	90.0	2.1
2	−1	0	2.24	**97.4**	96.4	95.6	63.6	92.0	3.8
3	−3	0	3.83	99.6	**99.7**	99.1	95.1	93.0	0.2
3	−2	0	3.44	99.1	**99.2**	98.5	86.5	94.0	1.1
3	−1.5	0	3.36	**99.2**	**99.2**	98.8	79.3	99.0	2.1
3	−1	0	3.38	**99.4**	99.1	98.7	71.1	96.0	3.6
3	−0.5	0	3.49	**99.6**	99.1	98.3	68.9	94.0	6.0
3	0.5	0	3.95	**99.8**	99.5	97.4	81.4	92.0	14.5
3	1	0	4.27	**99.9**	99.7	97.2	90.0	92.0	19.4

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE A10 Two-shift successful identification percentage (%) for the second scenario $\rho = 0.2$ with 3 variables

***K*1**	***K*2**	***K*3**	***δ***	**MYT**	**MUR**	**DFT**	**MAB**	**ANN**	**rPCA**
−3	1	0	3.47	**99.7**	99.4	99.0	82.7	91.0	15.3
−3	1.5	0	3.74	**99.8**	99.6	99.0	89.6	94.0	20.2
−3	2	0	4.05	**99.9**	**99.9**	99.4	95.2	90.0	23.6
−3	2.5	0	4.40	**99.9**	**99.9**	99.5	98.1	99.0	25.0
−3	3	0	4.78	**100.0**	99.9	99.8	99.4	99.0	25.8
−2	−3	0	3.35	**99.5**	99.3	99.4	94.6	99.0	0.2
−2	−2	0	2.61	**98.5**	97.9	98.1	87.7	98.0	0.4
−2	−1.5	0	2.33	97.9	97.1	**98.0**	80.5	**98.0**	1.1
−2	−1	0	2.13	97.6	96.5	97.6	71.7	**98.0**	1.3
−2	1	0	2.49	98.4	97.4	95.9	77.2	**99.0**	15.1
−2	1.5	0	2.82	**99.0**	98.3	96.8	85.9	98.0	19.6
−2	2	0	3.19	**99.3**	99.1	97.6	92.4	97.0	20.6
−2	2.5	0	3.60	99.6	99.4	98.5	96.0	**100.0**	18.4
−2	3	0	4.03	**99.8**	99.7	99.3	97.8	94.0	15.8
−1.5	1	0	2.03	**96.9**	95.7	94.3	77.1	92.0	15.5
−1.5	1.5	0	2.39	97.7	96.8	94.9	84.3	**99.0**	16.9
−1.5	2	0	2.80	**98.6**	98.0	96.1	90.0	97.0	15.9
−1.5	2.5	0	3.24	**99.4**	99.2	98.0	94.1	96.0	13.9
−1.5	3	0	3.70	**99.6**	**99.6**	98.9	96.3	95.0	10.1
−1	−3	0	3.03	**99.2**	99.0	99.0	86.9	98.0	0.6
−1	−2	0	2.10	**97.4**	96.5	97.0	80.7	97.0	1.1
−1	−1.5	0	1.67	95.6	94.3	95.5	77.1	**100.0**	1.6
−1	−1	0	1.31	92.0	89.7	**92.4**	73.5	90.0	2.9
−1	1	0	1.59	**94.3**	92.2	90.5	75.3	94.0	13.7
−1	1.5	0	2.01	**96.6**	95.3	93.2	82.5	95.0	14.0
−1	2	0	2.47	**97.9**	97.1	95.4	86.6	96.0	12.5
−1	2.5	0	2.94	**98.9**	98.6	97.2	90.7	93.0	10.2
−1	3	0	3.42	**99.5**	99.5	98.8	94.1	95.0	7.1
−0.5	−3	0	3.00	99.0	98.9	99.0	84.9	**100.0**	1.5
0.5	3	0	3.00	**99.2**	99.1	99.1	84.7	92.0	1.5
1	−3	0	3.42	**99.6**	**99.6**	99.0	92.2	90.0	8.0
1	−2	0	2.47	**97.9**	97.4	95.7	83.5	95.0	12.0
1	−1.5	0	2.01	**96.7**	95.2	93.6	75.3	95.0	14.1
1	−1	0	1.59	**93.6**	91.9	90.5	64.1	93.0	14.2
1.5	−3	0	3.70	**99.7**	**99.7**	98.8	95.1	94.0	11.5
1.5	−2	0	2.80	98.6	98.1	96.3	85.8	**99.0**	15.8
1.5	−1.5	0	2.39	**97.9**	97.0	94.7	76.7	91.0	17.3
1.5	−1	0	2.03	**96.0**	95.0	93.2	66.0	93.0	15.7
2	−3	0	4.03	**99.8**	**99.8**	99.2	96.7	94.0	15.8
2	−2	0	3.19	99.2	99.1	97.4	88.0	**100.0**	20.4
2	−1.5	0	2.82	**99.0**	98.3	96.6	80.2	98.0	18.7
2	−1	0	2.49	**97.9**	97.0	96.1	68.6	90.0	16.4
3	−3	0	4.78	**100.0**	**100.0**	99.9	98.6	97.0	26.8
3	−2	0	4.05	**99.9**	99.8	99.5	92.6	96.0	24.2
3	−1.5	0	3.74	**99.8**	99.6	99.1	86.9	94.0	20.5
3	−1	0	3.47	**99.5**	99.3	99.0	78.5	91.0	15.1
3	−0.5	0	3.26	**99.5**	99.1	99.3	68.9	98.0	10.4
3	0.5	0	3.06	**99.3**	98.9	**99.3**	64.2	94.0	3.1
3	1	0	3.09	99.5	99.1	99.6	69.5	**100.0**	1.4

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE A11 Two-shift successful identification percentage (%) for the second scenario $\rho = 0.8$ with 3 variables

$K1$	$K2$	$K3$	δ	MYT	MUR	DFT	MAB	ANN	rPCA
−3	1	0	6.55	**100.0**	**100.0**	96.7	99.9	96.0	28.4
−3	1.5	0	7.43	**100.0**	100.0	98.9	**100.0**	98.0	34.9
−3	2	0	8.36	**100.0**	100.0	99.8	99.9	90.0	43.9
−3	2.5	0	9.32	**100.0**	100.0	**100.0**	99.9	94.0	50.0
−3	3	0	10.30	**100.0**	100.0	**100.0**	99.9	**100.0**	55.5
−2	−3	0	5.04	98.1	**100.0**	88.9	91.9	98.0	4.6
−2	−2	0	3.40	97.1	99.7	74.5	84.4	90.0	5.5
−2	−1.5	0	2.83	97.1	98.7	76.5	87.7	90.0	5.5
−2	−1	0	2.60	97.9	98.0	79.7	84.2	98.0	7.5
−2	1	0	4.96	**100.0**	100.0	74.8	99.0	99.0	27.0
−2	1.5	0	5.89	**100.0**	100.0	86.6	99.4	93.0	33.1
−2	2	0	6.86	**100.0**	100.0	94.6	99.7	94.0	39.0
−2	2.5	0	7.86	**100.0**	100.0	98.8	99.5	99.0	42.9
−2	3	0	8.88	**100.0**	100.0	99.9	99.6	98.0	46.0
−1.5	1	0	4.18	99.9	99.9	58.8	97.1	90.0	24.0
−1.5	1.5	0	5.15	**100.0**	100.0	71.8	99.1	97.0	30.2
−1.5	2	0	6.15	**100.0**	100.0	87.0	99.4	99.0	33.8
−1.5	2.5	0	7.17	**100.0**	100.0	95.4	99.3	90.0	37.4
−1.5	3	0	8.20	**100.0**	100.0	99.2	99.3	97.0	40.9
−1	−3	0	5.52	99.0	**100.0**	88.0	93.4	90.0	10.7
−1	−2	0	3.48	97.7	99.5	64.6	67.7	99.0	10.1
−1	−1.5	0	2.52	96.4	98.0	58.8	58.1	92.0	8.7
−1	−1	0	1.70	94.3	94.2	59.5	63.1	92.0	7.1
−1	1	0	3.43	99.5	99.5	46.8	93.2	**100.0**	21.1
−1	1.5	0	4.44	99.8	99.9	58.3	97.6	99.0	25.7
−1	2	0	5.47	99.9	**100.0**	75.0	98.9	99.0	30.6
−1	2.5	0	6.51	**100.0**	100.0	90.0	98.9	96.0	32.1
−1	3	0	7.56	**100.0**	100.0	97.1	99.0	90.0	33.7
−0.5	−3	0	5.92	99.7	**100.0**	88.5	94.9	96.0	15.9
0.5	3	0	5.92	99.7	**100.0**	89.0	97.4	93.0	15.4
1	−3	0	7.56	**100.0**	100.0	97.3	97.3	93.0	33.2
1	−2	0	5.47	99.9	**100.0**	75.3	94.3	98.0	29.7
1	−1.5	0	4.44	99.8	**100.0**	58.2	92.8	93.0	26.5
1	−1	0	3.43	99.3	99.4	46.6	87.0	92.0	21.3
1.5	−3	0	8.20	**100.0**	100.0	99.2	98.3	98.0	39.7
1.5	−2	0	6.15	**100.0**	100.0	86.0	96.7	92.0	35.5
1.5	−1.5	0	5.15	**100.0**	100.0	72.2	95.9	97.0	29.5
1.5	−1	0	4.18	99.9	99.9	59.1	93.4	90.0	24.0
2	−3	0	8.88	**100.0**	100.0	99.8	99.0	91.0	45.7
2	−2	0	6.86	**100.0**	100.0	94.7	98.5	92.0	39.5
2	−1.5	0	5.89	**100.0**	100.0	86.6	98.1	96.0	33.1
2	−1	0	4.96	**100.0**	100.0	75.2	96.9	95.0	27.2
3	−3	0	10.30	**100.0**	100.0	**100.0**	99.8	93.0	54.2
3	−2	0	8.36	**100.0**	100.0	99.9	99.8	91.0	42.2
3	−1.5	0	7.43	**100.0**	100.0	99.1	99.6	95.0	34.6
3	−1	0	6.55	**100.0**	100.0	97.0	99.2	95.0	28.0
3	−0.5	0	5.73	**100.0**	100.0	94.0	97.4	**100.0**	21.2
3	0.5	0	4.41	**100.0**	99.9	90.8	88.3	98.0	11.7
3	1	0	4.03	**99.8**	**99.8**	91.2	84.9	96.0	8.2

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE A12 Overall two-shift successful identification percentage (%) for the second scenario with 3 variables

ρ		MUR	DFT	MAB	ANN	rPCA
−0.45	97.5	**97.6**	94.0	82.1	95.1	6.0
0.20	98.5	97.9	97.2	84.5	**97.3**	12.7
0.80	**99.3**	97.6	93.9	82.2	95.0	23.3

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE A13 two shifts succesful identification percentage(%) for the 2nd Scenario $\rho = -0.2$ with 5 variables

$K1$	$K2$	$K3$	K4	K5	δ	MYT	MUR	DFT	MAB	ANN	rPCA
−3	1	0	0	0	3.03	**100.0**	97.3	98.5	33.9	99.1	3.9
−3	1.5	0	0	0	3.15	**100.0**	97.9	98.6	42.0	99.2	2.4
−3	2	0	0	0	3.35	**100.0**	98.2	98.7	49.4	99.5	0.6
−3	2.5	0	0	0	3.61	**100.0**	98.9	99.1	56.8	**100.0**	0.5
−3	3	0	0	0	3.92	**100.0**	99.3	99.5	64.4	99.1	0.4
−2	−3	0	0	0	4.05	**100.0**	99.2	97.8	62.0	99.9	21.4
−2	−2	0	0	0	3.19	**100.0**	96.6	94.2	45.6	99.5	19.2
−2	−1.5	0	0	0	2.80	**100.0**	93.9	91.1	38.2	99.8	17.5
−2	−1	0	0	0	2.47	**100.0**	91.7	89.9	33.9	99.0	15.1
−2	1	0	0	0	2.10	**100.0**	90.1	91.9	30.6	99.7	3.5
−2	1.5	0	0	0	2.31	**100.0**	92.8	94.0	36.1	99.6	2.2
−2	2	0	0	0	2.61	**100.0**	94.4	94.8	40.8	99.8	1.2
−2	2.5	0	0	0	2.98	**100.0**	96.5	97.0	46.0	99.2	1.1
−2	3	0	0	0	3.38	**100.0**	98.2	98.1	51.4	99.8	0.7
−1.5	1	0	0	0	1.67	**100.0**	84.1	86.8	28.8	99.4	4.4
−1.5	1.5	0	0	0	1.96	**100.0**	86.8	89.4	32.6	99.7	2.6
−1.5	2	0	0	0	2.33	**100.0**	92.6	93.4	35.6	99.7	2.7
−1.5	2.5	0	0	0	2.75	**100.0**	95.4	96.0	40.5	99.5	1.2
−1.5	3	0	0	0	3.20	**100.0**	97.5	97.7	45.5	99.7	1.0
−1	−3	0	0	0	3.47	**100.0**	98.2	96.8	46.2	99.2	14.1
−1	−2	0	0	0	2.49	**100.0**	92.3	89.3	34.0	99.7	14.3
−1	−1.5	0	0	0	2.03	**100.0**	87.6	83.0	29.4	99.1	15.0
−1	−1	0	0	0	1.59	**100.0**	80.1	76.5	27.4	**100.0**	13.0
−1	1	0	0	0	1.31	**100.0**	76.8	78.4	27.0	99.5	3.2
−1	1.5	0	0	0	1.69	**100.0**	85.3	86.5	27.2	99.2	3.0
−1	2	0	0	0	2.13	**100.0**	90.3	90.7	31.0	99.7	2.4
−1	2.5	0	0	0	2.60	**100.0**	94.0	94.8	34.0	99.5	2.3
−1	3	0	0	0	3.09	**100.0**	97.5	97.3	39.8	99.8	1.7
−0.5	−3	0	0	0	3.26	**100.0**	97.7	96.0	39.8	**100.0**	9.0
0.5	3	0	0	0	3.26	**100.0**	97.8	96.8	37.5	99.8	11.0
1	−3	0	0	0	3.09	**100.0**	97.3	96.7	37.9	99.1	2.0
1	−2	0	0	0	2.13	**100.0**	90.5	90.7	26.9	99.4	2.8
1	−1.5	0	0	0	1.69	**100.0**	83.3	84.9	22.3	99.5	3.2
1	−1	0	0	0	1.31	**100.0**	76.8	78.6	16.8	99.6	4.0
1.5	−3	0	0	0	3.20	**100.0**	98.0	97.6	42.6	99.1	1.1
1.5	−2	0	0	0	2.33	**100.0**	92.5	93.1	31.5	99.9	1.5
1.5	−1.5	0	0	0	1.96	**100.0**	88.0	90.2	26.1	99.9	2.8
1.5	−1	0	0	0	1.67	**100.0**	85.0	86.8	18.2	99.6	3.0

Continues

TABLE A13 Continued

*K*1	*K*2	*K*3	K4	K5	δ	MYT	MUR	DFT	MAB	ANN	rPCA
2	−3	0	0	0	3.38	**100.0**	98.4	98.5	49.0	99.7	0.4
2	−2	0	0	0	2.61	**100.0**	95.2	95.4	35.3	99.0	2.1
2	−1.5	0	0	0	2.31	**100.0**	92.9	94.0	27.5	99.8	1.8
2	−1	0	0	0	2.10	**100.0**	90.7	92.9	22.0	99.0	3.4
3	−3	0	0	0	3.92	**100.0**	99.3	99.4	62.1	99.2	0.7
3	−2	0	0	0	3.35	**100.0**	98.2	98.6	45.2	99.1	0.4
3	−1.5	0	0	0	3.15	**100.0**	98.0	98.8	35.4	99.8	1.3
3	−1	0	0	0	3.03	**100.0**	97.5	98.4	30.9	99.6	1.8
3	−0.5	0	0	0	3.00	**100.0**	97.1	98.0	25.5	99.5	2.5
3	0.5	0	0	0	3.20	**100.0**	97.7	97.9	28.5	99.9	7.7
3	1	0	0	0	3.42	**100.0**	98.3	97.4	36.4	99.6	12.4

TABLE A14 Two-shift successful identification percentage (%) for the second scenario $\rho = 0.5$ with 5 variables

*K*1	*K*2	*K*3	K4	K5	δ	MYT	MUR	DFT	MAB	rPCA	ANN
−3	1	0	0	0	4.20	99.6	99.3	93.1	57.1	**100.0**	10.3
−3	1.5	0	0	0	4.66	99.2	**99.8**	94.9	70.0	99.4	16.6
−3	2	0	0	0	5.16	98.2	**99.9**	96.4	81.9	99.3	17.9
−3	2.5	0	0	0	5.69	97.3	**99.9**	98.3	90.3	99.1	20.5
−3	3	0	0	0	6.25	97.6	**100.0**	99.1	95.9	99.7	26.4
−2	−3	0	0	0	3.51	**99.5**	98.4	94.7	47.1	99.1	1.1
−2	−2	0	0	0	2.58	97.1	93.9	90.3	39.2	**99.3**	1.8
−2	−1.5	0	0	0	2.25	96.3	92.0	90.2	36.2	**99.4**	1.8
−2	−1	0	0	0	2.08	97.0	90.4	88.8	28.5	**99.9**	3.4
−2	1	0	0	0	3.11	**99.4**	95.7	78.5	37.1	99.3	13.1
−2	1.5	0	0	0	3.62	**99.2**	97.6	80.8	46.2	**99.2**	15.2
−2	2	0	0	0	4.16	**99.2**	99.0	86.2	58.3	99.4	18.8
−2	2.5	0	0	0	4.73	99.0	99.8	91.3	70.3	**100.0**	20.2
−2	3	0	0	0	5.32	99.0	**99.9**	96.1	81.6	99.2	23.0
−1.5	1	0	0	0	2.58	99.3	91.8	69.1	30.1	**99.8**	13.7
−1.5	1.5	0	0	0	3.12	**99.5**	95.8	73.6	36.6	99.2	14.7
−1.5	2	0	0	0	3.70	99.4	98.0	80.4	47.5	**99.8**	16.4
−1.5	2.5	0	0	0	4.29	99.4	99.3	87.0	57.3	**99.5**	18.4
−1.5	3	0	0	0	4.90	99.5	**99.9**	93.0	70.5	99.5	19.7
−1	−3	0	0	0	3.51	**99.9**	97.9	92.5	32.5	99.2	3.2
−1	−2	0	0	0	2.31	99.2	91.6	82.4	24.0	**99.8**	5.7
−1	−1.5	0	0	0	1.76	98.2	84.4	76.4	25.1	**99.0**	3.3
−1	−1	0	0	0	1.29	98.3	78.6	72.2	24.5	**99.4**	3.7
−1	1	0	0	0	2.08	**99.6**	87.0	61.1	23.9	99.6	12.0
−1	1.5	0	0	0	2.66	**99.6**	92.9	67.2	29.4	99.3	15.5
−1	2	0	0	0	3.27	**99.5**	96.6	74.9	35.7	99.4	14.8
−1	2.5	0	0	0	3.88	**99.8**	98.5	82.9	45.0	99.5	16.8
−1	3	0	0	0	4.51	**99.8**	99.6	90.1	56.7	99.6	15.8
−0.5	−3	0	0	0	3.65	**100.0**	98.3	90.4	29.9	99.0	5.8
0.5	3	0	0	0	3.65	**100.0**	98.5	91.0	29.3	99.5	4.1
1	−3	0	0	0	4.51	**99.9**	99.5	91.0	53.0	99.0	15.8
1	−2	0	0	0	3.27	**99.5**	96.7	75.4	29.6	99.0	15.3
1	−1.5	0	0	0	2.66	99.4	92.7	67.1	21.2	**99.9**	13.7

Continues

TABLE A14 Continued

*K*1	*K*2	*K*3	K4	K5	δ	MYT	MUR	DFT	MAB	rPCA	ANN
1	−1	0	0	0	2.08	99.5	86.3	60.1	14.5	**99.8**	12.3
1.5	−3	0	0	0	4.90	99.5	**99.8**	93.1	65.9	99.2	19.6
1.5	−2	0	0	0	3.70	99.4	98.1	79.4	39.9	**99.8**	16.7
1.5	−1.5	0	0	0	3.12	**99.2**	95.9	72.5	29.4	**99.2**	15.7
1.5	−1	0	0	0	2.58	99.3	93.0	68.8	22.0	**99.1**	12.9
2	−3	0	0	0	5.32	99.1	**99.9**	96.2	78.4	99.6	22.7
2	−2	0	0	0	4.16	99.1	99.2	86.8	52.8	**99.4**	18.2
2	−1.5	0	0	0	3.62	**99.2**	98.0	80.9	39.8	**99.2**	15.5
2	−1	0	0	0	3.11	99.3	96.1	78.6	28.4	**99.7**	13.5
3	−3	0	0	0	6.25	97.8	**100.0**	99.3	93.9	**100.0**	25.1
3	−2	0	0	0	5.16	98.5	**99.9**	96.5	77.0	99.1	18.1
3	−1.5	0	0	0	4.66	98.9	99.8	95.1	65.7	**99.8**	16.4
3	−1	0	0	0	4.20	99.6	99.2	92.8	50.9	**99.7**	10.1
3	−0.5	0	0	0	3.80	**99.8**	98.7	93.4	39.2	99.4	8.9
3	0.5	0	0	0	3.23	99.3	97.5	95.5	27.2	**99.6**	5.6
3	1	0	0	0	3.11	98.7	97.5	97.0	31.8	**99.5**	2.0

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE A15 Two-shift successful identification percentage (%) for the second scenario $\rho = 0.8$ with 5 variables

*K*1	*K*2	*K*3	K4	K5	δ	MYT	MUR	DFT	MAB	rPCA	ANN
−3	1	0	0	0	6.55	97.0	**100.0**	91.4	85.3	99.4	25.8
−3	1.5	0	0	0	7.43	86.6	**100.0**	95.8	94.3	99.8	28.8
−3	2	0	0	0	8.36	77.8	**100.0**	98.0	98.3	99.7	32.1
−3	2.5	0	0	0	9.32	82.6	**100.0**	99.0	99.6	99.3	35.2
−3	3	0	0	0	10.30	91.3	**100.0**	99.0	99.5	99.7	38.3
−2	−3	0	0	0	5.04	**99.9**	99.2	83.9	63.1	99.1	11.9
−2	−2	0	0	0	3.40	93.6	**97.0**	69.8	47.9	99.1	10.9
−2	−1.5	0	0	0	2.83	86.7	**95.6**	71.6	44.3	99.2	10.6
−2	−1	0	0	0	2.60	88.2	93.5	72.5	35.8	**99.2**	12.8
−2	1	0	0	0	4.96	99.1	**99.8**	62.5	63.7	99.2	20.6
−2	1.5	0	0	0	5.89	96.1	**100.0**	72.6	82.4	99.5	23.5
−2	2	0	0	0	6.86	90.1	**100.0**	86.0	93.7	99.6	28.5
−2	2.5	0	0	0	7.86	90.4	**100.0**	94.4	97.1	99.3	30.2
−2	3	0	0	0	8.88	94.8	**100.0**	97.6	98.3	**100.0**	33.6
−1.5	1	0	0	0	4.18	99.3	**99.3**	45.4	45.4	99.1	18.1
−1.5	1.5	0	0	0	5.15	98.6	**99.8**	56.4	66.7	99.0	21.1
−1.5	2	0	0	0	6.15	96.5	**100.0**	73.5	83.0	99.1	24.6
−1.5	2.5	0	0	0	7.17	95.3	**100.0**	87.3	92.0	99.3	27.6
−1.5	3	0	0	0	8.20	97.0	**100.0**	95.2	95.4	99.4	30.9
−1	−3	0	0	0	5.52	**100.0**	99.6	82.4	44.7	99.4	14.8
−1	−2	0	0	0	3.48	**99.9**	96.7	58.2	22.8	**99.9**	12.4
−1	−1.5	0	0	0	2.52	98.7	91.3	50.2	20.9	**99.8**	11.1
−1	−1	0	0	0	1.70	93.6	83.0	49.4	23.8	**99.3**	9.2
−1	1	0	0	0	3.43	99.4	97.9	34.8	27.8	**99.3**	15.7
−1	1.5	0	0	0	4.44	99.5	**99.6**	43.6	44.4	99.5	18.9

Continues

TABLE A15 Continued

$K1$	$K2$	$K3$	K4	K5	δ	MYT	MUR	DFT	MAB	rPCA	ANN
−1	2	0	0	0	5.47	99.1	**99.9**	60.1	64.7	99.5	21.9
−1	2.5	0	0	0	6.51	98.7	**100.0**	79.1	79.1	99.4	25.1
−1	3	0	0	0	7.56	99.1	**100.0**	91.2	87.6	99.1	29.2
−0.5	−3	0	0	0	5.92	**100.0**	99.9	81.8	44.1	99.5	17.3
0.5	3	0	0	0	5.92	**100.0**	99.9	82.2	45.8	99.5	16.5
1	−3	0	0	0	7.56	99.0	**100.0**	90.7	82.0	99.5	28.7
1	−2	0	0	0	5.47	99.3	**100.0**	61.3	57.0	99.5	21.8
1	−1.5	0	0	0	4.44	99.6	99.5	43.5	36.8	**99.9**	19.0
1	−1	0	0	0	3.43	99.3	97.7	35.6	21.9	**99.0**	16.4
1.5	−3	0	0	0	8.20	97.1	**100.0**	95.1	90.3	99.7	31.1
1.5	−2	0	0	0	6.15	96.2	**100.0**	72.8	77.1	99.5	25.1
1.5	−1.5	0	0	0	5.15	98.6	**99.9**	56.5	59.7	99.5	22.3
1.5	−1	0	0	0	4.18	**99.6**	99.3	45.7	37.2	99.1	18.2
2	−3	0	0	0	8.88	94.8	**100.0**	97.8	95.1	99.5	33.8
2	−2	0	0	0	6.86	90.0	**100.0**	86.0	89.0	99.2	27.2
2	−1.5	0	0	0	5.89	95.8	**100.0**	73.5	78.0	99.9	24.2
2	−1	0	0	0	4.96	99.2	**99.9**	62.2	57.5	99.5	20.6
3	−3	0	0	0	10.30	91.4	**100.0**	99.0	99.1	99.1	38.0
3	−2	0	0	0	8.36	78.7	**100.0**	98.2	96.8	99.0	32.0
3	−1.5	0	0	0	7.43	87.1	**100.0**	95.9	91.8	99.4	28.4
3	−1	0	0	0	6.55	96.5	**100.0**	91.6	82.4	99.9	25.2
3	−0.5	0	0	0	5.73	99.6	**100.0**	86.4	65.6	99.9	22.9
3	0.5	0	0	0	4.41	99.1	**99.4**	85.6	38.7	99.3	18.2
3	1	0	0	0	4.03	93.1	99.1	88.7	40.0	**99.9**	15.5

Abbreviations: ANN, artificial neural network; DFT, Doganaksoy-Faltin-Tucker; MAB, Maravelakis and Bersimis; MUR, Murphy; MYT, Mason-Young-Tracy; rPCA, Maravelakis-Bersimis-Panaretosa-Psarakis principal component analysis.

TABLE A16 Overall two shifts succesful identification percentage(%) for the 2nd scenario with 5 variables

ρ		MUR	DFT	MAB	ANN	rPCA
−0.2	100.0	90.6	90.0	34.4	**99.5**	8.9
0.50	98.7	92.6	82.9	36.9	**99.5**	9.0
0.80	95.3	96.6	69.7	49.5	**99.7**	17.9

APPENDIX B

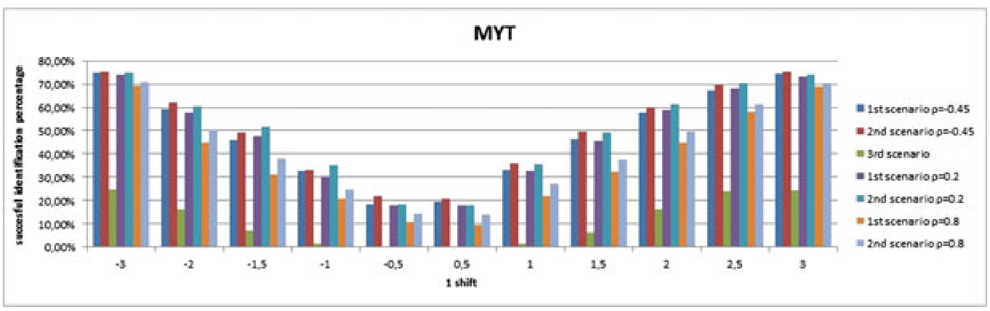

FIGURE B1 One-shift Mason, Young, Tracy's (MYT) successful identification ratio for the 5 variables case [Colour figure can be viewed at wileyonlinelibrary.com]

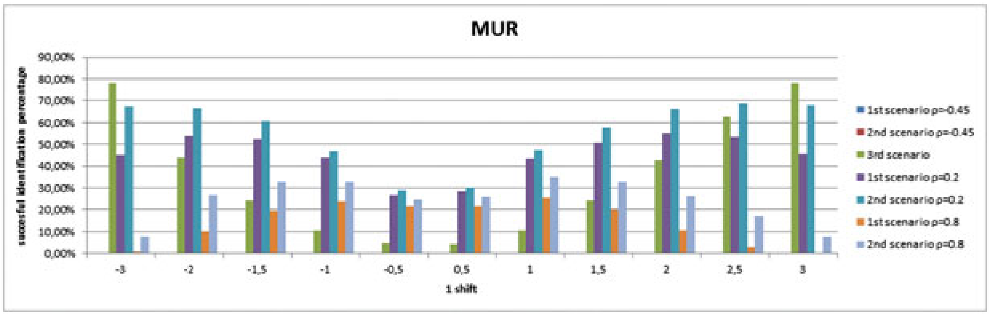

FIGURE B2 One-shift Murphy's (MUR) successful identification ratio for the 5 variables case [Colour figure can be viewed at wileyonlinelibrary.com]

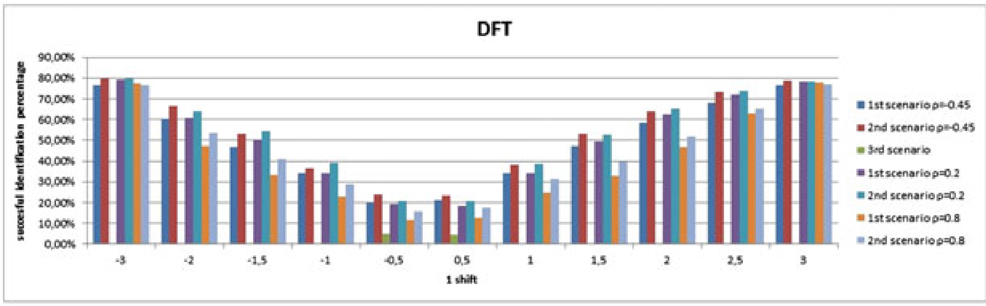

FIGURE B3 One-shift Doganaksoy, Faltin, Tucker's (DFT) successful identification ratio for the 5 variables case [Colour figure can be viewed at wileyonlinelibrary.com]

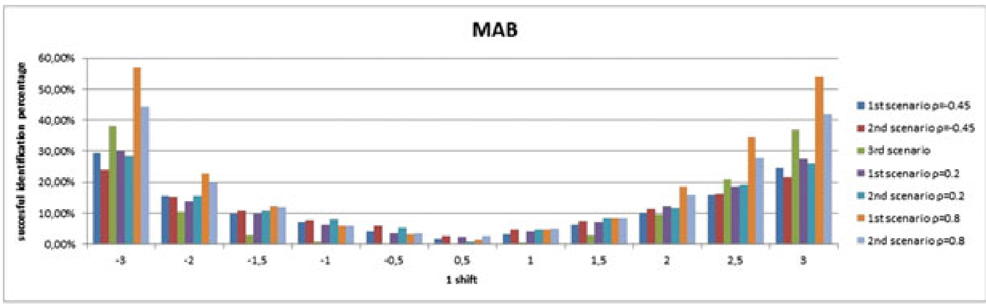

FIGURE B4 One-shift Maravelakis and Bersimis' (MAB) succesful identification ratio for the 5-variable case [Colour figure can be viewed at wileyonlinelibrary.com]