MAT 271E Probability and Statistics

Final Exam

Due: Sunday, January 31, 2021, 11:30 AM (upload to Ninova)

No late exam solutions will be accepted!

You will upload your solutions to the exam on **Sunday, January 31** before 11:30 ***AM***!

Exam format and academic honesty:

- Students are expected to continue to observe University Rules, treat all exams as if they were sitting in class in person to complete them, and maintain the highest standards of academic integrity during this period of online learning. You must answer questions on your own without help from others. In case of any plagiarism/cheating, disciplinary regulations of the University will be applied.
- Forms of cheating include (but are not limited to):
  a. communicating with any other student during the exam
  b. copying from any other student during the exam
  c. communicating with a third party to work on some or all parts of the exam
- Do not post the exam questions on chegg.com or any other websites.
- The exam will be given on **January 31, 2021**. The questions will be posted as an assignment on Ninova **at 09:00**.
- **NO LATE EXAM PAPERS WILL BE ACCEPTED**. The **total duration** of the exam is **2.5 hours**. This duration includes the time you need to solve the questions, scan the papers, and upload your files to Ninova. Two hours is more than enough to complete the exam. We are giving you significantly more time (two and a half hours) only to avoid any technical difficulties (e.g., power outages, Internet connection problems, etc.) you may encounter when trying to upload the exam. Upload your submission much earlier than the deadline. Be aware that Ninova's system clock may not be synchronized with your watch, your cell phone, or your computer's clock, etc. Do not risk leaving your submission to the last few minutes. **You must upload your solutions to Ninova before 11:30**.
- **Do not send your solutions by e-mail.** We will only accept files that you have uploaded to Ninova (the official e-learning system) before the deadline.

Preparing the solutions:

1. For each question, use a separate sheet of **plain white** paper. Please **do not use ruled paper, quad paper, or graph paper;** they do not scan well and make your solutions harder to read. You may write solutions to different parts (e.g., a, b, c) of the same question on the same sheet. You may also use multiple sheets for the parts of a question. Do not write solutions to different questions (e.g., Q1 and Q2) on the same sheet.
2. In the upper right-hand corner of each sheet, write the following information:
   Student ID:
   First Name and Last Name:
3. Write (and draw) the solutions using a computer program or by hand on A4 paper.
4. If you draw figures by hand, use a ruler.
5. PLEASE BE NEAT! If we cannot read or follow your solution, you will receive no partial credit.

6. Please **show ALL work**. Answers with no supporting explanations or work **will not receive any partial credit**. Your exam paper is **not just a final report** of your results; we want to see your steps. Upload all the papers you worked on to get to the solution.

Creating the files:

7. Create a separate file in PDF format for each solution. The file names should be Qx, where x is the respective question number, such as Q1.pdf, Q2.pdf, etc. If a solution to a question spans multiple sheets, you may create multiple files for the question and name them Q1a.pdf, Q1b.pdf, etc.
8. Combine all of your solution files for each question into a separate ZIP file, e.g., solution1.zip, solution2.zip, etc. Before submitting the ZIP files, check them to make sure that they extract correctly and contain all parts of the solution.
9. **Submit your ZIP files to Ninova before 11:30**. Do not send your solutions by e-mail. We will only accept files that you have uploaded to Ninova before the deadline.
   Upload your solutions using a computer. Please upload your solutions using a computer, not a mobile platform. Please do not use İTÜMOBİL to access Ninova, as this may cause problems with your submission.
10. **If you prepared the solutions using a computer**, create a pdf file for each solution and follow steps 7, 8, and 9.
11. **If you prepared the solutions by hand**, scan your papers and create a pdf file for each solution.

    To scan your paper, you may use a desktop scanner or a scanner application on your cell phone.
    Do not take photographs of your papers directly with high-resolution cameras because they create large files. This may cause problems during submission to Ninova. A size of 200-300 KB for one sheet is acceptable. If the resulting file looks blurry, faint, or dark, please scan again.
    Please rotate your pdf file if necessary and save it in the upright orientation, so that when we click on the file to open it, we can immediately read it. It should be in the upright Portrait orientation, not 90 degrees rotated to the left or right.
    Then, follow steps 7, 8, and 9.

FULL NAME_____

STUDENT ID_____

MAT 271E – PROBABILITY AND STATISTICS
FINAL EXAM – JANUARY 31, 2020
9:00-11:30 AM
100 POINTS TOTAL

1. Please ***write*** your name in CAPITAL LETTERS at the top of your solution pages, as shown above.
2. Check that the exam contains 4 problems.
3. Partial credit will be based upon your written intermediate results.
4. You must **show ALL work**/steps on all problems for credit. You must give explanations where needed. Simply writing a number is not enough. Answers with no supporting explanations or work **will not receive any partial credit**. Your exam paper is **not just a final report** of your results; we want to see your steps. Upload **all the papers** you worked on to get to the solution.
5. Write proper equations.
6. PLEASE BE NEAT! If I cannot read or follow your solution, you will receive no partial credit.
7. Simplify your final answer as much as possible (simplify fractions, etc.).
8. This is an open book, open notes exam.
9. You should have ample time to finish the exam and upload it.
10. No questions may be asked during the exam. If you are not sure about a problem, answer based on what you understand and state your assumptions.

GOOD LUCK!

Problem	Description	Point Value	Score
1	Inequalities	25	
2	MGF	25	
3	Marginal PDF, Conditioning	25	
4	MMSE Estimate, Covariance	25	
Total		**100**	

Problem 1 (25 points) Inequalities

In a factory, 101 printers are waiting to be transferred from the factory floor to the warehouse. The factory uses 4 forklifts to move them. Each forklift arrives independently: the first forklift arrives at T_1, the second at $T_1 + T_2$, the third at $T_1 + T_2 + T_3$, and the fourth at $T_1 + T_2 + T_3 + T_4$. The T_i have a continuous uniform distribution on the interval between 0 and 30 minutes. Each forklift can carry 25 printers and takes 20 minutes from the time it arrives to reach the warehouse. Let W be the total amount of time that passes before all 101 printers arrive at the warehouse.

a) **[5 pts]** Calculate $E[T_i]$.
b) **[5 pts]** Calculate $Var[T_i]$.
c) **[4 pts]** Calculate $E[W]$.
d) **[3 pts]** Calculate $Var[W]$.
e) **[4 pts]** Apply the Markov inequality to restrict the probability that the total amount of time that passes before all 101 printers arrive at the warehouse exceeds 200 minutes.
f) **[4 pts]** Use the Chebyshev inequality to restrict the probability that the total amount of time that passes before all 101 printers arrive at the warehouse exceeds 200 minutes.

Problem 2 (25 points) MGF

Assume that X_1, X_2, and X_3 are independent and identically distributed random variables with the following PDF

$$f_X(x) = \begin{cases} 2x & , 0 \le x \le 1 \\ 0 & , \text{otherwise} \end{cases}$$

and $W = X_1 + X_2 + X_3$.

a) [8 pts] Find $\phi_X(s)$.

b) [8 pts] Find $\phi_W(s)$.

c) [9 pts] Suppose that $\phi_T(s) = se^{3s}/(s-2)$. Find $E[T]$.

Problem 3 (25 points) Marginal, Conditioning

Assume random variables U and V have the joint PDF given below:

$$f_{U,V}(u,v) = \begin{cases} \frac{6}{5}(u^2+v) & ,0 \le u \le 1;\ \ 0 \le v \le 1 \\ 0 & ,\text{otherwise} \end{cases}$$

a) [4 pts] Calculate the marginal pdf $f_U(x)$.

b) [4 pts] Calculate the expectation of U.

c) [4 pts] Let B represent the event $u \le 1/2$. Find $P[B]$.

d) [4 pts] Compute $f_{U,V|B}(u,v)$.

e) [5 pts] Evaluate $E[U|B]$.

Problem 4 (25 points) MMSE Estimate, Covariance
Note: Parts (a) and (b) below are not related; they can be solved independently.

a) [15 pts] Assume that the speed of a typical car on the highway is modeled by the random variable *S* that is uniformly distributed over the interval [50, 70]. Let *N* represent the noise in the radar, and assume that it is uniformly distributed over the interval [-5, 5]. Let us assume that *N* and *S* are independent. The value measured by the radar is *R*=*S*+*N*.

Calculate $\hat{s}_M(r)$, the minimum mean square error (MMSE) estimate of *S* given *R*, and draw its graph as a function of *r*.

b) [10 pts] Assume that we have two random variables L and M. We also have the following information regarding their variances: $Var[M] = 6$, $Var[L] = 3$, and $Var[L + M] = 12$. Compute $Cov[L, M]$.