MAT 271E Probability and Statistics

Midterm

Due: **Tuesday,** December 1, 2020, 4:30 PM (upload to Ninova)

No late exam solutions will be accepted!

You will upload your solutions to the exam on **Tuesday, December 1** before 4:30 ***PM***!

- Submit your solutions as a single PDF file.
- Write your full name and student number in the upper right-hand corner.
- Write the solutions using a computer program or by hand on A4 paper.
- Solve the problems should in the order they are given.
- **Upload your solutions using a computer.** Please upload your solutions using a computer, not a mobile platform. Please do not use İTÜMOBİL to access Ninova, as this may cause problems with your submission.
- Upload your submission much earlier than the deadline. Do not risk leaving your submission to the last few minutes or even the last hour (be aware that Ninova's system clock may not be synchronized with your own watch, laptop, or cell phone). If you feel you are running out of time, then submit whatever you have finished before the deadline to receive some partial credit.
- Do not send your solutions by e-mail. We will only accept files that you uploaded to Ninova before the deadline.
- Students are expected to continue to observe University Rules, treat all exams as if they were sitting in class in person to complete them, and maintain the highest standards of academic integrity during this period of online learning. You must answer questions on your own without help from others. Any plagiarism/cheating will result in disciplinary action.
- Do not post the exam questions on chegg.com or any other websites.

Note on solutions prepared by hand: If you choose to write your solutions on paper, scan your papers and create a pdf file. Check the file sizes.

To scan your paper, you may use a desktop scanner or a scanner application on your cell phone.

Do not take photographs of your papers directly with high-resolution cameras because they create large files. This may cause problems during submission to Ninova. A size of 200-300 KB for one sheet is acceptable.

Please rotate your pdf file if necessary and save it in the upright orientation, so that when we click on the file to open it, we can immediately read it. It should be in the upright Portrait orientation, not 90 degrees rotated to the left or right.

FULL NAME___________________________________

STUDENT ID____________________________________

MAT 271E – PROBABILITY AND STATISTICS
MIDTERM EXAM – DECEMBER 1, 2020
1:30-4:30 PM
100 POINTS TOTAL

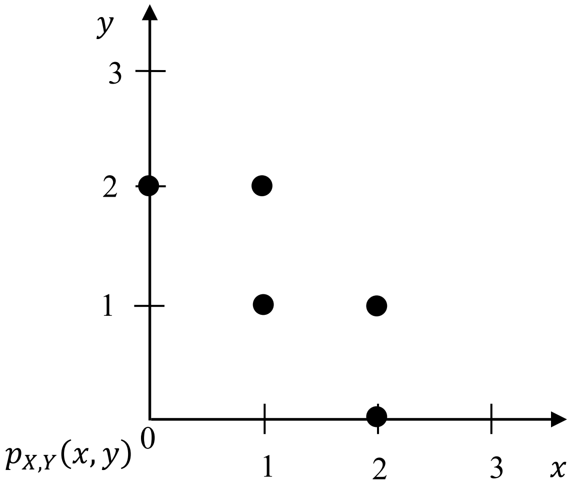
1. Please ***write*** your name in CAPITAL LETTERS at the top of your solution pages, as shown above.
2. Check that the exam has 6 pages (including this page) with a total of 5 problems.
3. Partial credit will be based upon your written intermediate results.
4. You must show your work/steps on all problems for credit. You must give explanations where needed. Simply writing a number is not enough.
5. Write proper equations.
6. PLEASE BE NEAT! If I cannot read or follow your solution, you will receive no partial credit.
7. Simplify your final answer as much as possible (simplify fractions, etc.).
8. This is an open book, open notes exam.
9. You should have ample time to finish the exam and upload it.
10. No questions may be asked during the exam. If you are not sure about a problem, answer based on what you understand and state your assumptions.

GOOD LUCK!

Problem	Description	Point Value	Score
1	Joint PMF, Exp., Var.	20	
2	CDF, Exp.	20	
3	Indep., Conditioning	20	
4	Indep., Variance of Func.	20	
5	Poisson	20	
Total		**100**	

<u>Problem 1 (20 points)</u> Joint PMF, Function of Two Variables
Note: Parts (a) and (b) below are <u>not</u> related.

a) [10 pts] The graph of the joint probability mass function $p_{X,Y}(x, y)$ is given below:



Assume that each of the five points on the graph are equally likely. We define a new variable $Z = X + 2Y$. <u>Find</u> the probability mass function of Z, $p_Z(z)$. Make sure that the PMF covers all possible cases.

b) [10 pts] For a random variable N, given that $E[N] = 6$ and $Var[N] = 3$, <u>find</u> $E[N^2]$.

<u>Problem 2 (20 points)</u> CDF, Expectation
Note: Parts (a) and (b) below are <u>not</u> related.

a) [10 pts] Random variable Y has the following cumulative distribution function (CDF):

$$F_Y(y) = \begin{cases} 1 - \left(\frac{1}{3}\right)^{y+1} & , y = 0, 1, 2, \ldots \\ 0 & , \text{otherwise} \end{cases}$$

<u>Find</u> the probability mass function (PMF) of Y. Make sure that the PMF covers all possible cases.

b) [10 pts] Suppose that n DVDs have been removed from their cases and placed in a drawer. You pull out k DVDs and randomly put them into their empty cases left on the desk (without looking at their labels). This results in X of the k DVDs being put back into their proper cases. <u>Find</u> $E[X]$, the expectation of random variable X.

<u>Problem 3 (20 points)</u> Independence, Conditioning
Note: Parts (a), (b), and (c) below are <u>not</u> related.

a) [6 pts] Assume that for events C and D , we have $P[C] = P[C|D]P[D]$. Does this imply that C and D are independent? <u>Explain</u>.

b) [8 pts] Assume that Y takes on the value -1 with probability 0.3; otherwise, it takes any of the values in the set {2, 3, 4, 6, 10} with equal likelihood. <u>Find:</u>

- **i)** $P[Y > 4]$,
- **ii)** $E[X]$, and
- **iii)** $Var[X]$.

c) [6 pts] Random variables K and M are such that $bK + M = c$, where b and c are real-valued constants. Given $E[K] = \mu$, $Var[K] = \sigma^2$, <u>find</u>:

- **i)** An expression for $E[M]$ in terms of μ and σ^2, and
- **ii)** An expression for $Var[M]$ in terms of μ and σ^2.

Problem 4 (20 points) Independence, Variance of a Function

Random variables X and Y are independent, and each can take on any value between 0 and a (inclusive), where a is a constant, with equal probability. We define a third random variable Z such that $Z = X - Y$.

a) [10 pts] Find the joint PMF of X and Y, $p_{X,Y}(x, y)$.

b) [10 pts] Find the variance of Z, $Var[Z]$.

Problem 5 (20 points) Poisson

Assume that the number of customers arriving at a bank in an hour has a Poisson distribution with parameter λ. Suppose that the queuing kiosk issuing tickets to arriving customers is not functioning properly and fails to issue a ticket to a customer with probability p.

a) **[10 pts]** Compute the probability that the kiosk will issue tickets to exactly k customers in an hour.

b) **[10 pts]** Given that the kiosk issued tickets to exactly k customers in an hour, find the probability mass function (PMF) of the actual number of customers that arrived at the bank during that hour. Make sure that the PMF you write covers all cases.