Influence Maximization

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Outline

- Introduction and motivation
- Stochastic diffusion models
- Influence maximization
- Algorithms
- Conclusion

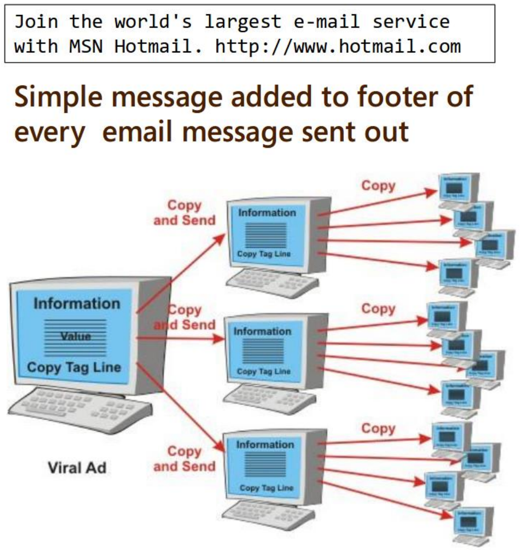
Social influence

- Wikipedia definition: Social influence occurs when one's opinions, emotions, or behaviors are affected by others, intentionally or unintentionally.

Booming of online social networks

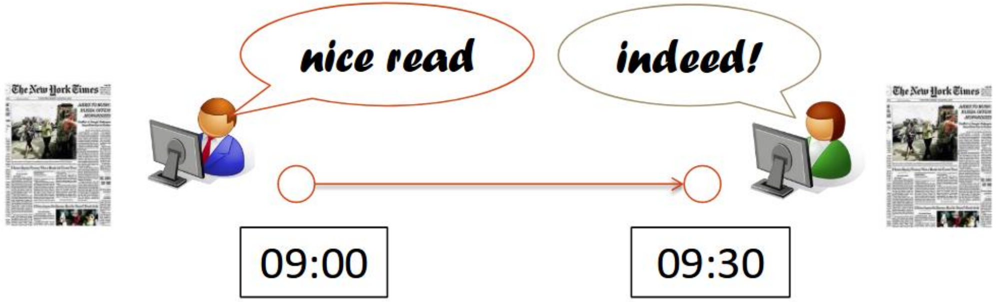
Hotmail: online viral marketing story

- Hotmail’s viral climbed to the top spot (90s): 8 million users in 18 months
- Boosted brand awareness
- Far more effective than conventional advertising by rivals
    - and far cheaper



Stochastic Diffusion Models

Influence propagation



People are **connected** and perform **actions**

friends, fans, followers, etc.

comment, link, rate, like, retweet, post a message, photo, or video, etc.

Terminologies

- Directed graph $G = (V, E)$
  - Node $v \in V$ represents an individual
  - Edge $(u, v) \in E$ represents the influence relationship
- Discrete time t: 0, 1, 2, …
- State of nodes: active or inactive
- S_t: set of active nodes at time t
  - S_0, seed set, initial nodes selected to start the diffusion

Stochastic diffusion models

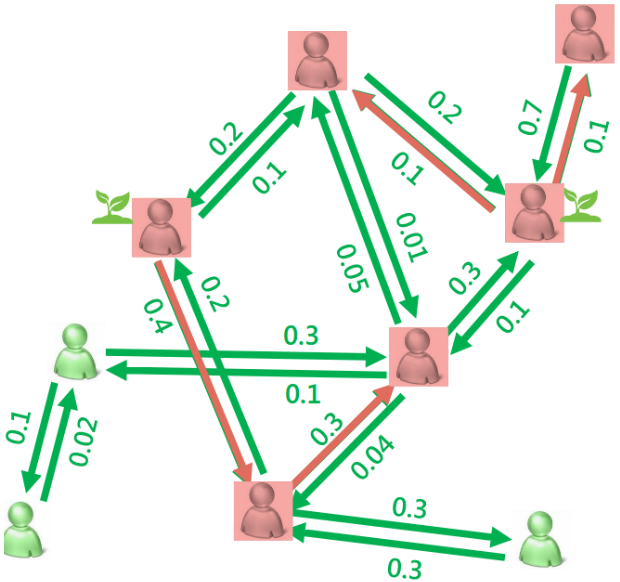
Definition: A *stochastic diffusion model (with discrete time steps)* for a social graph $G = (V, E)$ specifies the randomized process of generating active sets S_t for all $t > 0$ given the initial seed set S_0.

Progressive models: for all $t > 0$, $S_{t-1} \subseteq S_t$

- Once activated, always activated, e.g. once bought a product, cannot undo it.
- **Influence spread σ(S)** : expected number of activated nodes when the diffusion process starting from the seed set S ends

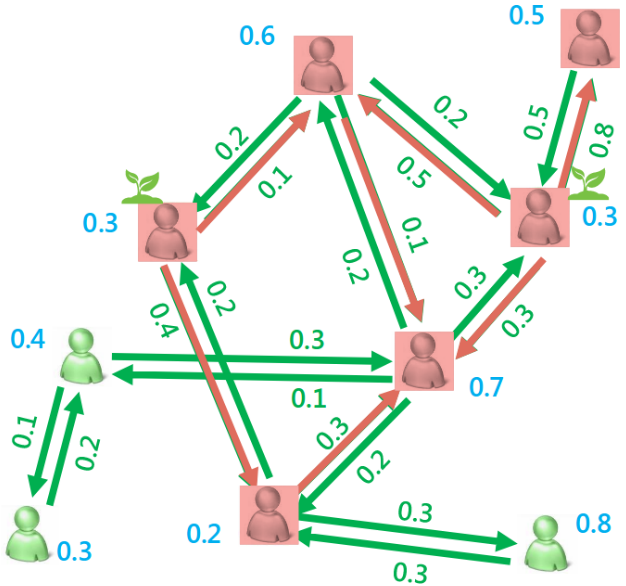
Independent cascade model

- Each edge (u, v) has a influence probability p(u, v)
- Initially seed nodes in S_0 are activated
- At each time step t, each node u activated at step t-1 activates its neighbor v independently with probability p(u, v)



Linear threshold model

- Each edge (u, v) has an influence weight w(u, v):
  - if $(u, v) \notin E$, $w(u, v)=0$
  - $\Sigma_u\, w(u, v) \leq 1$
- Each node v selects a threshold $\theta_v \in [0,1]$ uniformly at random
- Initially seed nodes in S_0 are activated
- At each step, node v checks if the weighted sum of its activated in-neighbors is greater than or equal to its threshold θ_v, if so v is activated



Interpretation of IC and LT models

- IC model reflects simple contagion, e.g. information, virus
- LT model reflects complex contagion, e.g. product adoption, innovations

  (activation needs social affirmation from multiple sources [1])

[1] Centola, Damon, and Michael Macy. "Complex contagions and the weakness of long ties." *American journal of Sociology 2007.*

General threshold model

- Each node v has a threshold function
$$f_v: 2^V \to [0,1]$$
- Each node v selects a threshold $\theta_v \in [0, 1]$ uniformly at random
- If the set of active nodes at the end of step t-1 is S, and $f_v(S) \geq \theta_v$, v is activated at step t
- Reward function r(A(S)): if A(S) is the final set of active nodes given seed set S, r(A(S)) is the reward from this set
- Generalized influence spread:
$$\sigma(S) = E[r(A(S))]$$

Submodularity in the general threshold model

- Properties: submodularity and monotonicity
- Theorem
  - In the general threshold model,
  - if for every v ∈ V, f is monotone and submodular with f(Ø)=0
  - and the reward function r is monotone and submodular
  - then the general influence spread funnction σ is monotone and submodular

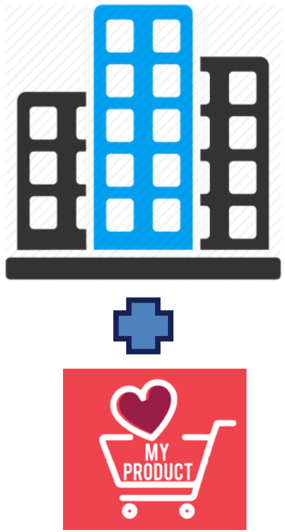
[2] Mossel, Elchanan, and Sebastien Roch. On the submodularity of influence in social networks. STOC, 2007.

Influence Maximization

Problem formulation

- Given a social network, a diffusion model with given parameters, and a number k, find a seed set S of at most k nodes such that the influence spread of S is maximized.
- May be further generalized:
  - Instead of k, given a budget constraint and each node has a cost of being selected as a seed
  - Instead of maximizing influence spread, maximizing a (submodular) function of the set of activated nodes.

Example



Hardness of influence maximization

- Influence maximization under both IC and LT models are NP hard
  - IC model: reduced from k-max cover problem
  - LT model: reduced from vertex cover problem
- Need approximation algorithms

Greedy-based Algorithm

Greedy-based Algorithm

- Influence Spread
  - Given a seed set S, the influence spread of S means that
    - the expected number of influenced users after the propagation terminates if the users in S were selected as seed users
  - $f(S)$
- Marginal gain
  - The influence from users has overlap
    - Some of the users influenced by user u may be the same as the users influenced by the seed set S, where $u \notin S$
  - The Marginal gain of a user u means that
    - If user u was selected into the seed set, how much extra influence spread it would contribute
  - $\Delta(u|S) = f(S \cup \{u\}) - f(S)$

Greedy-based Algorithm

- Iteratively selects a node that provides the maximum marginal gain in each round

Algorithm 1 Greedy(k, f)

```
initialize S = ∅
for i = 1 to k do
    select u = arg max_{w∈V\S} (f(S ∪ {w}) − f(S))
    S = S ∪ {u}
end for
output S
```

$$\text{select } u = \arg\max_{w \in V \setminus S} \underbrace{(f(S \cup \{w\}) - f(S))}_{\text{marginal gain}}$$

Greedy-based Algorithm

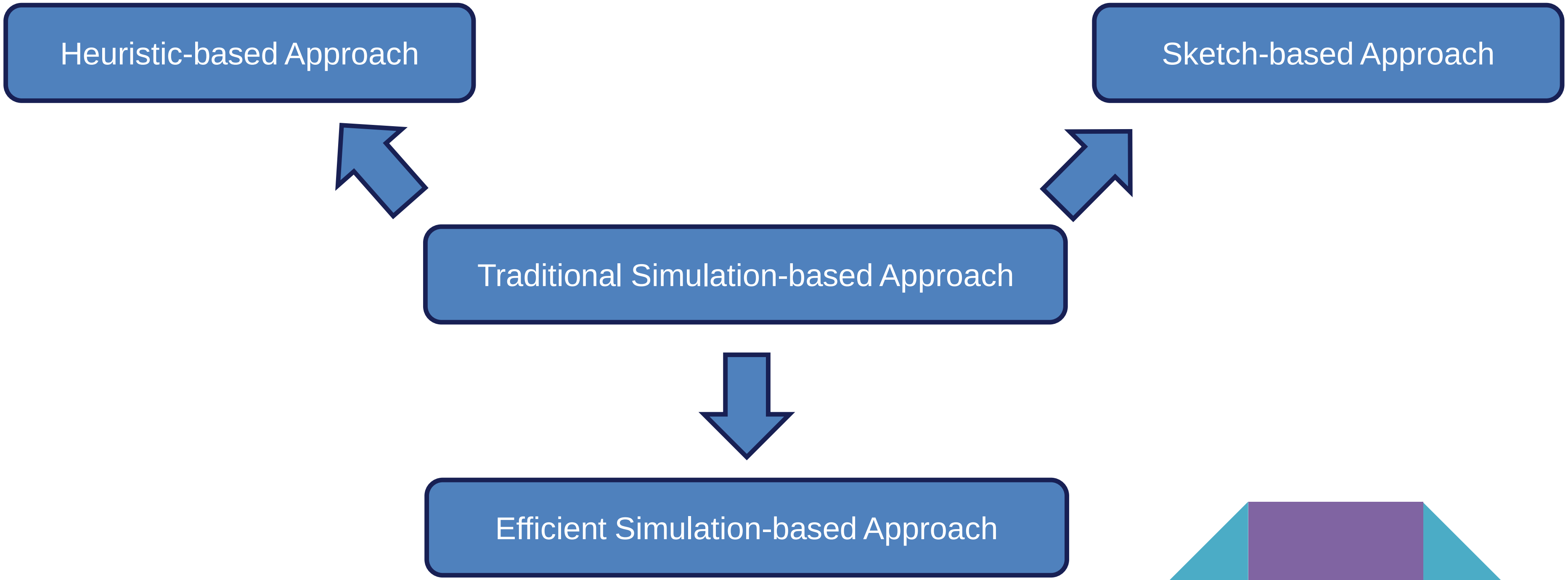
- Submodular
  - $f(S_j \cup \{u\}) - f(S_j) \leq f(S_i \cup \{u\}) - f(S_i)$ $where\ S_i \subseteq S_j$
- Monotone
  - $f(S_i) < f(S_j)$
- The greedy-based approach solves the influence maximization problem with an approximation ratio of 1 − 1/e.[2]

[2] D. Kempe, J. M. Kleinberg, and É. Tardos. Maximizing the spread of influence through a social network. KDD 2003.

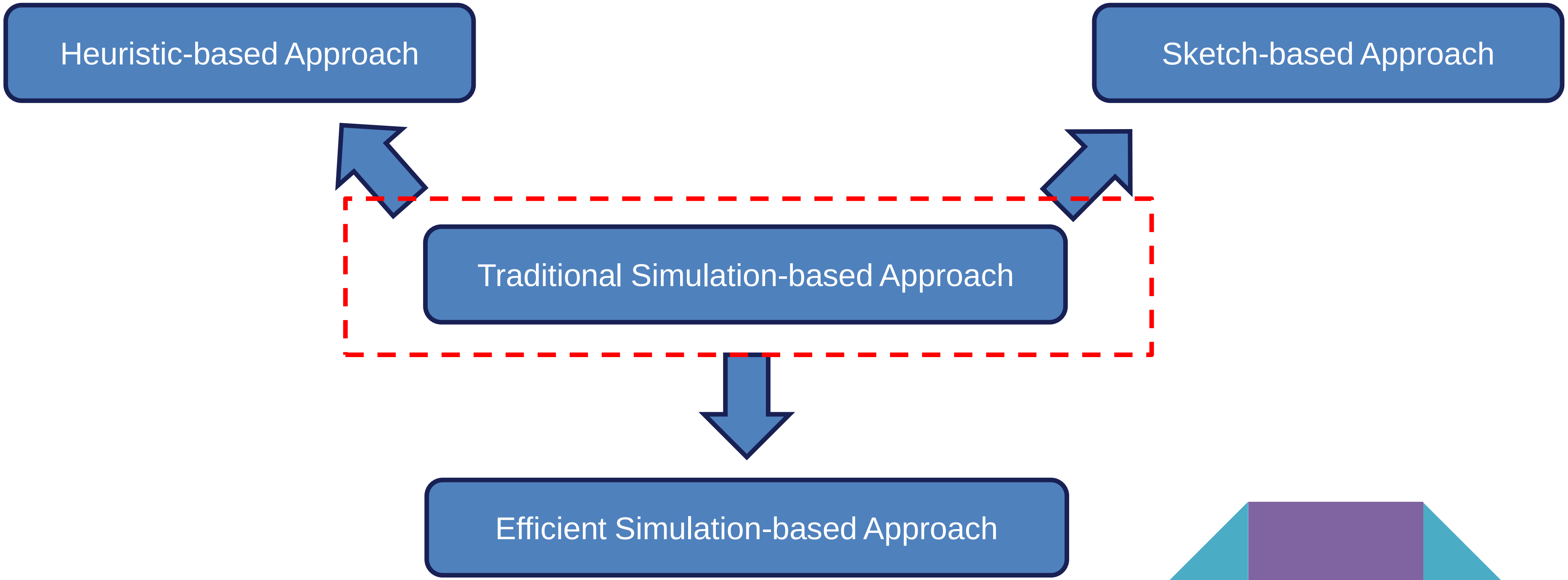
Greedy-based Algorithm

How to estimate the influence spread?

Overview


Heuristic-based Approach
Sketch-based Approach
Traditional Simulation-based Approach
Efficient Simulation-based Approach

Overview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Simulation-based approach

- The approach utilizes Monte Carlo(MC) simulations for estimating influence spread.
  - Starts from the seed set S
  - Simulates the activation process wrt. the corresponding diffusion model
  - Outputs the number of activated users

Traditional Simulation-based Approach

Algorithm 1 GeneralGreedy(G, k)

```
initialize S = ∅ and R = 20000
for i = 1 to k do
    for each vertex v ∈ V \ S do
        s_v = 0.
        for i = 1 to R do
            s_v += |RanCas(S ∪ {v})|
        end for
        s_v = s_v / R
    end for
    S = S ∪ {arg max_{v ∈ V \ S} {s_v}}
end for
output S.
```

Time complexity: O(knRm)

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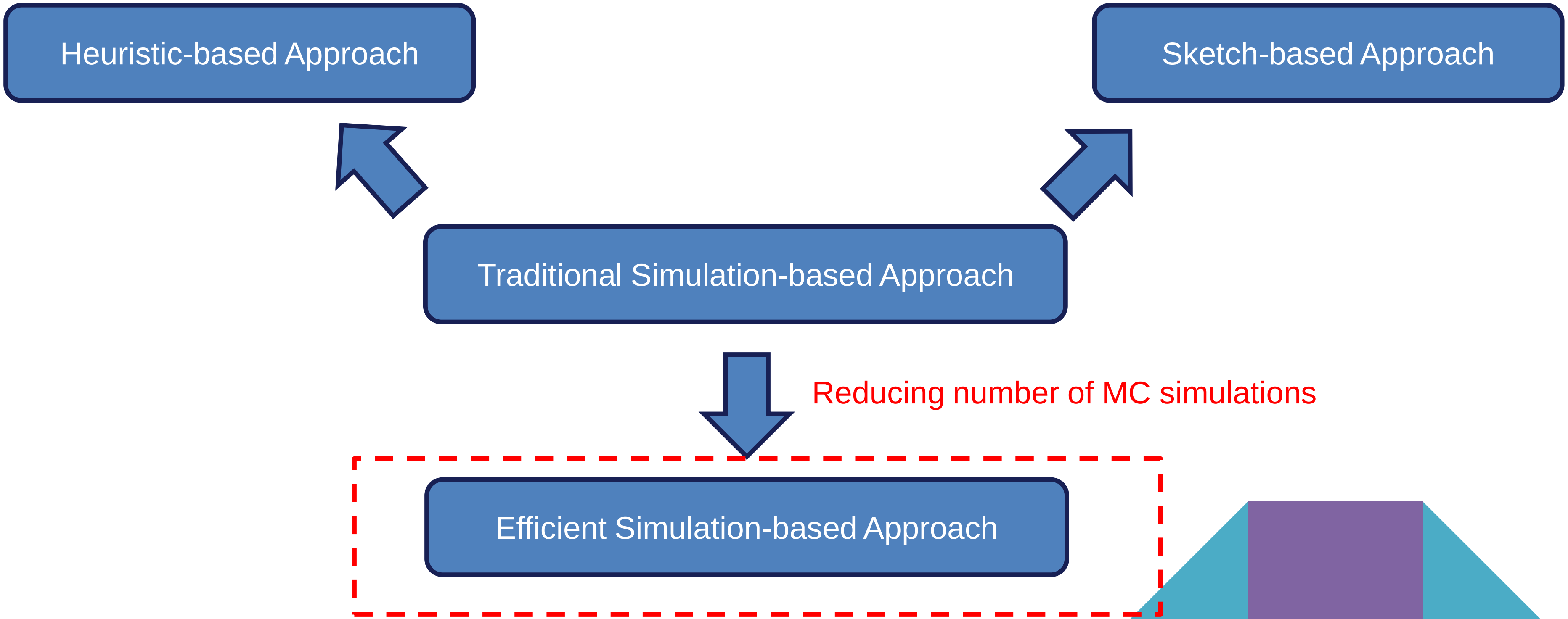
Heuristic-based Approach
Sketch-based Approach
Traditional Simulation-based Approach
Reducing number of MC simulations
Efficient Simulation-based Approach

Traditional Simulation-based Approach

Algorithm 1 GeneralGreedy(G, k)

initialize $S = \emptyset$ and $R = 20000$
for $i = 1$ to k **do**
  for each vertex $v \in V \setminus S$ **do**
    $s_v = 0.$
    for $i = 1$ to R **do**
      $s_v \mathrel{+}= |RanCas(S \cup \{v\})|$
    end for
    $s_v = s_v / R$
  end for
  $S = S \cup \{\arg\max_{v \in V \setminus S}\{s_v\}\}$
end for
output S.

Update the marginal gain of all unselected nodes

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Necessary?

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Update the marginal gain of all unselected nodes

Necessary?

No!!!

[2] D. Kempe, J. M. Kleinberg, and É. Tardos. Maximizing the spread of influence through a social network. KDD 2003.

CELF algorithm

- CELF[3] estimates an upper bound of the marginal gain to avoid updating the marginal gain of all nodes

[3] J. Leskovec, A. Krause, C. Guestrin, C. Faloutsos, J. VanBriesen, and N. S. Glance. Cost-effective outbreak detection in networks. KDD 2007

CELF algorithm

- Submodularity
  - $f(S_j \cup \{u\}) - f(S_j) \leq f(S_i \cup \{u\}) - f(S_i)$ where $S_i \subseteq S_j$
  - $\Delta(u|S_i) = f(S_i \cup \{u\}) - f(S_i)$ is an upper bound for any $\Delta(u|S_j)$ where $S_i \subseteq S_j$

- If the upper bound of node u is less than the updated marginal gain of node v,
  - the updated marginal gain of u is less than $v's$
  - no need to update the marginal gain of u

CELF algorithm

- Updates the marginal gain of nodes in descending order according to their upper bound
- Early terminates whenever the upper bound of a not updated node is less than the marginal gain of the updated node

CELF algorithm

- Updates the marginal gain of nodes in descending order according to their upper bound
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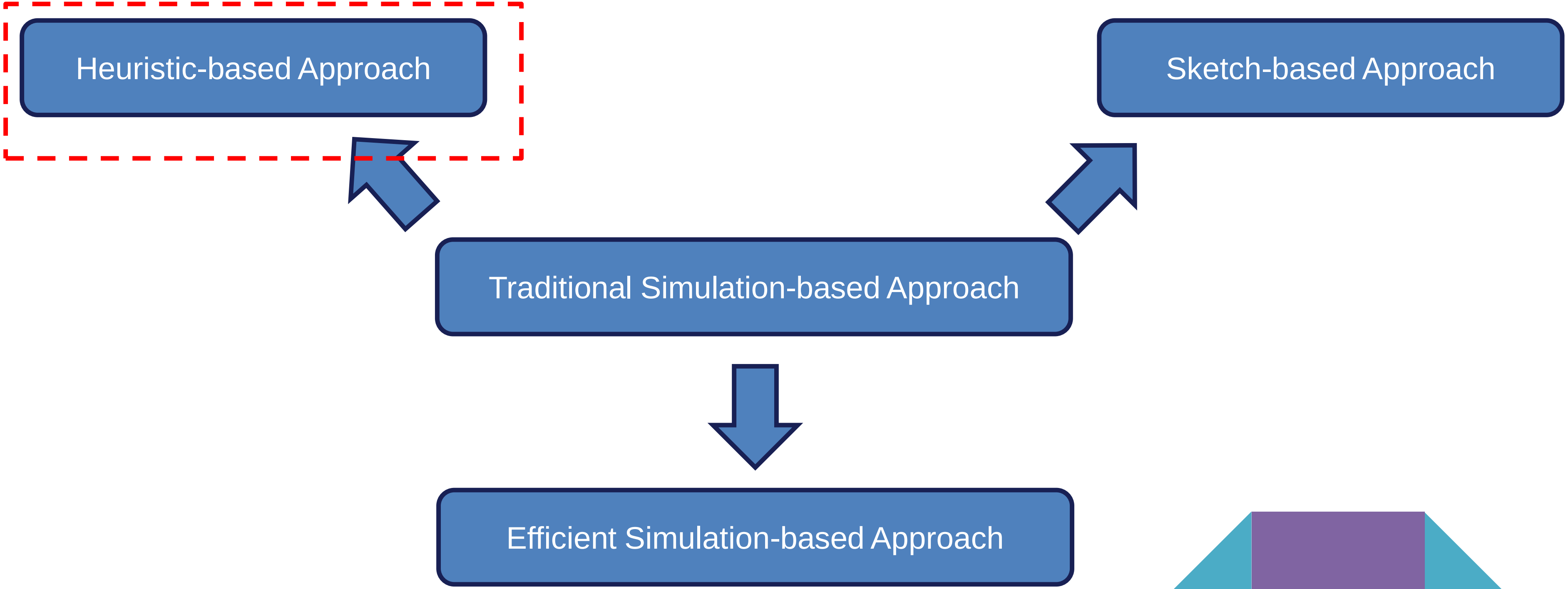
Enables an up to 700 times improvement compared with the traditional algorithm.

Overview

Estimate the influence spread in a heuristic way



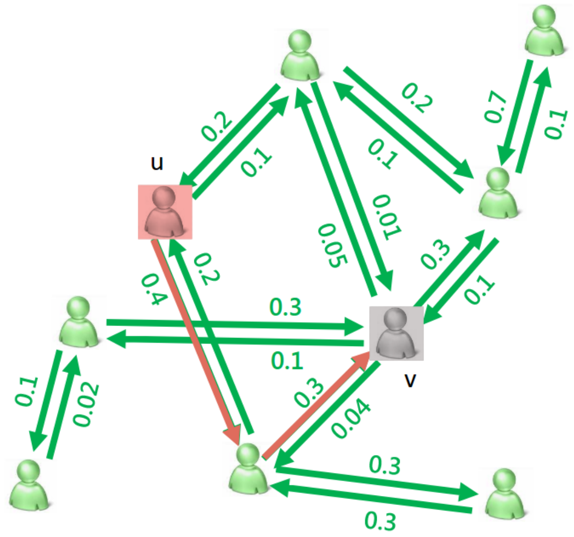
Heuristic approaches

- Exact influence computation is NP hard, for both IC and LT models -- computation bottleneck [4][5]
- Design new heuristics
  - MIA for general IC model [4]
  - LDAG for LT model [5]
  - IRIE for IC model [6]

[4] Chen, Wei, Chi Wang, and Yajun Wang. Scalable influence maximization for prevalent viral marketing in large-scale social networks. KDD 2010
[5] Chen, Wei, Yifei Yuan, and Li Zhang. Scalable influence maximization in social networks under the linear threshold model. ICDM 2010
[6] Jung, Kyomin, Wooram Heo, and Wei Chen. Irie: Scalable and robust influence maximization in social networks. ICDM 2012

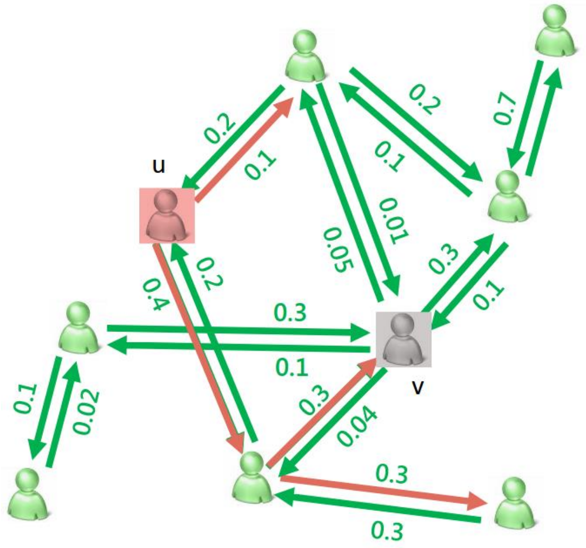
Maximum Influence Arborescence (MIA)

- For any pair of nodes u and v, find the maximum influence path (MIP) from u to v
- Ignore MIPs with too small probability (< parameter θ)



MIA (cont'd)

- Local influence regions
  - for every node v, all MIPs to v form its maximum influence in-arborescence (MIIA)
  - for every node u, all MIPs from u form its maximum influence out-arborescence (MIOA)
  - computing MIAs and the influence through MIAs is fast



MIA (cont’d)

- Recursive computation of activation probability ap(u) of a node u in its in-arborescence, given a seed set S

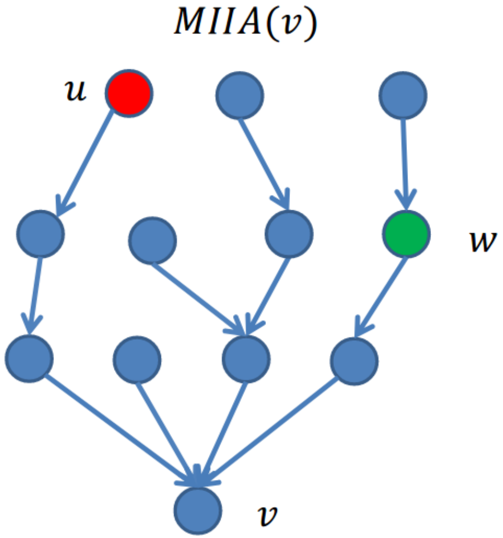
Algorithm 2 $ap(u, S, MIIA(v, \theta))$

if $u \in S$ **then**
    $ap(u) = 1$
else if $Ch(u) = \emptyset$ **then**
    $ap(u) = 0$
else
    $ap(u) = 1 - \Pi_{w \in Ch(u)}(1 - ap(w) \cdot pp(w, u))$
end if

- Can be us … eds, but not efficient enough

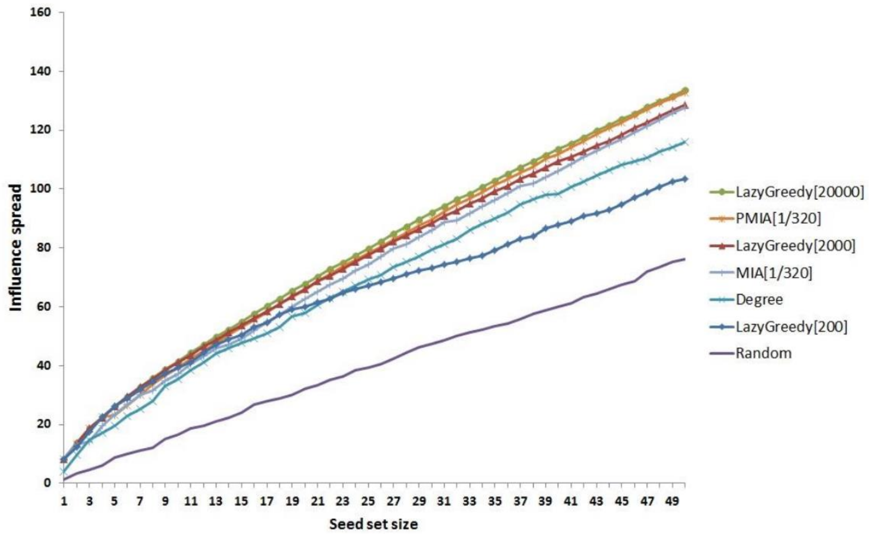
MIA (cont’d)

- u is the new seed in MIIA(v)
- Naive update: for each candidate W, redo the computation in the previous page to compute w’s incremental influence to v
  - O(|MIIA(v)|^2)
- Fast update: based on linear relationship of activation probabilities between any node w and root v, update incremental influence of all w’s to v in two passes
  - O(|MIIA(v)|)

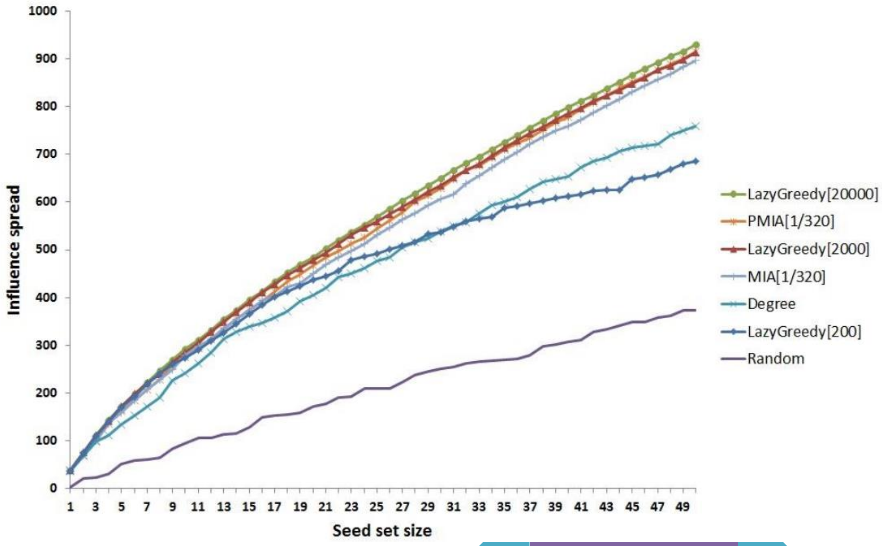


Experiment result

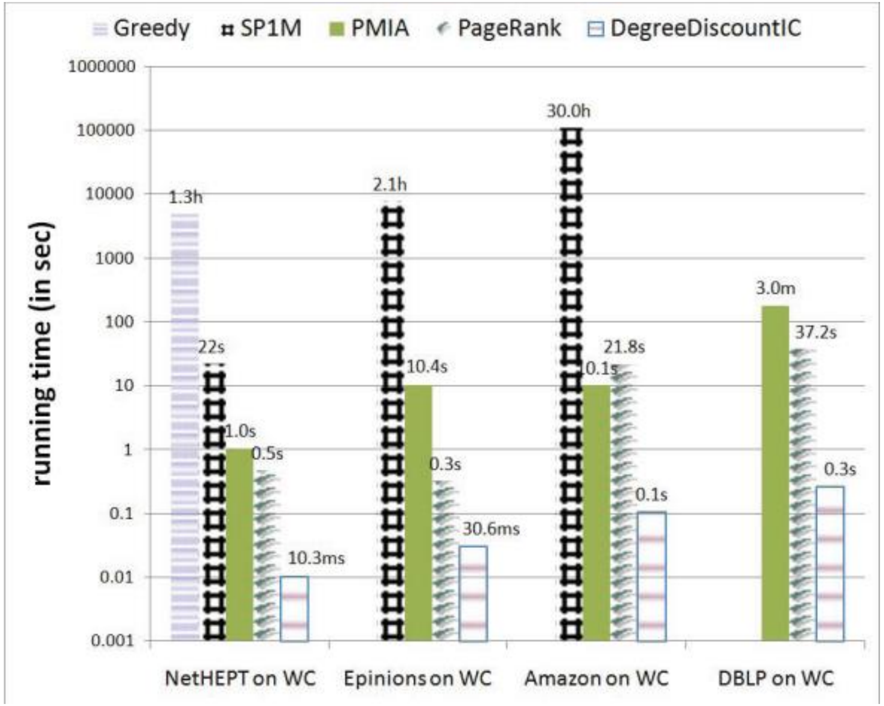
Influence spread in IC-UP[0.01] model



Influence spread in IC-WC model

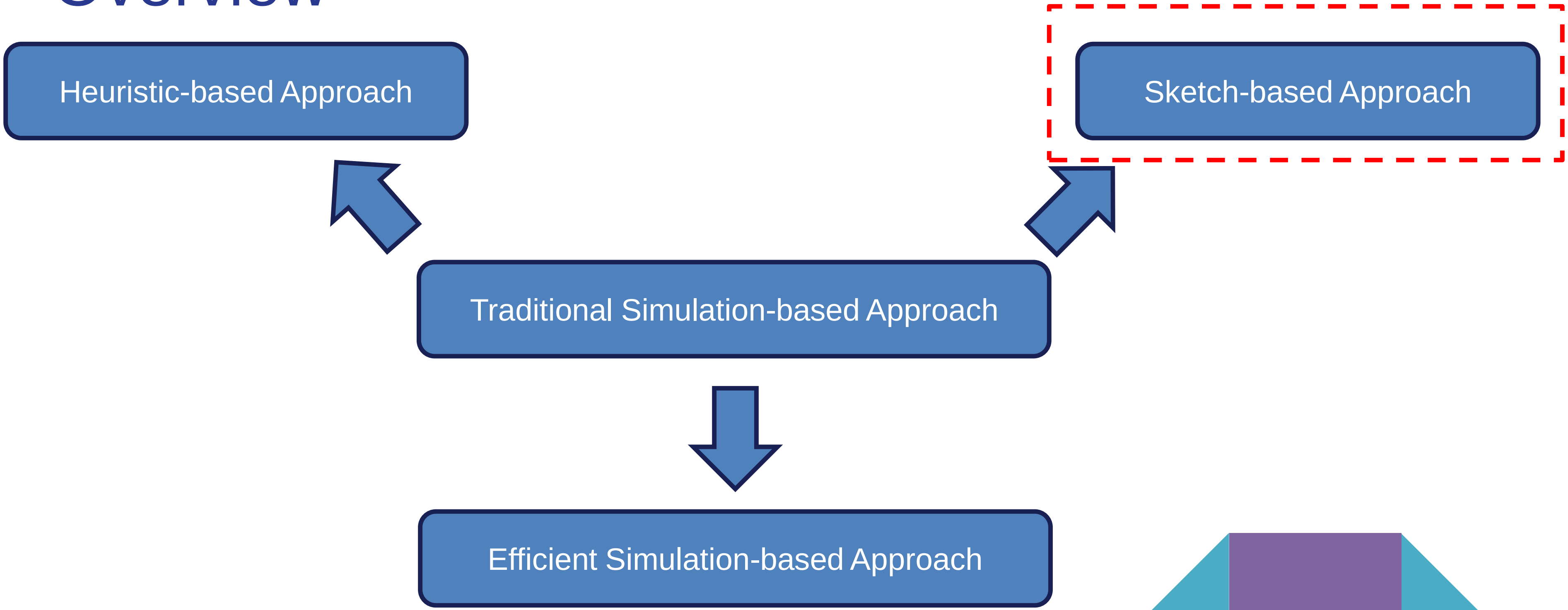


Experiment result



Estimate the influence spread based on the pre-computed sketch

Overview



Sketch-based Approach

- Let $v \in V$ be a node in G, and g be a graph obtained by removing each edge e in G with $1 - p(e)$ probability
  - The reverse reachable(RR) set
    - $RR(v)$: contains all nodes in g that can reach v
  - The Radom RR set
    - For $RR(v)$, v is randomly picked from V

C. Borgs, M. Brautbar, J. Chayes, and B. Lucier, "Maximizing social influence in nearly optimal time," in SODA, 2014
Y. Tang, X. Xiao, and Y. Shi, "Influence maximization: Nearoptimal time complexity meets practical efficiency," in SIGMOD, 2014, pp. 75–86.

Sketch-based Approach

- If a seed set S^* covers the maximal number of the RR set, S^* is likely to be the optimal seed set.

Algorithm 2: RR-Sketch (G, k, θ) **[100]**

Input : $G = (V, E)$: A social graph k: A number;
θ: Number of RR Sets.

Output: S: Seed Set.

$\mathcal{R} \leftarrow \emptyset, S \leftarrow \emptyset$
Generate θ random RR sets and insert them into $\mathcal{R}$
for $i = 1, \ldots, k$ **do**
    Pick node v_i that covers the most RR sets in $\mathcal{R}$
    Add v_i into S
    Remove from $\mathcal{R}$ all RR sets that are covered by v_i
return S

Conclusion

Pros: Efficient
Cons: Without theoretical guarantees

Heuristic-based Approach

Pros:
1. Efficient
2. Theoretical guarantees

Cons: Not general

Sketch-based Approach

Traditional Simulation-based Approach

Efficient Simulation-based Approach

Pros:
1. General
2. Theoretical guarantees

Cons: Inefficient

Future directions(I)

- Context-aware influence maximization
  - Topic-aware influence maximization[9]
  - Location-aware influence maximization[10][11]

[9] S. Chen, J. Fan, G. Li, J. Feng, K. Tan, and J. Tang, “Online topic-aware influence maximization,” PVLDB, 2015

[10] G. Li, S. Chen, J. Feng, K. Tan, and W. Li, “Efficient location-aware influence maximization,” in SIGMOD, 2014

[11] X. Wang, Y. Zhang, W. Zhang, and X. Lin, “Distance-aware influence maximization in geo-social network,” in ICDE, 2016

Future directions(II)

- Influence probability inference
  - Influence probability between users is fundamental for influence spread estimation[12][13]

[12] A. Goyal, F. Bonchi, and L. V. S. Lakshmanan. Learning influence probabilities in social networks. In ACM WSDM, 2010.

[13] W. Zhu, W. Peng, L. Chen, K. Zheng, and X. Zhou. Modeling user mobility for location promotion in location-based social networks. KDD 2015

Reference

[1] Centola, Damon, and Michael Macy. "Complex contagions and the weakness of long ties." American journal of Sociology 2007.

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[14] Li Y, Fan J, Wang Y, et al. Influence Maximization on Social Graphs: A Survey. TKDE 2018

Thanks!