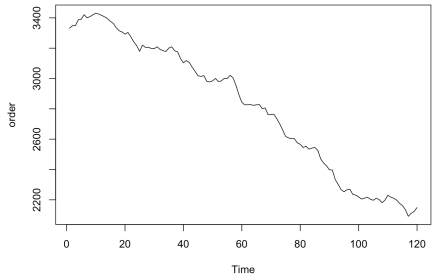
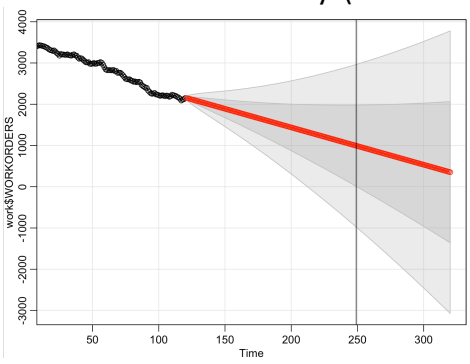
Report

1. Executive Summary
    - Objective—This report presents an analysis on the number of corrective work orders during the construction phase of a nuclear plant. And it provides an estimate for how many days it will take to reach the operational level of 1000 work orders.

    - Methods and Data—Here we mainly use the technology of time series, fitting an ARIMA model. The data include the work orders in the first 120 days of the construction phase of a nuclear plant.



    - Conclusion—We fit an ARIMA(1,2,1) to the 120 days data, it gradually declines finally reach 1000 work orders after 129 days(exclude the first 120 days).



2. Summary

- The provided dataset includes 120 days data, and the order begins at 3332, ends at 2149. The max data is 3431, the min data is 2091.
- We separate the data into two parts, 90% of the data used for training, and the rest data are used for testing.
- The first 120 data declines rather stably, so at first we fit some linear regression models (Linear Trend Model, Quadratic Trend Model, Cubic Trend Model). Although the model's R square is really high, the residuals act badly which means it would not do well in future forecasting.
- Then, we fit an ARIMA model, after checking the ACF&PACF graph and the residuals plot, we finally decide to choose ARIMA(1,2,1) and do the forecasting.
- The final model :$\nabla^2 x_t = 0.2528\nabla^2 x_{t-1} + w_t - 0.9687 w_{t-1}$, where $\mathrm{w_t}$ $is\ white\ noise$.

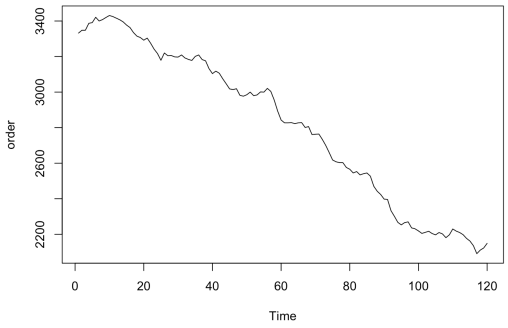
3. Other

All the update and document are available at https://github.com/yysyzz/hw5

Details:

Step 1:

At first, we plot the whole dataset and have a look.



Step 2:

Split the dataset into two parts, the train and the test. We use 90%of the whole dataset to build the train dataset, which contains 108 data. The remain data belong to the test dataset.

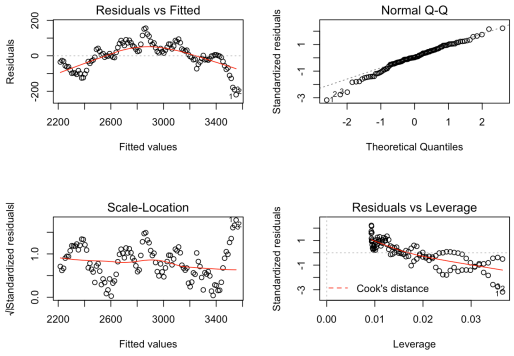
Step 3:

We firstly fit Linear Trend Regression model, $where\ w_t\ are\ i.i.d.N(0,\sigma^2)\ errors.$

K = 1, orders $= \beta_0 + \beta_1 * time + w_t.$

The fitted model is: orders $= 3564.1271 - 12.4828 * time.$

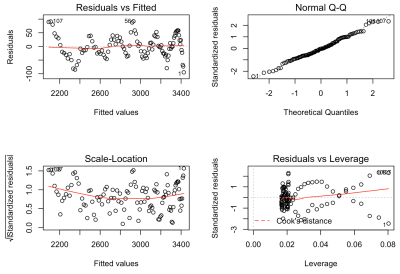
```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 3564.1271    13.6597  260.92   <2e-16 ***
timefit      -12.4828     0.2176  -57.38   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 70.49 on 106 degrees of freedom
Multiple R-squared:  0.9688,    Adjusted R-squared:  0.9685
F-statistic:  3292 on 1 and 106 DF,  p-value: < 2.2e-16
```



K = 2, orders $= \beta_0 + \beta_1 * time + \frac{\beta_2}{2!} * time^2 + w_t$

The fitted model is: orders $= 3432 - 5.293 * time - \frac{13.19}{2!} * time^2 + w_t$

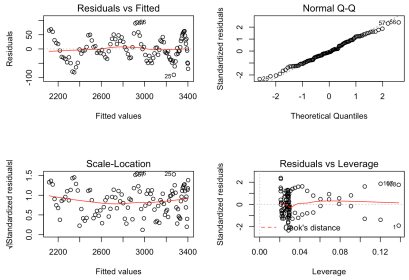
```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  3.432e+03  1.189e+01  288.69   <2e-16 ***
timefit     -5.293e+00  5.035e-01  -10.51   <2e-16 ***
tsqfit      -1.319e-01  8.951e-03  -14.74   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 40.43 on 105 degrees of freedom
Multiple R-squared:  0.9898,    Adjusted R-squared:  0.9896
F-statistic:  5113 on 2 and 105 DF,  p-value: < 2.2e-16
```



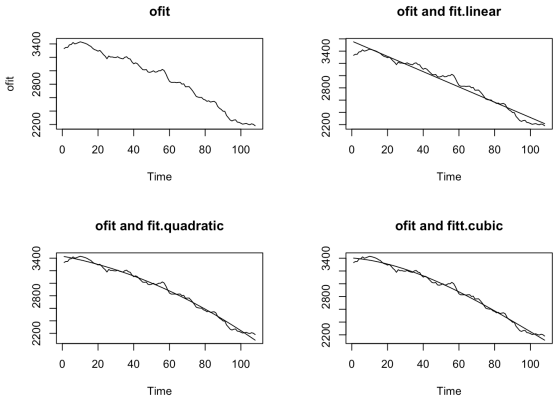
K = 3, orders $= \beta_0 + \beta_1 * time + \frac{\beta_2}{2!} * time^2 + \frac{\beta_3}{3!} * time^3 + w_t$

The fitted model is: orders $= 3404 - 2.26 * time - \frac{0.2704}{2!} * time^2 + \frac{0.00254}{3!} * time^3 + w_t$

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  3.404e+03  1.567e+01 217.271  < 2e-16 ***
timefit     -2.261e+00  1.239e+00  -1.824  0.07097 .
tsqfit      -2.704e-01  5.270e-02  -5.131 1.35e-06 ***
tcubfit      2.540e-03  9.537e-04   2.664  0.00896 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 39.3 on 104 degrees of freedom
Multiple R-squared:  0.9905,    Adjusted R-squared:  0.9902
F-statistic:  3609 on 3 and 104 DF,  p-value: < 2.2e-16
```



Plot time series with Fits from k=1,2,3 Models



Compare these three models(Linear Trend Model, Quadratic Trend Model, Cubic Trend Model, which one is preferred)

(1) Compare models using the R2 criterion

0.9688, 0.9898, 0.9905, Cubic Trend model.

(2) Compare models using Significance Tests

we use the Extra Sum of Squares F-test.

Cubic Trend Model or Linear Trend Model ? $H_0: \beta_2 = 0 \; and \; \beta_3 = 0. H_1: not \; all \; equal \; 0.$

Extra SS F-test= 118.4447, qf(0.95,2,104)= 3.083706, so reject H0.

Compare the Quadratic Trend Model to the Cubic Trend Model, $H_0: \beta_3 = 0, H_1: \beta_3 \neq 0$

Extra SS F-test= 7.0938, qf(0.95,1,104)=3.932438, so reject H0.

The significance tests prefer the Cubic Trend model.

(3) AIC

11.38555, 10.2827, 10.23522. The AIC prefer the Cubic Trend model.

(4) BIC

11.46006, 10.38204, 10.35939. The BIC prefer the Cubic Trend model.

(5) AICc

9.548908, 8.456612, 8.420159. The AICc prefer the Cubic Trend model.

(6)ME

31.98648, 206.1312, 158.2064. Linear Trend Model

(7)MPE

1.470221, 9.531554, 7.308646. Linear Trend Model

(8)MSE

1599.984, 44171.91, 25983.9. Linear Trend Model

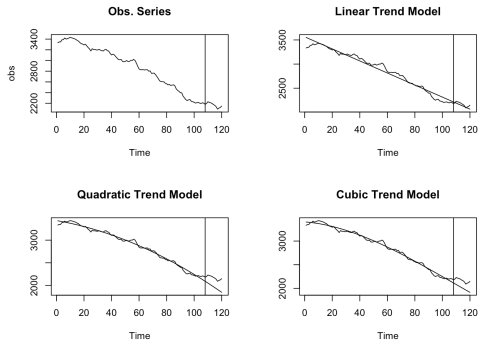
(9)MAE

35.17683, 206.1312, 158.2064. Linear Trend Model

(10)MAPE

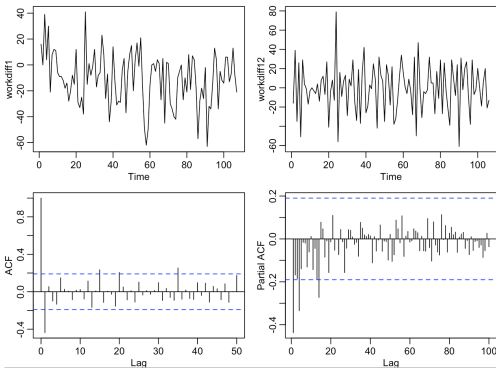
1.620296, 9.531554, 7.308646. Linear Trend Model

Finally, we choose the Linear Trend Model. Predict day is 86. (85 day: 1005.153268, 86 day: 992.670468)



Step 4: Fit a time series model.

We firstly plot the data, since the variance doesn't change much, we don't need to do log to the data. Obviously, there exists a trend, so we do two times differencing (df2) and the trend become stationary. Then we draw the ACF and PACF plot. The ACF cuts off at lag 1, and the PACF tails off, so we try to fit ARIMA(0,2,1) and similar ARIMA(1,2,1),ARIMA(2,2,1), ARIMA(3,2,1), ARIMA(4,2,1). Consider the AIC, AICc, BIC, we choose ARIMA(1,2,1). Also compare these index with the linear model, the ARIMA model is better.

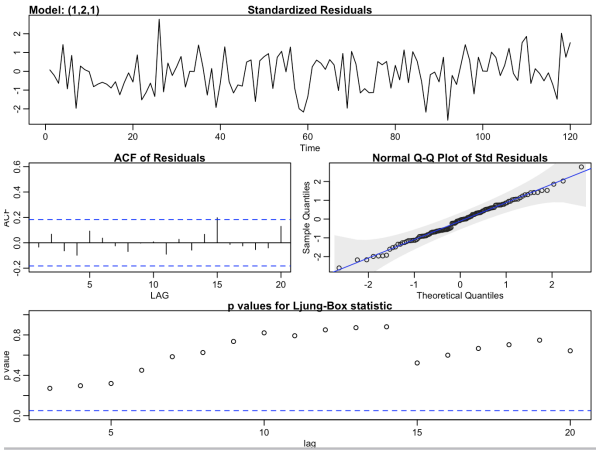


Consider that the last 12 day data might have a large influence to the model, we decide to fit the ARIMA model with the whole dataset. And the final model is ARIMA(1,2,1)

$$\nabla^2 x_t = 0.2528\nabla^2 x_{t-1} + w_t - 0.9687 w_{t-1}, \text{where } \mathrm{w_t}\ \textit{is white noise.}$$

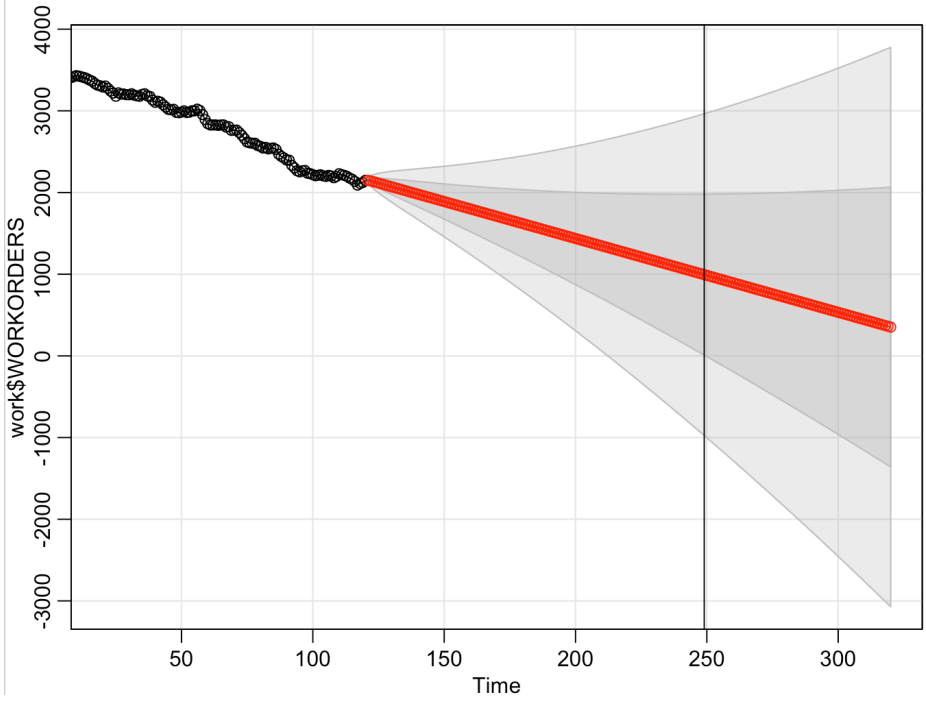
We check the residuals and find it acts much better than the linear regression, there is not any significant patterns and it is normal distributed. We conclude that the residual is similar to white noise which means the model fits good.

```
$ttable
    Estimate     SE  t.value p.value
ar1   0.2528 0.1033   2.4479  0.0158
ma1  -0.9687 0.0574 -16.8843  0.0000

$AIC
[1] 7.041322

$AICc
[1] 7.059713

$BIC
[1] 6.08778
```



Use the final model to estimate the day reaching 1000 order, the result is 129.(128

day:1003.6528 129 day: 994.6121)



Limitation:

As the time go by, the predicted interval actually is really wide. There is a big possibility that it would reach 1000 work orders much earlier or later. Also from previous experience, the series should reach a steady state which can't not be shown with our ARIMA model.

Appendix:

1. code

```
#read the data in cvs
work <- read.csv("workorders.csv",header = T)

#print the whole dataset
day <- work$DAY
order <- work$WORKORDERS
order <- ts(order)
ts.plot(order)
order

#split the data into train and test part.
```

```
ofit <- order[1:108]
nfit <- length(ofit)
timefit <- time(ofit)
otest <- order[109:120]
ntest <- length(otest)
timetest <- time(otest)

#fit linear regression model
#k = 1
mlr.lin <- lm(ofit~timefit)
summary(mlr.lin)
plot(mlr.lin)
anova(mlr.lin)
#k = 2
tsqfit <- timefit^2/factorial(2)
mlr.quad <- lm(ofit~timefit+tsqfit)
summary(mlr.quad)
plot(mlr.quad)
anova(mlr.quad)
#k = 3
tcubfit <- timefit^3/factorial(3)
mlr.cub <- lm(ofit~timefit+tsqfit+tcubfit)
summary(mlr.cub)
plot(mlr.cub)
anova(mlr.cub)
#plot fitted model
par(mfrow=c(2,2))
ts.plot(ofit,main="ofit") # Time Series Plot
# Plot of xfit vs mlr.lin$fitted
plin=cbind(ofit,mlr.lin$fitted)
ts.plot(plin,main="ofit and fit.linear")
pquad=cbind(ofit,mlr.quad$fitted)
ts.plot(pquad,main="ofit and fit.quadratic")
pcub=cbind(ofit,mlr.cub$fitted)
ts.plot(pcub,main="ofit and fitt.cubic")
#compare
sigsq.lin=anova(mlr.lin)[["Mean Sq"]][2]
sigsq.quad=anova(mlr.quad)[["Mean Sq"]][3]
sigsq.cub=anova(mlr.cub)[["Mean Sq"]][4]
# Akaike Information Criterion,AIC
(AIC.lin = AIC(mlr.lin)/nfit )
(AIC.quad = AIC(mlr.quad)/nfit)
(AIC.cub = AIC(mlr.cub)/nfit)
```

```
# Bayesian Information Criterion,BIC
(BIC.lin = BIC(mlr.lin)/nfit )
(BIC.quad = BIC(mlr.quad)/nfit)
(BIC.cub = BIC(mlr.cub)/nfit)
# Corrected AIC, i.e., AICc using formula
k = 1
(AICc.lin=log(sigsq.lin)+(nfit+k)/(nfit-k-2) )
k = 2
(AICc.quad=log(sigsq.quad)+(nfit+k)/(nfit-k-2))
k = 3
(AICc.cub=log(sigsq.cub)+(nfit+k)/(nfit-k-2))
#test
new <- data.frame(timefit=c(109:120))
pfore.lin <- predict(mlr.lin,new,se.fit = TRUE)
pfore.lin$fit
efore.lin=otest-pfore.lin$fit
(me.lin=mean(efore.lin) )#31.98648
(mpe.lin=100*(mean(efore.lin/otest)) )#1.470221
(mse.lin=sum(efore.lin**2)/ntest)#1599.984
(mae.lin=mean(abs(efore.lin)))#35.17683
(mape.lin=100*(mean(abs((efore.lin)/otest))))#1.620296
timefit=c(109:120)
tsqfit=timefit^2/factorial(2)
matq=matrix(c(timefit,tsqfit),ncol=2,dimnames = list(c(),c("timefit","tsqfit")))
matq
newnq <- data.frame(matq)
pfore.quad=predict(mlr.quad,newnq,se.fit = TRUE)
pfore.quad$fit # point predictions
(efore.quad=otest-pfore.quad$fit)
(me.quad=mean(efore.quad))#206.1312
(mpe.quad=100*(mean(efore.quad/otest)))#9.531554
(mse.quad=sum(efore.quad**2)/ntest)#44171.91
(mae.quad=mean(abs(efore.quad)))#206.1312
(mape.quad=100*(mean(abs((efore.quad)/otest))))# 9.531554
timefit=c(109:120)
tsqfit=tfit^2/factorial(2)
tcubfit=tfit^3/factorial(3)
matc=matrix(c(timefit,tsqfit,tcubfit),ncol=3,dimnames =
list(c(),c("timefit","tsqfit","tcubfit")))
newnc=data.frame(matc)
pfore.cub=predict(mlr.cub,newnc,se.fit = TRUE)
pfore.cub$fit
efore.cub=otest-pfore.cub$fit
```

```
(me.cub=mean(efore.cub))#158.2064
(mpe.cub=100*(mean(efore.cub/otest)))#7.308646
(mse.cub=sum(efore.cub**2)/ntest)#25983.9
(mae.cub=mean(abs(efore.cub)))#158.2064
(mape.cub=100*(mean(abs((efore.cub)/otest))))#7.308646

linff=c(mlr.lin$fitted,pfore.lin$fit)
quadff=c(mlr.quad$fitted,pfore.quad$fit)
cubff=c(mlr.cub$fitted,pfore.cub$fit)
obs=c(ofit,otest)
obslin=cbind(obs,linff)
obsquad=cbind(obs,quadff)
obscub=cbind(obs,cubff)
time=c(1:length(obs))
ts.plot(obs,main="Obs. Series")
abline(v=108)
ts.plot(obslin,main="Linear Trend Model")
abline(v=108)
ts.plot(obsquad,main="Quadratic Trend Model")
abline(v=108)
ts.plot(obscub,main="Cubic Trend Model")
abline(v=108)
#fit the linear model to find 1000
goal <- data.frame(timefit=c(121:300))
pfore.lin <- predict(mlr.lin,goal,se.fit = TRUE)
pfore.lin$fit
#fit the cub model to find 1000
timefit=c(121:300)
tsqfit=timefit^2/factorial(2)
tcubfit=timefit^3/factorial(3)
matc=matrix(c(timefit,tsqfit,tcubfit),ncol=3,dimnames =
list(c(),c("timefit","tsqfit","tcubfit")))
goalc=data.frame(matc)
pfore.cub=predict(mlr.cub,goalc,se.fit = TRUE)
pfore.cub$fit

#ARIMA model
workdiff1 <- diff(work$WORKORDERS[1:108])

ts.plot(workdiff1)
workdiff12 <- diff(workdiff1)
ts.plot(workdiff12)
```

```
acf(workdiff12,max(50))
pacf(workdiff12,max(100))
sarima(work$WORKORDERS[1:108],0,2,1)
foredata <- sarima.for(work$WORKORDERS[1:108], 12, 0, 1, 1)
tresidual <- work$WORKORDERS[109:120]-foredata$pred
sum(tresidual)/12#ME 57.50273 55.82046
100*mean(tresidual/work$WORKORDERS[109:120])#MPE 2.643544 2.565986
mean(tresidual^2)#3842.004 3651.492
par(mfrow=c(1,1))
sarima.for(work$WORKORDERS[1:108], 200, 0, 2, 1)

#use the whole dataset to bulid ARIMA model
finaldiff1 <- diff(work$WORKORDERS)
ts.plot(finaldiff1)
finaldiff2 <- diff(finaldiff1)
ts.plot(finaldiff2)
mean(finaldiff2)
acf(finaldiff2,max(100))
pacf(finaldiff2,max(50))
sarima(work$WORKORDERS,1,2,1)#6.104306 6.08778 6.191969
sarima.for(work$WORKORDERS, 200, 1, 2, 1)
abline(v=249)
summary(work$WORKORDERS)
```

2. Reference

Time Series Analysis , by R.H. Shumway & D.S. Sto er

Invoice

Detailed report

toggl

2018-04-09 - 2018-04-15

Total 06 h 26 min

consulting selected as projects

Date	Description	Duration	User
04-09	**getting data**	**0:19:02**	329881091
	consulting	03:02-03:21	
04-09	**research**	**0:50:17**	329881091
	consulting	17:00-17:50	
04-10	**model**	**0:34:49**	329881091
	consulting	02:31-03:06	
04-10	**model**	**0:20:24**	329881091
	consulting	03:18-03:38	
04-10	**model**	**2:52:11**	329881091
	consulting	04:10-07:02	
04-10	**report**	**1:29:56**	329881091
	consulting	12:40-14:10	

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