0.4 MATHS 207 2015/2016 EXAMINATION

1. A matrix with $a_{ij} = 0$ whenever $i < j$ is called ? (a) Identity matrix (b) Upper triangle matrix (c) Null matrix (d) Lower triangle matrix (e) None
   Solution
   D (Lower triangular matrix)

2. If in a Matrix A in which $R_j = R_k$, then $|A|$ is (a) 1 (b) 2 (c) 0 (d) k (e) $\frac{1}{k}$
   Solution

   C (0)

Let $A = (a_{ij}) = \begin{pmatrix} 1 & 3 & 6 & 2 \\ 4 & 0 & -2 & 4 \\ 2 & 7 & 6 & -1 \\ 0 & 1 & 5 & 3 \end{pmatrix}$, if $B = (b_{ij})$ and $C = (c_{ij})$ are symmetric anti (skew) symmetric matrices respectively, such as $A = B + C$, then use the information to answer the next five questions that follows

3. Which of the following is not true? (a) $B^T = B$ (b) $C^T = -C$ (c) $2B = A^T + A$ (d) $C^T = C$ (e) $2C = A - A^T$
   Solution
   D $(C^T = C)$

4. b_{22} is ? (a) $\frac{7}{2}$ (b) 0 (c) $\frac{5}{2}$ (d) 1 (e) 2
   Solution
   Since B is symmetric

$$B = \frac{A^T + A}{2}$$

$$A = \begin{bmatrix} 1 & 3 & 6 & 2 \\ 4 & 0 & -2 & 4 \\ 2 & 7 & 6 & -1 \\ 0 & 1 & 5 & 3 \end{bmatrix} \implies A^T = \begin{bmatrix} 1 & 4 & 2 & 0 \\ 3 & 0 & 7 & 1 \\ 6 & -2 & 6 & 5 \\ 2 & 4 & -1 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & \frac{7}{2} & 4 & 1 \\ \frac{7}{2} & 0 & \frac{5}{2} & \frac{5}{2} \\ 4 & \frac{5}{2} & 6 & 2 \\ 1 & \frac{5}{2} & 2 & 3 \end{bmatrix} \qquad \therefore \ b_{22} = 0 \qquad \text{(B)}$$

5. $b_{13} + c_{31}$ is ? (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

Solution

Since C is anti - symmetrical

$C = \frac{1}{2}(A - A^T)$

$$C = \begin{bmatrix} 0 & \frac{-1}{2} & 2 & 1 \\ \frac{1}{2} & 0 & \frac{-9}{2} & \frac{3}{2} \\ -2 & \frac{9}{2} & 0 & -3 \\ -1 & \frac{-3}{2} & 3 & 0 \end{bmatrix} \qquad b_{13} + b_{31} = 4 - 2 = 2$$

(B)

6. If $D = (d_{ij})$ is such that $B + D = D + B = I_4$, then d_{22} is ?
(a) 0 (b) 1 (c) -1 (d) 2 (e) -2

Solution

$B + D = D + B = I_4$

$\therefore \ D = I_4 - B$

$$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & \frac{-7}{2} & -4 & -1 \\ \frac{-7}{2} & 1 & \frac{-5}{2} & \frac{-5}{2} \\ -4 & \frac{-5}{2} & -5 & -2 \\ -1 & \frac{-5}{2} & -2 & -2 \end{bmatrix} \qquad d_{22} = 1 \qquad \text{(B)}$$

7. Suppose $E = (e_{ij})$ is such that, $E + C = C + E = 0$, then e_{43} is ? (a) -2 (b) 2 (c) 3 (d) -3 (e) 1

Solution

$E + C = C + E = 0$

$E = 0 - C$

$$E = \begin{bmatrix} 0 & \frac{1}{2} & -2 & -1 \\ \frac{-1}{2} & 0 & \frac{9}{2} & \frac{-3}{2} \\ 2 & \frac{-9}{2} & 0 & 3 \\ 1 & \frac{3}{2} & -3 & 0 \end{bmatrix} \qquad e_{43} = -3 \qquad \text{(D)}$$

8. A matrix **A** is such that $A^2 = A$ is called (a) Idempotent (b) Symmetric (c) Triangular (d) Scalar (e) Identity
Solution
A (Idempotent)

9. Let $A = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 1 & 0 \\ 6 & 2 & 0 \end{pmatrix}$ then $|A|$ is ? (a) 1 (b) 2 (c) -1 (d) 0 (e) 4
Solution
D (0)

10. The system $AX = 0$ will always have (a) Infinite solution (b) No solution (c) At least one solution (d) only one solution (e) None
Solution
(D) only one solution

11. A matrix **B** of order n with the property that, another matrix **A** of order n, $AB = BA = A$ is called? (a) Singular matrix (b) Inverse of A (c) Null matrix (d) Square matrix (e) Identity of A
Solution
E (Identity of A)

12. If **A** is symmetric, then (a) $A = -A^2$ (b) $A = A^T$ (c) $A = -A^T$ (d) $A = A^2$ (e) $A = -A$
Solution
B $A = A^T$

13. The inverse of **ABC** is? (a) $A^{-1}B^{-1}C^{-1}$ (b) $C^{-1}B^{-1}A^{-1}$ (c) $A^{-1}C^{-1}B^{-1}$ (d) $C^{-1}A^{-1}B^{-1}$ (e) $B^{-1}C^{-1}A^{-1}$
Solution
$E = B^{-1}C^{-1}A^{-1}$

Suppose $B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{5} & 0 \\ 0 & 0 & 1 \end{pmatrix}$, then use it to answer the following questions

14. $|B^{-1}|$ is? (a) 2 (b) 3 (c) 4 (d) 5 (e) 6
Solution
$|B^{-1}|$ The minor of B is

$$B_m = \begin{pmatrix} \frac{1}{5} & 0 & 0 \\ 0 & \frac{1}{5} & 0 \\ 0 & 0 & \frac{1}{5} \end{pmatrix} \quad |B| = \frac{1}{5}$$

$$B^{-1} = \frac{Adj B}{|B|} = \frac{1}{5}\begin{pmatrix} \frac{1}{5} & 0 & 0 \\ 0 & \frac{1}{5} & 0 \\ 0 & 0 & \frac{1}{5} \end{pmatrix} \qquad B^{-1} = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

$|B^{-1}| = 5$ (D)

15. If $A = (a_{ij}) = B - B^{-1}$, then a_{22} is? (a) $\frac{5}{24}$ (b) $\frac{-24}{5}$ (c) $\frac{24}{5}$ (d) $\frac{5}{24}$ (e) None

Solution

$$A = \begin{pmatrix} \frac{-24}{5} & 0 & 0 \\ 0 & \frac{-24}{5} & 0 \\ 0 & 0 & \frac{-24}{5} \end{pmatrix} \qquad a_{22} = \frac{-24}{5} \qquad \text{(B)}$$

16. If **B** is obtained from **A** by performing the operation $R_j \longleftrightarrow R_i$ on **A**, then (a) $|B| = |A|$ (b) $|B| = k|A|$ (c) $|B| = \frac{1}{k}|A|$ (d) $|A| = |B| = 0$ (e) $|B| = -|A|$

Solution

$E \quad (|B| = -|A|)$

17. Suppose **A** is of order n and the row reduced echeleon form of **A** has **r** non zero rows, then the rank of **A** is? (a) n (b) r (c) $n - r$ (d) $r - n$ (e) None

Solution

B (r)

Consider $A = \left(\begin{array}{ccc|c} 1 & 4 & 6 & k_1 \\ 0 & 1 & 3 & k_2 \\ 0 & 0 & \theta - 4 & k_3 \end{array}\right)$ use matrix **A** to answer the following three questions

18. If $\theta = 4$ and $k_3 \neq 0$, then the system represented by **A** has (a) Infinite solution (b) many solution (c) Single solution (d) No solution (e) Two solutions

Solution

D (No solution)

19. For what value of θ and k_3 would the system has infinite solutions (a) $\theta = 4 \quad k_3 = 0$ (b) $\theta = -4, \quad k_3 = 0$ (c) $\theta = 4, \quad k_3 = 4$ (d) $\theta = \infty, \quad k_3 = 0$ (e) $\theta = k_3$

Solution

$\theta = 4,\ k_3 = 0$ (A)

20. By letting $\theta = 1$ and solving the system, the value of $x_2 + 3x_3$ is? (a) k_3 (b) $k_2 - k_3$ (c) $-k_2$ (d) $k_3 - k_2$ (e) k_2

Solution

$$A = \begin{pmatrix} 1 & 4 & 6 & : & k_1 \\ 0 & 1 & 3 & : & k_2 \\ 0 & 0 & -3 & : & k_3 \end{pmatrix}$$

$-3x_3 = k_3,\quad x_3 = \dfrac{-k_3}{3}$
$x_2 + 3x_3 = k_2$
$x_2 + 3(\frac{-k_3}{3}) = k_2$
$x_2 - k_3 = k_2,\quad x_2 = k_2 + k_3$
$x_2 + k_3 \implies k_2 + k_3 + 3(\frac{-k_3}{3}) = k_2$
$k_2 + k_3 - k_3 = k_2$ (E)

21. A matrix obtained from I_n by performing a single row (column) is called ? (a) Row matrix (b) Elementary matrix (c) Singular matrix (d) Identity matrix (e) Row Identity

Solution

Elementary matrix (B)

Assume **A**,**B**,**C** are respectively $m \times n, n \times k\ and\ k \times l$ matrices, then use the information to answer the following two questions

22. Which of the following operation is not possible (a) AB (b) BC (c) $A + B\ if\ m = n = k$ (d) AC (e) All are possible

Solution

$A = m \times n$
$B = n \times k$
$C = k \times l$
A C (D)

23. Suppose $m = k$, then which of the following is true (a) AB, BA (b) AB, AC (c) BC, CB (d) BC, BA (e) AB, CB

Solution

$m = k$
$AB,\ BA$ (A)

24. A matrix in which a_{ij} are equal whenever $i = j$ and $a_{ij} = 0$ whenever $i \neq j$ is called (a) Singular matrix (b) Identity matrix (c) Null matrix (d) Square matrix (e) scalar matrix

Solution

Null matrix (C)

25. A matrix **A** is inverse of **B** if (a) $A - B = I_n$ (b) $AB = A$ (c) $BA = AB = 0$ (d) $A - B = 0$ (e) $AB = BA = I_n$

Solution

$AB = BA = I_n$ (E)

Use $\begin{array}{ccccccc} x & + & y & + & z & = & 3 \\ 2x & - & y & + & 3z & = & 5 \\ x & - & 3y & + & 4z & = & 6 \end{array}$ to answer the three questions that follow.

26. Using cramer's rule, the value of **z** is (a) $\frac{5}{13}$ (b) $\frac{-5}{13}$ (c) $\frac{-13}{5}$ (d) $\frac{13}{5}$ (e) 13

Solution

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -2 & 3 \\ 1 & -3 & 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 6 \end{pmatrix}$$

$|A| = 1(-4+9) - 1(8-3) + 1(-6+1) = 5 - 5 - 5$

$|A| = -5$

$$\begin{vmatrix} 1 & 1 & 3 \\ 2 & -1 & 5 \\ 1 & -3 & 6 \end{vmatrix}$$

$|Z| = 1(-6+15) - 1(12-5) + 3(-6+1)$

$|Z| = 9 - 7 - 15 = 13$

$$Z = \frac{|Z|}{|A|} = \frac{|-13|}{|-5|}$$

$$Z = \frac{13}{5} \quad \text{(D)}$$

27. The minors of a_{22} is ? (a) 1 (b) -1 (c) 2 (d) -2 (e) 3

Solution

Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -2 & 3 \\ 1 & -3 & 4 \end{pmatrix}$

Minor of $a_{22} = (1 \times 4) - (1 \times 1)$
$a_{22} = 4 - 1 = 3$ (E)

28. The cofactor of a_{32} is ? (a) 1 (b) −1 (c) 2 (d) −2 (e) 0
Solution

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -2 & 3 \\ 1 & -3 & 4 \end{pmatrix}$$

$(1 \times 3)(2 \times 1) = 3 - 2 = 1$
Cofactor $= (-1)^{i+j}(m_{ij})$
$= (-1)^{2+1}(1) = -1$ (B)

29. From the properties of adjoint of a matrix **A**, the inverse of $A = A^{-1}$ is ? (a) $|A| \times Adj A$ (b) $\frac{Adj A}{|A|}$ (c) $\frac{Adj A}{A}$ (d) $A \times Adj A$ (e) None
Solution

$$\frac{Adj A}{|A|} = A^{-1} \qquad \text{(B)}$$

30. If M_{ij} is the minor of a_{ij}, then it,s cofactors is defined as (a) $-M_{ij}$ (b) $a_{ij}M_{ij}$ (c) $-a_{ij}M_{ij}$ (d) $(-1)^{i+j}M_{ij}$ (e) $-(-1)^{i+j}M_{ij}$
Solution
Cofactor $= (-1)^{i+j}(m_{ij})$ (D)

0.5 MATHS 207 2016/2017 EXAMINATION

1. If the equations $x + 3y + z = 0$, $2x - y - z = 0$ *and* $kx + 2y + 3z = 0$ have non trivial solution then **k=** (a) $\frac{13}{2}$ (b) $\frac{9}{2}$ (c) $\frac{-15}{2}$ (d) $\frac{-13}{2}$

   Solution

   $x + 3y + z = 0$
   $2x - y - z = 0$
   $kx + 2y + 3z = 0$

   $$\begin{pmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ k & 2 & 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{Find the determinant of A}$$

   $|A| = 1(-3+2) - 3(6+k) + 1(4+k)$
   $= -1 - 18 - 3k + 4 + k$
   $= -15 - 2k$

   $$\begin{pmatrix} 0 & 3 & 1 \\ 0 & -1 & -1 \\ 0 & 2 & 3 \end{pmatrix}$$

   $|x| = 0$
   $$x = \frac{|A|}{|x|} = \frac{-15-2k}{0} = 0$$
   $-15 - 2k = 0$
   $-15 = 2k$
   $$k = \frac{-15}{2} \qquad \text{(C)}$$

2. The equation $x + 2y + 3z = 1$, $2x + y + 3z = 2$ *and* $5x + 5y + 9z = 4$ have (a) No solution (b) Unique solution (c) Infinite solution (d) cannot say anything

3. if $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{pmatrix}$, $A^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & c \\ \frac{5}{3} & \frac{-3}{2} & \frac{1}{2} \end{pmatrix}$, then

   (a) $a = 2$, $c = \frac{-1}{2}$ (b) $a = 1$ $c = -1$ (c) $a = -1, c = 1$ (d) $a = \frac{1}{2}$ $c\frac{1}{2}$

   Solution

$$I_3 = A.A^{-1} = \begin{bmatrix} 0 - 4 + \frac{10}{3} & 0 + 3 - 3 & 0 + c + 1 \\ \frac{1}{2} - 8 + 5 & \frac{-1}{2} + 6 - \frac{9}{2} & \frac{1}{2} + 2c + \frac{3}{2} \\ \frac{3}{2} - 4a + \frac{5}{3} & \frac{-3}{2} + 3a - \frac{3}{2} & \frac{3}{2} + ac + \frac{1}{2} \end{bmatrix}$$

$$= \begin{pmatrix} \frac{-2}{3} & 0 & c + 1 \\ \frac{-5}{2} & 1 & \frac{4+4c}{2} \\ \frac{19-24a}{6} & \frac{-6+6a}{2} & \frac{4+2ac}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$c = -1$

$$\frac{4 + 2ac}{2} = 1$$

$4 + 2ac = 2$

$4 - 2a = 2$

$2a = 2$

$a = 1$

$\therefore \; c = -1 \;\; and \;\; a = 1$ (B)

4. Consider the following system of equation

$$\begin{array}{ccccc} x_1 & + & x_3 & = & 5 \\ x_1 & - & x_2 & - & x_3 = 6 \\ x_2 & + & x_3 & = & 7 \end{array}$$

The above system of equation is (a) Inconsistent (b) Consistent with a unique solution (c) Consistent with infinitely many solutions (d) None of the above

Solution

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & -1 & -1 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \\ 7 \end{pmatrix}$$

$|A| = 0 - 0 + 1 = 1$

$$\begin{pmatrix} 5 & 0 & 1 \\ 6 & -1 & -1 \\ 7 & 1 & 1 \end{pmatrix}$$

$|x| = 5(-1 + 1) - 0(6 + 7) + 1(6 + 7) = 13$

$$x = \frac{|A|}{|x|} = \frac{1}{13} \quad \text{(B)}$$

It is consistent with a unique solution, because the number of equations must be at least equal to the number of variables.

5. $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0,$ then x =

(a) $\frac{3}{2}, \frac{3}{11}$ (b) $\frac{3}{2}, \frac{11}{3}$ (c) $\frac{2}{3}, \frac{11}{3}$ (d) $\frac{2}{3}, \frac{3}{11}$

Solution

Finding the determinant

$(3x-8)[(3x-8)^2-9]-3[(3x-8)(3)-9]+3[9-3(3x-8)]$

$(3x-8)(9x^2-48x+64-9)-3(9x-24-9)+3(9-9x+24)$

$(3x-8)(9x^2-48x+55)-3(9x-33)+3(33-9x)$

$= 27x^3-144x^2+165x-27x^2+384x-440-27x+99+99-27x$

$= 27x^3-216x^2+495x-242$

$$\therefore \quad x = \frac{2}{3}, \ \frac{11}{3} \qquad \text{(C)}$$

6. Let $A = \begin{pmatrix} 4 & 4k & k \\ 0 & k & 4k \\ 0 & 0 & 4 \end{pmatrix}$ If $det(A)^2 = 16$ then $|K|$ is

(a) 1 (b) $\frac{1}{4}$ (c) 4 (d) 4^2

Solution

$$A \times A = \begin{pmatrix} 16+0+0 & 16k+4k^2+0 & 4k+16k^2+4k \\ 0+0+0 & 0+k^2+0 & 0+4k^2+16k \\ 0+0+0 & 0+0+0 & 0+0+16 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 16 & 4k^2+16k & 16k^2+8 \\ 0 & k^2 & 4k^2+16k \\ 0 & 0 & 16 \end{pmatrix}$$

$= 16(16k^2) - 4k^2 - 16k(0)$

$$= k^2 = \frac{1}{16} = \frac{1}{4} \qquad \text{(B)}$$

7. If the equation $x-2y+3z=0$, $-2x+3y+2z=0$ *and* $-8x+\lambda y=0$ have non-trivial solution then $\lambda =$ (a) 18 (b) 13 (c) -10 (d) 4

Solution

$$\begin{pmatrix} 1 & -2 & 3 \\ -2 & 3 & 2 \\ -8 & \lambda & 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

To get the value, you have to take the determinant.

$0 = 1(0\times3)-(2\times\lambda)-(-2)(0\times(-2))-(-8\times2)+3(\lambda\times-2)-(-8\times2)$

$= -2\lambda + 32 - 6\lambda + 72 = 0$

$-8\lambda = -72 - 32$

$-8\lambda = -104$

$\lambda = \dfrac{-104}{-8} = 13$ (B)

8. Consider the following matrix $A = \begin{pmatrix} 0 & 0 & 2 & 0 \\ 2 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 1 \end{pmatrix}$ which of the following is true

(a) The columns are linearly dependent (b) The matrix is not invertible (c) The matrix has determinant -2 (d) None of the above

Solution

$$\begin{array}{ccc} 0 & 0 & 2 \\ 2 & -1 & 0 \\ 1 & -1 & 0 \\ \hline \end{array}$$

$0 + 2$ (D)

9. The value of **q** which the matrix $A = \begin{pmatrix} 1-q & 2 \\ 3 & 2-q \end{pmatrix}$ is singular are

(a) $1, 2$ (b) $1, 4$ (c) $1, -2$ (d) $-1, 4$

Solution

$(1-q)(2-q) - 6 = 0$

$2 - q - 2q + q^2 - 6 = 0$

$2 - 3q + q^2 - 6 = 0$

$q^2 - 3q - 4 = 0$

$q(q-4) + (q-4)$

$(q-4)(q+1) = 0$

$q = 4 \quad or \quad q = -1$ (D)

10. Given the matrix $A = \begin{pmatrix} 2 & -1 & 2 \\ 1 & 2 & 3 \\ 3 & -4 & 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, If $P =$

$(1\ 0\ 1)^T$
and $X = (x_1\ x_2\ x_3)^T$

Solving the system $AX = P$ using Cramer's rule $x_1 = \frac{\delta_1}{\delta}. \delta_3$ is equal to
(a) -5 (b) 4 (c) -2 (d) $\frac{-2}{3}$
Solution
$2(2+12)+1(1-9)+2(-4-6)$
$28-8-20=0$

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 0 \\ 3 & -4 & 1 \end{vmatrix}$$

$\delta_3 = 2(2)+1(1)+1(-4-6)$
$= 4+1-10 = 4-9 = -5$ (A)

11. If A and B are both invertible $n \times n$ matrices, then AB is invertible
(a) True (b) False
Solution
True (A)

12. Let A and B be $n \times n$ matrices assume that $AB = I_n$ then $BA = I_n$.
(a) True (b) False
Solution
True (A)

13. Let D be the diagonal matrix, then D^T is symmetric (a) True (b) False
Solution
True (A)

14. To multiple two matrices A and B, the numbers of rows of A must be equal to the number of columns of B. (a) True (b) False
Solution
True (A)

15. The j^{th} column of the matrix product AB is equal to the matrix product ABj where bj is the j^{th} column of B (a) True (b) False

16. An $m \times n$ matrix is said to be in reduced row echelon form if the rows I and $i+1$ are to successive rows that do not consist entirely

of zeros, then leading entry of the row $i+1$ is to the right of the leading entry row i_0.
(a) True (b) False

Solution

True (A)

17. A matrix in reduced row echelon form must have at least one row that consist entirely of zeros. (a) True (b) False

Solution

False (B)

18. A matrix is said to be upper - triangular matrix if $a_{ij} = 0 \; for \; i < j$ (a) True (b) False

Solution

False (B)

19. If A and B are two matrices that are inverse of one another, then they have unequal determinant. (a) True (b) False

Solution

True (A)

20. The determinant of a matrix is expressed as the sum of the product of the elements of any row/column by their corresponding cofactors (a) True (b) False

Solution

True (A)

21. Given $A = \begin{pmatrix} 6 & 2x+3 \\ 3x & 3x+3 \end{pmatrix}$ If A is symmetric, then $x =$ ———————— **Solution**

$A = A^T$

$$A^T = \begin{pmatrix} 6 & 3x \\ 2x+x & 3x+3 \end{pmatrix}$$

$3x = 2x + 3 \implies x = 3$
$3x + 3 = 3x + 3$
$= 0 = 0$
$\therefore \quad x = 3$

22. Given that $A = \begin{pmatrix} -2 & 3 \\ 4 & 5 \end{pmatrix}$ then $A^{-1} =$ ————————————

Solution

Firstly $|A| = -10 - 12 = -22$
$|A| = -22$

$$\frac{-1}{22}\begin{bmatrix} 5 & -3 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} \frac{-5}{22} & \frac{3}{22} \\ \frac{-2}{11} & \frac{1}{11} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{-5}{22} & \frac{3}{22} \\ \frac{-2}{11} & \frac{1}{11} \end{bmatrix}$$

23. If the matrix $\begin{pmatrix} x & y+3 \\ 2z & 8 \end{pmatrix} = \begin{pmatrix} 12 & 5 \\ 6 & 8 \end{pmatrix}$ then $2x + y - z =$

Solution
$x = 12 \quad 2z = 6$
$z = 3$
$y + 3 = 5$
$y = 5 - 3$
$y = 2$
$\therefore \quad x = 12, \quad y = 2 \text{ and } z = 3$
Substitute into the equation
$2(12) + 2 - 3 = 23$

Use the given information to answer questions 24 - 26

Given the matrices $A = \begin{pmatrix} 1 & 0 & 7 \\ 3 & 2 & -1 \\ -5 & -2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -6 & 7 \\ 8 & 9 & 10 \\ 0 & 1 & -3 \end{pmatrix}$

24. The determinant of A is ______________________

Solution
Det(A)
$|A| = 1(10 - 2) - 0 + 7(-6 + 10)$
$= 8 + 28 = 36$

25. If AB=D, where $D = (d_{ij})$, the entry d_{23} is ______________________

Solution

$$D = \begin{bmatrix} 1 & 0 & 7 \\ 3 & 2 & -1 \\ -5 & -2 & 5 \end{bmatrix}\begin{bmatrix} 4 & -6 & 7 \\ 8 & 9 & 10 \\ 0 & 1 & -3 \end{bmatrix}$$

$$D = \begin{bmatrix} 4+0+0 & -6+0+7 & 7+0-21 \\ 12+16+0 & -18+18-1 & 21+20+3 \\ -20-16+0 & 30-18+5 & -35-20-15 \end{bmatrix}$$

$$D = \begin{bmatrix} 4 & -1 & -14 \\ 28 & -1 & 44 \\ -36 & 17 & -70 \end{bmatrix} \qquad d_{23} = 44$$

26. Given that $S = A^T + B^T$ and $S = (S_{ij})$ then S_{32} is ______

Solution

$$A^T = \begin{bmatrix} 1 & 3 & -5 \\ 0 & 2 & -2 \\ 7 & -1 & 5 \end{bmatrix} \quad B^T = \begin{bmatrix} 4 & 8 & 0 \\ -6 & 9 & 1 \\ 7 & 10 & -3 \end{bmatrix}$$

$$S = A^T + B^T = \begin{bmatrix} 1+4 & 3+8 & -5+0 \\ 0-6 & 2+9 & -2+1 \\ 7+7 & -1+10 & 5-3 \end{bmatrix}$$

$$S = \begin{bmatrix} 5 & 11 & -5 \\ -6 & 11 & -1 \\ 14 & 9 & 2 \end{bmatrix} \qquad S_{32} = 9$$

27. The augmented matrix of the system

$$\begin{aligned} x + 3y &= 7 \\ y - 2z &= 10 \\ x - 5z &= 16 \end{aligned}$$

is ______

Solution

$$\begin{bmatrix} 1 & 3 & 0 & : & 7 \\ 0 & 1 & -2 & : & 10 \\ 1 & 0 & -5 & : & 16 \end{bmatrix}$$

28. If $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ Find the matrix B such that $AB = \begin{pmatrix} 2 & 0 \\ 4 & 3 \end{pmatrix}$

Solution

$$A^{-1} = \begin{bmatrix} 0 & +1 \\ +1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix} \times A^{-1}$$

$$B = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2\times 0 + 0 \times 1 & 2 \times 1 + 0 \times -1 \\ 4 \times 0 + 3 \times 1 & 4 \times 1 + 3 \times -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}$$

29. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix}$ Find a matrix C such that $A - B = 2C$

Solution
$A - B = 2C$

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} - \begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix} = \begin{pmatrix} -3 & -3 \\ -3 & -3 \end{pmatrix}$$

$$2C = \begin{pmatrix} -3 & -3 \\ -3 & -3 \end{pmatrix}$$

Dividing all matrix members by 2

$$\therefore \; c = \begin{pmatrix} \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & \frac{-3}{2} \end{pmatrix}$$

30. If $A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 3 & 4 \\ 3 & 1 & 2 \end{pmatrix}$ Given that $A + B$ is an identity matrix, find **B**

Solution
$B = I_3 - A$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 2 \\ 2 & 3 & 4 \\ 3 & 1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & -2 \\ -2 & -2 & -4 \\ -3 & -1 & -1 \end{bmatrix}$$