Network-Node Stochastic Programming

Python Modeling Code for (c) and (d)

```
import itertools
from pyomo.environ import *
from pyomo.core import *
from pyomo.pysp.annotations import
(PySP_ConstraintStageAnnotation,
StochasticConstraintBoundsAnnotation,
StochasticConstraintBodyAnnotation)

theta1_table = [1,1,1,1,1,1,1,0,0,0]
theta2_table = [1,1,1,1,1,1,1,0,0,0]
d1_rhs_table = [0,1,2,3,5]
d2_rhs_table = [0,1,2,3,5]
d3_rhs_table = [0,1,2,3]

model = ConcreteModel()

#-------C--------
# model.constraint_stage =
PySP_ConstraintStageAnnotation()
# model.stoch_matrix =
StochasticConstraintBodyAnnotation()
# num_scenarios = len(theta1_table)*len(theta2_table)
# scenario_data = dict(('Scenario'+str(i),
(theta1val, theta2val))
#                       for i, (theta1val, theta2val)
in
#                       enumerate
(itertools.product(theta1_table, theta2_table),1))
#-------D--------
model.constraint_stage =
PySP_ConstraintStageAnnotation()
model.stoch_rhs =
StochasticConstraintBoundsAnnotation()
model.stoch_matrix =
StochasticConstraintBodyAnnotation()

num_scenarios =
len(theta1_table)*len(theta2_table)*len(d1_rhs_table)
* len(d2_rhs_table) * len(d3_rhs_table)

scenario_data = dict(('Scenario'+str(i), (theta1val,
theta2val, d1_rhs_val, d2_rhs_val, d3_rhs_val))
                     for i, (theta1val, theta2val,
d1_rhs_val, d2_rhs_val, d3_rhs_val) in
                     enumerate
(itertools.product(theta1_table, theta2_table,
d1_rhs_table, d2_rhs_table, d3_rhs_table),1))
#----------------
model.d1_rhs = Param(mutable=True, initialize=0.0)
model.d2_rhs = Param(mutable=True, initialize=0.0)
model.d3_rhs = Param(mutable=True, initialize=0.0)

model.x1 = Var(within=NonNegativeReals)
model.x2 = Var(within=NonNegativeReals)
model.x3 = Var(within=NonNegativeReals)
model.f11 = Var(within=NonNegativeReals)
model.f12 = Var(within=NonNegativeReals)
model.f21 = Var(within=NonNegativeReals)
model.f22 = Var(within=NonNegativeReals)
model.f31 = Var(within=NonNegativeReals)
model.f32 = Var(within=NonNegativeReals)
model.s1 = Var(within=NonNegativeReals)
model.s2 = Var(within=NonNegativeReals)
```

```
model.s3 = Var(within=NonNegativeReals)
model.theta1 = Param(mutable=True, initialize=0)
model.theta2 = Param(mutable=True, initialize=0)

model.FirstStageCost = Expression(initialize=0)
model.SecondStageCost =
Expression(initialize=model.s1+model.s2+model.s3)

model.budget =
Constraint(expr=model.x1+model.x2+4*model.x3 <= 10)
model.constraint_stage.declare(model.budget,1)

#-------C--------
# model.d1 =
Constraint(expr=model.f11+model.f12+model.s1 == 2.05)
# model.constraint_stage.declare(model.d1,2)

# model.d2 =
Constraint(expr=model.f21+model.f22+model.s2 == 2.05)
# model.constraint_stage.declare(model.d2,2)

# model.d3 =
Constraint(expr=model.f31+model.f32+model.s3 == 1.5)
# model.constraint_stage.declare(model.d3,2)
#-------D--------
model.d1 =
Constraint(expr=model.f11+model.f12+model.s1 ==
model.d1_rhs)
model.constraint_stage.declare(model.d1,2)
model.stoch_rhs.declare(model.d1)

model.d2 =
Constraint(expr=model.f21+model.f22+model.s2 ==
model.d2_rhs)
model.constraint_stage.declare(model.d2,2)
model.stoch_rhs.declare(model.d2)

model.d3 =
Constraint(expr=model.f31+model.f32+model.s3 ==
model.d3_rhs)
model.constraint_stage.declare(model.d3,2)
model.stoch_rhs.declare(model.d3)
#----------------
model.c1 =
Constraint(expr=model.f11+model.f22+model.f32-model.t
heta1*model.x1 <= 0)
model.constraint_stage.declare(model.c1,2)
model.stoch_matrix.declare(model.c1,
variables=[model.x1])

model.c2 =
Constraint(expr=model.f12+model.f21+model.f32-model.t
heta2*model.x2 <= 0)
model.constraint_stage.declare(model.c2,2)
model.stoch_matrix.declare(model.c2,
variables=[model.x2])

model.c3 =
Constraint(expr=model.f12+model.f22+model.f31-model.x
3 <= 0)
model.constraint_stage.declare(model.c3,2)

model.obj =
Objective(expr=model.FirstStageCost+model.SecondStage
Cost)
```

```
def pysp_scenario_tree_model_callback():
    from pyomo.pysp.scenariotree.tree_structure_model
import \
        CreateConcreteTwoStageScenarioTreeModel

    st_model =
CreateConcreteTwoStageScenarioTreeModel(num_scenarios
)

    first_stage = st_model.Stages.first()
    second_stage = st_model.Stages.last()

    # First Stage
    st_model.StageCost[first_stage] =
'FirstStageCost'
    st_model.StageVariables[first_stage].add('x1')
    st_model.StageVariables[first_stage].add('x2')
    st_model.StageVariables[first_stage].add('x3')

    # Second Stage
    st_model.StageCost[second_stage] =
'SecondStageCost'
    st_model.StageVariables[second_stage].add('f11')
    st_model.StageVariables[second_stage].add('f12')
    st_model.StageVariables[second_stage].add('f21')
    st_model.StageVariables[second_stage].add('f22')
    st_model.StageVariables[second_stage].add('f31')
    st_model.StageVariables[second_stage].add('f32')
    st_model.StageVariables[second_stage].add('s1')
    st_model.StageVariables[second_stage].add('s2')
    st_model.StageVariables[second_stage].add('s3')
    return st_model

def pysp_instance_creation_callback(scenario_name,
node_names):
    #
    # Clone a new instance and update the stochastic
    # parameters from the sampled scenario
    #
    instance = model.clone()
    theta1_val, theta2_val, d1_rhs_val, d2_rhs_val,
d3_rhs_val = scenario_data[scenario_name]
    instance.theta1.value = theta1_val
    instance.theta2.value = theta2_val
 #-------D--------
    instance.d1_rhs.value = d1_rhs_val
    instance.d2_rhs.value = d2_rhs_val
    instance.d3_rhs.value = d3_rhs_val
 #----------------
    return instance
```

(c) Total execution time=3.67 seconds

(d) Total execution time=370.37 seconds

(C-1)SD input smps files

network_c.cor

```
* Source:    Pyomo MPS Writer
* Format:    Free MPS
*
NAME Scenario1
OBJSENSE
 MIN
ROWS
 N  obj
 L  c_u_budget_
 E  c_e_d1_
 E  c_e_d2_
 E  c_e_d3_
 L  c_u_c1_
 L  c_u_c2_
 L  c_u_c3_
 E  c_e_ONE_VAR_CONSTANT
COLUMNS
   x1 c_u_budget_ 1
   x1 c_u_c1_ -1
   x2 c_u_budget_ 1
   x2 c_u_c2_ -1
   x3 c_u_budget_ 4
   x3 c_u_c3_ -1
   f11 c_e_d1_ 1
   f11 c_u_c1_ 1
   f12 c_e_d1_ 1
   f12 c_u_c2_ 1
   f12 c_u_c3_ 1
   f21 c_e_d2_ 1
   f21 c_u_c2_ 1
   f22 c_e_d2_ 1
   f22 c_u_c1_ 1
   f22 c_u_c3_ 1
   f31 c_e_d3_ 1
   f31 c_u_c3_ 1
   f32 c_e_d3_ 1
   f32 c_u_c1_ 1
   f32 c_u_c2_ 1
   s1 obj 1
   s1 c_e_d1_ 1
   s2 obj 1
   s2 c_e_d2_ 1
   s3 obj 1
   s3 c_e_d3_ 1
   ONE_VAR_CONSTANT obj 0
   ONE_VAR_CONSTANT c_e_ONE_VAR_CONSTANT 1
RHS
   RHS c_u_budget_ 10
   RHS c_e_d1_ 2.0499999999999998
   RHS c_e_d2_ 2.0499999999999998
   RHS c_e_d3_ 1.5
   RHS c_u_c1_ 0
   RHS c_u_c2_ 0
   RHS c_u_c3_ 0
   RHS c_e_ONE_VAR_CONSTANT 1
BOUNDS
 LO BOUND x1 0
 LO BOUND x2 0
 LO BOUND x3 0
 LO BOUND f11 0
 LO BOUND f12 0
 LO BOUND f21 0
 LO BOUND f22 0
 LO BOUND f31 0
 LO BOUND f32 0
 LO BOUND s1 0
 LO BOUND s2 0
 LO BOUND s3 0
ENDATA
```

network_c.tim

```
TIME network
PERIODS IMPLICIT
  x1 obj TIME1
```

```
   f11 c_e_d1_ TIME2
ENDATA
```

network_c_revised.sto

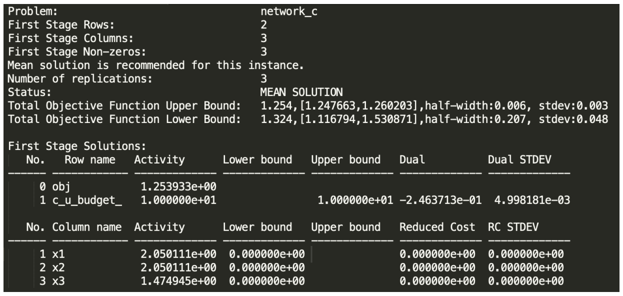
```
STOCH retail
INDEP        DISCRETE
   x1        c_u_c1_ -1       0.7
   x2        c_u_c2_ 0       0.3
   x1        c_u_c1_ 0       0.3
   x2        c_u_c2_ -1       0.7
ENDATA
```

(C-2) Output Console

```
Yului-MacBook:Debug yul$ ./sd

Beginning SD...

Please enter the name of the problem files (eg. `exags1p'):
network_c
mkdir: ./sdoutput: File exists
mkdir: ./sdoutput/network_c: File exists
Reading problems from ./sdinput/network_c/network_c.cor
the filename is : ./sdinput/network_c/network_c.cor
Specified objective sense: MINIMIZE
Selected objective  name:  obj
Selected RHS        name:  RHS
Selected bound      name:  BOUND
finished loading core file via external Solver.
-0.700000
-0.700000
Vector x_k from the mean problem ::
16.59999999913667068085 4.99999999949012696732
4.99999999935742867052 0.00000000028811110114
2.04999999981604874577 0.00000000003835065432
2.04999999981604652532 0.00000000003835065432
0.00000000021140979251 1.44999999969580239956
0.00000000014588982868 0.00000000014588982868
0.05000000009321703409 1.00000000000000000000
Objective from the mean problem : 0.050000

num_rv = 2;
num_cipher = 1;

-------------------- Replication No.0 --------------------
+++++++++++++++++>************Max_pi***************:
6.000000


-------------------- Replication No.1 --------------------
++++++Warning:  Output names have been modified to
conform to LP format.
Warning:  Output names have been modified to conform to LP
format.
+++Warning:  Output names have been modified to conform
to LP format.
+Warning:  Output names have been modified to conform to
LP format.
+++++++>************Max_pi***************: 6.000000


-------------------- Replication No.2 --------------------
++++++++++++++++++>
```

```
 Begin evaluation of average solution
........................................

obs:10000 mean:1.250491   error: 0.041535
 0.95 CI: [1.224521 , 1.276461]
........................................

(ETC…)

Final Estimate
 obs:172378 mean:1.253933
 0.95 C.I.: [1.247663 , 1.260203]

************Max_pi***************: 6.000000

Ending SD...

Total running time: 7512.242000 ms
```

(C-3) Output Result



```
Problem:                              network_c
First Stage Rows:                     2
First Stage Columns:                  3
First Stage Non-zeros:                3
Mean solution is recommended for this instance.
Number of replications:               3
Status:                               MEAN SOLUTION
Total Objective Function Upper Bound:   1.254,[1.247663,1.260203],half-width:0.006, stdev:0.003
Total Objective Function Lower Bound:   1.324,[1.116794,1.530871],half-width:0.207, stdev:0.048

First Stage Solutions:
   No.   Row name   Activity      Lower bound   Upper bound   Dual          Dual STDEV
------ ------------ ------------- ------------- ------------- ------------- -------------
     0 obj           1.253933e+00
     1 c_u_budget_   1.000000e+01                1.000000e+01 -2.463713e-01  4.998181e-03

   No. Column name  Activity      Lower bound   Upper bound   Reduced Cost  RC STDEV
------ ------------ ------------- ------------- ------------- ------------- -------------
     1 x1            2.050111e+00  0.000000e+00               0.000000e+00  0.000000e+00
     2 x2            2.050111e+00  0.000000e+00               0.000000e+00  0.000000e+00
     3 x3            1.474945e+00  0.000000e+00               0.000000e+00  0.000000e+00
```

Hence, we get the optimal value of

x1 = 2.0501, x2 = 2.0501, x3 = 1.4750

(D-1) smps files

network_d.cor

```
* Source:    Pyomo MPS Writer
* Format:    Free MPS
*
NAME Scenario1
OBJSENSE
 MIN
ROWS
 N  obj
 L  c_u_budget_
 E  c_e_d1_
 E  c_e_d2_
 E  c_e_d3_
 L  c_u_c1_
 L  c_u_c2_
 L  c_u_c3_
 E  c_e_ONE_VAR_CONSTANT
COLUMNS
    x1 c_u_budget_ 1
    x1 c_u_c1_ -1
    x2 c_u_budget_ 1
    x2 c_u_c2_ -1
    x3 c_u_budget_ 4
    x3 c_u_c3_ -1
    f11 c_e_d1_ 1
    f11 c_u_c1_ 1
    f12 c_e_d1_ 1
    f12 c_u_c2_ 1
    f12 c_u_c3_ 1
    f21 c_e_d2_ 1
    f21 c_u_c2_ 1
    f22 c_e_d2_ 1
    f22 c_u_c1_ 1
    f22 c_u_c3_ 1
    f31 c_e_d3_ 1
    f31 c_u_c3_ 1
    f32 c_e_d3_ 1
    f32 c_u_c1_ 1
    f32 c_u_c2_ 1
    s1 obj 1
    s1 c_e_d1_ 1
    s2 obj 1
    s2 c_e_d2_ 1
    s3 obj 1
    s3 c_e_d3_ 1
    ONE_VAR_CONSTANT obj 0
    ONE_VAR_CONSTANT c_e_ONE_VAR_CONSTANT 1
RHS
    RHS c_u_budget_ 10
    RHS c_e_d1_ 0
    RHS c_e_d2_ 0
    RHS c_e_d3_ 0
    RHS c_u_c1_ 0
    RHS c_u_c2_ 0
    RHS c_u_c3_ 0
    RHS c_e_ONE_VAR_CONSTANT 1
BOUNDS
 LO BOUND x1 0
 LO BOUND x2 0
 LO BOUND x3 0
 LO BOUND f11 0
 LO BOUND f12 0
 LO BOUND f21 0
 LO BOUND f22 0
 LO BOUND f31 0
 LO BOUND f32 0
 LO BOUND s1 0
 LO BOUND s2 0
 LO BOUND s3 0
ENDATA
```

network_d.tim

```
TIME network
PERIODS IMPLICIT
   x1 obj TIME1
   f11 c_e_d1_ TIME2
ENDATA
```

network_d.sto

```
STOCH retail
INDEP        DISCRETE
   RHS         c_e_d1_ 0        0.05
   RHS         c_e_d1_ 1        0.2
   RHS         c_e_d1_ 2        0.5
   RHS         c_e_d1_ 3        0.2
   RHS         c_e_d1_ 5        0.05
   RHS         c_e_d2_ 0        0.05
   RHS         c_e_d2_ 1        0.2
   RHS         c_e_d2_ 2        0.5
   RHS         c_e_d2_ 3        0.2
   RHS         c_e_d2_ 5        0.05
   RHS         c_e_d3_ 0        0.1
   RHS         c_e_d3_ 1        0.4
   RHS         c_e_d3_ 2        0.4
   RHS         c_e_d3_ 3        0.1
   x1        c_u_c1_ -1       0.7
   x1        c_u_c1_ 0        0.3
   x2        c_u_c2_ -1       0.7
   x2        c_u_c2_ 0        0.3
ENDATA
```

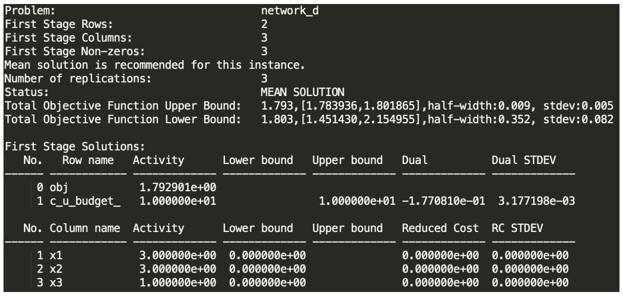
(E-1) Output Console

```
Yului-MacBook:Debug yul$ ./sd

Beginning SD...

Please enter the name of the problem files (eg. `exags1p'):
network_d
mkdir: ./sdoutput: File exists
Reading problems from ./sdinput/network_d/network_d.cor
the filename is : ./sdinput/network_d/network_d.cor
Specified objective sense: MINIMIZE
Selected objective  name:  obj
Selected RHS        name:  RHS
Selected bound      name:  BOUND
finished loading core file via external Solver.
2.050000
2.050000
1.500000
-0.700000
-0.700000
Vector x_k from the mean problem ::
16.5999999913668489171 4.9999999949014206635
4.9999999935741357149 0.0000000028811109845
```

```
2.0499999981605007804 0.0000000003835065390
2.0499999981605185440 0.0000000003835065390
0.0000000021140979064 1.4499999969578707848
0.0000000014588982748 0.0000000014588982748
0.0500000009324059858 1.0000000000000000000
Objective from the mean problem : 0.050000

num_rv = 5;
num_cipher = 1;



-------------------- Replication No.0 --------------------
+++++++++++++++Warning:  Output names have been
modified to conform to LP format.
Warning:  Output names have been modified to conform to LP
format.
+>************Max_pi***************: 6.000000


-------------------- Replication No.1 --------------------
+++++++++++++Warning:  Output names have been modified
to conform to LP format.
Warning:  Output names have been modified to conform to LP
format.
+Warning:  Output names have been modified to conform to
LP format.
>************Max_pi***************: 6.000000


-------------------- Replication No.2 --------------------
++++++++++++++Warning:  Output names have been
modified to conform to LP format.
>

 Begin evaluation of average solution
........................................

obs:10000 mean:1.810400   error: 0.039467
 0.95 CI: [1.774675 , 1.846125]
........................................

obs:20000 mean:1.801300   error: 0.027890
 0.95 CI: [1.776181 , 1.826419]
........................................

obs:30000 mean:1.803500   error: 0.022772
 0.95 CI: [1.782965 , 1.824035]
........................................

obs:40000 mean:1.800275   error: 0.019738
 0.95 CI: [1.782508 , 1.818042]
........................................

obs:50000 mean:1.796300   error: 0.017678
 0.95 CI: [1.780422 , 1.812178]
........................................

obs:60000 mean:1.792733   error: 0.016168
 0.95 CI: [1.778241 , 1.807226]
........................................

obs:70000 mean:1.794871   error: 0.014958
 0.95 CI: [1.781448 , 1.808295]
........................................

obs:80000 mean:1.793413   error: 0.014011
 0.95 CI: [1.780849 , 1.805976]
```

```
........................................

obs:90000 mean:1.795678   error: 0.013189
 0.95 CI: [1.783836 , 1.807519]
........................................

obs:100000 mean:1.797750   error: 0.012518
 0.95 CI: [1.786498 , 1.809002]
........................................

obs:110000 mean:1.794745   error: 0.011945
 0.95 CI: [1.784026 , 1.805465]
........................................

obs:120000 mean:1.794275   error: 0.011441
 0.95 CI: [1.784010 , 1.804540]
........................................

obs:130000 mean:1.792977   error: 0.011004
 0.95 CI: [1.783112 , 1.802842]
........................................

obs:140000 mean:1.791564   error: 0.010600
 0.95 CI: [1.782069 , 1.801060]
........................................

obs:150000 mean:1.793207   error: 0.010239
 0.95 CI: [1.784026 , 1.802387]
.............................

Final Estimate
 obs:157253 mean:1.792901
 0.95 C.I.: [1.783936 , 1.801865]

************Max_pi***************: 6.000000

Ending SD...

Total running time: 6902.213000 ms
```

(E-2) Output Result



```
Problem:                             network_d
First Stage Rows:                    2
First Stage Columns:                 3
First Stage Non-zeros:               3
Mean solution is recommended for this instance.
Number of replications:              3
Status:                              MEAN SOLUTION
Total Objective Function Upper Bound:   1.793,[1.783936,1.801865],half-width:0.009, stdev:0.005
Total Objective Function Lower Bound:   1.803,[1.451430,2.154955],half-width:0.352, stdev:0.082

First Stage Solutions:
   No.   Row name   Activity      Lower bound   Upper bound   Dual          Dual STDEV
------ ------------ ------------- ------------- ------------- ------------- -------------
     0 obj           1.792901e+00
     1 c_u_budget_   1.000000e+01                1.000000e+01 -1.770810e-01  3.177198e-03

   No. Column name  Activity      Lower bound   Upper bound   Reduced Cost  RC STDEV
------ ------------ ------------- ------------- ------------- ------------- -------------
     1 x1            3.000000e+00  0.000000e+00               0.000000e+00  0.000000e+00
     2 x2            3.000000e+00  0.000000e+00               0.000000e+00  0.000000e+00
     3 x3            1.000000e+00  0.000000e+00               0.000000e+00  0.000000e+00
```

Hence, we get the optimal value of

x1 = 3.00, x2 = 3.00, x3 = 1.00