# LEO-Wyndor : Advertising and Production

1. Introduction

The "Wyndor Glass Co." is outsourcing the resource utilization project for the production of high quality glass doors A and B: some with aluminum frames (A), and others with wood frames (B). These doors are produced in three plants, named 1, 2, and 3. The data for the production resources are shown in Table 2-1. The product mix to maximize the total revenue will not only be decided by production plan, but also by the potential future sales. Sales, however, is uncertain and depends on the marketing strategy adopted. Given 200 advertising time slots, the marketing strategy involves choosing a mix of advertising outlets through which can reach out to potential consumers efficiently. We assume that the advertising dataset reflects sales resulted from advertising campaigns undertaken by Wyndor Glass.

| Plant | Prod.time for A | Prod.time for B | Total Hours |
|---|---|---|---|
| 1 | 1 | 0 | 8 |
| 2 | 0 | 2 | 24 |
| 3 | 3 | 2 | 36 |
| Profit per Batch | $3,000 | $5,000 | |

Table 2-1. Data for the Wyndor Glass Problem

The advertisement uses two different media, TV and radio. As in the original dataset, advertising strategy is represented as budgeted dollars for each media type. Thus, in our statistical model, sales predictions are based on the dollar amount of TV and radio advertising. In our interpretation, product-sales reflect total number of doors sold ($\{W_i\}$) when advertising expenditure for TV is $X_{i,1}$and that for radio is $X_{i,2}$. (The data set has 200 data points, that is, $i = 1, \ldots, 200$). Let $x_1$ denote the TV advertising dollar amount, and $x_2$ denote the radio advertising dollar amount.

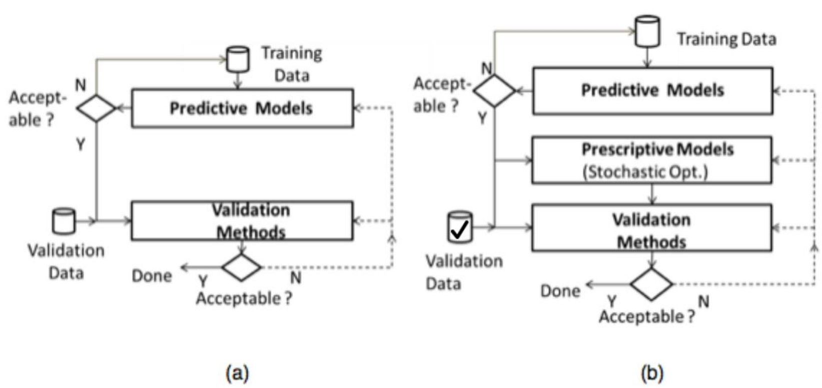


Figure 2-1. Statistical Learning and Learning Enabled Optimization

This project aims to maximize the total revenue of the production with respect to advertisement plans. We will compare the statistical learning model and learning enabled model to get reliable and realizable predictions of the future revenue.

2. Methodology

The process summarized in Figure 2-1 in which the entire data set is divided into two parts (Training and Validation). The former is used to learn model parameters (for a predictive model), and the latter data set is used for model assessment and selection. Once a model is selected, it can be finally tested by either simulation or by the additional “test dataset” before adoption.

The SL and LEO framework is used for two mathematical programming models which are deterministic linear programming and stochastic right-hand-side programming. The deterministic linear programming model uses pyomo for the formulation and the stochastic programming model uses PySP. The empirical additive error, denoted $\xi$, is the difference between the actual value and predicted value. Empirical additive errors is used to represent the randomness in the stochastic modeling. Each model corresponds to Statistical Learning, Figure 2-1 (a) and Learning Enabled Optimization, (b), respectively.

3. Predictive Model

The statistical learning model uses deterministic multiple linear programming with the advertising effect on sales. The linear regression model is used to measure the total sales on each possible pair amounts of the TV and Radio advertisement. For the regression model, the data is splitted into training data and validation data, 50% each with random seed 1. The train dataset is used for the predictive multiple linear regression model. The model with the training data is as below.

$$y_{Sales} = 3.74982951 + 0.05370186 \times x_{TV} + 0.22231025 \times x_{Radio}$$

Using the predictive regression model, we found the difference between the actual sales data and predicted sales data. We set $\{Z_i, W_i\}$ where $Z_i$ is predicted sales amount and the $W_i$ is the actual sales amount. We denote the error terms $\xi_{i,T} = Z_{i,T} - W_{i,T}$ for all $i \in Training$ and $\xi_{j,V}$ as $\xi_{j,V} = Z_{j,V} - W_{j,V}$ for all $j \in Validation$. Since $\xi_{i,T}$ and $\xi_{j,V}$ represents the normalized errors between the two datasets, qq-Plot is used to determine the error trend of the two datasets. The minimum and maximum data points were found as the outliers in the training dataset. Hence, we remove the two points and refit the multiple linear regression model. The model without any outlier is below.

$$y_{Sales} = 4.07459792 + 0.05119161 \times x_{TV} + 0.22734845 \times x_{Radio}$$

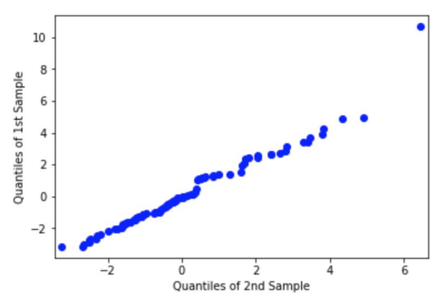


Figure 2-2. qq-Plot for $\xi_{i,T}$ and $\xi_{j,V}$

4. Statistical Learning

The deterministic linear programming model for the statistical learning is formulated as model 2-1.1 Total sales constraint has upper bound of predicted sales from the advertisement effect. This model can be treated as a single linear program with 4 decision variables ($x_1, x_2, y_a, y_b$) and 9 constraints. We solve the model with training dataset to derive the optimal solution for $x_1, x_2$ denoted as $(x_{1,D}, x_{2,D})$. The optimal solution is in the Table 2-2.

$$
\begin{aligned}
Maximize &: -0.1 \times x_1 - 0.5 \times x_2 + 3 \times y_a + 5 \times y_b \\
subject\ to &: y_a \leq 8 \\
& y_b \leq 24 \\
& 3 \times y_a + 2 \times y_b \leq 36 \\
& y_a + y_b \leq \beta_0 + \beta_1 \times x_1 + \beta_2 \times x_2 \\
& x_1 + x_2 \leq 200 \\
& x_1 - 0.5 \times x_2 \geq 0 \\
& x_1,\ x_2,\ y_a,\ y_b\ \geq 0 \\
& l_1 \leq x_1 \leq u_1 \\
& l_2 \leq x_2 \leq u_2
\end{aligned}
$$

Model 2-1.1. Deterministic MLP

| | Optimal Solution |
|---|---|
| $Objective$ | 48.16914111658245 |
| $x_{1,D}$ | 190.42285279145614 |
| $x_{2,D}$ | 9.577147208543863 |
| $y_a$ | 4.0 |
| $y_b$ | 12.0 |

Table 2-2. Optimal Solution for Statistical Learning

Now, we validate the statistical learning model with the possible errors derived from the validation dataset. The optimal solution $(x_{1,D}, x_{2,D})$ is applied to the model 2-1.2 and the $\xi_{j,V}$ is introduced as the possible 100 errors during the validation procedure. The validation model with two decision variables $(y_a, y_b)$ is formulated as below.

$$
\begin{aligned}
Maximize &: -0.1 \times x_{1,D} - 0.5 \times x_{2,D} + 3 \times y_a + 5 \times y_b \\
subject\ to &: y_a \leq 8 \\
& y_b \leq 24 \\
& 3 \times y_a + 2 \times y_b \leq 36 \\
& y_a + y_b - \beta_0 - \beta_1 \times x_{1,D} - \beta_2 \times x_{2,D} \leq \xi_{j,V} \\
& y_a,\ y_b\ \geq 0
\end{aligned}
$$

Model 2-1.2. Validation model for Deterministic MLP

The model 2-1.2 is solved for each $\xi_{j,V}, j \in Validation$. Thus, we have 100 optimal objective values from the model 2-1.2. The 95% confidence interval for the optimal objective value is (45.56898, 46.57599)

5. Learning Enabled Optimization

The stochastic programming model for the learning enabled optimization is formulated as model 2-2.1 Total sales constraint has the probabilistic right-hand-side of the empirical additive errors. This model can be solved by the SD solver with PySP formulation of 4 decision variables ($x_1, x_2, y_a, y_b$) and 9 constraints. To prevent the objective becoming negative value, we added big M to the objective function and set M as 200. We solve the model with training dataset errors to derive the optimal solution for $x_1, x_2$ and denoted the solution as $(x_{1,S}, x_{2,S})$. The optimal status is in the Table 2-3.

$$
\begin{aligned}
&Minimize: 0.1 \times x_1 + 0.5 \times x_2 - 3 \times y_a - 5 \times y_b + M \\
&subject\ to: y_a \le 8 \\
&\quad y_b \le 24 \\
&\quad 3 \times y_a + 2 \times y_b \le 36 \\
&\quad y_a + y_b - \beta_1 \times x_1 - \beta_2 \times x_2 \le \beta_0 + \xi_i \\
&\quad x_1 + x_2 \le 200 \\
&\quad x_1 - 0.5 \times x_2 \ge 0 \\
&\quad x_1,\ x_2,\ y_a,\ y_b\ \ge 0 \\
&\quad l_1 \le x_1 \le u_1 \\
&\quad l_2 \le x_2 \le u_2
\end{aligned}
$$

Model 2-2.1 Stochastic MLP

| | Optimal Solution |
|---|---|
| $Objective$ | 45.9991 |
| $x_{1,S}$ | 1.983564e+02 |
| $x_{2,S}$ | 1.643590e+00 |

Table 2-3. Optimal Solution for Learning Enabled Optimization

Now, we validate the learning enabled model with the possible errors derived from the validation dataset. While the deterministic model needs to confirm the derived objective value is in between the 95% confidence interval, the stochastic model needs to confirm if the probabilistic distribution for the training objective values corresponds with the distribution of validation objective values. The optimal solution $(x_{1,S}, x_{2,S})$ is applied to the model 2-2.2 and the $\xi_{i,T}, \xi_{j,V}$ is introduced as the possible errors. The validation model with two decision variables $(y_a, y_b)$ is formulated as below.

$$
\begin{aligned}
&Maximize: -0.1 \times x_{1,D} - 0.5 \times x_{2,D} + 3 \times y_a + 5 \times y_b \\
&subject\ to: y_a \le 8 \\
&\quad y_b \le 24 \\
&\quad 3 \times y_a + 2 \times y_b \le 36 \\
&\quad y_a + y_b - \beta_0 - \beta_1 \times x_{1,D} - \beta_2 \times x_{2,D} \le \xi_{i,T}/\xi_{j,V} \\
&\quad y_a,\ y_b\ \ge 0
\end{aligned}
$$

Model 2-2.2. Training and Validation model for Deterministic MLP

The model 2-2.2 is solved for each $\xi_{i,T}, i \in Training$ and $\xi_{j,V}, j \in Validation$ for 100 times, respectively. Thus, we have 100 optimal objective values for each training and validation errors. We conducted the Kruskal Wallis H-test to validate the difference between two groups. The test results are below.

H-statistic: 0.3128535655961006, P-Value: 0.5759343796987677
Accept NULL hypothesis - No significant difference between groups.

6. Conclusion

We observed the difference between the deterministic statistical learning model and learning enabled optimization model. We first confirmed the insignificant difference between the training and validation dataset and conducted multiple linear regression of advertisement effect on sales. The 95% confidence interval for the revenue of the validation dataset when considering the possible EAE of the regression model is ($45.56898, $46.57599) (in 1000s). The statistical model with the MLP overestimated the revenue as $48.17 (in 1000s) which is reasonable in that the regression model did not consider error terms which will depreciate the expected revenue. However, the learning enabled optimization model estimated the total revenue as $46.00 (in 1000s). The total revenue by LEO model is in the 95% confidence interval and the probabilistic distribution for the training objective values corresponds with the validation objective values. Hence, we concluded that the LEO model gives more probable and reliable outcome for the prediction than the statistical learning model itself.