Controlling Fake Reviews

Yuta Yasui

August 25, 2020

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  - Hollenbeck et al (2019): ratings vs advertisement in hotel industry
  - Reimers and Waldfogel (2020): ratings vs professtional reviews for books

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- Effort to reduce fake reviews:
  - Amazon is strictly prohibiting incentivized reviews since 2016.
  - In 2019, FTC filed the first case challenging fake paid reviews: Cure Encapsulations:
    - selling a weight loss pill ($12.8 million in sales on Amazon)
    - paid AmazonVerifiedReview.com for fake reviews

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  - Costly fake reviews might work as a signal of good quality.
    - It might pay off only for high quality sellers through future sales. (Nelson; 70,74)
- Instruments of the platform:
  1. intensity of censorship on fake reviews
  2. weights on previous reviews and new reviews,

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- The stringent censorship reduces
  - the number of fake reviews in expectation,
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- For credulous consumers:
  - the stringent censorship reduces bias for the credulous consumers as long as positive number of fake reviews are observed.

Literature

Design of Rating Systems

- **[certification]** Lizzeri (1999), Harbaugh and Rasmusen (2018), DeMarzo, Kremer, Skrzypacz (2019), Hopenhayn and Saeedi (2019), Hui et al (2018), Zapechelnyuk (2020)
- **[scoring][one-shot]** Ball (2019), **[dynamic]** Vellodi (2019): entry/exit, directed search;

  Horner and Lambert (2018), Bonatti and Cisternas (2020), **this paper**: **signal jamming**

	platform controls..		eqm action	credulous consumer
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Promotion and Signaling (Q: The higher quality, the more promotion?)

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- Horstmann and MacDonald (1994), Saraiva (2020) [numerical/empirical], and **this paper**: **Repeated promotions**; **true quality** and **reputation** play different/interactive roles

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Fake reviews with "verified purchase" on Amazon

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2. The platform takes a **transaction fee** from each transaction
3. The platform can detect a part of fake reviews.

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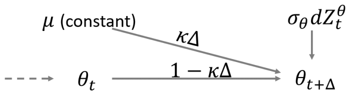
Model (2/3)

- State transition:
  - quality θ_t follows

$$\theta_{t+dt} = \theta_t (1 - \kappa dt) + \mu\kappa dt + \sigma_\theta dZ_t^\theta$$

exogenous for players (seller/buyers) and for the platform



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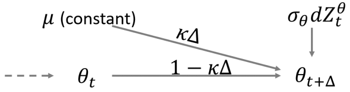

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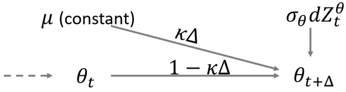

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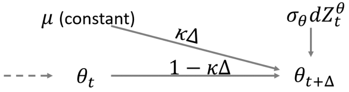

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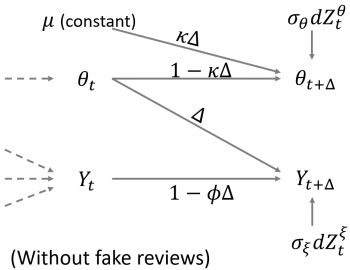

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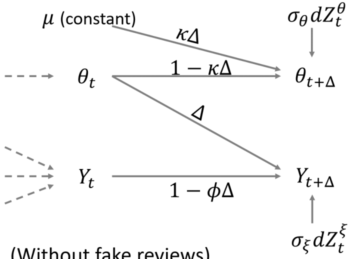

(Without fake reviews)

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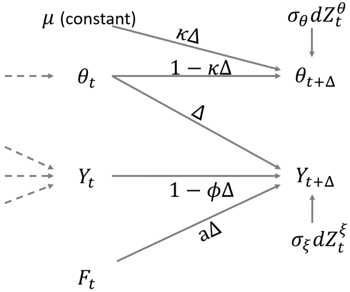
    $a > 0$: effectiveness of fake reviews. (low a = stringent censorship)



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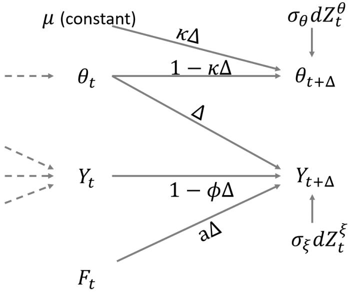

Model (2/3)

- State transition:
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$$d\theta_t = -\kappa\left(\theta_t - \mu\right)dt + \sigma_\theta dZ_t^\theta$$

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$$dY_t = -\phi Y_t dt + aF_t dt + \theta_t dt + \sigma_\xi dZ_t^\xi$$

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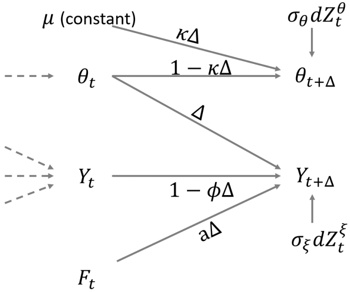

Model (3/3)

- Seller's instantaneous payoff:

$$\pi_t = \underbrace{(1-\tau)\, p_t\, (1+F_t)}_{\text{revenue}} - \underbrace{p_t \cdot F_t}_{\text{costs of refund}} - \underbrace{\frac{c}{2} F_t^2}_{\text{additional costs}}$$

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  - τ: transaction fee imposed by the platform.
- The market determines $p_t = E\left[\theta_t | Y_t\right] \equiv M_t$

$$\pi_t = (1-\tau)\, M_t - \tau M_t \cdot F_t - \frac{c}{2} F_t^2$$

  - $\tau = 0$: a. la. Holmstrom (1999), a special case of Horner and Lambert (2018)

Definition of Equilibrium

Stationary Linear Markov equilibrium

Definition

A linear Markov strategy $F = (F_t)_{t \geq 0}$ s.t. $F_t = \alpha\theta_t + \beta Y_t + \gamma$ is a stationary linear Markov equilibrium if

1. $F = \arg\max_{\left(\tilde{F}_t\right)_{t \geq 0}} E_0 \left[\int_0^\infty e^{-tr} \left((1-\tau) M_t - \tau M_t \cdot \tilde{F}_t - \frac{c}{2} \tilde{F}_t^2 \right) \right]$
2. $M_t = E^F \left[\theta_t | Y_t \right]$
3. $(\theta_t,\ Y_t)_{t \geq 0}$ induced by F is stationary Gaussian

Stationarity of Equilibrium

- Transition of $(\theta_t,\ Y_t)$ (in discrete analogue):

$$\theta_{t+dt} = \theta_t \left(1 - \kappa dt\right) + \mu\kappa dt + \sigma_\theta dZ_t^\theta$$

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  - $(\theta_t,\ Y_t)$ is stationary Gaussian if $\phi - a\beta > 0$

Stationarity of Equilibrium

- Transition of (θ_t, Y_t) (in discrete analogue):

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  - (θ_t, Y_t) is stationary Gaussian if $\phi - a\beta > 0$
- When (θ_t, Y_t) is stationary Gaussian, then

$$M_t \equiv E[\theta_t | Y_t] = \underbrace{E[\theta_t]}_{\equiv\mu} + \underbrace{\frac{Cov(\theta_t, Y_t)}{Var(Y_t)}}_{\equiv\lambda} [Y_t - \underbrace{E[Y_t]}_{\equiv\bar{Y}}]$$

Characterize Equilibrium

- HJB equation:

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\begin{aligned}
rV(\theta,\ Y) = \sup_{F \in \mathbb{R}} &(1-\tau)\, M \cdot q - \tau M \cdot F - \frac{c}{2} F^2 \\
&- \kappa\,(\theta - \mu)\, V_\theta + \{-\phi Y_t + aF + \theta\}\, V_Y \\
&+ \frac{\sigma_\theta^2}{2} V_{\theta\theta} + \frac{\sigma_\xi^2}{2} V_{YY} \\
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- Note: θ appears in the transition of states
- The equilibrium is characterized by guess-and-verify of
  - $F = \alpha\theta + \beta Y + \gamma$ (linear strategy)
  - $V = v_0 + v_1\theta + v_2 Y + v_3\theta^2 + v_4 Y^2 + v_5 Y\theta$ (quadratic value function)
  - $\phi - a\beta > 0$ (stationarity)

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There exists a stationary linear Markov equilibrium.

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  - Higher quality, more **cost-saving** in the future
    - Once its rating is boosted, the future self will reduce the fake reviews.
    - This effect remains for a long time, given high θ_t. ▸ Details

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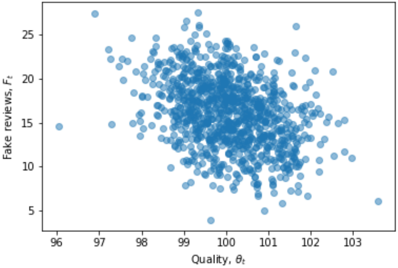

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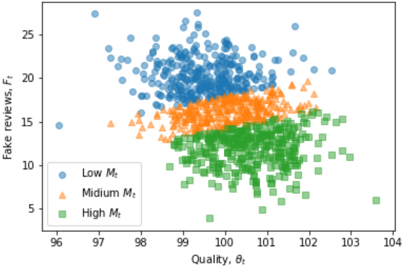

Assume that the platform can change a and ϕ

- Recall:
  - Rating: $Y_{t+dt} = Y_t \times (1 - \phi dt) + d\xi_t$ (higher ϕ, a higher weight on today's review & faster transition)
  - New review:$d\xi_t = aF_t dt + \theta_t dt + \sigma_\xi dZ_t^\xi$ (smaller a, more stringent filtering)
- [In the paper, I am working on comparative statics about τ and σ_ξ]

Proposition

$E[F_t]$ is increasing in a. $E[F_t] \geq 0$ for sufficiently large a.

Proposition (The effects of fake reviews)

$a \cdot \alpha$, $a \cdot \beta$, $a \cdot \gamma$ goes to zero as $a \to 0$.

- Reminder: $aF_t = a\alpha\theta_t + a\beta Y_t + a\gamma =$ the effect of fake reviews
- Stringent censorship can **reduce** the **expected amount** and the **effects** of fake reviews.
- Note: $(\alpha, \beta, \gamma) \nrightarrow 0$ even when $E[F_t] \to 0$ or $(a\alpha, a\beta, a\gamma) \to 0$

Q: **Should** the platform reduce fake reviews?

Criteria: $\rho^2 = \frac{Cov(\theta_t, Y_t)^2}{Var(\theta_t)Var(Y_t)}$

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- For the platform, if the rating system is not informative, the sellers and buyers might move out to other platforms.
  - Maximization of ρ^2 is equivalent to minimizing $Var(\theta|Y)$

$$Var(\theta|Y) = \underbrace{Var(\theta)}_{\text{exogenous}}\left(1-\rho^2\right)$$

  - Note: M_t is an unbiased estimate of θ_t $(E[E[\theta_t|Y_t]] = E[\theta_t])$

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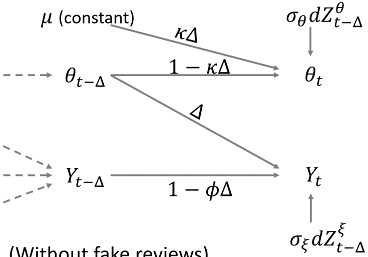

(Without fake reviews)

Q: Should the platform reduce fake reviews?

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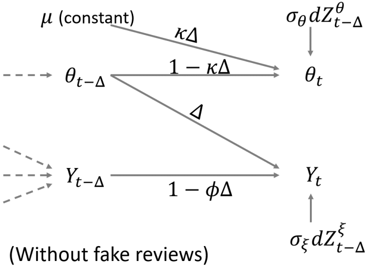

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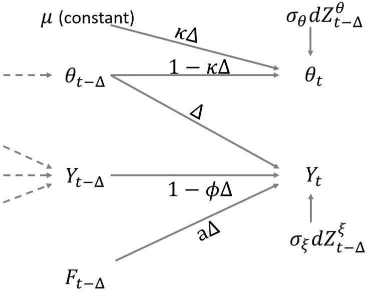

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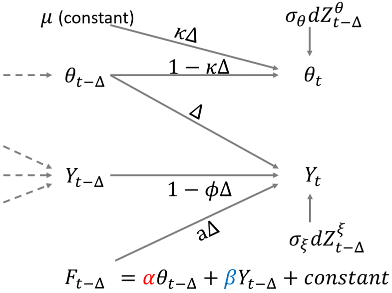

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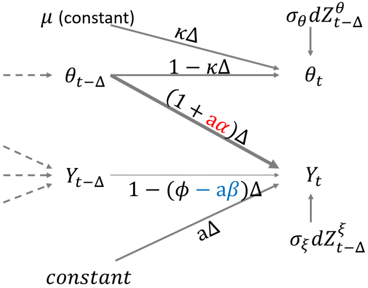

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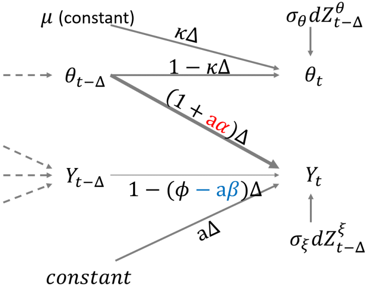

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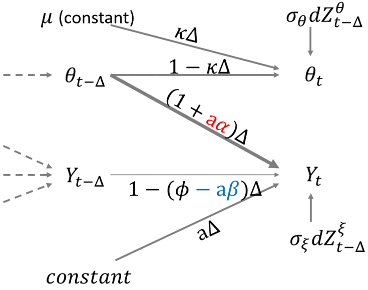

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Sketch of the proof:

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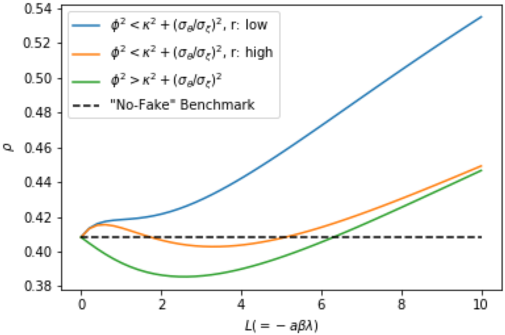

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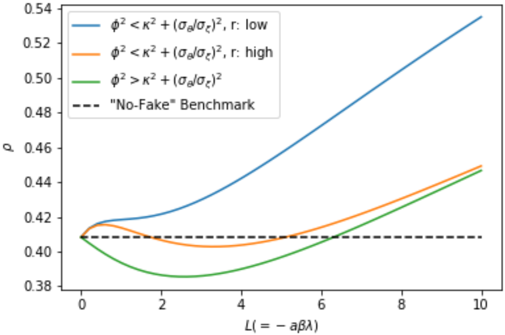

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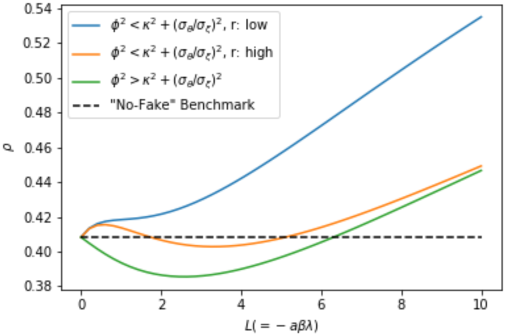

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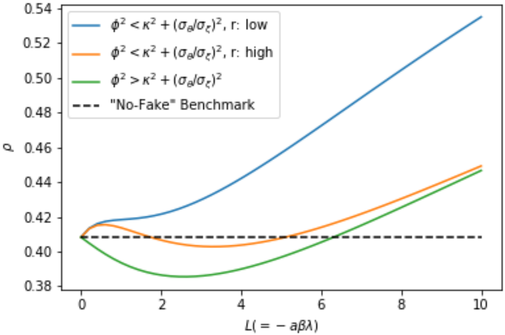

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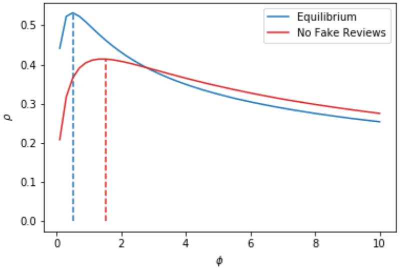

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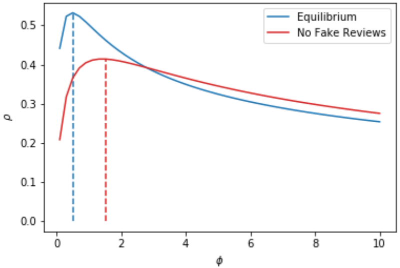
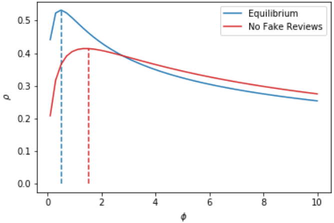
Furthermore, $\rho^2(0, \phi^0) < \rho^2(L(\phi^*), \phi^*)$ for sufficiently small r.





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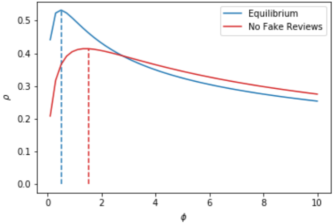

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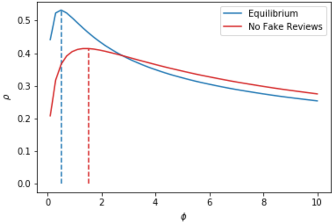

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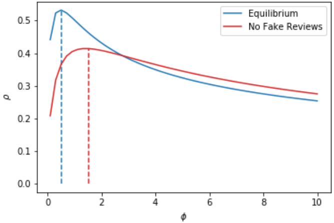

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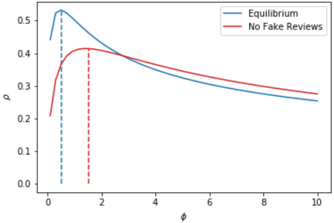

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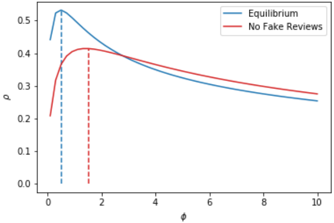

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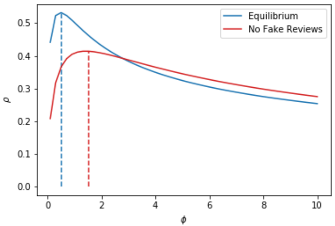

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Existence and uniqueness given the same condition as the baseline model

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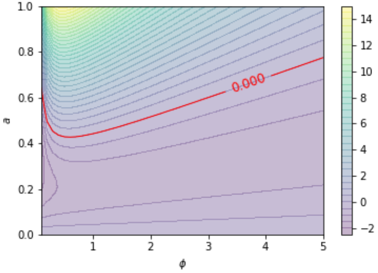
- Intuition:
  - Credulous consumers are less sensitive to the rating than rational consumers.
    - Rational consumers regard the rating infomative because of $a\alpha > 0$.
  - Less marginal benefit with credulous consumers.
  - Less fake reviews with credulous consumers.

Criteria for the credulous consumers:

$$Bias = E\left[\widetilde{E}\left[\theta_t | Y_t\right] - \theta_t\right]$$



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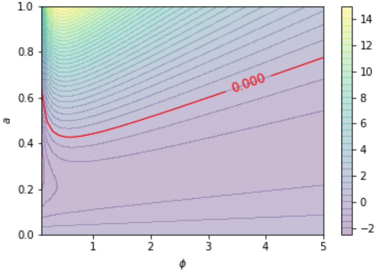

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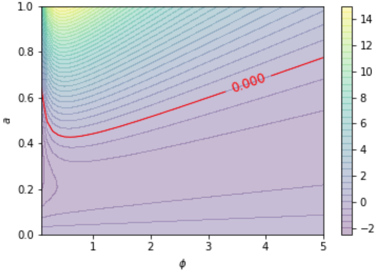

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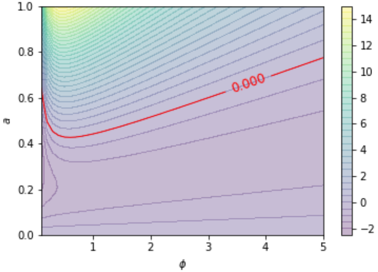

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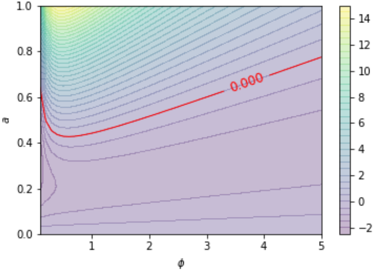

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- For credulous consumers:
  - As long as $E[F_t] \geq 0$, the more stringent censorship, the less bias for the credulous consumers.

Intuition of the Equilibrium Strategy

▸ Back to Theorem

- Reminder: $V = v_0 + v_1\theta + v_2 Y + v_3\theta^2 + v_4 Y^2 + v_5\theta Y$
- FOC: $F_t = -\frac{\tau}{c} M_t + \frac{a}{c}\{v_2 + 2Y_t v_4 + \theta v_5\}$
- $\beta < 0$
  - $\beta = -\frac{\tau}{c}\lambda + 2\frac{a}{c} v_4 = -\frac{\tau}{c}\lambda + \frac{a}{c}\frac{-\beta\lambda\tau}{(-a\beta+r+2\phi)}$
  - $\beta < 0$ since today's cost saving incentive dominates.
- $v_4 = \frac{-\beta\lambda\tau}{2(-a\beta\lambda+r+2\phi)} > 0$
  - Higher reputation, less promotion, less costly.
- $\alpha > 0$
  - $\alpha = \frac{a}{c} v_5$
  - $v_5 = \frac{1}{\kappa+r+\phi}\{\underbrace{2(a\alpha + bq)}_{\text{feedback to future } Y_t} \quad v_4 - \alpha\lambda\tau\}$

    - Driving Force: Higher quality, higher reputation in the future, cost reduction in the future.
    - Counteracting effect: Higher quality, more promotion today/in the near future (if $\alpha > 0$)

Microfoundation of the price: $p_t = M_t$

Back to Model

- (Reminder: $M_t \equiv E[\theta_t | Y_t]$)
- Suppose there is a mass (2) of buyers.
- Consumer $i \in [0, 2]$ feels

$$u_i = \begin{cases} \theta + \epsilon_i - p & \text{if the consumer purchase the product} \\ 0 & \text{otherwise} \end{cases}$$

  - $\epsilon_i \sim_{i.i.d.} F(\cdot)$ where $F(\cdot)$ is symmetric around zero
- Given Y, rational consumer purchases iff $M + \epsilon_i - p \geq 0$
- Market clearing

$$\begin{aligned} 1 &= 2q \cdot (1 - F(p - M)) \\ \Leftrightarrow p &= M \end{aligned}$$

Mixture of the rational/credulous consumers

Back to Model

- $M = E[\theta|Y]$: rational consumer's belief (on the seller's quality)
- $\widetilde{M} = \widetilde{E}[\theta|Y]$: credulous consumer's belief (on the seller's quality)

Ratinale:

- 2η rational consumers and $2(1-\eta)$ credulous consumers in mkt
- Consumer $i \in [0, 2]$ feels

$$u_i = \begin{cases} \theta + \epsilon_i - p & \text{if the consumer purchase the product} \\ 0 & \text{otherwise} \end{cases}$$

  - $\epsilon_i \sim U(-C, C)$: iid over the consumer types.
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- Credulous consumer purchases iff $\widetilde{M}_t + \epsilon_i - p \geq 0$
- Market clearing

$$1 = 2\eta \cdot (1 - F(p - M)) + 2(1-\eta) \cdot \left(1 - F\left(p - \widetilde{M}\right)\right)$$

$$\Leftrightarrow p = \eta M + (1-\eta)\widetilde{M}$$