Controlling Fake Reviews

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October 13, 2020

Introduction

- **Customer ratings** play key roles in platform markets:
  - Hollenbeck et al (2019): ratings vs advertisement in hotel industry
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- Platform markets are **growing**,
  - so does the **incentive** to make **fake reviews**.

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  - Amazon strictly prohibits incentivized reviews since 2016.
  - In 2019, FTC filed the first case challenging fake paid reviews
- We can still find fake reviews

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    - It might pay off only for high quality sellers through future sales. (similarly to Nelson; 70,74)

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- Instruments of the platform:
  1. filtering policy on suspicious reviews
  2. weights on old/new reviews

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- #(fake reviews) is **increasing in the quality** and **decreasing in the rating**.
  - Implications for empirical analysis

Literature

Design of Rating Systems

- **[certification]** Lizzeri (1999), Harbaugh and Rasmusen (2018), DeMarzo, Kremer, Skrzypacz (2019), Hopenhayn and Saeedi (2019), Hui et al (2018), Zapechelnyuk (2020)
- **[scoring][one-shot]** Ball (2019), **[dynamic]** Vellodi (2019): entry/exit, directed search;

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  This paper:
  - Fake reviews **with refunds**
  - Impact of a filtering policy on the rating's precision
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Promotion and Signaling (Q: The higher quality, the more promotion?)

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- [**One shot promotion**] Nelson (1970, 1974), Kihlstrom and Riordan (1984), Milgrom and Roberts (1986), Horstmann and Moorthy (2003), Mayzlin (2006), Dellarocas (2006):
- [**Repeated promotion**] Horstmann and MacDonald (1994):
  This paper:
  - New source of signaling promotion caused by dynamics
  - Promotion's dependence on the rating/reputation
  - Implication on quality and reputation in empirical research

Motivating example

Fake reviews with "verified purchase" on Amazon

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  - (Revenue from the fake sales) $<$ (Refund of the fake sales)

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- Time: $t \in [0, \infty)$

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  - Buyers:
    - buy the product, or not
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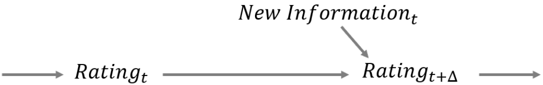

- State transition:
  - Rating Y_t follows $Y_{t+dt} = Y_t(1 - \phi dt) + (\text{new reviews})$
  - Quality θ_t follows $\theta_{t+dt} = \theta_t(1 - \kappa dt) + \mu\kappa dt + \sigma_\theta dZ_t^\theta$
    - exogenous for players (seller/buyers) and for the platform

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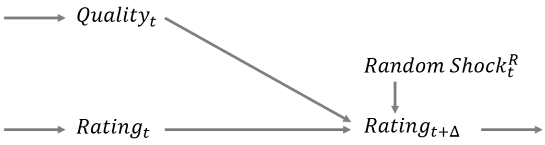

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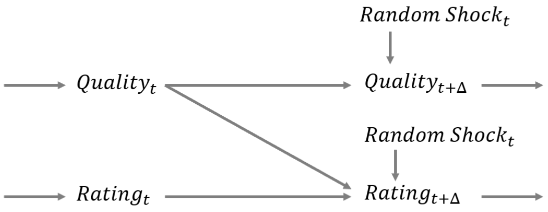

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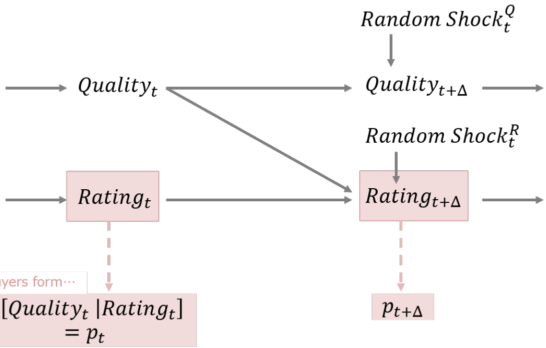

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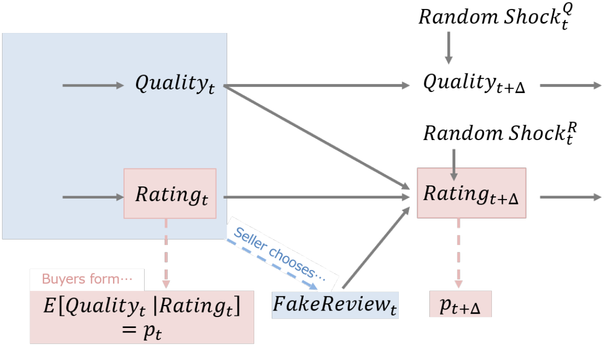

- State transition:
  - Rating Y_t follows $Y_{t+dt} = Y_t(1 - \phi dt) + aF_t dt + \theta_t dt + \sigma_\xi dZ_t^\xi$
    - $a > 0$: effectiveness of fake reviews. (low a = stringent censorship)
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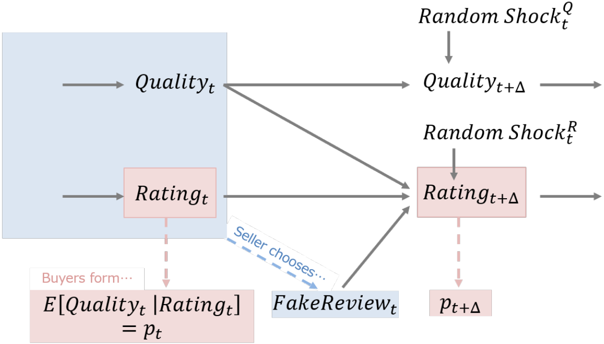

- State transition:
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Model (3/3)

- Seller's instantaneous payoff:

$$\pi_t = \underbrace{(1-\tau)\,p_t\,(1+F_t)}_{\text{revenue}} - \underbrace{p_t \cdot F_t}_{\text{refund}} - \underbrace{\frac{c}{2}F_t^2}_{\text{other costs}}$$

$$= (1-\tau)\,p_t - \tau p_t \cdot F_t - \frac{c}{2}F_t^2$$

  - τ: transaction fee imposed by the platform.

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  - $\tau = 0$: a. la. Holmstrom (1999); $F_t = \bar{F} > 0$ for all t, at eqm.
  - $\tau > 0$: F_t depends on θ_t and M_t
    - Key: $M_t \cdot F_t$ in the cost term.
      [An alternative micro-foundation is in the paper]

Definition of Equilibrium

Definition (Stationary Linear Markov equilibrium)

A linear Markov strategy $F = (F_t)_{t \geq 0}$ s.t. $F_t = \alpha\theta_t + \beta Y_t + \gamma$ is a stationary linear Markov equilibrium if

1. Buyers take the seller's strategy into account
2. Seller maximizes its own expected discounted value
3. $(\theta_t,\ Y_t)_{t \geq 0}$ induced by F is stationary Gaussian

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1. $M_t = E^F\left[\theta_t | Y_t\right]$
2. $F = \arg\max_{\left(\tilde{F}_t\right)_{t\geq 0}} E_0\left[\int_0^\infty e^{-tr}\left(\left(1-\tau\right)M_t - \tau M_t \cdot \tilde{F}_t - \frac{c}{2}\tilde{F}_t^2\right)dt\right]$
3. $\left(\theta_t,\ Y_t\right)_{t\geq 0}$ induced by F is stationary Gaussian

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3. $(\theta_t,\ Y_t)_{t \geq 0}$ induced by F is stationary Gaussian

- Note: The last condition is not exogenously given.

Stationarity of Equilibrium

- Transition of $(\theta_t,\ Y_t)$ (in discrete analogue):

$$\theta_{t+dt} = \theta_t\left(1 - \kappa dt\right) + \mu\kappa dt + \sigma_\theta dZ_t^\theta$$

$$Y_{t+dt} = Y_t\left(1 - \phi dt\right) + aF_t dt + \theta_t dt + \sigma_\xi dZ_t^\xi$$

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Characterize Equilibrium

- HJB equation:

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\begin{aligned}
rV(\theta,\ Y) = \sup_{F \in \mathbb{R}} &(1-\tau) M \cdot q - \tau M \cdot F - \frac{c}{2} F^2 \\
&- \kappa (\theta - \mu) V_\theta + \{-\phi Y_t + aF + \theta\} V_Y \\
&+ \frac{\sigma_\theta^2}{2} V_{\theta\theta} + \frac{\sigma_\xi^2}{2} V_{YY} \\
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- The equilibrium is characterized by guess-and-verify of
  - $F = \alpha\theta + \beta Y + \gamma$ (linear strategy)
  - $V = v_0 + v_1\theta + v_2 Y + v_3\theta^2 + v_4 Y^2 + v_5 Y\theta$ (quadratic value function)
  - $\phi - a\beta > 0$ (stationarity)

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There exists a stationary linear Markov equilibrium.

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- $\beta < 0$:
  - Driving force: $Y_t \uparrow \Rightarrow p_t \uparrow \Rightarrow$ marginal cost of fake reviews $\uparrow$
  - Countervailing effect: $Y_t \uparrow \Rightarrow Y_{t+\Delta} \uparrow \Rightarrow \frac{\partial V}{\partial Y_{t+\Delta}} \uparrow$ by $\frac{\partial^2 V}{\partial Y^2} > 0$ ▸ Details
- $\alpha > 0$:
  - $\theta_t \uparrow \Rightarrow Y_{t+\Delta} \uparrow \Rightarrow \frac{\partial V}{\partial Y_{t+\Delta}} \uparrow$ by $\frac{\partial^2 V}{\partial Y^2} > 0$

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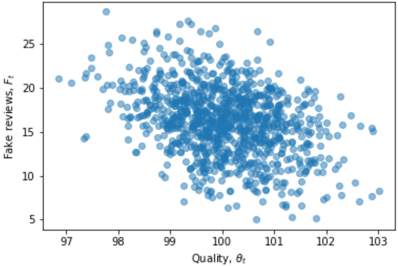

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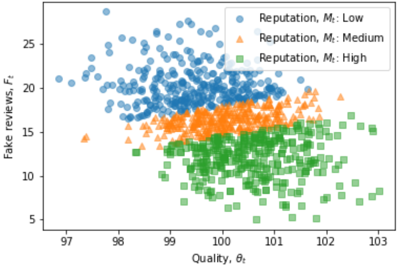
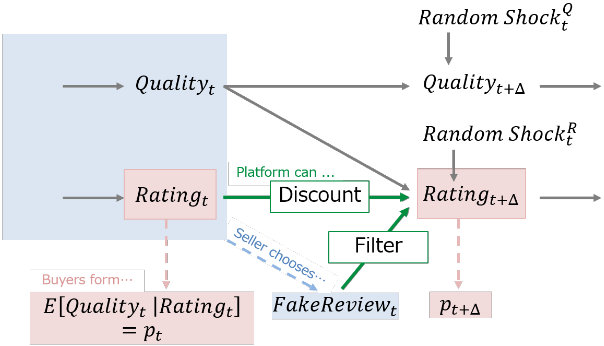


Comparative Statics

- Assume that the platform can change
  - a (filtering policy) and
  - ϕ (weights on old/new review)



$$Y_{t+dt} = Y_t(1 - \phi dt) + aF_t dt + \theta_t dt + \sigma_\xi dZ_t^\xi$$

- [Comparative statics about τ and σ_ξ is found in the paper]

Proposition

(i) $E[F_t]$ is increasing in a.
(ii) $a \cdot \alpha$, $a \cdot \beta$, and $a \cdot \gamma$ go to zero as $a \to 0$.

- Reminder: $aF_t = a\alpha\theta_t + a\beta Y_t + a\gamma =$ the effect of fake reviews
- Stringent censorship can **reduce** the **expected amount** and the **effects** of fake reviews.
  - Note: $(\alpha, \beta, \gamma) \nrightarrow 0$ even when $E[F_t] \to 0$ or $(a\alpha, a\beta, a\gamma) \to 0$

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- For the platform, if the rating system is not informative, the sellers and buyers might move out to other platforms.
  - Maximization of ρ^2 is equivalent to minimizing $E\left[(p_t - \theta_t)^2\right]$

$$E\left[(p_t - \theta_t)^2\right] = \underbrace{Var(\theta)}_{\text{exogenous}}\left(1 - \rho^2\right)$$

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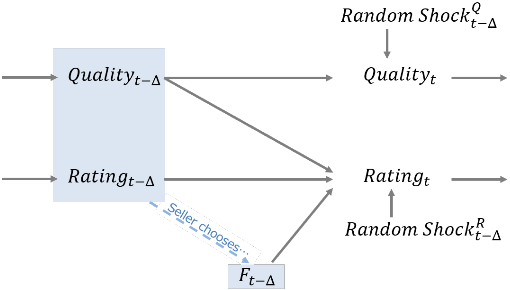

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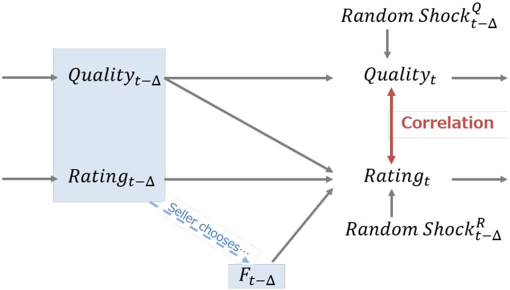

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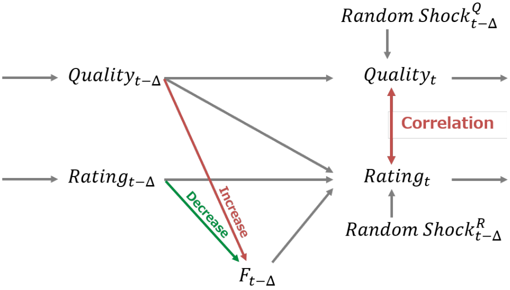

- Impacts of the fake reviews:
  1. $a \cdot \alpha > 0$ enhances the positive relationship between the true quality θ_t and the rating Y_t.
  2. $a \cdot \beta < 0$ cancels out the variation in the old rating, $Y_{t-\Delta}$.
     - More discount on old reviews. (ie, Faster transition of the rating)

Proposition (Informativeness of fake reviews)

*The rating **with fake reviews** is **more informative** than one without when*

1. *a is sufficiently large, or*
2. *(i) a is sufficiently small and*
   (ii) ϕ is too small

- The first effect (from $a \cdot \alpha > 0$) dominates for large a, and
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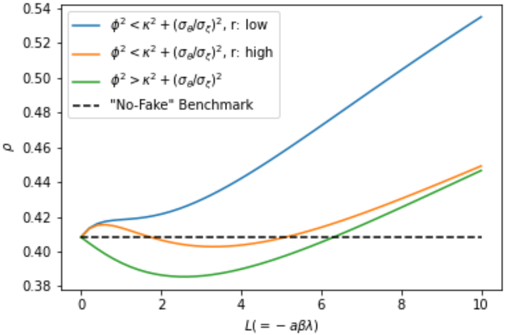
Sketch of the proof:

- $\rho^2 = \frac{(\phi - a\beta)}{(\kappa + \phi - a\beta)} \frac{(a\alpha+1)^2}{((a\alpha+1)^2 + \kappa(\sigma_\xi/\sigma_\theta)^2(\kappa+\phi-a\beta))}$ (given any α, β, δ)



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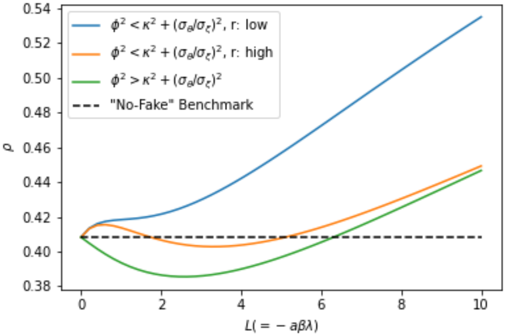

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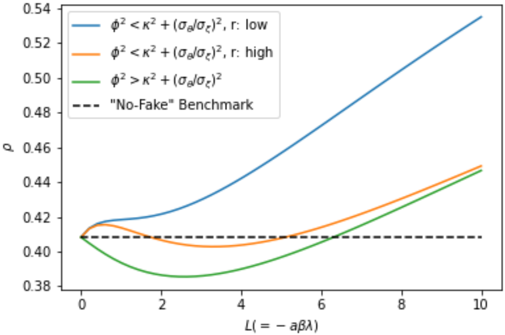

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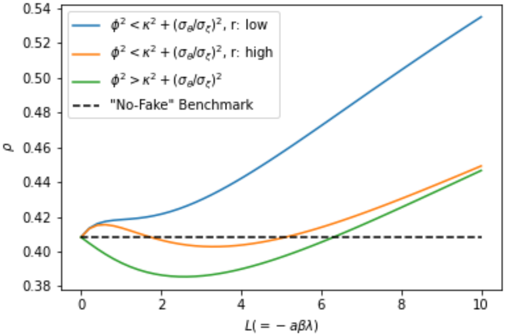

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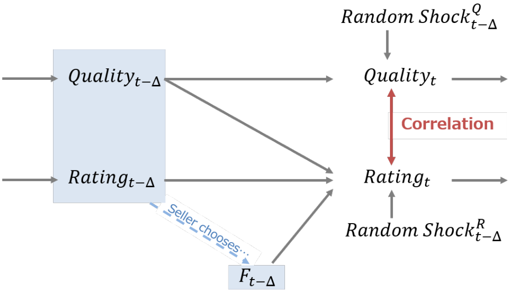

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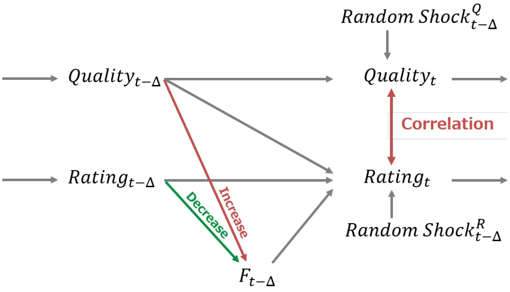

Proposition

(i) At eqm (with fake reviews), ρ^2 is decreasing in ϕ at $\phi = \phi^0$.

- (i) w/ fake reviews: effective transition speed is $\phi - a\beta$
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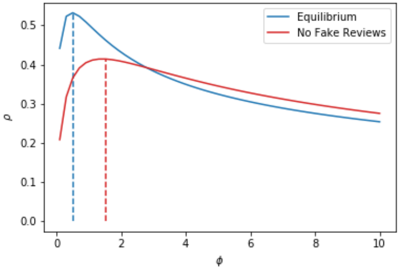
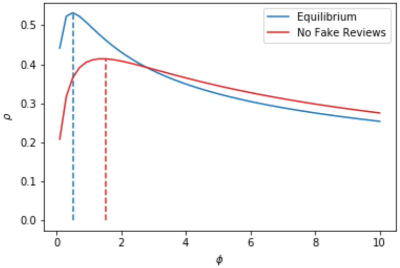


Proposition

(i) At eqm (with fake reviews), ρ^2 is decreasing in ϕ at $\phi = \phi^0$.
(ii) Furthermore, for sufficiently small r, the maximum of ρ^2 with fake reviews is higher than without.

- (i) w/ fake reviews: effective transition speed is $\phi - a\beta$
  - $\rightarrow$ the platform should adjust ϕ downward.
- (ii) small $r \Rightarrow$ high weight on the future$\Rightarrow$ high $\alpha \Rightarrow$ rating is informative with fake review, given ϕ^0
  - The platform can further adjust ϕ from ϕ^0 .



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    - η captures the rationality of each consumer

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    - belief based an wrong joint distribution of (θ_t, Y_t)
- Seller's payoff:
  - $\pi_t = (1 - \tau) p_t - \tau p_t \cdot F_t - \frac{c}{2} F_t^2$
  - $p_t = \eta M_t + (1 - \eta)\widetilde{M}_t$ where $\eta \in [0, 1]$
  - Interpretation:
    - η captures the rationality of each consumer
    - η is the ratio of rational consumers in the market. ▸ Details

Theorem

Existence and uniqueness given the same condition as the baseline model

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Proposition

Existence of the naive consumers decreases $E[F_t]$.

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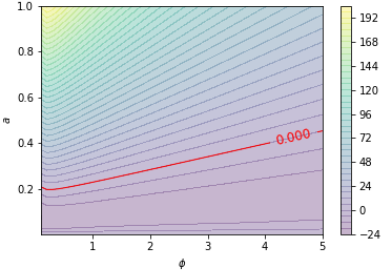
- Intuition:
  - Naive consumers set higher price, but
  - **Rational** consumers are **more sensitive to the rating** than naive consumers.
    - Rational consumers takes $a\alpha > 0$ into account.
  - Less marginal benefit with naive consumers.
  - Less fake reviews with naive consumers.

Criteria for the naive consumers:

$$Bias = E\left[\widetilde{E}\left[\theta_t | Y_t\right] - \theta_t\right]$$



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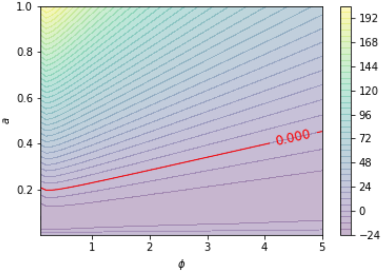

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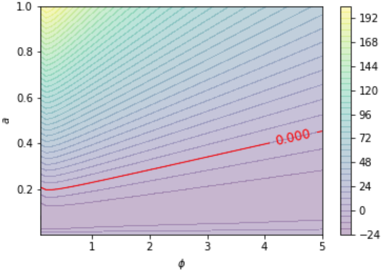

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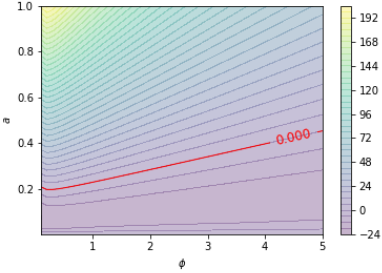
Proposition

A strict filtering policy reduces $Bias$ as long as $Bias \geq 0$.



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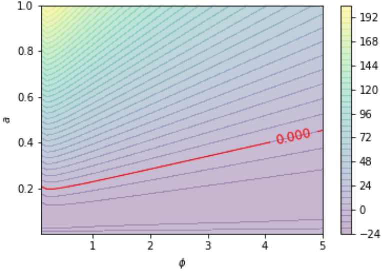

Summary

Positive Analysis:

Normative Analysis:

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- The number of fake reviews is increasing in quality, decreasing in reputation.
- The stringent censorship
  - reduces fake reviews in expectation, but
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- For rational consumers:
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- The number of fake reviews is increasing in quality, decreasing in reputation.
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Normative Analysis:

- For rational consumers:
  - a rating with fake reviews can be more informative than without fake reviews
  - Transition speed of the rating should be slower than the optimal level without fake reviews.
- For naive consumers:
  - As long as $E[F_t] \geq 0$, the more stringent censorship, the less bias for the naive consumers.

Intuition of the Equilibrium Strategy

Back to Theorem

- Reminder: $V = v_0 + v_1\theta + v_2 Y + v_3\theta^2 + v_4 Y^2 + v_5\theta Y$
- FOC: $F_t = -\frac{\tau}{c}M_t + \frac{a}{c}\{v_2 + 2Y_t v_4 + \theta v_5\}$
- $\beta < 0$
  - $\beta = -\frac{\tau}{c}\lambda + 2\frac{a}{c}v_4 = -\frac{\tau}{c}\lambda + \frac{a}{c}\frac{-\beta\lambda\tau}{(-a\beta+r+2\phi)}$
  - $\beta < 0$ since today's cost saving incentive dominates.
- $v_4 = \frac{-\beta\lambda\tau}{2(-a\beta\lambda+r+2\phi)} > 0$
  - Higher reputation, less promotion, less costly: $\tau M_t F_t = \tau\alpha\theta_t Y_t + \tau\beta Y_t^2 + \text{constant}$
- $\alpha > 0$
  - $\alpha = \frac{a}{c}v_5$
  - $v_5 = \frac{1}{\kappa+r+\phi}\{ \underbrace{2(a\alpha + bq)}_{\text{High High future } Y_t} v_4 - \alpha\lambda\tau\}$
    - Driving Force: Higher θ today, higher Y in the future, value is quadratically increasing in Y.
    - Counteracting effect: Higher quality, higher $F_{t+\Delta}$(if $\alpha > 0$). Less complementarity from $-\tau M_t F_t$.

Microfoundation of the price: $p_t = M_t$

Back to Model

- (Reminder: $M_t \equiv E\left[\theta_t | Y_t\right]$)
- Suppose there is a mass (2) of buyers.
- Consumer $i \in [0, 2]$ feels

$$u_i = \begin{cases} \theta + \epsilon_i - p & \text{if the consumer purchase the product} \\ 0 & \text{otherwise} \end{cases}$$

  - $\epsilon_i \sim_{i.i.d.} F(\cdot)$ where $F(\cdot)$ is symmetric around zero
- Given Y, rational consumer purchases iff $M + \epsilon_i - p \geq 0$
- Market clearing

$$\begin{aligned} 1 &= 2q \cdot (1 - F(p - M)) \\ \Leftrightarrow p &= M \end{aligned}$$

Mixture of the rational/naive consumers

Back to Model

- $M = E[\theta|Y]$: rational consumer's belief (on the seller's quality)
- $\widetilde{M} = \widetilde{E}[\theta|Y]$: naive consumer's belief (on the seller's quality)

Rationale:

- 2η rational consumers and $2(1-\eta)$ naive consumers in mkt
- Consumer $i \in [0, 2]$ feels

$$u_i = \begin{cases} \theta + \epsilon_i - p & \text{if the consumer purchase the product} \\ 0 & \text{otherwise} \end{cases}$$

  - $\epsilon_i \sim U(-C, C)$: iid over the consumer types.
- Rational consumer purchases iff $M_t + \epsilon_i - p \geq 0$
- naive consumer purchases iff $\widetilde{M}_t + \epsilon_i - p \geq 0$
- Market clearing

$$1 = 2\eta \cdot (1 - F(p - M)) + 2(1-\eta) \cdot \left(1 - F\left(p - \widetilde{M}\right)\right)$$

$$\Leftrightarrow p = \eta M + (1-\eta)\widetilde{M}$$