3D Rendering: Intro & Ray Casting

COS 426, Fall 2022

PRINCETON UNIVERSITY

Syllabus

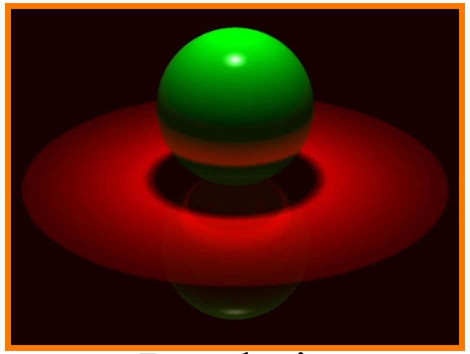
I. Image processing

II. Modeling

III. Rendering

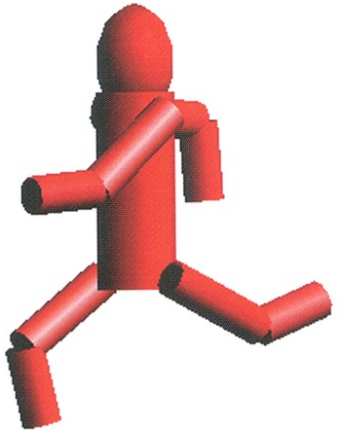
IV. Animation

Image Processing
(Rusty Coleman, CS426, Fall99)



Rendering
(Michael Bostock, CS426, Fall99)

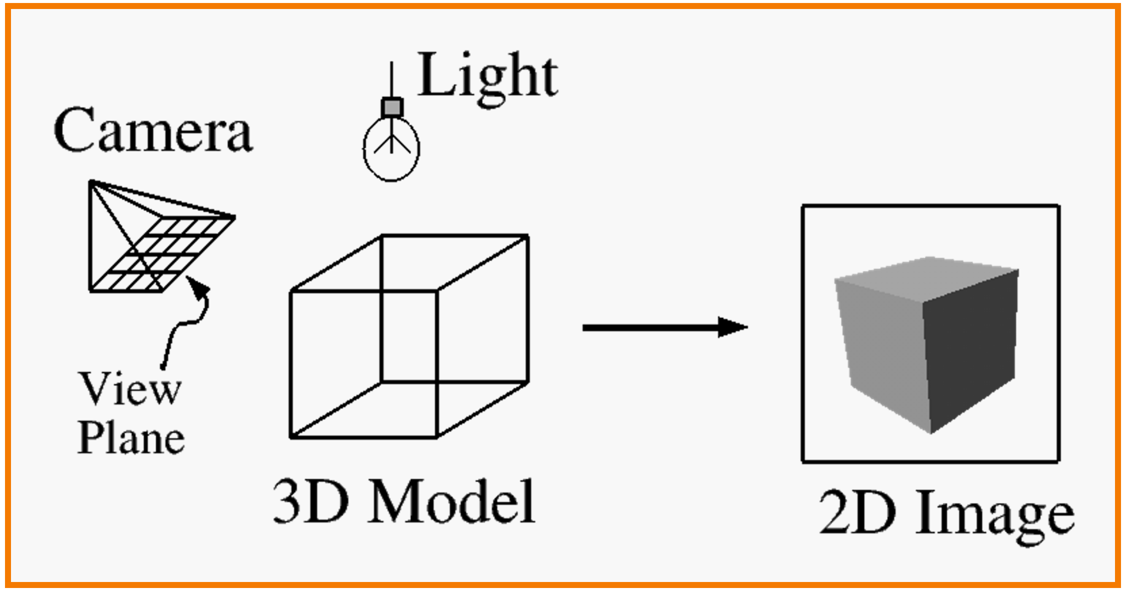
Modeling
(Dennis Zorin, CalTech)



Animation
(Angel, Plate 1)

What is 3D Rendering?

- Topics in computer graphics
  - Imaging = *representing 2D images*
  - Modeling = *representing 3D objects*
  - Rendering = *constructing 2D images from 3D models*
  - Animation = *simulating changes over time*

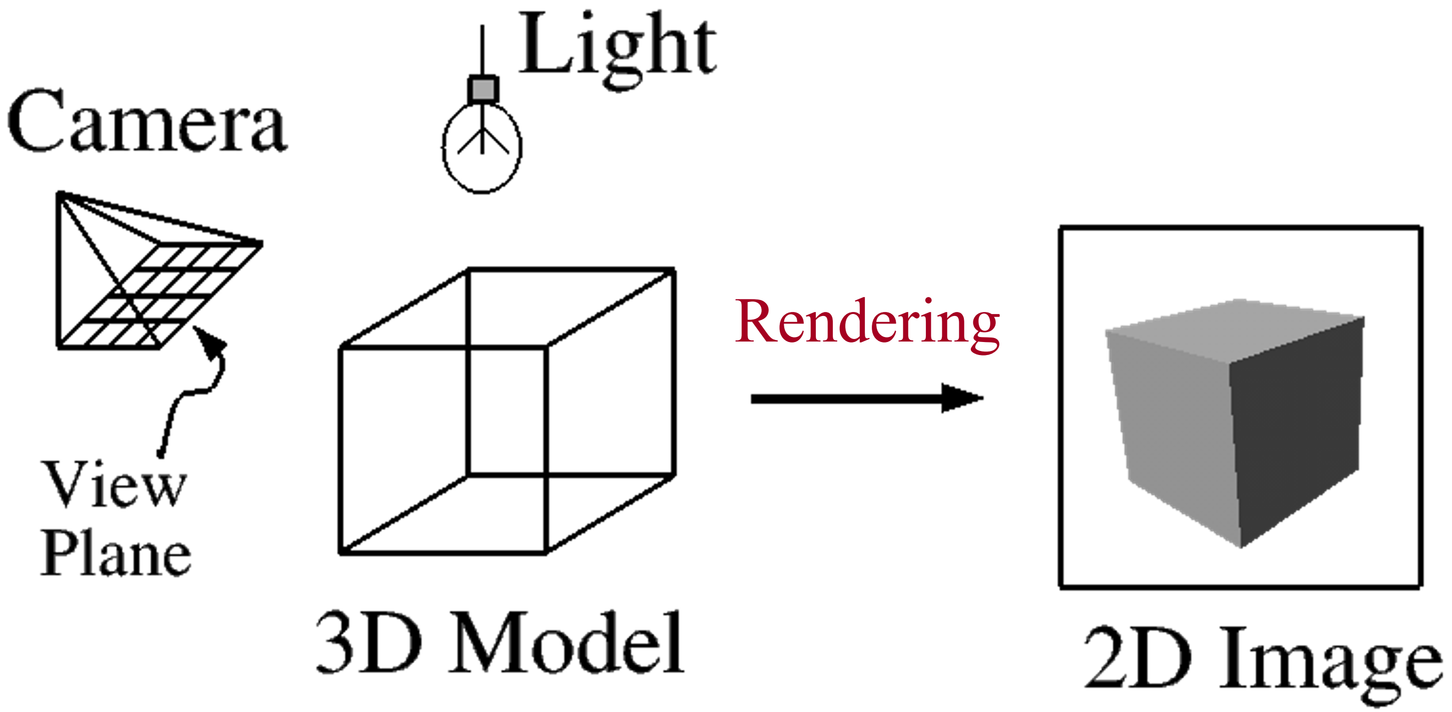


Rendering: Inspiration

Source: (Project Sol Part 2) https://www.youtube.com/watch?v=pNmhJx8yPLk

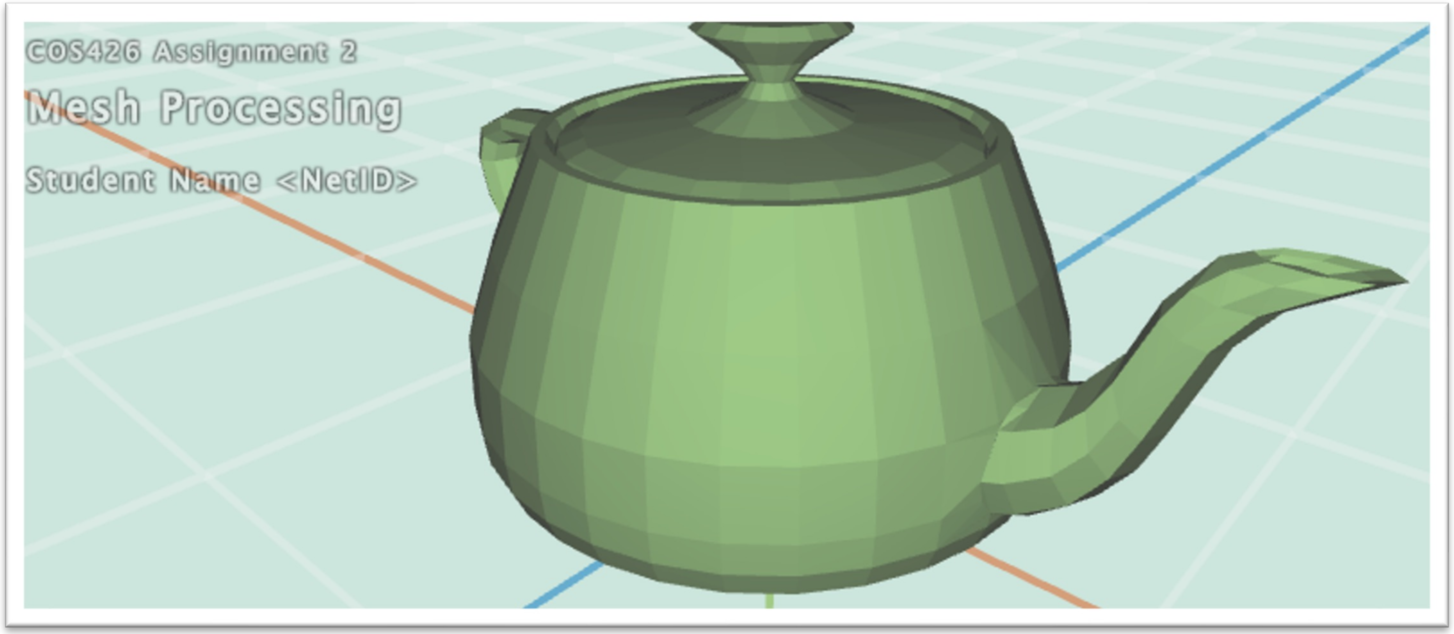
What is 3D Rendering?

- Construct image from 3D model



Interactive 3D Rendering

- Images generated in fraction of a second (e.g., 1/30) as user controls rendering parameters (e.g., camera)
  - Achieve highest quality possible in given time
  - Useful for visualization, games, etc.



Offline 3D Rendering

- One image generated with as much quality as possible for a particular set of rendering parameters
  - Take as much time as is needed (minutes, hours…)
  - Photorealism: movies, cut scenes, etc.

Avatar

3D Rendering Issues

- What issues must be addressed by a 3D rendering system?

Pixar

3D Rendering Issues

- What issues must be addressed by a 3D rendering system?
  - Camera
  - Visible surface determinaton
  - Lights
  - Reflectance
  - Shadows
  - Indirect illumination
  - Sampling
  - etc.

3D Rendering Issues

- What issues must be addressed by a 3D rendering system?
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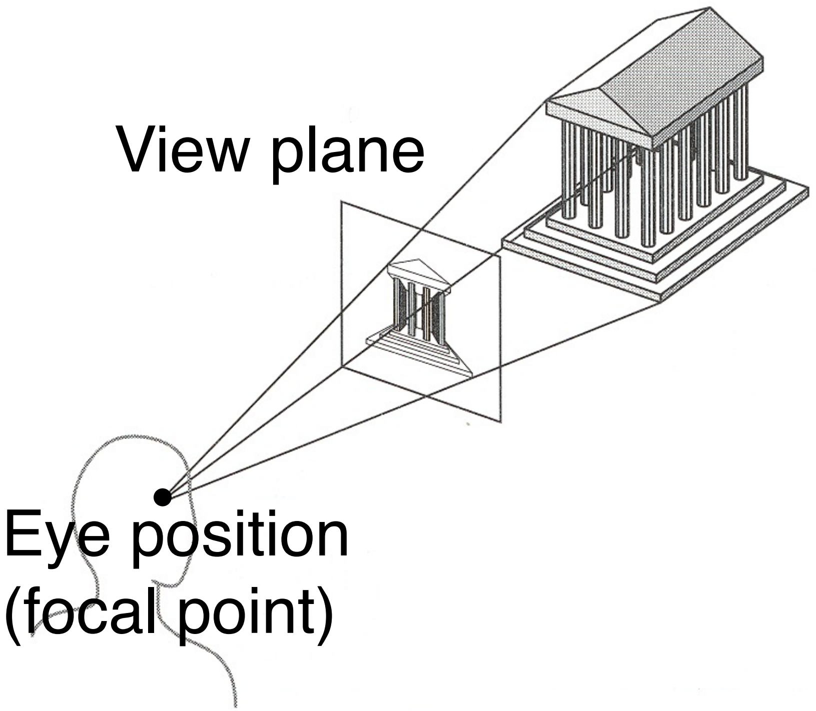
  - etc.

Camera Models

- The most common model is pin-hole camera
  - Light rays arrive along paths toward focal point
  - No lens effects (e.g., everything in focus)



Other models consider ...

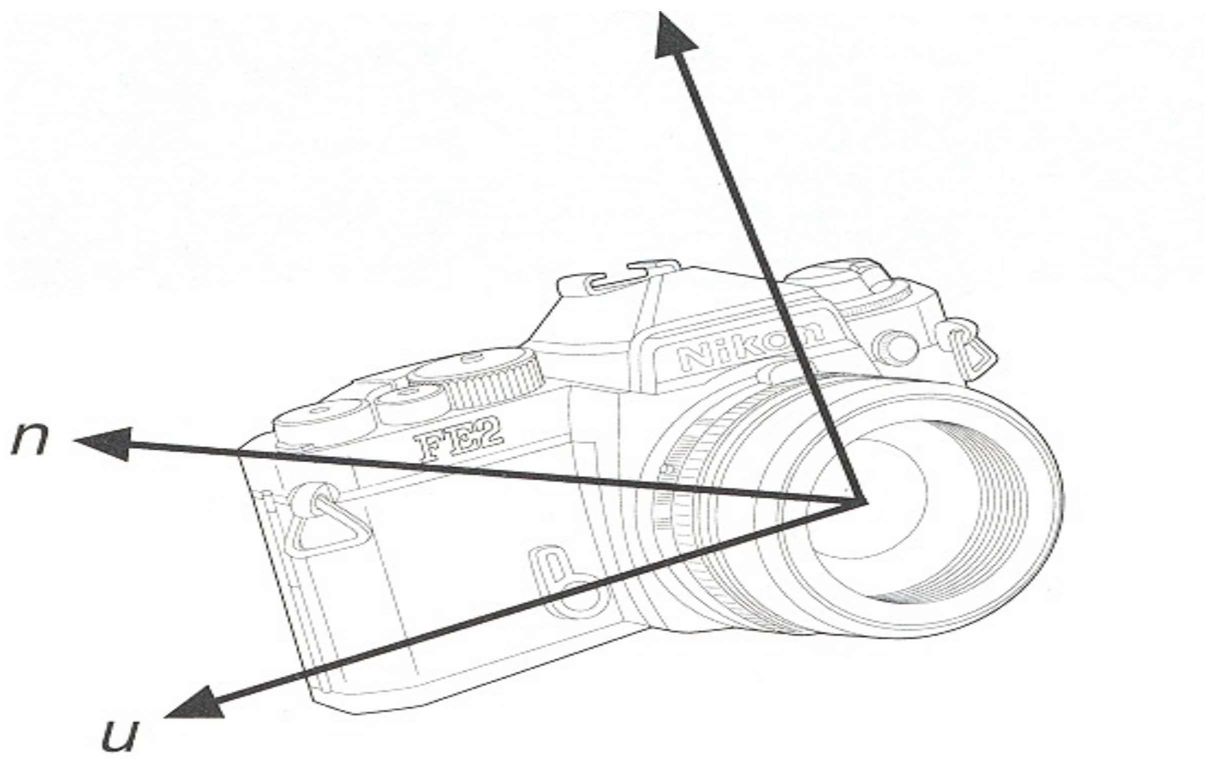
Depth of field

Motion blur

Lens distortion

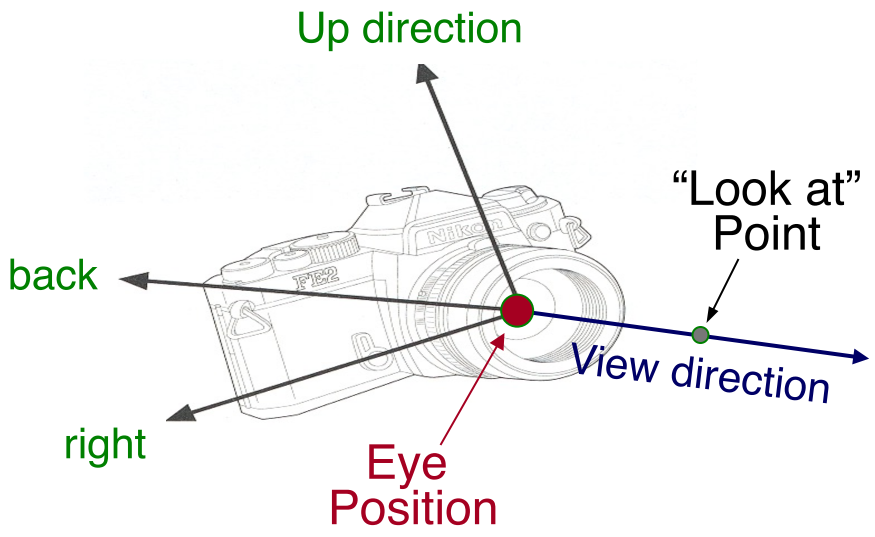
Camera Parameters

- What are the parameters of a camera?

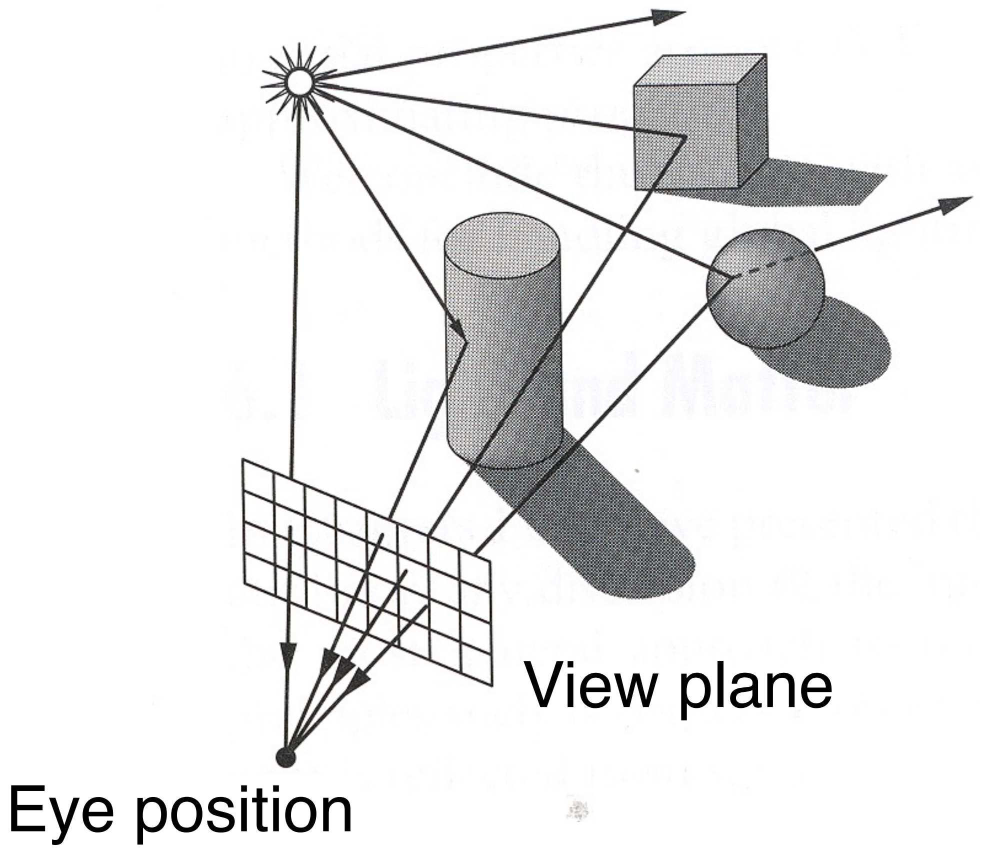


Pinhole Camera Parameters

- Position
  - Eye position (p_x, p_y, p_z)
- Orientation
  - View direction (d_x, d_y, d_z) or "look at" point
  - Up direction (u_x, u_y, u_z)
- Coverage
  - Field of view (fov_x, fov_y)
- Resolution
  - x and y



View Plane



3D Rendering Issues

- What issues must be addressed by a 3D rendering system?
  - Camera
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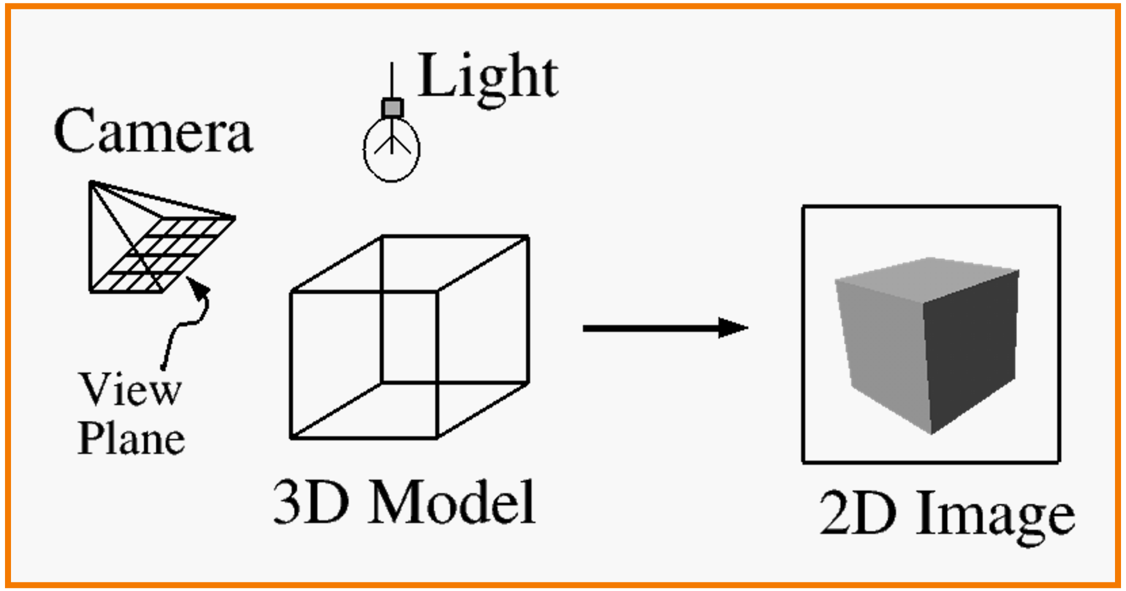
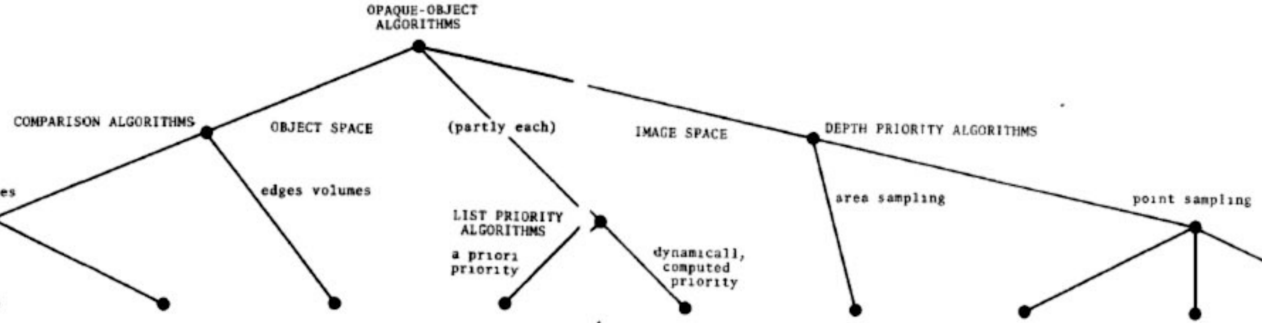
  - Shadows
  - Indirect illumination
  - Sampling
  - etc.

Visible Surface Determination

- The color of each pixel on the view plane depends on the radiance ("amount of light") emanating from visible surfaces

How find visible surfaces?





	APPEL 1967	GALIMBERTI, et al 1969	LOUTREL 1967	ROBERTS 1963	SCHUMACKER, et al 1969
RESTRICTIONS	TP, NP	TP, NP	TP, NP	TP, CC, CF, NP	CF, NP, LS (TP)
COHERENCE	Promote visibility of a vertex to all edges at vertex	Promote visibility of a vertex to all edges at vertex	Promote visibility of a vertex to all edges at vertex		Frame coherence in depth No X coherence used
SORTING (1) What, what property (2) Method	Back Edge Cull 1) Edges separating back-facing planes 2) Dot product with normals & topology 3) Cull 4) List of edges, E_s 5) 1, E_t	Back Edge Cull 1) Edges separating back-facing planes 2) Dot product with normals & topology 3) Cull 4) List of edges, E_s 5) 1, E_t	Back Edge Cull 1) Edges separating back-facing planes 2) Dot product with normals & topology 3) Cull 4) List of edges, E_s 5) 1, E_t	Back Edge Cull 1) Edges separating back-facing planes 2) Dot product with normals & topology 3) Cull 4) List of edges, E_s 5) 1, E_t	Intra-Cluster Priority 1) Faces - visibility 2) Dot product with normals 3) Exhaustive search 4) Ordered table 5) 0, (off-line)
(3) Type (4) Result structure (5) Number per frame, number of objects	Contour Edge Cull 1) Edges separating front & back faces 2) Dot product with normals & topology 3) Cull 4) List, E_c 5) 1, E_t	(Omitted)	(Omitted)	Clipping Cull 1) Intersect edge with visible volume 2) 3) Cull 4) E_s 5) 1, E_s	Inter-Cluster Priority 1) Clusters 2) Dot product with separating planes 3) Prefix scan binary tree 4) ordered table 5) 1, C_t
(merge) Number of new entries per frame, length of list	Initial Visibility 1) Ray to vertex against all faces 2) Depth, Surroundedness 3) Exhaustive search 4) Quantitative visibility of vertex 5) # Objects, F_r	Initial Visibility 1) Ray to vertex against all faces 2) Depth, surroundedness 3) Exhaustive search 4) Quantitative visibility of vertex 5) #objects, F_r	Initial Visibility 1) Ray to vertex against all faces 2) Betweenness, surroundedness 3) Exhaustive search 4) Quantitative visibility of vertex 5) #objects, F_r	Edge/Volume Test 1) Edges, visibility relative to volumes 2) Linear Programming 3) Mini-max sort 4) Answer 5) E_s + split edges, #objects	Back-Face Cull 1) Faces 2) Dot product with face normal 3) Cull 4) Smaller ordered table 5) 1, F_t
(search) Number of searches, length of list	Edge Intersection 1) Intersect one E_s with all E_c 2) Penetration with sweep triangle 3) Cull (unordered) 4) Intersection list 5) E_s, E_c	Edge Intersection 1) Intersect one E_s with all E_c 2) Intersect in picture plane, depth 3) Cull (unordered) 4) Intersection list 5) E_s, E_s - 1	Edge Intersection 1) Intersect one E_s with all E_c 2) Intersect in picture plane, depth 3) Cull (unordered) 4) Intersection list 5) E_s, E_s - 1		Y Cull 1) Faces by Y extent 2) Mini-max on X intercepts 3) Cull (unordered) 4) X intercepts of relevant segments 5) n, E_s
	Sort Along Edge 1) Intersections on edge, ordering 2) Comparison 3) Bubble 4) Answer 5) E_s, X_v/E_s (Omit if well hidden)	Sort Along Edge 1) Intersections on edge, ordering 2) 3) 4) Answer 5) E_s, X_v/E_s (must be done)	Sort Along Edge 1) Intersections on edge, ordering 2) 3) 4) Answer 5) E_s, X_v/E_s (Omit if well hidden)		X Sort 1) Segments 2) Counters 3) Hardware 4) Segments at this X 5) nm, S_ℓ
					Priority Search 1) Segments, priority 2) Logic network 3) Logic network 4) visible segment 5) nm, S_ℓ

	NEWELL, et al 1972	WARNOCK 1968	WATKINS 1970	ROMNEY, et al 1967	BOUKNIGHT 1969
RESTRICTIONS	None	(TR) None	None	TR, CF, NP	
COHERENCE	None used	Area coherence	Scanline X coherence	Scanline Depth Coherence	Scanline X Coherence
SORTING (1) What, what property (2) Method	Z Sort 1) Faces, max Z 2) Comparison of max points 3) n logm 4) Ordered table 5) 1, F_r	Z Sort (Opt.) 1) Faces, max Z 2) Comparison of max points 3) n log m 4) Ordered table 5) 1, F_r	Y Sort 1) Edges, min Y 2) Comparison 3) Bucket 4) Table of Lists 5) 1, E_r	Y Sort 1) Polygons, Y endpoints 2) Comparison 3) 2 bucket 4) Table of lists 5) 1, F_r	Y Sort 1) Edges, Min Y 2) Comparison 3) Bucket 4) Table of lists 5) 1, E_r
(3) Type (4) Result structure (5) Number per frame, number of objects	Newell Special 1) Faces, pairwise visibility 2) Depth, bounding boxes, separation 3) Bubble, splitting 4) Ordered table 5) 1, F_r+split faces	Warnock Special 1) Faces with window 2) Depth, mini-max in X and Y, sum of angles 3) Radix 4 subdivision with overlap 4) Stacks of unordered tables 5) L_v, F_r/factor 1	X Merge 1) Edges, X value 2) Comparison 3) Merge (ordred) 4) 2-way linked list 5) E_r, S_ℓ	X Sort 1) Edges, X value 2) Comparison 3) 2 bucket 4) Table of lists 5) n, S_ℓ	X Merge 1) Edges, X value 2) Comparison 3) Merge (ordered) 4) Linked list 5) E_r, $2S_\ell$ (edges)
(merge) Number of new entries per frame, length of list	Y Sort 1) Face segment by Y range 2) Y intercept 3) Bucket 4) None 5) F_r + split faces H_f^r	Depth Search 1) Surrounder faces 2) 4-corner compare 3) Exhaustive 4) Answer/failure 5) L_v, F_r/factor 2	X Sort 1) Segments, X left 2) Comparison 3) Bubble 4) 2-way linked list 5) n, S_ℓ	X Priority Search 1) Edges, X value 2) Comparison 3) Priority search 4) Active segment list 5) n, m	X Sort 1) Edges, X value 2) Comparison 3) Bubble 4) 1-way linked list 5) N, $2S_\ell$ (edges)
(search) Number of searches, length of list	X Merge 1) Segments, X intercept 2) Comparison 3) Ordered merge 4) Ordered list 5) S_r, $S_v/2$	TV Sort (Opt.) Sort windows into scan-line order if needed	Span Cull 1) Segments, overlap with sample span 2) Double comparison 3) Cull ordered list 4) Active list 5) $n*S_v*f$ (>1), S_ℓ	Z Search 1) Segments, depth 2) Linear equations and comparison 3) Search (unordered) 4) Visible segment 5) $n*2S_\ell$, D_c (Omitted if X priorities same as last time)	Z Search 1) Segments, depth 2) Linear equations and comparison 3) Search of unordered active list 4) Visible segment 5) $n*2S_\ell$, D_c
			Z Search 1) Segments, Z 2) Depth by logarithmic search 3) Search (unordered) 4) Visible segment 5) $n*S_v*f(>1)$, D_c		

Figure 29. Characterization of ten opaque-object algorithms b. Comparison of the algorithms.

In Practice… Brute Force

- **Ray tracing**
  - **for each** pixel: determine closest object hit by ray
  - compute color

- **Rasterization**
  - **for each** object: enumerate pixels it hits
  - keep track of color, depth of current-best surface at each pixel

3D Rendering Issues

- What issues must be addressed by a 3D rendering system?
  - Camera
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  - Lights
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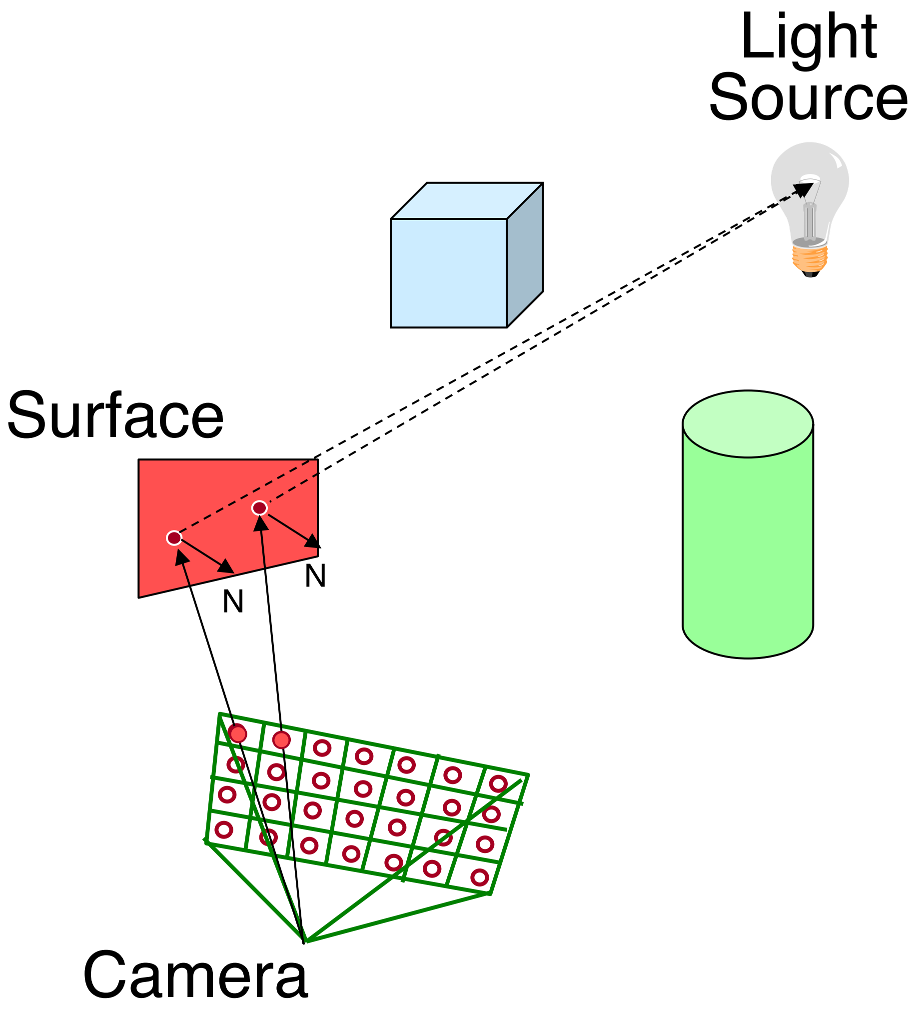
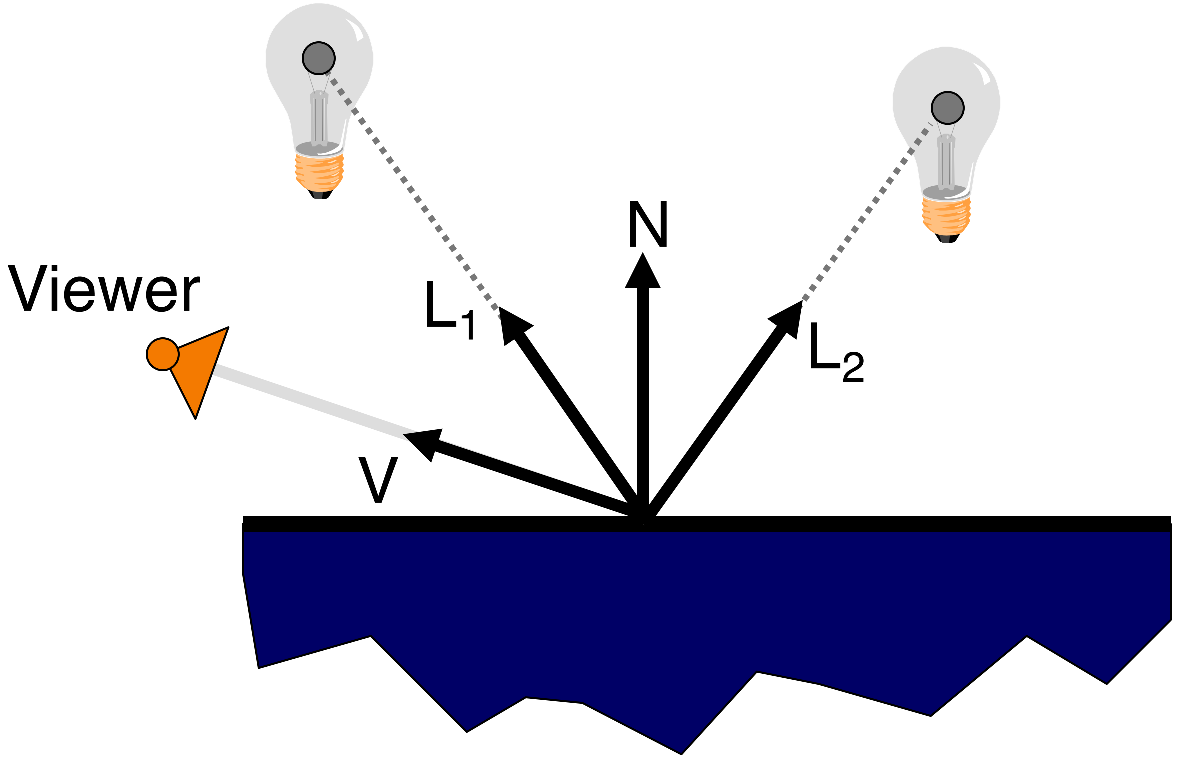
  - Shadows
  - Indirect illumination
  - Sampling
  - etc.

Lighting Simulation

- Lighting parameters
  - Light source emission
  - Surface reflectance
  - Atmospheric attenuation
  - Camera response



Lighting Simulation



3D Rendering Issues

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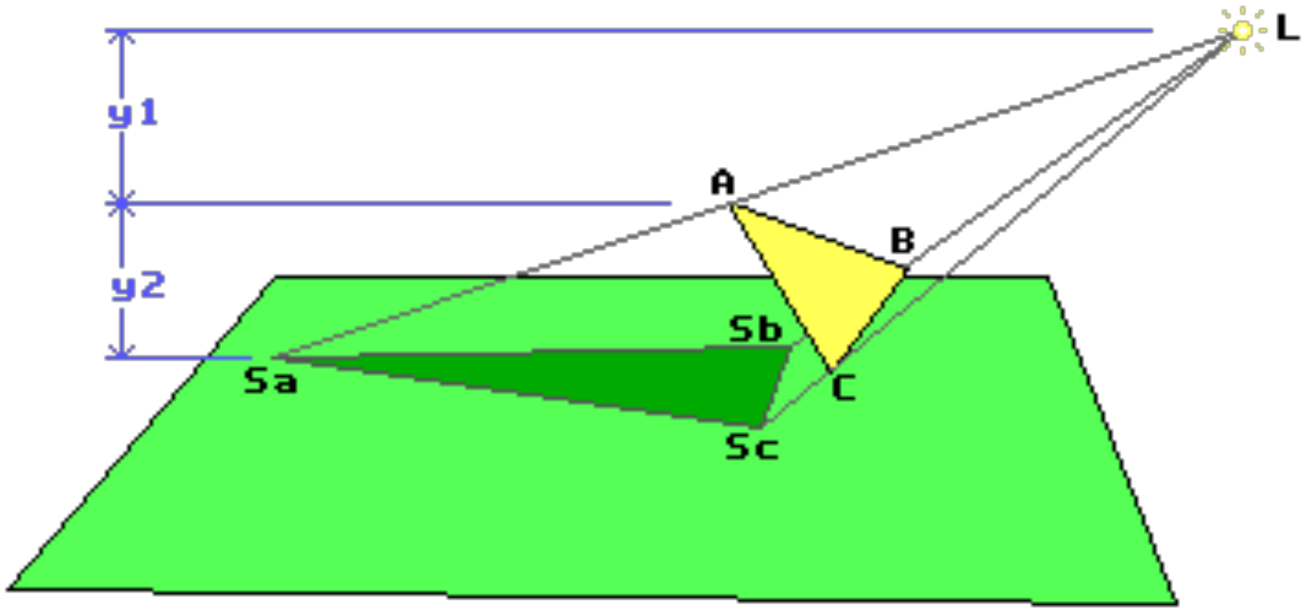
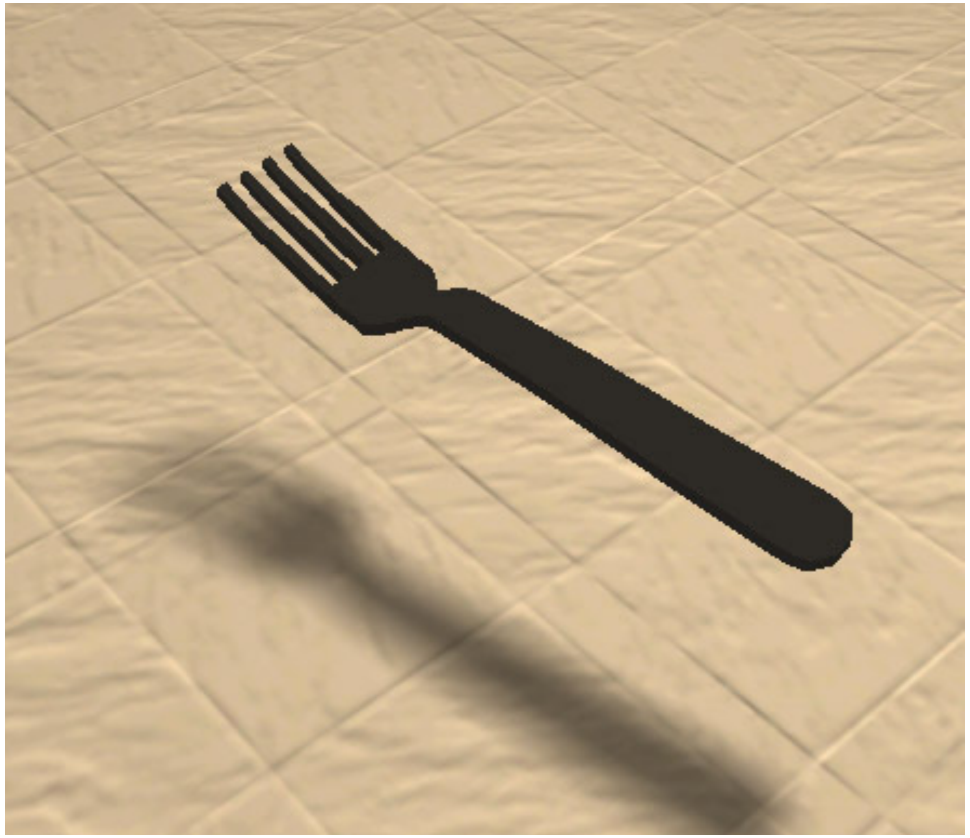
Shadows

- Occlusions from light sources



Shadows

- Occlusions from light sources
  - Soft shadows with area light source



Moller

Shadows

Herf

3D Rendering Issues

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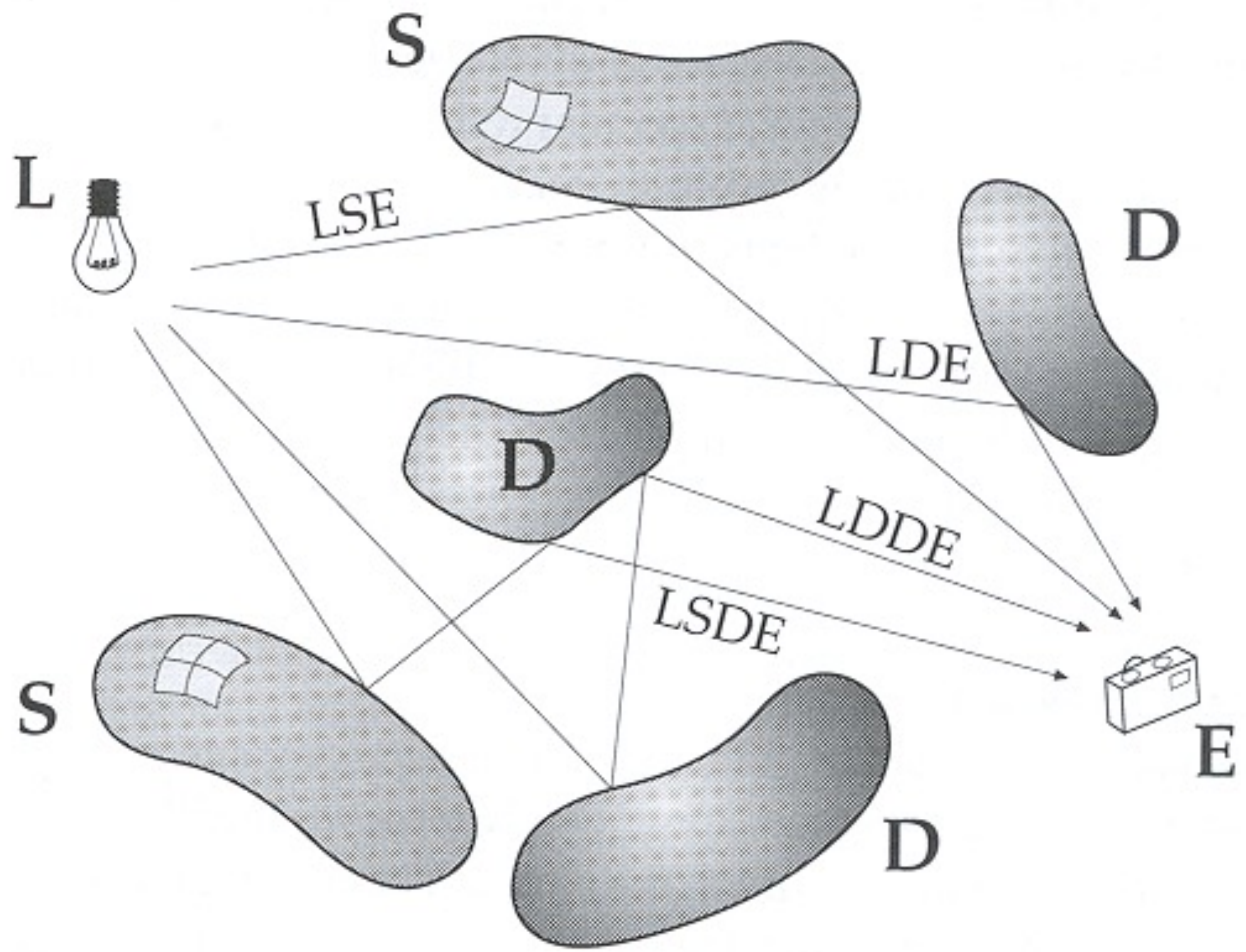
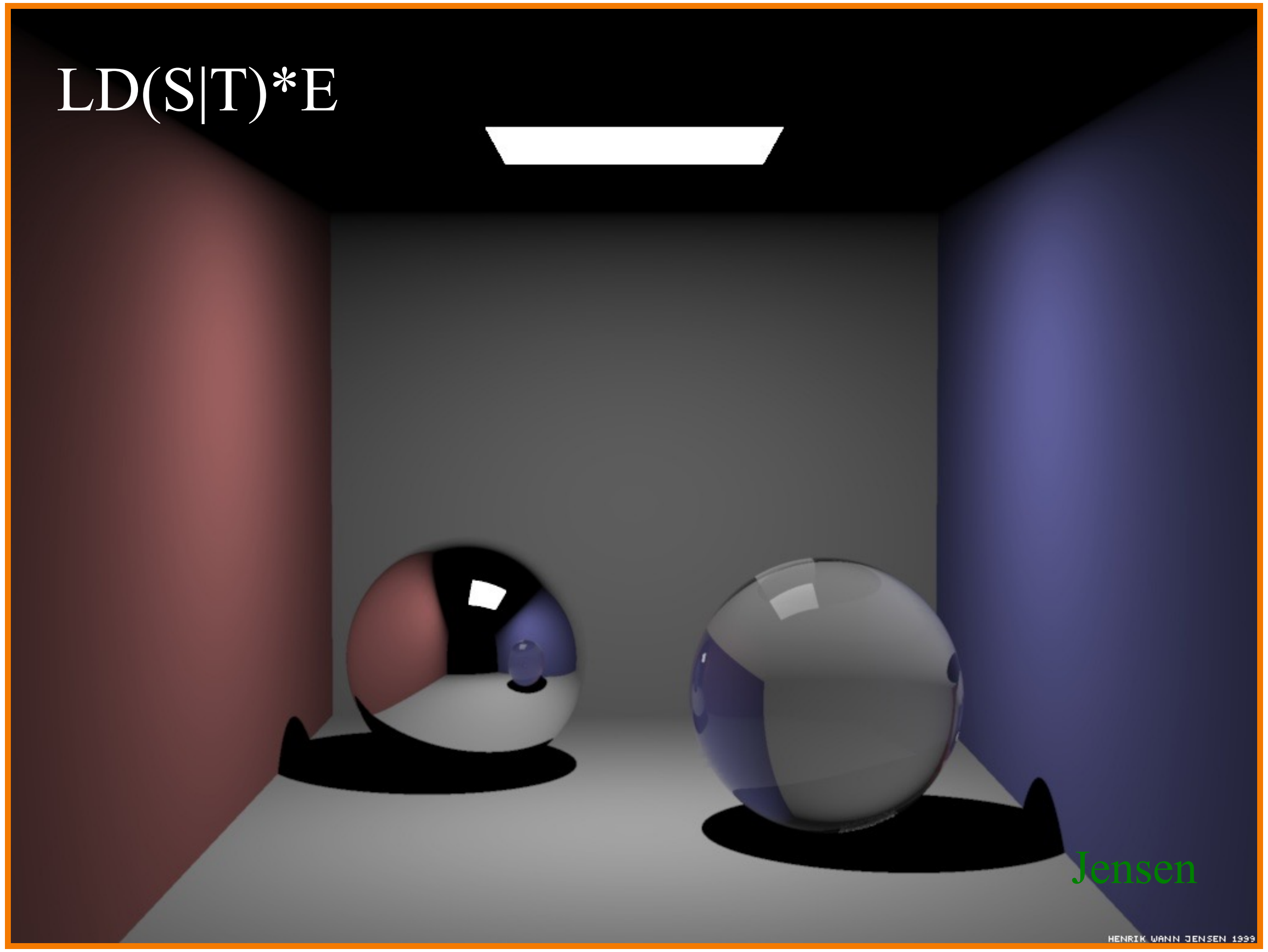
Path Types



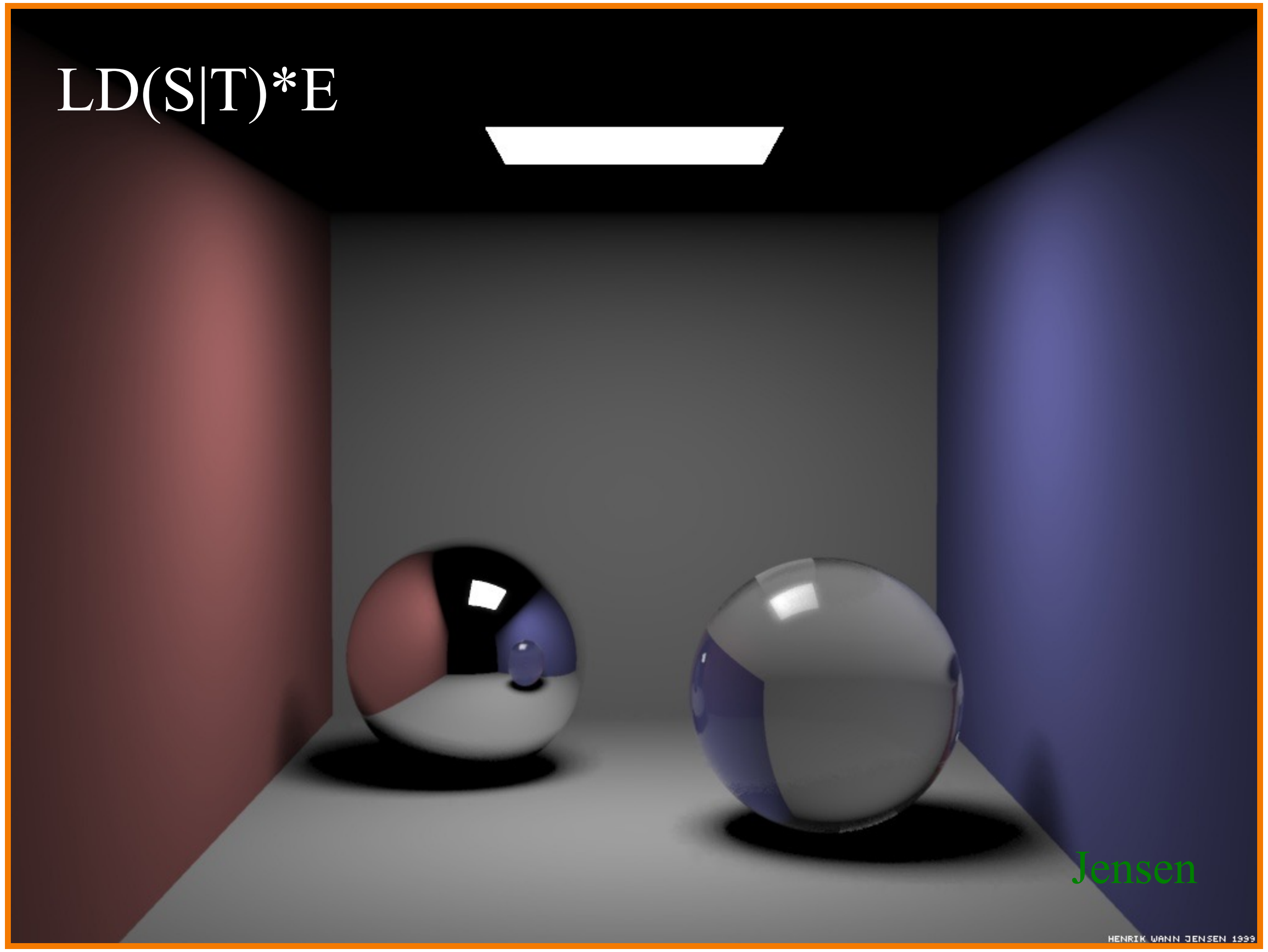
Path Types



direct diffuse + indirect specular and transmission

Henrik Wann Jensen

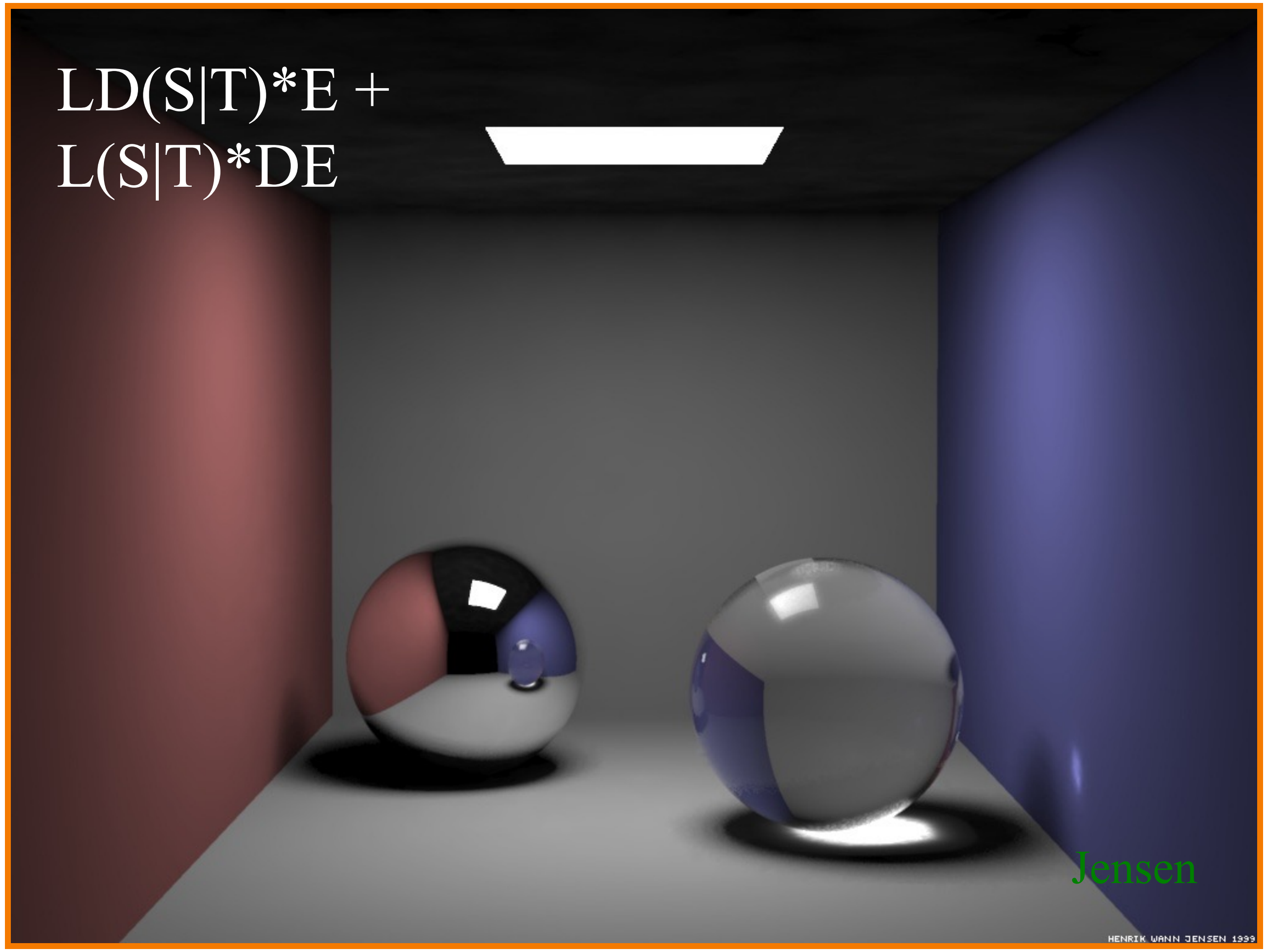
Path Types



+ soft shadows

Henrik Wann Jensen

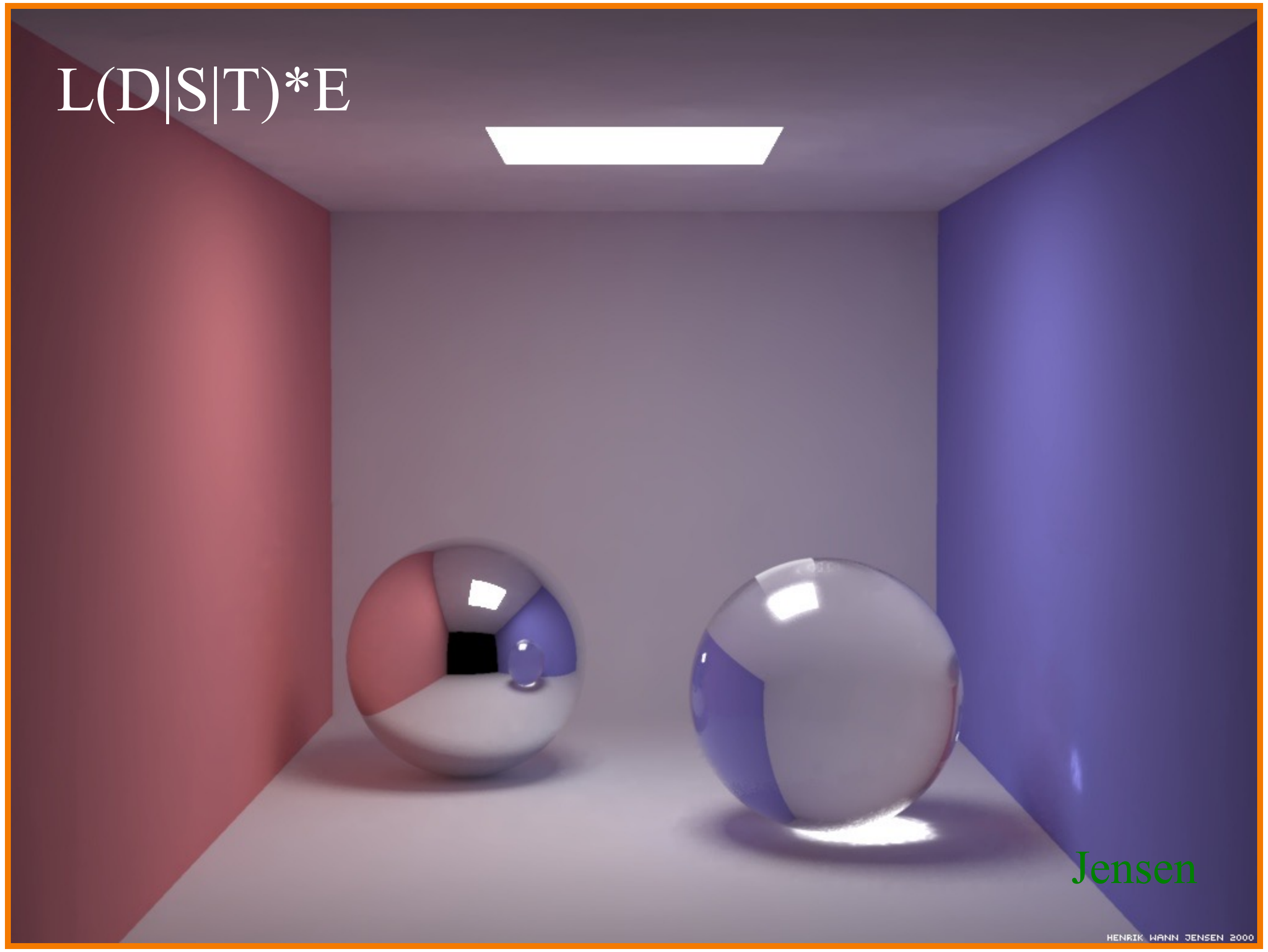
Path Types



+ caustics

Henrik Wann Jensen

Path Types



+ indirect diffuse illumination

Henrik Wann Jensen

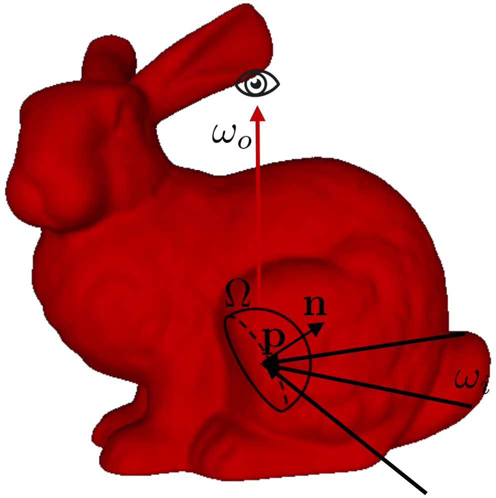
Rendering Equation

$$\boxed{L_o(\mathbf{p},\, \omega_o)} = \boxed{L_e(\mathbf{p},\, \omega_o)} + \boxed{\int_\Omega \boxed{L_i(\mathbf{p},\, \omega_i)}\, \boxed{f_r(\mathbf{p},\, \omega_i,\, \omega_o)\, (\omega_i \cdot \mathbf{n})}\, d\,\omega_i}$$

Outgoing radiance

Incident radiance

BRDF



3D Rendering Issues

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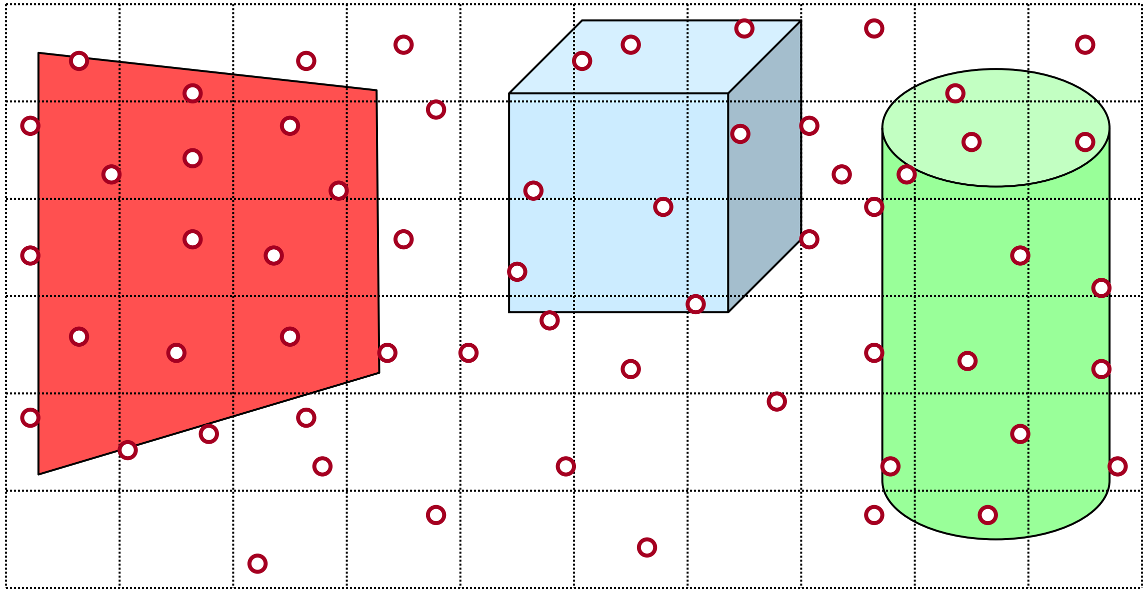
  - etc.

Sampling

- Scene can be sampled with any ray
  - Rendering is a problem in sampling and reconstruction



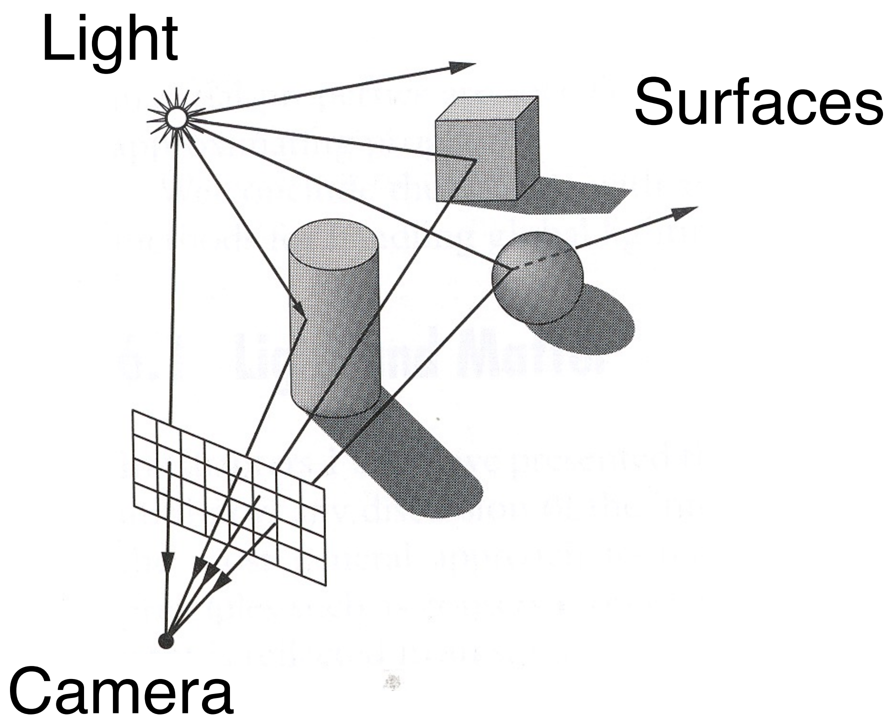
Rendering Method I:
Ray Casting

Ray Casting

- Primitive operation for one class of renderers:
  - **Given** a ray (origin, direction)
  - **Find** point of first intersection with scene
- May return:
  - Whether intersection occurs
  - Point of intersection (x,y,z)
  - Parameters of intersection on object
- Used for:
  - Camera (primary) rays: backwards ray tracing
  - Accumulate brightness from lights: forwards ray tracing
  - Shadow rays
  - Indirect illumination (path tracing)

Traditional (Backwards) Ray Tracing

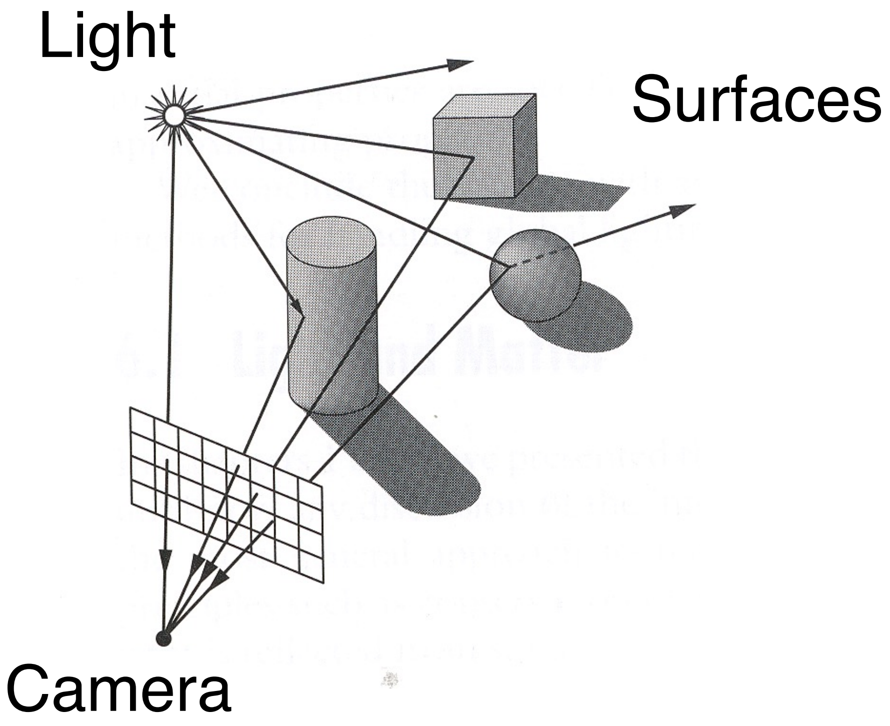
- The color of each pixel on the view plane depends on the radiance emanating along rays from visible surfaces in scene



Scene

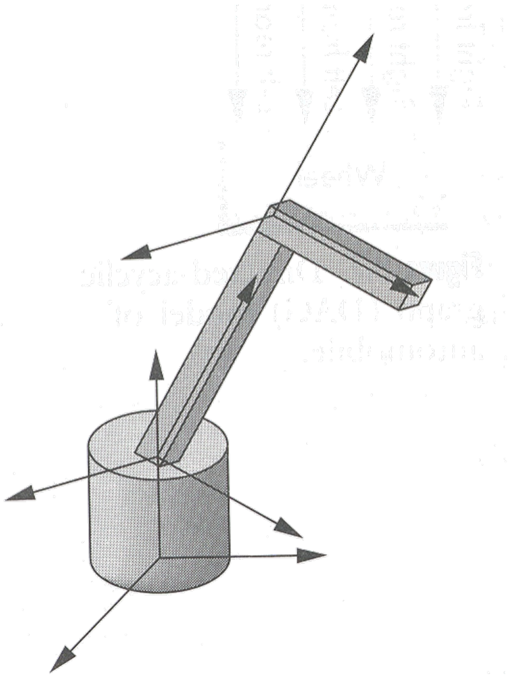
- Scene has:
  - Scene graph with surface primitives
  - Set of lights
  - Camera

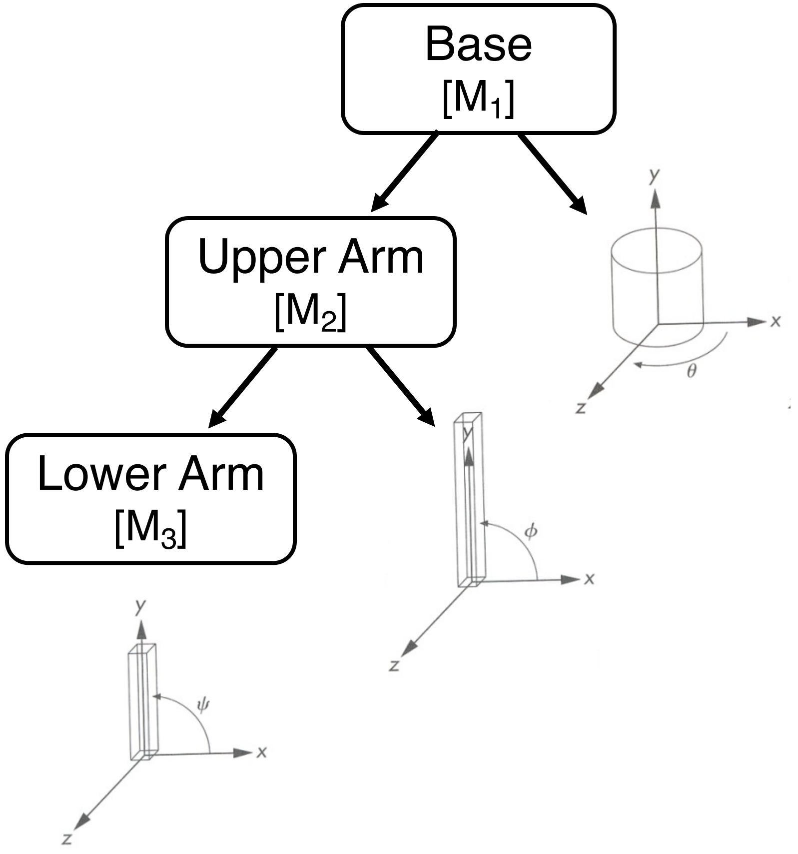
```
struct R3Scene {
        R3Node *root;
        vector<R3Light *> lights;
        R3Camera camera;
        R3Box bbox;
        R3Rgb background;
        R3Rgb ambient;
};
```



Scene Graph

- Scene graph is hierarchy of nodes, each with:
  - Bounding box (in node's coordinate system)
  - Transformation (4x4 matrix)
  - Shape (mesh, sphere, … or null)
  - Material (more on this later)





Scene Graph

- Simple scene graph implementation:

```
struct R3Node {
    struct R3Node *parent;
    vector<struct R3Node *> children;
    R3Shape *shape;
    R3Matrix transformation;
    R3Material *material;
    R3Box bbox;
};
```

```
struct R3Shape {
    R3ShapeType type;
    R3Box *box;
    R3Sphere *sphere;
    R3Cylinder *cylinder;
    R3Cone *cone;
    R3Mesh *mesh;
};
```

Ray Casting

- Simple implementation:

```
R2Image *RayCast(R3Scene *scene, int width, int height)
{
    R2Image *image = new R2Image(width, height);
    for (int i = 0; i < width; i++) {
        for (int j = 0; j < height; j++) {


            image->SetPixel(i, j, radiance);
        }
    }
    return image;
}
```

Ray Casting

- Simple implementation:

```
R2Image *RayCast(R3Scene *scene, int width, int height)
{
    R2Image *image = new R2Image(width, height);
    for (int i = 0; i < width; i++) {
        for (int j = 0; j < height; j++) {
            R3Ray ray = ConstructRayThroughPixel(scene->camera, i, j);

            image->SetPixel(i, j, radiance);
        }
    }
    return image;
}
```

Ray Casting

- Simple implementation:

```
R2Image *RayCast(R3Scene *scene, int width, int height)
{
    R2Image *image = new R2Image(width, height);
    for (int i = 0; i < width; i++) {
        for (int j = 0; j < height; j++) {
            R3Ray ray = ConstructRayThroughPixel(scene->camera, i, j);
            R3Rgb radiance = ComputeRadiance(scene,  &ray);
            image->SetPixel(i, j, radiance);
        }
    }
    return image;
}
```

Ray Casting

- Simple implementation:

```
R2Image *RayCast(R3Scene *scene, int width, int height)
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        }
    }
    return image;
}
```
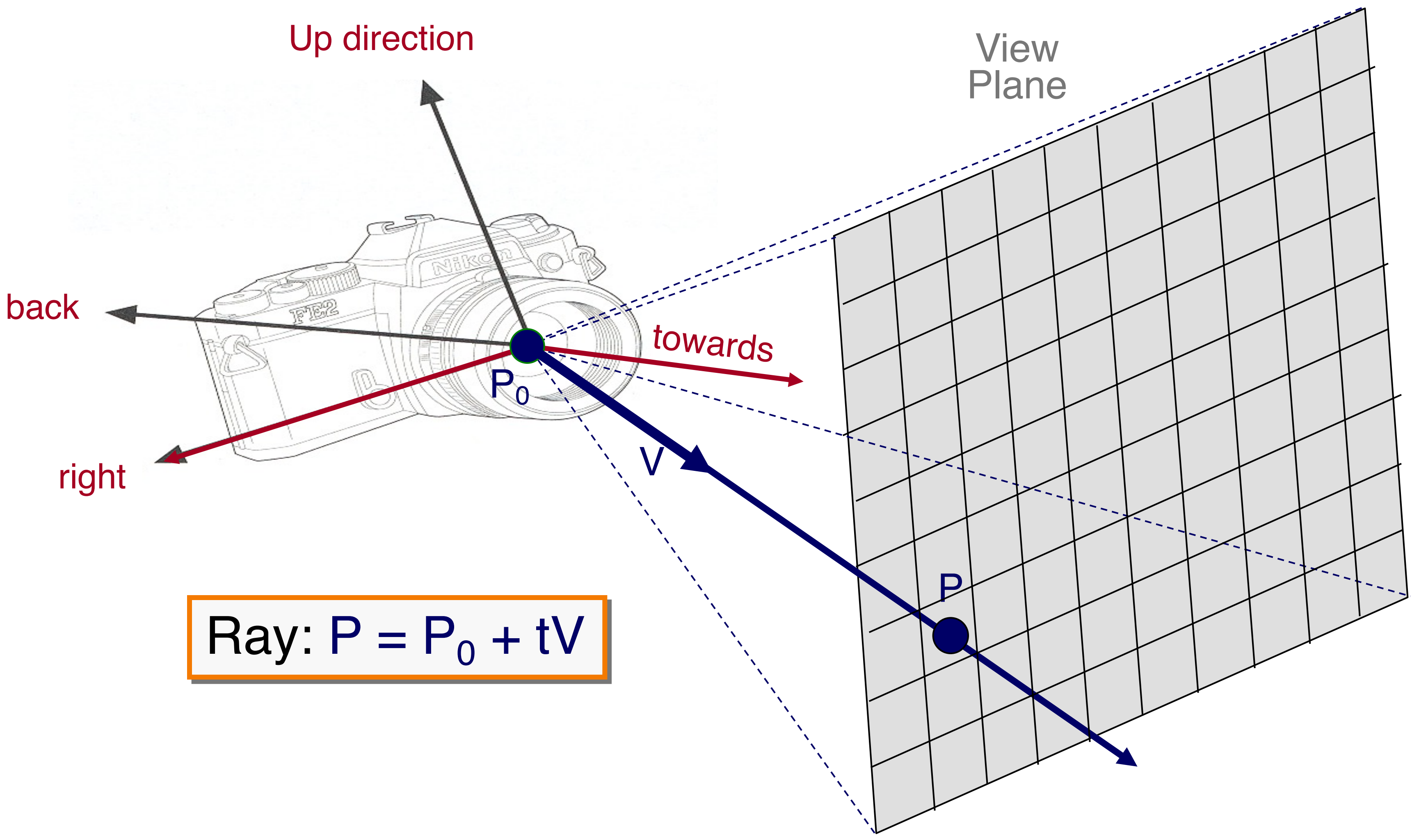

Constructing Ray Through a Pixel



Ray: $P = P_0 + tV$

Constructing Ray Through a Pixel

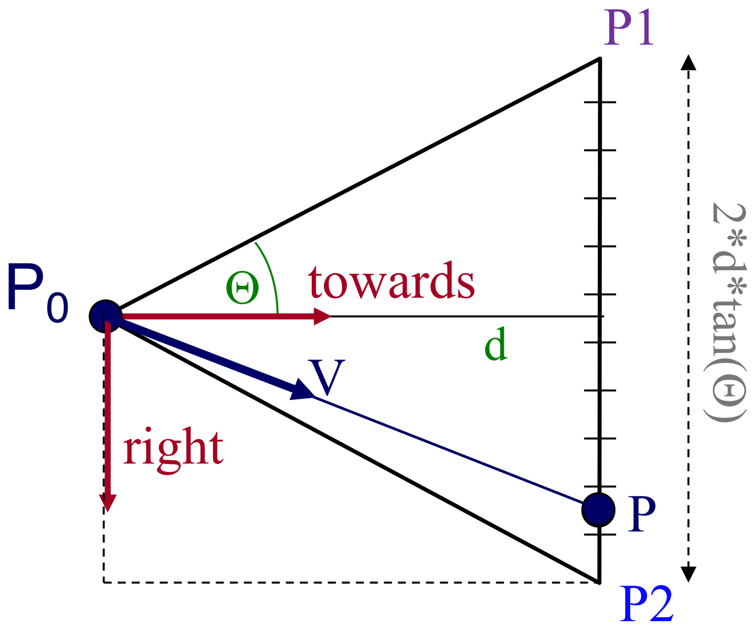
- 2D Example

Θ = frustum **half**-angle
d = distance to view plane

right = towards $\times$ up

$P1 = P_0 + d*towards - d*\tan(\Theta)*right$
$P2 = P_0 + d*towards + d*\tan(\Theta)*right$

$P = P1 + ((i + 0.5) / width) * (P2 - P1)$
$V = (P - P_0) / ||P - P_0||$
(d cancels out…)



Ray: $P = P_0 + tV$

Ray Casting

- Simple implementation:

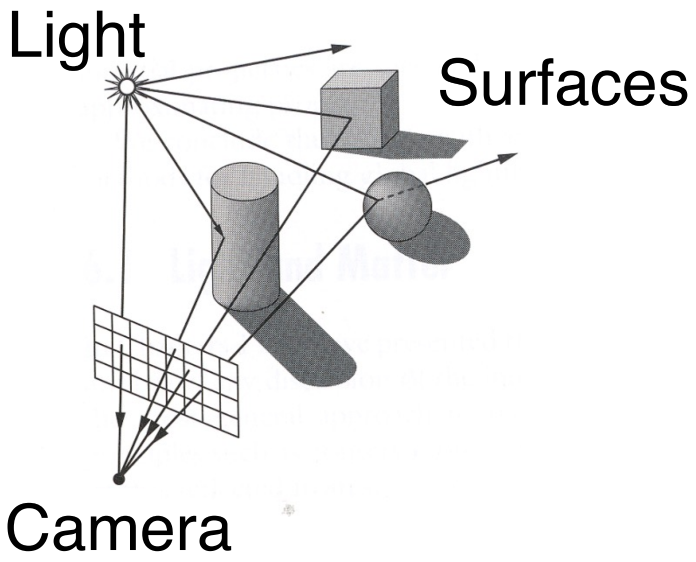
```
R2Image *RayCast(R3Scene *scene, int width, int height)
{
    R2Image *image = new R2Image(width, height);
    for (int i = 0; i < width; i++) {
        for (int j = 0; j < height; j++) {
            R3Ray ray = ConstructRayThroughPixel(scene->camera, i, j);
            R3Rgb radiance = ComputeRadiance(scene,  &ray);
            image->SetPixel(i, j, radiance);
        }
    }
    return image;
}
```

Ray Casting

- Simple implementation:

```
R3Rgb ComputeRadiance(R3Scene *scene, R3Ray *ray)
{
    R3Intersection intersection = ComputeIntersection(scene, ray);

}
```

```
struct R3Intersection {
    bool hit;
    R3Node *node;
    R3Point position;
    R3Vector normal;
    double t;
};
```

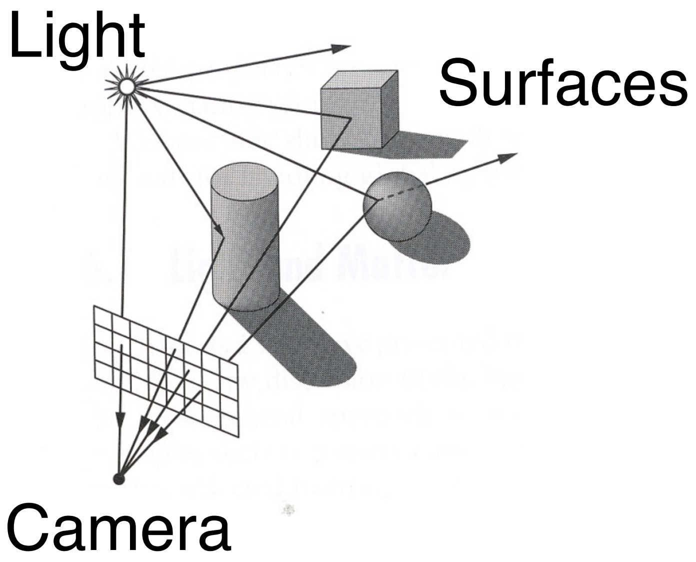


Ray Casting

- Simple implementation:

```
R3Rgb ComputeRadiance(R3Scene *scene, R3Ray *ray)
{
    R3Intersection intersection = ComputeIntersection(scene, ray);
    return ComputeRadiance(scene, ray, intersection);
}
```

```
struct R3Intersection {
    bool hit;
    R3Node *node;
    R3Point position;
    R3Vector normal;
    double t;
};
```

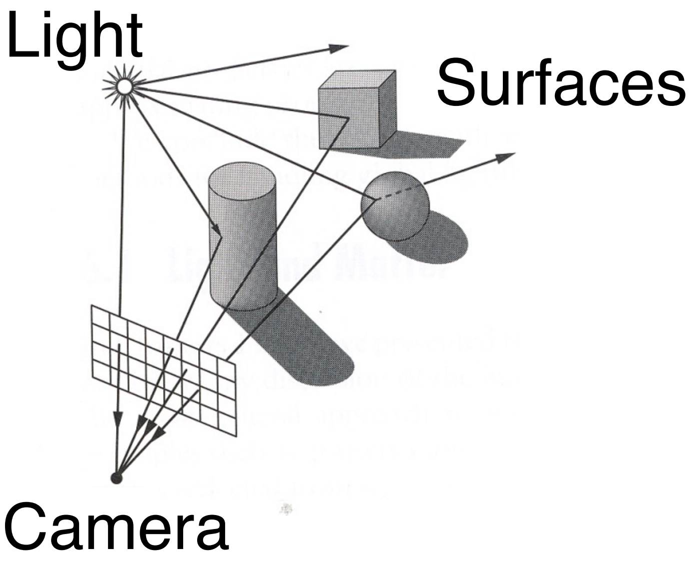


Ray Casting

- Simple implementation:

```
R3Rgb ComputeRadiance(R3Scene *scene, R3Ray *ray)
{
    R3Intersection intersection = ComputeIntersection(scene, ray);
    return ComputeRadiance(scene, ray, intersection);
}
```

```
struct R3Intersection {
    bool hit;
    R3Node *node;
    R3Point position;
    R3Vector normal;
    double t;
};
```



Ray Intersection

- Ray Intersection
  - Sphere
  - Triangle
  - Box
  - Scene

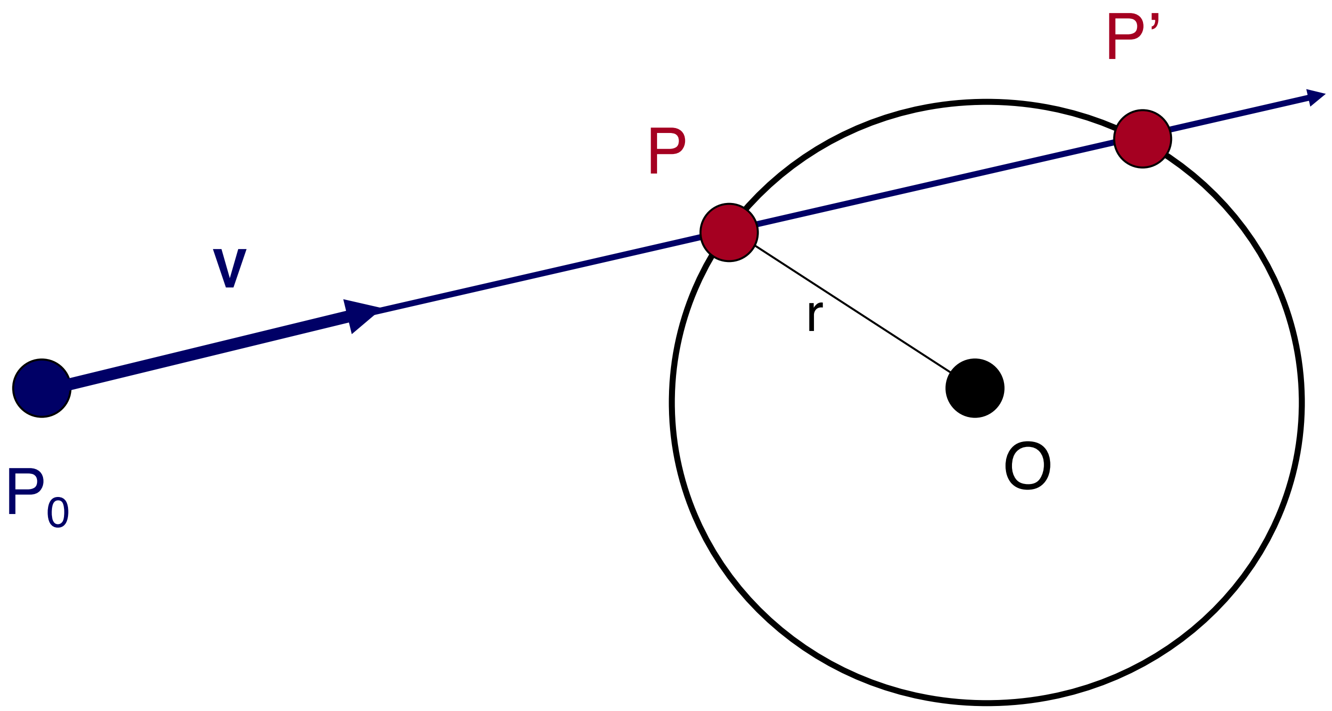
- Ray Intersection Acceleration
  - Bounding volumes
  - Uniform grids
  - Octrees
  - BSP trees

Ray Intersection

- Ray Intersection
  - Sphere
  - Triangle
  - Box
  - Scene

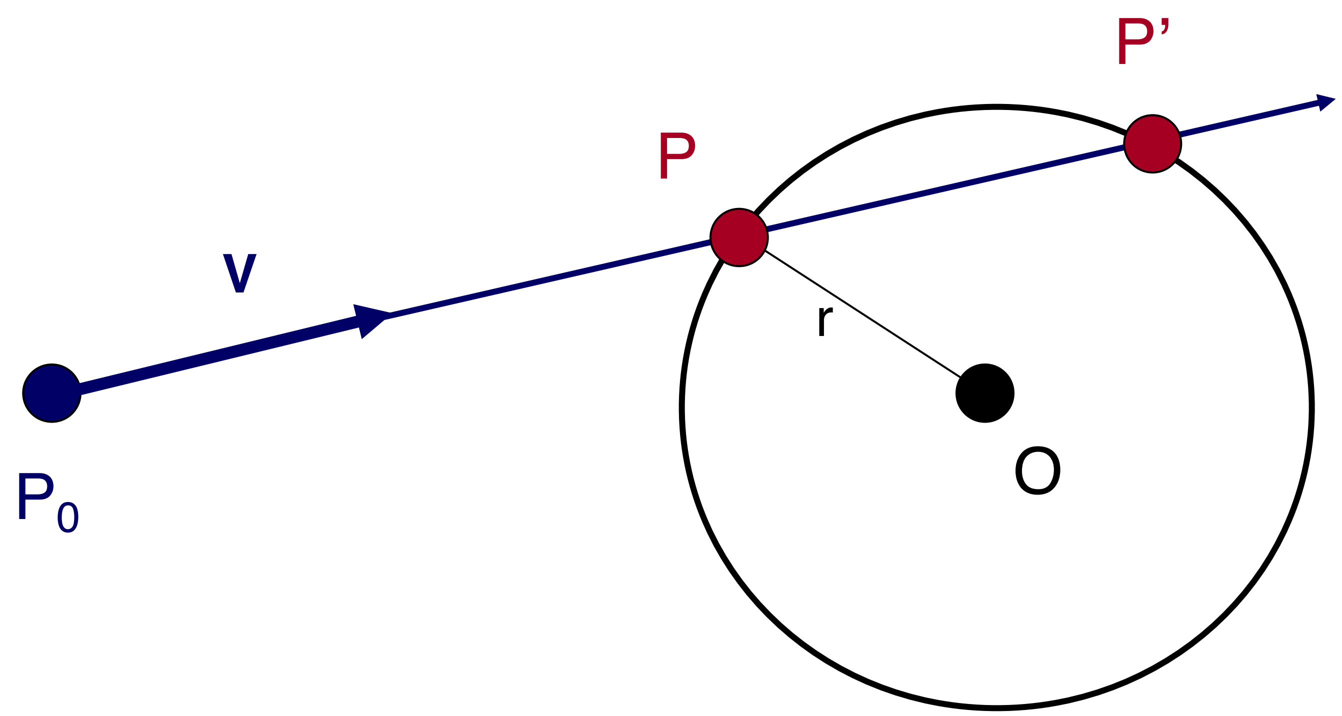
- Ray Intersection Acceleration
  - Bounding volumes
  - Uniform grids
  - Octrees
  - BSP trees

Ray-Sphere Intersection



Ray-Sphere Intersection

Ray: $P = P_0 + tV$
Sphere: $|P - O|^2 - r^2 = 0$



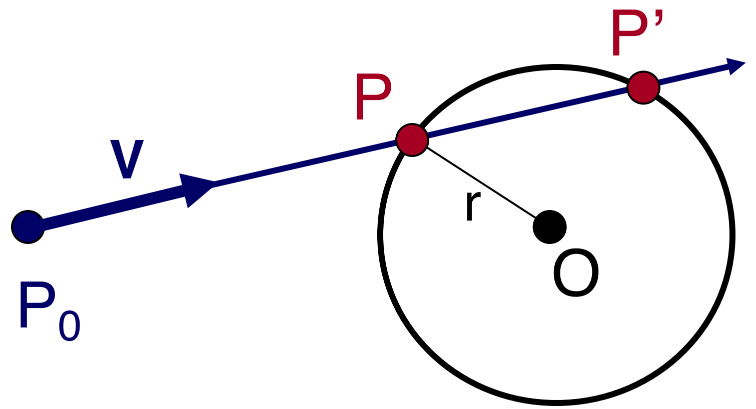
Ray-Sphere Intersection I

Ray: $P = P_0 + tV$
Sphere: $|P - O|^2 - r^2 = 0$

Algebraic Method

Substituting for P, we get:

$$|\mathbf{P_0 + tV} - O|^2 - r^2 = 0$$



$P = P_0 + tV$

Ray-Sphere Intersection I

Ray: $P = P_0 + tV$
Sphere: $|P - O|^2 - r^2 = 0$

Algebraic Method

Substituting for P, we get:

$$|\mathbf{P_0 + tV} - O|^2 - r^2 = 0$$

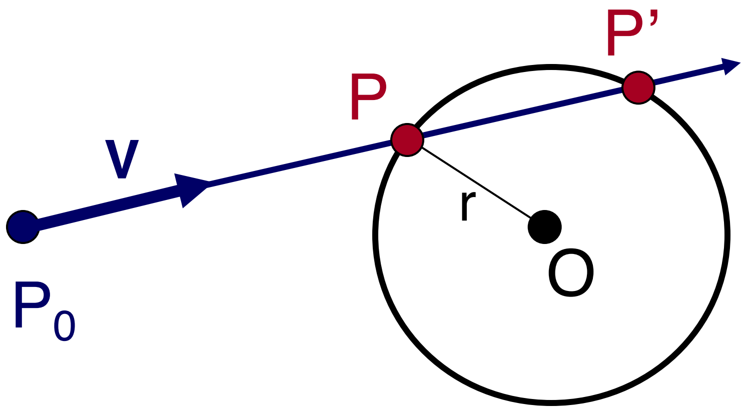
Solve quadratic equation:

$$at^2 + bt + c = 0$$

where:

$$a = V^2$$
$$b = 2\, V \cdot (P_0 - O)$$
$$c = |P_0 - C|^2 - r^2 = 0$$

$P = P_0 + tV$



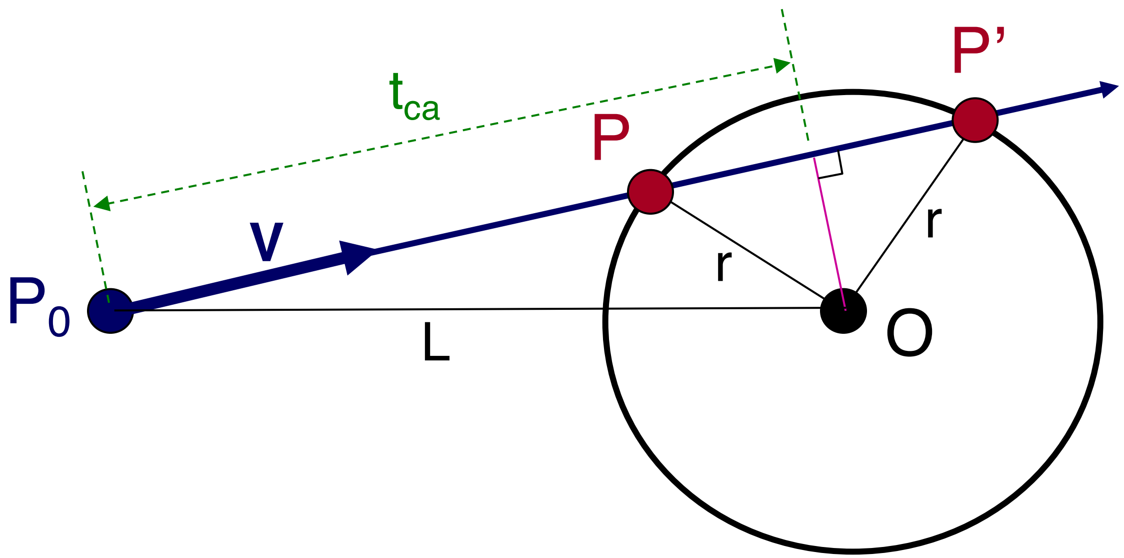
Ray-Sphere Intersection II

Ray: $P = P_0 + tV$
Sphere: $|P - O|^2 - r^2 = 0$

Geometric Method

$L = O - P_0$

$t_{ca} = L \cdot V$
if ($t_{ca} < 0$) return INF



$P = P_0 + tV$

Ray-Sphere Intersection II

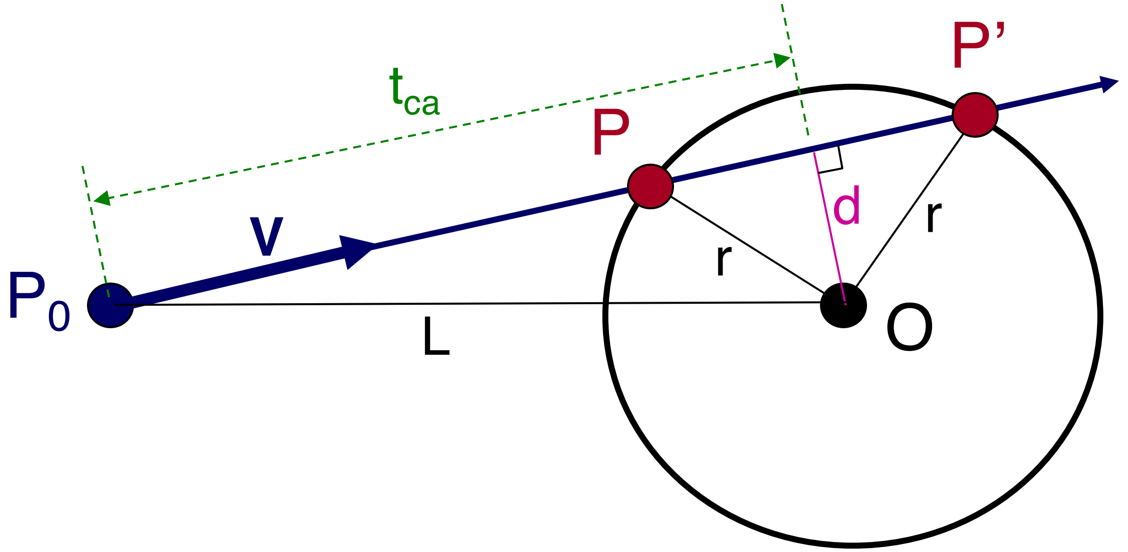
Ray: $P = P_0 + tV$
Sphere: $|P - O|^2 - r^2 = 0$

Geometric Method

$L = O - P_0$

$t_{ca} = L \cdot V$
if ($t_{ca} < 0$) return INF

$d^2 = L \cdot L - t_{ca}^2$
if ($d^2 > r^2$) return INF



$P = P_0 + tV$

Ray-Sphere Intersection II

Ray: $P = P_0 + tV$
Sphere: $|P - O|^2 - r^2 = 0$

Geometric Method

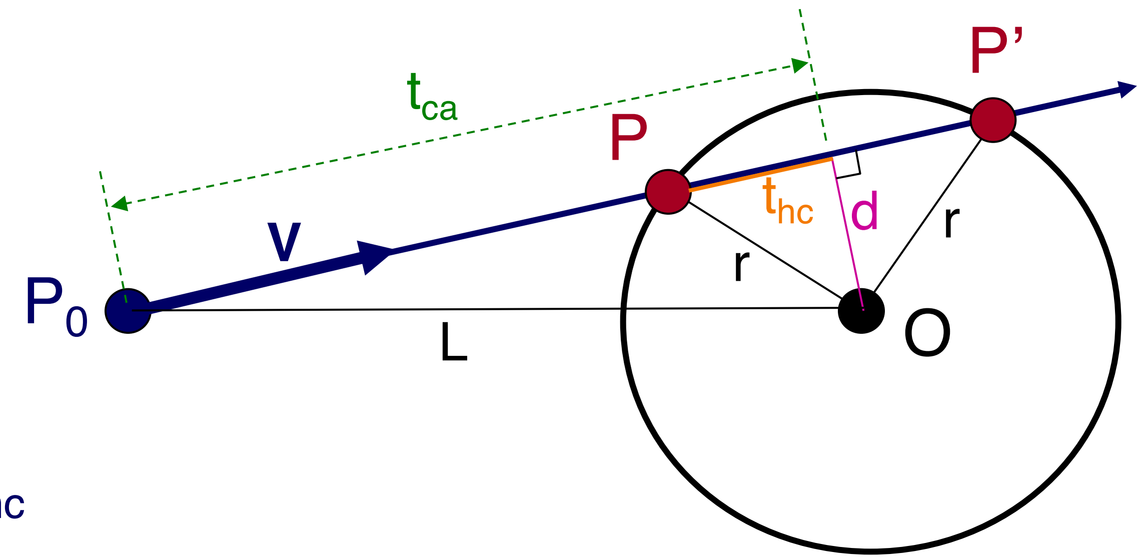
$L = O - P_0$

$t_{ca} = L \cdot V$
if ($t_{ca} < 0$) return INF

$d^2 = L \cdot L - t_{ca}^2$
if ($d^2 > r^2$) return INF

$t_{hc} = \text{sqrt}(r^2 - d^2)$
$t = t_{ca} - t_{hc}$ and $t_{ca} + t_{hc}$

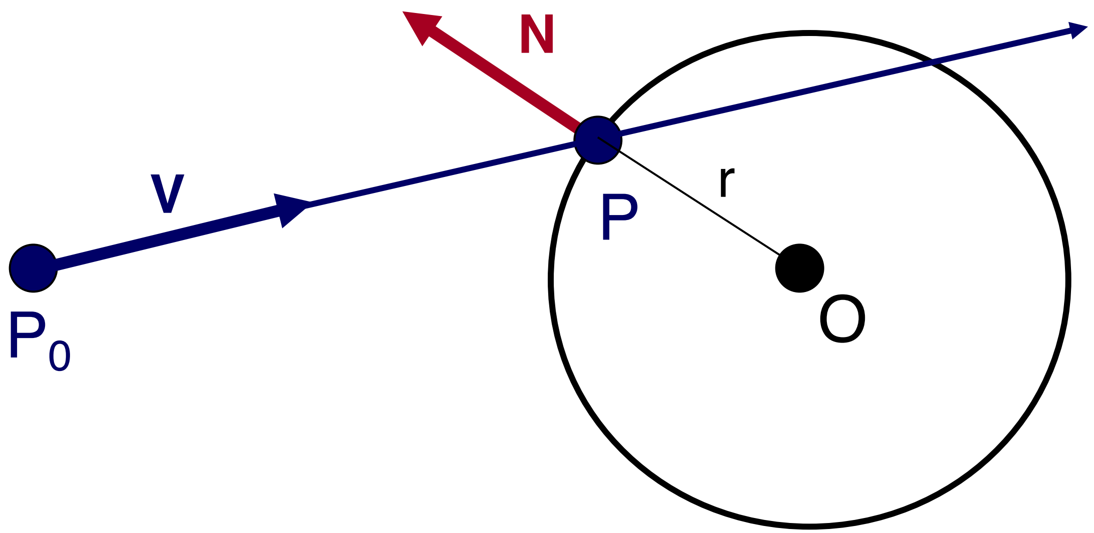
$P = P_0 + tV$



Ray-Sphere Intersection

- Need normal vector at intersection for lighting calculations (next lecture)

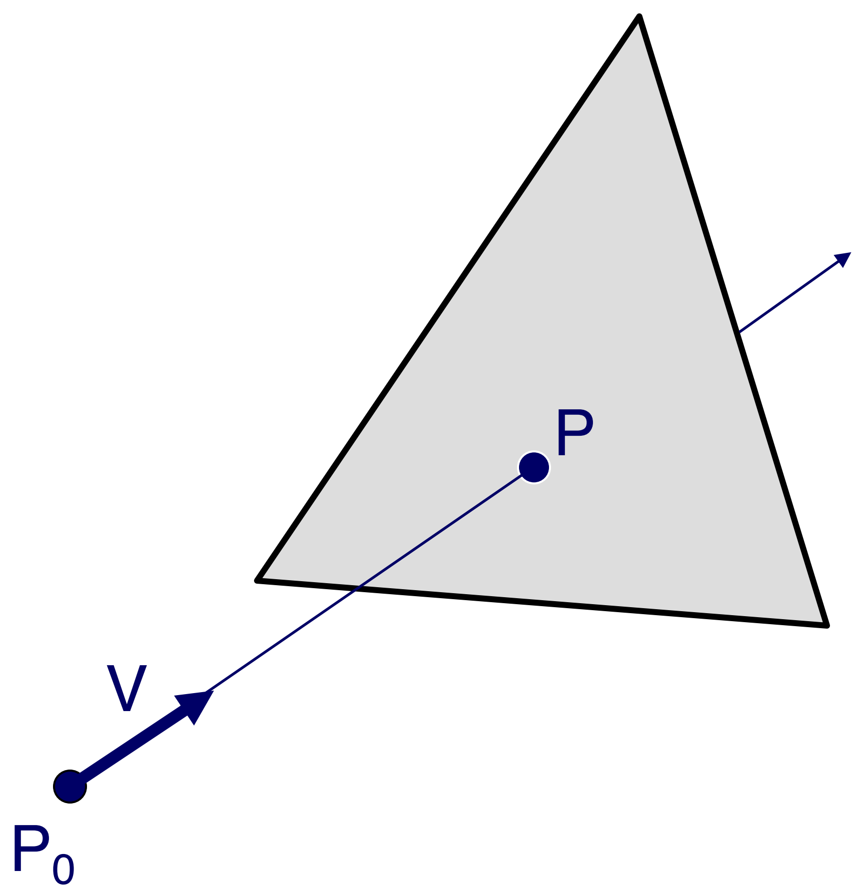
$$N = (P - O) / ||P - O||$$



Ray Intersection

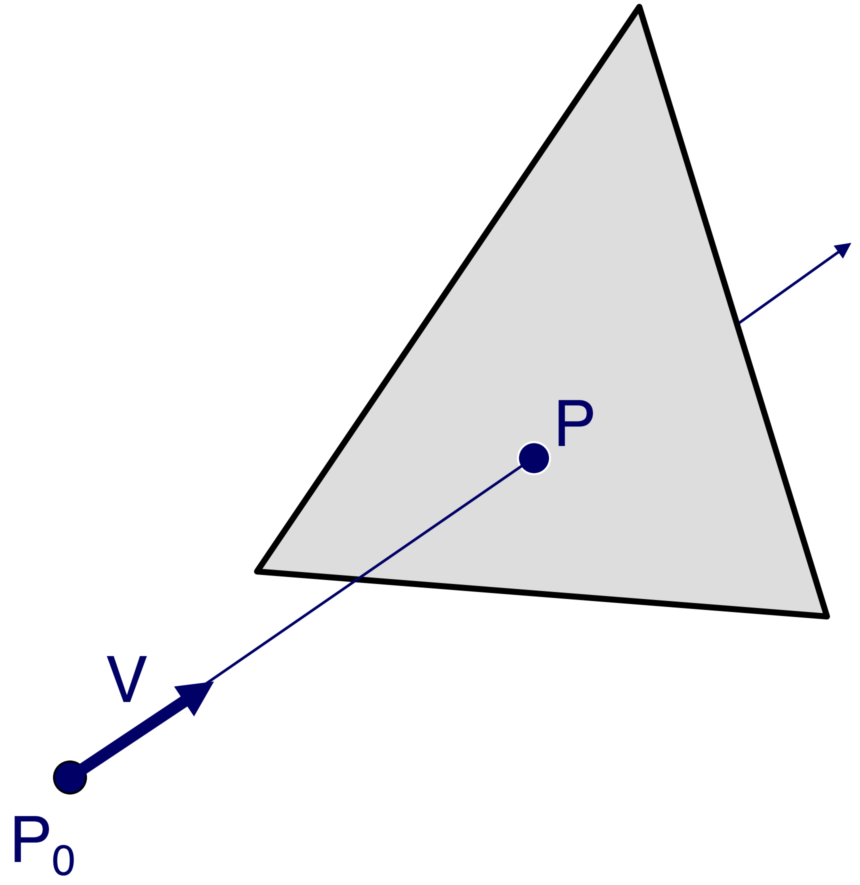
- Ray Intersection
    - Sphere
    - Triangle
    - Box
    - Scene
- Ray Intersection Acceleration
    - Bounding volumes
    - Uniform grids
    - Octrees
    - BSP trees

Ray-Triangle Intersection



Ray-Triangle Intersection

- First, intersect ray with plane
- Then, check if intersection point is inside triangle



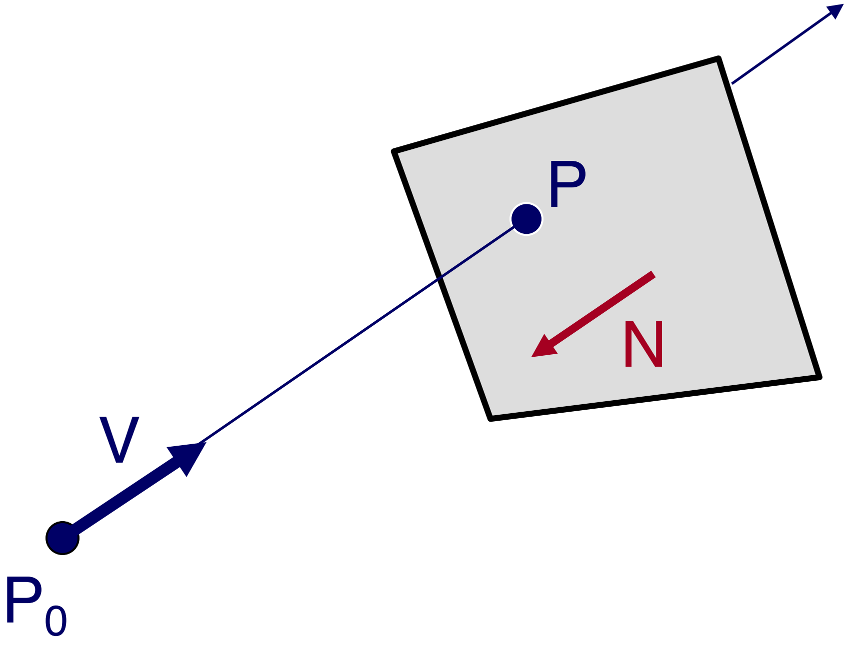
Ray-Plane Intersection

Ray: $P = P_0 + tV$
Plane: $P \cdot N + d = 0$

Substituting for P, we get:

$$(\mathbf{P_0 + tV}) \cdot N + d = 0$$

Algebraic Method



Ray-Plane Intersection

Ray: $P = P_0 + tV$
Plane: $P \cdot N + d = 0$

Algebraic Method

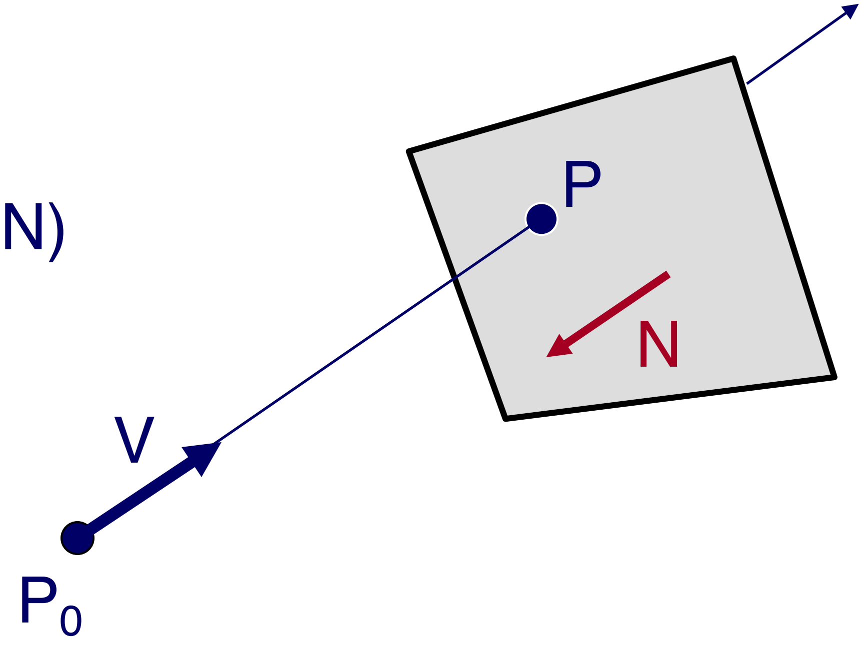
Substituting for P, we get:

$$(\mathbf{P_0 + tV}) \cdot N + d = 0$$

Solution:

$$t = -(P_0 \cdot N + d) / (V \cdot N)$$

$$P = P_0 + tV$$



Ray-Triangle Intersection I

- Check if point is inside triangle algebraically

For each side of triangle

$V_1 = T_1 - P_0$

$V_2 = T_2 - P_0$

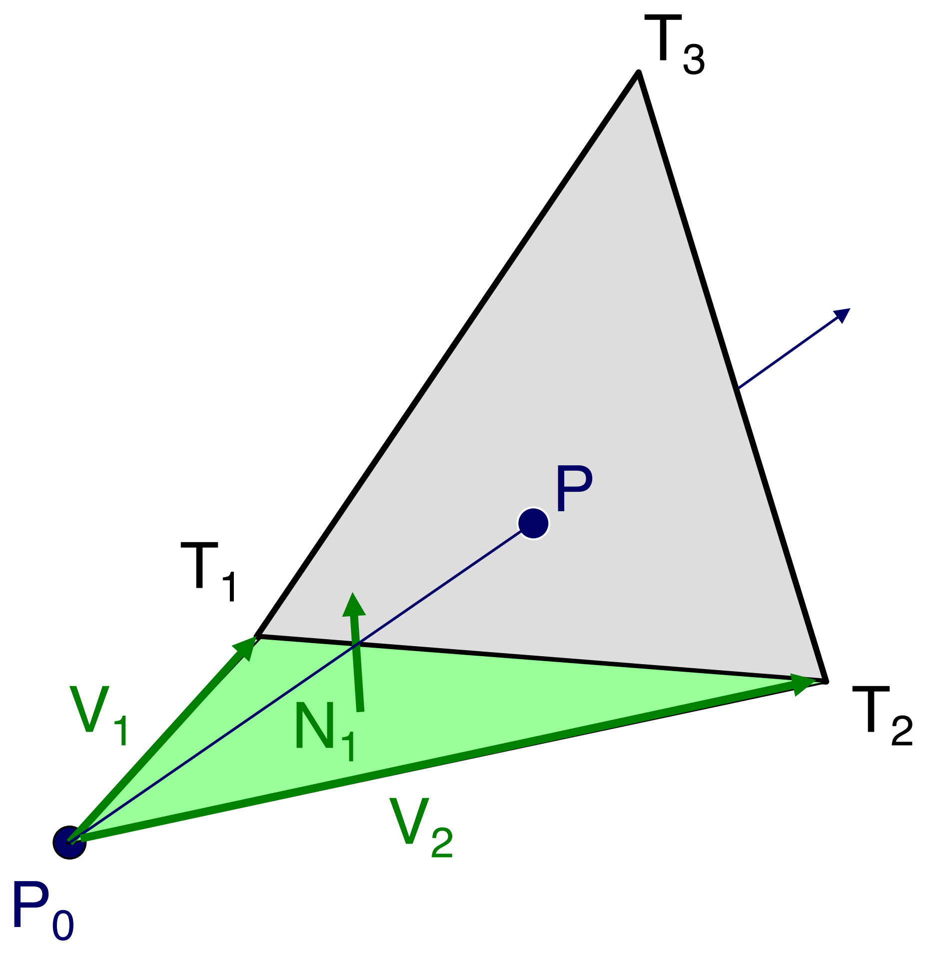
$N_1 = V_2 \times V_1$

Normalize N_1

Plane $p(P_0, N_1)$

end

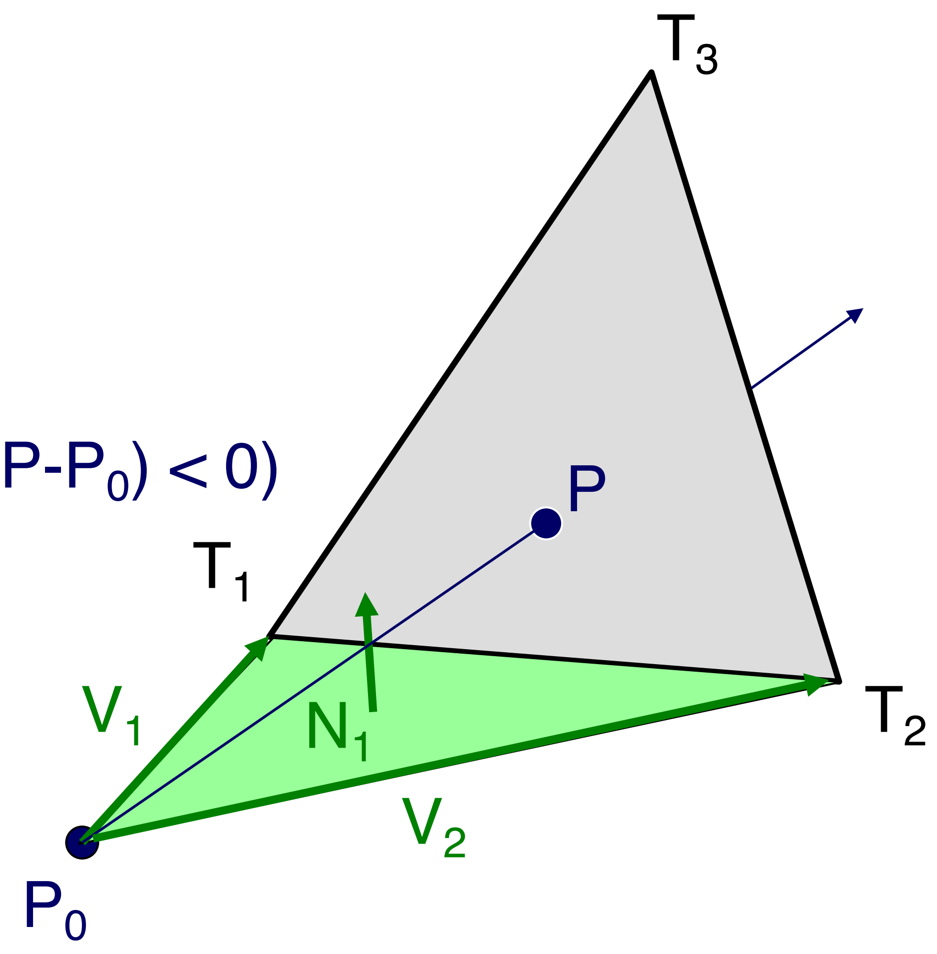
return TRUE



Ray-Triangle Intersection I

- Check if point is inside triangle algebraically

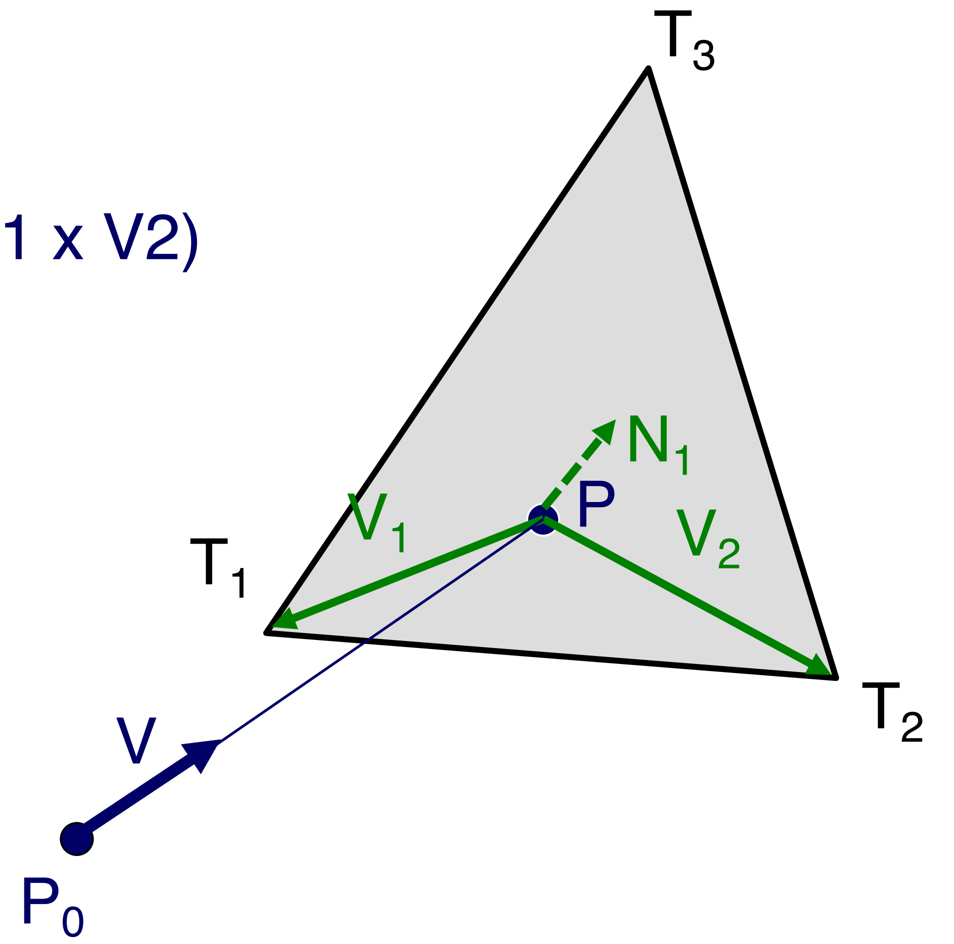
For each side of triangle
    $V_1 = T_1 - P_0$
    $V_2 = T_2 - P_0$
    $N_1 = V_2 \times V_1$
    Normalize N_1
    Plane $p(P_0, N_1)$
    if (SignedDistance(p, P-P_0) < 0)
        return FALSE
end
return TRUE



Ray-Triangle Intersection II

- Check if point is inside triangle algebraically

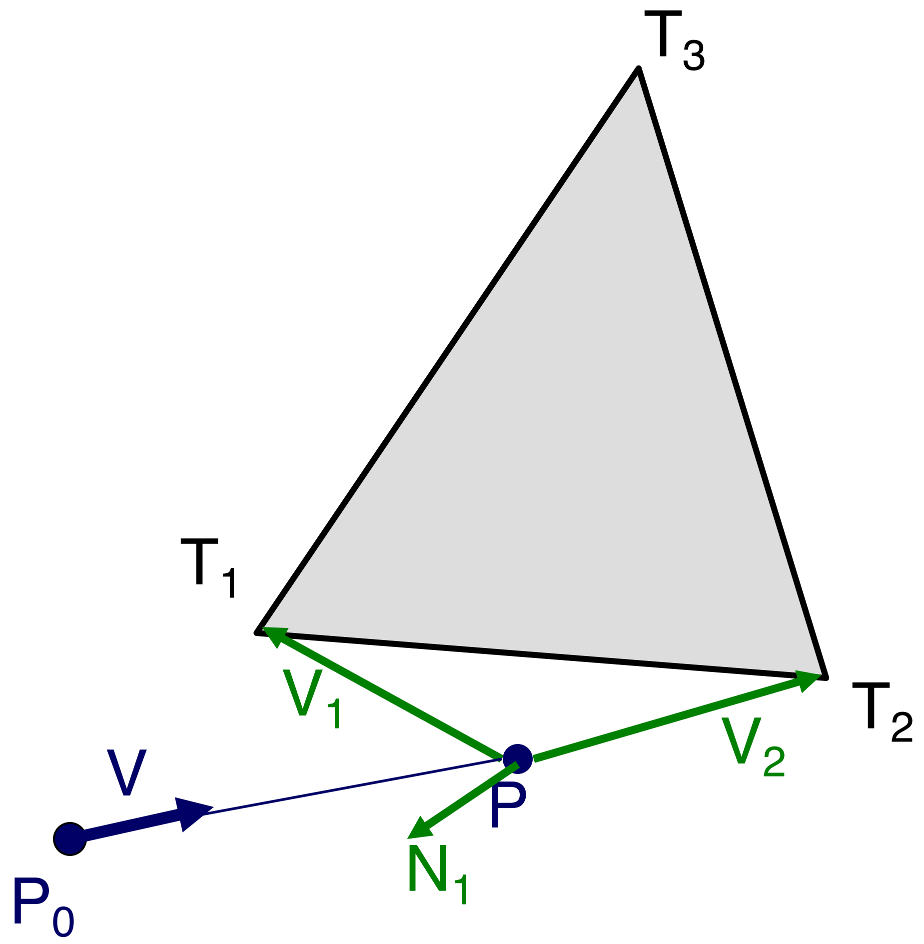
For each side of triangle
    $V_1 = T_1 - P$
    $V_2 = T_2 - P$
    $N_1 = \mathbf{V_2 \times V_1}$ (but not V1 x V2)
    if $(V \cdot N_1 < 0)$
        return FALSE
end
return TRUE



Ray-Triangle Intersection II

- Check if point is inside triangle algebraically

For each side of triangle
    $V_1 = T_1 - P$
    $V_2 = T_2 - P$
    $N_1 = \mathbf{V_2 \times V_1}$
    if $(V \cdot N_1 < 0)$
        return FALSE
end
return TRUE



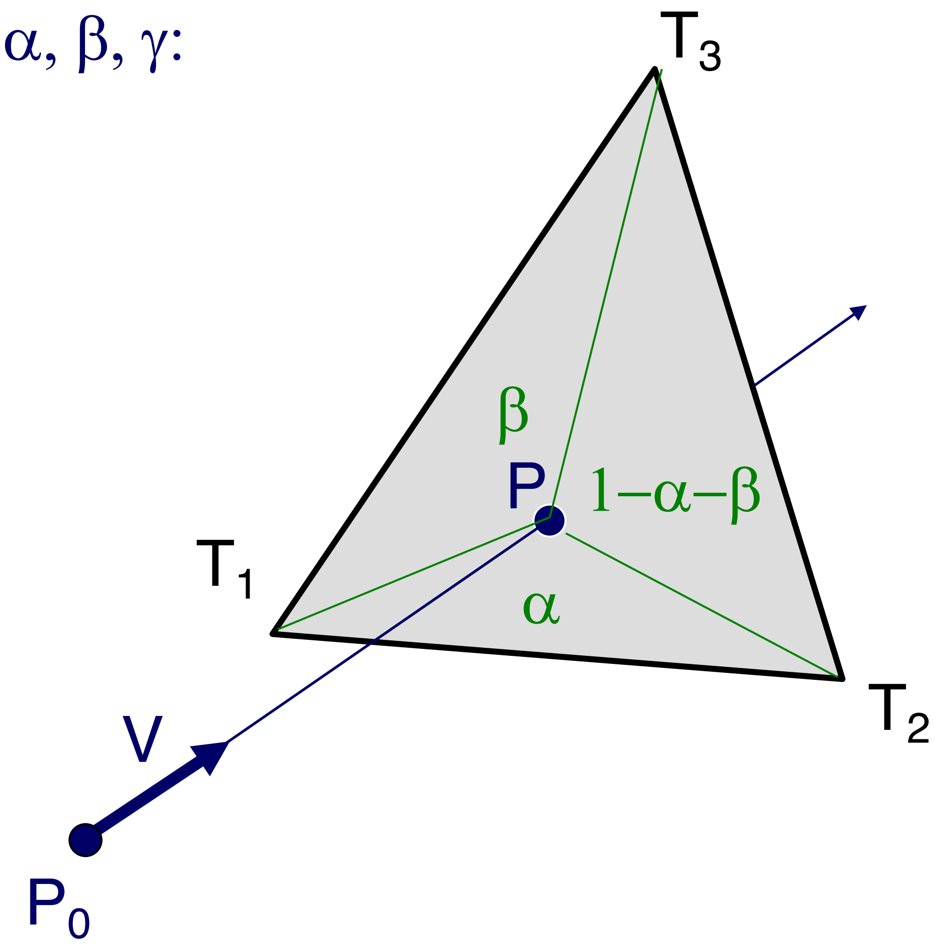
Ray-Triangle Intersection III

- Check if point is inside triangle parametrically

"Barycentric coordinates" α, β, γ:

$$P = \alpha T_3 + \beta T_2 + \gamma T_1$$

where $\alpha + \beta + \gamma = 1$



Ray-Triangle Intersection III

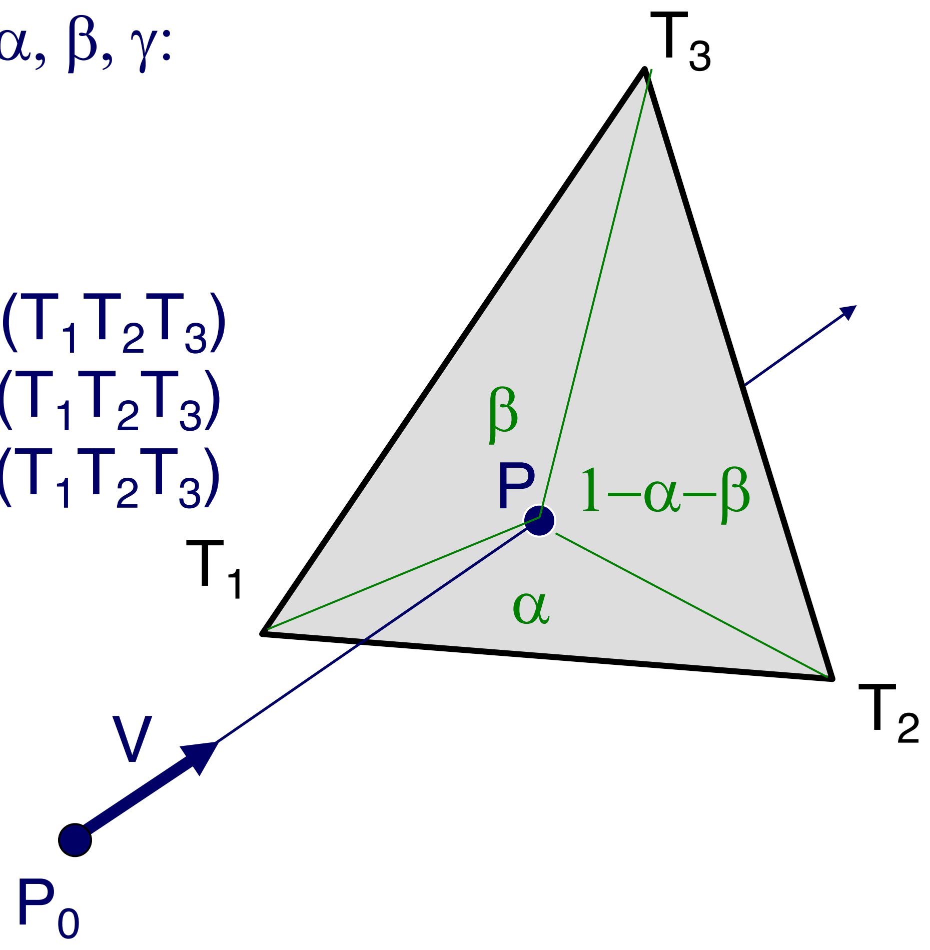
- Check if point is inside triangle parametrically

"Barycentric coordinates" α, β, γ:

$$P = \alpha T_3 + \beta T_2 + \gamma T_1$$

where $\alpha + \beta + \gamma = 1$

$$\alpha = \text{Area}(T_1 T_2 P) \,/\, \text{Area}(T_1 T_2 T_3)$$
$$\beta = \text{Area}(T_1 P T_3) \,/\, \text{Area}(T_1 T_2 T_3)$$
$$\gamma = \text{Area}(P T_2 T_3) \,/\, \text{Area}(T_1 T_2 T_3)$$
$$= 1 - \alpha - \beta$$



Ray-Triangle Intersection III

- Check if point is inside triangle parametrically

Compute "barycentric coordinates" α, β:

$\alpha = \text{Area}(T_1T_2P) / \text{Area}(T_1T_2T_3)$

$\beta = \text{Area}(T_1PT_3) / \text{Area}(T_1T_2T_3)$

$\text{Area}(T_1T_2T_3) = ½ \| (T2-T1) \times (T3-T1) \|$

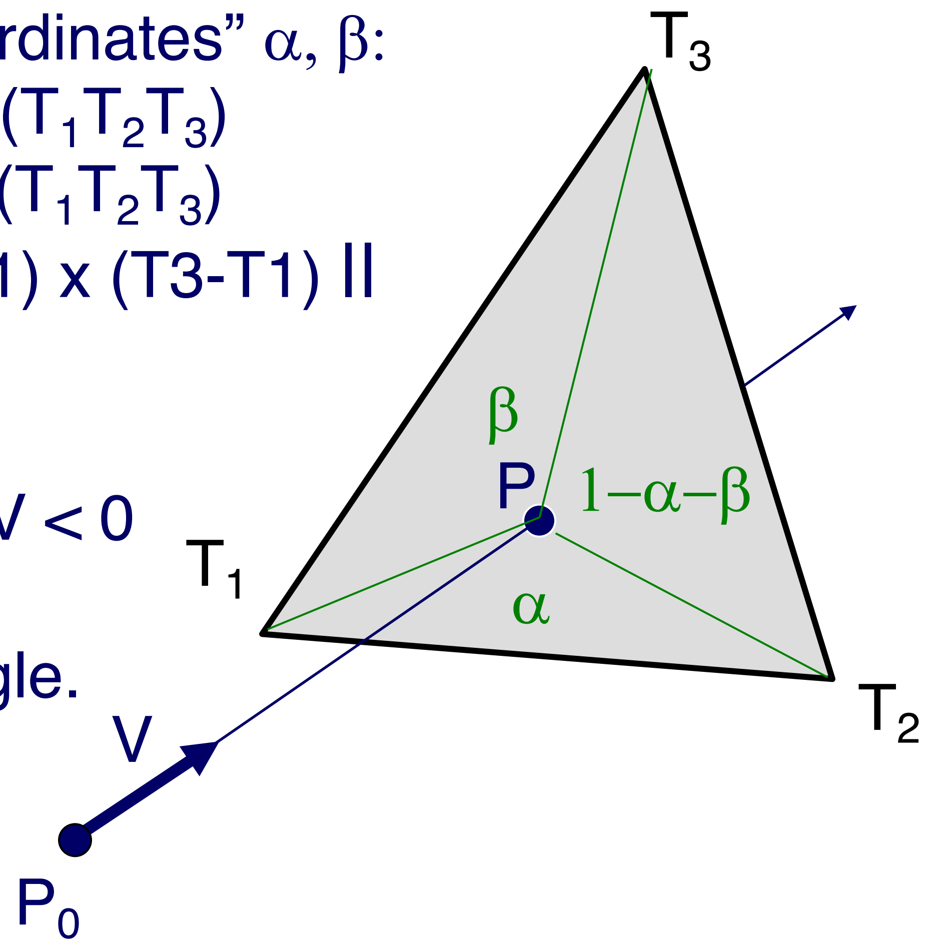
check if backfacing:

$((T2-T1) \times (T3-T1)) \cdot V < 0$

Check if point inside triangle.

$0 \le \alpha \le 1$ and $0 \le \beta \le 1$

and $\alpha + \beta \le 1$



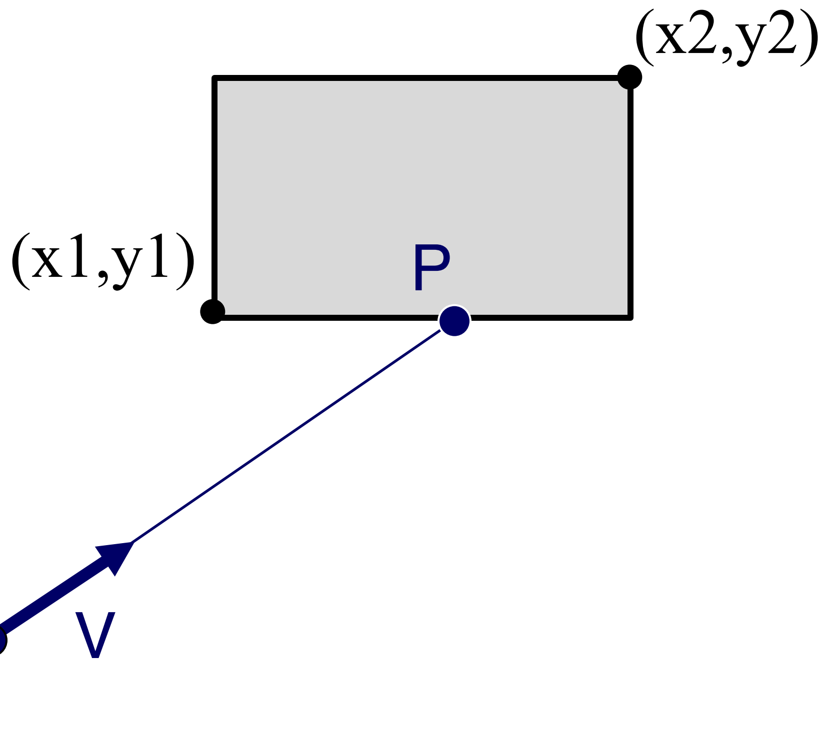
Ray Intersection

- Ray Intersection
  - Sphere
  - Triangle
  - Box
  - Scene

- Ray Intersection Acceleration
  - Bounding volumes
  - Uniform grids
  - Octrees
  - BSP trees

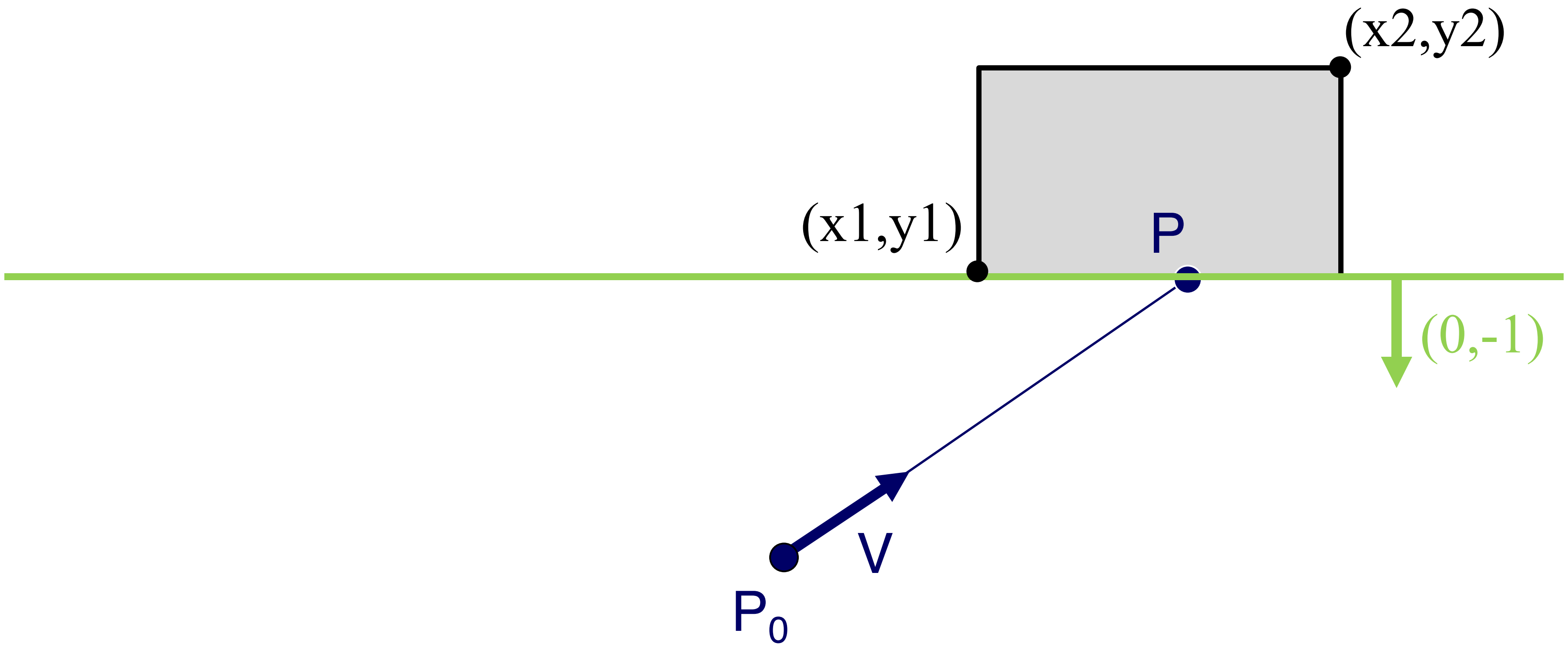
Ray-Box Intersection

- Check front-facing sides for intersection with ray and return closest intersection (least t)



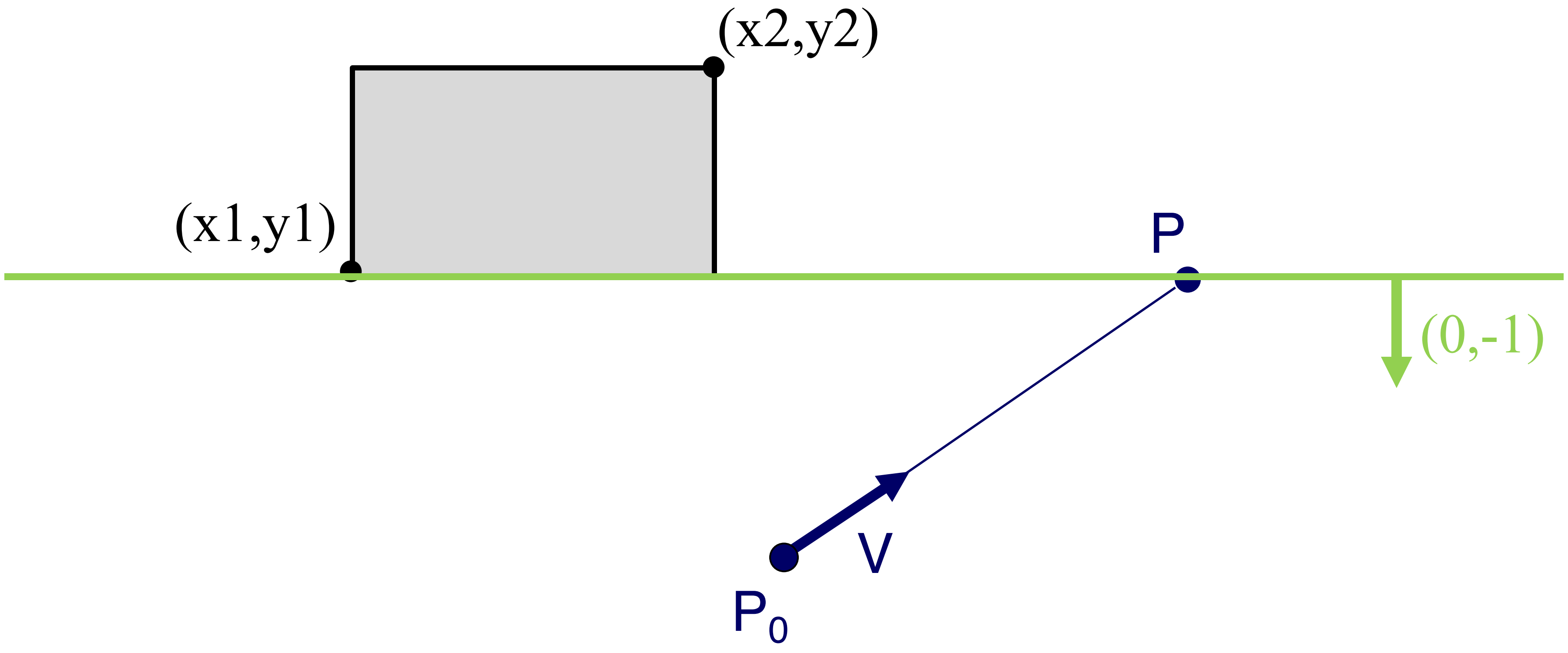
Ray-Box Intersection

- Check front-facing sides for intersection with ray and return closest intersection (least t)
  - Find intersection with plane
  - Check if point is inside rectangle



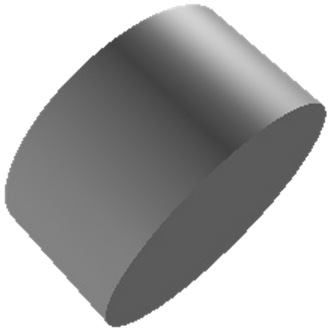
Ray-Box Intersection

- Check front-facing sides for intersection with ray and return closest intersection (least t)
  - Find intersection with plane
  - Check if point is inside rectangle



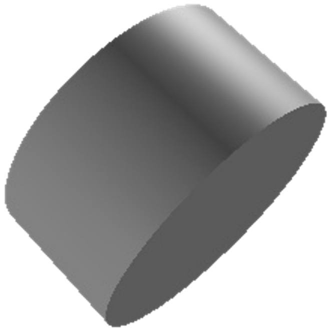
Other Ray-Primitive Intersections

- Cone, cylinder:
  - Similar to sphere
  - Must also check end caps



Other Ray-Primitive Intersections

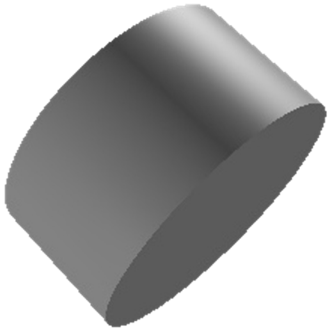
- Cone, cylinder:
  - Similar to sphere
  - Must also check end caps



- Convex polygon
  - Same as triangle (check point-in-polygon algebraically)
  - Or, decompose into triangles, and check all of them

Other Ray-Primitive Intersections

- Cone, cylinder:
  - Similar to sphere
  - Must also check end caps



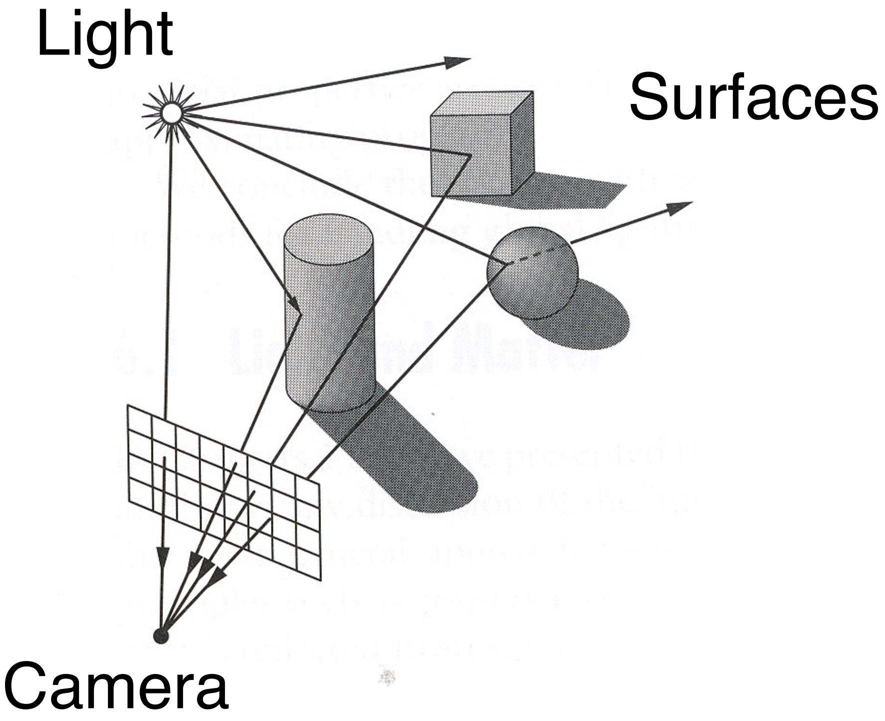
- Convex polygon
  - Same as triangle (check point-in-polygon algebraically)
  - Or, decompose into triangles, and check all of them
- Mesh
  - Compute intersection for all polygons
  - Return closest intersection (least t)

Ray Intersection

- Ray Intersection
  - Sphere
  - Triangle
  - Box
  - Scene
- Ray Intersection Acceleration
  - Bounding volumes
  - Uniform grids
  - Octrees
  - BSP trees

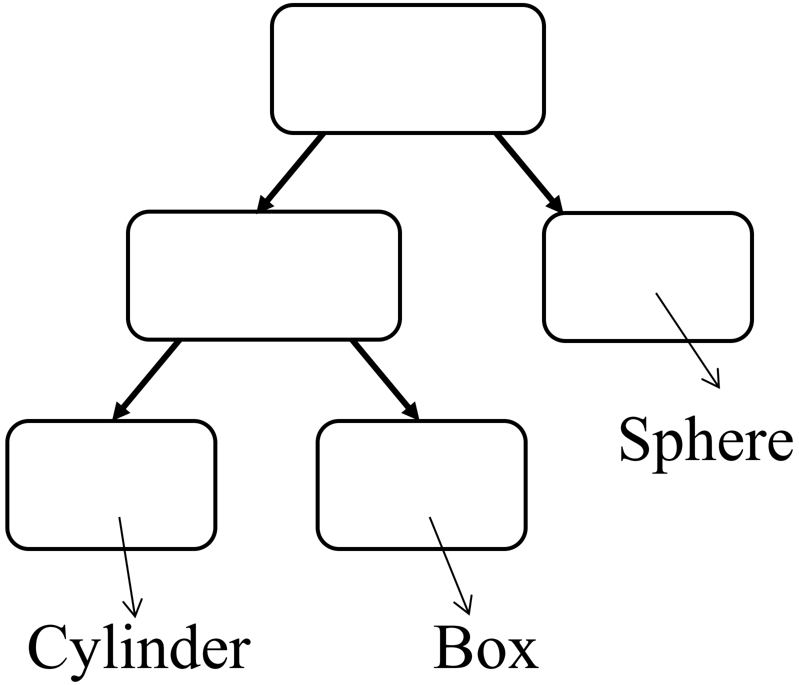
Ray-Scene Intersection

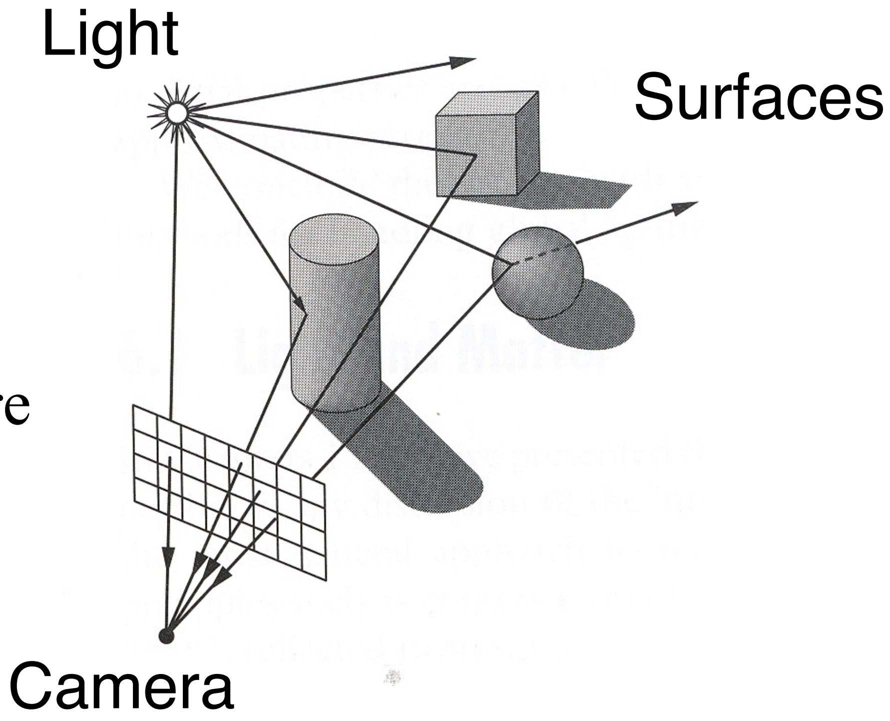
- Intuitive method
  - Compute intersection for **all** nodes of scene graph
  - Return closest intersection (least t)



Ray-Scene Intersection

- Scene graph is a DAG
  - Traverse with recursion





Ray-Scene Intersection I

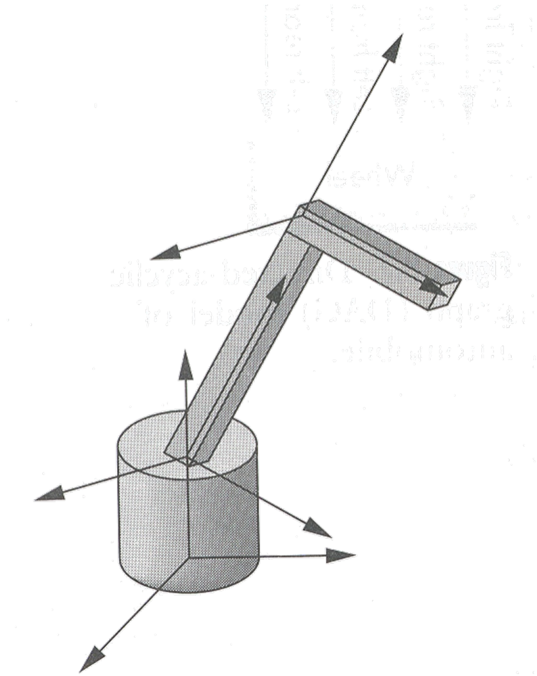
```
R3Intersection ComputeIntersection(R3Scene *scene, R3Node *node, R3Ray *ray)
{
    // Check for intersection with shape
    shape_intersection = Intersect node's shape with ray
    if (shape_intersection is a hit) closest_intersection = shape_intersection
    else closest_intersection = infinitely far miss
















}
```

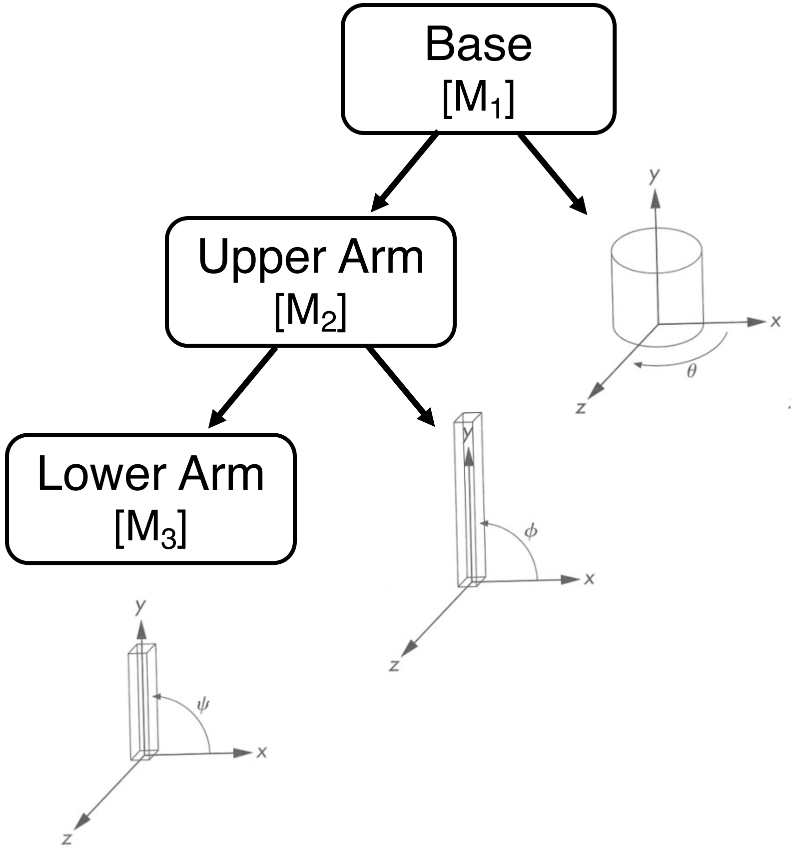
Ray-Scene Intersection I

```
R3Intersection ComputeIntersection(R3Scene *scene, R3Node *node, R3Ray *ray)
{
    // Check for intersection with shape
    shape_intersection = Intersect node's shape with ray
    if (shape_intersection is a hit) closest_intersection = shape_intersection
    else closest_intersection = infinitely far miss

    // Check for intersection with children nodes
    for each child node
        // Check for intersection with child contents
        child_intersection = ComputeIntersection(scene, child, ray);
        if (child_intersection is a hit and is closer than closest_intersection)
            closest_intersection = child_intersection;

    // Return closest intersection in tree rooted at this node
    return closest_intersection
}
```

Ray-Scene Intersection

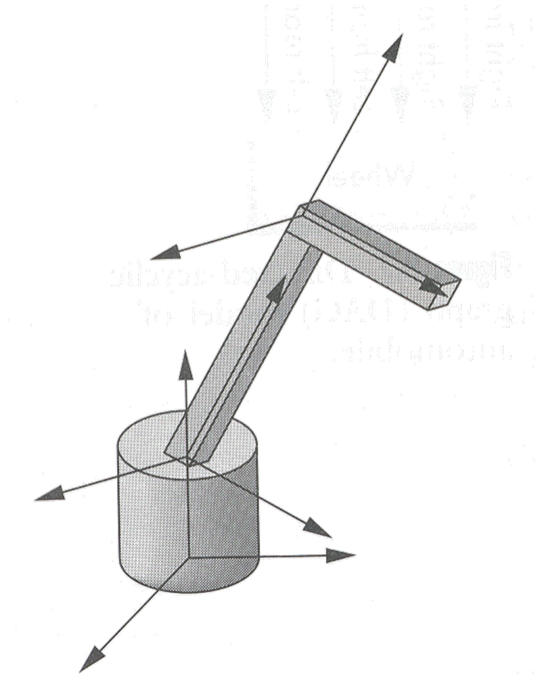
- Scene graph can have transformations

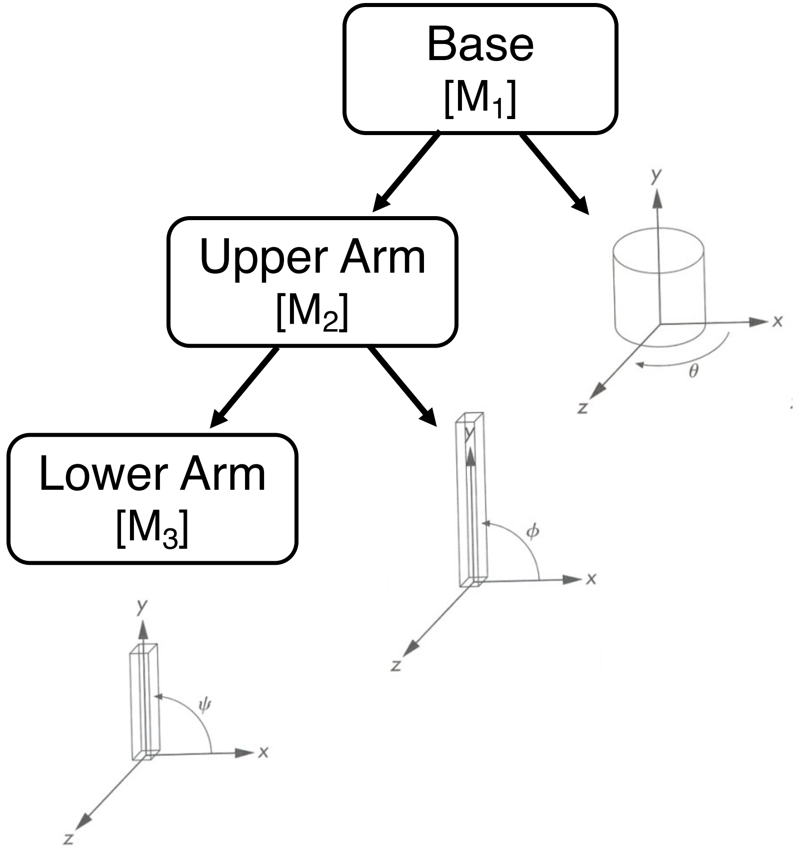




Ray-Scene Intersection

- Scene graph node can have transformations
  - Transform ray (not primitives) by inverse of M
  - Intersect in coordinate system of node
  - Transform intersection by M





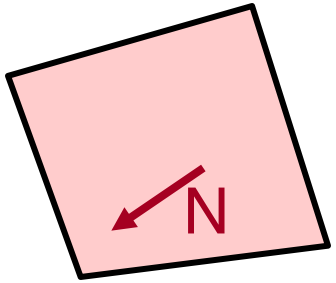
Ray-Scene Intersection II

```
R3Intersection ComputeIntersection(R3Scene *scene, R3Node *node, R3Ray *ray)
{
    // Transform ray by inverse of node's transformation

    // Check for intersection with shape

    // Check for intersection with children nodes

    // Transform intersection by node's transformation

    // Return closest intersection in tree rooted at this node
}
```

Ray-Scene Intersection II

```
R3Intersection ComputeIntersection(R3Scene *scene, R3Node *node, R3Ray *ray)
{
    // Transform ray by inverse of node's transformation

    // Check for intersection with shape

    // Check for intersection with children nodes

    // Transform intersection by node's transformation

    // Return closest intersection in tree rooted at this node
}
```

Note: directions must be transformed by inverse of M

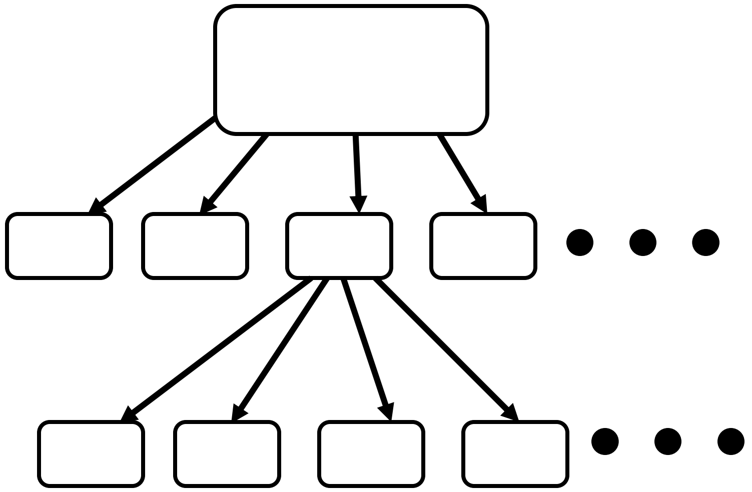


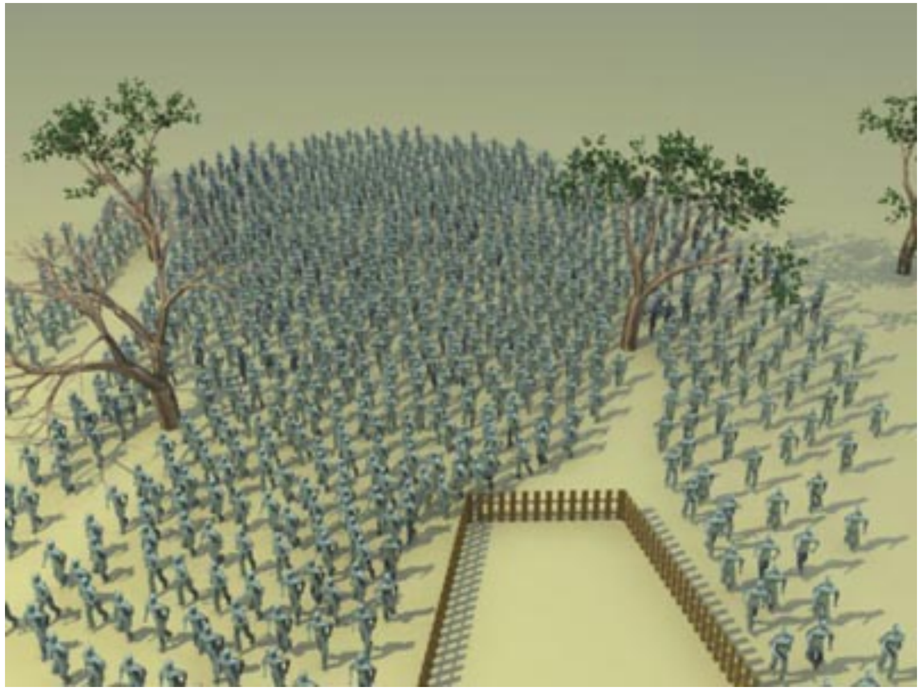
Ray Intersection

- Ray Intersection
  - Sphere
  - Triangle
  - Box
  - Scene
- Ray Intersection Acceleration
  - Bounding volumes
  - Uniform grids
  - Octrees
  - BSP trees

Ray Intersection Acceleration

- What if there are a lot of nodes?

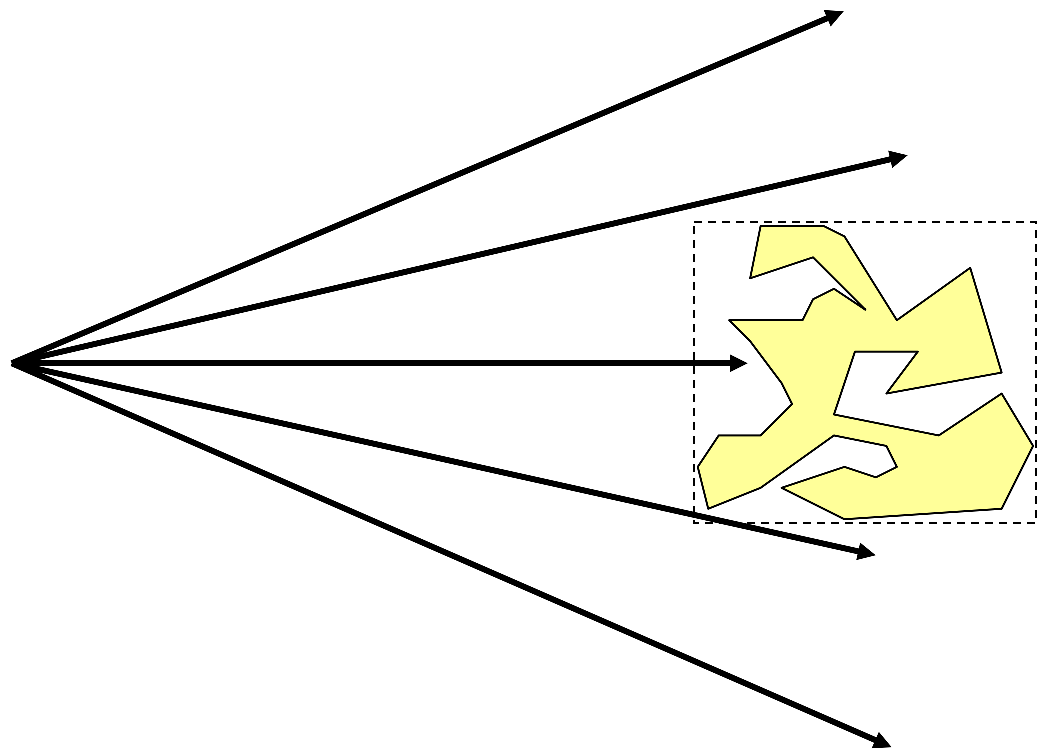




http://www.3dm3.com

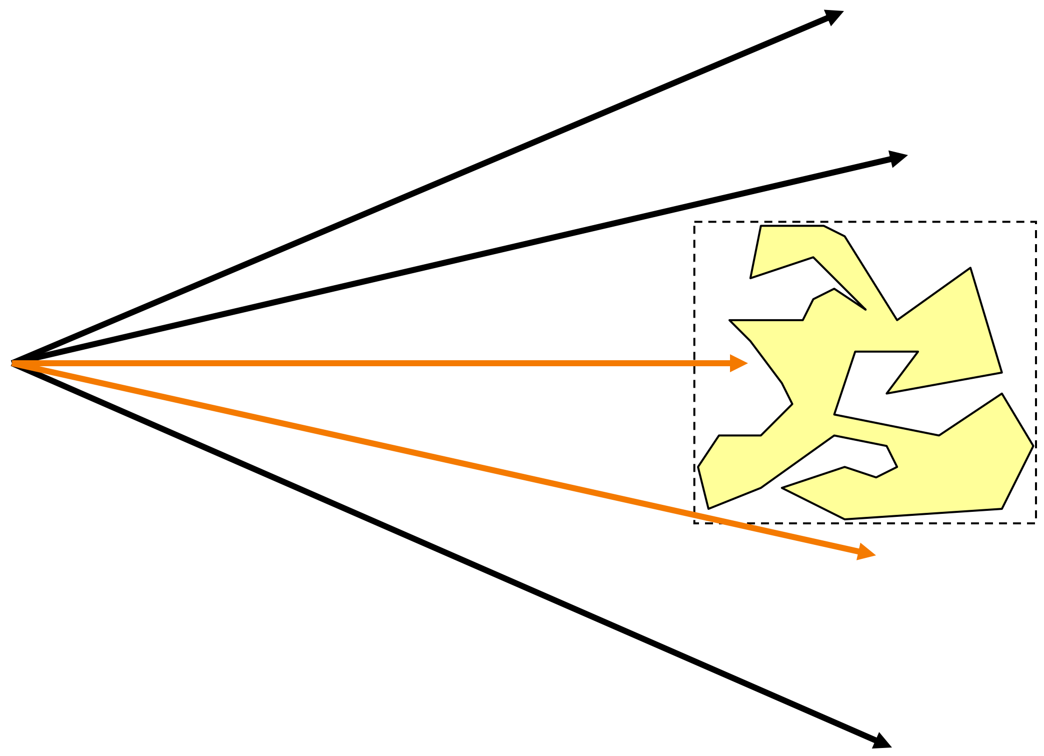
Bounding Volumes

- Check for intersection with simple bounding volume first



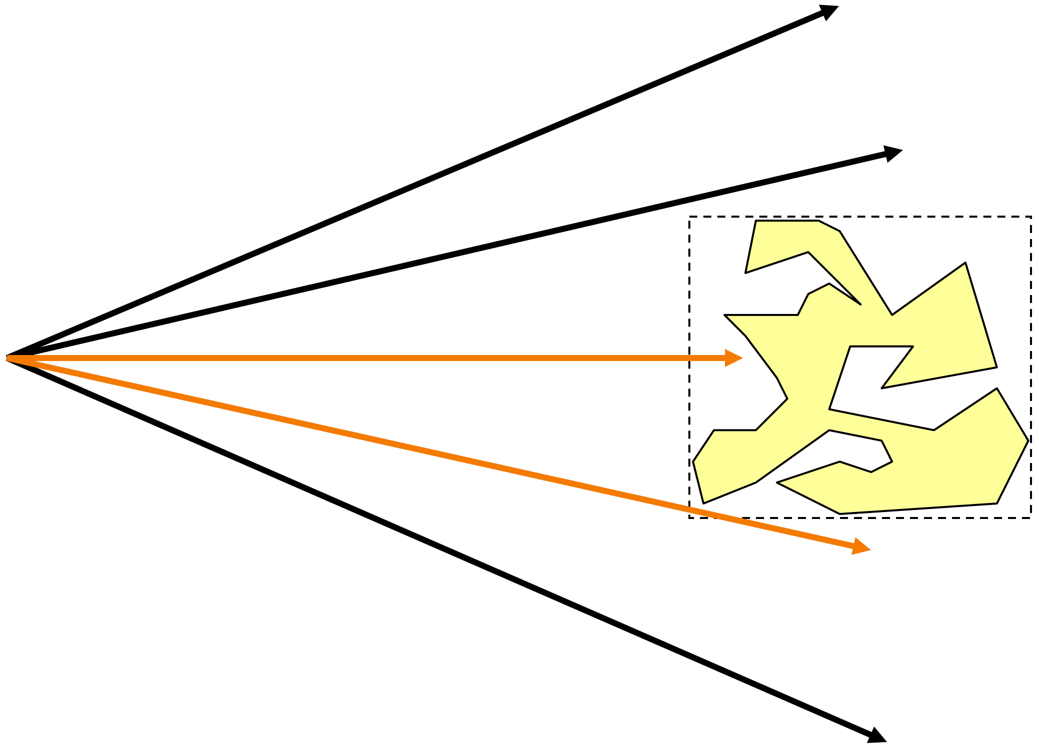
Bounding Volumes

- Check for intersection with bounding volume first



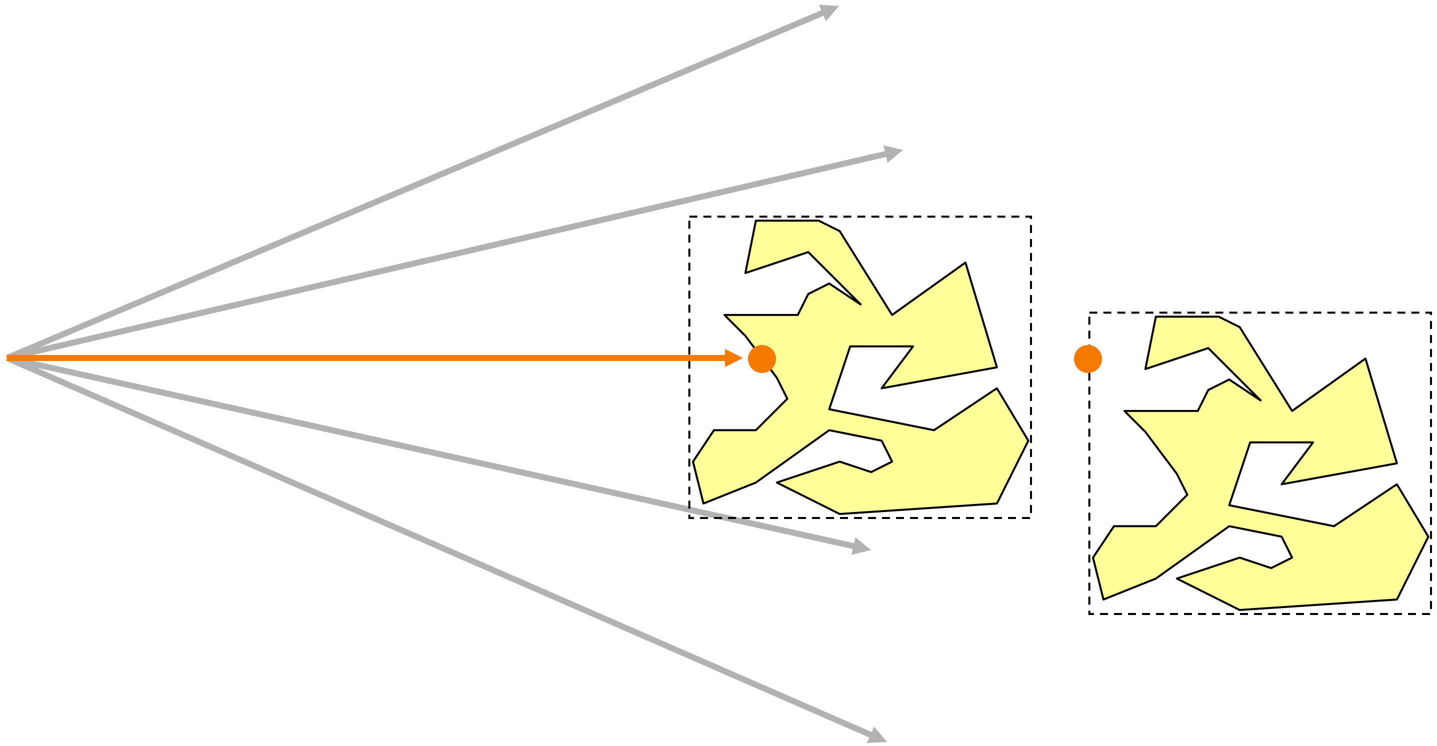
Bounding Volumes

- Check for intersection with bounding volume first
  - If ray doesn't intersect bounding volume, then it can't intersect its contents



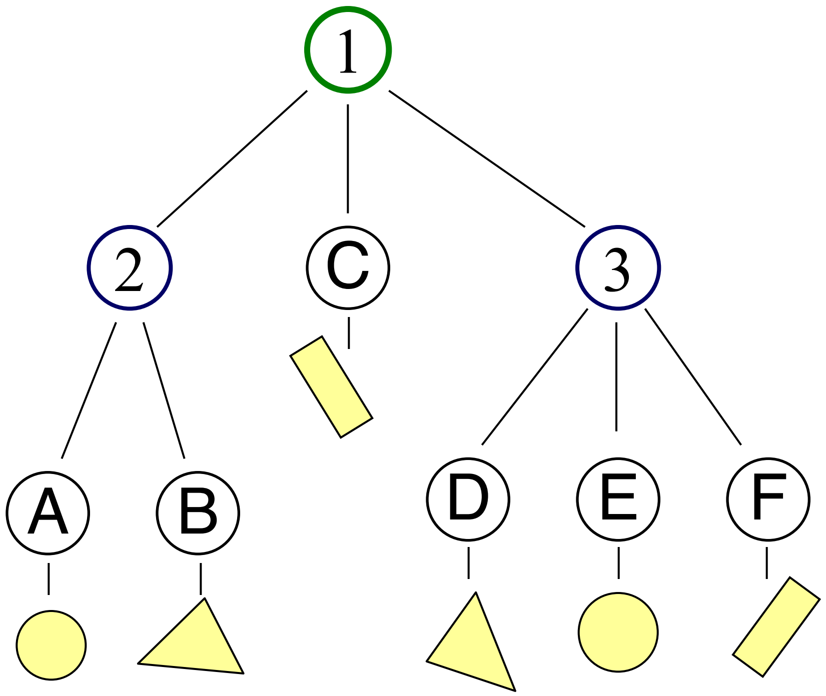
Bounding Volumes

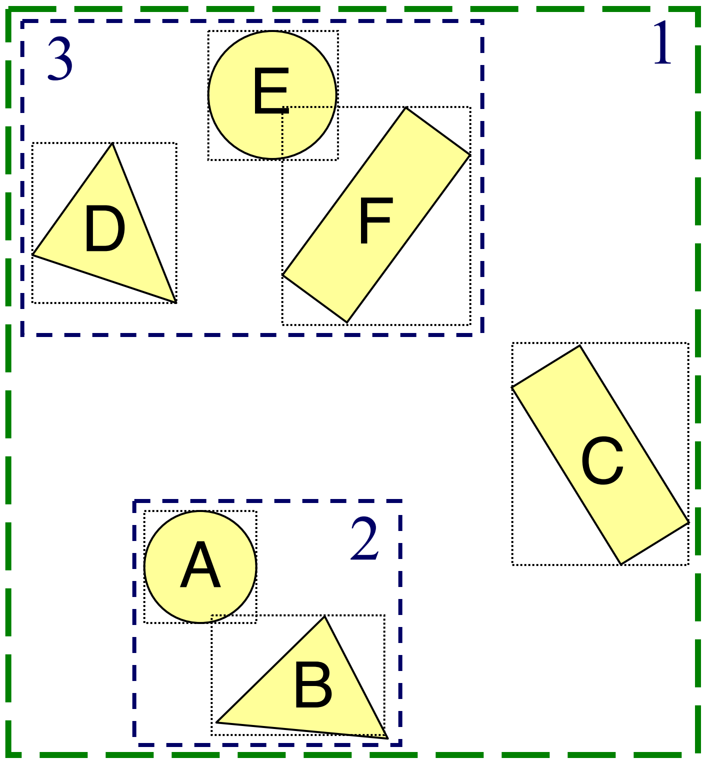
- Check for intersection with bounding volume first
    - If already found a primitive intersection closer than intersection with bounding box, then skip checking contents of bounding box



Bounding Volume Hierarchies

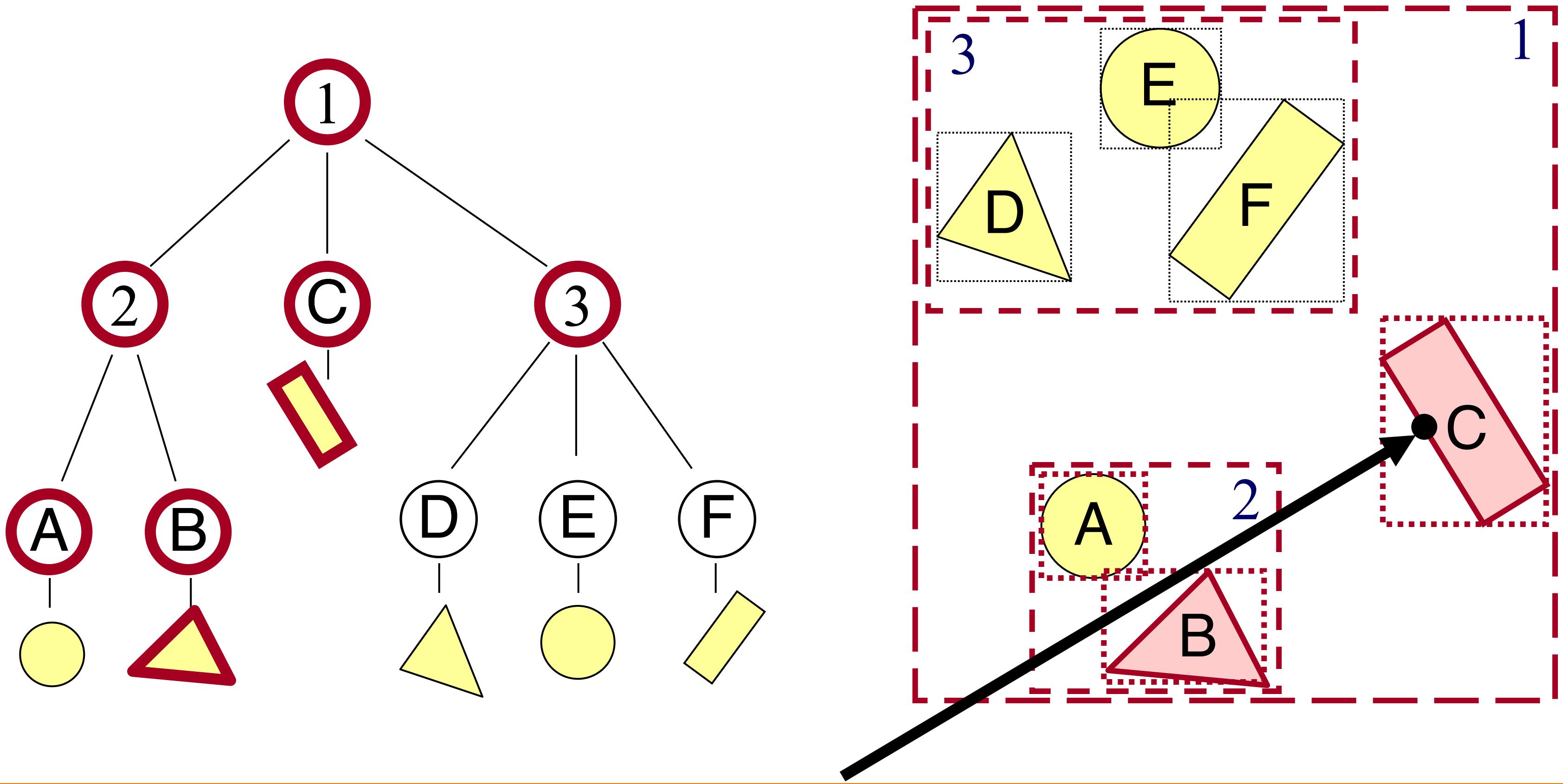
- Scene graph has hierarchy of bounding volumes
  - Bounding volume of interior node contains all children





Bounding Volume Hierarchies

- Checking bounding volumes hierarchically (within each node) can greatly accelerate ray intersection

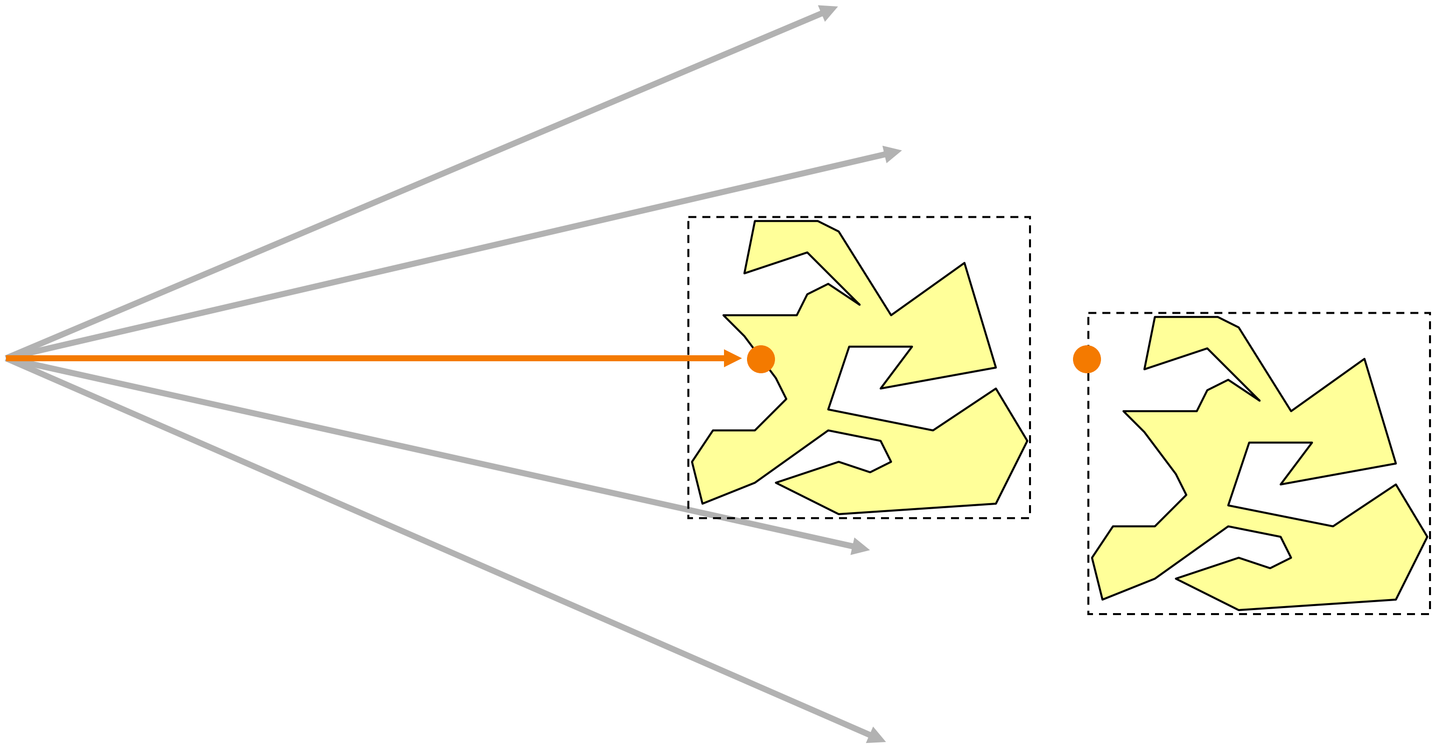


Bounding Volume Hierarchies

```
R3Intersection ComputeIntersection(R3Scene *scene, R3Node *node, R3Ray *ray)
{
    // Transform ray by inverse of node's transformation
    // Check for intersection with shape

    // Check for intersection with  children nodes
    for each child node
        // Check for intersection with child bounding box first
        bbox_intersection = Intersect child's bounding box with ray
        if (bbox_intersection is a miss or further than closest_intersection) continue

        // Check for intersection with child contents
        child_intersection = ComputeIntersection(scene, child, ray);
        if (child_intersection is a hit and is closer than closest_intersection)
            closest_intersection = child_intersection;

    // Transform intersection by node's transformation
    // Return closest intersection in tree rooted at this node
}
```

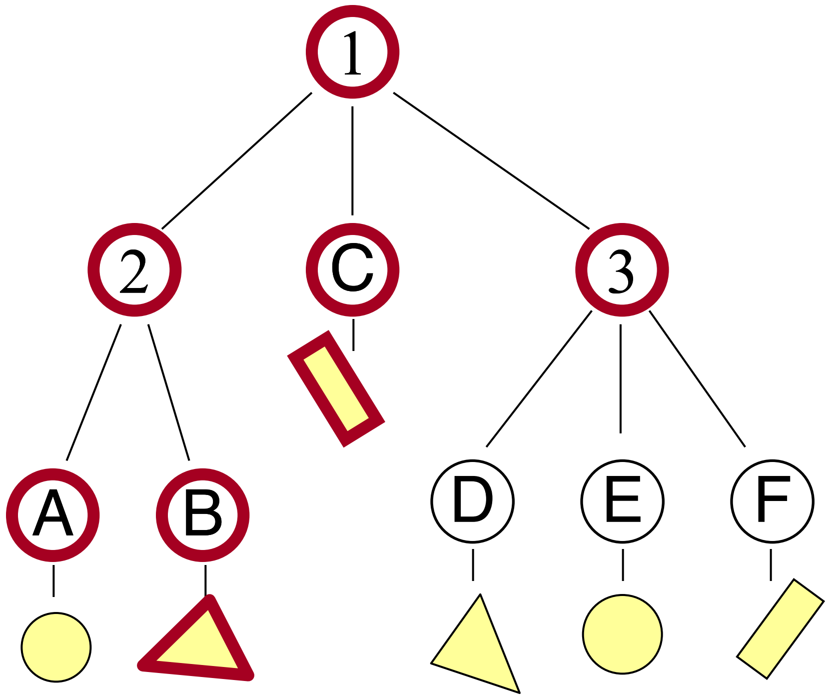
Sort Bounding Volume Intersections

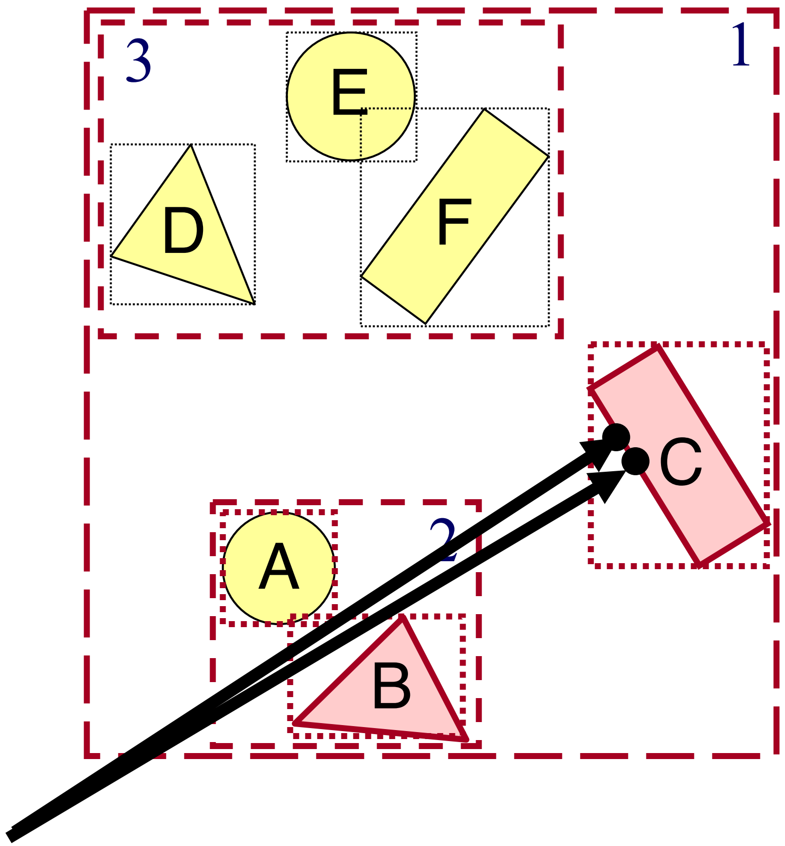
- Sort child bounding volume intersections and then visit child nodes in front-to-back order
- Why?



Cache Node Intersections

- For each node, store closest child intersection from previous ray and check that node first





Bounding Volumes

- Common primitives are:
  - Axis-aligned bounding box
  - Sphere
- What are the tradeoffs?
  - Sphere has simple/efficient intersection code
  - Bounding box is generally “tighter”

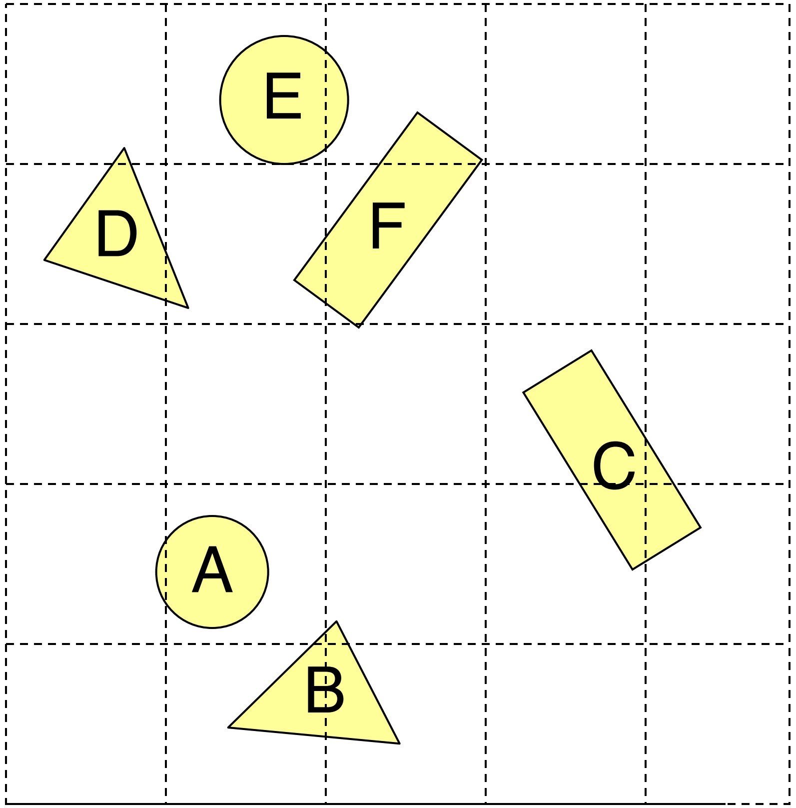
Ray Intersection

- Ray Intersection
  - Sphere
  - Triangle
  - Box
  - Scene

- Ray Intersection Acceleration
  - Bounding volumes
  - ➢ Uniform grids
  - Octrees
  - BSP trees

Uniform Grid

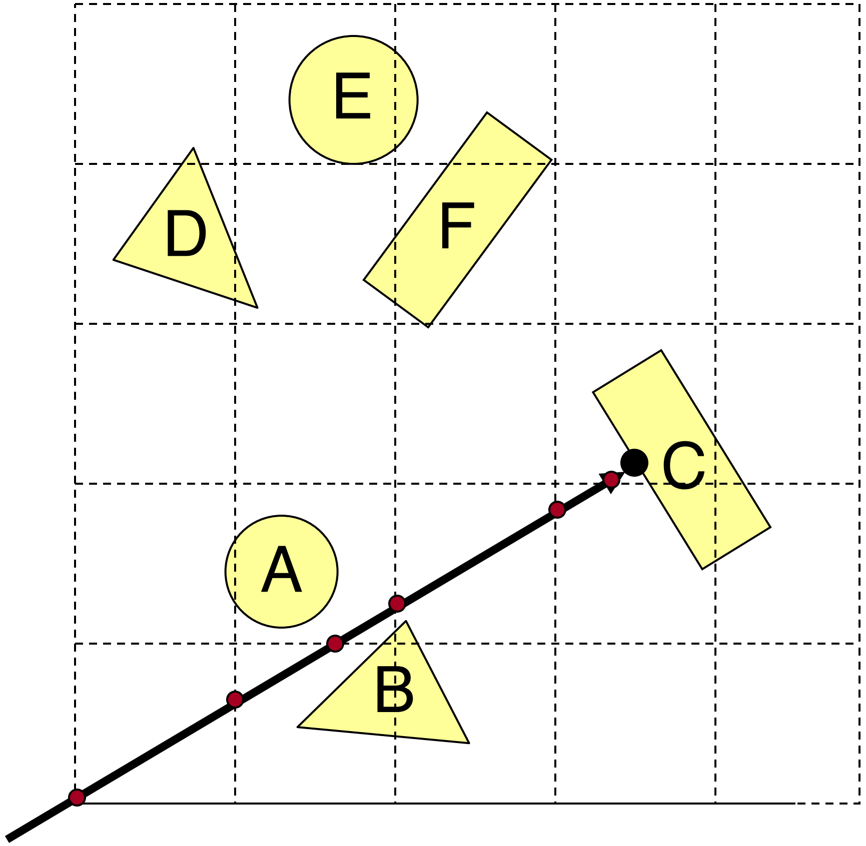
- Construct uniform grid over scene
  - Index primitives according to overlaps with grid cells



Uniform Grid

- Trace rays through grid cells
  - Fast
  - Incremental

Only check primitives in intersected grid cells

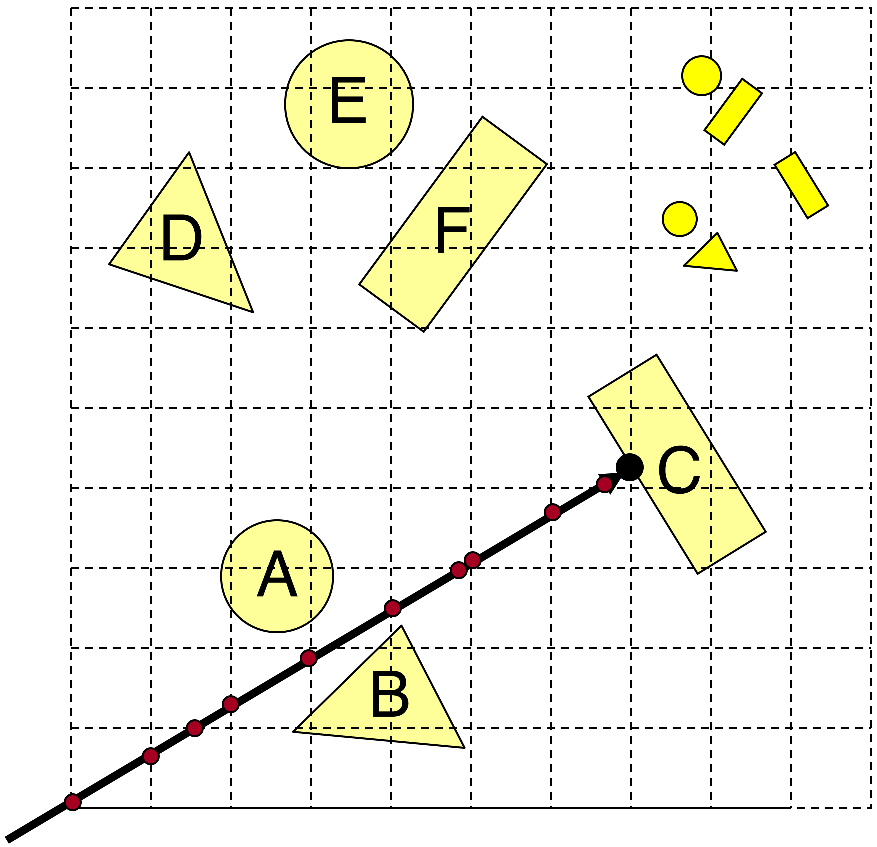


Uniform Grid

- Potential problem:
  - How choose suitable grid resolution?

Too little benefit
if grid is too coarse

Too much cost
if grid is too fine



Ray Intersection

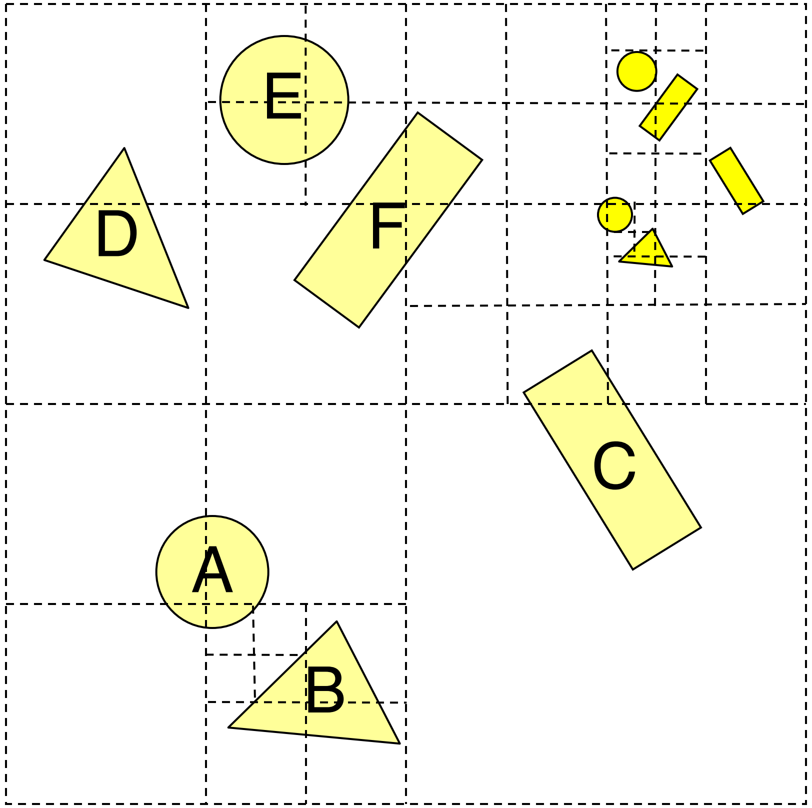
- Ray Intersection
  - Sphere
  - Triangle
  - Box
  - Scene

- Ray Intersection Acceleration
  - Bounding volumes
  - Uniform grids
  - ➢ Octrees
  - BSP trees

Octree

- Construct adaptive grid over scene
  - Recursively subdivide box-shaped cells into 8 octants
  - Index primitives by overlaps with cells

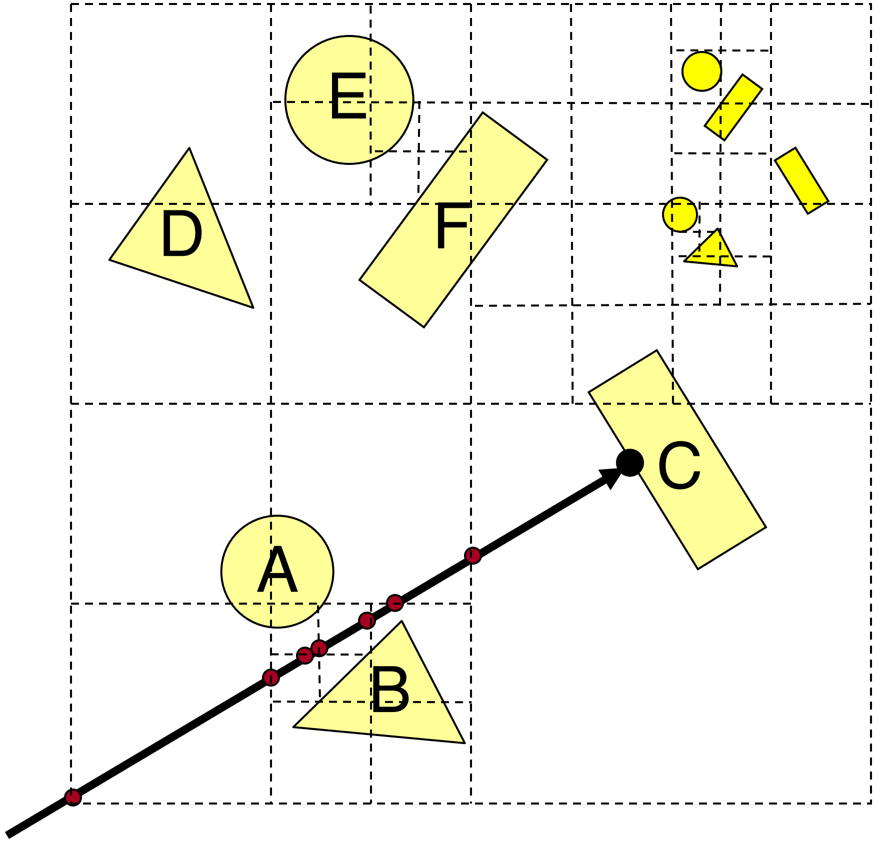
Generally fewer cells



Octree

- Trace rays through neighbor cells
  - Fewer cells

Trade-off fewer cells for more expensive traversal



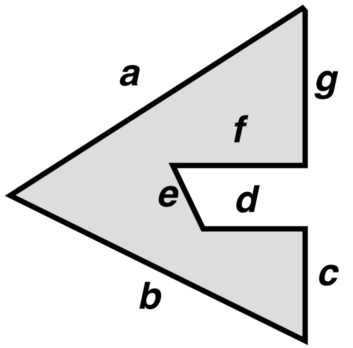
Ray Intersection

- Ray Intersection
  - Sphere
  - Triangle
  - Box
  - Scene

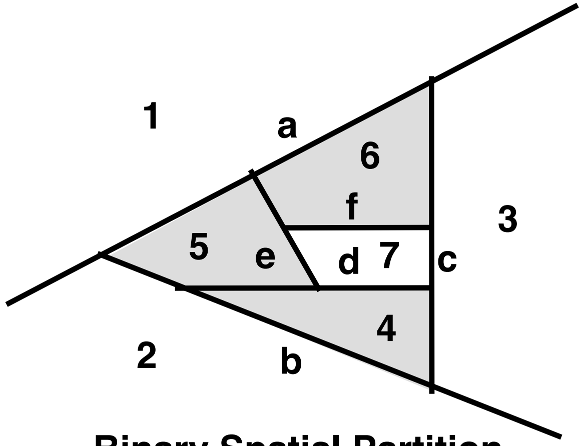
- Ray Intersection Acceleration
  - Bounding volumes
  - Uniform grids
  - Octrees
  - BSP trees

Binary Space Partition (BSP) Tree

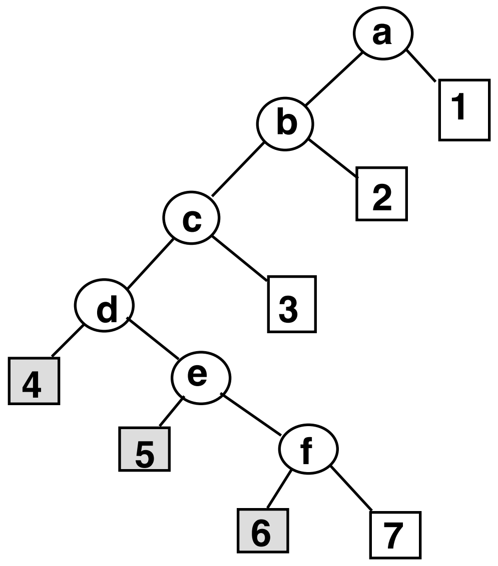
- Recursively partition space by planes
  - BSP tree nodes store partition plane and set of polygons lying on that partition plane
  - Every part of every polygon lies on a partition plane



Object



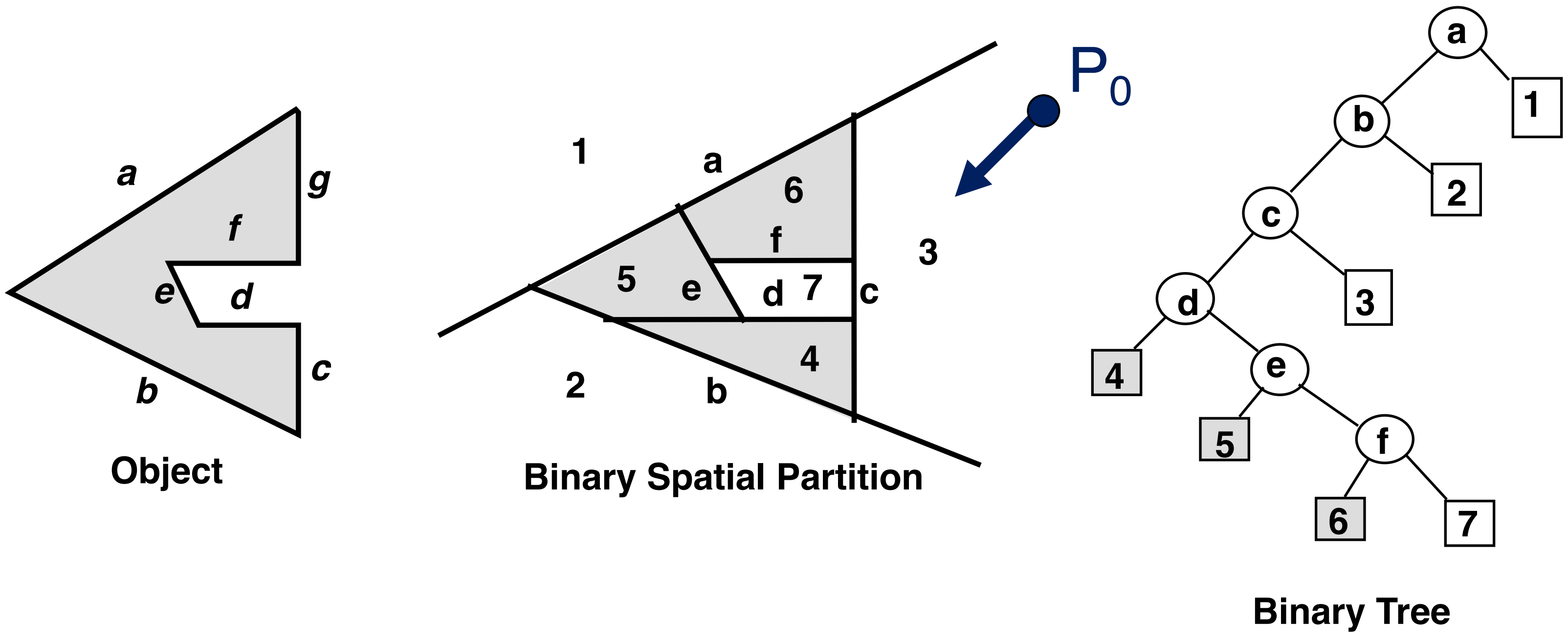
Binary Spatial Partition



Binary Tree

Binary Space Partition (BSP) Tree

- Traverse nodes of BSP tree front-to-back
  - Visit halfspace (child node) containing P_0
  - Intersect polygons lying on partition plane
  - Visit halfspace (other child node) not containing P_0



Binary Space Partition (BSP) Tree

```
R3Intersection
ComputeBSPIntersection(R3Ray *ray, BspNode *node, double min_t, double max_t)
{
    // Compute parametric value of ray-plane intersection
    t = ray parameter for intersection with split plane of node
    if  (t < min_t) || (t < max_t)) return no_intersection;

    // Compute side of partition plane that contains ray start point
    int side = (SignedDistance(node->plane, ray.Start()) < 0) ? 0 : 1;
    intersection1 = ComputeBSPIntersection(ray, node->child[side],  min_t, t);




}
```

Binary Space Partition (BSP) Tree

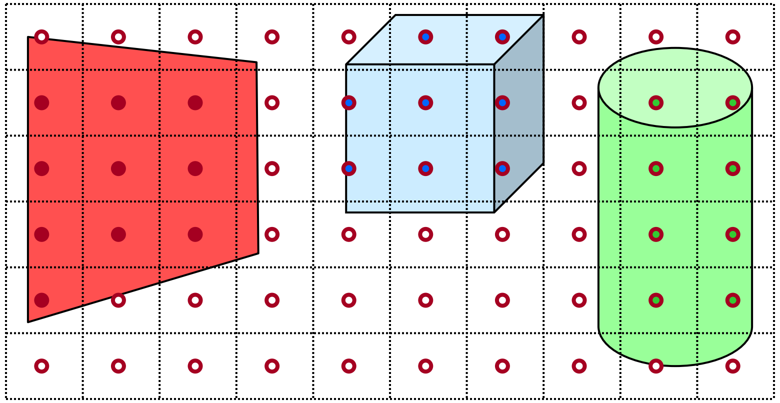
```
R3Intersection
ComputeBSPIntersection(R3Ray *ray, BspNode *node, double min_t, double max_t)
{
    // Compute parametric value of ray-plane intersection
    t = ray parameter for intersection with split plane of node
    if  (t < min_t) || (t < max_t)) return no_intersection;

    // Compute side of partition plane that contains ray start point
    int side = (SignedDistance(node->plane, ray.Start()) < 0) ? 0 : 1;
    intersection1 = ComputeBSPIntersection(ray, node->child[side],  min_t, t);
    if (intersection1 is a hit) return intersection1;
    intersection2 = ComputePolygonsIntersection(ray, node->polygons);
    if (intersection2 is a hit) return intersection2;


}
```

Binary Space Partition (BSP) Tree

```
R3Intersection
ComputeBSPIntersection(R3Ray *ray, BspNode *node, double min_t, double max_t)
{
    // Compute parametric value of ray-plane intersection
    t = ray parameter for intersection with split plane of node
    if  (t < min_t) || (t < max_t)) return no_intersection;

    // Compute side of partition plane that contains ray start point
    int side = (SignedDistance(node->plane, ray.Start()) < 0) ? 0 : 1;
    intersection1 = ComputeBSPIntersection(ray, node->child[side],  min_t, t);
    if (intersection1 is a hit) return intersection1;
    intersection2 = ComputePolygonsIntersection(ray, node->polygons);
    if (intersection2 is a hit) return intersection2;
    intersection3 = ComputeBSPIntersection(ray, node->child[1-side], t, max_t);
    return intersection 3;
}
```

Other Accelerations

- Screen space coherence – check > 1 ray at once
  - Beam tracing
  - Pencil tracing
  - Cone tracing



- Memory coherence
  - Large scenes
- Parallelism
  - Ray casting is “embarrassingly parallelizable”
  - Assignment 3 (raytracer) runs program per-pixel
- etc.

Acceleration

- Intersection acceleration techniques are important
  - Bounding volume hierarchies
  - Spatial partitions
- General concepts
  - Sort objects spatially
  - Make trivial rejections quick
  - Perform checks hierarchically
  - Utilize coherence when possible

Expected time is sub-linear in number of primitives

Summary

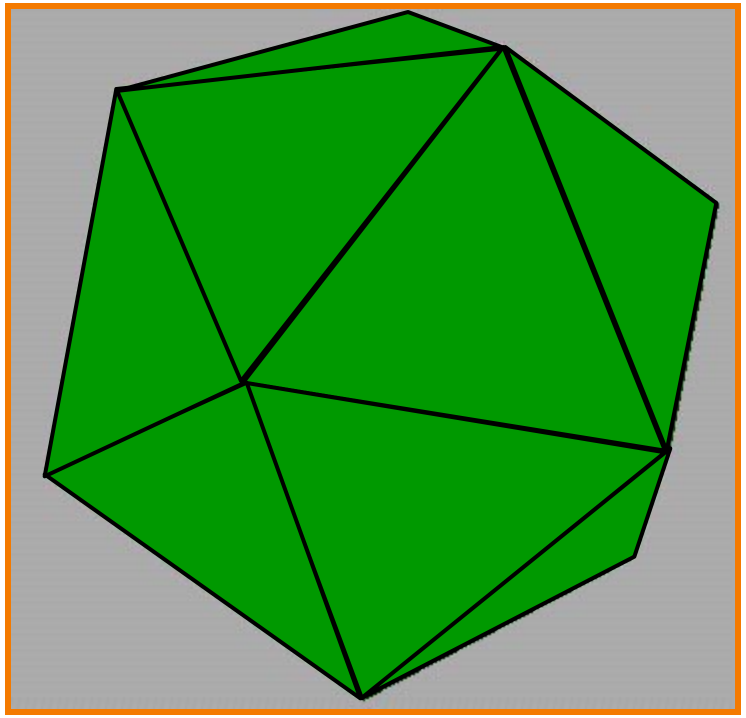
- Writing a simple ray casting renderer is easy
  - Generate rays
  - Intersection tests
  - Lighting calculations

```
R2Image *RayCast(R3Scene *scene, int width, int height)
{
    R2Image *image = new R2Image(width, height);
    for (int i = 0; i < width; i++) {
        for (int j = 0; j < height; j++) {
            R3Ray ray = ConstructRayThroughPixel(scene->camera, i, j);
            R3Rgb radiance = ComputeRadiance(scene,  &ray);
            image->SetPixel(i, j, radiance);
        }
    }
    return image;
}
```

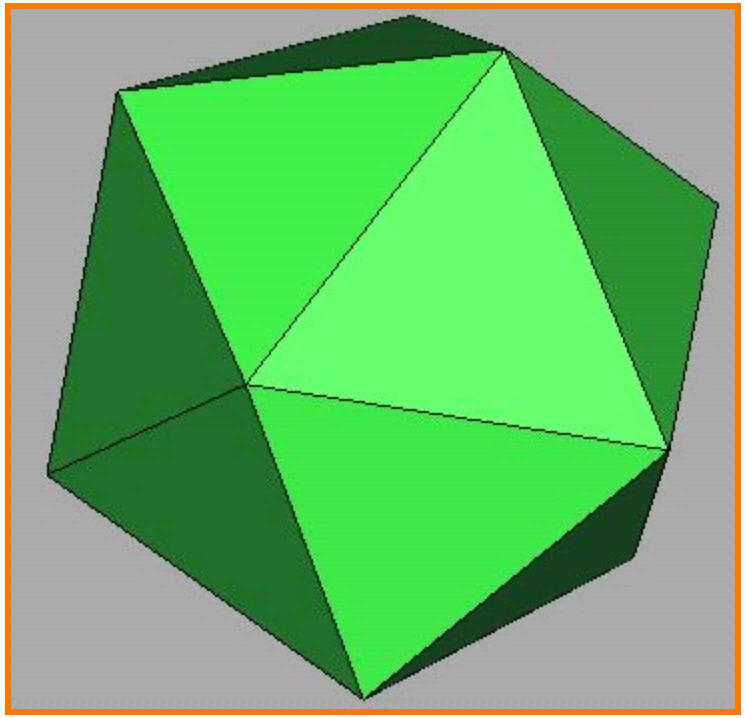
Heckbert's Business Card Ray Tracer

```
typedef struct{double x,y,z}vec;vec U,black,amb={.02,.02,.02};struct sphere{ vec cen,color;
double rad,kd,ks,kt,kl,ir}*s,*best,sph[]={0.,6.,.5,1.,1.,1.,.9, .05,.2,.85,0.,1.7,-1.,8.,-.5,1.,.5,.2,1.,
.7,.3,0.,.05,1.2,1.,8.,-.5,.1,.8,.8, 1.,.3,.7,0.,0.,1.2,3.,-6.,15.,1.,.8,1.,7.,0.,0.,0.,.6,1.5,-3.,-3.,12.,
.8,1., 1.,5.,0.,0.,0.,.5,1.5,};yx;double u,b,tmin,sqrt(),tan();double vdot(A,B)vec A ,B;{return A.x
*B.x+A.y*B.y+A.z*B.z;}vec vcomb(a,A,B)double a;vec A,B;{B.x+=a* A.x;B.y+=a*A.y;B.z+=a*A.z;
return B;}vec vunit(A)vec A;{return vcomb(1./sqrt( vdot(A,A)),A,black);}struct sphere*intersect
(P,D)vec P,D;{best=0;tmin=1e30;s= sph+5;while(s-->sph)b=vdot(D,U=vcomb(-1.,P,s->cen)),
u=b*b-vdot(U,U)+s->rad*s ->rad,u=u>0?sqrt(u):1e31,u=b-u>1e-7?b-u:b+u,tmin=u>=1e-7&&
u<tmin?best=s,u: tmin;return best;}vec trace(level,P,D)vec P,D;{double d,eta,e;vec N,color;
struct sphere*s,*l;if(!level--)return black;if(s=intersect(P,D));else return amb;color=amb;eta=
s->ir;d= -vdot(D,N=vunit(vcomb(-1.,P=vcomb(tmin,D,P),s->cen )));if(d<0)N=vcomb(-1.,N,black),
eta=1/eta,d= -d;l=sph+5;while(l-->sph)if((e=l ->kl*vdot(N,U=vunit(vcomb(-1.,P,l->cen))))>0&&
intersect(P,U)==l)color=vcomb(e ,l->color,color);U=s->color;color.x*=U.x;color.y*=U.y;color.z
*=U.z;e=1-eta* eta*(1-d*d);return vcomb(s->kt,e>0?trace(level,P,vcomb(eta,D,vcomb(eta*d-
sqrt (e),N,black))):black,vcomb(s->ks,trace(level,P,vcomb(2*d,N,D)),vcomb(s->kd, color,vcomb
(s->kl,U,black))));}main(){printf("%d %d\n",32,32);while(yx<32*32) U.x=yx%32-32/2,U.z=32/2-
yx++/32,U.y=32/2/tan(25/114.5915590261),U=vcomb(255., trace(3,black,vunit(U)),black),printf
("%.0f %.0f %.0f\n",U);}/*minray!*/
```

Next Time is Illumination!



Without Illumination



With Illumination