# ASTR2013 Problem Set 4 - due **27 Sep 2019**

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1. (Based on Textbook Q5.3) A new star lights up inside a cloud of atomic hydrogen with a constant number density of $n$ atoms per unit volume. The star emits ionizing photons at a rate of $Q_*$ photons per unit time. The ionizing photons begin carving out a growing "Strömgren sphere" of ionized gas inside the neutral gas.

   (a) At a distance $r$ from the star, assuming that the radiation from the star isn't attenuated, what is the flux (photons per unit area per unit time) of ionizing photons?

   (b) At this location, what is the rate of photons ionizing the H atoms, writing the ionization cross-section as $\sigma_{\rm ion}$. With some representative numbers of $\sigma_{\rm ion} = 10^{-18}\,{\rm cm}^2$, $Q_* = 10^{48}$ ionizing photons/s and $r = 0.1\,{\rm pc}$, what is the typical time that it takes a H atom to be ionized?

   (c) When electrons and protons recombine, the most important velocity is the electron velocity. What is this typical velocity in any direction for a temperature of 7000 K? (you can assume that the kinetic energy per degree of freedom is 0.5 $k_B T$).

   (d) Assume that the recombination cross-section for an electron-proton collision is the size of an atom, i.e. (1 Angstrom)$^2 \approx 10^{-16}\,{\rm cm}^2$. With a gas density of $10\,{\rm cm}^{-3}$, what is the recombination timescale? [hint - the key parameters are cross section, velocity and gas density]

   (e) What is the equilibrium radius of the Strömgren sphere under these assumptions?

2. (Based on Textbook Q7.3) In the solar neighborhood, the Milky Way has a flat rotation curve, with $v(r) = v_c$, where $v_c$ is a constant, implying a mass density profile of $\rho(r) \sim r^{-2}$.

   (a) Assume there is a cutoff radius, R, beyond which the mass density is zero. Prove that the velocity of escape from the galaxy from any radius $r < R$ is:

   $$v_{\rm esc}^2 = 2v_c^2 \left(1 + \ln \frac{R}{r}\right) \tag{1}$$

   (b) Based on this density profile, a value of $v_c = 220\,{\rm km/s}$ a value of $R$ more than 16 kpc, what is the total mass of the Galaxy interior to $r = 16\,{\rm kpc}$?