Australian National University

ASTR2013 – *Foundations of Astrophysics*

Week 2: Physics of Stars

Following Dan Maoz – Astrophysics in a Nutshell

Mike Ireland

Recap: Specific Intensity to Flux

- If we have a specific intensity of an object being viewed at near-normal incidence to an aperture, simply multiply by the solid angle being integrated over to get flux. i.e.

$$F_\nu = \int I_\nu d\Omega \approx I_\nu \Delta\Omega$$

- If we have an isotropic radiation field, e.g. a black-body, then we can find the specific intensity from the flux.
- See derivation PDF on wattle if the 1-liner in the textbook was too brief or you didn't follow the tutorial on the board.

$$I_\nu = F_{\nu,\mathrm{isotropic}}/\pi$$

$$f_\nu = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_\nu \cos\theta \sin\theta d\theta\, d\phi = I_\nu 2\pi \frac{1}{2} = \pi I_\nu = \frac{c}{4} u_\nu = \frac{2\pi h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}. \qquad (2.5)$$

Energy Density and Specific Intensity

- We can also think of a radiation field as local energy flow at the speed of light.
- Then I_ν/c is the energy density of radiation for a small solid angle $d\Omega$, and we can find the total energy density by:

$$u_\nu = \frac{1}{c}\int I_\nu d\Omega$$

(I find this more intuitive than the textbook's differential form)

- For an *isotropic* radiation field:

$$u_\nu = \frac{4\pi}{c} I_\nu$$

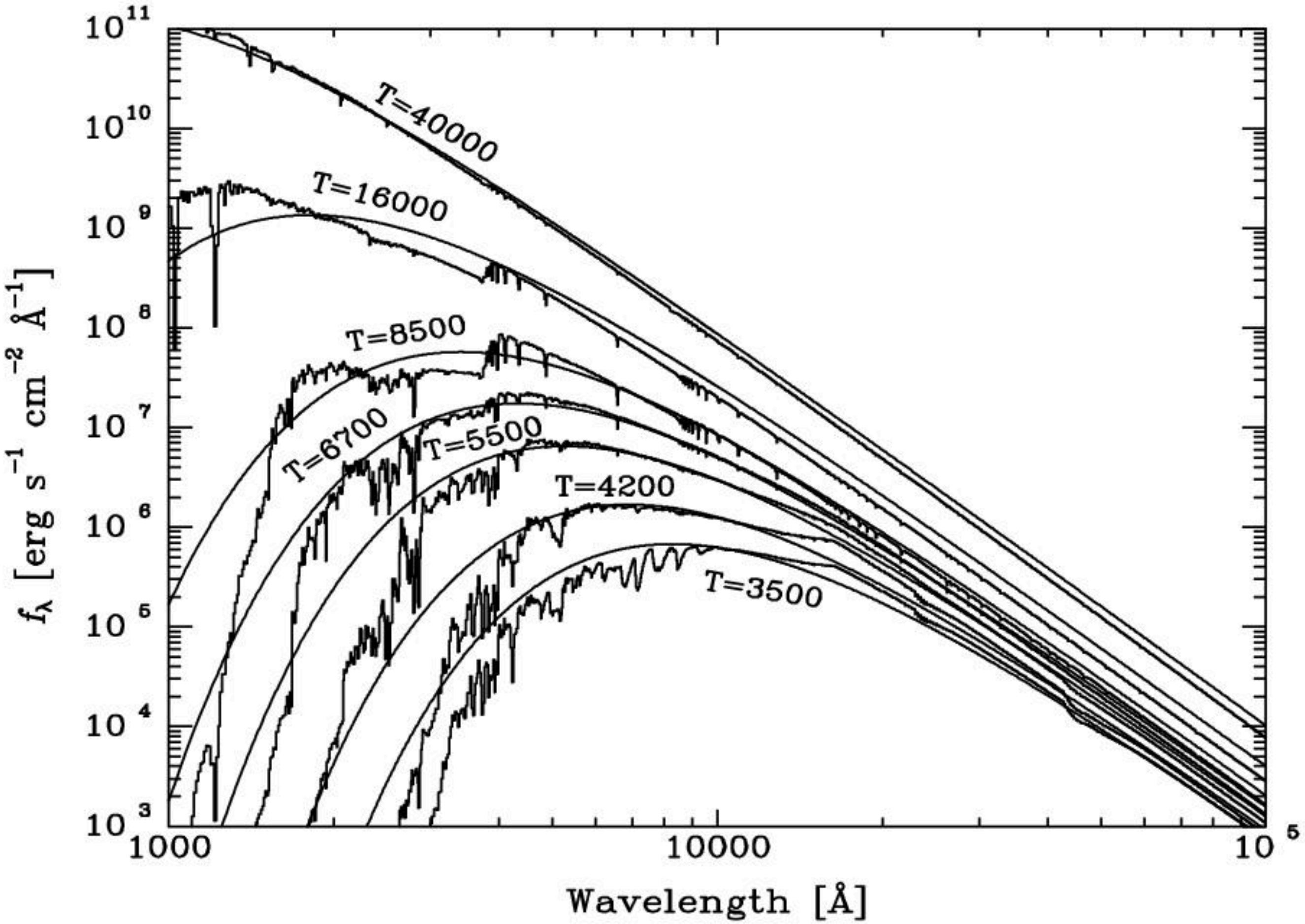
Week 2 Summary

Textbook: Sections Finishing 2.2, 2.3, 3.1, 3.8, 3.9 (not including Gamow energy).

1. Stellar Spectral Types
2. Orbits, Kepler's Law and Measuring Stellar Masses.
3. The Main Sequence and the HR Diagram.
4. Hydrostatic Equilibrium and the Virial Theorem.
5. The Kelvin-Helmholtz Timescale.

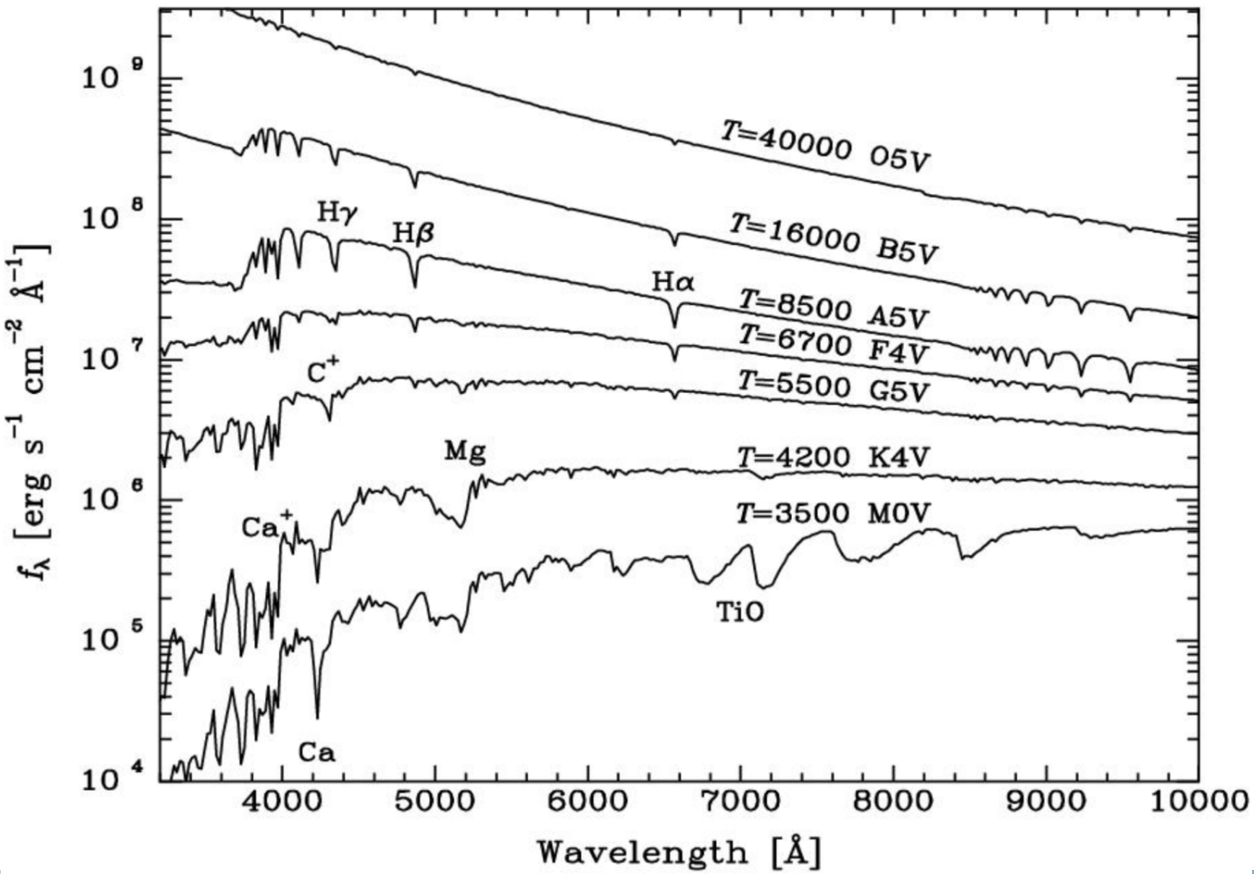
Stellar Spectral Types

Plots of flux density (spectra) broadly resemble blackbody curves, but have additional spectral features due to atoms/molecules in cool upper layers absorbing light.

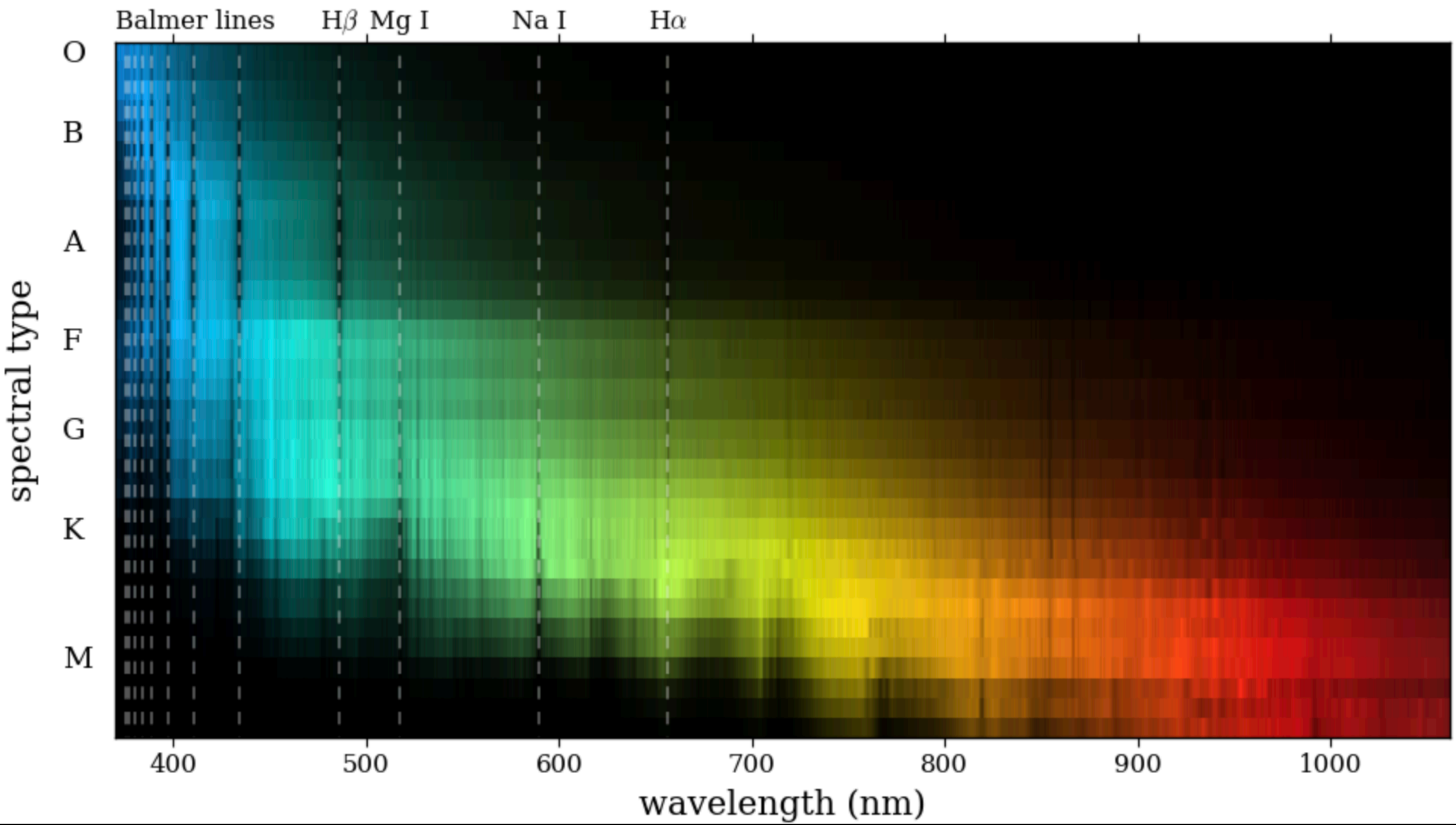


Stellar Spectral Types

Historically, spectral types were created before there was a deep understanding of what caused the spectral lines. My favourite modern reference on spectral type conversions is Erik Mamajek's home page (mostly in refereed papers): http://www.pas.rochester.edu/~emamajek/EEM_dwarf_UBVIJHK_colors_Teff.txt



Stellar Spectral Types

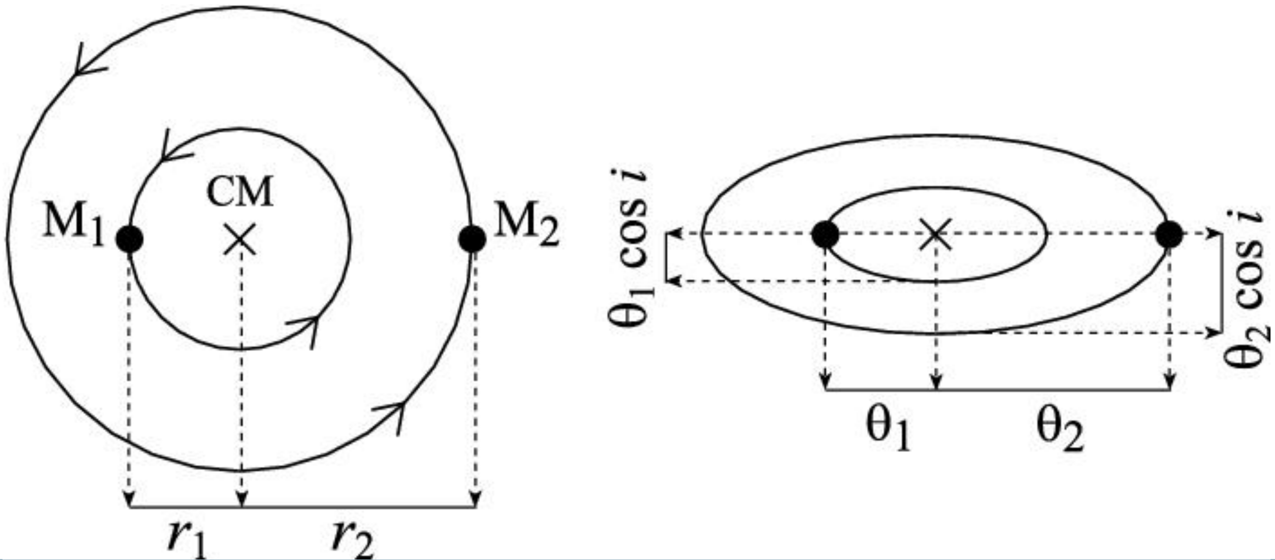


Kepler's Law and Mass Measurement

- Newton's law of Gravitation for circular orbits can easily be used to derive Kepler's law:

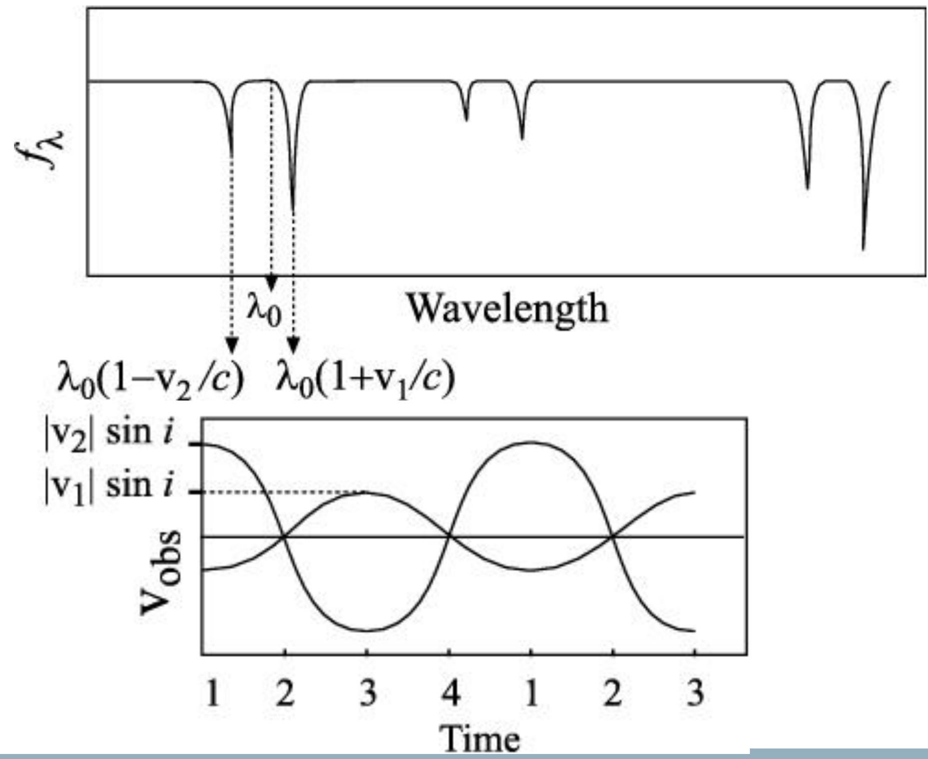
$$\omega^2 = \frac{G(M_1 + M_2)}{a^3}$$

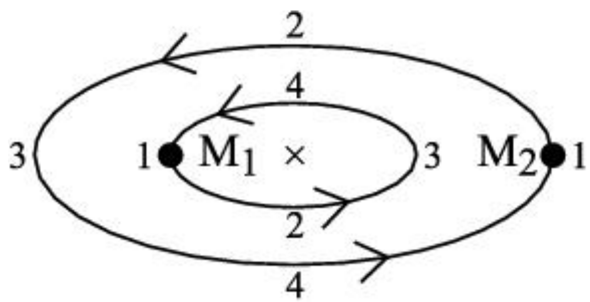
- Here a is the radius of the circular orbit. It turns out that this can be generalised for elliptical orbits, with a being the semi-major axis of an elliptical orbit, with the center of mass at the focus.



Kepler's Law and Mass Measurement

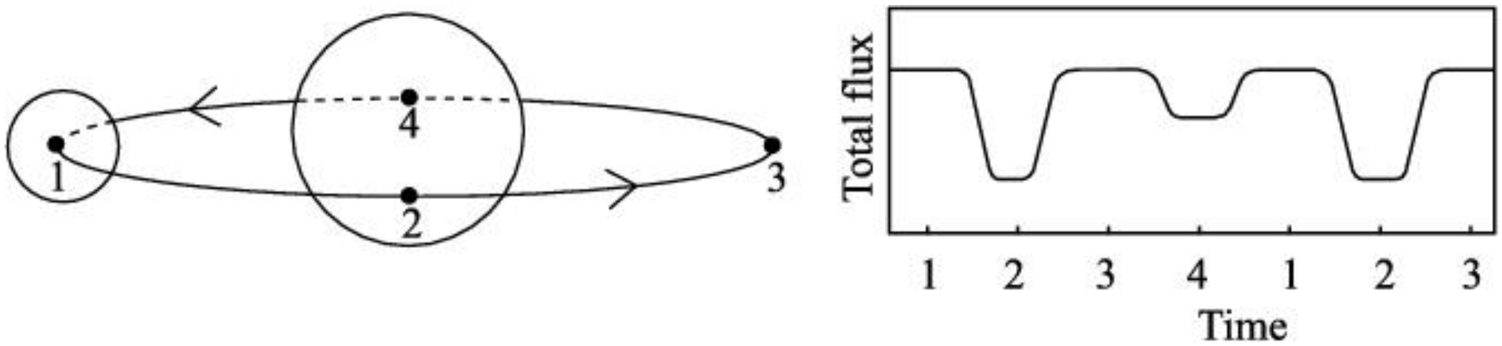
- In *Solar-Units*, we can write Kepler's law as: $\left(\frac{M_{\rm tot}}{M_\odot}\right)\left(\frac{T}{1\,{\rm yr}}\right)^2 = \left(\frac{a}{1\,{\rm au}}\right)^3$
- In practice, we can measure all orbital parameters other than inclination and position angle on the sky (e.g. orbit oriented North or East?) using Doppler-shifts and a *spectroscopic binary orbit.*
- Spectroscopic binaries can be *Single-lined* or *Double-lined,* depending on how bright the *secondary* is.





Measuring Inclination

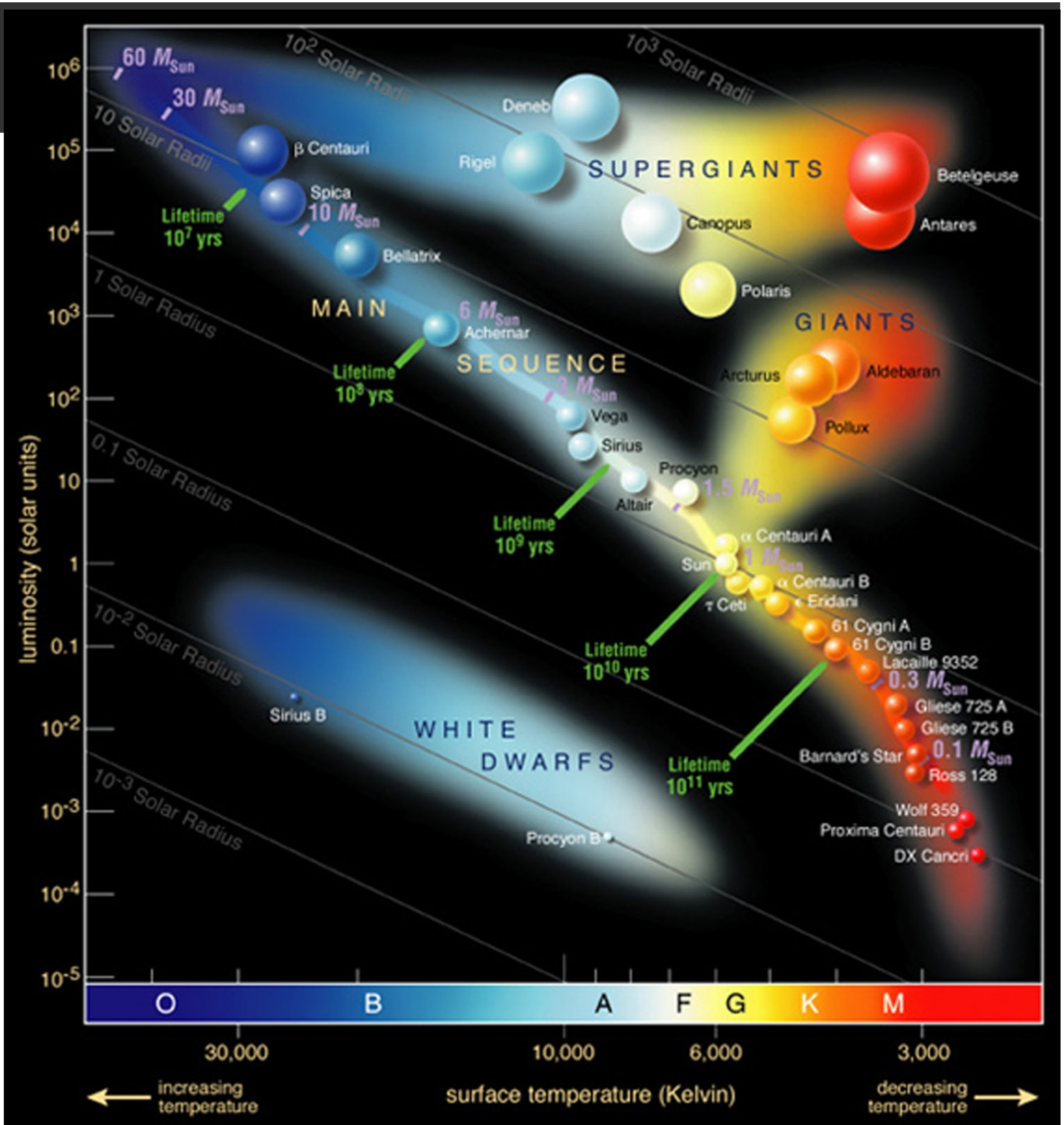
- Integrating the velocity in a spectroscopic binary gives the semi major axis, multiplied by sin(*i*).
- We can get the inclination *i* by visually resolving the orbit (e.g. with adaptive optics), or by observing a lucky orientation – an *eclipsing binary*.



The Main Sequence

Without any knowledge of stellar physics, we can plot luminosity and temperature, adding mass measurements, and observe that most stars are on the *Main Sequence*, although there are many exceptions.

Especially when plotted as colour and magnitude (week 6), this is a Hertzsprung-Russell (HR) diagram.



Hydrostatic Equilibrium

- Like the earth's atmosphere, stars are typically in a stable state, governed by *hydrostatic* equilibrium. This is the momentum balance equation – forces on a parcel of gas neither accelerate it upwards or downwards.

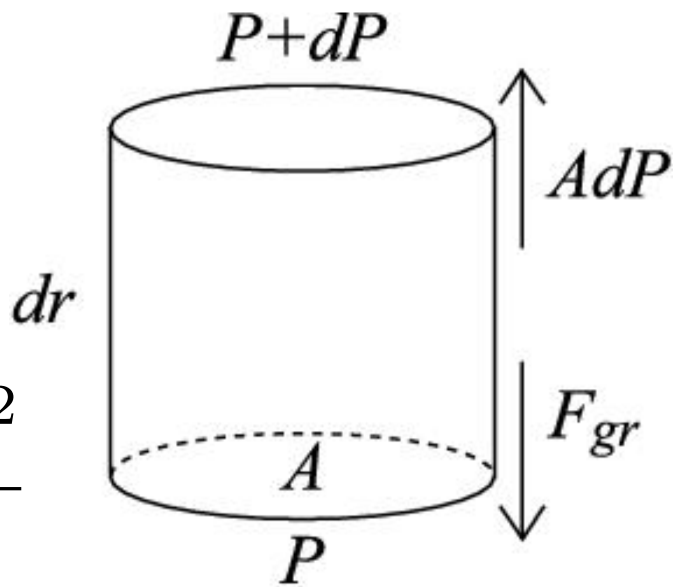
$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}.$$

- We can integrate this equation through the star:

$$\int_0^{r_*} 4\pi r^3 \frac{dP}{dr} dr = -\int_0^{r_*} \frac{GM(r)\rho(r)4\pi r^2 dr}{r}.$$

- LHS: $[P(r)4\pi r^3]_0^{r_*} - 3\int_0^{r_*} P(r)4\pi r^2 dr.$
- RHS: this is the integral of potential energy in shells, resulting in the total gravitational energy E_{gr}.
- Approximating M(r)~r, the RHS is:

$$E_{\mathrm{gr}} \approx \frac{-GM^2}{R}$$



Virial Theorem

- Noting that pressure is zero at the surface, denoting the total star volume by *V*, we get an expression for the mean stellar pressure (by volume) as a function of the gravitational energy:

$$\bar{P} = -\frac{1}{3}\frac{E_{\rm gr}}{V}.$$

- From the ideal gas law and and internal energy of a monatomic gas, we have:

$$PV = NkT,$$
$$E_{\rm th} = \tfrac{3}{2}NkT.$$
$$P = \frac{2}{3}\frac{E_{\rm th}}{V},$$

- Applying this over the whole volume of the star, we equate two equations for mean pressure and get:

$$E_{\rm th}^{\rm tot} = -\frac{E_{\rm gr}}{2}$$

The Virial Theorem for Stars (written in 3 forms)

$$E_{\rm gr} = -2E_{\rm th}^{\rm tot}$$

$$E^{\rm tot} = -\frac{1}{2}E_{\rm gr}$$

Stellar Timescales

- The *dynamical timescale* for any body of mass M and radius R is:

$$\tau_{\rm ff} \approx \tau_{\rm dyn} = \sqrt{\frac{R^3}{GM}}$$

~1 hour

$$\propto \bar{\rho}^{-1/2}$$

- This is approximately the *free-fall* timescale, 1/2π times the orbital period of a body almost scraping the surface, or the sound-crossing time.
- We can take the ratio of the *total internal energy* to the *stellar luminosity* to get the Kelvin-Helmholtz timescale:

$$\tau_{\rm KH} \approx \frac{1}{2}\frac{GM^2}{R}\frac{1}{L}$$

~10^7 years

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