ASTR2013 Problem Set 2 - due **16 Aug 2019**

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Certain stars are fully convective. In this case, we can approximate the relationship between pressure and density as:

$$\frac{d\log(P)}{d\log(\rho)} = \gamma \tag{1}$$

The adiabatic index γ is in general a function of thermodynamic variables, e.g. P and ρ, but can be approximated as constant and equal to 5/3 for a gas with a fixed ionisation fraction.

1. For a constant γ, show that $P(\rho)$ obeys a power-law for some constant K:

$$P = K\rho^{\gamma} \tag{2}$$

2. For a fully-convective star where dT/dr is determined by this adiabatic index γ and not radiation, find alternative equations for stellar structure based on the three derivatives dM'/dr', $d\log(\rho)/dr'$ and $d\log(T)/dr'$, where $M' = M/M_{\odot}$ and $r = r/R_{\odot}$. Show all working. [Hint - first convert the dP/dr' equation to $d\log(P)/dr'$]

3. The resulting equations have been coded up into a python script (in raw python and a Jupyter notebook) on the course wattle page. The functions in `saha.py` can be treated as a "black box" unless you are particularly keen, but the functions in `hayashi.py` should be easy to understand. Run the function `convective_star` for a core density of 1 g/cm^3 and a core temperature of 3 million K, with both the default settings and `simplified_EOS=True`. The `simplified_EOS=True` option assumes H and He are fully ionised throughout the star. Plot the mass in solar masses versus radius, and the temperature versus radius. Plot both equation of state (EOS) approximations on the same plot. Zoom in to the near-surface layers. Are the models significantly different? What is the greatest cause of the difference?

4. This star is not realistic in the surface layers, where the radiative flux limited temperature gradient becomes lower than the convective temperature gradient. The opacity in these layers is dominated by the H- ion, which has a complex opacity law well approximated for solar abundances by:

$$\kappa(\rho, T) = 2 \times 10^{-18} T^6 \rho^{0.8} \text{ cm}^2/\text{g}, \tag{3}$$

with T in K and ρ in g/cm^3. Find the temperature at which the atmosphere becomes sufficiently transparent to radiation for convection to stop. You'll have to approximate the star's luminosity as being equal to $4\pi r^2 \sigma_{\rm SB} T^4$, where T is the near-surface layer where convection stops. Express your answer to the nearest 500 K, e.g. by choosing a single layer in the model output. Show working, including the value of $d\log(T)/dr$ that applies when convection stops.

5. Place this star on a HR diagram. If you didn't complete the previous question, assume the effective temperature is 4000 K. Where is the star situated on this diagram? Describe its subsequent evolution over the next GYr.