Combined Reference Notes for All the Lessons

These references notes are provided to help you quickly refer back to points raised in the videos. We supply all of them combined here in pdf format for your convenience, and you can also find them within the individual lessons.

LESSON 1: SIRIUS B

DISCOVERY

In the early 19th century, Bessell used the world's best telescope (designed by Fraunhofer) to discover that the star Sirius was wobbling backwards and forwards every 50 years. He concluded that it must have a massive but invisible companion.

The mass of this companion can be estimated by working out how much centripetal force is needed to make Sirius move in the observed circle. If you observe a star of mass M_1 moving in a circle of radius r_1, then the gravity from its unseen companion (of mass M_2, moving in a circle of radius r_2) must supply the necessary centripetal force. This tells us that:

$$M_2 = \frac{4\pi^2}{GP^2} r_1 (r_1 + r_2)^2$$

where P is the orbital period. We also know that both the star and its unseen companion must be orbiting around their common centre of mass, which tells us that $M_1 r_1 = M_2 r_2$.

If we plot both these curves as a function of r_2 and look for where they intersect, we can determine the mass M_2 of the unseen companion.

In the case of Sirius, this companion has a mass of roughly one solar mass.

IT IS SMALL!

50 years later, the companion (known as Sirius B) was seen for the first time. Curiously, it is around 1000 times fainter than the main star Sirius (now called Sirius A), despite having half its mass. It was initially assumed that its faintness must be due to its cool temperature, but early in the 20th century, spectroscopy showed that Sirius B was actually extremely hot - 2.5 times hotter than Sirius A.

If it was so hot, why was it so small? The only possibility was if it was extremely small. We can estimate its size using the Stefan-Boltzmann equation: $L = A\sigma T^4$, where L is the luminosity (power radiated), A the surface area, $\sigma = 5.67 \times 10^{-8}\mathrm{W\ m^{-2}\ K^{-4}}$ is the Stefan-Boltzmann constant, and T is the temperature (in K). Take the ratio of the luminosities of the two stars, and we find that:

$$r_B = r_A \sqrt{\frac{L_B}{L_A}} \left(\frac{T_A}{T_B}\right)^2$$

which comes out as around 6000 km for Sirius B, i.e. slightly smaller than the Earth! Despite weighing as much as the Sun.

PRESSURE AND GRAVITY

Sirius B is so small and so heavy that the force of gravity near its surface must be enormous: you can work it out using Newton's law of gravity:

$$F = \frac{GMm}{r^2}$$

We can estimate the pressure in the centre of a star using a bit of calculus, if we assume that the density of the star is constant. In this case, we find the pressure gradient by balancing forces on a cylinder of star material (the pressure on the inside must exceed the pressure on the outside by enough to balance gravity), and we find that the pressure gradient is:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{4}{3}\pi r G \rho^2$$

where P is the pressure and ρ the density. If we integrate this inwards, staring at a pressure of zero on the outside of the star, we find that the central pressure is:

$$p_c = \frac{2}{3}\pi G\rho^2 R^2$$

Using this, we see that the pressure in the centre of Sirius B is vastly greater than that in the middle of the Sun.

THE PROBLEMS

All of which leaves us with two problems. Firstly, given that Sirius B is much denser than the Sun, and hotter two, it should be doing nuclear fusion at a much faster rate than the Sun. But it is clearly not - or it would be more luminous than the Sun. So why isn't it undergoing fusion?

Secondly, the gravity on Sirius B is so intense that no material could withstand it. So why doesn't it's surface collapse inwards? In the Sun, heat from fusion stops this collapse. But there is no such heat in Sirius B - so what is holding its surface up?

LESSON 2: White Dwarfs

QUANTUM MECHANICS

To explain how white dwarf stars can support their immense pressure, we need quantum mechanics.

The first clues to quantum mechanics came from the discovery of emission- and absorption-lines in gas spectra. Why do gasses emit and absorb at particular narrow wavelengths and not over a wide range of wavelengths?

One possible answer came from an analogy with sound waves. Musical instruments also emit sound waves only at particular narrow frequencies, not at all frequencies. This is because the sound in generated by waves (string vibrations in the case of a guitar) but these waves are confined. In the case of a guitar, the waves must have zero amplitude at both ends of the string. This means that only certain discrete wavelengths are allowed (the harmonics).

WAVES OR PARTICLES?

If the electrons in an atom were actually waves, then the fact that they are trapped in the atom would confine them, and you would get certain distinct wavelengths (energy levels), just like you do for sound waves on a guitar string.

But how could an electron be a wave? We know that if you fire a beam of electrons at a phosphorescent plate, you get a series of discrete pin-point flashes, not a spread-out wave pattern. That sounds like a particle, not a wave.

PROBABILITY WAVES

The very strange answer proposed by quantum mechanics is that electrons (and photons, protons, and pretty much everything else) are actually probability waves.

If (and only if) you don't measure exact positions, then electrons behave like waves, and do all the wave-like things, such as have discrete energy levels, interfere with themselves, diffract etc. But if you make a precise position measurement (say by having a phosphorescent plate), the "wave function collapses" and all of a sudden the electron has a definite position. What is this position? Well - it's random, but the odds of it being in a particular region are proportional to the square of the amplitude of the probability wave at that location.

EXCLUSION PRINCIPLE AND UNCERTAINTY PRINCIPLE

Two more quantum-mechanics laws:*

*The Pauli Exclusion Principle. No two fermions (particles like electrons and neutrons) can be in the same state. So you can't have all the 26 electrons in an iron atom in the ground state - once it fills up, further electrons have to go into progressively higher energy levels.

*The Heisenberg Uncertainty Principle. $\Delta p \Delta x = \frac{\hbar}{2}$, where p is momentum and x the position. $\hbar = \frac{h}{2\pi}$ where h is Planck's constant. So the more accurately you know one of these pair of variables, the less accurately you can know the other.

WHITE DWARF PHYSICS

In the core of a white dwarf you have vast numbers of electrons compressed into a small space. Because of the Pauli exclusion principle, they cannot share space but must be in different states. This means each is confined into a small volume, which means (because of the Heisenberg uncertainty principle) that their momenta become quite uncertain (and hence large on average).

Because they have large momenta, the electrons bash into things quite hard and fast, so they exert quite a strong pressure - the so-called "Degeneracy pressure", which is given by:

$$\text{Pressure} = \frac{(3\pi^2)^{2/3}}{5} \frac{\hbar^2}{m_e} \left[\left(\frac{Z}{A} \right) \frac{\rho}{m_H} \right]^{5/3}$$

where m_e is the mass of an electron, m_H is the mass of a hydrogen atom, ρ is the density of the white dwarf, Z is the average atomic number of whatever the white dwarf is made of, and A is the atomic mass of whatever it's made of.

If we set this equal to the central pressure of a white dwarf (which we derived in Section 1), we get an equation for the radius of a white dwarf:

$$R_{\text{WD}} = \frac{(18\pi)^{2/3}}{10} \frac{\hbar^2}{G m_e M_{WD}^{1/3}} \left[\left(\frac{Z}{A} \right) \frac{1}{m_H} \right]^{5/3}$$

where M_{WD} is the mass of the white dwarf.

Remarkably, this equation, based on huge approximations and basic physics, gets the size of a white dwarf correct to within a factor of two!

ORIGIN OF WHITE DWARFS

So white dwarfs are held up by amazing combination of gravity and quantum mechanics.

But that still leaves the problem of how they formed in the first place, and why they are made of something other than Hydrogen and Helium, despite the fact that the universe is overwhelming made of Hydrogen and Helium.

Most likely, white dwarfs start off as the cores of normal stars, like our Sun. In its centre, the star ends up burning hydrogen and helium to form carbon and oxygen. Eventually, the star swells up to become a red giant. Red giants are so large and low density that they only have a tenuous hold on their outer layers, and eventually most of the gas blows away into space, producing a planetary nebula, surrounding the naked core (a carbon-oxygen white dwarf).

LESSON 3: Dwarf Novae

DWARF NOVAE

How can white dwarfs be violent? The first clue came from the 19th century discovery of certain stars that occasionally become 100 times brighter - so-called “Dwarf Novae”.

When spectra were first taken of these stars, it was found that they showed emission lines when faint, but during their explosions, they showed absorption lines.

EMISSION AND ABSORPTION LINES

To explain this change in their spectra, we need to think about how matter and radiation interact. If a cloud of gas is tenuous, then any radiation generated by it will easily escape. In this case, you tend to get emission lines in a spectrum. This is called an “optically thin” gas cloud.

If, on the other hand, some object out in space is so dense that any photon emitted deep in its interior will be absorbed before it reaches the surface, then the object is called “optically thick”. In this case, it will show a “black body” spectrum, given by the Planck law:

$$f_\lambda = \frac{2\pi hc}{\lambda^5 e^{hc/\lambda kT}}$$

where f_λ is the flux per unit wavelength λ, h is Planck’s constant, c is the speed of light, k is Boltzmann’s constant and T the temperature (in K).

The peak of this emission can be found by differentating this equation, and occurs at a wavelength given by the Wien displacement law:

$$\lambda = \frac{0.00290}{T}$$

for temperature T in Kelvin.

If you integrate the Planck law, you can find the total radiation emitted by a black body, which is given by the Stefan-Boltzmann equation:

$$L = A\sigma T^4$$

where L is the luminosity (total power of radiation emitted), A is the surface area, σ is the Stefan-Boltzmann constant and T the temperature.

If you have some cool gas in front of a black body, then it will cause absorption lines. The cool gas can just be a cooler part of the same object that is causing the black body radiation - as is the case in stellar atmospheres.

So - we have optically thin gas when the dwarf nova is quiet, which becomes optically thick (but surrounded by cooler gas) during an erruption.

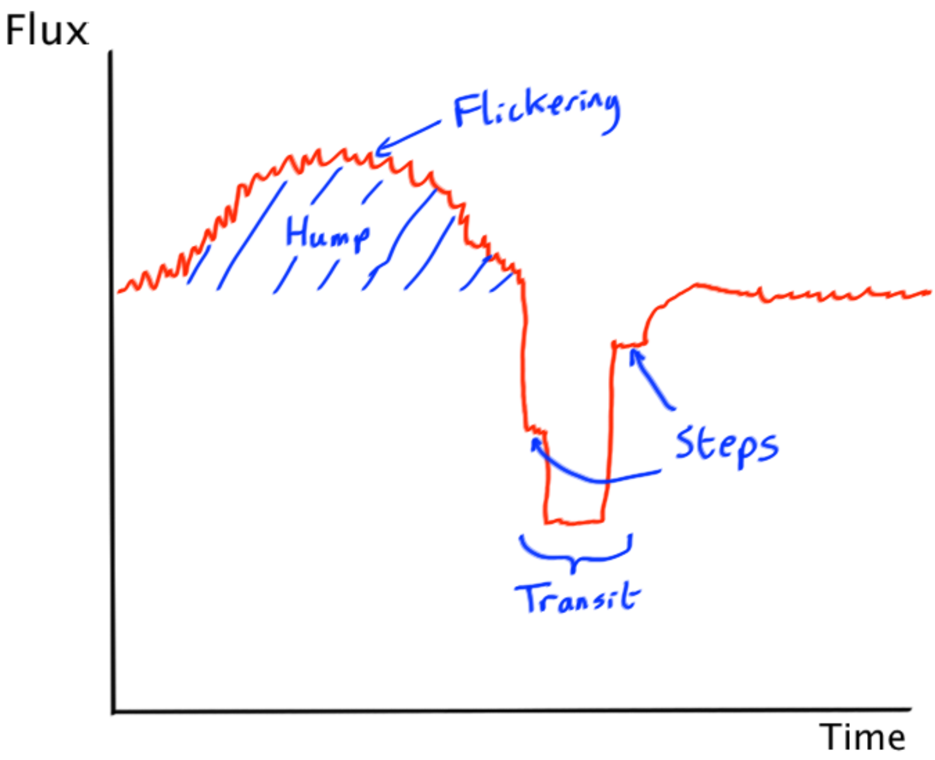
MORE CLUES

If you look at the spectrum, you can also see the spectrum of a red star combined with that of the gas. The red star and gas show oscillating wavelength shifts, indicating that they are both orbiting around their common centre of mass. The gas cloud must be as heavy as a star to make the star wobble like this. The speeds are around 120 km/s, which allows you to estimate the separation of the star and gas cloud by balancing centrifugal force and gravity, which gives you the equation:

$$r = \frac{GM}{4v^2}$$

where r is the distance of each object from their common centre of mass: they are typically only a couple of million kilometres apart - so very close, almost touching!

If we repeatedly measure the brightness of these objects, we see that in many cases there are eclipses as the gas cloud moves behind the red star.



Light curve explained

This graph shows some features of a typical light curve explained. The transit is when the gas cloud moves behind the star. The steps in the side of the transit indicate that there are at least two bright point-like objects within the gas cloud. The hump occurs because one of these point-like objects only shines in one direction - acting like a glowing paint on an opaque surface. The flicking indicates that the brightness of this “paint” varies very rapidly, which means it must come from a tiny area.

A final clue is that the gas spectra show emission lines with double peaks. During the transit, first one peak weakens then the other. This indicates that the gas is rotating in a disk.

ROCHE SURFACE

We try to come up with a physical model that can explain all these puzzling observations. The basic idea is that we have a red star and a white dwarf in orbit around their common centre of mass. The red star has come to the end of its life and is swelling up, but its gas is spilling over onto the white dwarf.

We can calculate the gravitational potential of a small object moving through a system dominated by two massive objects - this is called the “Roche surface”. Gas flows from the red star through a saddle point, then forms a disk around the white dwarf.

ACCRETION DISK

The spinning disk of gas around the white dwarf gets its power from the gravitational potential energy as the gas moves closer to the white dwarf. The power radiated can be calculated from the energy lost and comes out as:

$$P = \frac{GM_{WD}\dot{m}}{r_{WD}}$$

where $\dot{m}$ is the rate of mass transfer, M_{WD} the mass of the white dwarf and r_{WD} the radius of the white dwarf.

We find mass transfer rates of around $\dot{m} \sim 3 \times 10^{13}\text{kg s}^{-1}$, which is a tiny fraction of the red star mass. This means that the transfer can continue for a very long time.

We can estimate the temperature of the gas in the accretion disk by requiring it to radiate this amount of energy from its area. We get a temperature given by:

$$T = \sqrt[4]{\frac{GM_{WD}\dot{m}}{\pi r_D^2 r_{WD}\sigma}}$$

where r_D is the disk radius. This comes out as roughly 9000 K. In practice, however, the inner parts of the disk will be much hotter and will dominate the total emission, producing a spectrum smoothly rising towards shorter wavelengths.

MAGNETIC WHITE DWARFS

A small fraction of white-dwarfs are magnetic, and in these cases, the accretion disk cannot reach the white dwarf surface. Instead, the infalling gas is threaded onto the magnetic field lines and slides along them down to the surface. The charged particles doing spirals around the magnetic field lines can produce cyclotron radiation.

ORIGIN AND OUTSTANDING MYSTERIES

These dwarf novae are very common. Most stars are binaries, and the more massive star will die first and swell up. At this point, tidal drag often brings the two stars close together. The more massive star then turns into a white dwarf, and at some later stage the less massive star begins to swell up. At that point, you get a dwarf nova as gas from the second star spills over onto the white dwarf.

We don’t really know, however, what causes the big outbursts. Most likely it is some form of instability in the accretion disk - matter builds up but cannot move inwards, until some threshold is reached, at which point a huge amount of matter is dumped onto the white dwarf.

LESSON 4: Classical Novae and the Chandrasekhar Limit

CLASSICAL NOVAE

Classical novae are explosions much brighter than the dwarf nova explosions we’ve been talking about so far. They too repeat, but often on much longer timescales (technically we have different names for those that have and have not been seen to repeat, but they are really the same thing). Unlike dwarf novae, these classical novae actually blow material out into space.

Curiously, at least one of the classical novae (Nova Persei 1901) has subsequently turned into a dwarf nova! So classical novae seem to involve binary systems where a red star is transferring mass to a white dwarf companion - just like dwarf novae.

In this case, the energy source is fusion, not gravity. As gas falls to the surface of the white dwarf, a thin shell of hydrogen builds up (on top of the carbon-oxygen core). Because the gravity of the white dwarf is so immense, the pressure at the base of this thin hydrogen level becomes enormous - and can get up to the same pressure as the middle of our Sun. When this happens, nuclear fusion will begin.

Normally, when some gas gets hotter, its pressure rises and so it expands and cools down. This keep the fusion in the middle of our Sun nice and steady - if the fusion rate increases for any reason, the temperature and pressure will increase, which will drop the density of the core of the sun and reduce the fusion rate to balance things out. But the base of the hydrogen layer on the white dwarf is degenerate - supported by quantum mechanics, not heat. So the pressure does *not* increase as it gets hotter. But as it gets hotter, and the pressure remains the same, the fusion rate will get bigger and bigger, causing an explosion. All sorts of highly radioactive elements are produced, which get convected to the surface where they radiate away their prodigious energy.

EDDINGTON LIMIT

So - we have floods of photons pouring out. These photons have momentum, and if they hit an electron, they give it a push. We can think of an electron as a target of area equal to the Thomson cross section $\sigma_T = 6.7 \times 10^{-29}\mathrm{m}^2$. Each photon has a momentum equal to its energy divided by the speed of light. The cumulative force of all the photons flooding out from an object of luminosity L and hitting one electron a distance D away is

$$F = \frac{L}{4\pi D^2 c}\,\sigma_T$$

If this is greater than gravity, the radiation will blow matter out into space. This occurs when the Eddington Luminosity L_E is exceeded:

$$L_E = \frac{4\pi G M_{WD} m_p c}{\sigma_T}$$

where m_p is the mass of a proton (this equation assumes pure hydrogen gas).

It turns out that classical novae exceed this limit - so their radiation is sufficient to blow gas out into space, as observed.

TYPES OF CLASSICAL NOVA

Some classical novae have big rare explosions, while others have more frequent smaller explosions. Most likely, the novae with big rare explosions are the less massive ones - they can accumulate more hydrogen on their surface before fusion starts, due to their lower gravity.

Some novae probably burn hydrogen to helium in the gas as it falls onto the surface. They may then have a burning shell of helium - but as this is not degenerate it would burn steadily rather than exploding.

STABLITY OF A WHITE DWARF

As a white dwarf gets more massive, its radius decreases. Can this continue for ever?

Consider a layer of gas somewhere in a star. Imagine that for some reason it moves inwards. Two competing effects are at work. As it moves in, the pressure inside rises, and pushes back. But as it moves in, it is closer to the centre of mass, so gravity increases. If the first effect wins, the star is stable, but if the second effect dominates, there is nothing to stop the star collapsing.

Which effect wins depends on the properties of the gas in the star. We approximate the behavior of the gas with the equation of state $PV^{\gamma} = c$, where P is the pressure, V the volume, c a constant and γ is known as the equation of state parameter.

If $\gamma > 4/3$, a star is stable, but if $\gamma \leq 4.3$ a star is not stable.

THE CHANDRASEKHAR LIMIT

For a normal white dwarf, $\gamma = 5/3$, so they are stable. But if a white dwarf has a mass of over 1.44 times the mass of the Sun, the electrons are moving at nearly the speed of light. This means that the number of electrons hitting a given surface per unit time can no longer increase as their momentum gets larger, and this changes the equation of state such that $\gamma = 4/3$. So the star is unstable and collapses!

How would a white dwarf get this big? When sun-like stars die, they blow away around 1/3 of their mass in the red giant/planetary nebula phaase, leaving white dwarfs of around 0.7–0.8 solar masses. More massive stars blow off a larger fraction of their mass, so even a 7 solar mass star would only leave behind a white dwarf of around 1.2 solar masses. So white dwarfs are rarely born this big.

One possible way to make them more massive would be if suceeding classical nova explosions left a little extra mass behind each time (unclear if this actually happens). A helium burning shell could deposit carbon and oxygen. Or if you have a white-dwarf orbiting another white-dwarf in a close enough orbit, they would radiate energy in the form of gravity waves, which would eventually cause them to spiral in and merge.

What would happen then? The star would collapse, until the density became high enough to fuse carbon and oxygen.

NUCLEAR PHYSICS

A large part of astrophysics depends on the shape of the curve of binding energy versus number of nucleons (protons and neutrons). It explains why you get energy from fusion by combining light elements, why Iron 56 is the most stable element (and hence so common) and why heavier elements liberate energy when they break down. But why does this curve have the observed shape?

Normally, you would expect an atomic nucleus to fly apart, because of the mutual electrostatic repulsion of the protons (like charges repel). Nuclei are held together by a much stronger force - the imaginatively named “strong force”. But this force is short ranged, so once a nucleus becomes large, it can be overwhelmed by the electrostatic repulsion and hence break apart.

You expect a roughly equal mix of protons and neutrons due to quantum mechanics - if you have a lot more of one or another, they will be forced into higher energy level and the overall energy goes up.

LESSON 5: Thermonuclear Supernovae

ENERGY FROM A COLLAPSING WHITE DWARF

If a white dwarf that was made of carbon and oxygen exceeded its Chandrasekhar limit and collapsed, how much energy would be released in nuclear fusion?

You can look up the mass of a carbon or oxygen nucleus (12.01079 atomic mass units for carbon) and compare it to the mass of an Iron nucleus. You will find that the mass per nucleon of Iron is 0.3% less than that of carbon or oxygen. So if you combine enough carbon and oxygen nuclei to form iron, you’ve lost mass somewhere.

This “missing mass” has been converted into energy, and you can work out how much using $E = mc^2$. This comes out as around $3.3 \times 10^{14} \mathrm{J\ kg^{-1}}$, giving a total energy of around $10^{45} J$ from complete fusion of a collapsing white dwarf.

This is much greater than the energy liberated from a classical nova - so we need a new category, a “supernova”.

HISTORICAL SUPERNOVA EXPLOSIONS

In the last 1000 or so years, there have been four incredibly bright explosions - “new stars” that apeared as bright as the moon for a few weeks. But were these really more energetic that classical novae, or just closer?

In August 1885, one of these explosions was seen in the Andromeda nebula. But back then, people did not know that this was a distant galaxy, so it still didn’t help work out if these explosions were very luminous or just close. In the 1920s, however, Edwin Hubble managed to show that spiral nebulae like Andromeda were actually millions of light years away. This meant that these explosions must be unbelievably luminous!

What causes these explosions? One clue is to look at the locations of the bright historical supernovae and see what is left behind. Bright nebulae are seen - most famously the Crab Nebula, at the site of the 1054AD explosion. These are expanding shells of gas, so evidence that these bright “new stars” really were explosions.

To learn more, we would need to study more supernovae, but this is hard if you only get one every 250 years or so. Fritz Zwicky, however, figured out that if you could survey enough galaxies, you could find supernovae every year. He built a new telescope with a very wide field of view, and over several decades discovered and studied many supernovae.

The supernovae seemed to split into two classes on the basis of their spectra:

Type 1: which show no hydrogen in their spectra
Type 2: with spectra dominated by hydrogen.

A collapsing carbon-oxygen white dwarf would have little or no hydrogen, so perhaps these can explain the Type 1 supernovae?

NUCLEAR REACTIONS

If you calculate the nuclear physics involved, you find that carbon and oxygen will mostly fuse to form ^{56}Ni. Not much energy is released in this process. The energy is released later, as the ^{56}Ni decays to ^{56}Co (half-life of 6.1 days, releases 1.7 MeV per nucleon) and then as ^{56}Co decays to ^{56}Fe (half life of 77 days, releases 3.7 MeV per nucleon).

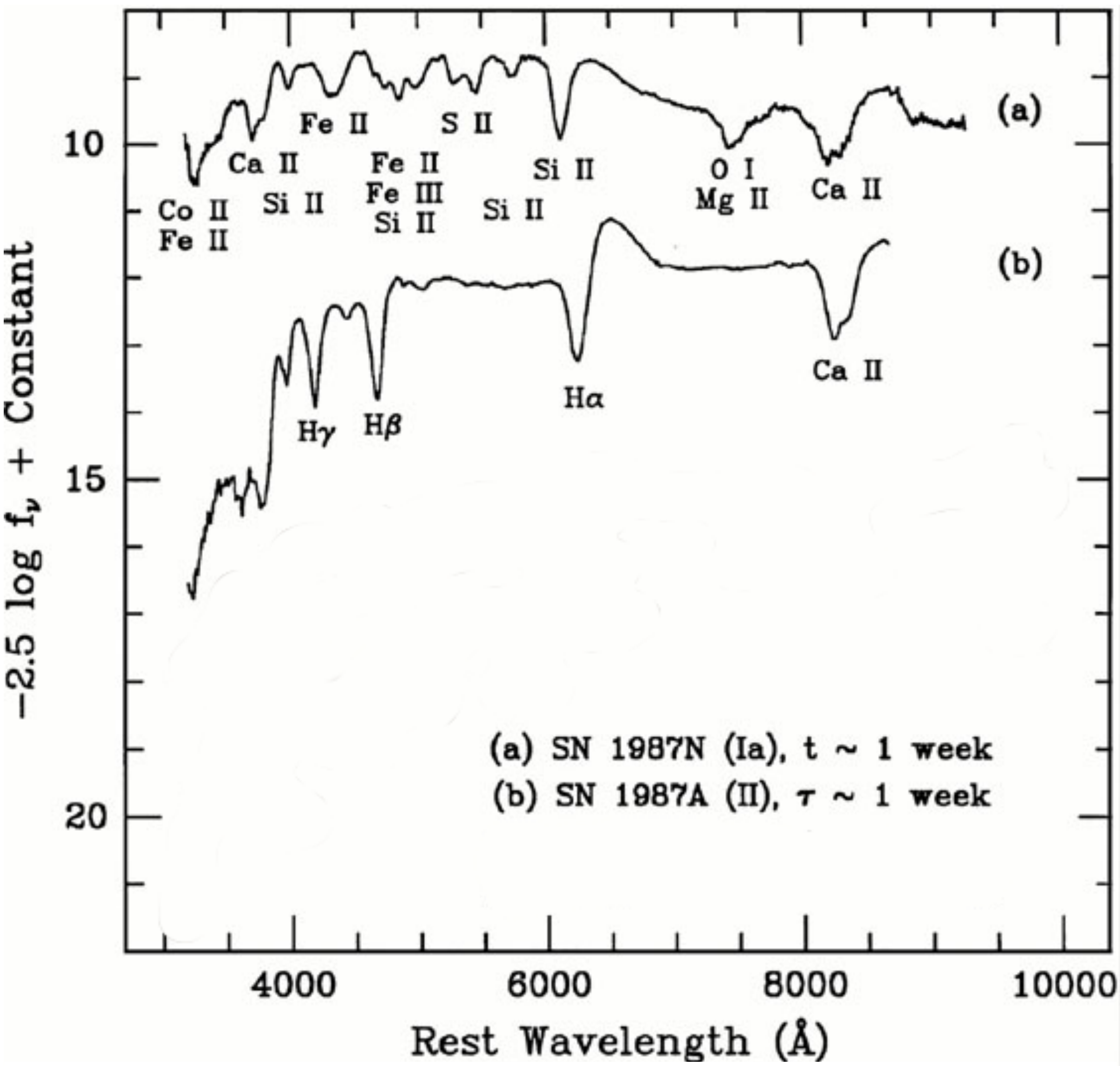
We can estimate how much ^{56}Ni must have been produced in one of these supernovae. The brightness peak (caused by the decay of ^{56}Ni) has a luminosity of around $10^{36}W$ and lasts for a couple of weeks. The total energy released in this phase is just the luminosity times the time - i.e. around $2.4 \times 10^{42}J$.

We know that 1.7 MeV ($2.7 \times 10^{-13}J$) is released per nucleon, so we need around 9×10^{54} atoms, with a total mass of around $10^{30}kg$ (half a solar mass).

Which is curiously small - it seems that less than half of the initial mass of a Chandrasekhar limit white dwarf has been converted to ^{56}Ni.

SUPERNOVA SPECTRA

Another mystery comes from the spectra of these Type 1a supernovae. If you take a spectrum when the supernova is at maximum brightness, it is dominated by lines of calcium, silicon and oxygen (not iron, nickel and cobalt). The lines show an emission peak at the expected wavelength of the element, but in addition, an absorption trough at a shorter wavelength. This type of line profile is called a “P Cygni” profile and is caused by the scattering and absorption of light in an outflowing explosion. The lines are very wide due to the enormously high velocities of the outflowing gas.



Spectra of young supernovae

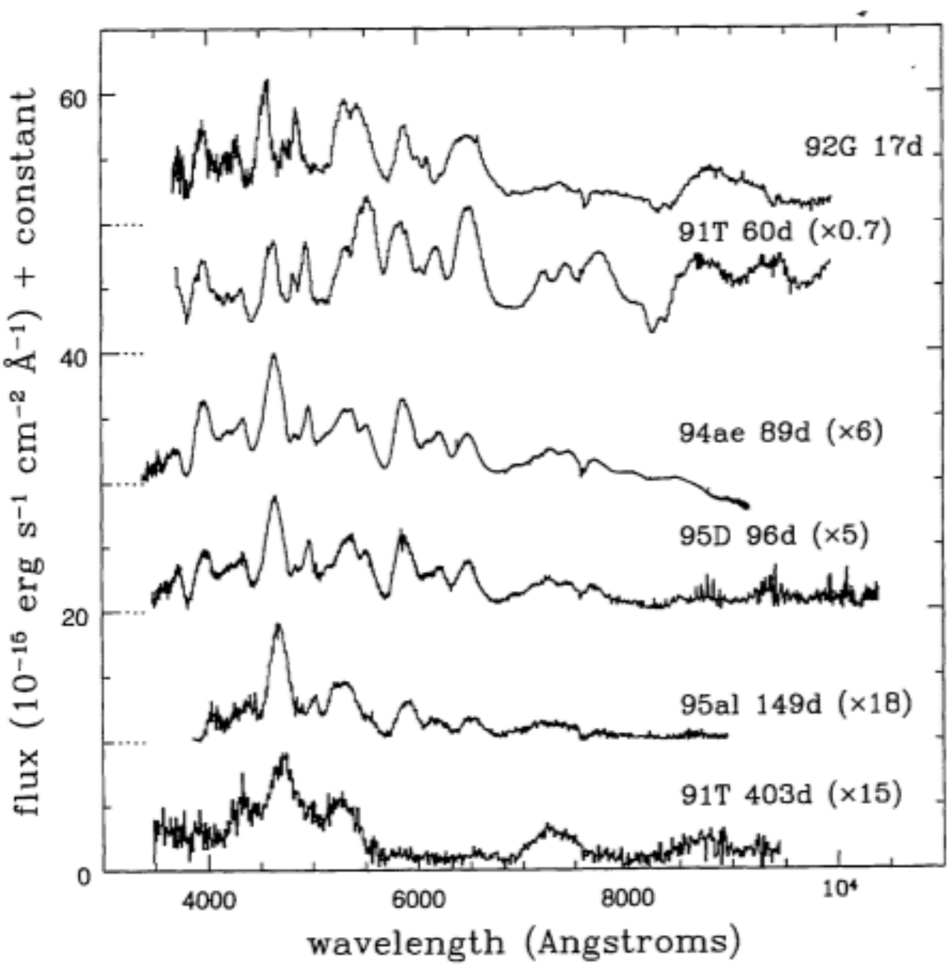
This diagram shows spectra of a young Type 1 supernova (top) and a young Type 2 supernova (bottom). From Filippenko et al. 1997, Ann Rev Astron Astrophy 35, 309.

Young type 1 supernovae typically show the following spectral lines:

Si II (singly ionised Silicon) at 385.8, 413.0, 505.1, 597.2 and 635.5 nm
Ca II (singly ionised Calcium) at 393.4, 396.8 and 857.9 nm
Mg II (singly ionised Magnesium) at 448.1 nm
S II (singly ionised Sulphur) at 546.8, 561.2 and 565.4 nm
O I (neutral oxygen) at 777.3 nm

By contrast, Type 2 supernovae spectra are dominated by hydrogen lines: the Balmer series at 656.3nm (H-alpha), 486.1 nm (H-beta), 434.1nm (H-gamma), and 410.2nm, 397.0nm and 388.9 nm.

If you get a spectrum several months after peak brightness, however, the spectra of type 1a supernovae look quite different:



Late spectra of Type 1 supernovae

Late time spectra of a number of Type 1a supernovae, from Bowers et al. 1997, MNRAS 290, 663.

200 days after maximum light, we can see right through deeper into the exploding gas, and now we see lower velocities (the gas near the centre is moving slower) and strong iron lines. So the iron (from the decay of nickel 56) is there, but buried below lots of lighter elements.

REVISION - SPECTRAL SHIFTS

When interpreting details of the spectra, you will need to remember some equations covered in the first course. We include them again here for the benefit of those who didn't do the first course, and as revision.

If an object is far away, the expansion of the universe will redshift its spectrum. Redshift z is defined as

$$z = \frac{\Delta\lambda}{\lambda}$$

where $\Delta\lambda$ is the change in wavelength (observed minus lab) and λ is the lab wavelength of the line. Lab wavelengths can be found in the reference notes.

If, in addition, the gas is moving along the lines of sight at a velocity v, the doppler effect will cause its observed wavelength will move by an an amount $\Delta\lambda$ given by the equation

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

where c is the speed of light.

See the worked example for how to use these equations.

MODELS

So - it seems that a collapsing white dwarf is a reasonable candidate for explaining these Type 1 supernovae, but we have a puzzle - only some of the mass has been converted to ^{56}Ni . This is puzzling - if the white dwarf is degenerate, pressure will not depend on temperature, so once fusion begins there is nothing to stop it until everything has been converted. It seems we need the white dwarf to be only partially degenerate before the explosion begins.

Three possibilities are currently being considered:

Perhaps we are looking at the merger of two white dwarfs? A close white-dwarf/white dwarf binary would slowly radiate energy via gravity waves, and eventually merge. This would push the combined object over the Chandrasekhar mass and cause an explosion - perhaps a very messy one as it would take place at the same time as the merger.
Perhaps, as mass is slowly added to the surface of a white dwarf from its binary companion, it starts to simmer, rather than burning all at once. This simmering slowly heats things up over decades to centuries, partially removing the degeneracy, before going exponential and ripping everything apart in less than a second.
Perhaps the white dwarf is well below the Chandrasekhar limit, and is being supplied helium onto its surface from a white dwarf binary partner. This helium layer could detonate on the surface (similar to a classical nova) and send a shock wave in to the centre. The compression caused by thus shock wave could increase the density and temperature enough to trigger fusion despite the white dwarf being below the Chandrasekhar limit.

TESTING THE MODELS

Which (if any) of these models is correct? We don’t know.

One clue is the seemingly symmetrical nature of observed supernova remnants, which might suggest a more symmetrical origin, such as the simmering model or the helium model.

On the other hand, both of these models imply that a binary companion should be left behind by the explosion. This binary companion would be moving away quite fast (at its orbital speed) and so should be quite easy to find, but none have been found. This would suggest the binary white-dwarf merger model, were it not for the problem of the symmetrical remnants. In addition, the binary companion should be severely traumatised immediately after the explosion, and hence so bright that it should be detectable in the very early light curve. But this contribution is not seen.

Is there a way out of this seeming paradox?

One possibility is that despite the messy geometry of a white-dwarf collision, the explosion actually regularises everything, so you can get a nice spherical remnant.

Another possibility is that when you dump matter onto a white dwarf from its binary companion, it produces a very rapidly spinning, highly flattened white dwarf. The centrifugal force in something like this might reduce the pressure so much that it can survive despite being above the Chandrasekhar mass. It might stay in this state for a billion years, while its companion star dwindles into a fain white dwarf. Its rotation will very slowly decrease, until eventually centrifugal force no longer can prevent the collapse and explosion. In this model, there will still be a fast moving secondary star, but it is too small and too faint to discover.

These thermonuclear Type 1a supernovae are vital for cosmology, as their standard luminosities mean that they can be used to measure distances very accurately. So it is somewhat worrying that we don't know what causes them. This is not a problem for the discovery of dark energy - we know statistically that they are good enough standard candles. But it may be a problem for future surveys which seek to use samples of thousands of supernovae to measure cosmological parameters to high precision.

Perhaps we will only solve this mystery when we next have a supernova in our own galaxy or a very nearby one, where we will have records of what was there before the explosion.

LESSON 6: Core Collapse Supernovae

TYPE 2 SUPERNOVAE

We've explained the Type 1 (thermonuclear) supernovae, but what of the supernovae with hydrogen in their atmospheres - the Type 2 supernovae?

Luckily for us, one of these exploded nearby (in the Magellanic Clouds) in 1987: Supernova 1987A, which was intensively studied with pretty much every telescope in the southern hemisphere. By comparing its position with previous images of this part of the sky, we were able to determine that the star that exploded was a massive one - around 15 times the mass of the Sun. So unlike Type 1 supernovae, the type 2's seem to involve the explosion of massive stars.

How can we get a massive star to explode? Less massive stars stop fusion reactions once their cores have been converted into carbon and oxygen, but more massive stars can continue fusion - burning silicon to iron and nickel. This whole process is very quick - only taking a few weeks. Once their cores are made of iron and nickel, fusion stops. Gravity takes over and causes the cores to shrink. The cores are too massive to end up as iron-nickel white dwarfs, so they shrink further still.

Their shrinking is finally stopped by neutron degeneracy pressure. This is exactly the same quantum-mechanical effect we discussed when caused by electrons in white dwarf stars, but because neutrons are more massive, they can exert more pressure, and hence support more massive cores. If we take the equation for the radius of a white dwarf, and replace the mass of an electron with the mass of a neutron (1840 times larger), we get a radius 1840 times smaller - i.e. around 3km. In fact, the cores supported by neutron degeneracy pressure (which are called neutron stars) are around 10km in size.

GRAVITATIONAL COLLAPSE AS AN ENERGY SOURCE

If the core of a massive star has collapsed to form a neutron star, the rest of the star will fall onto this core. Because the gravity is so intense, this fall will liberate immense amounts of energy. ROughly speaking, the energy released is

$$E = \frac{GM_{NS}M_s}{r}$$

where M_{NS} is the mass of the neutron star, M_s the mass of the rest of the star, and r the radius of the neutron star. This energy comes out as around $2 \times 10^{47} J$ - i.e. more than 1000 times greater than type 1 supernovae!

But why does this fall cause an explosion? One idea is that the matter falls in and bounces off the surface of the neutron star. But normally when you drop something and it bounces, it never comes back to the same height, let alone goes further.

A possible solution is that the star collapses from the inside out. Inner layers bounce off the neutron star and relatively slow speeds, while higher levels bounce off the inner layers (which are already moving out). This can, at least in principle, lead to a tiny fraction of the mass flying out at enormous velocities. But only a small fraction of the energy can be released this way.

These predictions can be compared to the observed energy output. The energy radiated can be estimated by multiplying the peak luminosity by the time it stays bright, which gives an energy of around $10^{42} J$. The kinetic energy of the outflowing material can be estimated using the kinetic energy equation $KE = \frac{1}{2} mv^2$ and comes out as around $10^{44} J$.

So - the bulk of the observed energy cones out as kinetic energy, and this is about what you might expect from a bounce. But it's still less than 0.1% of the total expected energy - where does the rest go?

NEUTRINOS

Let's look at the collapse of the core in more detail. As the density increases, the temperature rises to around $10^{10}K$. At these temperatures, the black body photons are gamma rays, with eneough energy to rip the iron and nickel nuclei apart, breaking them down into their component protons and neutrons, reversing in a fraction of a second all the fusion over the whole lifetime of the star.

Under these conditions of incredible temperature and pressure, the electrons combine with the protons to produce a neutrons and neutrinos. An amazing 10^{57} neutrinos are produced. It is these neutrinos that carry away most of the energy.

Neutrinos only interact very weakly with matter - the interaction with an atom has a cross-sectional area of around $10^{-47}m^2$ - i.e. a neutrino has to hit a tiny target of this area to interact with a given atom.

Luckily there are a very large number of atoms in the infalling outer layers of the star - enough to intercept around 1% of the neutrinos. This 1% dumps its energy into the gas and causes the observed explosion. The rest escapes into space.

The flux of neutrinos from a supernova at distance D reaching a given square metre on the Earth is given by the same inverse square law that applies to light:

$$F_{obs} = \frac{n_{em}}{4\pi D^2}$$

From supernova 1987A, this comes out as a whopping $3 \times 10^{13}\nu m^{-2}$ - i.e. thirty trillion neutrinos hitting every square metre. However, you can factor in the very small probability of each neutrino interacting with anything - you'd need around 10^5 cubic metres of detector to get one interaction.

Luckily, some very large neutrino detectors existed, and they picked up a handful of neutrinos from the explosion, confirming this theory.

PUZZLES

In principle, we have a good story of how these core-collapse supernovae explode. Simulations including all this physics, however, tend not to explode - they actually stall. The physics that goes into these simulations is, however, really complicated, so most likely future modelling improvements will explain the observed supernovae

Observationally, we can often work out what exploded in nearby core collapse supernovae by using archival data taken before the explosion, and this suggests that stars in the mass range 8.5–16.5 solar masses explode. But there are many more massive stars out there which don't seem to produce supernovae. Perhaps they just collapse to form black holes. It would look as if they just disappeared! But alternatively, it could be that some of the stars that produce supernovae were just partially obscured by dust, so we underestimate their true luminosity and hence their true mass.

Future observations will find rarer types of supernovae In particular, supernovae in the very early universe may belong to a whole new class (pair-instability supernovae).

LESSON 7: Neutron Stars

INTRODUCTION

Core collapse supernovae should leave neutron stars behind - balls of neutrons with about 1.5 times the mass of the Sun, but a radius of only a few kilometres, and hence absolutely amazing gravity.

From what we know about the rates of core-collapse supernovae, there should be around a hundred million of them in our own galaxy. But surely they would be hard to see?

X-RAY ASTRONOMY

X-rays are blocked by the atmosphere, so X-ray astronomy needs to be done from space. The first experiments used sounding rockets, and found that the Sun was an X-ray source. For reasons that are still not very well understood, the outermost layers of the Sun are much hotter than lower down, and X-rays from here produce the Earth's ionosphere. It was felt that X-rays from anything other than the Sun were probably going to be forever too faint to see.

But to everyone's surprise, a whole bunch of incredibly strong X-ray sources were detected once detectors became better (using anti-coincidence to eliminate cosmic rays). These sources were so strong that they could appreciably ionise the Earth's upper atmosphere! Only rough positions could be determined for these sources, but they weren't coming from anything obvious. The first of these sources was called Scorpius X–1 (being the brightest X-ray source in the constellation of Scorpius).

NEUTRON STAR CONNECTION?

What could these things be? If their radiation comes from a black body, then we can use the Wien displacement law to estimate their temperature - and it comes out as tens of millions of degrees. If you can estimate the distance to one of these objects, you can use the observed flux and the inverse square law to calculate the luminosity L. This comes out as a few by $10^{31} W$ for Scorpius X–1. Which is high - but actually not as high as you might expect given the amazing temperature. Once again assuming it radiates as a black body, you can use the Stefan-Boltzmann equation to calculate the radius needed to generate this luminosity:

$$r = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

which comes out as around 6km - the size of a neutron star!

But why would a neutron star be so hot? It has come from the centre of a supernova, and so should start extremely hot! But its immense radiation should soon cool it down. So you might only see these X-ray emitting neutron stars for a short time after the explosion, in the middle of the supernova remnant.

The second X-ray source detected was somewhere in the Crab nebula - the remains of the 1054AD supernova, so this seems to fit. But unfortunately, detailed measurements of the position (made possible when the Moon covered the source) showed that a large part of the X-rays are actually coming from the extended nebula, and not from a compact object in its centre.

One possibility - neutron stars could be extremely magnetic, if they've trapped some of the magnetic field from the star that formed them. If they are spinning fast, this magnetic field will whip through the surrounding gas, accelerating charges and producing a very strong ultra-low frequency radio signal. These radio waves won't escape the nebula but they can spread the decaying rotational energy out, heating the gas enough to produce X-rays.

PULSARS

The next clues was a totally unexpected one - the discovery of pulsars. They were picked up accidently, as a by-product of a radio astronomy search for interstellar scintillation. They were radio sources pulsing around once a second!

An object of a given mass cannot rotate too fast as otherwise its gravity is insufficient to hold objects on its surface. For a spherical object of radius r, mass M and spin period P, the maximum possible radius is:

$$r = \sqrt[3]{\frac{GMP^2}{4\pi^2}}$$

which gives an upper limit on the size of a pulsar that is much smaller than a white dwarf. So neutron stars seemed to be the only possibilities.

And sure enough - a pulsar was detected in the centre of the Crab nebula. It was strongly magnetic, and its period was decreasing at a rate that was consistent with the magnetic field carrying away its energy to power the X-ray emission from the rest of the nebula!

So pulsars seem to be magnetised neutron stars emitting a spinning beam of radio waves.

X-RAY BINARIES

But what of the other X-ray sources like Scorpius X–1, which are not asociated with supernova remnants, and do not pulse at radio wavelengths? WIth steadily improving X-ray telescope technology, it became possible to pin down more precisely where the X-rays were coming from. And they seemed to becoming from fairly normal looking stars!

Closer observations indicated that these stars were wobbling backwards and forwards, and so are presumably part of a binary system - the X-rays are coming from their neutron star companion. So we seem to be seeing a neutron-star equivalent of the dwarf novae: a star feeding matter via the Roche lobe to its binary companion - in this case, a neutron star. Because the companion is smaller and denser, the radiation comes out at X-ray wavelengths rather than UV wavelengths. The matter falling on the surface of the neutron star must spin it up to extremely high speeds.

SS433

Perhaps the strangest of these X-ray binaries is SS433, which has a spectrum containing satellite emission lines that move in and out from a central line. In this case, the disk around the neutron star is somehow producing a jet, which moves out at a good fraction of the speed of light. This disk and the jet precess, producing the mobile satellite emission lines.

USING PULSARS

In addition to their instrinsic interest, pulsars have become crucial probes of all sorts of physics. Millisecond pulsars (old neutron stars that have been spun up by a binary companion to phenomenal rotation speeds) act as clocks with precision rivalling the world's best atomic clocks. Having these phenomenally accurate clocks spread around the galaxy allows all sorts of neat physics. In the exoplanets course we talked about using the to find orbiting planets.

One famous case is a binary pulsar, which is radiating away its potential energy in the form of gravity waves, providing the best tests of General Relativity to date. By monitoring networks of pulsars, it may even be possible to detect long wavelength X-rays as they sweep through the solar system.

LESSON 8: Special Relativity

Hundreds of years ago, philosophers speculated tha "dak stars" might exist. These would be stars much like our own, glowing brightly, but with gravity so intense that their escape velocity was greater than the speed of light, so photons flying up from the surface would turn around and fall back down. Thus these stars would be invisible.

Our actual picture of black holes is very different - and to understand this, we have to learn about Einstein's theory of Special Relativity.

ELECTROMAGNETIC WAVES

In the 19th century, it was discovered that a changing electric field can produce a magnetic field, and a changing magnetic field can produce an electric field. Putting these two together, Maxwell showed that if you created a changing electric field, it would generate a changing magnetic field which would in turn produce a changing electric field and so on - the pair of oscillating fields could move through space all by themselves, quite self-sustaining, with no need for wires! And the speed of this "electro-magnetic" wave was

$$v = \sqrt{\frac{1}{\epsilon_0 \mu_0}}$$

where the two terms on the bottom are constants used in electic and magnetic fields, both of which can be measured in the lab.

And curiously, when this speed was worked out, it came out as 300,000 km/s - a remarkably familiar number, the speed of light! This was one of the great triumphs of science, and led to the whole development of radio communications.

PARADOX

This was all very exciting - but it caused a serious philosophical problem. Ever since Galileo, it had been believed that position and velocity were purely relative - i.e. there was no experiment you could do to tell where you were or how fast you were moving. You could only tell your position or velocity *relative* to something else. But the equation above for the speed of light was not relative to anything. It said that this was the speed of light. End of story.

So imagine if you were moving at half the speed of light and shone a light beam forward. Would this light then be moving at 1.5 times the speed of light? Or just the normal speed of light, in which case relative to us it would only be moving at half the speed of light? If we were flying along-side a light beam at the speed of light, then to us it would appear to be stationary (which according to Maxwell was impossible - fields need to move to exist)? Either seems problematic.

If there actually was some universal “standard of rest” which light moved relative to, we would use this to do an experiment to measure (say) the true velocity of the Earth. If we were moving in a particular direction, light shone in that direction would have to move a little bit slower *relative to us*, while light shone in the opposite direction would have to move apparently a bit faster.

Michaelson and Morley did this experiment and discovered, amazingly, that the speed of light, relative to the Earth, was the same in all directions. How could this possibly be true, given that we know the Earth is orbiting the Sun, and the Sun is orbiting the galaxy? The only possibility seemed to be that the speed of light was relative to some invisible “ether”, and that we lived in a chunk of ether that was following the Earth around the Sun. But if that were the case, we should see distortion of light when it crossed the boundary between one chunk of ether and another. We did not. So what could possible be going on?

FRAMES OF REFERENCE

Einstein’s solution to this paradox was to mess with something so fundamental that people tended to take it for granted - the transform between different frames of reference.

Whenever you measure positions and velocities, you are implictly measuring them relative to a frame of reference. Let’s say one person measures positions and times relative to themselves (position components x, y and z, time t). A second person is moving at speed v along the x-axis relative to the first person, and measures x‘, y’, z’ and t’.

Common sense would tell us that:

$$x' = x + vt$$

$$z' = z$$

$$t' = t$$

These equations are the “Galilean Transform” - they show us how to transform coordinates from one frame of reference to another, assuming Galileo was correct about position and velocity being purely relative.

But could we come up with an alternative transform, that kept the speed of light constant in all frames of reference? There is only one possibility that does this, the Lorentz Transform:

$$x' = \gamma(x + vt)$$

$$z' = z$$

$$t' = \gamma\left(t + \frac{vx}{c^2}\right)$$

where

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Note that for $v \ll c, \gamma \sim 1$ so if you are moving much slower than light, the Lorentz transform behaves just like Galileo's transform. But things are very different as you get closer to the speed of light.

This transform means that no matter how fast you move, light is always travelling c faster than you. This is shown in the appendix (below).

CAN YOU GO FASTER THAN LIGHT?

What happens if you try to go faster than light?

From your own point of view, no matter how fast you travel, if you shine light in front of you, it will still move away from you at c. If you accelerate hard and try again, light still moves away in front at this exact same speed.

From the point of view of someone watching you, you are nearly at the speed of light, but time has slowed down for you and your length has been compressed, which means that you measure the speed of light wrongly. Your mass has apparenly increased, making it harder and harder for you to accelerate further.

So the speed of light seems to be a really fundamental barrier.

APPENDIX (REPEATED FROM BELOW VIDEO 8.5)

As mentioned in the video, we will here show how the Lorentz Transform gives you the same speed of light in all frames of reference, and we will also talk about time, space and velocity distortion. These notes are purely for the benefit of those who are interested.

VELOCITY OF LIGHT IN DIFFERENT FRAMES

Imagine that at time 0, Brian turns on a light. A distance L away (along the positive x axis) is a detector, which will record when the light reaches it, a time L/c later (light travels a distance L in a time L/c).

So in Brian's frame of reference, there are two events. Let's define Brian's position as X=0.

1 First event - light turned on. x=0, t=0
2 Second event - light reaches detector. x=L, t=0

Brian will work out the speed of light by dividing the distance by the time, so the measured velocity will be

$$\frac{L-0}{\frac{L}{c}-0}=c$$

But now consider Paul's frame of reference. Paul is moving in the positive x direction with velocity v, and measures position relative to him (x' = 0). At time 0 (t' = 0), Paul is next to Brian.

So Paul also sees two events. But to work out what position and time Paul sees them at, we will have to use the Lorentz transform.

First event - light turned on. We know that it Brian's frame of reference, x=0 and t=0. So what does Paul see?

The Lorentz Transform tells us that

$$x' = \gamma(x + vt) = \gamma(0 + v \times 0) = 0$$

and

$$y' = \gamma(t + \frac{vx}{c^2}) = \gamma() = 0$$

So nothing strange there. But what about the second event?

Second event - light reaches detector. Once again, let's use the Lorentz transform:

$$x' = \gamma(x + vt) = \gamma\left(L + \frac{vL}{c}\right)$$

and

$$y' = \gamma(t + \frac{vx}{c^2}) = \gamma\left(\frac{L}{c} + \frac{vL}{c^2}\right)$$

So from Paul's point of view, the velocity of the light is the distance travelled minus the time - i.e.

$$\frac{x' - 0}{t' = 0} = \frac{\gamma\left(L + \frac{VL}{c}\right)}{\gamma\left(\frac{L}{c} + \frac{VL}{c^2}\right)} = c$$

So both Brian and Paul measure the same speed of light - if the light is going in the plus x direction!

The remarkable thing is that this works regardless of the direction the light is going in. You can try it for yourself - sent out a light-beam at any arbitrary angle in Brian's frame of reference, and work out the x, y, z and t coordinates of the two events (light emitted and light detected). Then use the Lorentz transform to convert all these coordinates into Paul's frame (x', y', z', t'). In both frames of reference you will get the same velocity of light.

OTHER EFFECTS

You can use the Lorentz transform to work out many other strange effects. For example, consider a clock in Brian's frame of reference. Let's take as our two events the moments when the clock says zero and one second later. So the first event is at t=0, x=0 and the second event is at t = 1, x=0.

Now convert the two events, using the Lorentz Transform, into Paul's frame of reference. You will find that the time interval is now γ seconds. So every tick of Brian's clock is separated, in Paul's frame of reference, by longer than one second.

Now consider two events that happened at the same time but different locations (like two lights turning on simultaneously at opposite ends of Brian's barge). They might have positions x=0 and x=L, and t=0 for both. But when you use the Lorentz transform to see where they appear from Paul's point of view, you will find that they don't both turn on at the same time - one is now before the other!

You can do a similar calculation for lengths. In Brian's frame of reference the two events would be the two ends of a stick (say). Once again, use the Lorentz transform. But this time, you have to be careful, because when Paul measures the length, he will want to measure the difference in position of the two ends of the stick at the same moment *in his frame of reference*. But this will not be the same times in Brian's frame of reference.

So things get really complicated! As you can imagine, using the Lorentz transform will mess up conservation of energy and momentum, if you use the standard equations (e.g. kinetic energy $E = 1/2mv^2$). So you need new equations - most famously $E^2 = p^2c^2 + m_0^2c^4$ (where p is the momentum and m_0 the rest mass) which for objects at rest, boils down to the famous $E = m_0c^2$.

That's all beyond the scope of this course, but if you are interested, it needs no more than high-school algebra and a lot of very hard thinking to work this all out.

LESSON 9: Black Holes

SPACE-TIME DIAGRAMS

Relativity is often best understood by means of space-time diagrams: diagrams with a coupld of spatial coordinates shown horizontally (e.g. x and y) but with time shown vertically. In a diagram like this, a stationary object is a vertical line. The faster something moves, the more slanted the line is.

As nothing can move faster than light, there is a maximum possible angle any motion can take, corresponding to the speed of light. This means that for any point in space and time, you can define a "light cone", indicating where light emitted at that point could get to. As any real object can only travel at the speed of light or less, this means that our point can only influence things inside this cone, and that if you were at that point in space and time, you can only ever reach regions within this cone.

Gravity causes these light-cones to tilt over (by distorting space-time). If the light cones are tilted over so much that the far side of the cone is vertical, then staying stationary becomes impossible - you have to move inwards. This is a black hole.

They thus cannot have solid surfaces - matter on the surface would have to move faster than light just to stay put. Everything must thus collapse down to a singularity - a point of huge mass but zero size, and hence infinite density.

EVENT HORIZON

While the actual black hole has a radius of zero, you can define an “event horizon” radius, also known as the Schwarzschild radius - the radius at which the edge of the light cones becomes vertical, and hence the radius from within which nothing can escape. This radius is given by the equation

$$r_s = \frac{2GM}{c^2}$$

If you fell into a black hole, you would feel nothing as you passed the event horizon - it is not a solid barrier. From the outside, however, the light you emitted just before crossing the event horizon would need a very long time to struggle back out of the gravity field - so distant observers would see an enormously redshifted and slowed down version of you last moments before crossing the event horizon drawn out for ever.

In practice, black hole event horizons are very small - most objects with any appreciable angular momentum (which means virtually everything in the universe) will orbit around black holes rather than falling in.

For physically extended objects (like astronauts), however, even if they do not fall into the event horizon, they can still be seriously distorted if they come near, as some parts of the object will be exposed to much stronger gravity (and hence require faster orbits) than others. This distortion is called “Spaghettification”.

FORMATION OF BLACK HOLES

How do you form a black hole? One possibility is through the collapse of a very massive star. Normally massive stars produce neutron stars, but neutron stars (just like white dwarfs) have radii that decrease as their mass increases. If they are massive enough, their radius will be smaller than their event horizon, and then nothing can save them from collapsing down to a singularity. The limiting mass is probably around 2 solar masses. These collapses could produce one type of gamma-ray burst.

In principle, the merger of two neutron stars (a binary pair, radiating energy via gravity waves) could produce a black hole. In practice, it is not clear if this really happens - the merger would be extremely messy and maybe spinning too fast to collapse. But if it works, this might be another type of gamma-ray burst.

A third possibility is for slow but steady accretion onto a neutron star surface from a binary companion to push the neutron star over the mass limit and cause it to collapse. In practice this too may be hard - any gas falling onto a neutron star becomes so hot that its radiation can blow away any further infalling gas.

OBSERVING BLACK HOLES

Do they actually exist? An isolated black hole will be almost impossible to detect, but if a black hole is easting nearby gas or stars, it could become very bright.

One of the best candidates is Cygnus X–1: a massive blue star orbiting very rapidly in a circle around something dark. As the dark "something" has 10–20 times the mass of a Sun, it is probably a black hole.

In the centre of our own galaxy, we see stars accelerating around elliptical orbits: orbits around something dark, that must have around four million times the mass of our Sun, and yet be very small and dark. That too sounds very much like a black hole.

Virtually all galaxies seem to have big black holes in their centres, and the mass of the black hole correlates remarkably tightly with the mass of the central bulge of the galaxy. This is presumably telling us that it some way the formation and evolution of the black hole and its surrounding galaxy are tightly coupled. The biggest black holes are in the centres of giant elliptical galaxies.

ACTIVE GALACTIC NUCLEI

A tiny fraction of these galactic centre black holes are incredibly bright and are called active galactic nuclei (AGNs). Some of the brightest, which also emit radio jets, are called quasars. They shine by tearing apart matter falling in towards them, and hence liberating some fraction of their gravitational potential energy as heat before the matter disappears into the event horizon (see course 1).

Most galaxies, however, do not have anything this bright in their centres. In some cases this is because the central black hole is obscured by dust, but in most cases it is probably because the black hole has already eaten everything close to it, and hence cannot shine any more.

It is commonly believed that galaxy collisions feed gas into the central regions and hence cause AGN to light up. Unfortunately, galaxies in the middle of collisions seem to be no more likely than other glaxies to host an active nucleus - this is a puzzle.

BLAZARS AND SUPERLUMINAL MOTION

One of the strangest types of active galaxy are blazars. In these objects, a jet of material is travelling close to our line of sight at very near the speed of light.

Blobs in these jets appear to move away from the nucleus at faster than the speed of light. This is an optical illusion - the apparent speed of these jets is accelerated because they almost catch up with their own light. If a jet is moving at a fraction β of the speed of light at an angle of θ to the line of sight, the apparent velocity will be:

$$v_{app} = \frac{\beta \sin \theta}{1 - \beta \cos \theta}$$

CONCLUSIONS

While not totally certain, it now seems very likely that black holes exist. In the future it may be possible to actually see the shadow of the black hole in the galactic centre.

There are many speculations of strange physics around black holes (such as radiation from their surfaces, or falling through them into another dimension). These rely on quantum gravity theories, which are not currently experimentally testable.