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First Semester 2019
Mathematical Sciences Institute
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Assignment 2

This assignment must be submitted by **5th April 5pm**. Late Submissions will incur a 5% penalty per working day. Assignment submissions will close on the **12th April 5pm**. Submissions after this time will be invalid.

Question 1 (Gaussian Elimination)

1a. Compute a vector $\mathbf{u}$ such that $E = I - ue_2^T$ satisfies

$$Ex = E(x_1, x_2, x_3, x_4)^T = (x_1, x_2, 0, 0)^T.$$

$\mathbf{e}_2 = (0, 1, 0, 0)^T$

1b. Show that

$$M = (I - \alpha e_3 e_2^T)(I - \beta e_4 e_2^T)$$

is an elementary matrix.

1c. Find a vector x for which $Mx = (0, 4, 0, 0)^T$.

Question 2 (Linear Iterative Methods)

2a. Given a linear system of equations $Ax = b$ with a symmetric positive definite matrix $A \in \mathbb{R}^{4\times 4}$ which has eigenvalues $1, 1/4, 1/9, 1/16$. Consider the iterative method defined by

$$x^{(k+1)} = x^{(k)} - \omega(Ax^{(k)} - b).$$

Can you choose ω such that method is convergent? If so, what is the best possible ω?

2b. Discuss the convergence of the Jacobi method for $Ax = b$ with the tridiagonal matrix

$$A = \begin{bmatrix} 4 & -1 & & \\ -1 & 5 & -1 & \\ & -1 & 6 & -1 \\ & & -1 & 3 \end{bmatrix}.$$

- Does the Jacobi method converge for this matrix?
- What is the convergence rate?

Question 3 (Gradient Descent and CG)

3a. Show that for the steepest descent method the direction vectors $d^{(k)}$ and $d^{(k+1)}$ are orthogonal.

3b. Let $A = \begin{bmatrix} 2 & -1 & & \\ -1 & 5 & -1 & \\ & -1 & 6 & -1 \\ & & -1 & 3 \end{bmatrix}$. Show that A is positive definite. Discuss the convergence of the conjugate gradient method for $Ax = b$ with this matrix A and arbitrary vectors b.

Question 4 (Nonlinear Equations and Bisection)

4a. Show that $f(x) = x^4 - 8x - 2$ has a zero in the intervall $[0, 4]$. 4b. How many iteration steps would you require with bisection to get an error $|e^{(k)}| < 0.01$?

Question 5 (Fixed Point Methods)

Consider $f(x) = x^4 - 8x - 2$. How would you choose α such that

$$x^{(k+1)} = F(x^{(k)})$$

is convergent where $F(x) = x - \alpha f(x)$ for $x \in [0, 4]$.

Question 6 (Newton's Method)

Does the Newton's method for $f(x) = x^4 - 8x - 2$ converge for any point.

Find the point for which the Newton's method does not converge for this function