Australian
National
University

ASTR2013 – *Foundations of Astrophysics*

Week 1: Making Astronomical Observations. Basic observations of Stars.

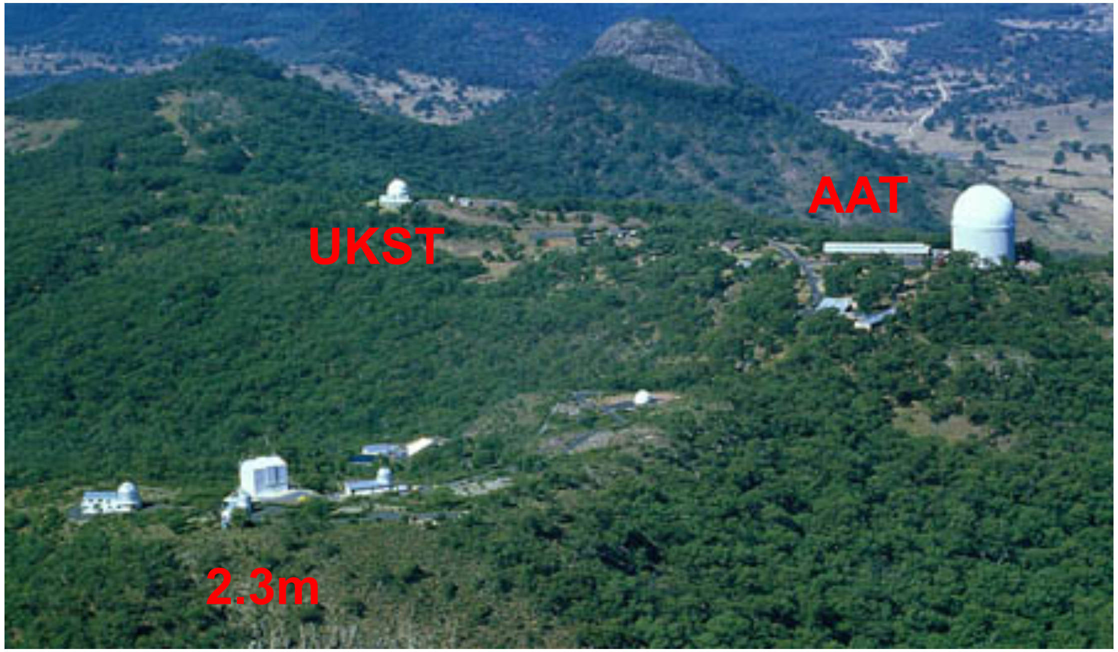
Mike Ireland

Intro to RSAA

- Established as Commonwealth Solar Observatory in 1924
- Became part of the Australian National University in 1954
- Established Siding Spring Observatory in 1960
- We are ranked as one of the top 10 research institutes in the world for astronomy and space science.
- Our staff have been recognized with major prizes including the 2011 Nobel Prize for Physics.

Offices, Labs, Observatories

- As Canberra grew, "dark sky" astronomy moved to Siding Spring observatory.
- At 6.5 hours driving time, you can only go there on a vacation project or… the ASTR2013 field trip!



New Astrophysics Major

- ASTR2013 leads into ASTR3002, ASTR3007, ASTR3013, ASTR8011 and other honours/Masters subjects.
- Not a pre-requisite yet, but will almost certainly become one in time (esp. for ASTR3007).
- Most important for a successful 3rd year research project! (ASTR3005 or PhB)

36 units from completion of the following courses:

PHYS1101 Physics 1

PHYS1201 Physics 2

PHYS2013 Quantum Mechanics

PHYS2020 Thermal and Statistical Physics

ASTR2013 Foundations of Astrophysics

ASTR3013 Astrophysical Processes

A minimum of 6 units from completion of one of the following courses:

ASTR3002 Galaxies and Cosmology

ASTR3007 Stars

A maximum of 6 units from completion of one of the following courses:

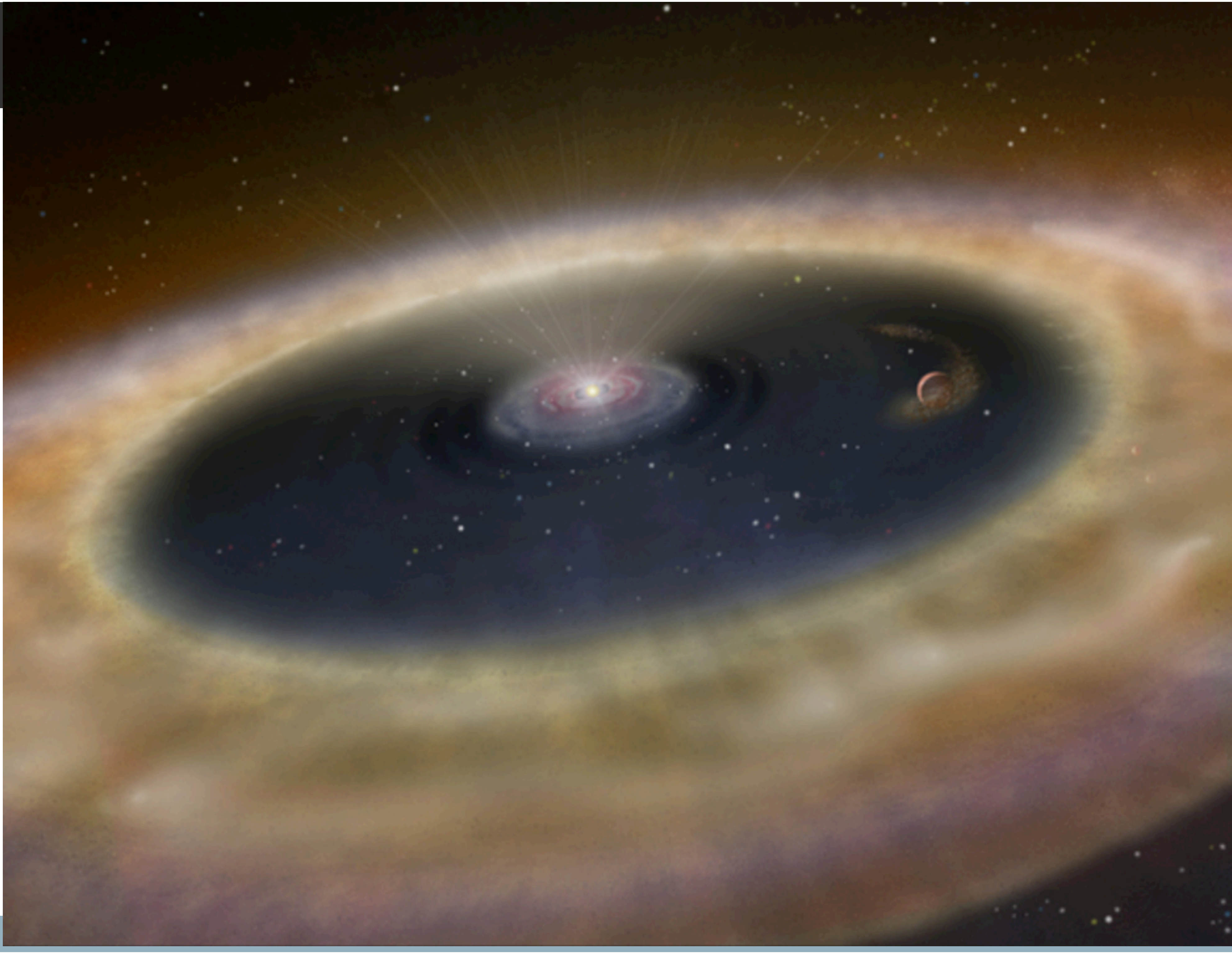
ASTR3005 Astrophysics Research Topic

EMSC3022 Planetary Science

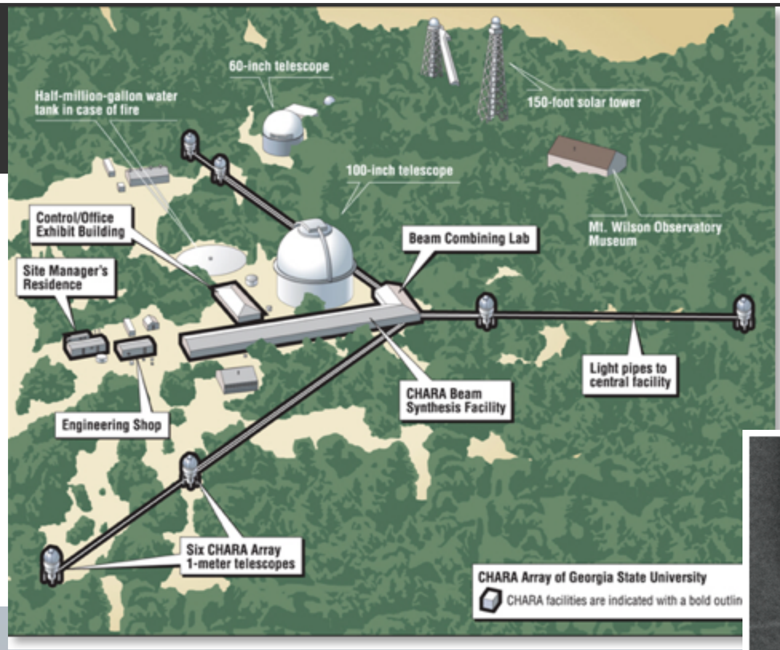
PHYS3057 Optical Physics

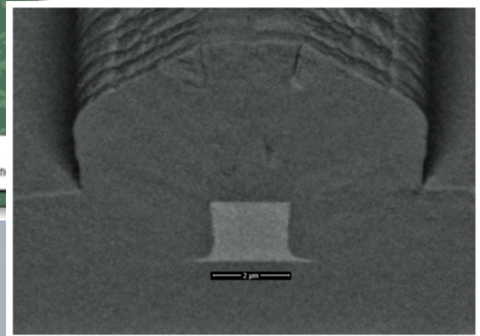
Intro to Me and your Tutors

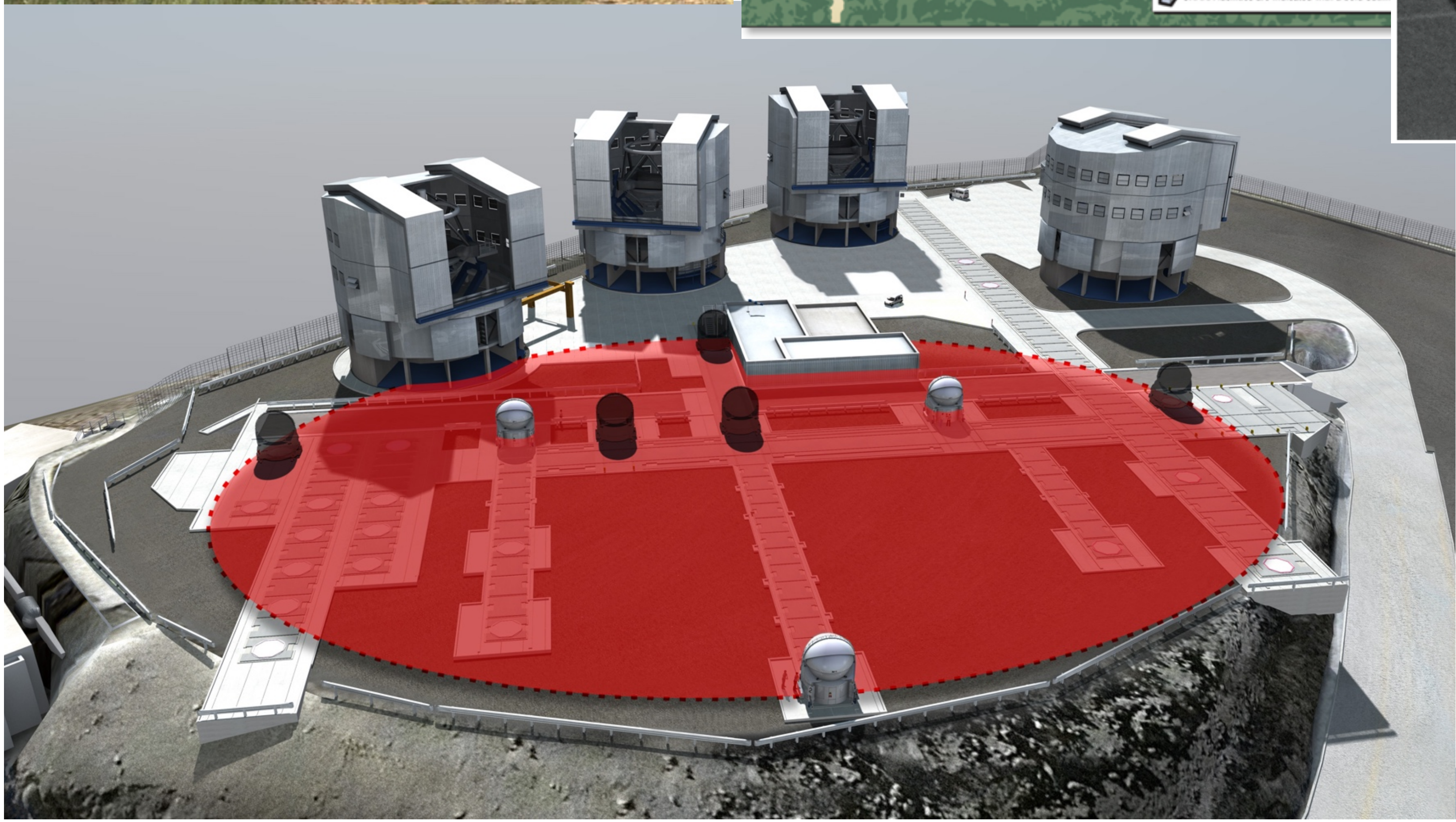
- I’m ~40% an observational stellar and planetary astrophysicist, 40% a scientist developing new instruments for telescopes, and 20% "other”.
- See http://www.mso.anu.edu.au/~mireland, Google Scholar and NASA ADS.
- Rotating Tutors are current PhD students:
  - Jack Livingston (radio observations of diffuse gas in the Galaxy and environs)
  - Stephanie Monty (galactic archaeology, metal poor stars and new instruments)
  - Abdu Abohalima (3D computational models of stars)
  - Jamie Soon (instrumentation and infrared bright supernovae)




SYDNEY UNIVERSITY STELLAR INTERFEROMETER
SUSI
AUSTRALIA


60-inch telescope
Half-million-gallon water tank in case of fire
150-foot solar tower
100-inch telescope
Control/Office Exhibit Building
Beam Combining Lab
Mt. Wilson Observatory Museum
Site Manager's Residence
Light pipes to central facility
CHARA Beam Synthesis Facility
Engineering Shop
Six CHARA Array 1-meter telescopes
CHARA Array of Georgia State University
CHARA facilities are indicated with a bold outlin





Course Structure and Expectations

- See the "Class Summary" for Key details of timeline, assessment and policies…
- Schedule largely follows the text:
  Dan Maoz: Astrophysics in a Nutshell.

Class Schedule

WEEK/SESSION	SUMMARY OF ACTIVITIES	ASSESSMENT
1	Week 1 Making Astronomical Observations. Basic observations of Stars.	
2	Week3 2-3 Stellar Physics	
3	Weeks 4-5 Stellar Evolution	
4	Week 6 Practical Astronomical Observations and Coordinate Systems	
5	Week 7 The Interstellar Medium and Star Formation	
6	Week 8 Structure and Dynamics of the Milky Way	
7	Week 9 Other Galaxies and Active Galactic Nuclei	
8	Week 11 Big Bang Cosmology (NB Week 10 a Public Holiday)	
9	Week 12 Special Topic in Astrophysics	

Course Structure and Expectations

- Assessment is split between by problem sets (first one given out later today), final exam and the field trip.
- Field trip will be ~9am on 2 Sep to 6pm on 5 Sep, and includes using the ANU 2.3m telescope in small groups. Cost is $200, with more official details later this week (but please ask questions now!)

Assessment Summary

ASSESSMENT TASK	VALUE	DUE DATE	RETURN OF ASSESSMENT
Problem Sets	40 %	02/08/2019	01/11/2019
Field Trip Report	20 %	20/09/2019	11/10/2019
Participation and Performance during Field Trip	10 %	05/09/2019	13/09/2019
End of Semester exam	30 %	31/10/2019	28/11/2019

Course Structure and Expectations

- The 3 hours per week face to face plus ~14 hours compulsory assessable activities on the field trip makes up less than half the time you're expected to spend on this course.
- Each week, you'll should spend at least an hour reading the applicable textbook sections, some time revising and the rest on the problems sets and assessments.
- Every second tutorial will provide and opportunity to discuss the assessable problem sets with tutors and peers. Work is expected to be your own, but we will cover related problems in tutorials and you're certainly free to ask questions.
- We will also need 2 class representatives to provide feedback, and I'll be hoping for volunteers during tutorial.

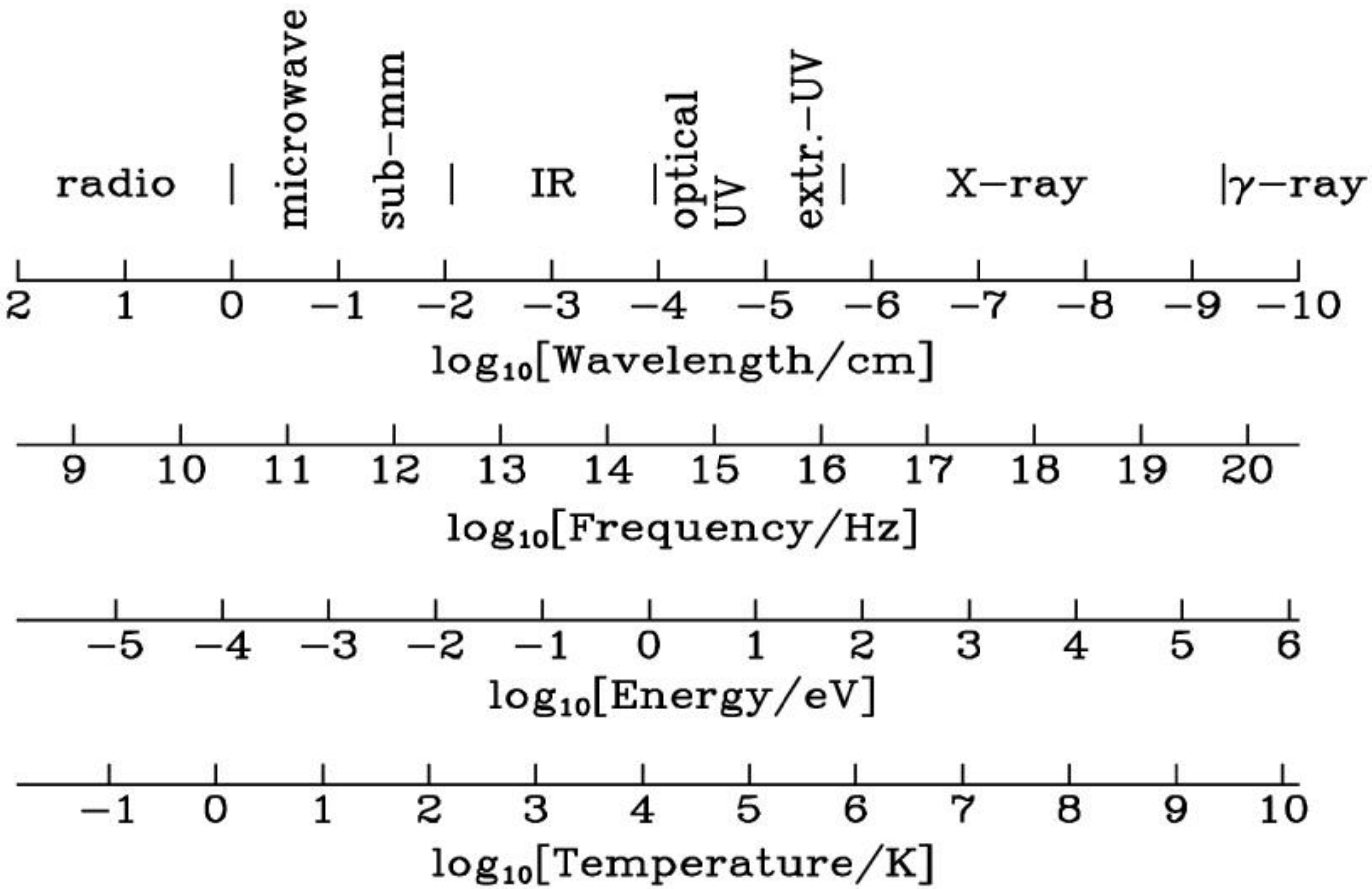
Questions now?

Week 1 Summary

Textbook: Sections 1.1, 2.1 and 2.2.

1. Angular resolution and Diffraction Limit.
2. Telescopes as photon counters.
3. Blackbody radiation
   a) Energy density.
   b) Specific Intensity and Flux.
   c) Asymptotic forms (Wien and Rayleigh-Jeans)
4. Three main observation types:
   a) Photometry: counting photons.
   b) Spectroscopy: measuring photon wavelength or frequency.
   c) Astrometry: measuring photon angle of arrival.

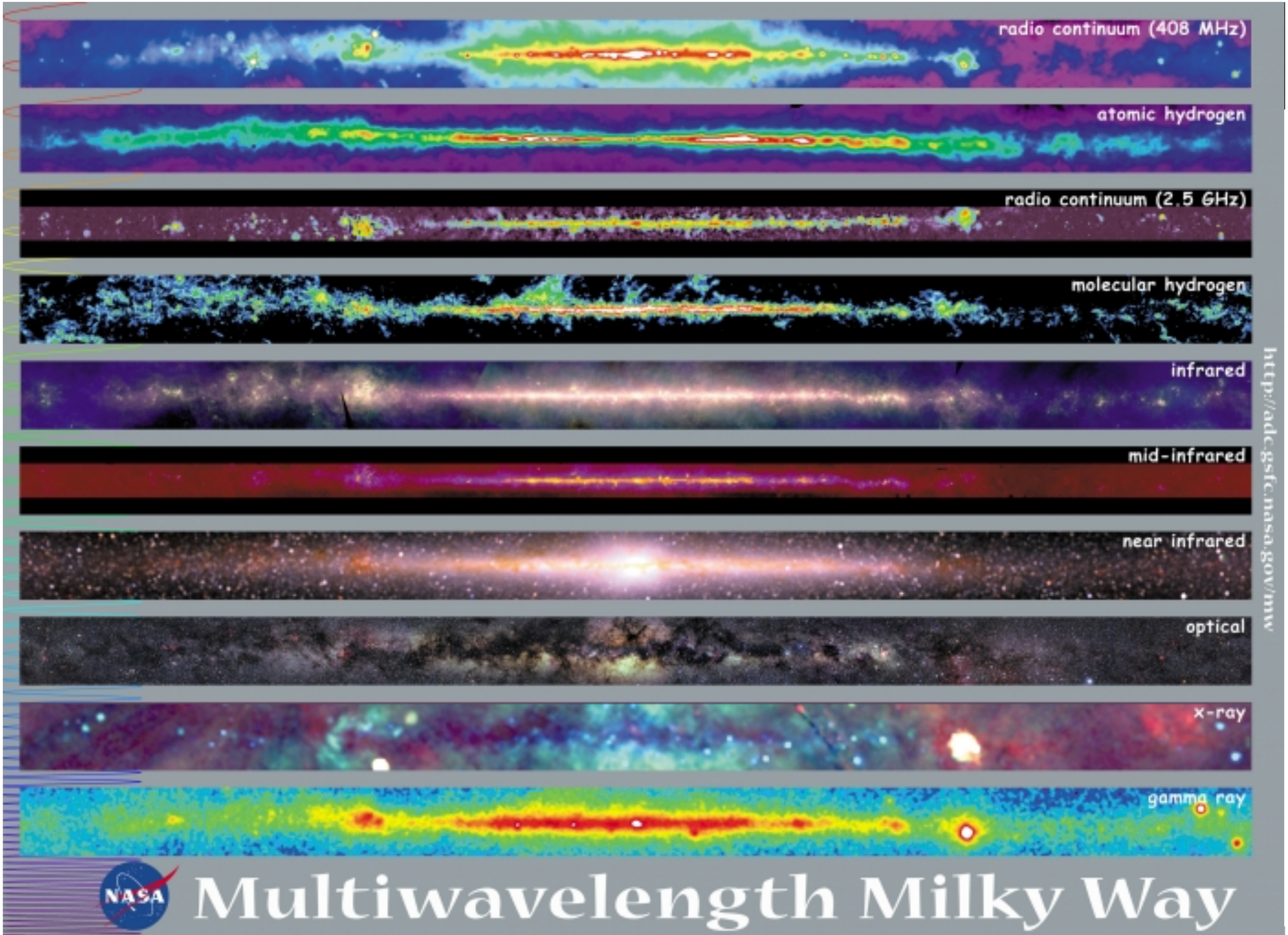
Electromagnetic Waves as Primary Probes



$$\lambda = \frac{c}{\nu}$$

$$E = h\nu$$

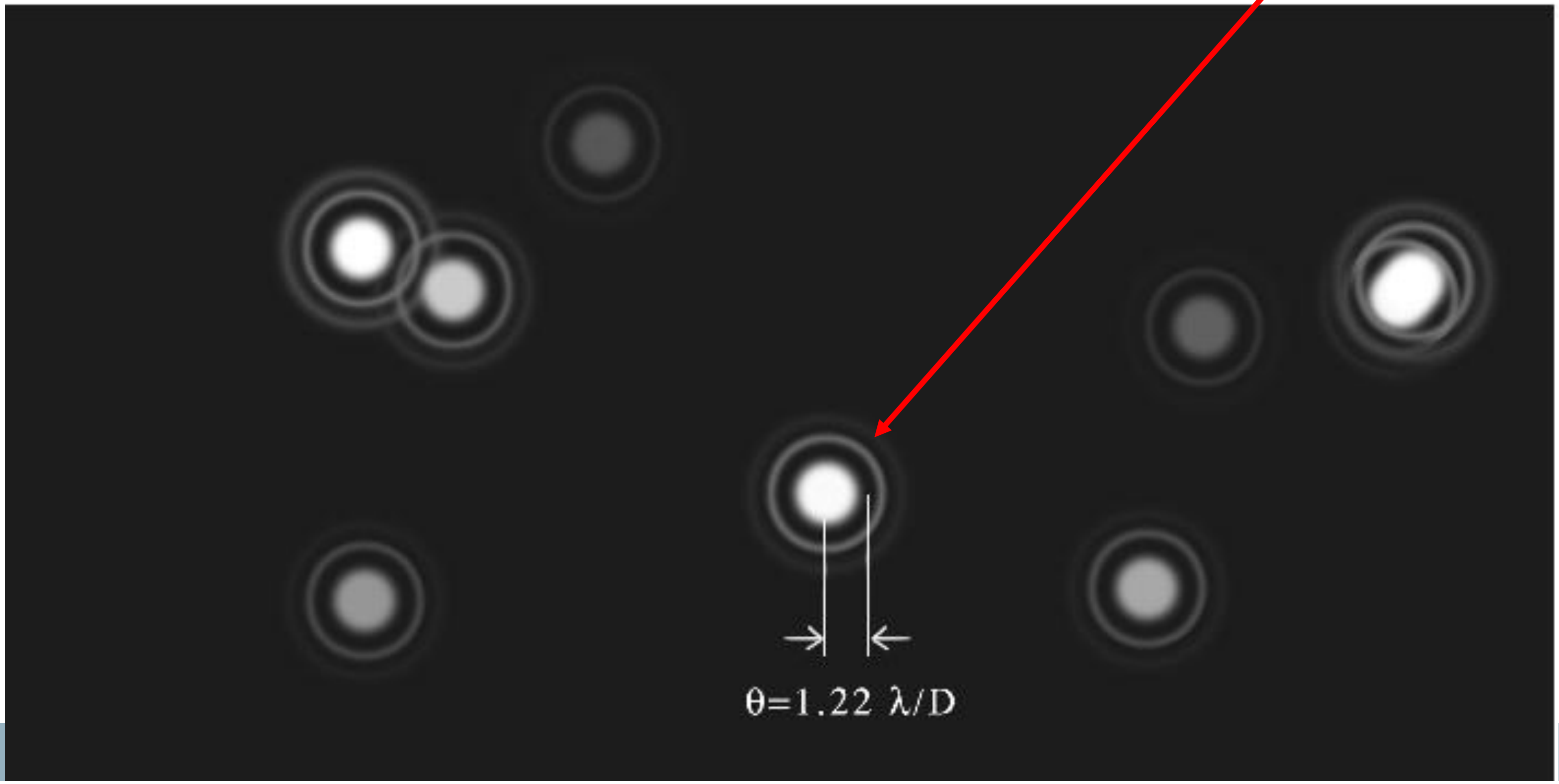
$$E = k_B T$$


radio continuum (408 MHz)
atomic hydrogen
radio continuum (2.5 GHz)
molecular hydrogen
infrared
mid-infrared
near infrared
optical
x-ray
gamma ray
http://adc.gsfc.nasa.gov/mw
NASA
Multiwavelength Milky Way

Diffraction Limit

- Wave nature of light means that an aperture of size ~λ diffracts light by about a radian.
- Larger circular apertures are described by *Fraunhofer Diffraction.*

“Airy Disk” (described by Bessel Functions)



Photons and Quantum Mechanics

- Whenever we want to add up the energy in an electromagnetic wave, we can only do so in discrete units - photons. Each photon has an energy equal to:

$$\begin{aligned} E &= h\nu \\ &= \frac{hc}{\lambda}. \end{aligned}$$

- Many astronomical measurements are about counting photons - the rate of photons arriving in a range of wavelengths (or frequency) tells us the about the energy output of the astronomical source at that wavelength (frequency) range.
- Although photons are the observed units of energy, there are other more general definitions we have to consider…

Poisson Statistics In More Detail

- Applies only to light in the particle approximation…
- If the number of photons expected to arrive in a particular aperture from a particular range of angles in a particular wavelength (frequency) range is *r*, then the actual number of detected photons has a Poisson distribution:

$$P(N = k) = \frac{r^k e^{-r}}{k!}$$

- Importantly, the variance of *N* is *r*, the standard deviation is $\sqrt{r}$, so:

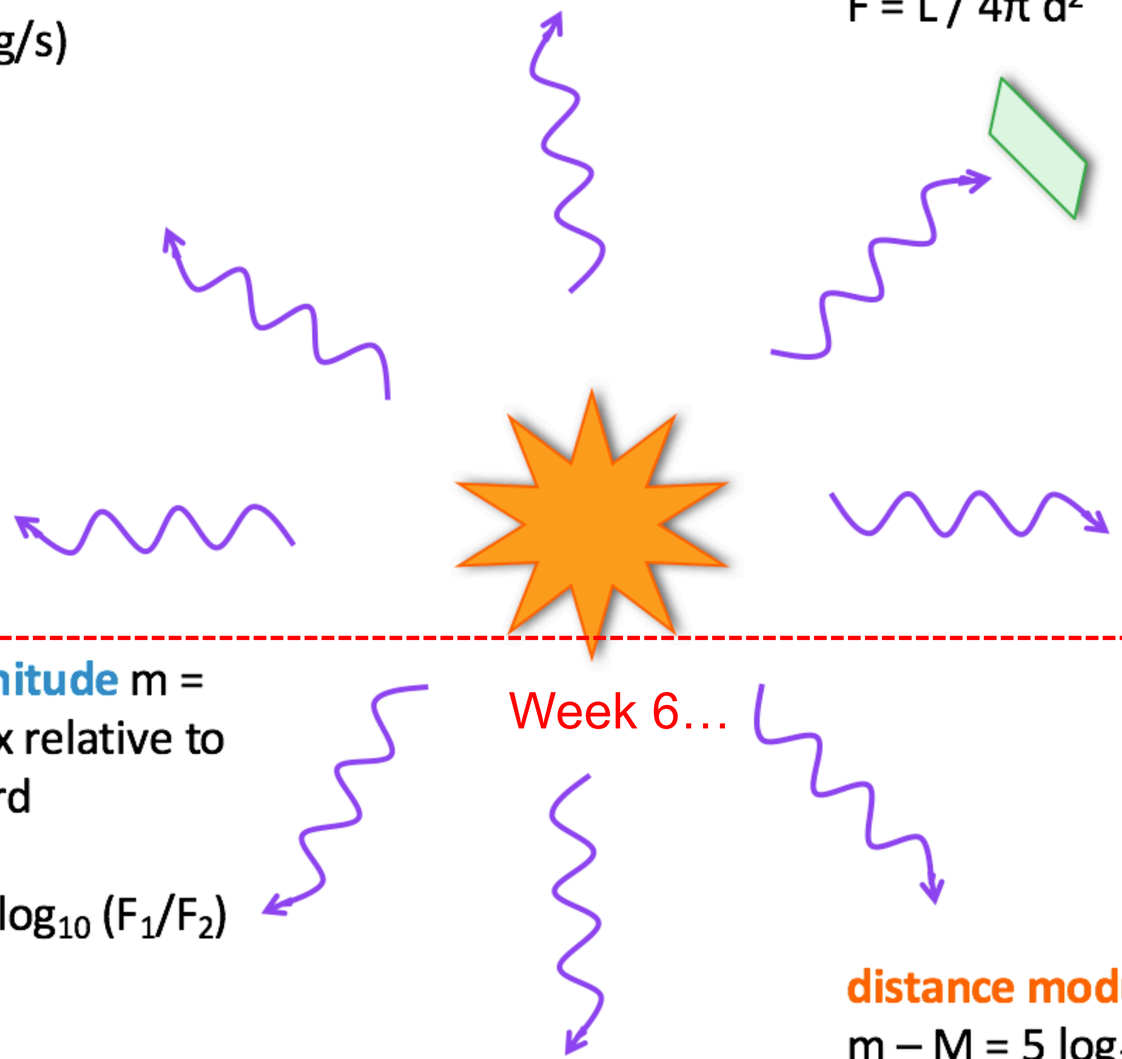
$$S/N = \sqrt{r} \approx \sqrt{N}$$

See python example

Luminosity and Flux

intrinsic **luminosity**
L = energy emitted
per second (erg/s)

flux F = energy received
per unit area
$F = L / 4\pi d^2$ (erg/s/cm^2)



apparent magnitude m =
measure of flux relative to
chosen standard

$m_1 - m_2 = -2.5 \log_{10} (F_1/F_2)$

distance modulus
$m - M = 5 \log_{10} (d/10pc)$

absolute magnitude M = measure of flux
you would see if you were 10 pc away

Luminosity and Flux

intrinsic **luminosity**
L = energy emitted
per second (erg/s)

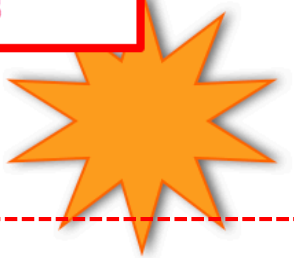
Luminosity can be either in a given band or "**bolometric**" = covering all wavelengths

But remember these are all a function of wavelength!

flux F = energy received per unit area
$F = L / 4\pi d^2$ (erg/s/cm²)

flux density = flux per unit wavelength or per unit frequency

F_λ = erg/s/cm²/Å
F_ν = erg/s/cm²/Hz or Jansky (Jy) = 10^{-26} W/m²/Hz



Week 6…

apparent magnitude m = measure of flux relative to chosen standard

$m_1 - m_2 = -2.5 \log_{10} (F_1/F_2)$

distance modulus
$m - M = 5 \log_{10} (d/10\text{pc})$

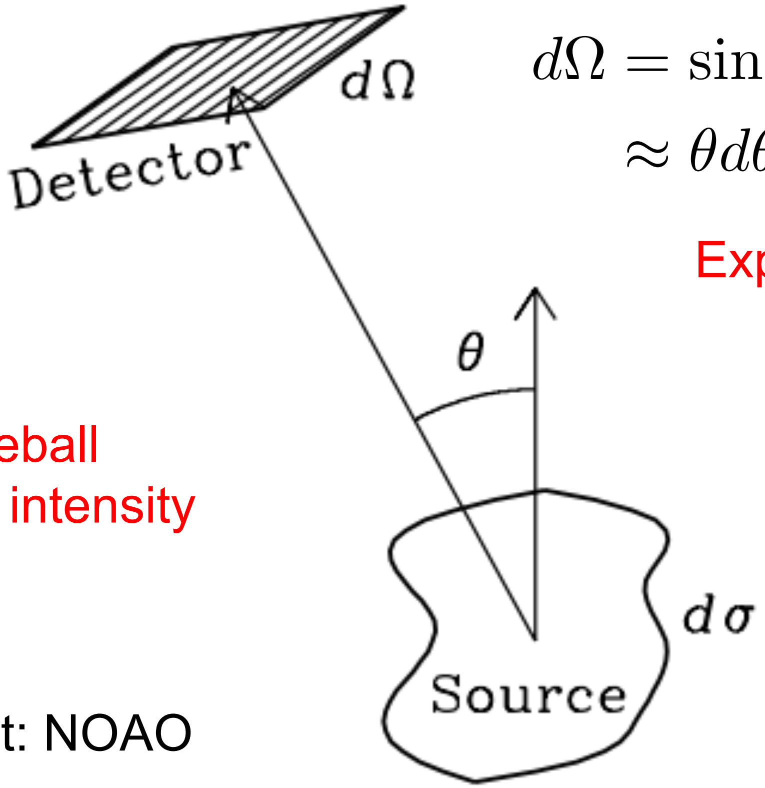
absolute magnitude M = measure of flux you would see if you were 10 pc away

Flux Units

- Units are either based on SI, or “cgs” (centimeters-grams-seconds). I will try to *mostly* stick with SI units, but will not be completely consistent.
- 1 erg = 10^{-7} Joule.
- Energy received through a telescope from a given source depends on wavelength (frequency), bandpass and telescope size.
- f_λ is the energy per unit time, per unit telescope area per unit wavelength.
- f_ν is the energy per unit time, per unit telescope area per unit frequency.
- The Jy = 10^{-26} $Wm^{-2}Hz^{-1}$, a standard unit of f_ν , originally from radio astronomy.

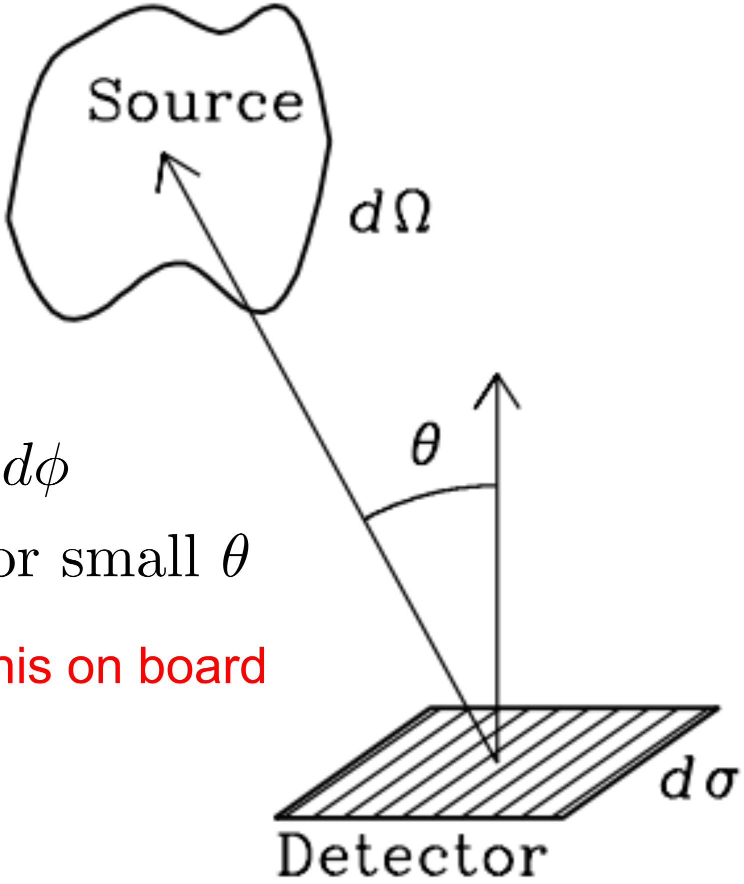
Specific Intensity or Surface Brightness

- Specific intensity is flux density per unit solid angle.
- Specific intensity is conserved in a lossless optical system (including free space).



$$d\Omega = \sin(\theta)d\theta d\phi$$
$$\approx \theta d\theta d\phi \text{ for small } \theta$$

Explain this on board



$$dW_\nu = \overbrace{\underbrace{I_\nu(\nu)}_{\text{emitted}}}^{\text{received}} \, d\Omega \, \cos\theta \, d\sigma \, d\nu$$

Play eyeball specific intensity game

Credit: NOAO

Specific Intensity to Flux

- If we have a specific intensity of an object being viewed at near-normal incidence to an aperture, simply multiply by the solid angle being integrated over to get flux. i.e.

$$F_\nu = \int I_\nu d\Omega \approx I_\nu \Delta\Omega$$

- If we have an isotropic radiation field, e.g. a black-body, then we can find the specific intensity from the flux (tutorial question or problem for board):

$$I_\nu = F_{\nu,\mathrm{isotropic}}/\pi$$

Energy Density and Specific Intensity

- We can also think of a radiation field as local energy flow at the speed of light.
- Then I_ν/c is the energy density of radiation for a small solid angle $d\Omega$, and we can find the total energy density by:

$$u_\nu = \frac{1}{c}\int I_\nu d\Omega$$

(I find this more intuitive than the textbook's differential form)

- For an *isotropic* radiation field:

$$u_\nu = \frac{4\pi}{c} I_\nu$$

Blackbody Radiation

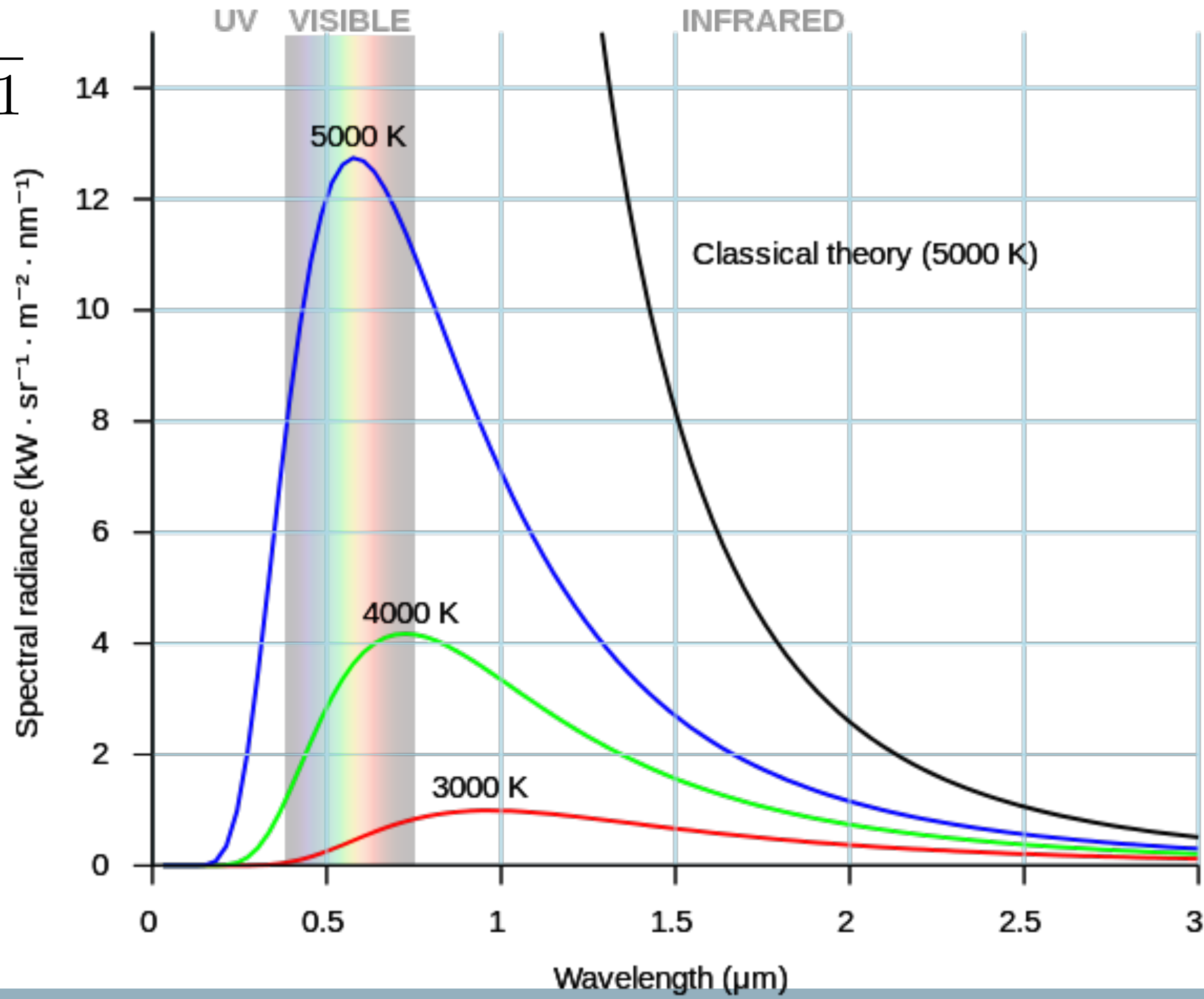
- One of the easiest ways to determine an objects temperature is to look at the radiation it emits. *Blackbody radiation* is the radiation emitted from a body that has perfect emissivity.
- One way to get near-perfect blackbody radiation is to punch a small hole in a cavity of uniform temperature. This evens out the effects of albedo variations.
- Astrophysics doesn't have perfect cavities... but we can talk about:
  - The *effective temperature* of a surface is the temperature of an blackbody that would emit the same total radiation.
  - The *brightness temperature* of a surface at wavelength λ is the temperature of a blackbody that would emit the same specific intensity at that wavelength.

Planck's Formula – Blackbody Radiation

$$\frac{dn}{d\nu} = \frac{1}{\exp(h\nu/k_B T) - 1} \text{ photons/s/Hz/mode}$$

$$f_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{\exp(hc/\lambda k_B T) - 1}$$

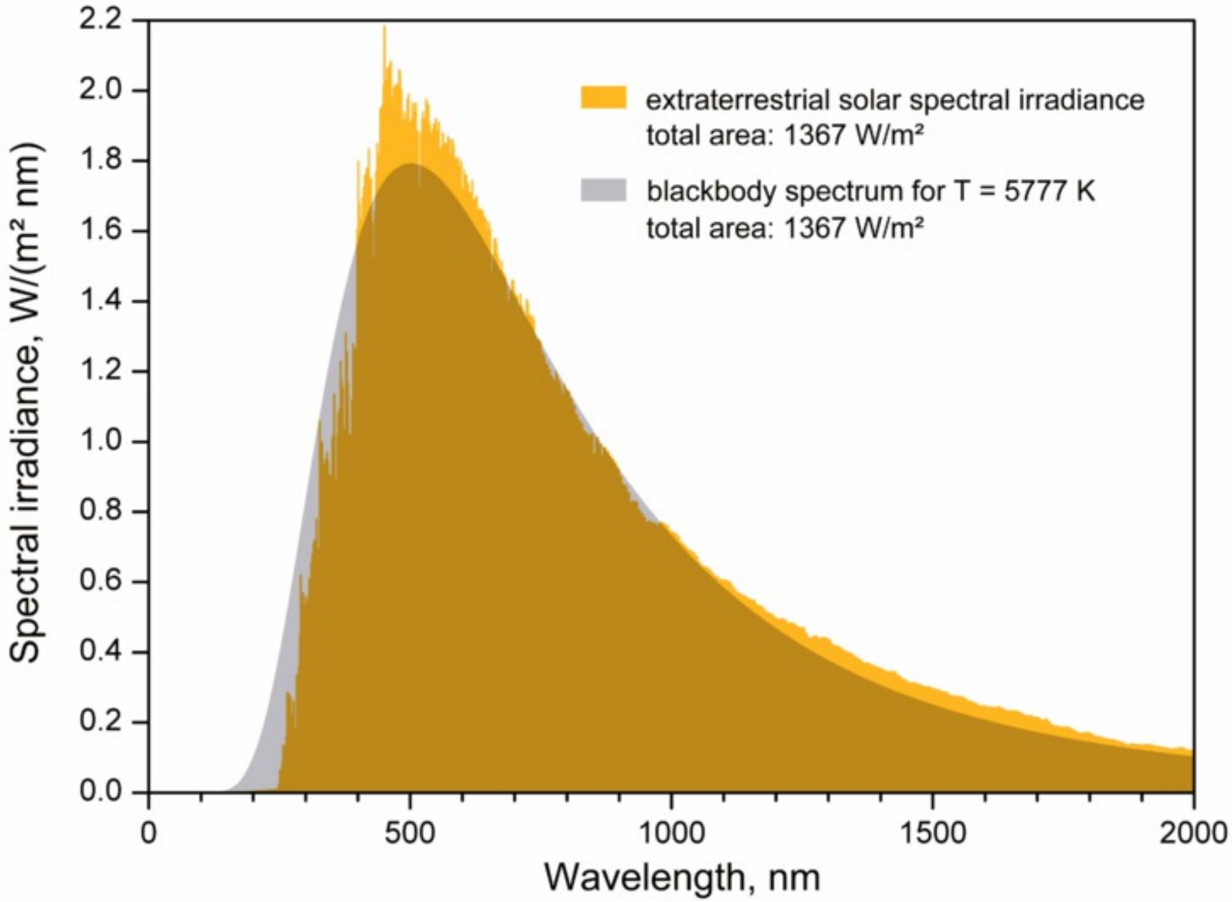
$$f_\nu = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/k_B T) - 1}$$



More on this in the tutorial…

Solar Radiation

- Given that the sun varies, we now define *nominal* values for the sun's properties (IAU):



$$T^{N}_{\mathrm{eff}\odot} = 5772\,\mathrm{K}$$

$$S^{N}_{\odot} = 1361\,\mathrm{W\,m^{-2}}$$

Week 1 Summary

Textbook: Sections 1.1, 2.1 and 2.2.

1. Angular resolution and Diffraction Limit.
2. Telescopes as photon counters.
3. Blackbody radiation
   a) Energy density.
   b) Specific Intensity and Flux.
   c) Asymptotic forms (Wien and Rayleigh-Jeans)
4. Three main observation types:
   a) Photometry: counting photons.
   b) Spectroscopy: measuring photon wavelength or frequency.
   c) Astrometry: measuring photon angle of arrival.

Inspiring Picture: Zodiacal Light (APOD 0902)

Australian National University

ASTR2013 – *Foundations of Astrophysics*

Week 2: Physics of Stars

Following Dan Maoz – Astrophysics in a Nutshell

Mike Ireland

Recap: Specific Intensity to Flux

- If we have a specific intensity of an object being viewed at near-normal incidence to an aperture, simply multiply by the solid angle being integrated over to get flux. i.e.

$$F_\nu = \int I_\nu d\Omega \approx I_\nu \Delta\Omega$$

- If we have an isotropic radiation field, e.g. a black-body, then we can find the specific intensity from the flux.
- See derivation PDF on wattle if the 1-liner in the textbook was too brief or you didn't follow the tutorial on the board.

$$I_\nu = F_{\nu,\mathrm{isotropic}}/\pi$$

$$f_\nu = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_\nu \cos\theta \sin\theta d\theta \, d\phi = I_\nu 2\pi \frac{1}{2} = \pi I_\nu = \frac{c}{4} u_\nu = \frac{2\pi h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}. \qquad (2.5)$$

Energy Density and Specific Intensity

- We can also think of a radiation field as local energy flow at the speed of light.
- Then I_ν/c is the energy density of radiation for a small solid angle $d\Omega$, and we can find the total energy density by:

$$u_\nu = \frac{1}{c}\int I_\nu d\Omega$$

(I find this more intuitive than the textbook's differential form)

- For an *isotropic* radiation field:

$$u_\nu = \frac{4\pi}{c} I_\nu$$

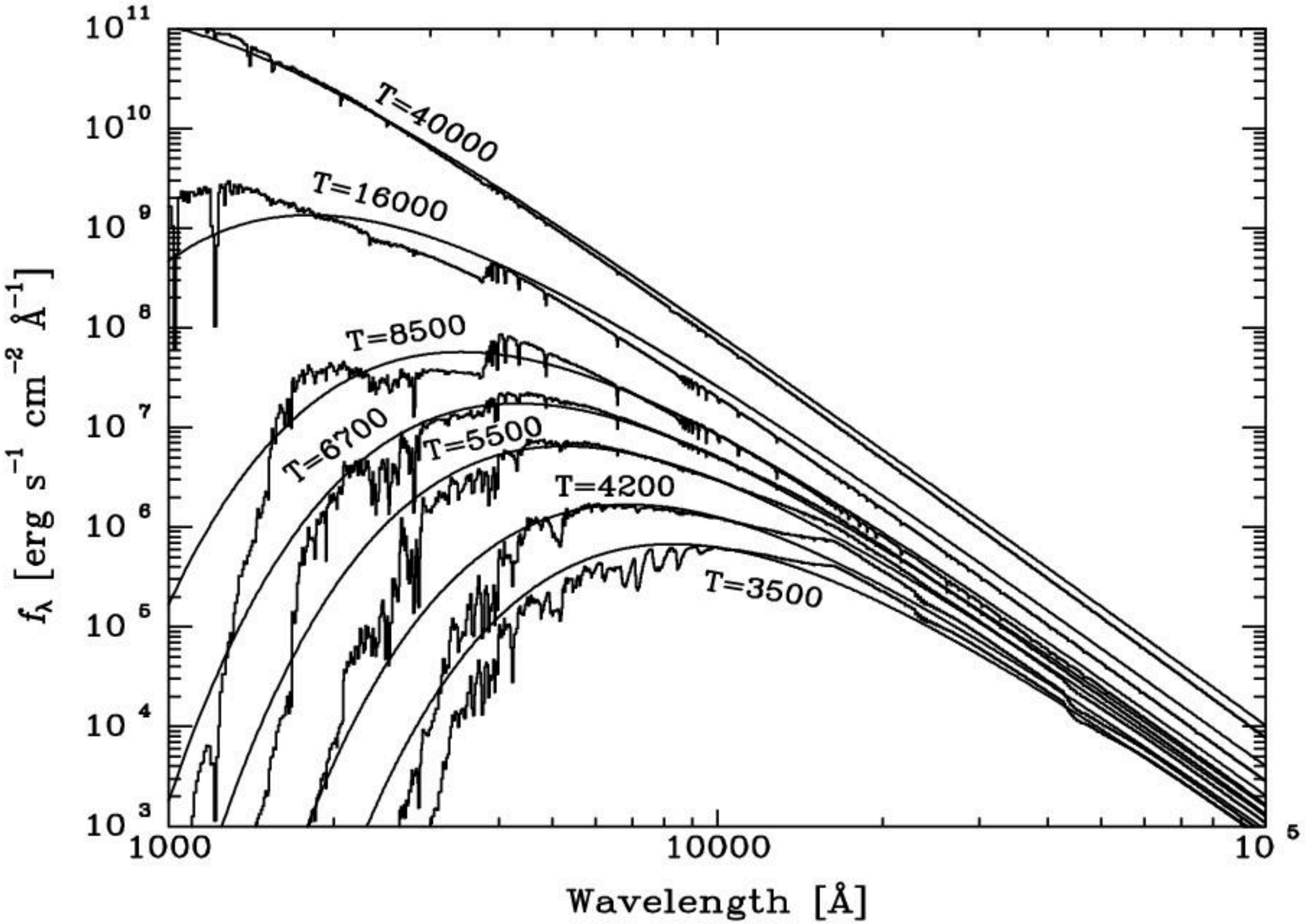
Week 2 Summary

Textbook: Sections Finishing 2.2, 2.3, 3.1, 3.8, 3.9 (not including Gamow energy).

1. Stellar Spectral Types
2. Orbits, Kepler's Law and Measuring Stellar Masses.
3. The Main Sequence and the HR Diagram.
4. Hydrostatic Equilibrium and the Virial Theorem.
5. The Kelvin-Helmholtz Timescale.

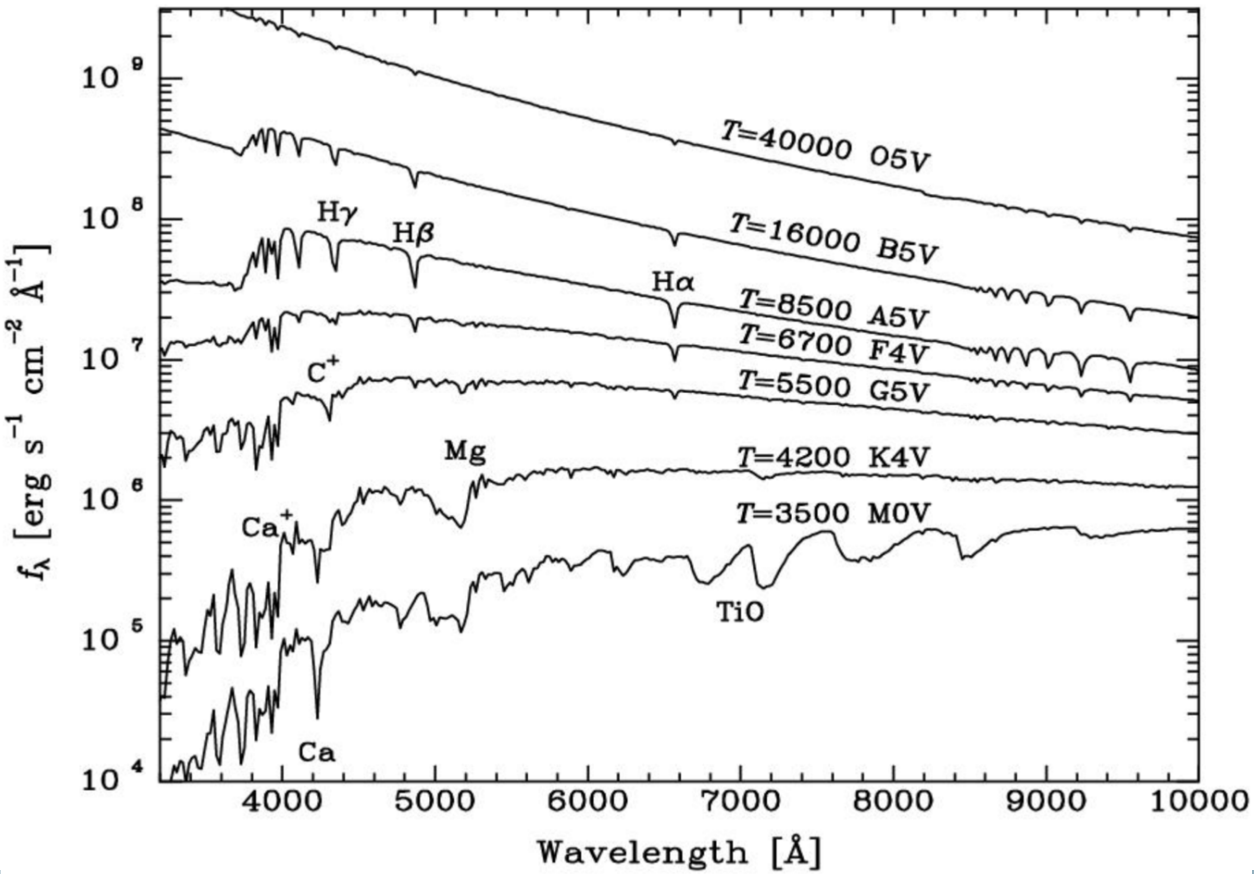
Stellar Spectral Types

Plots of flux density (spectra) broadly resemble blackbody curves, but have additional spectral features due to atoms/molecules in cool upper layers absorbing light.

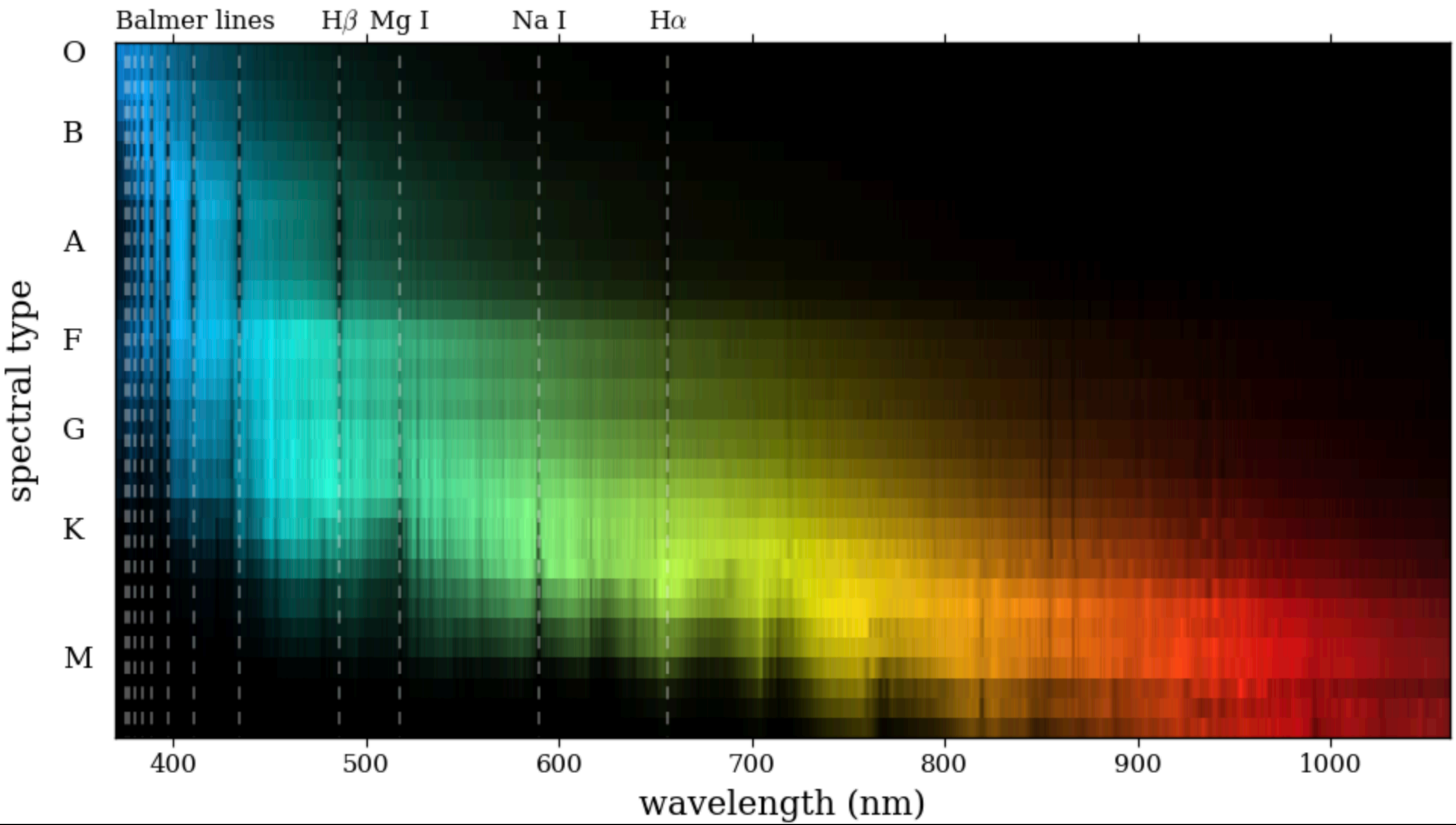


Stellar Spectral Types

Historically, spectral types were created before there was a deep understanding of what caused the spectral lines. My favourite modern reference on spectral type conversions is Erik Mamajek's home page (mostly in refereed papers): http://www.pas.rochester.edu/~emamajek/EEM_dwarf_UBVIJHK_colors_Teff.txt

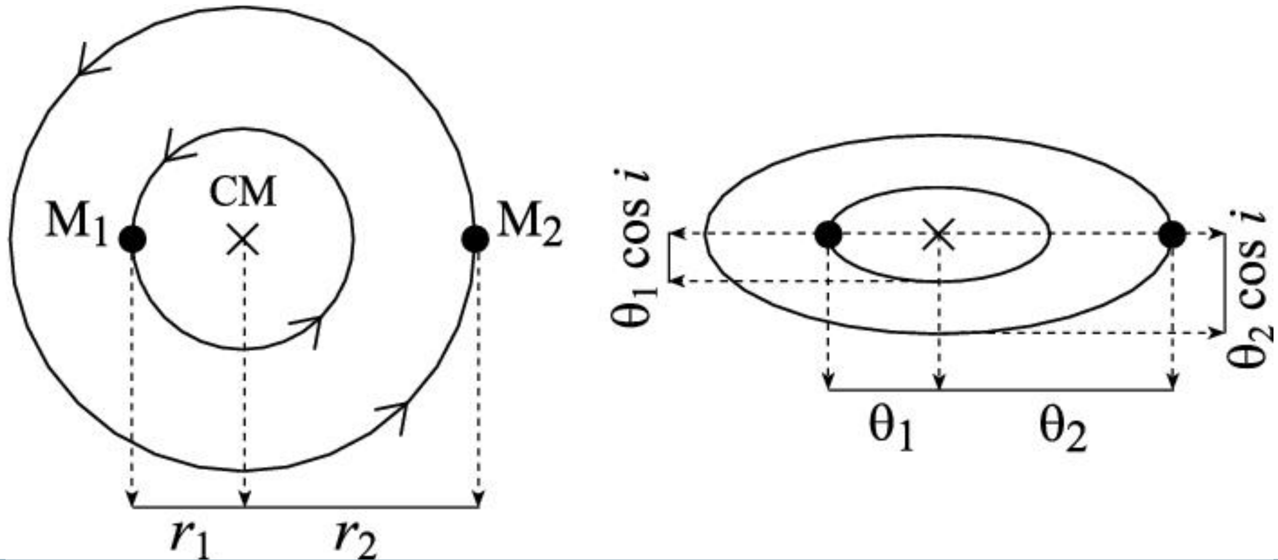


Stellar Spectral Types



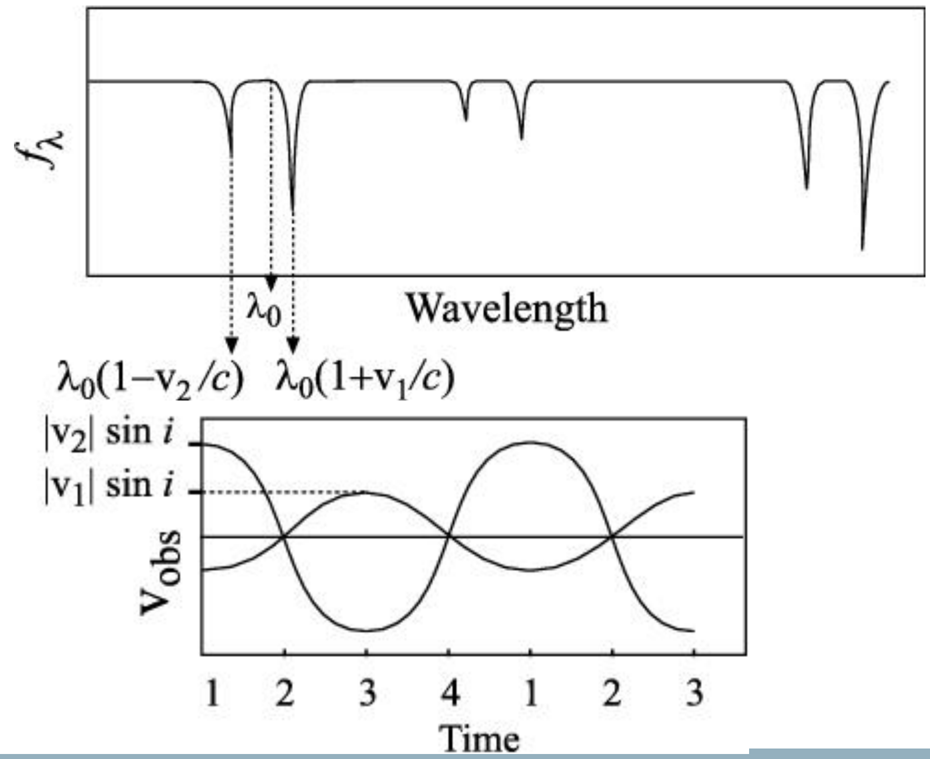
Kepler's Law and Mass Measurement

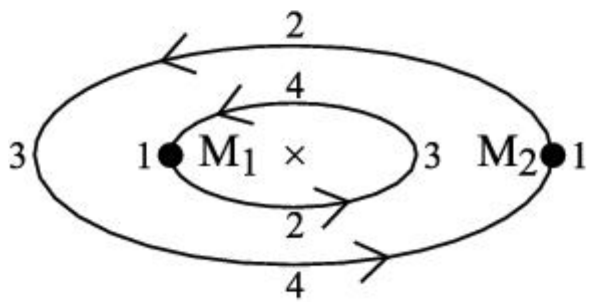
- Newton's law of Gravitation for circular orbits can easily be used to derive Kepler's law: $$\omega^2 = \frac{G(M_1 + M_2)}{a^3}$$
- Here a is the radius of the circular orbit. It turns out that this can be generalised for elliptical orbits, with a being the semi-major axis of an elliptical orbit, with the center of mass at the focus.



Kepler's Law and Mass Measurement

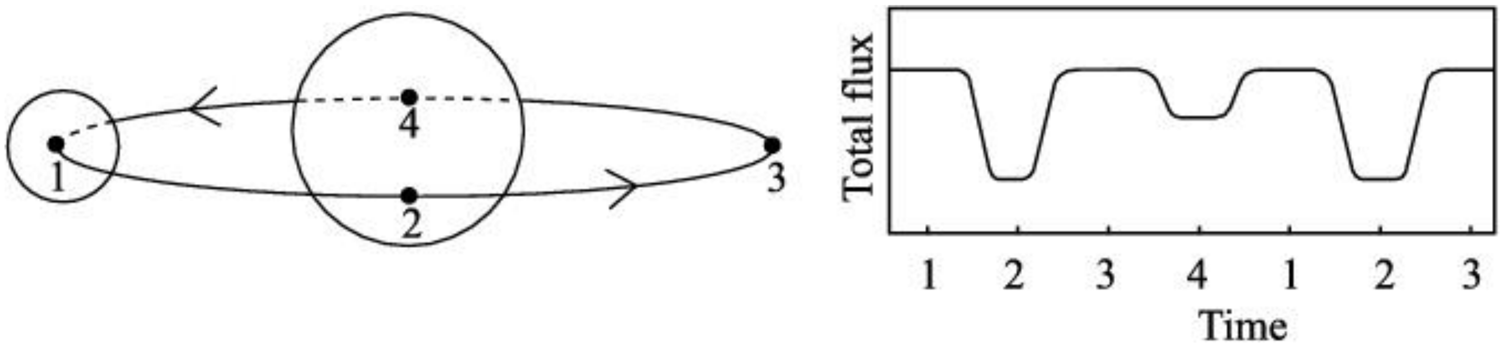
- In *Solar-Units*, we can write Kepler's law as: $$\left(\frac{M_{\mathrm{tot}}}{M_\odot}\right)\left(\frac{T}{1\,\mathrm{yr}}\right)^2 = \left(\frac{a}{1\,\mathrm{au}}\right)^3$$
- In practice, we can measure all orbital parameters other than inclination and position angle on the sky (e.g. orbit oriented North or East?) using Doppler-shifts and a *spectroscopic binary orbit.*
- Spectroscopic binaries can be *Single-lined* or *Double-lined,* depending on how bright the *secondary* is.





Measuring Inclination

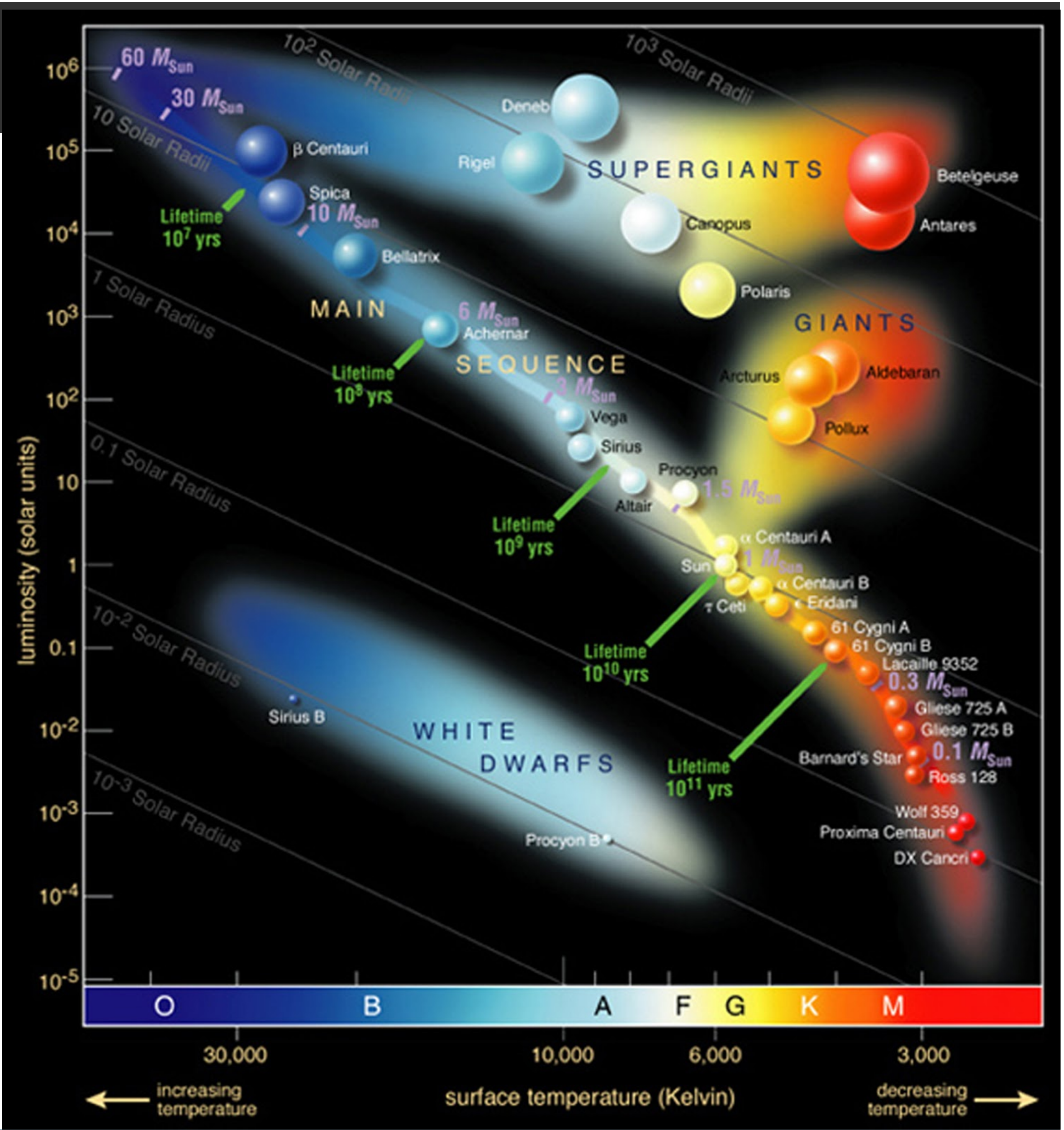
- Integrating the velocity in a spectroscopic binary gives the semi major axis, multiplied by sin(*i*).
- We can get the inclination *i* by visually resolving the orbit (e.g. with adaptive optics), or by observing a lucky orientation – an *eclipsing binary*.



The Main Sequence

Without any knowledge of stellar physics, we can plot luminosity and temperature, adding mass measurements, and observe that most stars are on the *Main Sequence*, although there are many exceptions.

Especially when plotted as colour and magnitude (week 6), this is a Hertzprung-Russell (HR) diagram.



Hydrostatic Equilibrium

- Like the earth's atmosphere, stars are typically in a stable state, governed by *hydrostatic* equilibrium. This is the momentum balance equation – forces on a parcel of gas neither accelerate it upwards or downwards.

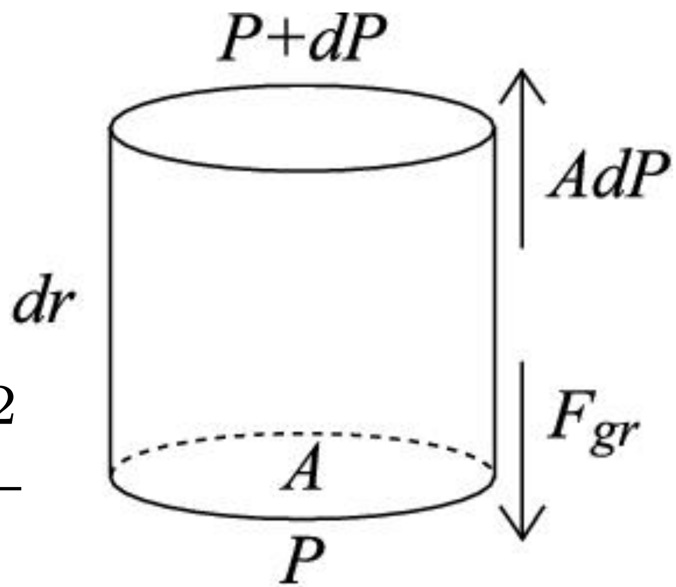
$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}.$$

- We can integrate this equation through the star:

$$\int_0^{r_*} 4\pi r^3 \frac{dP}{dr} dr = -\int_0^{r_*} \frac{GM(r)\rho(r)4\pi r^2 dr}{r}.$$

- LHS: $$[P(r)4\pi r^3]_0^{r_*} - 3\int_0^{r_*} P(r)4\pi r^2 dr.$$
- RHS: this is the integral of potential energy in shells, resulting in the total gravitational energy E_{gr}.
- Approximating M(r)~r, the RHS is:

$$E_{\mathrm{gr}} \approx \frac{-GM^2}{R}$$



Virial Theorem

- Noting that pressure is zero at the surface, denoting the total star volume by *V*, we get an expression for the mean stellar pressure (by volume) as a function of the gravitational energy:

$$\bar{P} = -\frac{1}{3}\frac{E_{\rm gr}}{V}.$$

- From the ideal gas law and and internal energy of a monatomic gas, we have:

$$PV = NkT, \qquad E_{\rm th} = \tfrac{3}{2}NkT. \qquad P = \frac{2}{3}\frac{E_{\rm th}}{V},$$

- Applying this over the whole volume of the star, we equate two equations for mean pressure and get:

$$E_{\rm th}^{\rm tot} = -\frac{E_{\rm gr}}{2}$$

The Virial Theorem for Stars (written in 3 forms)

$$E_{\rm gr} = -2E_{\rm th}^{\rm tot}$$

$$E^{\rm tot} = -\frac{1}{2}E_{\rm gr}$$

Stellar Timescales

- The *dynamical timescale* for any body of mass M and radius R is:

$$\tau_{\rm ff} \approx \tau_{\rm dyn} = \sqrt{\frac{R^3}{GM}}$$

~1 hour

$$\propto \bar{\rho}^{-1/2}$$

- This is approximately the *free-fall* timescale, $1/2\pi$ times the orbital period of a body almost scraping the surface, or the sound-crossing time.
- We can take the ratio of the *total internal energy* to the *stellar luminosity* to get the Kelvin-Helmholtz timescale:

$$\tau_{\rm KH} \approx \frac{1}{2}\frac{GM^2}{R}\frac{1}{L}$$

~10^7 years

Week 2 Summary

Textbook: Sections Finishing 2.2, 2.3, 3.1, 3.8, 3.9 (not including Gamow energy).

1. Stellar Spectral Types
2. Orbits, Kepler's Law and Measuring Stellar Masses.
3. The Main Sequence and the HR Diagram.
4. Hydrostatic Equilibrium and the Virial Theorem.
5. The Kelvin-Helmholtz Timescale.

Australian National University

ASTR2013 – *Foundations of Astrophysics*

Week 3: Physics of Stars

Following Dan Maoz – Astrophysics in a Nutshell

Mike Ireland

Recap – The Virial theorem and Stellar Timescales

$$E_{th}^{tot} = -\frac{E_{gr}}{2}$$

The Virial Theorem for Stars (written in 3 forms)

$$E_{gr} = -2E_{th}^{tot}$$

$$E^{tot} = -\frac{1}{2}E_{gr}$$

- The *dynamical timescale* for any body of mass M and radius R is:

~1 hour

$$\tau_{ff} \approx \tau_{dyn} = \sqrt{\frac{R^3}{GM}}$$

$$\propto \bar{\rho}^{-1/2}$$

- This is approximately the *free-fall* timescale, 1/2π times the orbital period of a body almost scraping the surface, or the sound-crossing time.
- We can take the ratio of the *total internal energy* to the *stellar luminosity* to get the Kelvin-Helmholtz timescale:

~10^7 years

$$\tau_{KH} \approx \frac{1}{2}\frac{GM^2}{R}\frac{1}{L}$$

Australian National University

Week 3 Summary

Textbook: Sections Finishing Chapter 3 (other than quantum mechanics in 3.9 and 3.10).

1. Main Sequence Scaling Relations
2. Equations of Stellar Structure.
3. Equation of State
4. Convective instability and convective energy transport.
5. Nuclear generation by the proton-proton chain and the CNO cycle.

Australian National University

Solar Parameters

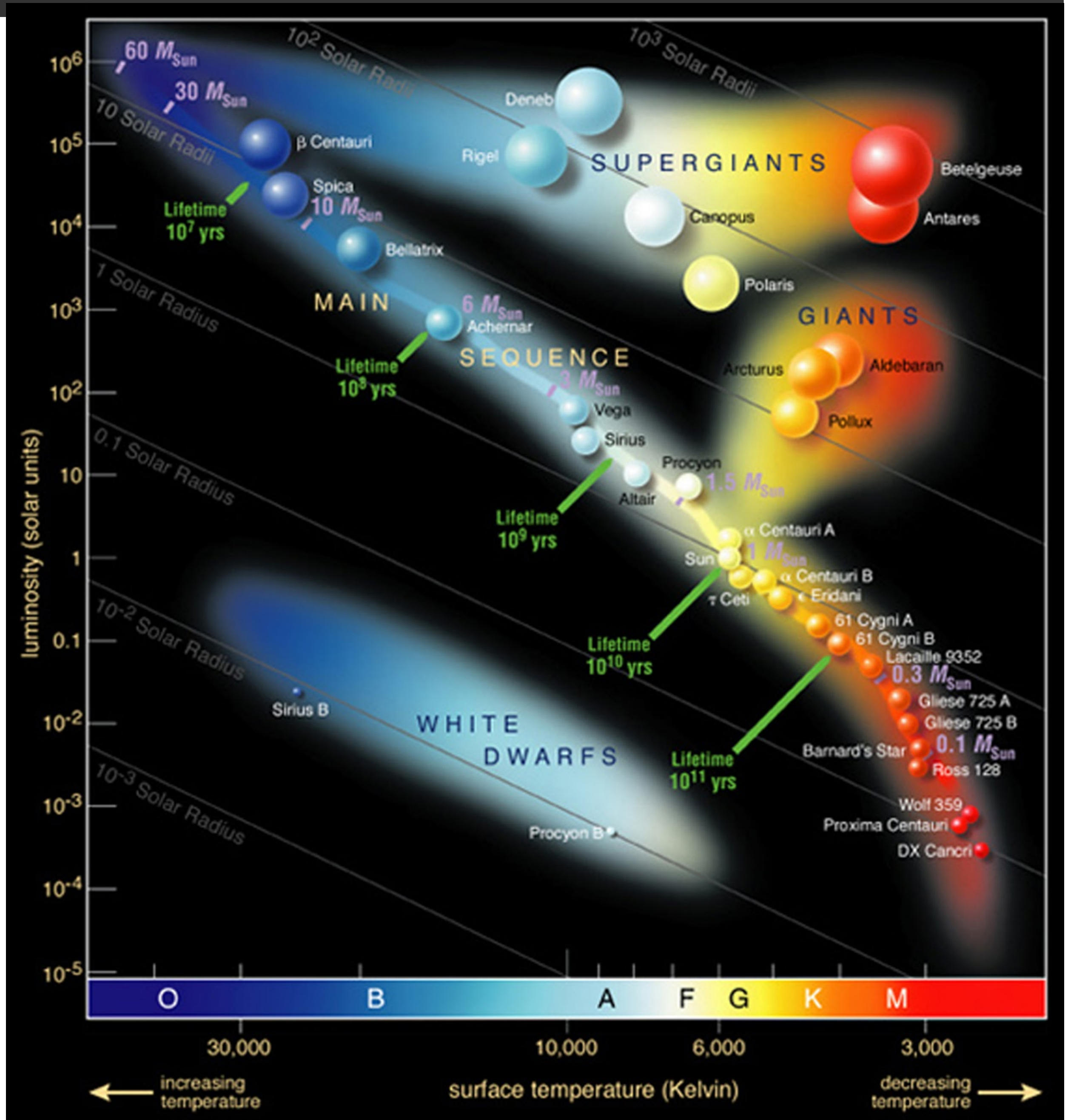
- The sun has an effective temperature of 5772K, and a radius of 6.957 x 10^8W, resulting in a Luminosity of 3.828 x 10^{26}W. Why not a different radius?

$$L = 4\pi R^2 \sigma_{\rm SB} T_{\rm eff}^4$$

- Radioactive dating of rocks in the solar system given an age of 4.6 Gyr, and the sun appears middle-aged compared to other 1 M_{sun} stars. This means its lifetime is 10Gyr. Why not a different age?

Scaling Relations

- Luminosity is roughly ~M^4 for intermediate mass stars, ~M^5 for low mass stars and ~M for high mass stars. Why?
- Life time goes as M3 for intermediate mass stars – assuming nuclear burning you can figure this out for yourself based on the above…
- Mass appears to relate to radius as roughly M~R. Again… why?



Stellar Structure Equations

- Conservation of mass, momentum and energy are the underlying principles, resulting in 4 differential equations as a function of radius coordinate *r* or in other text and codes, mass coordinate M_r.
- Mass Continuity:

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

- Hydrostatic Equilibrium:

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

Pressure in a little more detail

- Pressure actually as 3 components:
  - Ideal gas pressure: $$P_{\text{gas}} = nk_BT = \frac{\rho k_B}{\mu u}T$$
  - Radiation pressure: $$P_{\text{rad}} = \frac{u}{3} = \frac{4\sigma_{\text{SB}}}{3c}T^4$$
  - Degeneracy pressure… important as the electron density approaches 1 electron per cube of the de Broglie wavelength: $$\lambda_e = \frac{h}{mv_e}$$

Energy Transport

- If radiation transports energy, then the derivative of the radiative energy density is proportional to luminosity.

$$\frac{du(r)}{dr} \propto \frac{dT(r)^4}{dr} \propto F(r) = \frac{L(r)}{4\pi R^2}$$

- If internal kinetic energy of gas transports energy perfectly, then the temperature gradient with pressure is the adiabatic gradient:

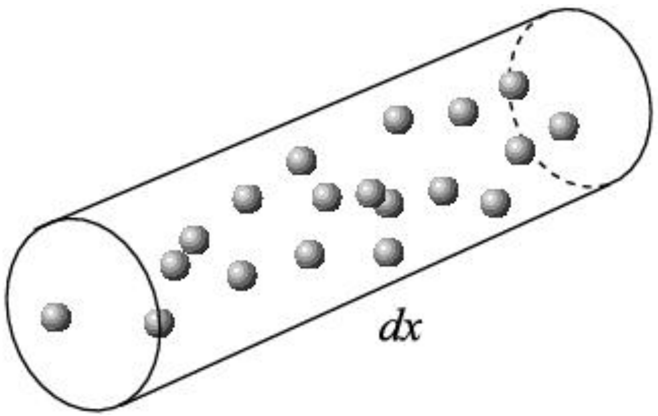
$$\frac{dT}{dr} = \frac{\gamma - 1}{\gamma} \frac{T}{P} \frac{dP}{dr} \qquad \text{from } PV^\gamma = \text{const}$$

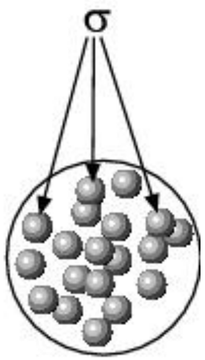
more on this later…

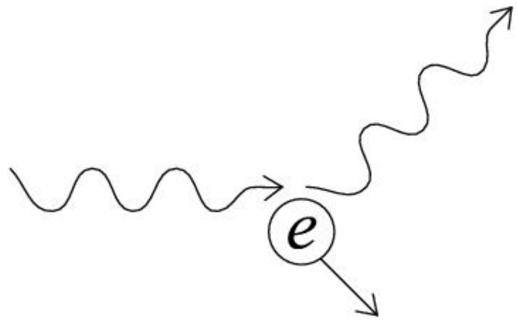
Opacity and Radiative Diffusion

- Electrons, atoms and ions absorb or scatter photons inside a star.
- Each particle has a cross-section $\sigma(\lambda)$ to absorbing radiation of wavelength λ.
- Given a particle number density *n*, we can define a mean-free path:

$$l = \frac{1}{n\sigma}$$







Opacity and Radiative Diffusion

- A mean free path is inversely proportional to density, so we instead typically talk about *opacity:*

$$\kappa = \frac{\Sigma n_i \sigma_i}{\rho}$$

- For mostly ionized gas, Thompson scattering provides a lower limit to opacity:

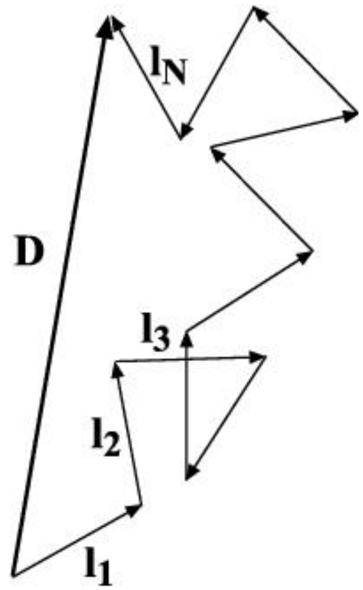
$$\sigma_{\rm T} = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = 6.7 \times 10^{-25}\,{\rm cm}^2.$$

- *Free-Free*, and *bound-free* opacities are proportional to density. *Bound-bound* opacities matter much less in stellar interiors.

Australian National University

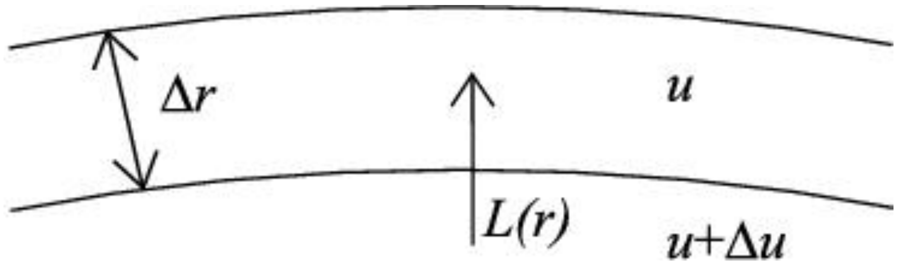
Opacity and Radiative Diffusion

- Photons diffuse through the star, with an average mean free path of 2cm.
- Considering neighboring shells with radiative energy density *u*, changing with *r*, we can write:



$$\frac{L(r)}{4\pi r^2} = -\frac{cl}{3}\frac{du}{dr}$$

- In a more rigorous derivation, the factor of 3 is called the *Eddington Approximation*, and is accurate deep in the star.



Opacity and Radiative Diffusion

- The energy density derivative is:

$$\frac{du}{dr} = \frac{du}{dT}\frac{dT}{dr}$$
$$= \frac{4\sigma_{\mathrm{SB}}}{c}T^3\frac{dT}{dr}$$

- We can write the mean free path in terms of the *Rosseland Mean Opacity* as:

$$l = (\kappa_R \rho)^{-1}$$

- This gives the radiative energy flow equation:

$$\frac{dT(r)}{dr} = -\frac{3L(r)\kappa_R(\rho, T)\rho(r)}{64\pi r^2 \sigma_{\mathrm{SB}} T^3(r)}$$

(textbook uses *a* instead of σ_{SB})

Energy Generation

- On the *nuclear timescale*, we can approximate a star as being in steady state, with constant energy generation from nuclear reactions.
- Energy generation is conventionally parameterized by ε, which is the power generated per unit mass.
- Energy conservation gives:

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho(r) \epsilon(\rho, T)$$

Equations of Stellar Structure

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}, \tag{3.56}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r), \tag{3.57}$$

$$\frac{dT(r)}{dr} = -\frac{3L(r)\kappa(r)\rho(r)}{4\pi r^2 4acT(r)^3}, \tag{3.58}$$

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r). \tag{3.59}$$

Non-relativistic equations here only !

The opacity, nuclear generation rate and pressure are really a complex function of density and temperature.

Equations of Stellar Structure

- Pressure as a function of density and temperature is and *equation of state*, e.g:

$$P_{\rm gas} = nk_BT = \frac{\rho k_B}{\mu u}T$$

- [noting that $\mu = \mu(\rho, T)$ in general]
- We also need boundary conditions including the obvious ones and something like:

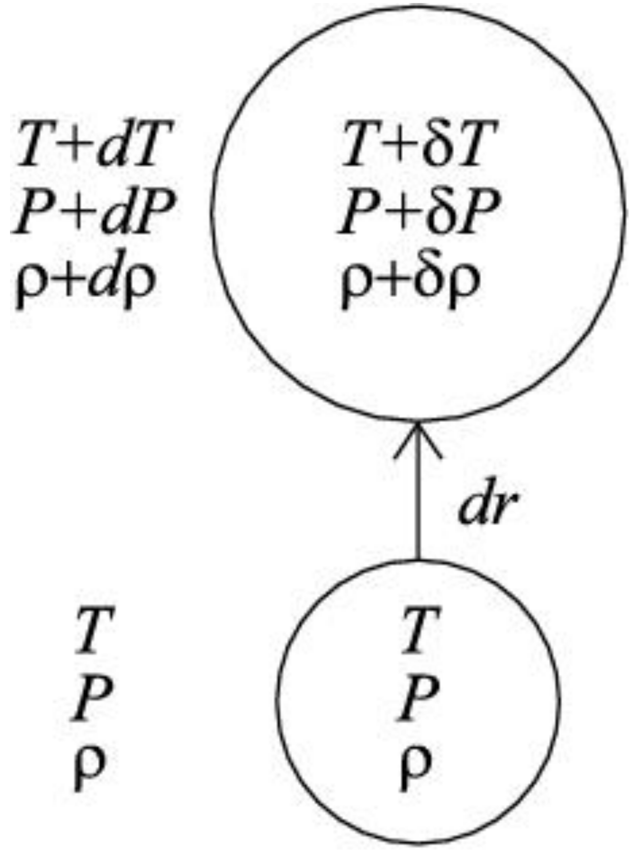
$$L(r_*) = 4\pi R^2 \sigma_{\rm SB} T^4 (r \text{ near surface})$$

- Here "near surface" is often 2/3 of a photon mean-free path from r_*, but needs a multi-wavelength photospheric model to be accurate.

Australian National University

Convective Instability

- A parcel of gas that moves upwards without exchanging energy with its surroundings is *expanding adiabatically*.
- The gradient dP/dT, or the gradient of logarithms for this parcel, is called the *adiabatic gradient*.
- If the parcel expands and cools more than the surrounding gas, it becomes buoyant and accelerates upwards, i.e. it is *unstable*.



Convective Instability

- The adiabatic gradient is therefore the steepest possible temperature gradient in a star. If γ is too small, or κ too large (or both) the star has a convective region.

$$PV^{\gamma} = \text{const.}$$

γ=5/3 for a monatomic gas
γ~1.1 when hydrogen is 50% ionized

$$P \propto \rho^{\gamma}$$

$$\frac{d\log(P)}{d\log(\rho)} = \gamma$$

Maximum luminosity carried by convection for a gas with energy density u_{gas} = is of order:

$$L_{\text{conv.}} < 4\pi r^2 u_{\text{gas}} v_s$$

$$\nabla_{\text{ad}} = \frac{d\log(T)}{d\log(P)} = \frac{\gamma - 1}{\gamma}$$

Nuclear Generation

- Nuclear energy generation in the main sequence comes from two key reaction chains. The proton-proton chain:

$$p + p \rightarrow d \ (1.4 \text{ MeV})$$
$$p + d \rightarrow {}^3_2\text{He} \ (5.5 \text{ MeV})$$
$${}^3_2\text{He} + {}^3_2\text{He} \rightarrow {}^4_2\text{He} + 2p \ (12.9\text{MeV})$$

- … and the CNO cycle:

$$p + {}^{12}\text{C} \rightarrow {}^{13}\text{N} + \gamma$$
$${}^{13}\text{N} \rightarrow {}^{13}\text{C} + e^+ + \nu_e,$$
$$p + {}^{13}\text{C} \rightarrow {}^{14}\text{N} + \gamma,$$
$$p + {}^{14}\text{N} \rightarrow {}^{15}\text{O} + \gamma,$$
$${}^{15}\text{O} \rightarrow {}^{15}\text{N} + e^+ + \nu_e,$$
$$p + {}^{15}\text{N} \rightarrow {}^{12}\text{C} + {}^4\text{He}.$$

More on this with numbers in the labs!

Week 3 Summary

Textbook: Sections Finishing Chapter 3 (other than quantum mechanics in 3.9 and 3.10).

1. Main Sequence Scaling Relations
2. Equations of Stellar Structure.
3. Equation of State
4. Convective instability and convective energy transport.
5. Nuclear generation by the proton-proton chain and the CNO cycle.

Australian National University

ASTR2013 – *Foundations of Astrophysics*

Week 3: Stellar Evolution

Following Dan Maoz – Astrophysics in a Nutshell

Mike Ireland

Equations of Stellar Structure

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}, \tag{3.56}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r), \tag{3.57}$$

$$\frac{dT(r)}{dr} = -\frac{3L(r)\kappa(r)\rho(r)}{4\pi r^2 4acT(r)^3}, \tag{3.58}$$

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r). \tag{3.59}$$

Non-relativistic equations here only !

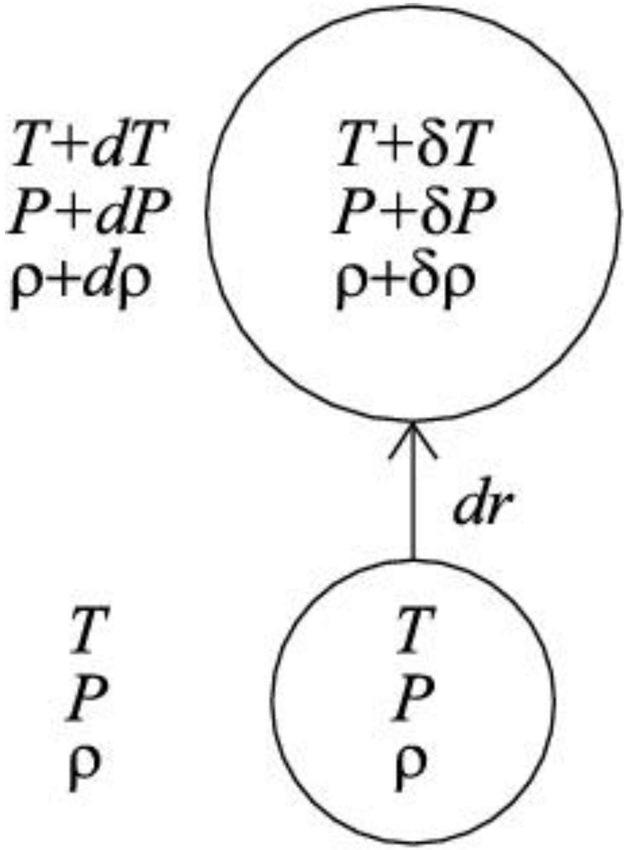
The opacity, nuclear generation rate and pressure are really a complex function of density and temperature.

$$P_{\rm gas} = nk_B T = \frac{\rho k_B}{\mu u} T$$

EOS (gas only)

Convective Instability

- A parcel of gas that moves upwards without exchanging energy with its surroundings is *expanding adiabatically*.



$$PV^{\gamma} = \text{const.}$$

$$P \propto \rho^{\gamma}$$

$$\frac{d\log(P)}{d\log(\rho)} = \gamma$$

$$\nabla_{\text{ad}} = \frac{d\log(T)}{d\log(P)} = \frac{\gamma - 1}{\gamma}$$ Monatomic Gas

Week 4 Summary

Textbook: Sections 3.8, 4.1, 4.2, 4.3.

1. Theory of Scaling Relations.
2. Key observed stages of stellar evolution.
3. Evolutionary tracks and key theoretical stages of stellar evolution.
4. White dwarfs.

Scaling Relations

Work through on board

- Take 3 first equations of Stellar Structure, and turn them into equations of proportionality with average quantities:

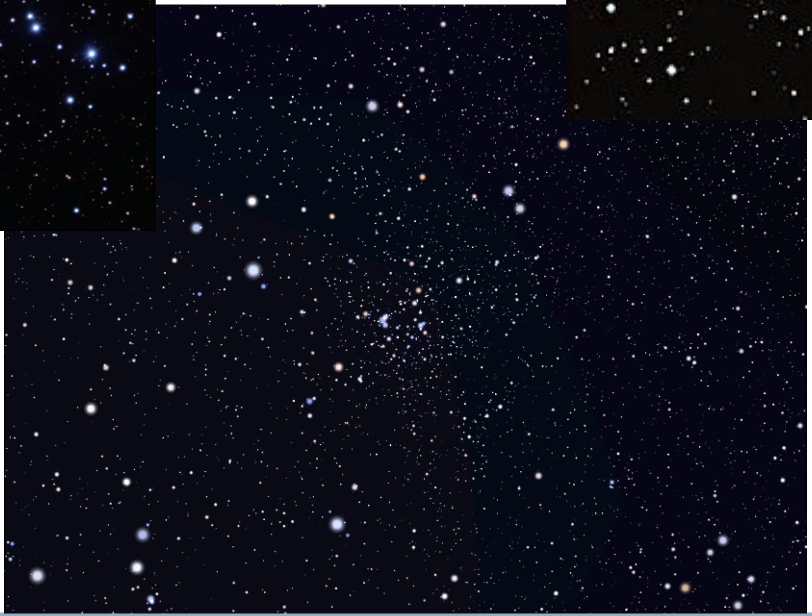
$$M \sim R^3 \rho, \quad P \sim \frac{M\rho}{R}, \quad L \sim \frac{T^4 R}{\kappa \rho}$$

- Assuming gas pressure dominates (P~ρT), we re-derive the Virial theorem: T~M/R. If there is a strong nuclear thermostat on the main sequence, this means R~M.
- If Thompson scattering dominates (κ=const) then the luminosity relationship gives L~M^3.
- If radiation pressure dominates, P~T^4, and we put the 3 equations together to get L~M.
- See textbook for L~M^4 for low mass stars.

Observations of Stellar Evolution

Given scaling relations, we know high mass stars have shorter lifetimes, and use observations of stellar clusters to observationally determine what happens to stars as they age.

NGC4755



NGC 6819

M67

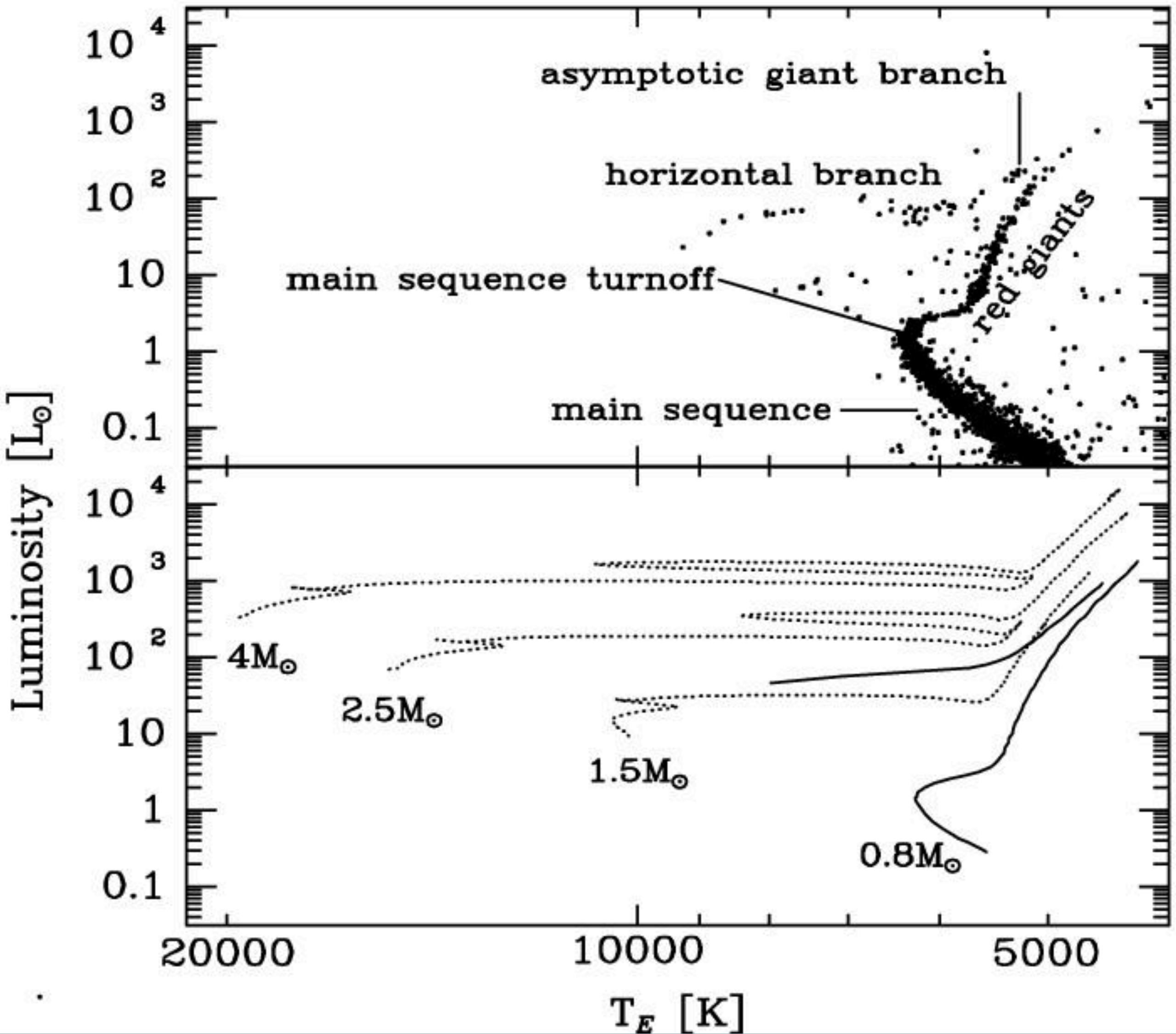
M15

See Wikipedia for image credits

Isochrone and Evolutionary Tracks

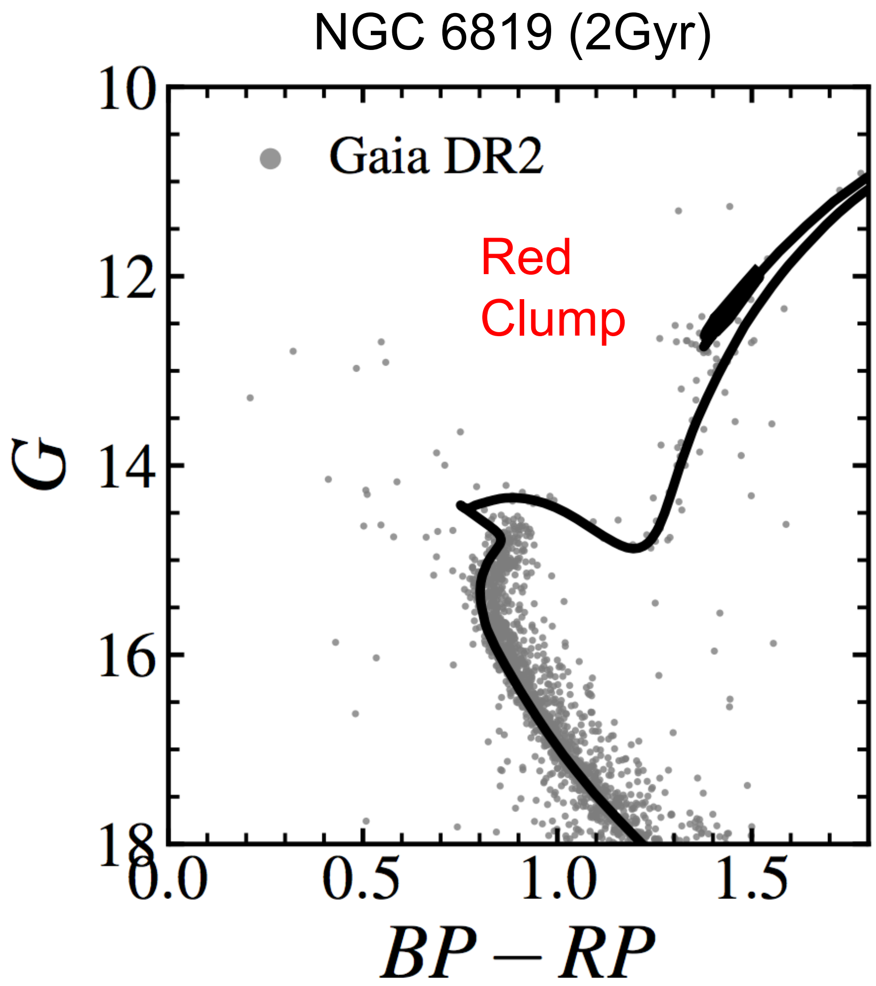
Cluster (~10 Gyr, metal-poor)

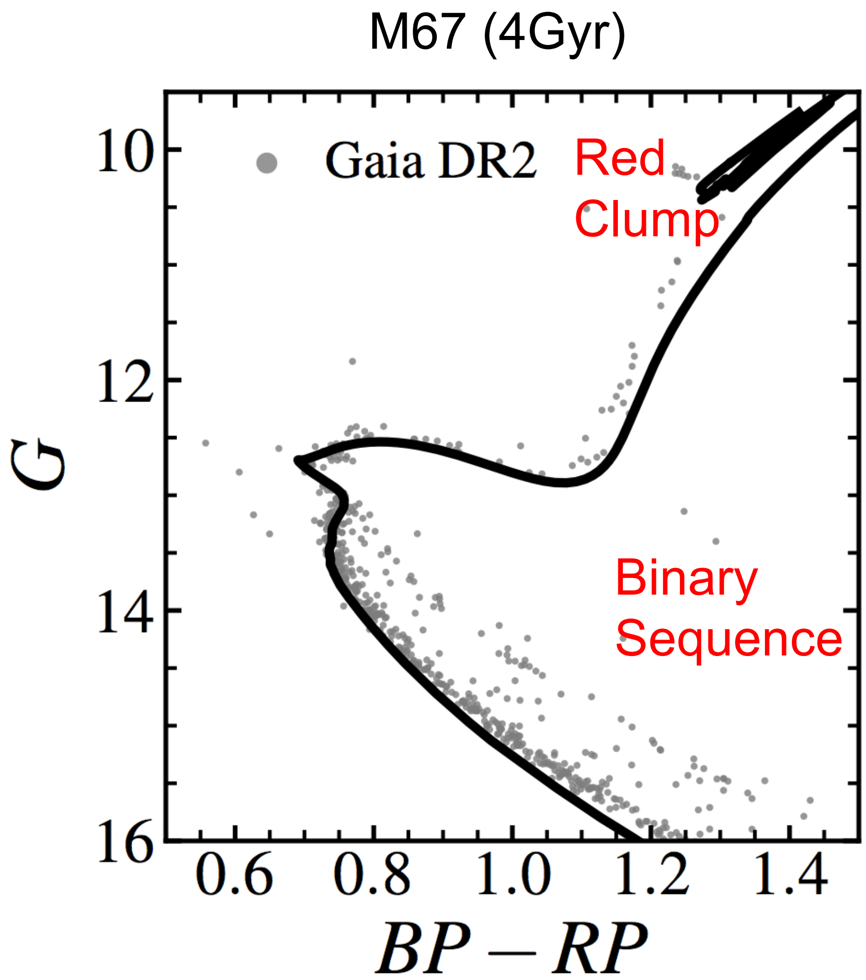
Isochrones (also metal-poor)



Open Clusters – Gaia Data

Even the best data affected by fits for *distance* and *dust reddening*.
G measures luminosity (2.5 = factor of 10) and BP-RP measures temperature.





Choi et al. (2018, ApJ, 863, 65)

General Stages of Stellar Evolution (near solar mass)

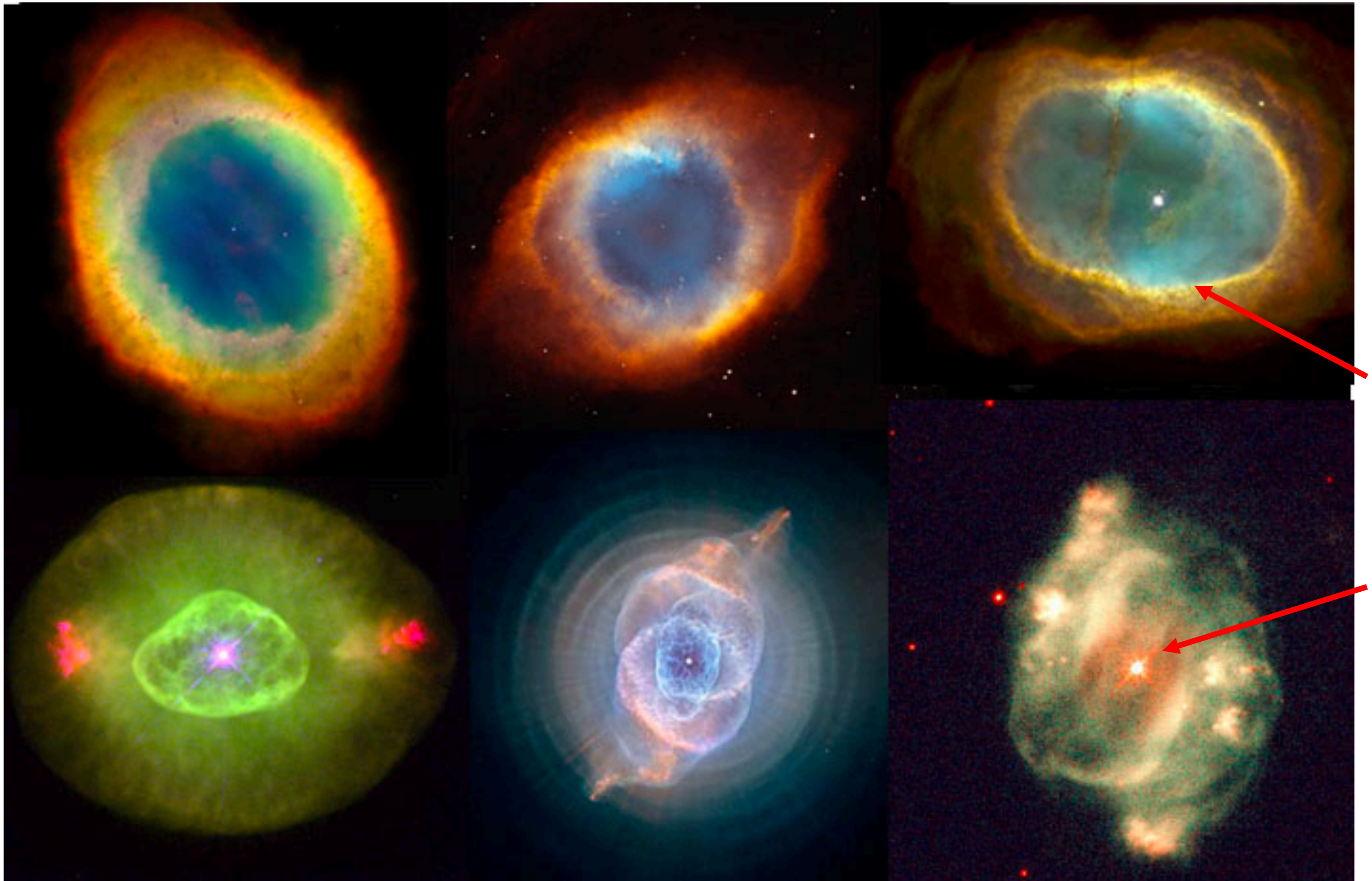
Add Whiteboard Pictures Here…

- A pre-main sequence star is fully convective, doesn't obey the 4th equation of stellar structure and contracts on the Kelvin-Helmholtz timescale.
- The main sequence ends when the core is supported by electron degeneracy pressure and not gas pressure.
- This star becomes a *red giant*, on the K-H timescale if more massive than ~1.3 M_{sun}, or more gradually via the *subgiant stage* for solar or lower masses.
- The giant star has *hydrogen shell burning.*

General Stages of Stellar Evolution (near solar mass)

- When the core temperature and density is high enough, *helium burning* begins in the core.
- [As this is a degenerate core, the reaction runs away as a helium flash]
- The helium-burning star is either a *horizontal-branch* or *red clump* star, depending on metallicity.
- Once He burning to (to C, or even O) is complete, the core contracts until supported by degeneracy pressure.
- Hydrogen burning resumes, and the star and the star resumes the giant branch where it left off (roughly), now an *asymptotic giant branch* star. Occasional substantive He burning is called a *thermal pulse.*
- The resulting high luminosity drives the surface layers off, leaving behind an inert C,N,O core called a *white dwarf*. 1M_{sun} star : 0.6 M_{sun} white dwarf
- [the intermediate, rapid phases are called post-AGB, pre-planetary nebula and planetary nebula phases]

Planetary Nebula Phase



Ejected Stellar Atmosphere

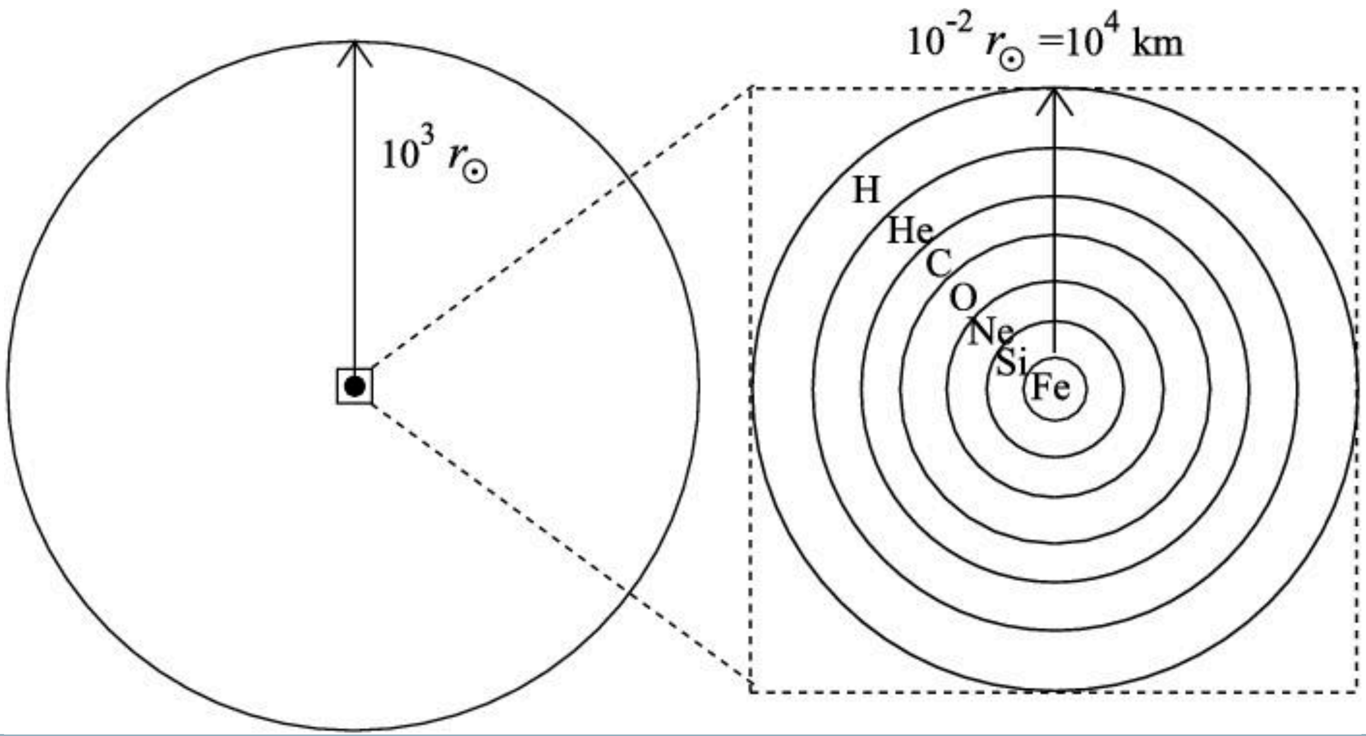
Cooling (still very hot) White Dwarf

Nuclear Generation – Energy Available

- Nuclear reactions produce most their energy in photons (γ rays) and kinetic energy of daughter products including annihilating positrons.
- A smaller but still significant (several percent) energy fraction comes in the form of Neutrinos which typically leave stars.
- Energy available is computed by $E=mc^2$.
  - Proton: 1.00728 amu.
  - Helium-4: 4.0026 amu (1.00065 amu per nucleon)
  - Carbon-12: 12.0000 (1.0000 amu per nucleon)
  - Iron-56: 55.9349 (1 - 0.00116 per nucleon)
  - Gold-197: 196.9666 (1 - 0.00017 per nucleon)
- 0.65% of mass energy to H to He, 0.065% He to C and 0.001% to Fe.

Massive Star Evolution

- For a star more massive than about 8 M_{sun}, the core is so hot that electron degeneracy pressure is never important enough compared with kinetic gas and photon pressure.
- An inert core never forms, and the center of the star has an “onion” structure.
- If it actually looks like the structure below, then it is within a day away from becoming a supernova! (more on this next week)



White Dwarfs

- Electron degeneracy pressure can be derived from simple quantum particle-in-a-box considerations. In the non-relativistic case, this is:

$$P_e = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20 m_e m_p^{5/3}} \left(\frac{Z}{A}\right)^{5/3} \rho^{5/3}.$$

- Note that it has *exactly the same form* as a convective star consisting of a monatomic ideal gas: a white dwarf is the low entropy limit of this functional form.
- If there were no nuclear reactions, this relationship between pressure and density would be all you have to know [called a polytrope with index 1.5].
- If P approaches, rc2, we have to use the relativistic degenerate EOS.....

White Dwarfs

- Electron degeneracy pressure can be derived from simple quantum particle-in-a-box considerations. In the non-relativistic case, this is:

$$P_e = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20 m_e m_p^{5/3}} \left(\frac{Z}{A}\right)^{5/3} \rho^{5/3}.$$

- Note that it has *exactly the same form* as a convective star consisting of a monatomic ideal gas: a white dwarf is the low entropy limit of this functional form.
- If there were no nuclear reactions, this relationship between pressure and density would be all you have to know [called a polytrope with index 1.5].
- If P approaches, ρc^2, we have to use the relativistic degenerate EOS.....

White Dwarfs

- For the ultra-relativistic equation of state where electron degeneracy pressure dominates, the rest mass of the electron no longer matters in the equation of state:

$$P_e = \left(\frac{3}{8\pi}\right)^{1/3} \frac{hc}{4m_p^{4/3}} \left(\frac{Z}{A}\right)^{4/3} \rho^{4/3}.$$

- The scaling relations no longer work with a 4/3 exponent (try it – radius cancels out).
- The electron equation of state becomes relativistic as the mass approaches the Chandrasekhar mass (below) and the radius of the white dwarf approaches 0.

More on this in lab!

$$M_{\rm ch} = 0.21 \left(\frac{Z}{A}\right)^2 \left(\frac{hc}{Gm_p^2}\right)^{3/2} m_p.$$

Week 4 Summary

Textbook: Sections 3.8, 4.1, 4.2, 4.3.

1. Theory of Scaling Relations.
2. Key observed stages of stellar evolution.
3. Evolutionary tracks and key theoretical stages of stellar evolution.
4. White dwarfs.

Australian National University

ASTR2013 – *Foundations of Astrophysics*

Week 5: Stellar Evolution

Following Dan Maoz – Astrophysics in a Nutshell

Mike Ireland

Field Trip!!!

- This is a compulsory part of the course comprising 30% of the assessment.
- Payment and pre-departure form due this week.
- Next week, rooms will be assigned for those who haven’t chosen to share with friends.
- Week 6 Lecture and tutorial is prep for field trip.

Revision - Stellar Evolution

- A 1 M_{sun} star goes through the following stages. Lets see who remembers their key features:
  - Pre-main sequence contraction
  - Main sequence
  - Sub-giant phase
  - First ascent giant branch
  - Red clump
  - Asymptotic Giant Branch
  - White Dwarf

Revision - White Dwarfs

- Electron degeneracy pressure can be derived from simple quantum particle-in-a-box considerations. In the non-relativistic case and relativistic cases respectively, this is:

$$P_e = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20 m_e m_p^{5/3}} \left(\frac{Z}{A}\right)^{5/3} \rho^{5/3}.$$

$$P_e = \left(\frac{3}{8\pi}\right)^{1/3} \frac{hc}{4 m_p^{4/3}} \left(\frac{Z}{A}\right)^{4/3} \rho^{4/3}.$$

- As the electrons approach relativistic speeds throughout the white dwarf, the star approaches the limiting mass called the Chandrasekhar mass.

What happens if a Fe core white dwarf reaches this limit?

$$M_{\mathrm{ch}} = 0.21 \left(\frac{Z}{A}\right)^2 \left(\frac{hc}{G m_p^2}\right)^{3/2} m_p.$$

Week 5 Summary

Textbook: Sections 4.3, 4.4, 4.5.

1. End states of massive stars.
2. Neutron Stars and Black Holes.
3. Supernovae Ia – exploding white dwarfs
4. Accretion disk brief intro.

The End of a Massive Star

- As the inert Fe core approaches the Chandrasekhar mass and collapses, it compresses and the nuclear reactions go backwards.
- Total energy available in complete collapse to a black hole is *much* larger than that available from nuclear reactions [exercise for board – derive that the radius at which collapse to release 0.65% of the mass energy is ~160 km]
- When electrons are absorbed in *neutronization* processes, the electron degeneracy pressure goes away.

$$e^- + p \rightarrow n + \nu_e,$$

$$e^- + {}^{56}\mathrm{Fe} \rightarrow {}^{56}\mathrm{Mn} + \nu_e,$$

$$e^- + {}^{56}\mathrm{Mn} \rightarrow {}^{56}\mathrm{Cr} + \nu_e.$$

- Collapse continues until a neutron star is formed.

Neutron Stars

- We can find the radius of an object supported by *neutron degeneracy pressure* rather than electron degeneracy pressure by the EOS of a nonrelativistic electron gas (4.27) and the virial theorem, to get a relationship between degenerate particle mass *m* and radius:

$$R \propto \frac{1}{m}\left(\frac{Z}{A}\right)^{5/3}$$

- This implies neutron stars are 574 times smaller than a white dwarf at the same mass in the non-relativistic limit, giving a mass (see textbook) of:

$$r_{ns} \approx 2.3 \times 10^9 \text{ cm} \frac{m_e}{m_n}\left(\frac{Z}{A}\right)^{5/3}\left(\frac{M}{M_\odot}\right)^{-1/3} \approx 11 \text{ km}\left(\frac{M}{1.4 M_\odot}\right)^{-1/3}.$$

Neutron Stars

- Just like the Chandrasekhar limit for white dwarfs, there is an equivalent limit for Neutron stars:

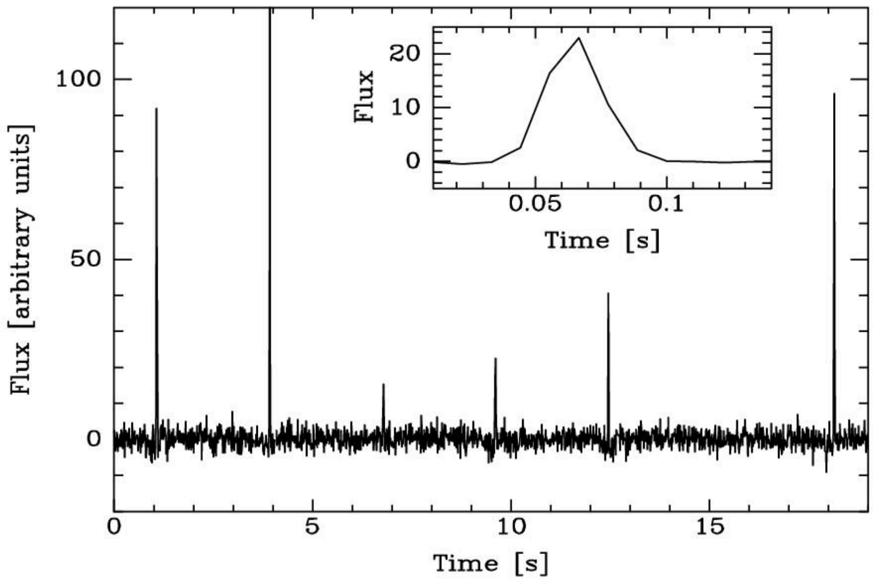
$$M_{\text{ch}} = 0.21 \left(\frac{Z}{A}\right)^2 \left(\frac{hc}{Gm_p^2}\right)^{3/2} m_p$$

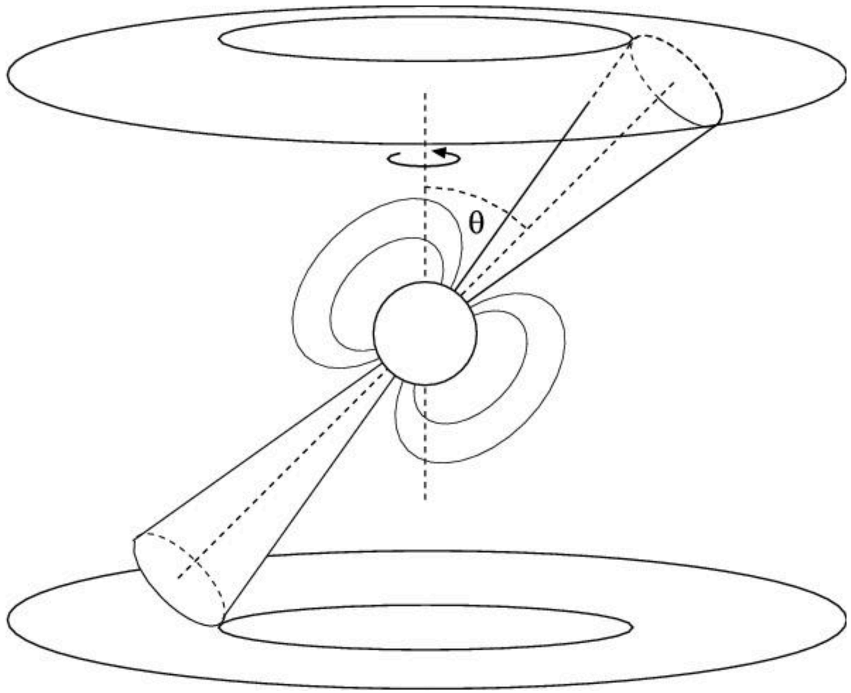
Replace this with 1.0, and get 4 times the mass?

- However, general relativity becomes important (GM/R close to c^2), really complicating this scenario.
- Neutrons are also *not* point masses – the strong nuclear force comes into play, and the equation of state is a subject of active research.
- The maximum mass of a neutron star is agreed to be around $2M_{\text{sun}}$.

Pulsars

- The dynamical timescale of a neutron star is ~1ms.
- Most of the neutron stars we observe are as *pulsars*, with periods of a few ms to many seconds.
- Interpreted as where electrons caught in the magnetic field of a neutron star emit radio waves brightly whenever the N or S pole points in our direction.





Black Holes

- This isn't a General Relativity course, so we'll only introduce black holes roughly.
- If the potential energy of a compact object GM/R approaches c^2, then Newtonian gravity has to be replaced with general relativity.
- Once an object can no longer support itself, it collapses to a point-like object with an apparent radius of the Schwarzschild radius:

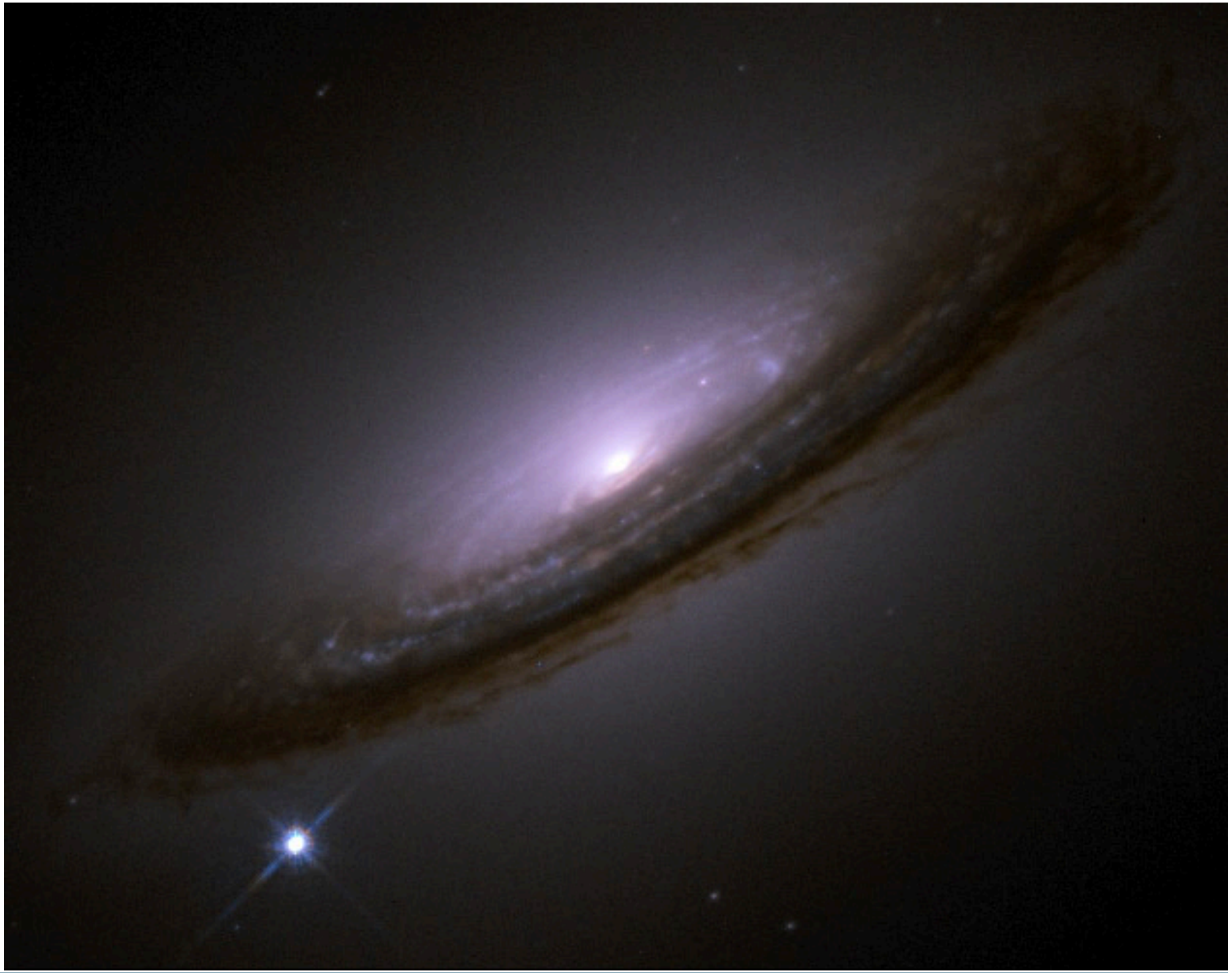
$$r_s = \frac{2GM}{c^2} = 3\,\mathrm{km}\,\frac{M}{M_\odot}.$$

- Photons can not escape inside this radius, and approach infinite redshift (and infinite time dilation) as radius approaches this value.

Supernova Explosions

Textbook pic:

Actually a SNIa… not the kind we're talking about yet.

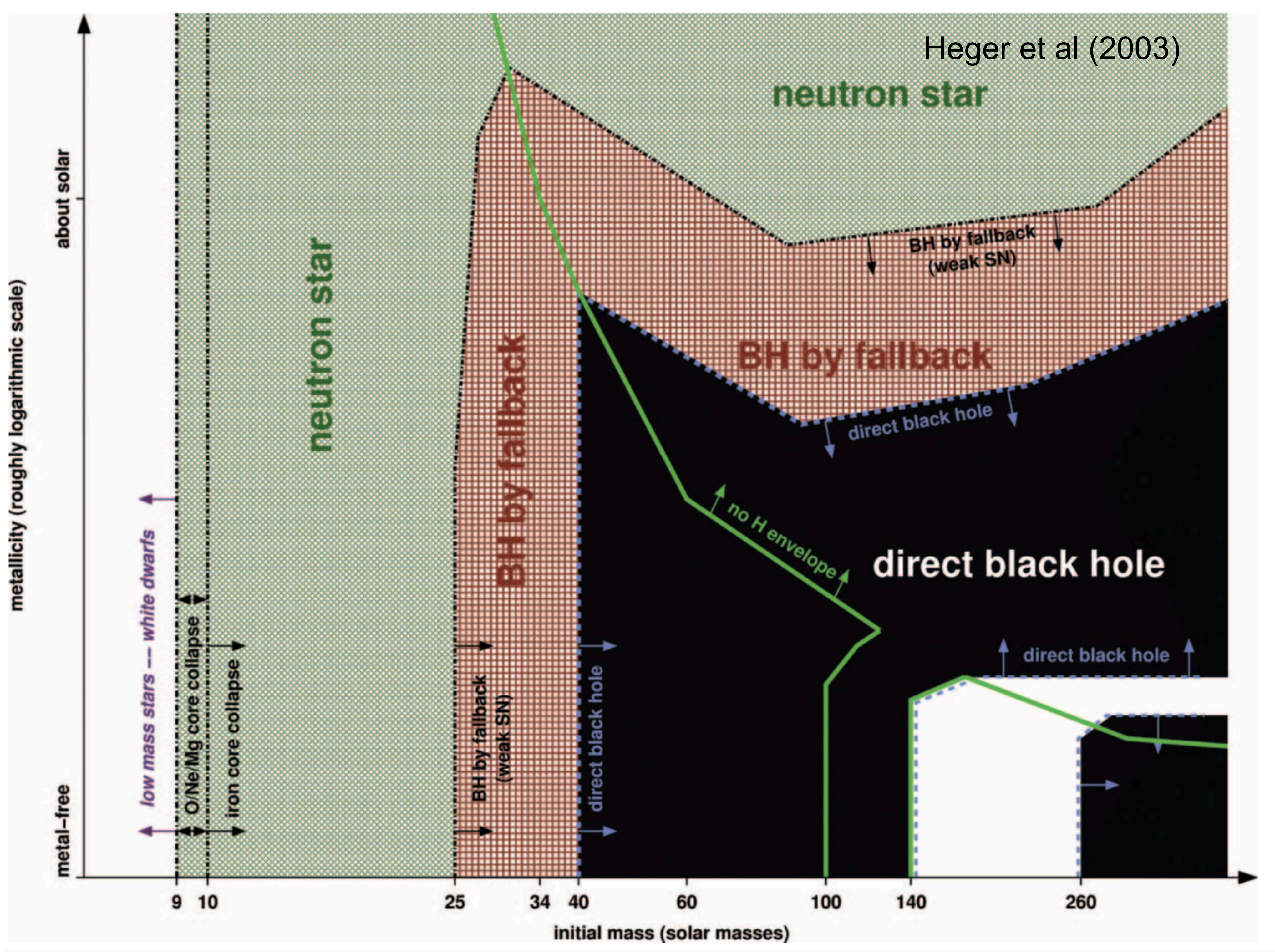


Supernova Explosions

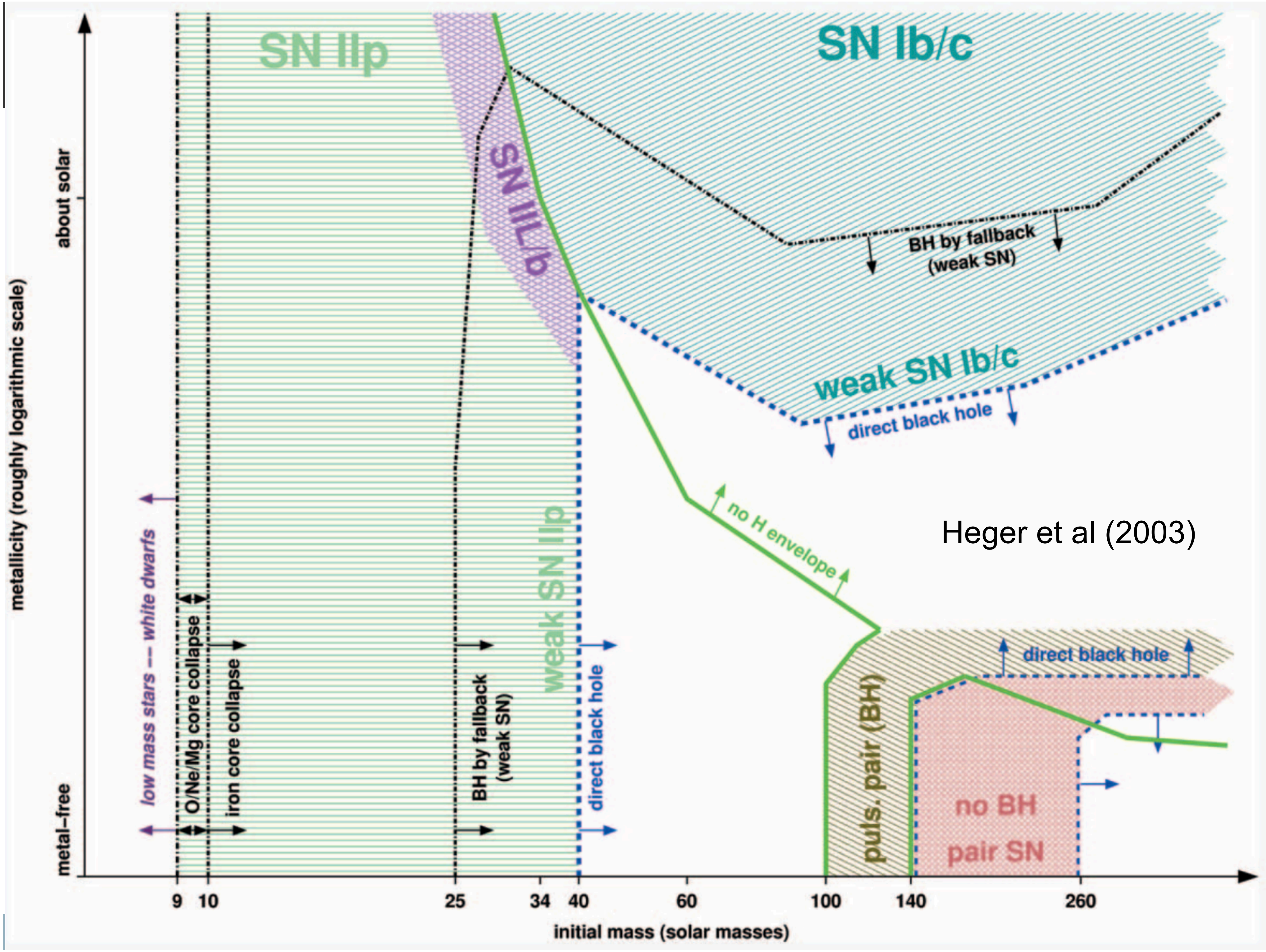
- Supernova 1987A: an exploding massive star.

Supernova Explosions

- It only takes a fraction of the energy released in the collapse to a neutron star ($\sim GM_{core}/R_{ns}$) to unbind the rest of a massive ($\sim 10M_{sun}$) star.
- This energetic argument led bright explosions, called supernovae, to be attributed to the end of massive stars.
- Energy leaves the collapsing core primarily via neutrinos, which can couple to the layers immediately outside the core.
- Neutrino luminosity was *directly observed* in the case of 1987A – supporting the accepted theory that this sort of explosion was due to a collapsing massive star core.
- However, there are many different kinds of supernovae and end states of massive stars…



Heger et al (2003)
neutron star
neutron star
BH by fallback
BH by fallback
BH by fallback (weak SN)
BH by fallback (weak SN)
direct black hole
direct black hole
direct black hole
direct black hole
no H envelope
low mass stars — white dwarfs
O/Ne/Mg core collapse
iron core collapse
metallicity (roughly logarithmic scale)
about solar
metal-free
9
10
25
34
40
60
100
140
260
initial mass (solar masses)



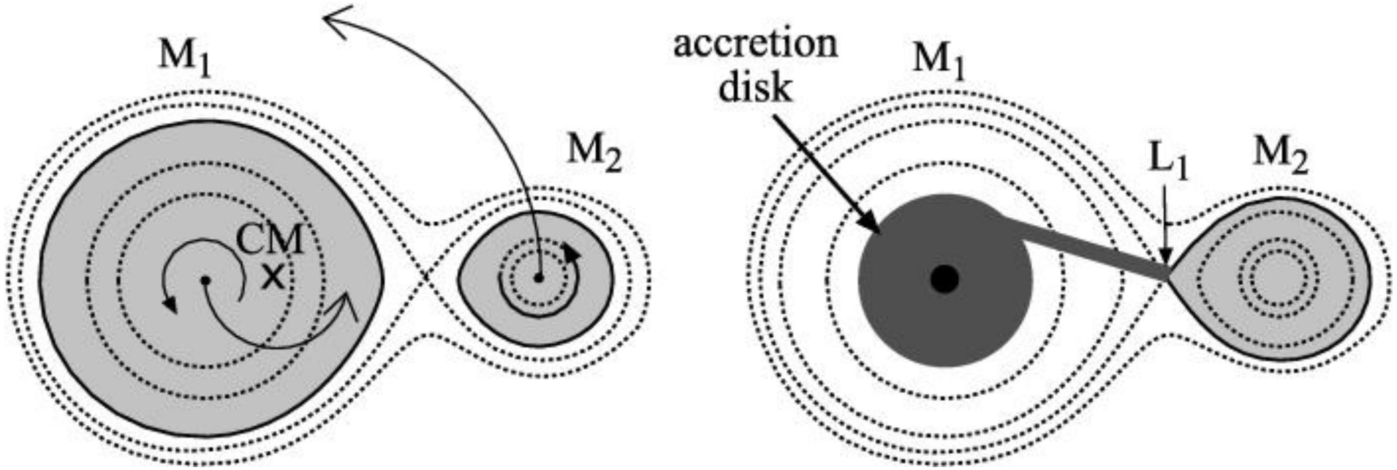
SN IIp
SN Ib/c
SN IIL/b
BH by fallback (weak SN)
weak SN Ib/c
direct black hole
weak SN IIp
no H envelope
direct black hole
puls. pair (BH)
no BH
pair SN
low mass stars -- white dwarfs
O/Ne/Mg core collapse
iron core collapse
BH by fallback (weak SN)
direct black hole
metallicity (roughly logarithmic scale)
about solar
metal-free
9
10
25
34
40
60
100
140
260
initial mass (solar masses)
Heger et al (2003)

Type Ia Supernovae

- Of the many, many kinds of supernovae, one type stands out – the SN Ia class. These have a characteristic spectrum which has no Hydrogen, and a characteristic maximum luminosity.
- It is generally accepted that this class come from a white dwarf that has reached the Chandrasekhar mass.
- Models predict that the energy released in fusing CNO to Fe is typically enough to unbind the white dwarf (Q: what does this depend on?)
- The key question is whether two smaller white dwarfs merge to exceed the Chandrasekhar mass (double degenerate scenario) or whether a single white dwarf can gradually accrete mass from a companion (made tricky by “novae” explosions).
- Textbook author (Dan Maoz) is a leading researcher in this area, and has published in support of the double degenerate scenario being dominant.

Accretion

- After starting with individual stars, we've also mentioned objects like white dwarfs that grow.
- Growth of old stellar mass objects typically occurs by overflow of mass from one star to another, in what is called an *interacting binary*.
- The overflow occurs through the 1st Lagrange point, where in the rotating reference frame, the gravity from the two stars balance.



Accretion

- One key limiting factor for accretion is called the *Eddington limit.* This will come up later in the course as well (quasars).
- We equate the radiative force on electrons from Thomson scattering:

$$F_{\rm rad} = \frac{L\sigma_{\rm T}}{4\pi r^2 c}$$

Derive this on board…

- … to the gravitational force:

$$F_{\rm grav} = \frac{GMm_p}{r^2}$$

- Arriving at a maximum possible accretion luminosity for a given mass:

$$L_{\rm E} = \frac{4\pi c GMm_p}{\sigma_{\rm T}}$$

Week 5 Summary

Textbook: Sections 4.3, 4.4, 4.5.

1. End states of massive stars.
2. Neutron Stars and Black Holes.
3. Supernovae Ia – exploding white dwarfs
4. Accretion disk brief intro.

ASTR2013 – *Foundations of Astrophysics*

Week 6: Practical Astronomical Observations and Coordinate Systems

An introduction to concepts that will be important on the field trip.

Mike Ireland

Revision - The End of a Massive Star

- As the inert Fe core approaches the Chandrasekhar mass and collapses, there is much more than enough energy to completely reverse nuclear fusion processes.
- If neutron stars were supported simply by neutron degeneracy pressure their radius would be:

$$r_{ns} \approx 2.3 \times 10^9 \text{ cm } \frac{m_e}{m_n}\left(\frac{Z}{A}\right)^{5/3}\left(\frac{M}{M_\odot}\right)^{-1/3} \approx 11 \text{ km}\left(\frac{M}{1.4M_\odot}\right)^{-1/3}.$$

- Forming a neutron star is a factor of ~10 more energetic than fusion in forming the Fe core, resulting in a massive supernova explosion, with luminosity dominated by Neutrinos.
- Most supernovae (type II, and also type Ibc) are caused by massive stars collapsing, but type Ia are caused by white dwarfs merging or otherwise approaching the Chandrasehkar mass and fusing CNO to Fe in an explosion.

Week 6 Summary

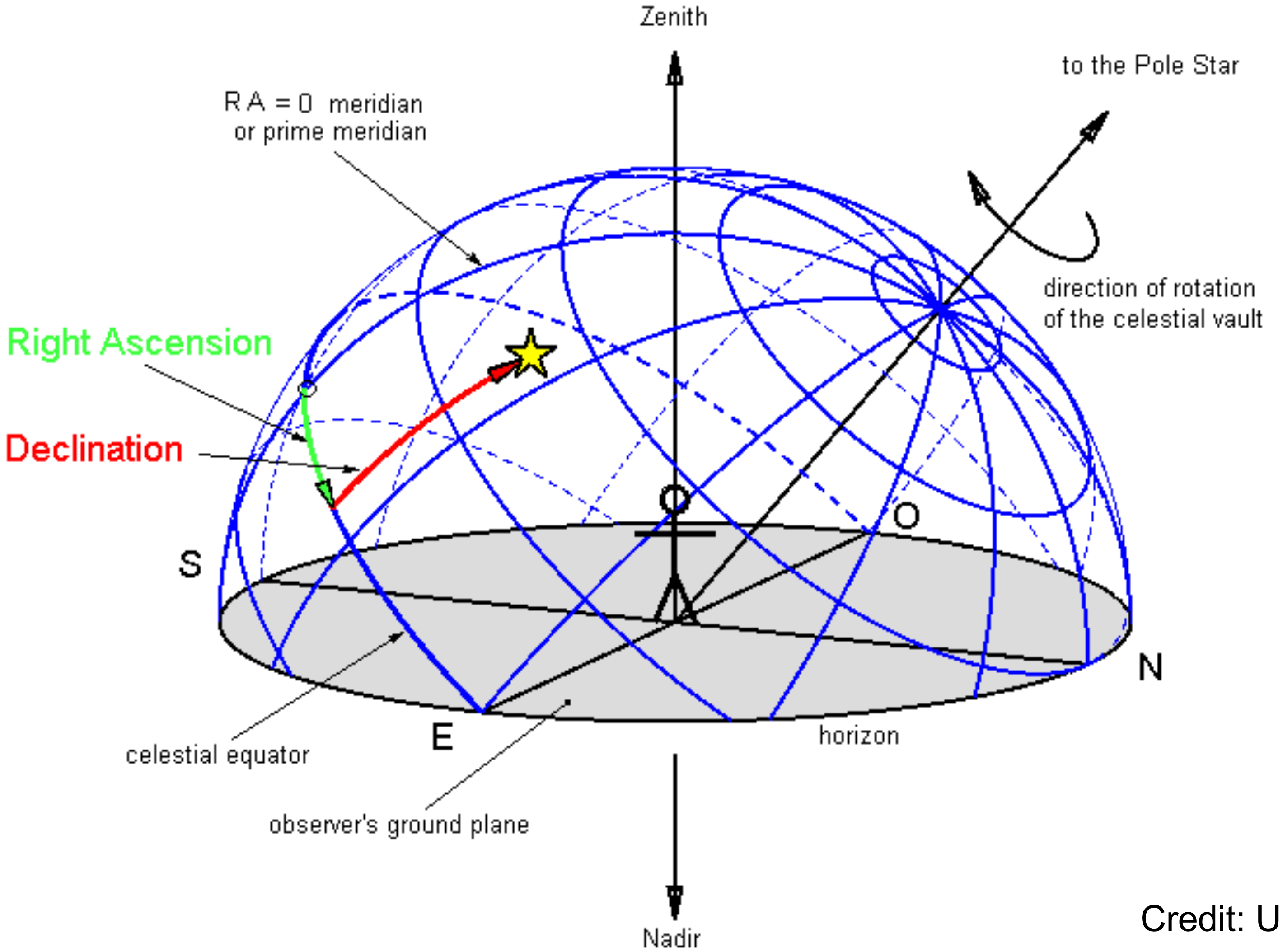
Textbook: Only Chapters 1 and 2 again, plus these notes, links and tutorial exercises.

1. Astronomical Coordinate Systems and catalogs.
2. The Magnitude Scale(s) and distance modulus.
3. Types of telescopes and instruments.
4. Signal to noise: beyond target shot noise.

Equatorial Co-Ordinates

- Imagine you're inside the globe looking out. *Right Ascension* (RA, α) is like longitude (in an East direction).
- RA is measured in hours or degrees : 1 hour = 15 degrees. From 0 to 24 hours, or 0 to 360°.
- Declination (Dec, δ) is like latitude, i.e. running from -90° to $+90^\circ$.
- The co-ordinate system precesses with the earth, so has to be defined at a given *equinox*. Modern convention is to use the equinox of the J2000.0 *epoch*.
- Zero degrees RA is (roughly) the sun at noon at the northern spring equinox, one of two points where the *ecliptic* and *equatorial* equators cross.

Equatorial Co-Ordinates

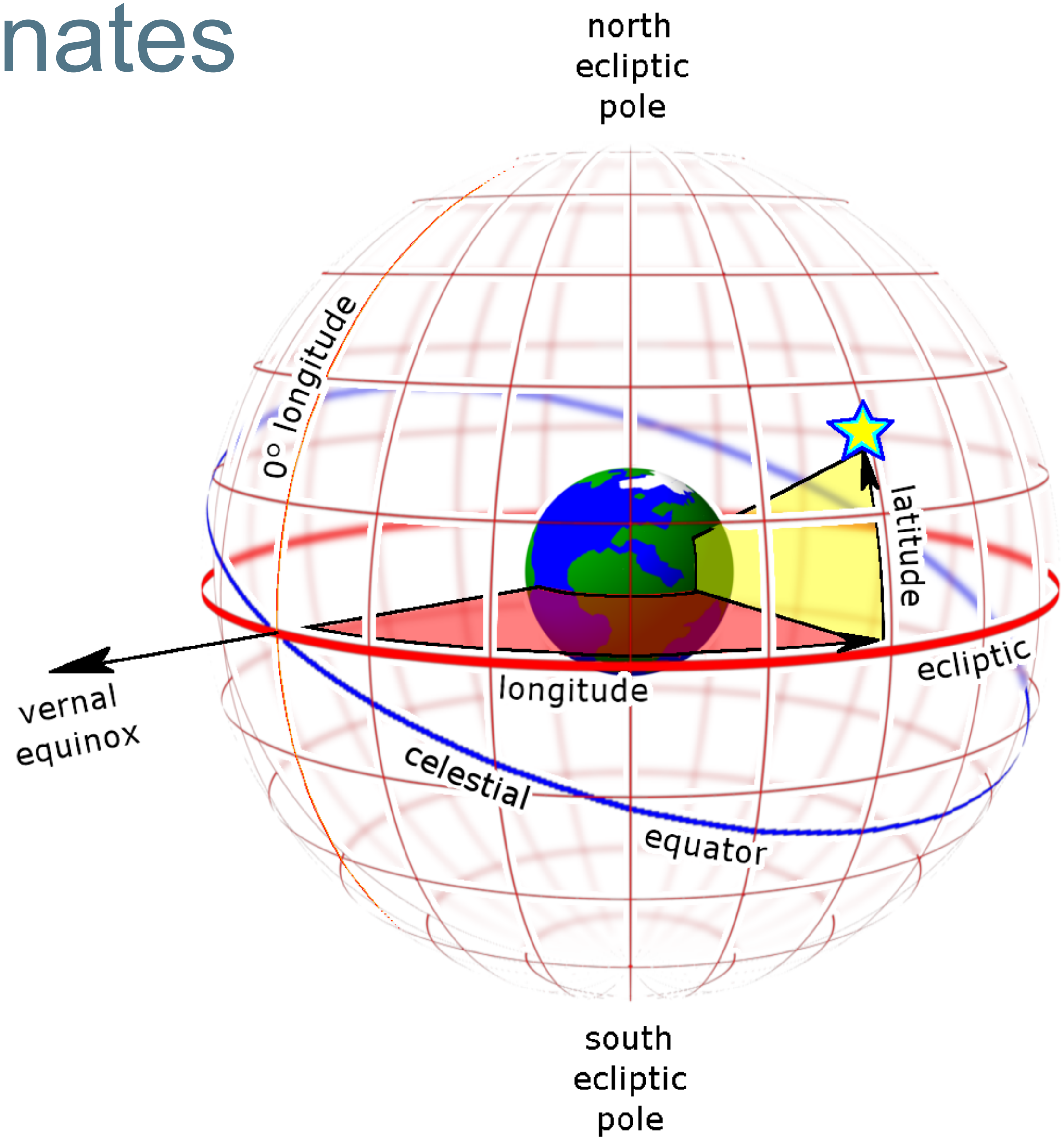


Go through this on board and with examples…

Credit: University of Michigan

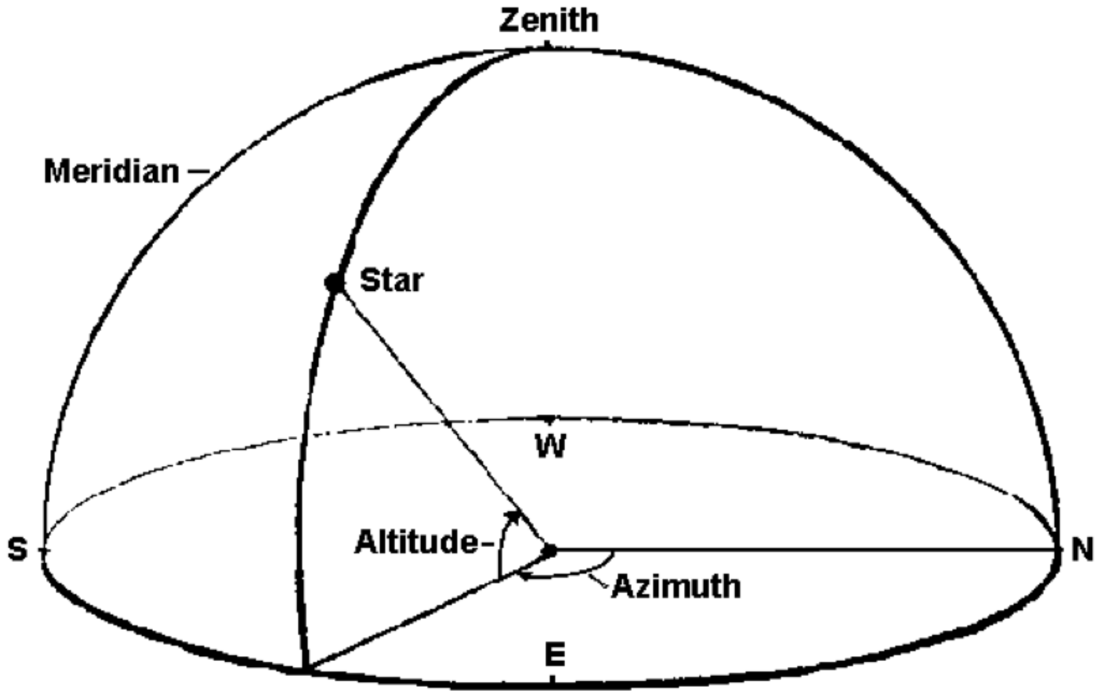
Ecliptic Co-Ordinates

Ecliptic Co-ordinates has its equator on the *ecliptic plane* (the earth's orbit) and not the equatorial plane.



Alt-Az Coordinates

- Easily understandable but always changing
- Azimuth: degrees E of N following the horizon.
- Altitude=90◦ is *zenith* i.e. straight up. 90◦-Altitude is the *zenith distance* (or zenith angle).
- *Airmass* ≈ secant (zenith distance).



Key Online Resources

- NED for extragalactic objects.
- Simbad for (bright) stars.
- SkyView and Aladin for images and images overlayed with object catalogs.
- StarAlt and the ESO observability tool for seeing where objects are at particular times.
- Astropy (python – in tutorial) does some co-ordinate transforms, as does the NED coordinate transform webpage.

Australian National University

The Magnitude Scale

- Many astronomers (e.g. me) use a historical scale called *Vega magnitudes*.
- These were once defined relative to the brightest star in the Northern hemisphere, α Lyrae or Vega. For any filter F , we have:

$$\begin{aligned} m_F &= -2.5\log_{10}\frac{f_F}{f_{F,\mathrm{Vega}}} \\ f_F &= f_{F,\mathrm{Vega}} 10^{-0.4 m_F}. \end{aligned}$$

- In practice, Vega doesn't have a Vega magnitude of exactly 0. It has a magnitude of 0.03 in the visible region of the spectrum [See Bessell and Murphy (2012) and Bessell (2005)]

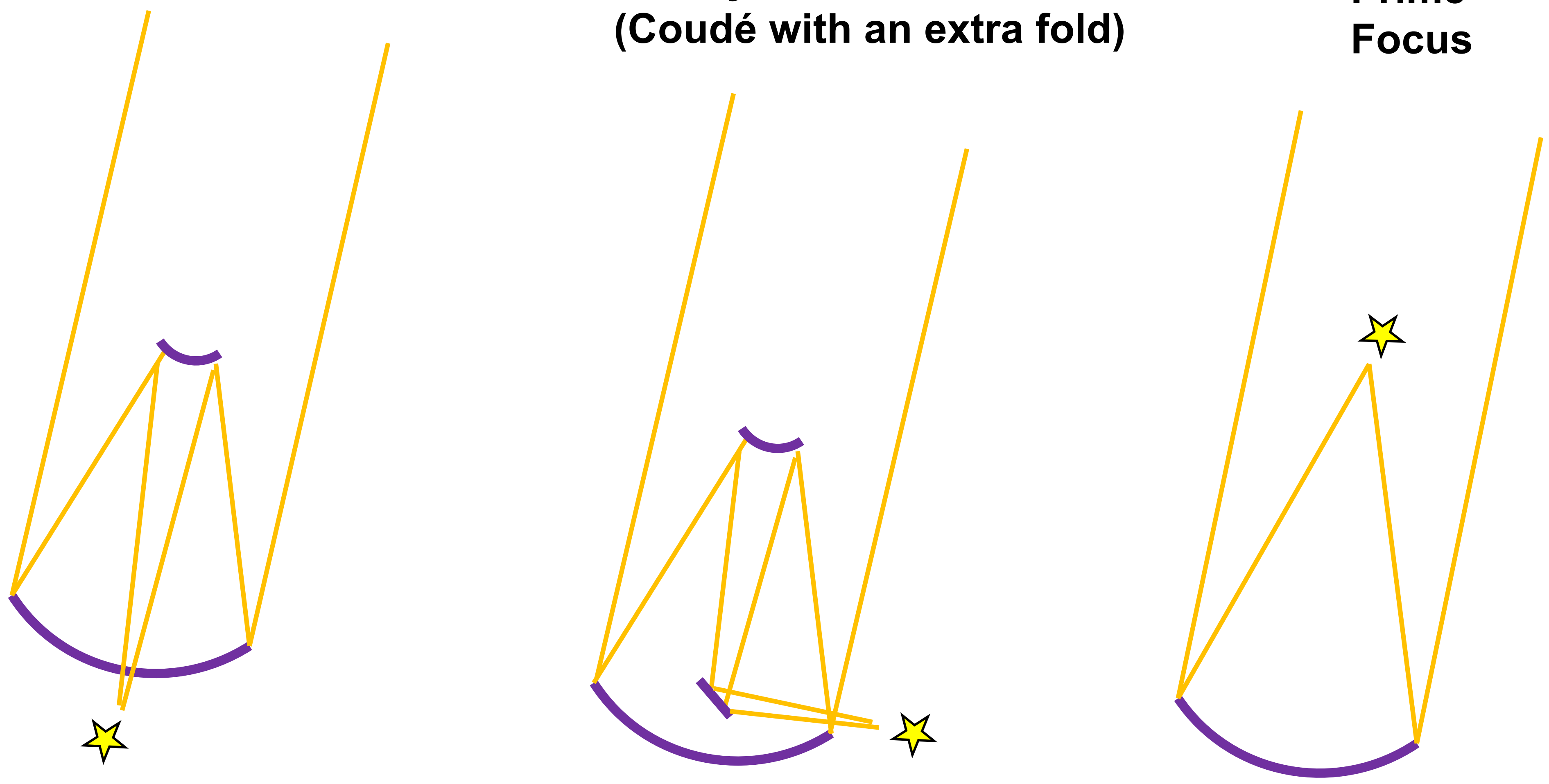
Absolute Magnitudes

- Observed magnitudes are called *apparent magnitudes, m*. The same object has a different magnitude at different distances.
- By convention, the *absolute magnitude, M* of an object is the magnitude it would have if placed at 10pc.
- The inverse square law enables us to define the *distance modulus:*

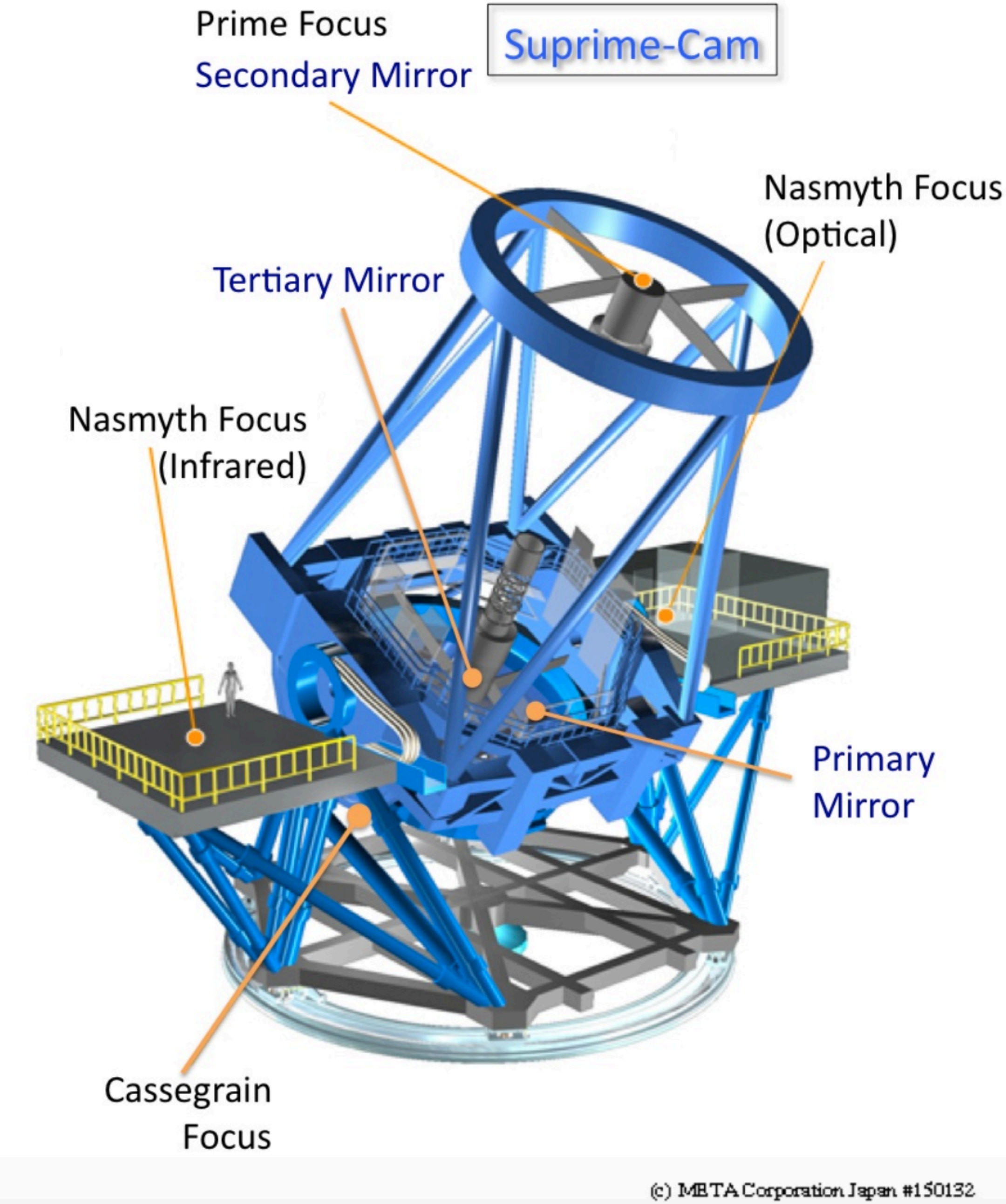
$$m - M = 5\log_{10}\left(\frac{d}{10\,\mathrm{pc}}\right)$$

Telescope Foci

Cassegrain **Nasmyth (Coudé with an extra fold)** **Prime Focus**

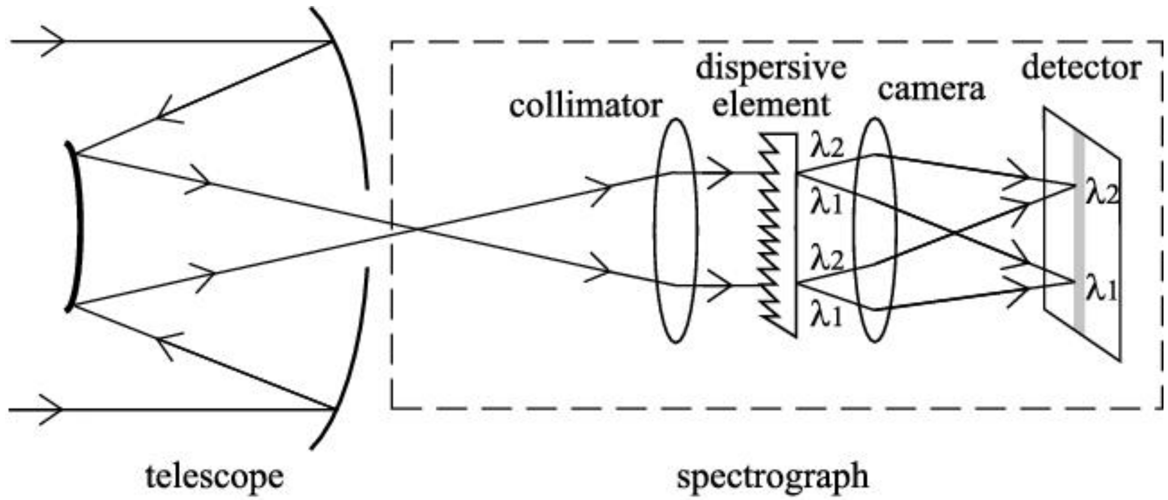


Example – Subaru has all 3 foci accessible

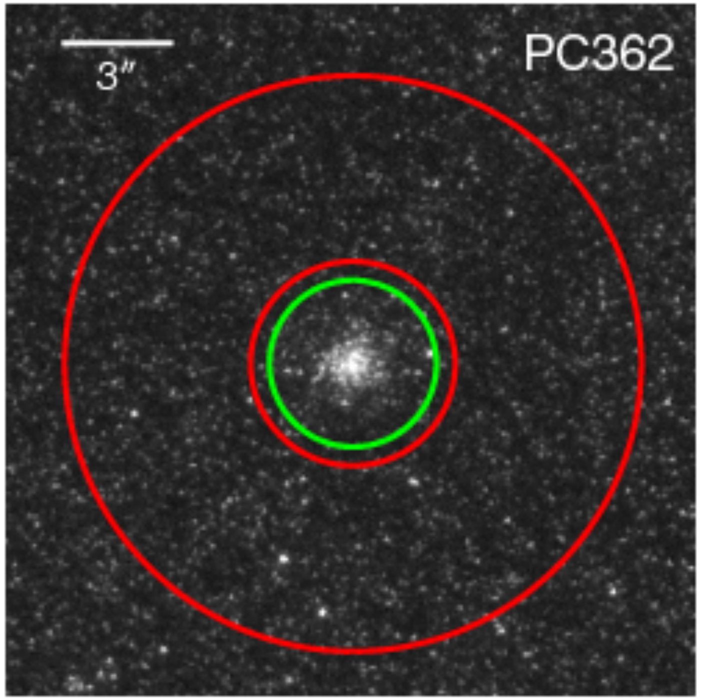


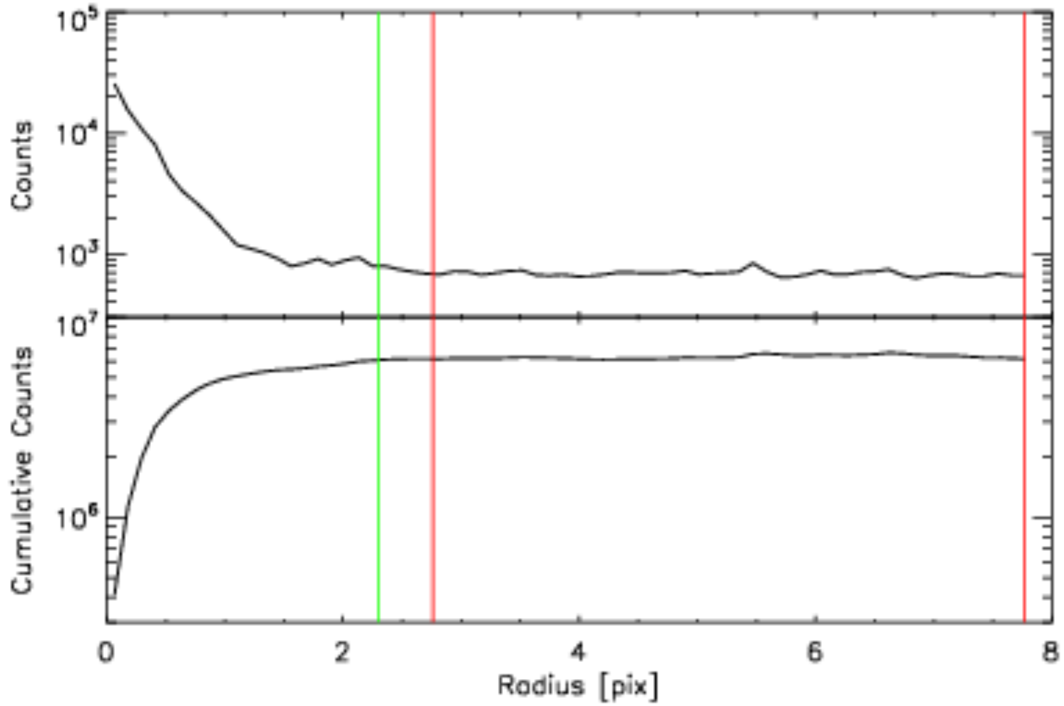
Different Foci for Different Purposes

- At Prime and Cassegrain focus, instruments have a variable gravity vector – not the best place for stable spectrographs.
- Spectrographs are best at Nasmyth, or fed by fibers. Some are *integral field units* – taking a spectrum of ever pixel, called a “spaxel”
- Refractive telescopes have a limited aperture size – only good for wide fields of view.



Aperture Photometry





- Measure the total counts ($\Sigma C(i.j)$) in green circle.
- Estimate sky background (sky/pixel) between two annulus

$$C = \Sigma C(i.j) - \text{Npixel} * \text{sky/pixel}$$

N_{pixel} is total pixel number in the green circle

Signal-to-noise

- Remember back in week 1: $S/N = \sqrt{r} \approx \sqrt{N}$
- In practice, counting photons from a target is not always the dominant source of noise.
  - Observations are *background limited* if the sky is brighter than the target, when averaged over the solid-angle *aperture* of the target.
  - If image size (”seeing”) is larger, then the aperture is larger and an background noise is larger.
  - Observations are *readout noise* limited if the detector noise, rather than target shot noise or background shot noise limits an observation.

Week 6 Summary

Textbook: Only Chapters 1 and 2 again, plus these notes, links and tutorial exercises.

1. Astronomical Coordinate Systems and catalogs.
2. The Magnitude Scale(s) and distance modulus.
3. Types of telescopes and instruments.
4. Signal to noise: beyond target shot noise.

Australian National University

ASTR2013 – *Foundations of Astrophysics*

Week 7: The Interstellar Medium and Star Formation

Mike Ireland

Field Trip Follow-Up

- Drop-in sessions. These have now been requested formally by your student representative. We can have them 19 Sep (Thursday), one of 26 or 27 Sep (Thurs/Fri before problem set and Field trip report due) and 10 or 11 Oct (Thurs/Fri before 2nd-last problem set due). VOTE NOW
- However *tutorials remain the most important times for getting help.*
- Overall, feedback would be useful – I've already mentioned that we'll do everything we can to accommodate the class of 2020 on the mountain (observatory purchasing extra beds etc).

Field Trip Follow-Up

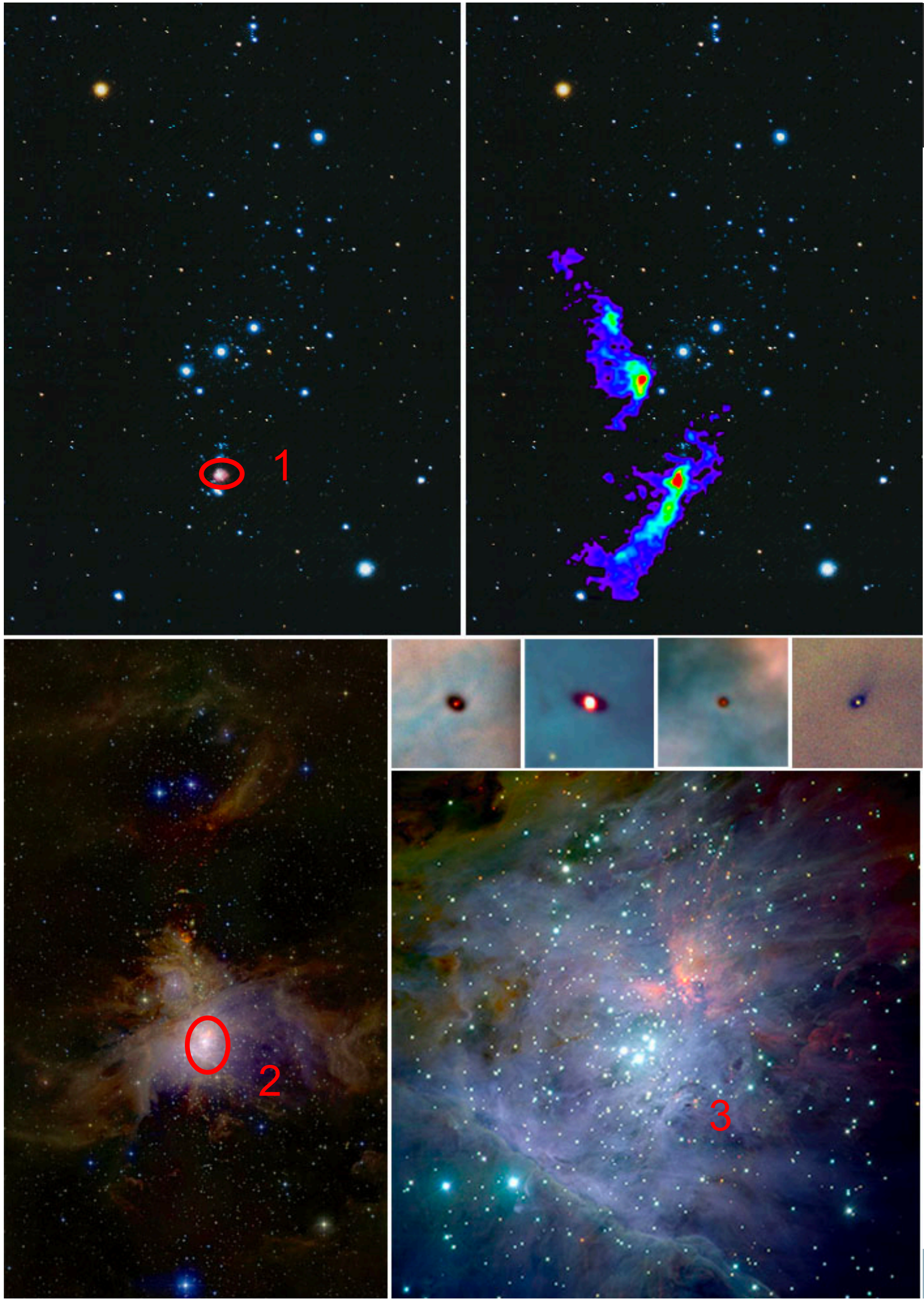
- For the field trip report, instructions and Due Date (31 Oct) are on Wattle – reports should be individual, but work should be as a group. Everyone has to say which part of the group work they did.
- Data reduction and analysis is ongoing. All groups should have enough to complete the data “reduction” and make plots by the end of today’s tutorial. Please:
  - Check both anu365 and Wattle.
  - Add your observing log and notes to Wattle
  - Meet with your group in addition to the tutorial times.

Week 7 Summary

Textbook: Sections 5.1, 5.2 and 5.3 (5.4 is also great, but Naomi will cover this in ASTR3013 next year)

1. The Jean's Mass.
2. The Strömgren Sphere.
3. Gas heating and cooling mechansms.
4. Components of the Interstellar Medium

Additional reference: Mark Krumholz's textbook available for free at https://arxiv.org/abs/1511.03457.



- Zoom in to a star forming region…(e.g. Orion)
- Cold gas collapses to form stars, which in turn locally heats the gas to ~7000K
- *How does the gas collapse? What determines its properties?*

Jean's Mass (side comment)

- The "Jean's Mass" is the mass of a cloud of gas that can collapse under its own gravity.
- There are many derivations of the "Jean's mass", partly because the 52 page paper published in 1902 is a very difficult read.
- Most modern derivations use partial differential equations (MATH2306) to derive the wave equation for a self-gravitating gas of (locally) uniform density:

$$\omega^2 = c_{\mathrm{is}}^2 k^2 - 4\pi G \rho_0$$

- Short wavelength waves are like sound waves, but long wavelength waves are unstable and grow.
- For this course, please focus on the textbook derivation…

Jean's Mass

- Assume there is a cloud of ideal, non-relativistic gas of uniform temperature and density.
- On the board, we can consider what happens to the internal gravitational and thermal energy of the cloud if the radius changes by a small amount.
- If the gravitational energy changes by more than the thermal energy, the cloud will be unstable to collapse, and we derive a critical mass:

$$M_J = \frac{3k_B T}{G\bar{m}} r = \frac{3c_s^2}{G} r$$

Jean’s Density and Scaling

- The critical density for collapse depends strongly on the cloud mass – solar mass clouds have to be much more dense:

$$\rho_J = \frac{M}{\frac{4}{3}\pi r_J^3} = \frac{3}{4\pi M^2}\left(\frac{3kT}{G\bar{m}}\right)^3 .$$

- Observed cold gas clouds have a mass of around 1000 M_{sun}, a temperature of around 20K and an measured density at least 10^2 particles/cm^3.
- The Jean’s density is then ~1 particle/cm3, meaning that the cold gas clouds should be unstable to collapse.
- Note that a 1 M_{sun} part of the cloud is stable – individual stars should not form from the collapse of clouds of typically observed densities and temperatures.

Cloud Collapse and Fragmentation

- Remember the free-fall timescale:

$$\tau_{\rm ff} \sim \left(\frac{3\pi}{32G\rho}\right)^{1/2}$$

- A uniform density cloud has a single timescale, and would collapse until it became optically-thick (and heated up).
- A cloud that starts off more dense in the center (or elsewhere) collapses faster in the more dense parts.
- As density increases, smaller parts of the cloud become Jeans unstable, resulting in collapse inside a collapsing cloud: *fragmentation.*

Cloud Collapse and Fragmentation

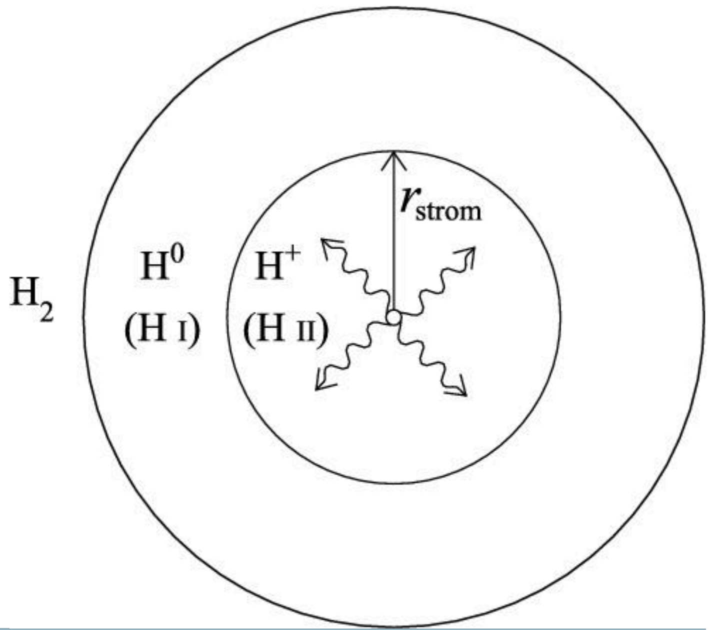
- The collapsing cloud is kept at a near-constant temperature by two key processes:
  - Radiative cooling, especially in optically-thin gas (reminder of what this is on board – what is the limiting column density for typical opacities?)
  - Dissociation processes in the gas that absorb energy.
- The energy involved in this dissociation is significant: 4.5 eV per molecule to dissociate H_2, and 13.6 eV per atom to ionize H.
- These processes alone can absorb the energy needed for a 1 M_{sun} star to collapse to 0.3 au (see textbook or equate the energy yourself!).

End Result – Star Clusters

- The outside-in collapse, or top-down fragmentation of gas clouds means that stars very rarely form alone - typically stars form in large associations.
- If the associations are still gravitationally bound after gas disperses, the result is a *star cluster* as seen on the field trip.

Strömgren Spheres

- Once a massive stars form, they can start emitting emitting strong UV radiation on a timescale even shorter than the cluster free-fall time.
- This radiation heats up local gas, preventing collapse in part of the cluster.
- The radius of ionized gas is called the Strömgren radius.
- r_{strom} is determined by the balance between photo-ionization and recombination (via collision)



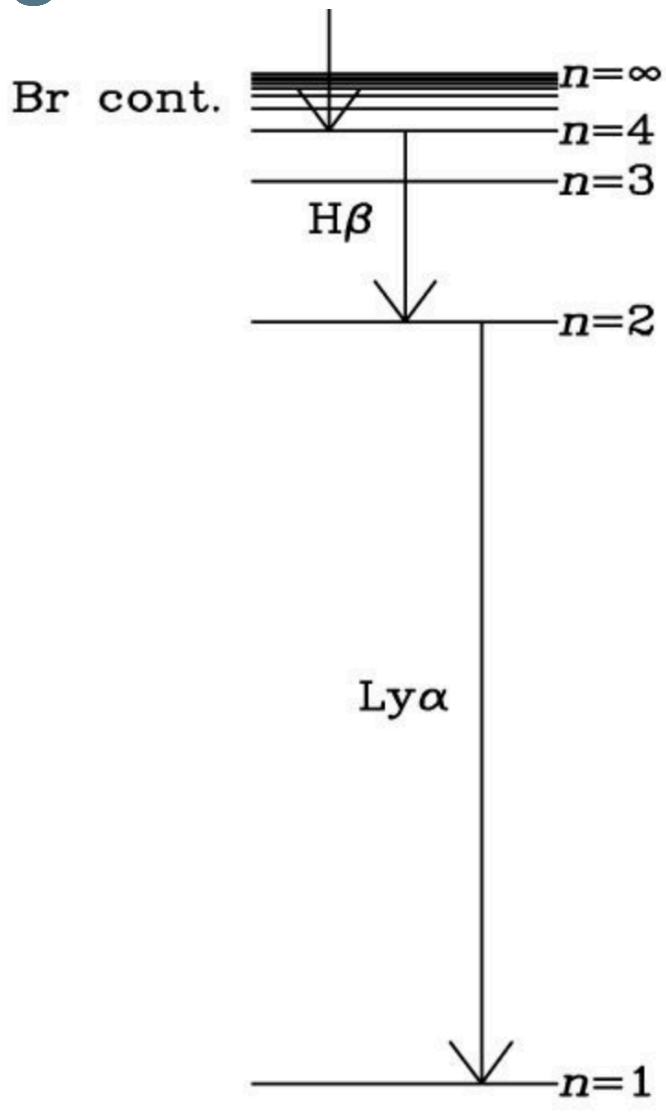
Australian National University

Strömgren Spheres

- Only photons with energies about 13.6eV can ionize H. We call the rate of emitted photons Q*, and the recombination rate R_{rec} per unit volume.
- Then we have: $$Q_* = \mathcal{R}_{\mathrm{rec}} \frac{4}{3} \pi r^3$$
- The recombination rate depends on temperature and the number of collisions:
$$\mathcal{R}_{\mathrm{rec}} = \alpha(T) n^2$$ (assuming nearly ionized gas)
- This results in: $$r_{\mathrm{strom}} = \left(\frac{3Q_*}{4\pi \alpha n^2} \right)^{1/3}$$

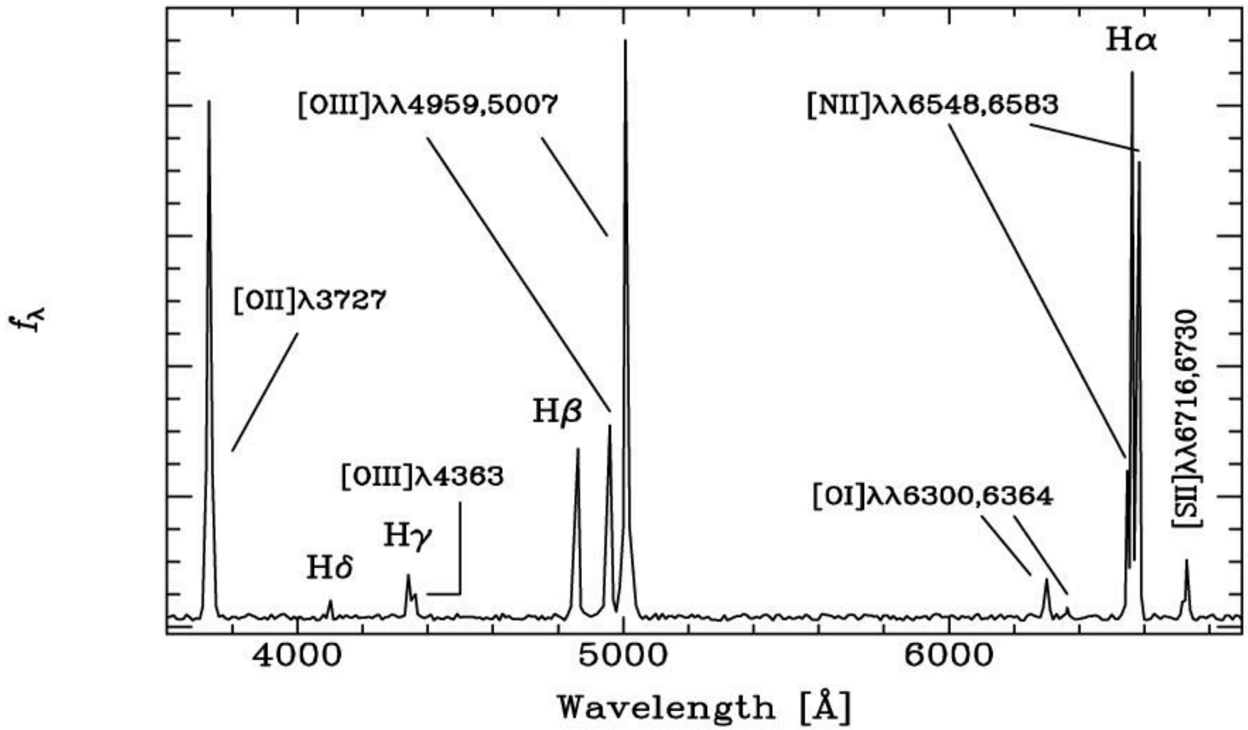
Heating and Cooling

- Outside the Strömgren sphere, photons are absorbed very rapidly, resulting in a sharp transition.
- Inside the Strömgren sphere, energy can be lost by:
  - Balmer series photons (e.g. “pink” Ha) that emit recombination energy but can’t be re-absorbed.
  - Collisionally excited heavier atoms (especially Oxygen, the 3rd most abundant element).
  - “Free-free” or “Bremsstrahlung” emission, which is most relevant at longer wavelengths and dense regions, or very high temperatures.



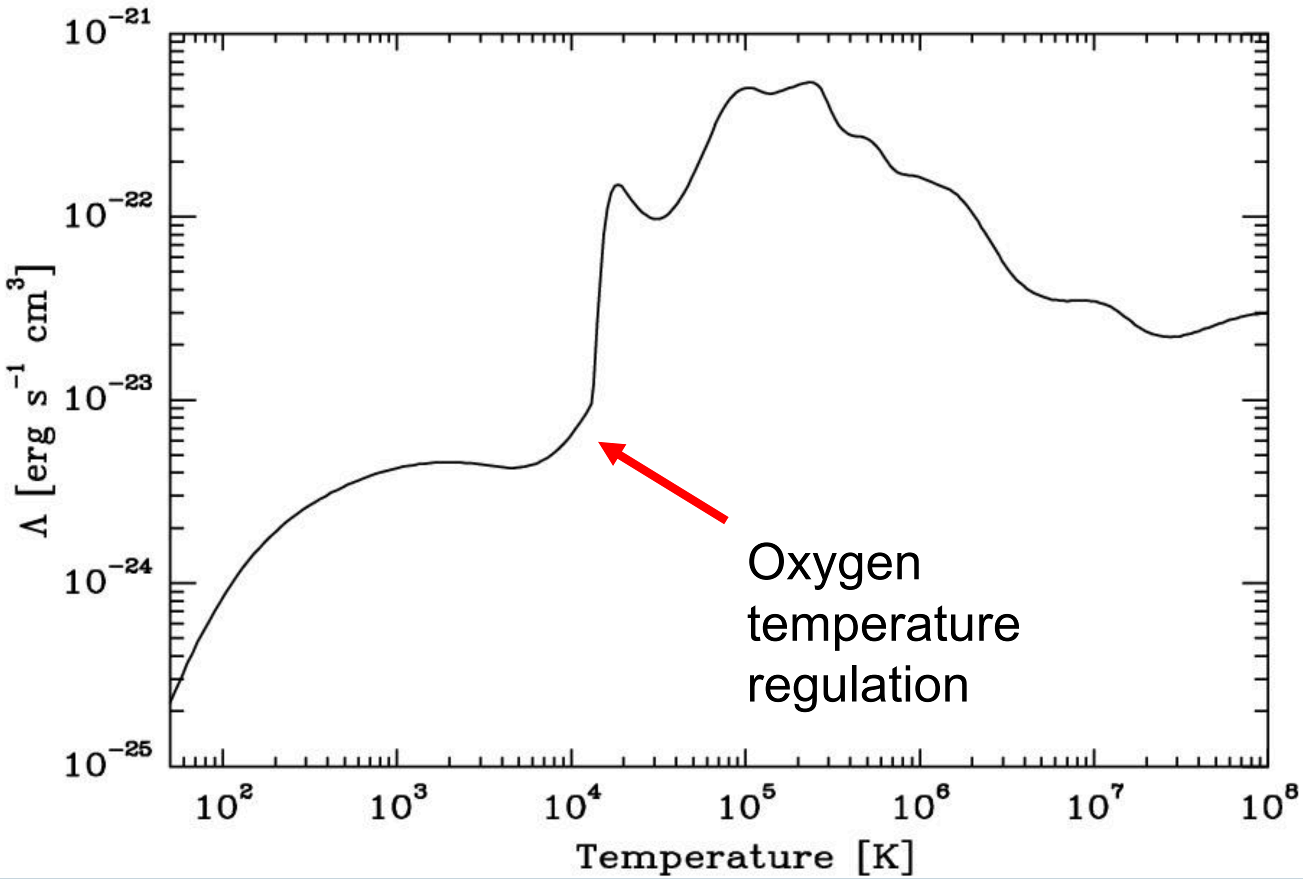
Heating and Cooling

- The observed spectrum of a HII region (ionized H, where neutral H is HI) is then dominated by photons that result in cooling.
- Many other photons are emitted and re-absorbed within the HII region, but only ones that can escape are seen.
- These are *forbidden* transitions, given square brackets in conventional spectroscopic notation, like [OII].



Heating and Cooling

- Heating of electrons happens e.g. when photons have more energy than they need to ionize hydrogen.
- The balance of heating with cooling determines the gas temperature. When cooling rises rapidly with temperature, there is a natural thermostat.



Components of the ISM

- The ISM is stable in 5 main phases:
    1. Molecular hydrogen (~30K) in cool, star forming regions (small fraction of the Galaxy) – efficient cooling e.g. by dust and ice.
    2. Cold neutral gas (100K), seen by the hyperfine transition of Hydrogen (next week)
    3. Warm neutral gas (7000K) which is harder to see.
    4. Warm ionized gas (10,000K), seen by Balmer series and [O] lines.
    5. Hot ionized gas (10^6K), where cooling by atoms is not efficient (e.g. intergalactic medium.
- Additionally, dust is important at absorbing light, especially in Molecular and Cold Neutral phases.

Week 7 Summary

Textbook: Sections 5.1, 5.2 and 5.3 (5.4 is also great, but Naomi will cover this in ASTR3013 next year)

1. The Jean’s Mass.
2. The Strömgren Sphere.
3. Gas heating and cooling mechansms.
4. Components of the Interstellar Medium

Additional reference: Mark Krumholz’s textbook available for free at https://arxiv.org/abs/1511.03457.

Australian National University

ASTR2013 – *Foundations of Astrophysics*

Week 8: Structure and Dynamics of the Milky Way

Mike Ireland

Jean’s Mass

- Assume there is a cloud of ideal, non-relativistic gas of uniform temperature and density.
- The *Jean’s Mass* is the mass as a function of radius (or density) that is unstable to gravitational collapse:

$$M_J = \frac{3k_B T}{G\bar{m}} r = \frac{3c_s^2}{G} r$$

Phases of ISM and Strömgren Spheres

- The Interstellar Medium (ISM) has a range of temperatures and densities, and a several stable phases.
- The transition between ionized gas around a hot star and surrounding neutral as is called the Strömgren sphere.
- For a rate of ionizing photons Q_*, a number density *n* and a recombination rate per unit volume α, the sphere radius is:

$$r_{\rm strom} = \left(\frac{3Q_*}{4\pi\alpha n^2} \right)^{1/3}$$

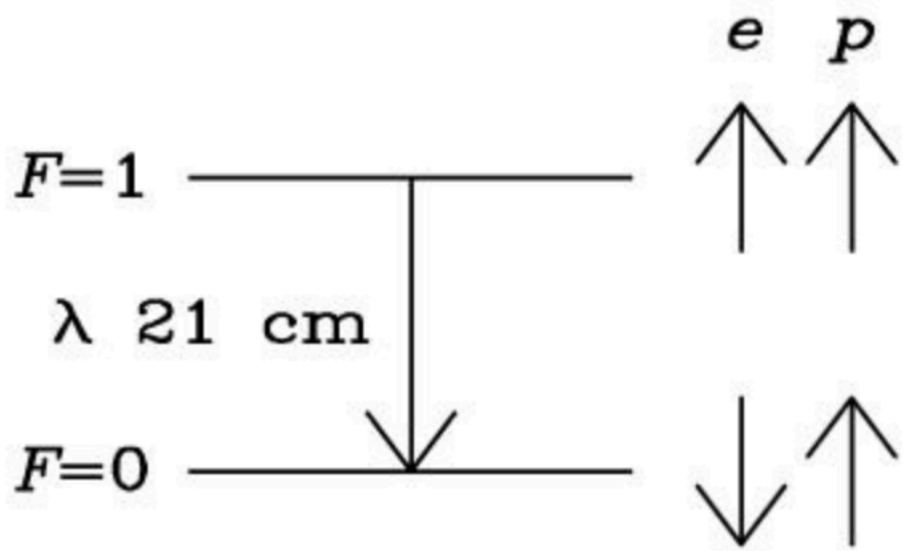
Week 8 Summary

Textbook: Sections 5.3, (2nd edition – 1st edition has 1 fewer chapters) and 7.1.

- HI radio emission at 21cm probes the mass of a key ISM component and structure of the Galaxy.
- The Galaxy has a Disk (young and old), Halo (old, metal poor), Bulge and bar.
- At the center is a 4 million solar mass black hole, which is relatively quiescent.
- The Galactic rotation curve is nearly flat at the location of the sun. Mass is dominated by dark matter – WIMPS not MACHOs.

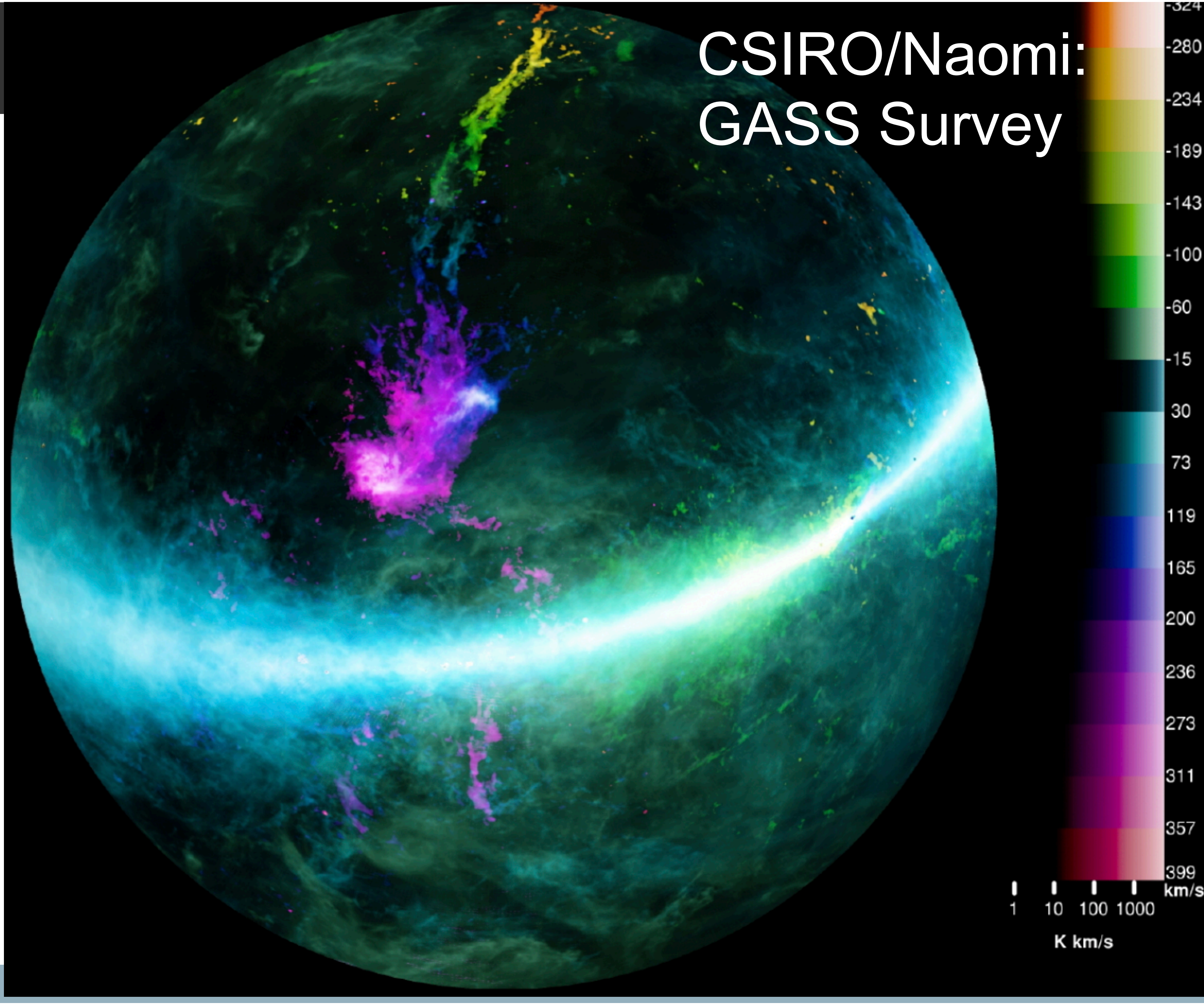
HI Emission

- Atomic transitions are *allowed* if e.g. electron spin doesn't change and angular momentum is conserved.
- Spin-orbit coupling splits energy levels, regarded as *fine* structure. E.g. The Sodium Doublet, or transitions between ground state level of neutral Carbon THz/sub-mm line of CI at 370μm.
- *Hyperfine* structure refers to transitions where the nuclear and electron spins are aligned or anti-aligned.
- Key examples are the Cs atomic clock transition at 9.19 GHz, and the 21cm line (1.42GHz) of H.



HI Emission

- HI is so important because all neutral hydrogen emits at this wavelength. Ionised and molecular hydrogen can be difficult to detect.
- The transition rate is 3×10^{-15} Hz from upper to lower energy level – i.e. 10^7 years on average to emit a photon.
- Fraction of HI in upper level depends on collision excitation only.
- Even at 30K, $k_B T$ is ~400 times lower than $h\nu$., so very close to 50% of H is always in upper state – *emission measures mass.*


CSIRO/Naomi:
GASS Survey
-324
-280
-234
-189
-143
-100
-60
-15
30
73
119
165
200
236
273
311
357
399
km/s
1
10
100
1000
K km/s

Milky Way as a Spiral Galaxy

- We can see the disk-like nature of our Galaxy on a dark night in Coonabarabran.
- The spiral structure is most easily inferred by looking at other, otherwise similar Galaxies…

M51
(Textbook – NASA, ESA and Hubble Heritage team)

Face-on Spiral Galaxy

Note – our galaxy is a barred spiral… clearest evidence only in last 18 months

Milky Way as a Spiral Galaxy

NGC 3370
(Textbook – NASA, ESA
and Hubble Heritage team)

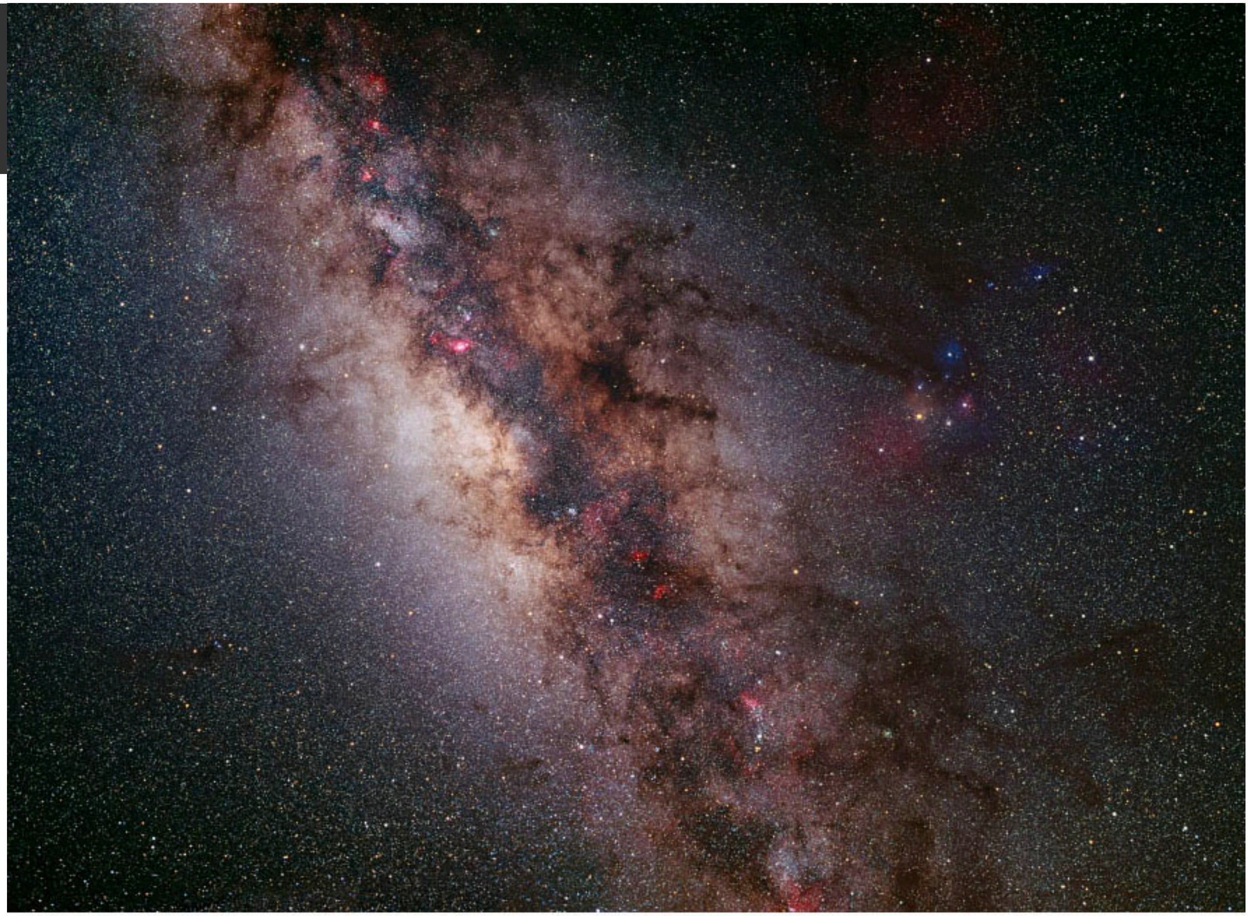
Moderately Inclined Galaxy

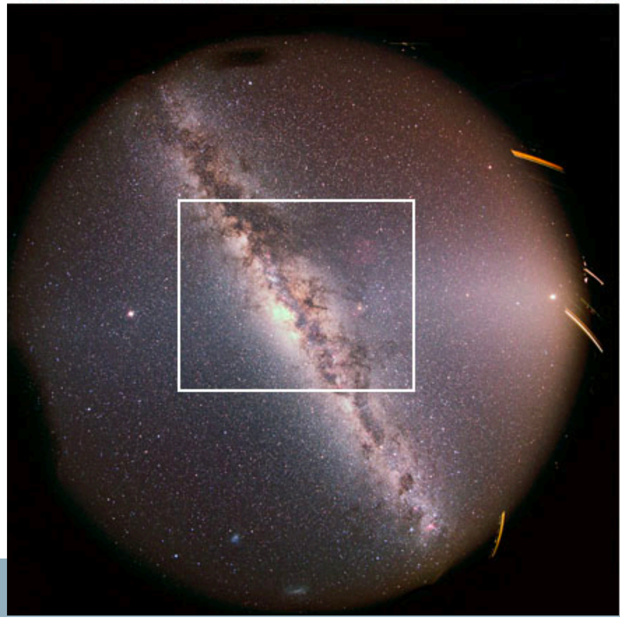
Milky Way as a Spiral Galaxy

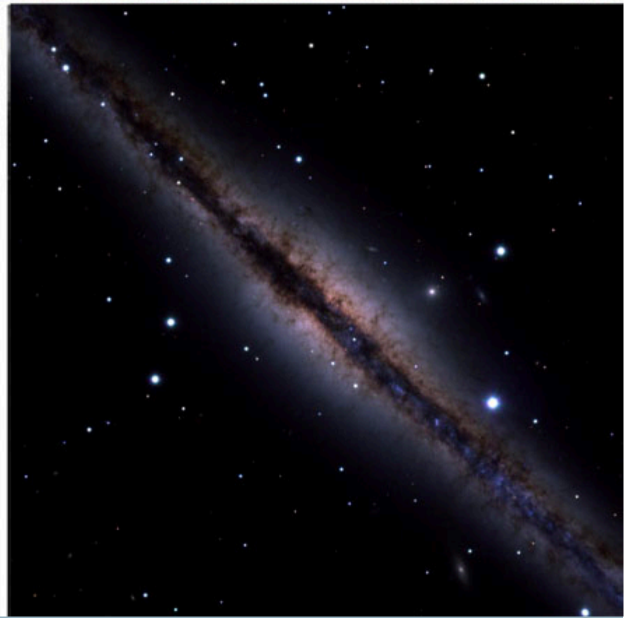
NGC 4594 *Near Edge On Galaxy*
(Textbook – NASA, ESA and Hubble Heritage team)

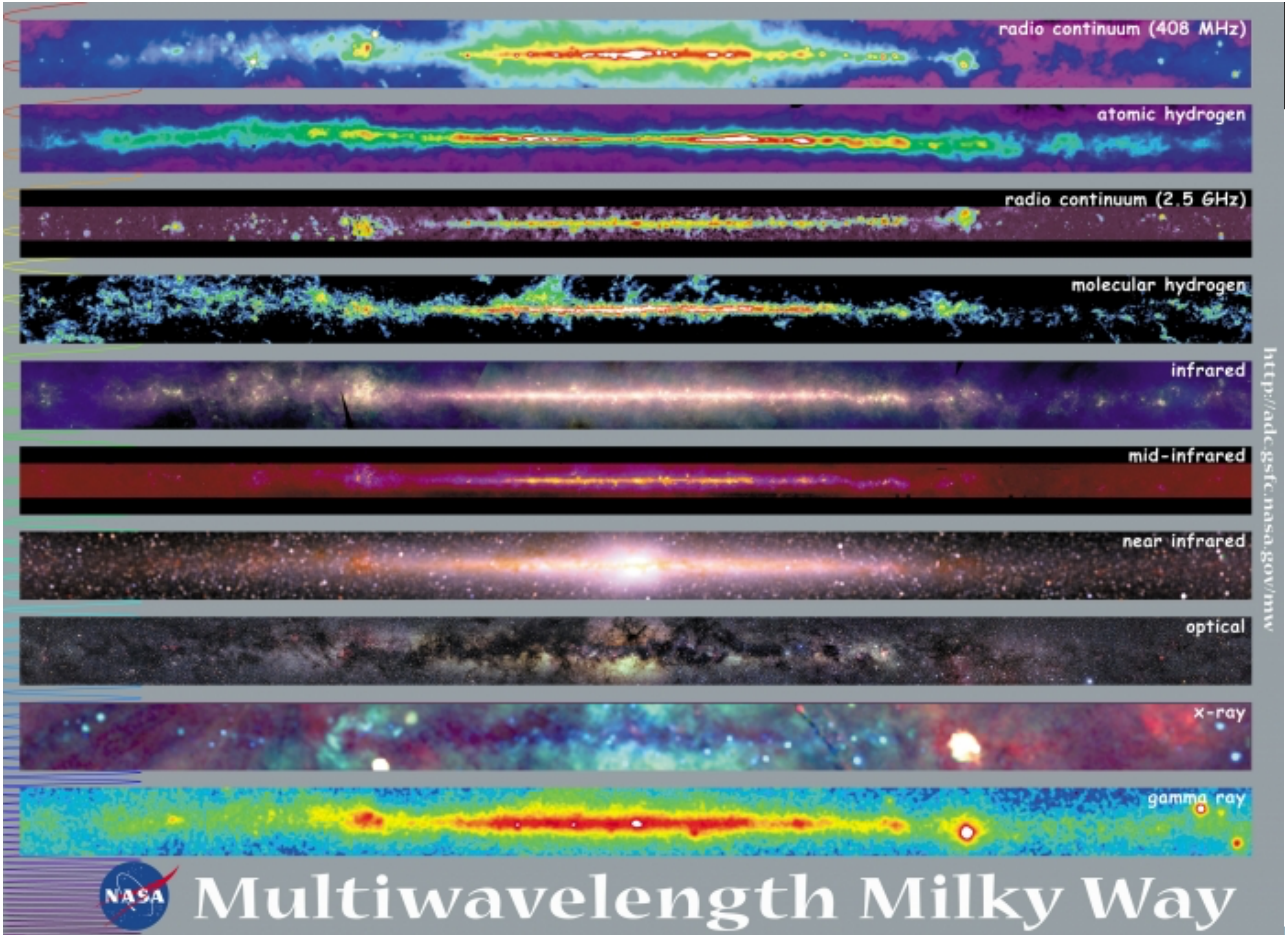
Australian National University

- Our view of the Galaxy in optical light (top and bottom-right)
- NGC 891 – an edge on spiral



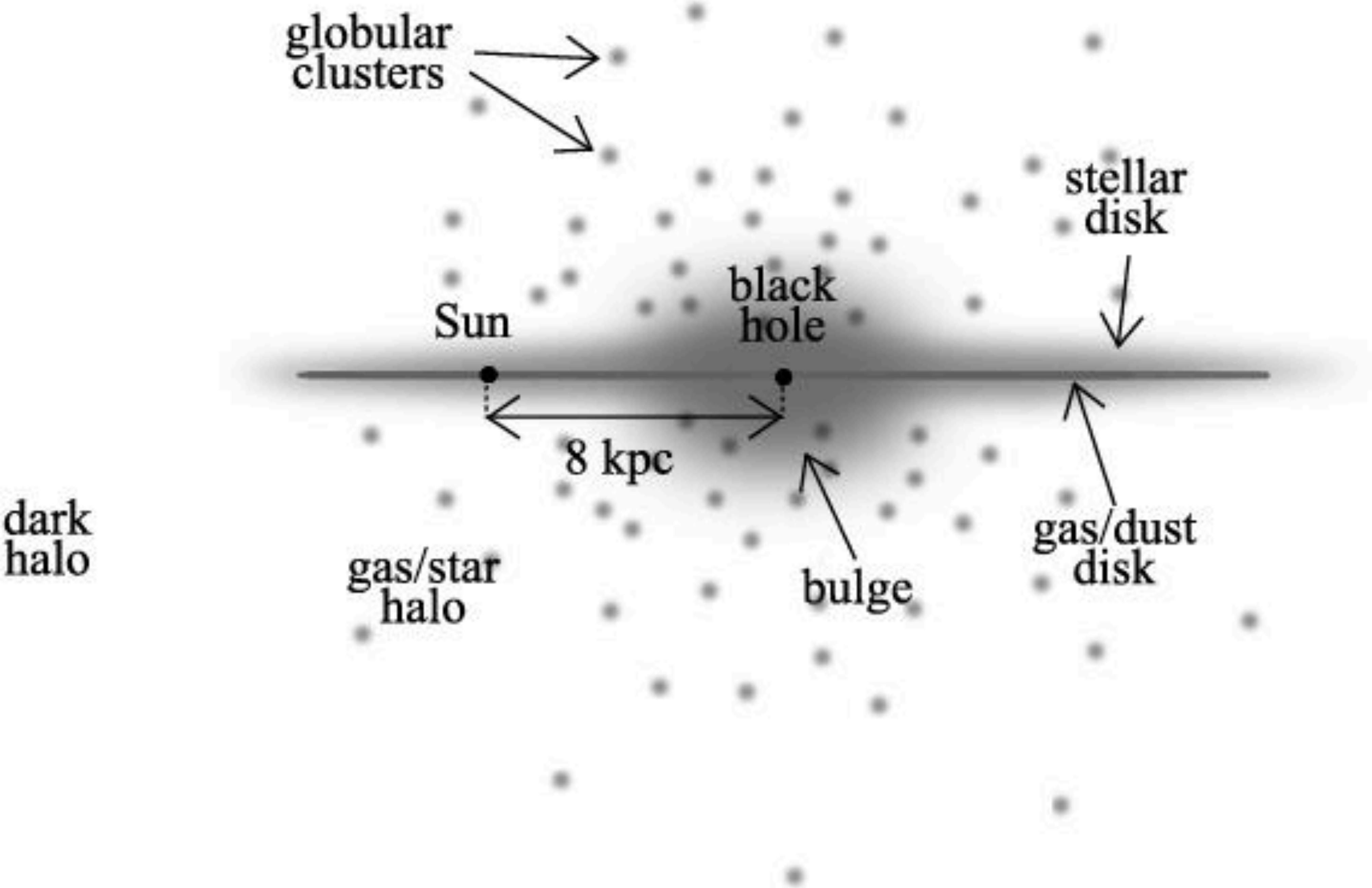





radio continuum (408 MHz)
atomic hydrogen
radio continuum (2.5 GHz)
molecular hydrogen
infrared
mid-infrared
near infrared
optical
x-ray
gamma ray
http://adc.gsfc.nasa.gov/mw
NASA
Multiwavelength Milky Way

Other Key Galactic Observations

- Even without parallax measurements, star counts (histogram of the number of stars of a given brightness as a function of angle) historically informed our Galactic structure models.
- Velocity measurements of stars (radial velocity or proper motion) and gas (radial velocity only) enable the Galactic potential to be modelled and statistical structures of stars to be found.
- For example, stars that have a similar spatial and velocity distribution to gas are likely young stars.


globular
clusters
stellar
disk
black
hole
Sun
8 kpc
dark
halo
gas/star
halo
bulge
gas/dust
disk

Key Galactic Parameters

- Distance to Galactic Center 8.18 +/- 0.03 kpc (star orbiting black hole , https://arxiv.org/abs/1904.05721)
- Circular rotational velocity at our Galactocentric radius: 234 +/- 3 km/s (https://arxiv.org/abs/1810.02131), which results in an orbital period of 2 x 10^8 years.
- For circular orbits, gravitational acceleration balances centripetal acceleration:

$$\frac{GM(M_\odot)}{R_\odot^2} = \frac{v_{\rm circ}^2}{R_\odot}$$

- This gives a mass of 1.1 x 10^{11} M_{sun}.

The Galactic Disks

- The density of various components of the Galactic disk approximately follows an exponential in radius and height in cylindrical coordinates.

$$\rho(r,z) = \rho_0 \left[\exp\left(-\frac{r}{r_d}\right)\right] \left[\exp\left(-\frac{|z|}{h_d}\right)\right]$$

- The scale height is 130 to 400pc for young to old stars in the thin disk, and ~1kpc for theso-called *thick* disk.
- The cold gas (HI, molecular clouds) has the same scale height as the young stars.

Stellar Interactions and Collisions

- As the disk consists of the highest densities of stars, we can ask the question: *how often to stars collide?*
- Like any group of particles, all we need to know is the number density *n*, the particle velocities *v* and the cross-section σ to result in a collisional time of:

$$\tau_{\rm coll} = \frac{l}{v_{\rm ran}} = \frac{1}{\bar{n}\sigma_{\rm geom} v_{\rm ran}}$$

- This is 10^{19} years with a naieve stellar cross-section, or 10^{16} years once gravitational focusing is taken into account (see textbook for details).
- On the other hand, stars do collide with outer planetary systems – a 50au radius planetary system is disrupted by a stellar collision every ~200 Gyr.

The Bulge and Halo

- The spherical distribution of stars in the Galaxy consists of the bulge (~1kpc in size) and the halo. The textbook has:

$$\rho \propto r^{-3}$$

- Other references, e.g. the often used Besançon model (Robin et al 2003) has:

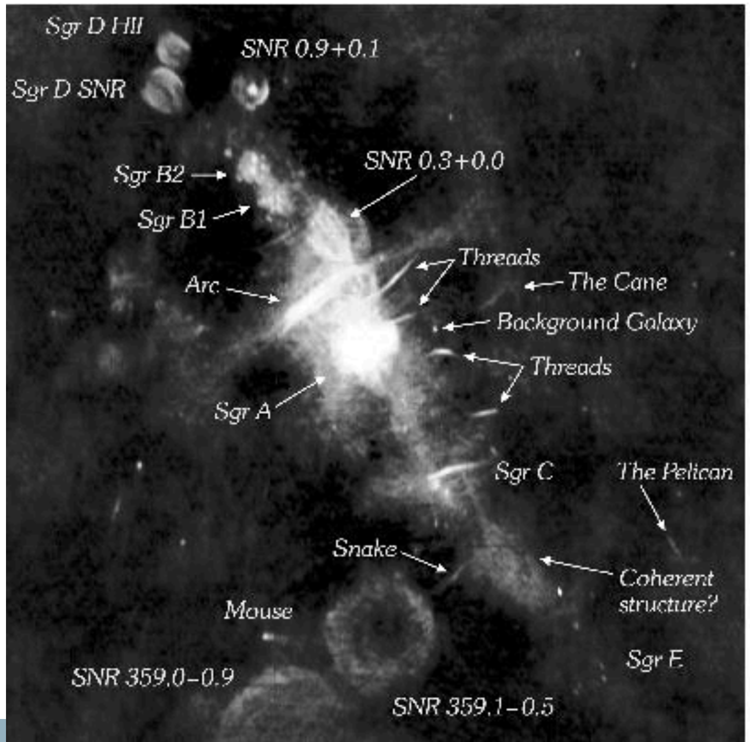
$$\rho \propto r^{-2.44}$$

- The halo/spheroid consists of old, metal poor stars.

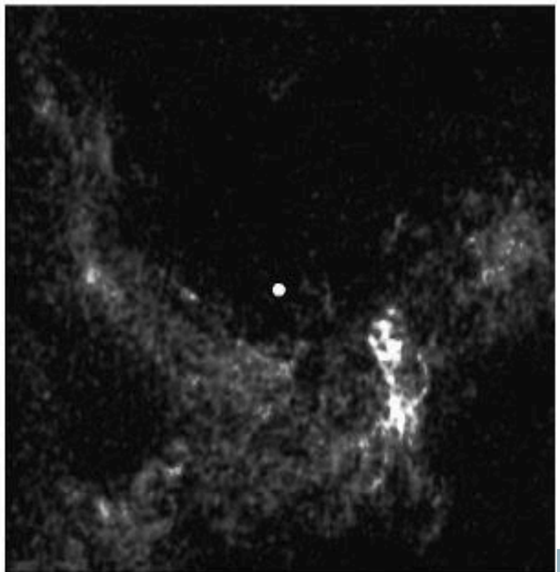
$$\frac{[\mathrm{Fe/H}]}{[\mathrm{Fe/H}]_\odot} = 10^{-4.5} \text{ to } 10^{-0.5}.$$

Galactic Center

- Sgr A (and Sgr A* for the very central source) was a radio source detected at the center of the Galaxy in the 1930s.
- X-ray variability and orbits of stars around the center both measured approximately 20 years ago made the identification as a supermassive black hole secure – $M=4 \times 10^6 M_{sun}$

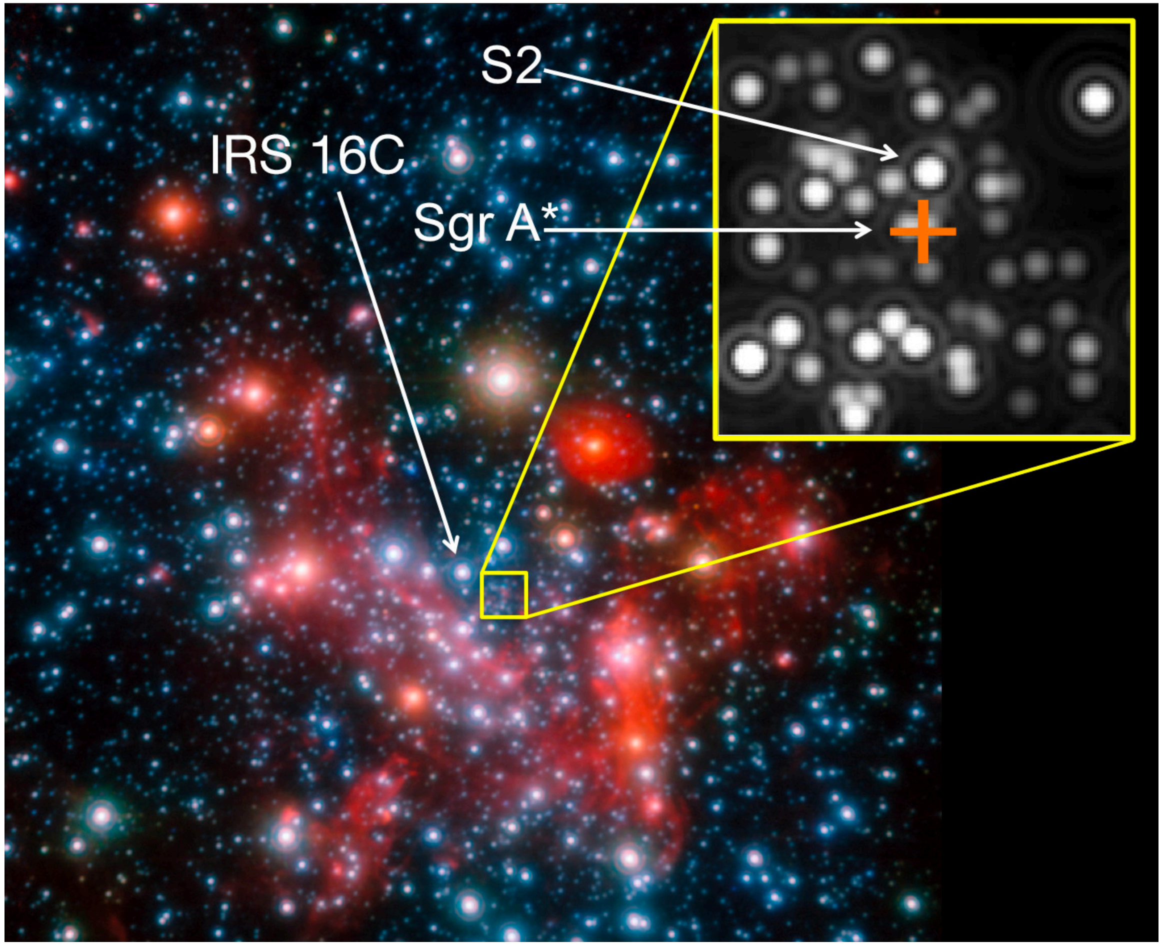


λ=90cm
1kpc



λ=1.3cm
1pc

Galactic Center

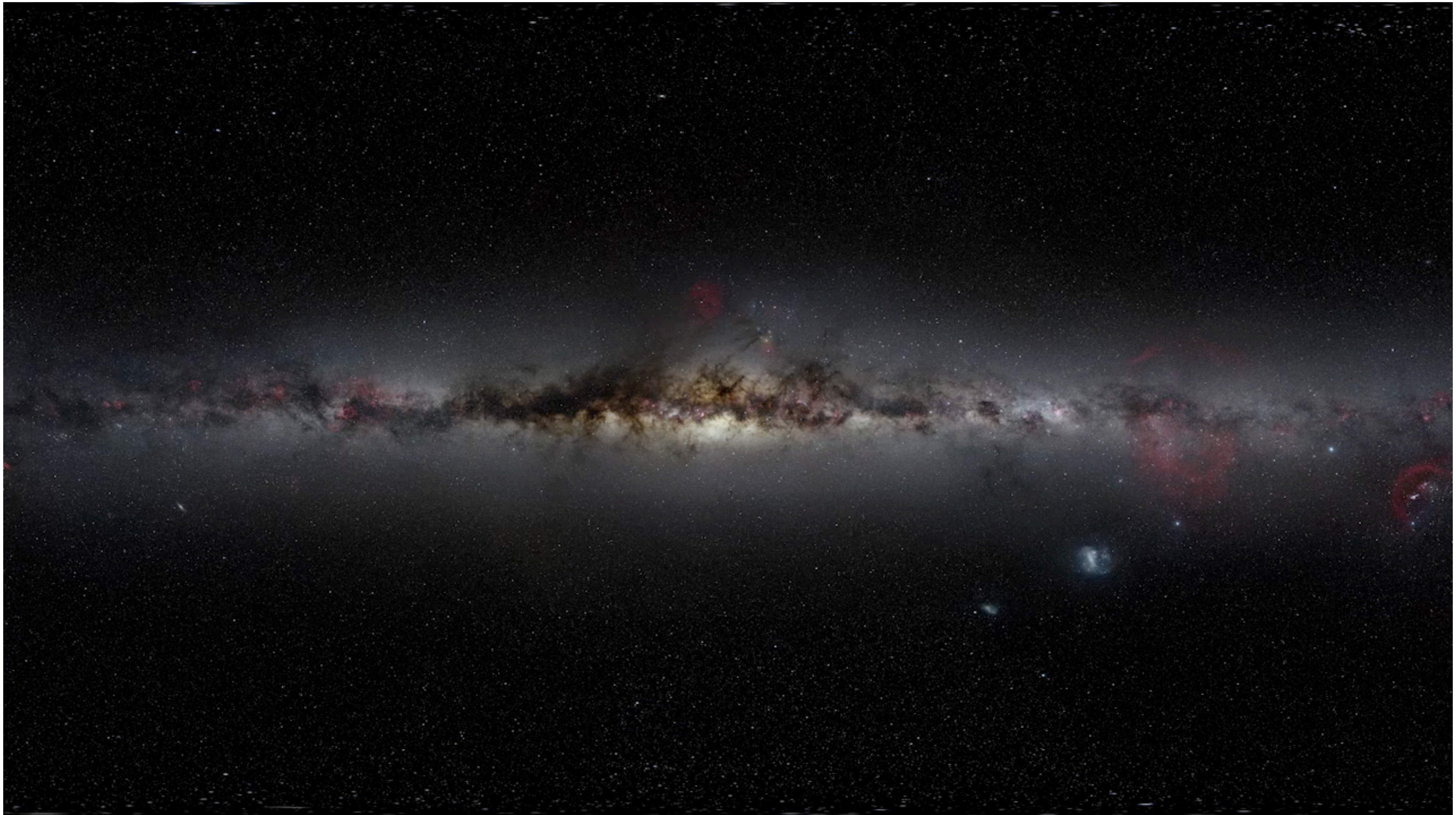


- Many bright sources near Sgr A* make stellar orbits complex
- US (Keck – Ghez/Lu textbook image) vs Europe (Genzel) competition.
- ESO's `Gravity' instrument (4x 8m telescopes) has best data now…

(https://arxiv.org/abs/1810.12641)

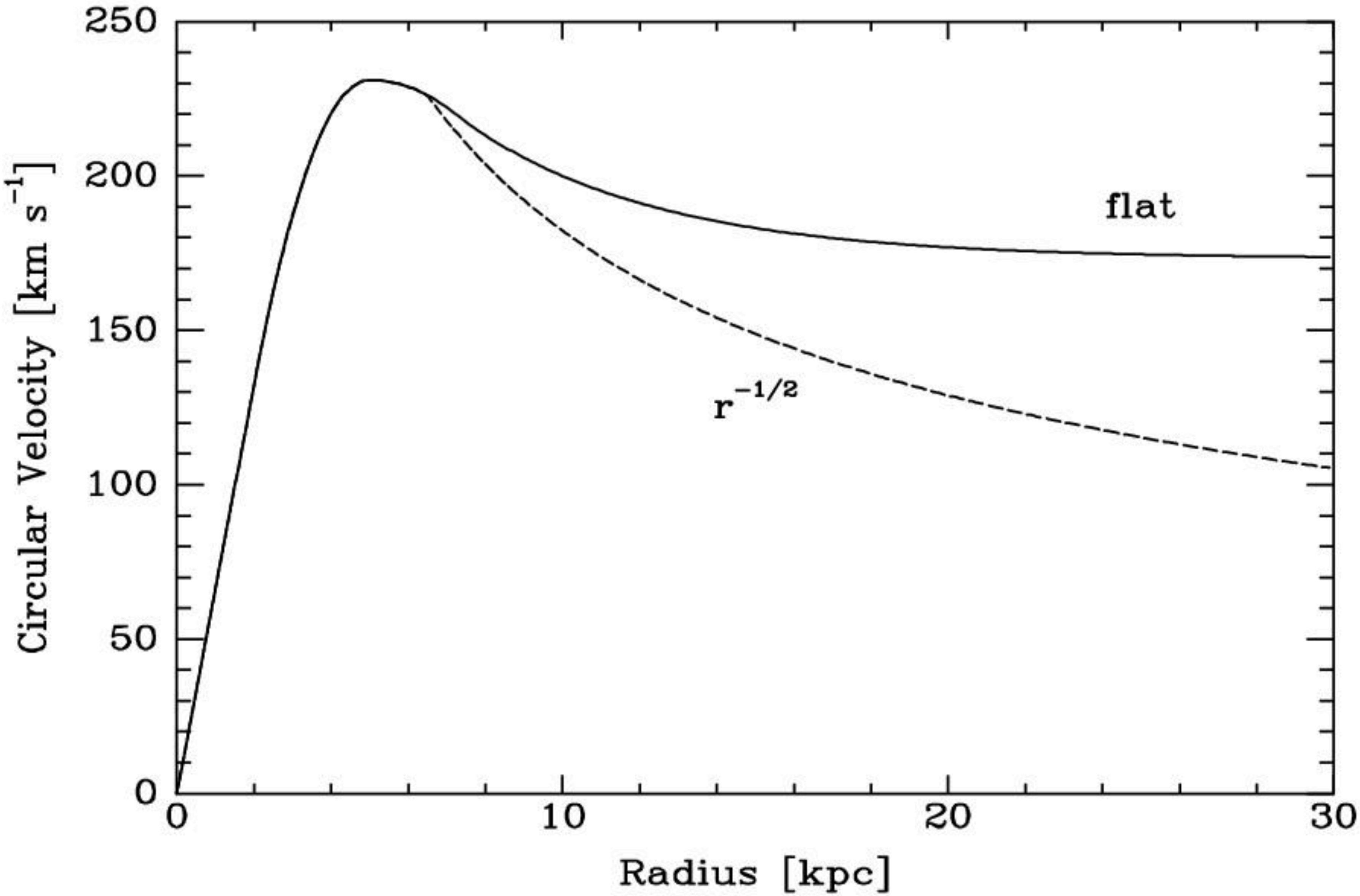
Galactic Center

ESO/MPE movie from NACO/VLT and Gravity
Discuss mass, flares, astrometric motion during flare



Dark Matter – the Dark Halo

- Locally, if we estimate the interior mass in the Galaxy (~10^{11} Msun), it is suspiciously higher (less than a factor of 2) than an estimate of the gas and stellar mass.
- However, most of the luminous stars are interior to the sun, so we'd naievely expect the rotation curve of the galaxy to approach Kepler's law.
- However, the rotation curve (measured from stars and HI) is flat – demonstrating there is unseen matter that dominates at the sun's radius and further out.



250
200
150
100
50
0
0
10
20
30
flat
r^{-1/2}
Circular Velocity [km s^{-1}]
Radius [kpc]

What is Dark Matter?

- 1970s to 1990s involved many attempts to find the source of dark matter.
- Best candidate: Cold dark matter consisting of massive particles that interact by gravity and nothing stronger than the electroweak force.
- The MACHO survey from Mt Stromlo eliminated massive objects due to the lack of gravitational lensing (many other microlensing surveys now, e.g. OGLE)
- Extremely cold molecular hydrogen clouds of just the right size not completely ruled out (as far as I’m aware), but there is no model of their formation.

Week 8 Summary

Textbook: Sections 5.3, (2nd edition – 1st edition has 1 fewer chapters) and 7.1.

- HI radio emission at 21cm probes the mass of a key ISM component and structure of the Galaxy.
- The Galaxy has a Disk (young and old), Halo (old, metal poor), Bulge and bar.
- At the center is a 4 million solar mass black hole, which is relatively quiescent.
- The Galactic rotation curve is nearly flat at the location of the sun. Mass is dominated by dark matter – WIMPS not MACHOs.

Australian National University

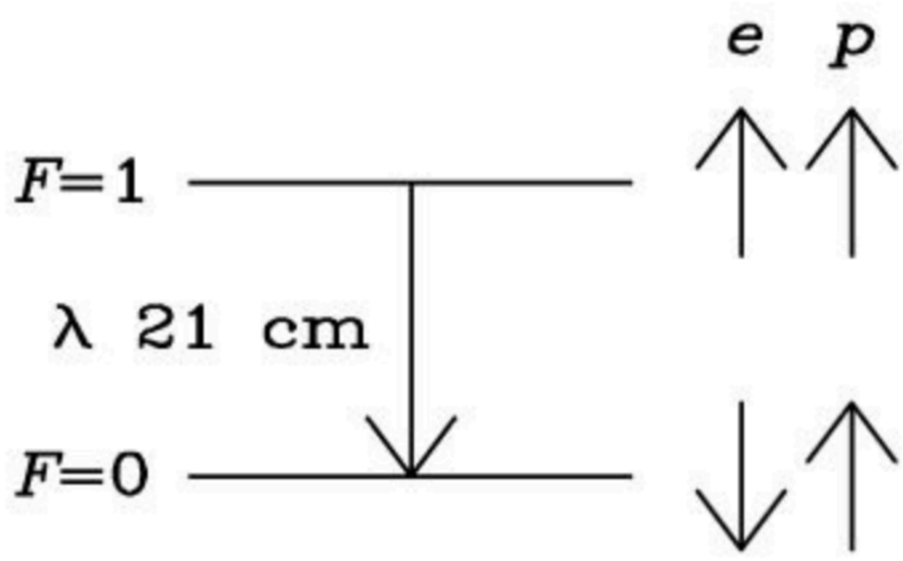
ASTR2013 – *Foundations of Astrophysics*

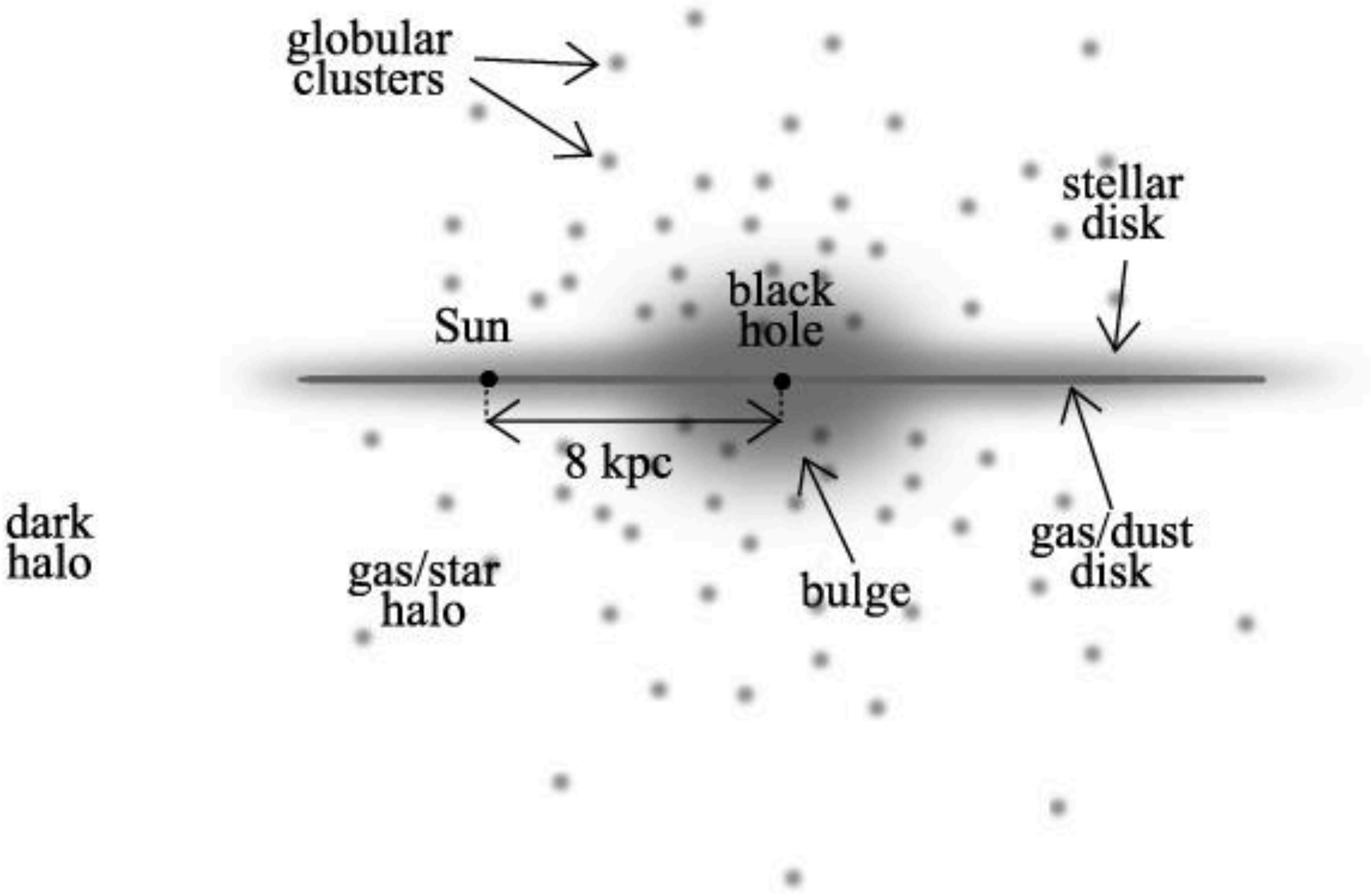
Week 9: Other Galaxies and Active Galactic Nuclei

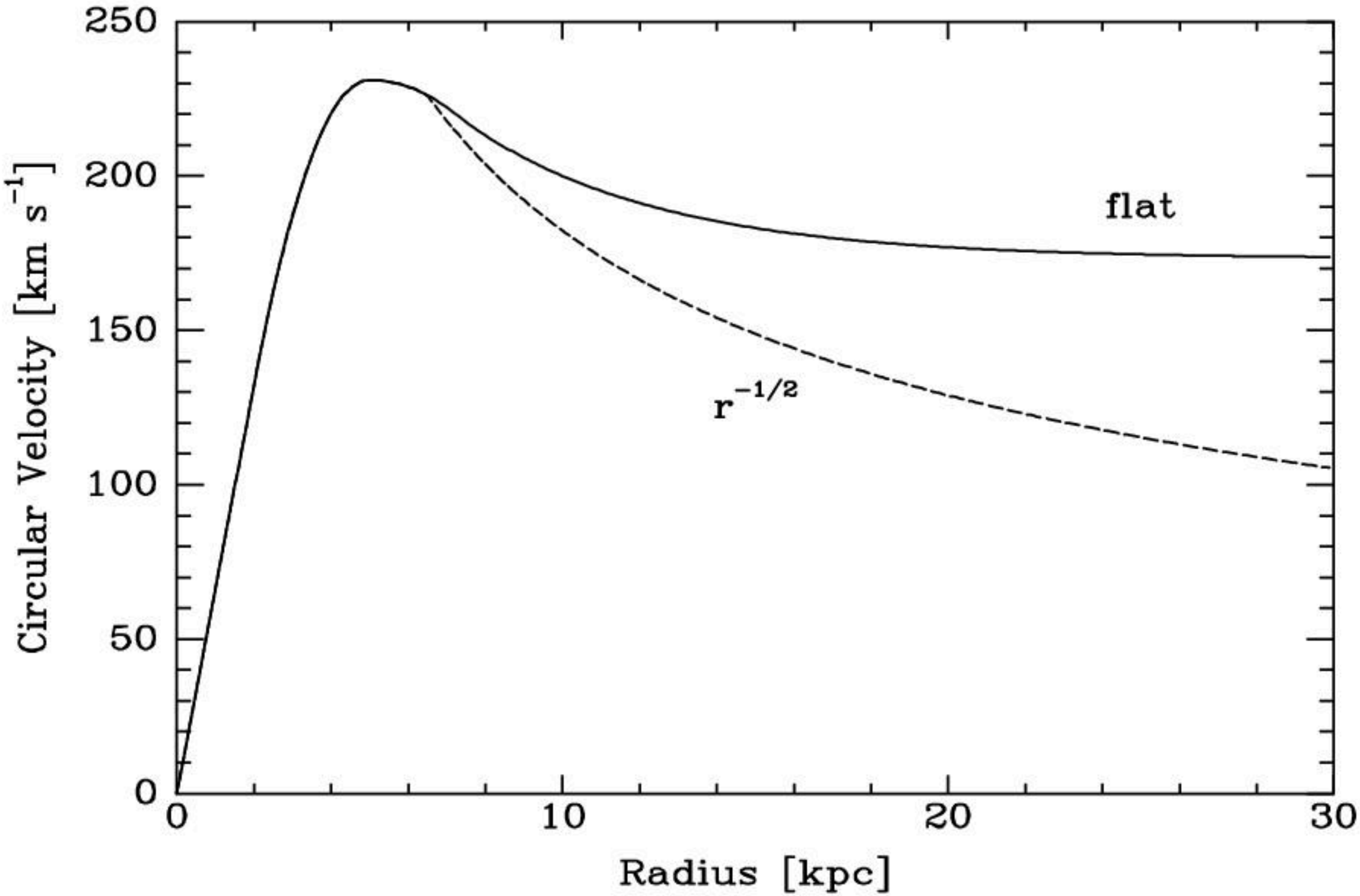
Mike Ireland

HI Emission

- HI is so important because all neutral hydrogen emits at this wavelength. Ionised and molecular hydrogen can be difficult to detect.
- The transition rate is 3 x 10^{-15} Hz from upper to lower energy level – i.e. 10^7 years on average to emit a photon.
- Very close to 50% of H is always in upper state – *emission measures mass.*




globular
clusters
stellar
disk
black
hole
Sun
8 kpc
dark
halo
gas/star
halo
bulge
gas/dust
disk


Circular Velocity [km s^{-1}]
Radius [kpc]
flat
r$^{-1/2}$
0
50
100
150
200
250
0
10
20
30

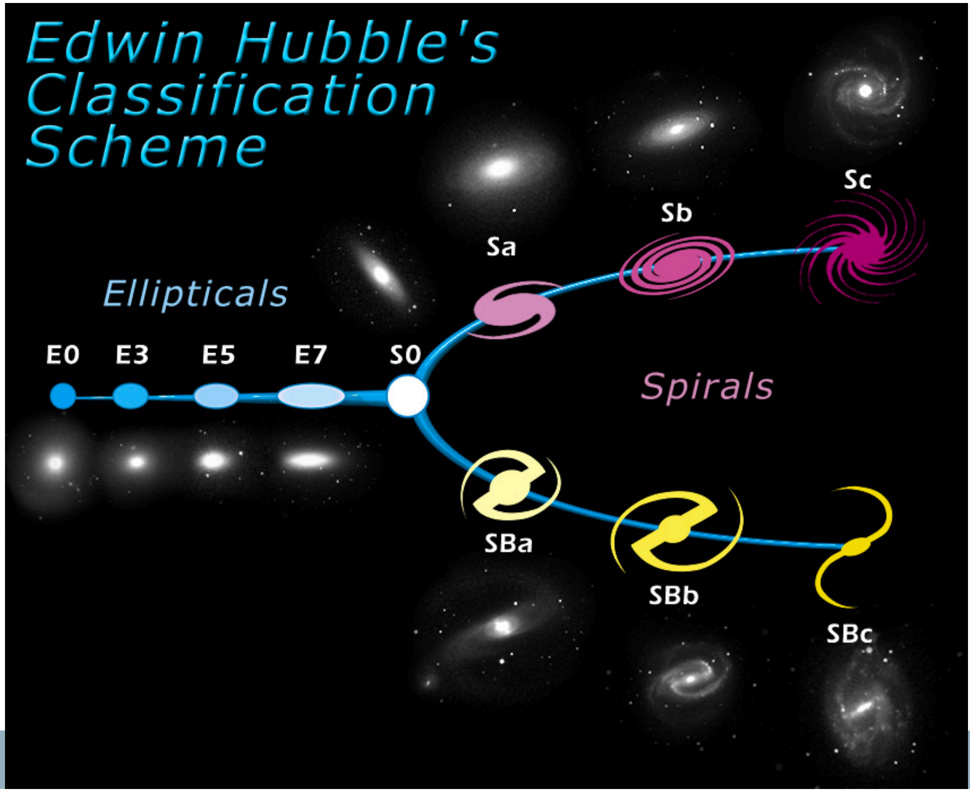
Week 9 Summary

Textbook: Sections 7.2, 4.6.1, 4.6.2, 7.3, 7.4 (2^{nd} edition – 1^{st} edition has 1 fewer chapters)

- Other Galaxies have historically been classified morphologically – irregular galaxies, spirals and ellipticals. The reason for morphology remains complex.
- Galaxy luminosity is described by a *Schechter* function, with our Milky Way being at around the exponential cutoff mass.
- Galaxy evolution is intimately tied in to evolution of their AGN/Quasars/Supermassive black holes. Accretion rates are (roughly) limited by the *Eddington limit.*
- The distribution of galaxies is observed to be highly structured, in *groups* and *clusters*.

Historical Classification

- Edwin Hubble created in 1926 a classification from *early* to *late* type galaxies: terms which have nothing to do with evolution.
- In ASTR2013, we will only consider the broad definitions of Irregular, Spiral and Elliptical Galaxies.



NASA/STSci/ Wikipedia

Irregular Galaxies

Irregular galaxies, with a combination of young and old stars and some gas/star formation are the most common Galaxy type: e.g. LMC and SMC.

Elliptical Galaxies

- Elliptical galaxies have no cool gas to collapse and form new stars: they have *old stellar populations.*
- Although not all elliptical galaxies are massive, the most massive galaxies are giant ellipticals.

Virgo cluster ellipticals: M49, M60 and M86

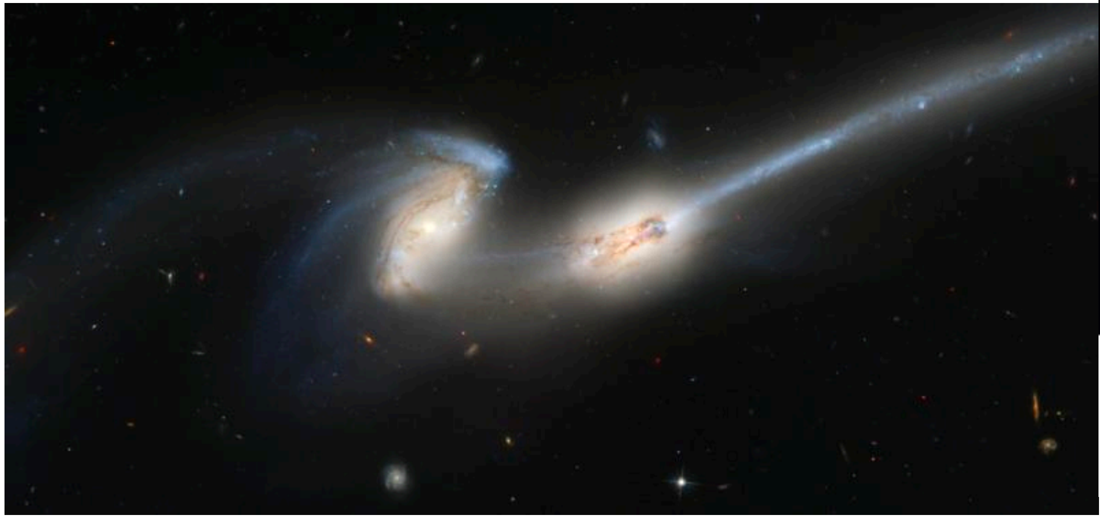
Galaxy Collisions

NGC 4038/4039 zoom in (right)

NGC 4676 (bottom)

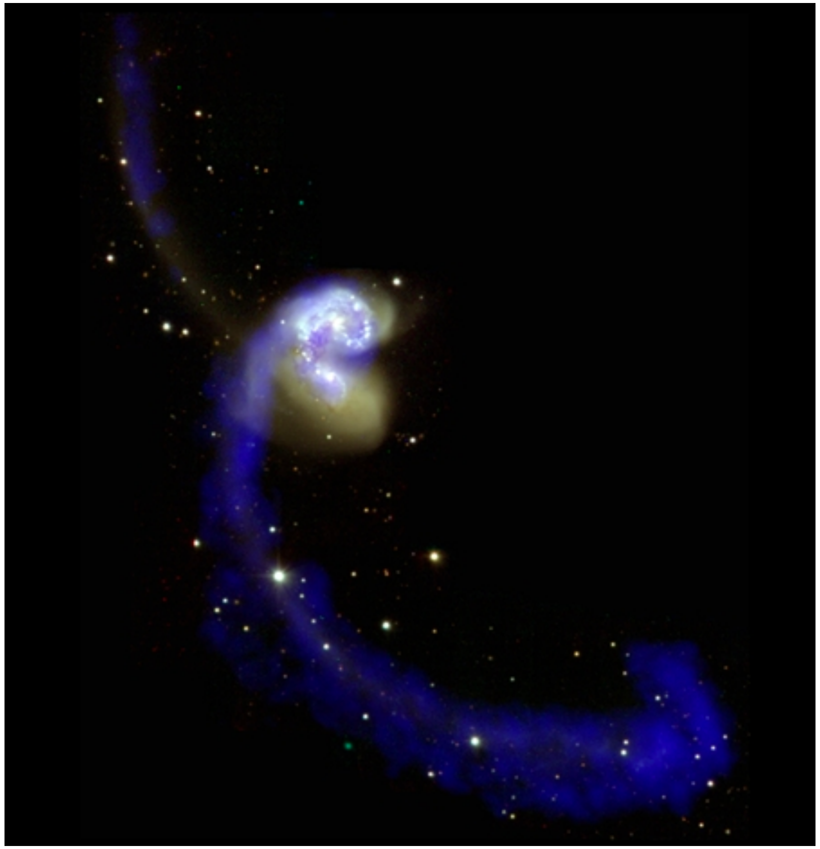
VIMOS Image of the Antennae Galaxies NGC 4038/39
(VLT MELIPAL + VIMOS)

ESO PR Photo 09a/02 (13 March 2002)



Galaxy Collisions

Interacting and colliding galaxies are likely important in forming massive ellipticals: gas collides but stars flow through each other (but tidally disrupt).



NGC 4038 and 4039

Colliding Spirals Simulation

Credit: Josh Barnes (University of Hawaii) and John Hibbard (National Radio Astronomy Observatory) [also NASA/Hubble press release in 2002]

Galaxy Luminosity Function

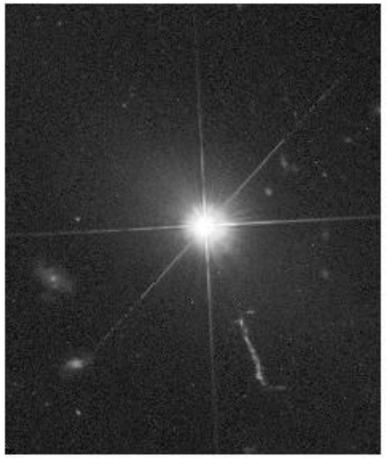
- The distribution of galaxy luminosities is approximately described by the *Schechter* function (at least within a factor of 100 of the Milky Way luminosity).
- [ϕ is the number per unit L and volume]

$$\phi(L)\,dL \approx \phi(L_*)\left(\frac{L}{L_*}\right)^{-1}\exp\left(-\frac{L}{L_*}\right)dL.$$

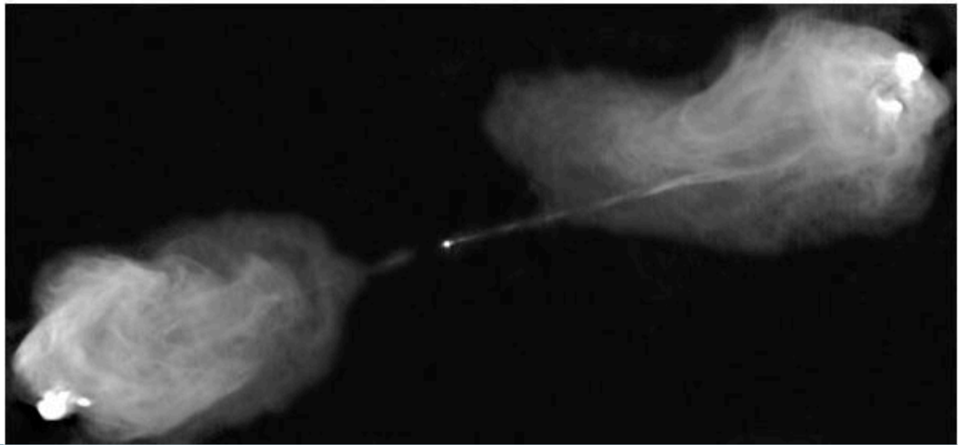
- The critical turning point in this distribution, L_*, is approximately the Milky Way.
- The distribution of lower luminosities can be described as *uniform in log(L).*

Quasars

- Quasars are "quasi-stellar" objects, observationally defined as being point-like but at moderate redshift (i.e. not in the Galaxy)
- They are generally associated with strong ratio emission from jets that travel close to the speed of light.
- Quasars are now known to be supermassive black holes accreting gas rapidly at the center of galaxies: *Active Galactic Nuclei (AGN)*

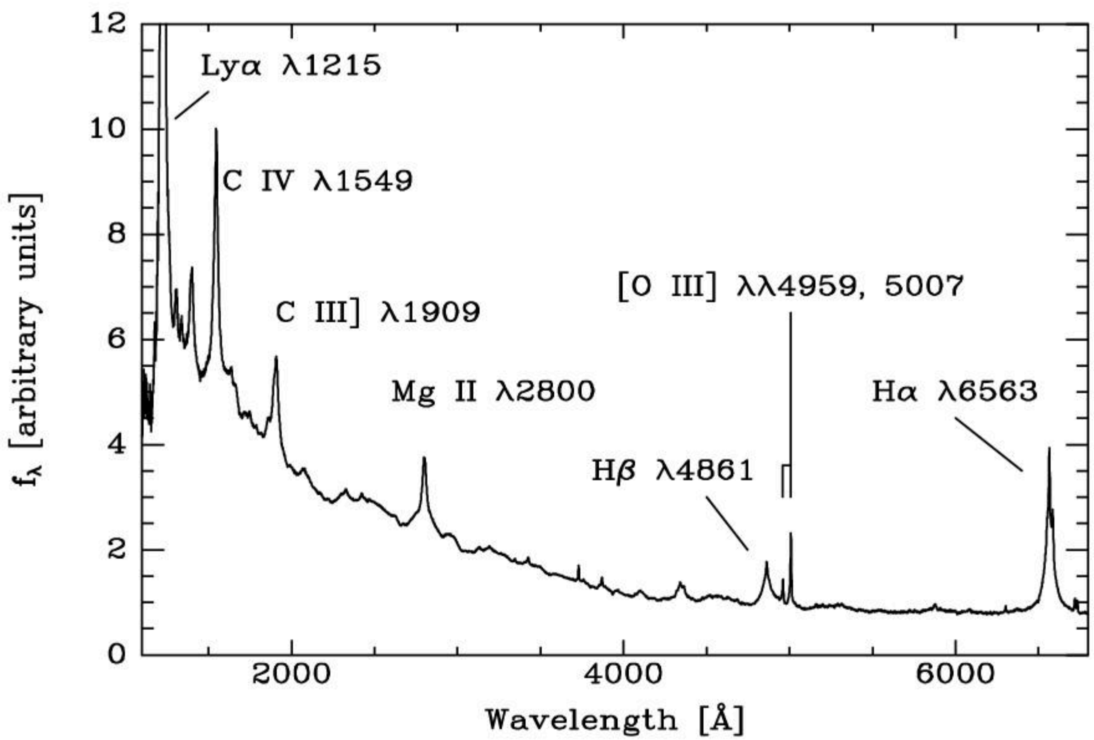


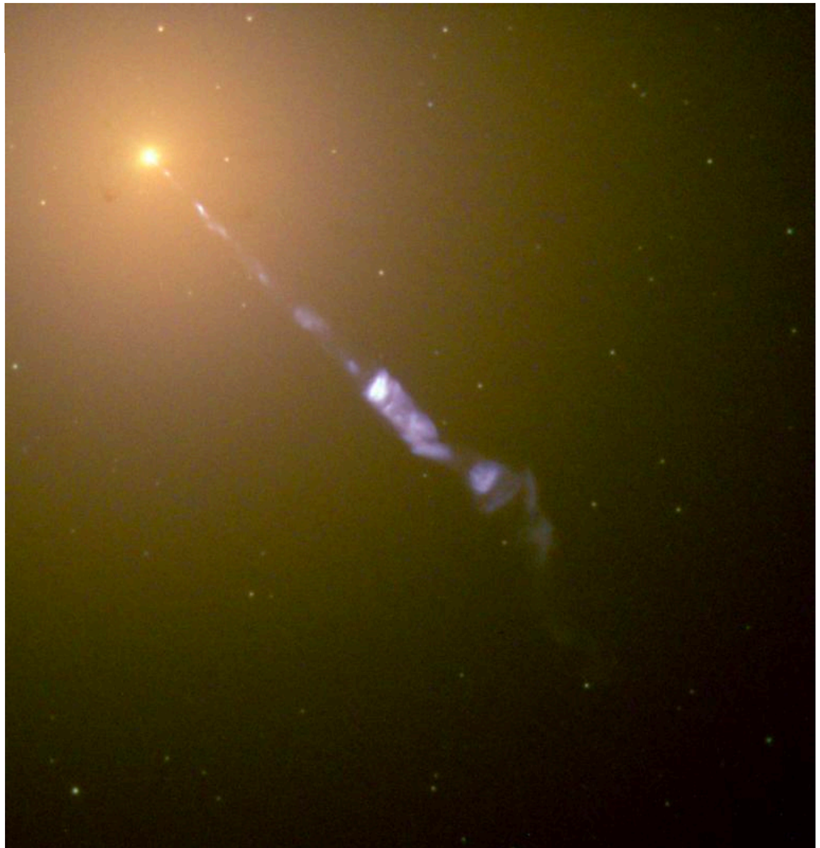
- A supermassive black hole is only an AGN if it is accreting rapidly.
- An AGN is only a quasar if it is seen from the right angle – if seen edge on, dust (e.g. in a Torus shape) gets in the way.



Quasars

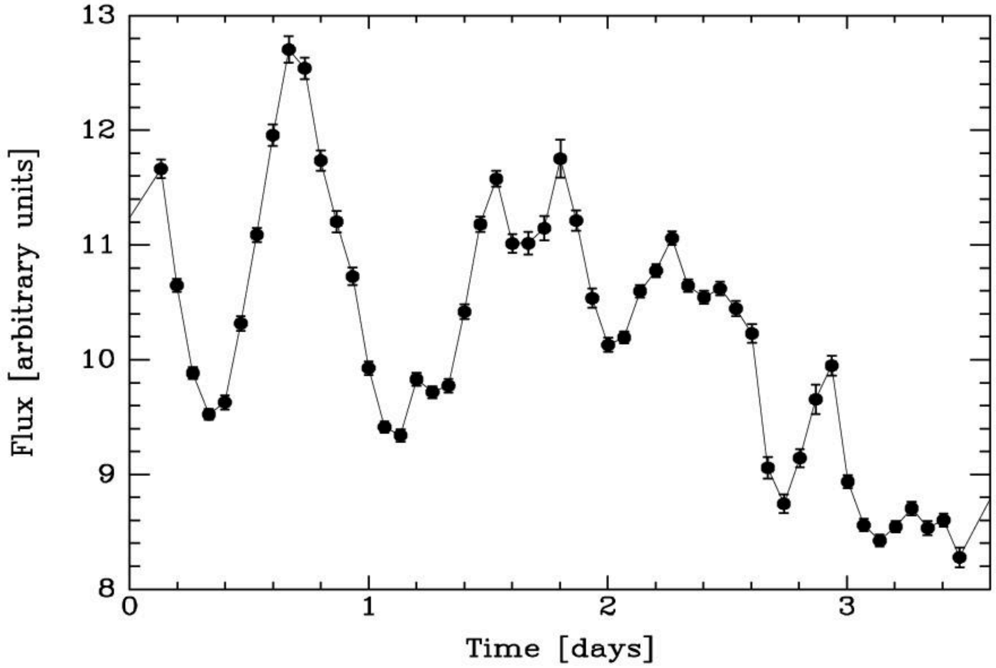
Quasars have a characteristic spectrum, with emission at all wavelengths as occurs in an accretion disk, and emission lines of highly ionized elements.

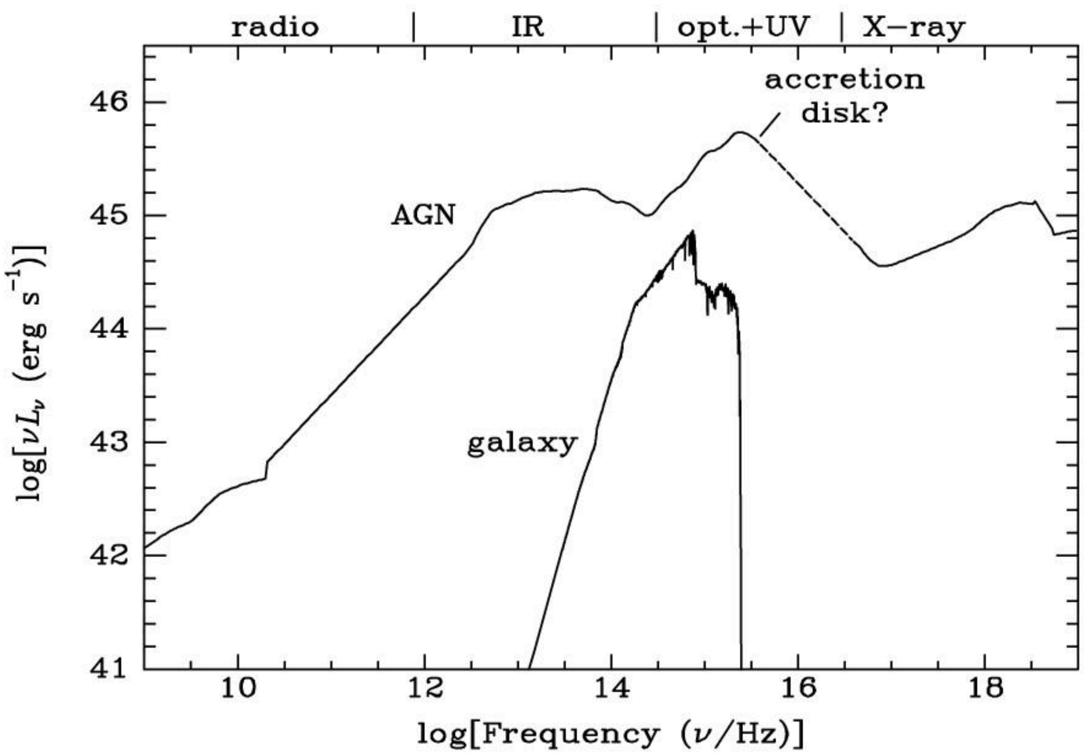




Quasar variability (top) and spectrum (bottom) are very different to a galaxy.

discuss





Eddington Luminosity

- Ignoring the geometry of the accretion disk, we can estimate the maximum possible accretion rate when the force per atom due the outgoing radiation due to accretion is equal to the gravitational force.
- We take the highly ionized opacity – the Thompson scattering cross section:

$$\frac{GMm_p}{r^2} = \frac{L_E \sigma_T}{4\pi r^2 c}$$

Derive on board!

$$L_E = \frac{4\pi c G M m_p}{\sigma_T}$$

Eddington-Limited Accretion

- Equate the energy radiated to the energy lost to the last stable orbit:

$$L \approx \frac{1}{2}\frac{GM\dot{M}}{r_{\text{in}}} = \frac{\dot{M}c^2}{12}$$

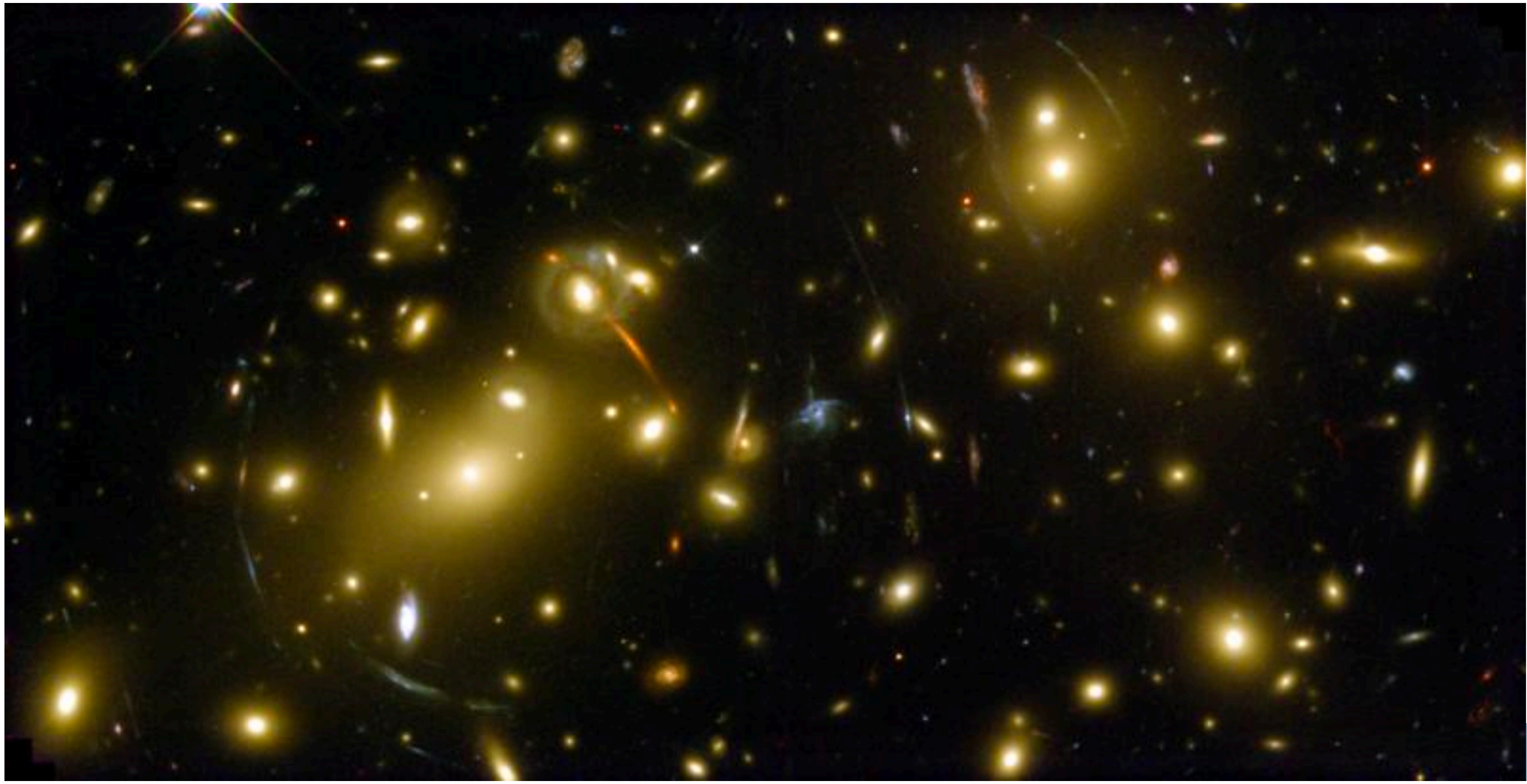
- After taking into account a small correction for general relativistic effects, we arrive at:

$$L = L_{\text{E}} = 1.3 \times 10^{38}\ \text{erg s}^{-1}\frac{M}{M_{\odot}}$$

- Accretion can be “super-Eddington” e.g. through a disk, but it is difficult to significantly exceed this limit.

Galaxy Clusters and Groups

- Galaxies are found in groups (e.g. Andromeda and the Milky Way) and in clusters (e.g. Virgo, Coma).
- The most massive galaxy clusters show strong gravitational lensing of background galaxies



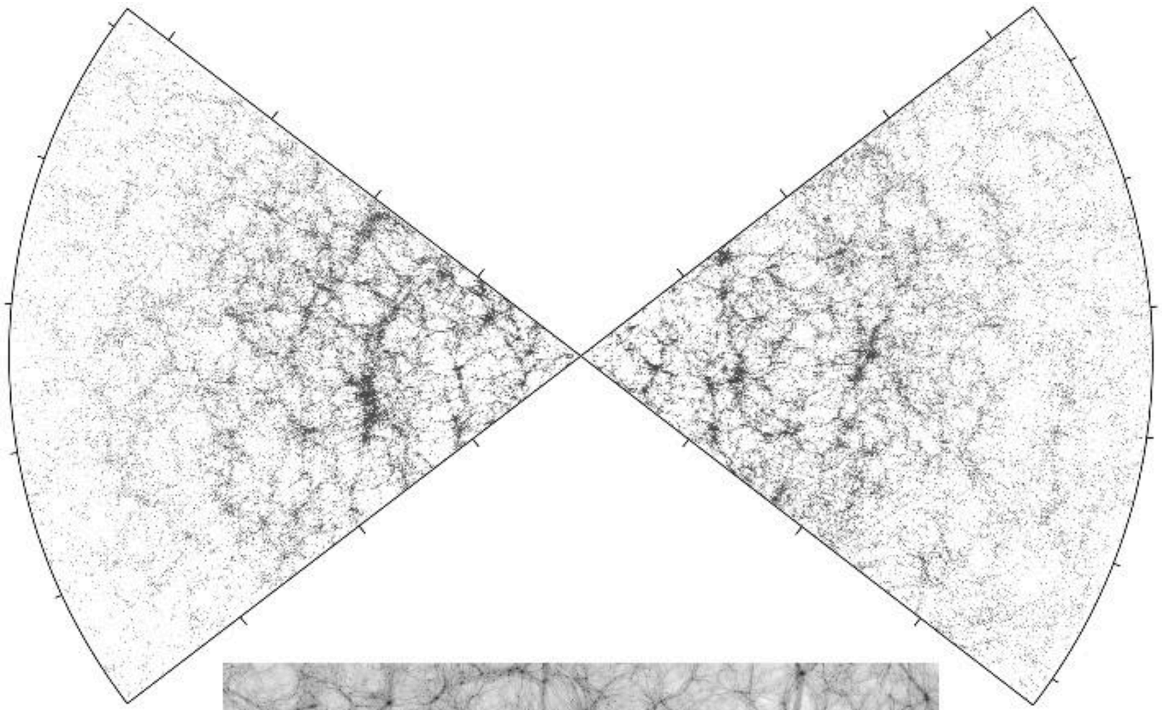
Abell 2218 (NASA/textbook)

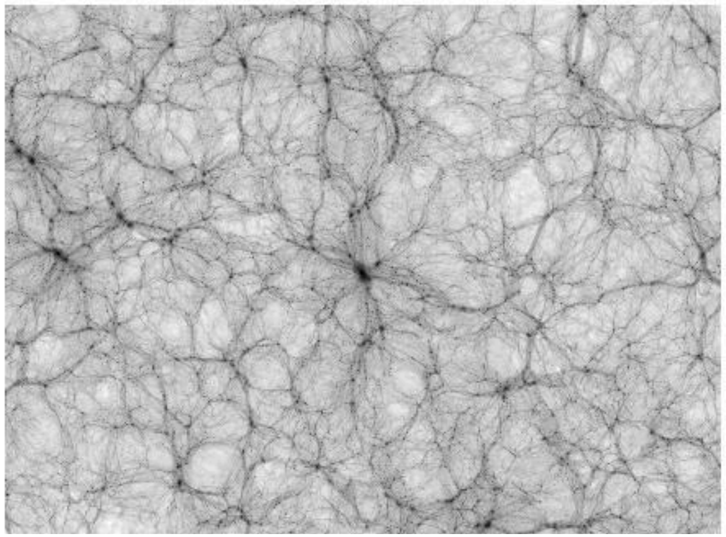
Galaxy Cluster Mass

- We can measure galaxy velocities within a cluster and its size.
- This gives a crossing timescale of ~1 Gyr – much less then the Universe age.
- This means that the cluster is *virialized*, i.e. that motions are random enough that the virial theorem should apply, implying cluster masses of order 10^{14} M_{sun}.

$$M \approx \frac{\sigma_v^2 r_{cl}}{G}$$

Large Scale Structure





Observed large scale structure of Galaxies (2dF, top) match simulations (bottom).

Very large voids i-between clusters and fillaments… more on this in 2 weeks!

Week 9 Summary

Textbook: Sections 7.2, 4.6.1, 4.6.2, 7.3, 7.4 (2nd edition – 1st edition has 1 fewer chapters)

- Other Galaxies have historically been classified morphologically – irregular galaxies, spirals and ellipticals. The reason for morphology remains complex.
- Galaxy luminosity is described by a *Schechter* function, with our Milky Way being at around the exponential cutoff mass.
- Galaxy evolution is intimately tied in to evolution of their AGN/Quasars/Supermassive black holes. Accretion rates are (roughly) limited by the *Eddington limit.*
- The distribution of galaxies is observed to be highly structured, in *groups* and *clusters*.

Australian National University

ASTR2013 – *Foundations of Astrophysics*

Week 11: Big Bang Cosmology

Mike Ireland

Course Overview

- Quality of Field trip reports (on a skim) excellent.
- One problem set (6.7%) and final exam (30%) the only assessments remaining.
- Final exam will have some relatively easy questions (e.g. based on concepts re-iterated at the start of each lecture), some based on tutorials, problem sets and field trip worksheet, and 1-2 challenge questions.
- Some feedback though Matthew Dotta. Please send final feedback to course representatives.

Properties of other galaxies

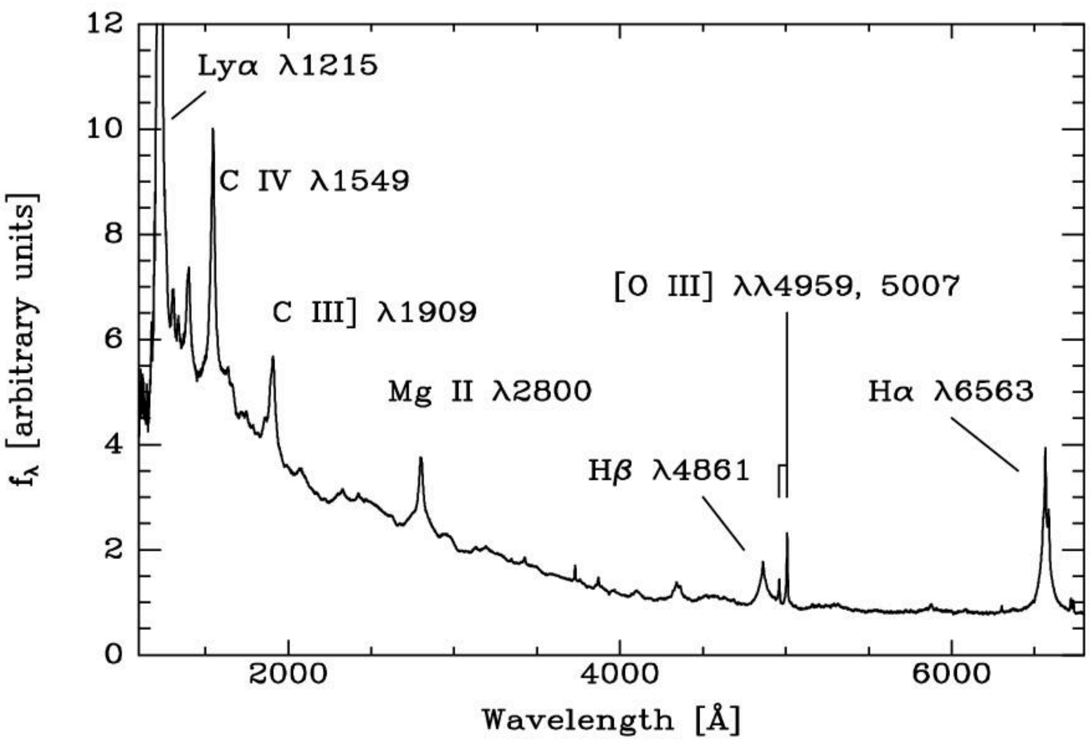
- The distribution of galaxy luminosities is approximately described by the *Schechter* function:

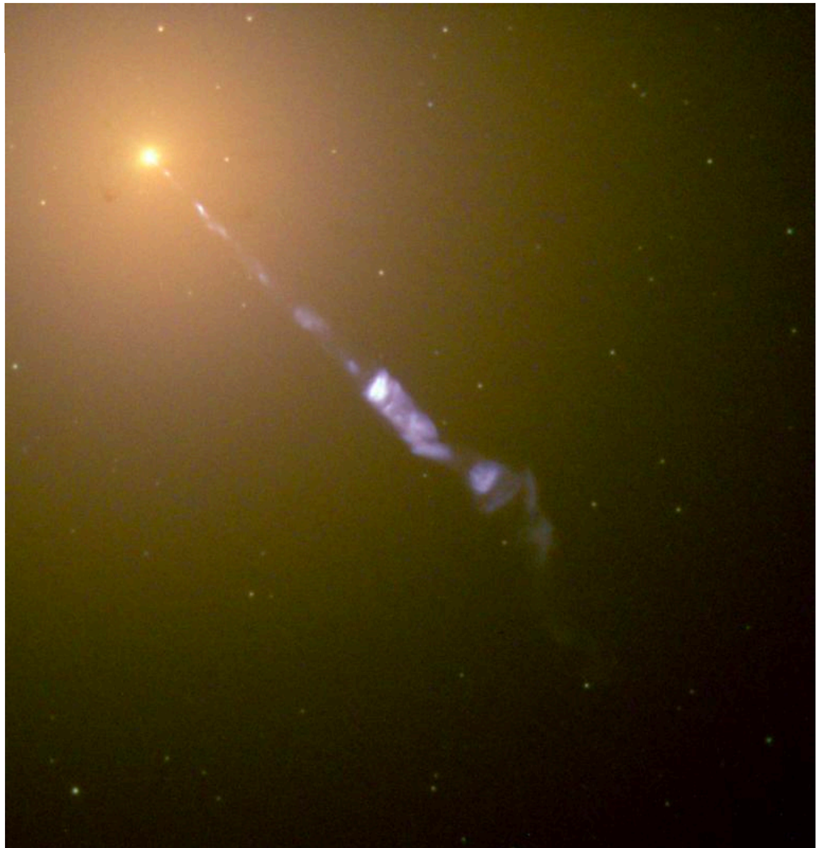
$$\phi(L)\,dL \approx \phi(L_*)\left(\frac{L}{L_*}\right)^{-1} \exp\left(-\frac{L}{L_*}\right) dL.$$

- The critical turning point in this distribution, L_*, is approximately the Milky Way.
- Galaxies can be irregular, spiral or elliptical.
- The most massive galaxies are elliptical, and may have resulted from galaxy mergers.

Quasars

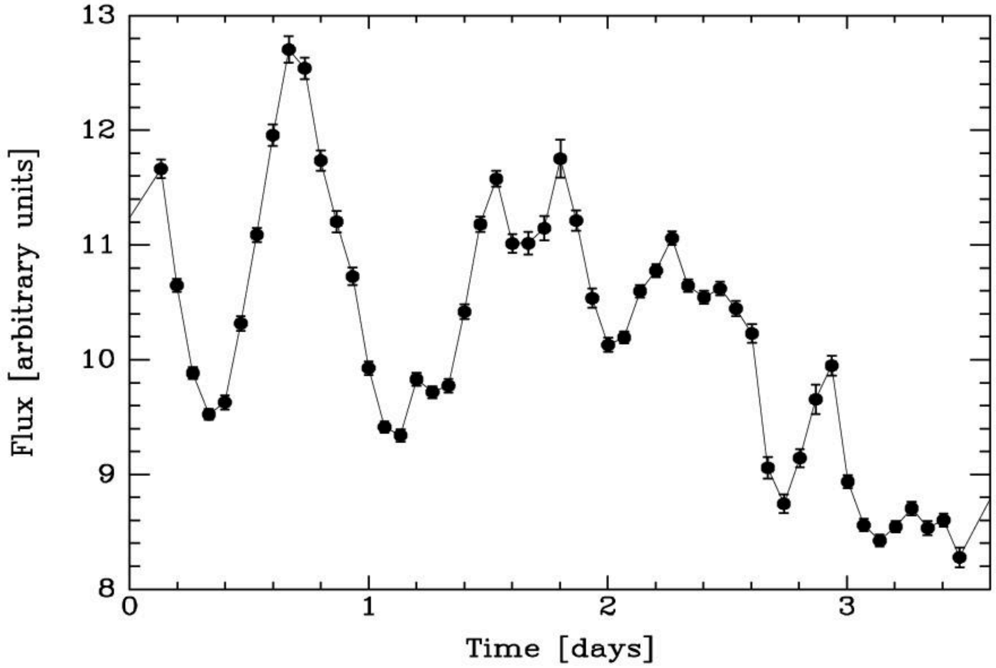
Quasars have a characteristic spectrum, with emission at all wavelengths as occurs in an accretion disk, and emission lines of highly ionized elements.

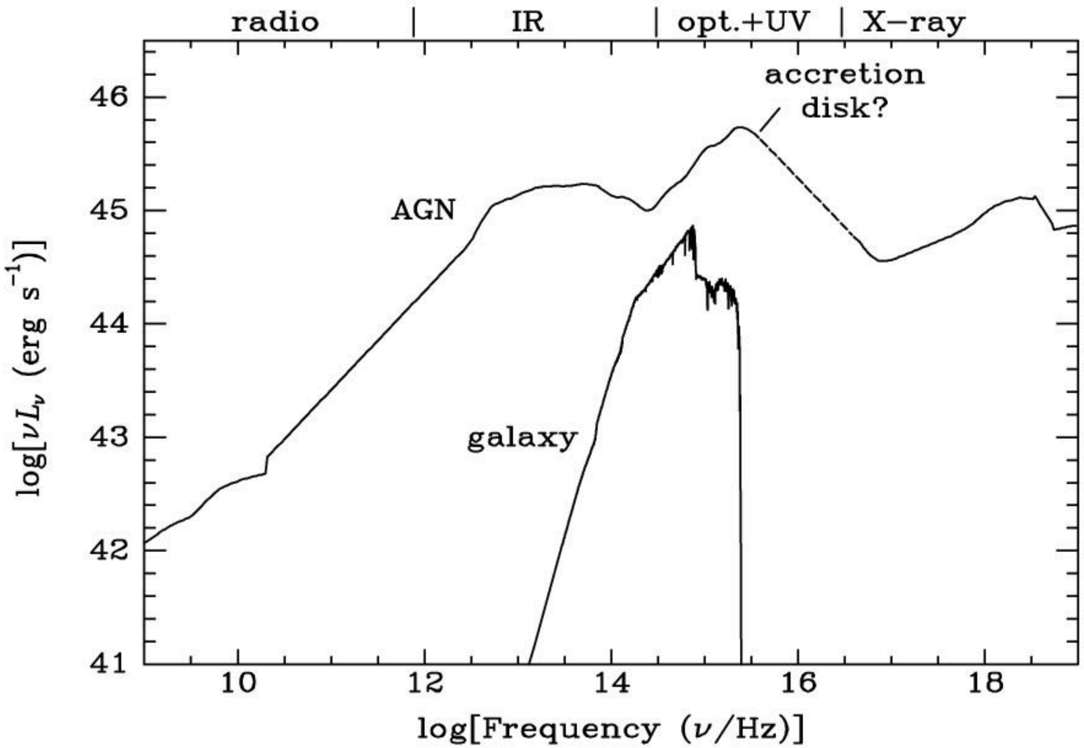




Quasar variability (top) and spectrum (bottom) are very different to a galaxy.

discuss





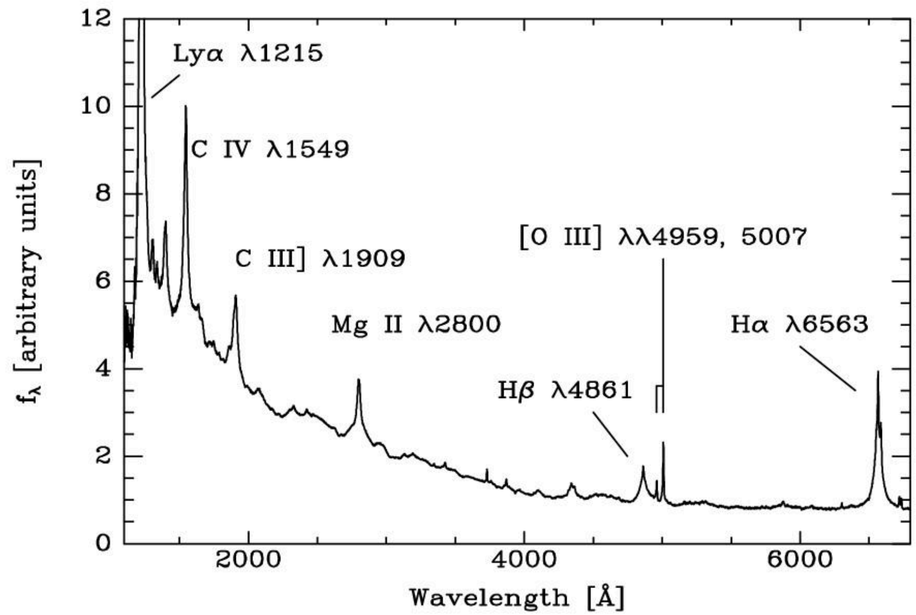
Quasars and Eddington Luminosity

- Quasars have luminosities limited only by the absorbed photon momentum (e.g. from Thompson scattering) equaling gravitational attraction – called Eddington-limited accretion.
- Key equations:

$$L = L_E = 1.3 \times 10^{38}\ \mathrm{erg\ s^{-1}}\ \frac{M}{M_\odot}$$

$$L = 0.057 \dot{M} c^2$$

- They have a characteristic spectrum when not obscured:



Week 11 Summary

Textbook: Sections 8.1, 8.3, 8.4, 8.5, equation 9.28, 9.55 and 9.96 (briefly only!), 10.1, 10.2 and 10.3. This is half of a 3rd year course – a super-brief intro only this week.

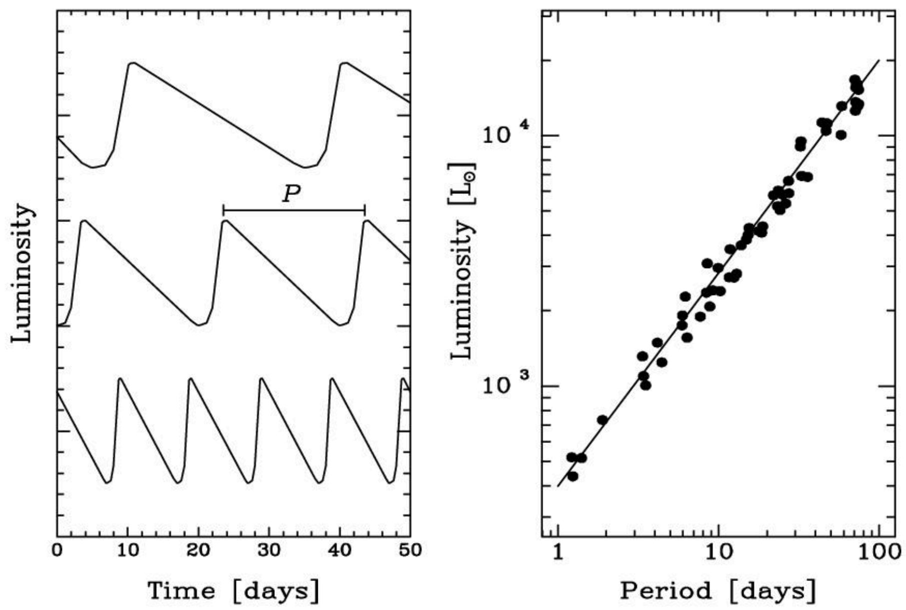
- Olbers Paradox – the Universe is not infinite.
- Hubble's law, age and isotropy of the Universe.
- Solution to the Friedman equations: what the scale parameter *R* means, and what the *critical density* is.
- Cosmological redshift from an observational perspective.
- The Cosmic Microwave Background.

Olbers Paradox

- The simplest cosmological observation is that the night sky (i.e. the space between the stars) is dark.
- In an (simplistic) infinite universe made up of space and stars, all sight lines would eventually hit a stellar surface.
- This means that either:
  1. The Universe is finite.
  2. The Universe has finite age.
  3. Something about physics removes photon energies when they travel large distances.
  4. More than 1 of the above.

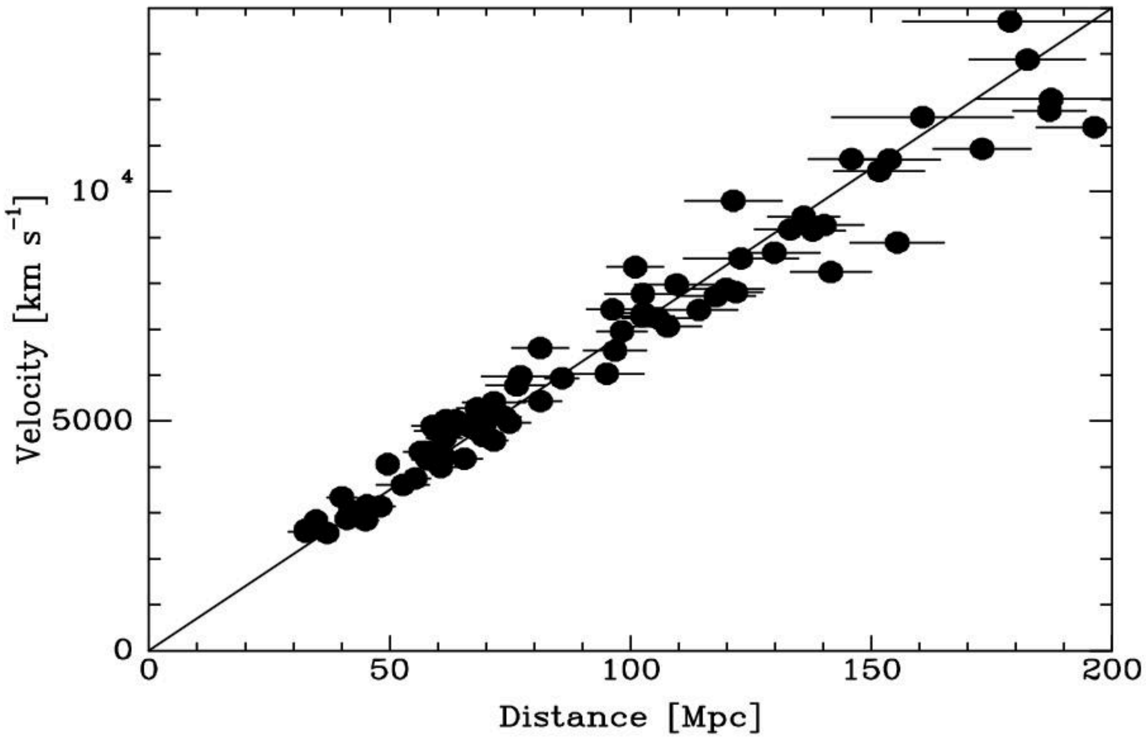
Extragalactic Distances

- Measuring the luminosity of Galaxies (last lecture) or inferring the structure of the universe needs *standard candles*.
- Measure luminosity and flux to derive distance.
- A huge literature exists on this topic – one example is the Cepheid period-luminosity relationship. Physics: period measures dynamical timescale and radius.



Hubble's Law

- As soon as the first (rather inaccurate) distances were available, Hubble (1929) measured galaxy velocities and published a trend – more distant galaxies were receding.



$$v = H_0 D.$$

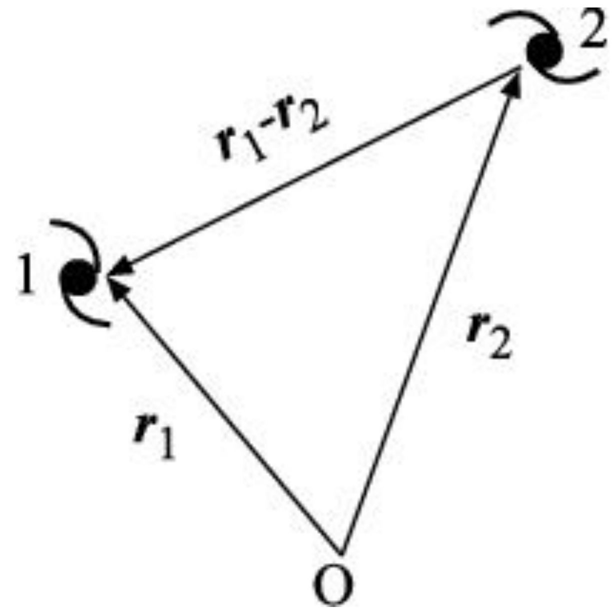
$$H_0 = 70 \pm 5\,\mathrm{km\,s^{-1}Mpc^{-1}}$$

Hubble "constant"

Consequences of Hubble's Law

- If H_0 was a constant, then all observers everywhere see the same Hubble flow (i.e. expansion of the local Universe)
- Tracing this uniform expansion backwards in time means the Universe has an age equal to the Hubble time (14 Gyr).

$$t_0 = \frac{1}{H_0} = \frac{1}{70\ \mathrm{km\ s^{-1} Mpc^{-1}}}$$



Universe Age and Isotropy

- The age of objects in the Universe provide a lower limit on its age.
- E.g. the solar system is 4.6 Gyr old. Based on nucleosynthesis calculations, the Uranium itself is 6.2 Gyr old.
- The oldest stars are ~13 Gyr old, and the oldest white dwarfs around 10 Gyr.

All this is consistent with a Big Bang cosmology, as long as the Universe hasn't been expanding nearly constantly.

- The Universe is also isotropic on large scales (e.g. 2dF from last lecture), meaning that it is worthwhile to consider the action of gravity on the Universe as a whole.

General Relativity and the Universe

- Several theorists attempted to apply Einstein's general relativity theory to the Universe as a whole.
- An isotropic universe is described by a *scale factor R*. This could be the Universe radius (e.g. a 3D universe that is like the surface of a 4D sphere)… but curved spacetime is no longer as relevant.
- For flat spacetime, for some co-moving (i.e. constant for an isotropic universe) coordinate *r*, we have lengths:

$$l = R(t)r$$

Draw picture on board

- The scale parameter *R* follows the Friedman equation:

$$\dot{H} = \frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}(\rho c^2 + 3P).$$

- If we ignore pressure (e.g. current Universe) there is a critical density where the Hubble parameter is always (just) constant:

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G}$$

- In the late 1900s, a lot of effort went in to determining the Hubble parameter derivative, and if the Universe was above or below this density. However, this didn't work, and a constant had to be added to the equation (dating back to Einstein):

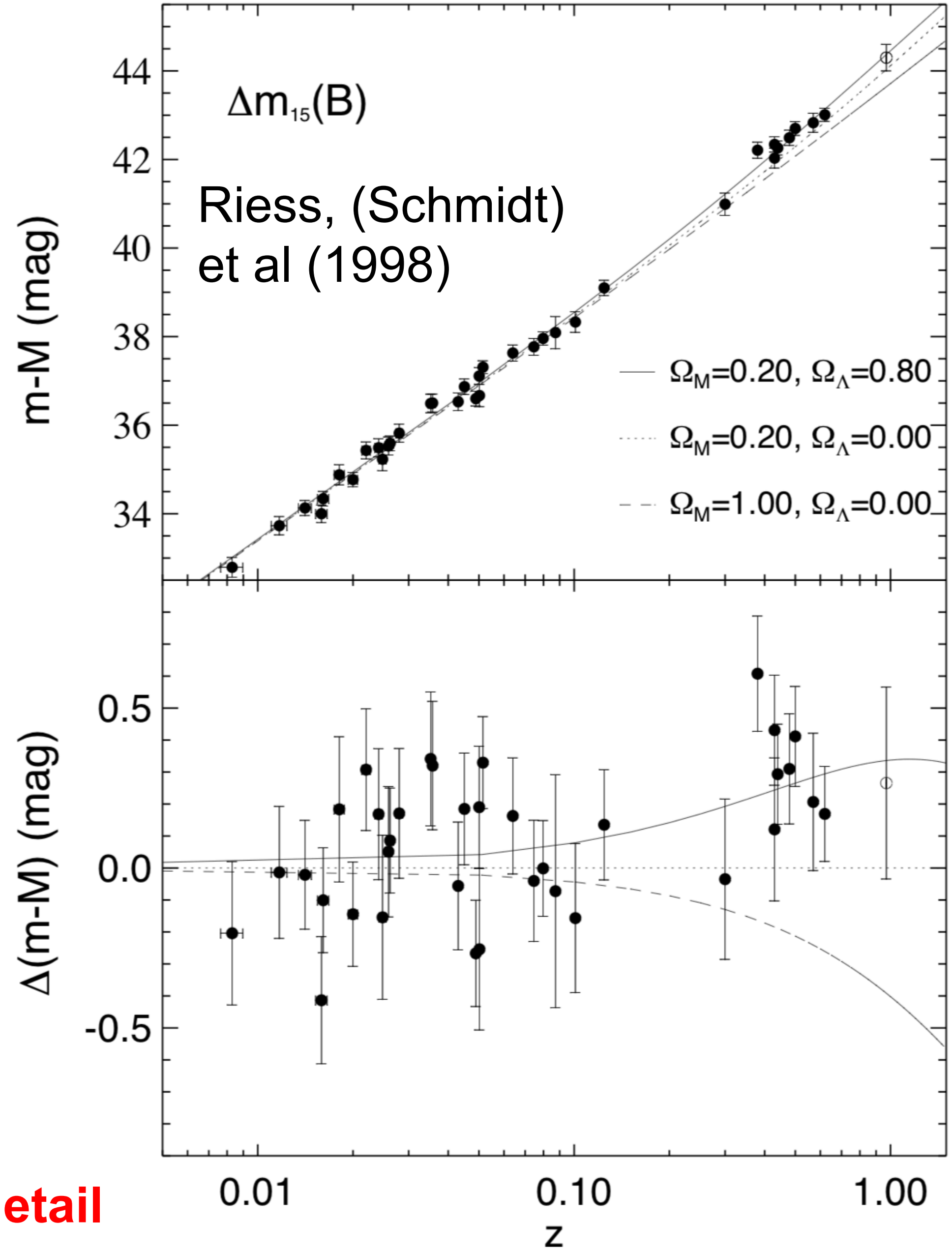
$$\dot{H} = \frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}(\rho c^2 + 3P) + \frac{\Lambda}{3}.$$

Cosmological Redshift

- As light travels through an expanding universe, the wavelength must increase as space expands… just like *any* comoving distance. This is fully consistent with time dilation, as would be expected in a relativistic theory: $$\frac{\Delta t_0}{\Delta t_e} = \frac{\lambda_0}{\lambda_e} = \frac{\nu_e}{\nu_0} = \frac{R(t_0)}{R(t_e)} \equiv 1 + z,$$
- Redshift *z* is defined so that the change in scale parameter *R* is (1+*z*).

“Dark Energy”

- Since 1998, it has been clear that a Universe decelerating by matter only doesn’t make sense.
- The equation with the cosmological constant Λ is needed.
- It is called “dark energy”, but is observationally just a functional form of redshift versus distance.

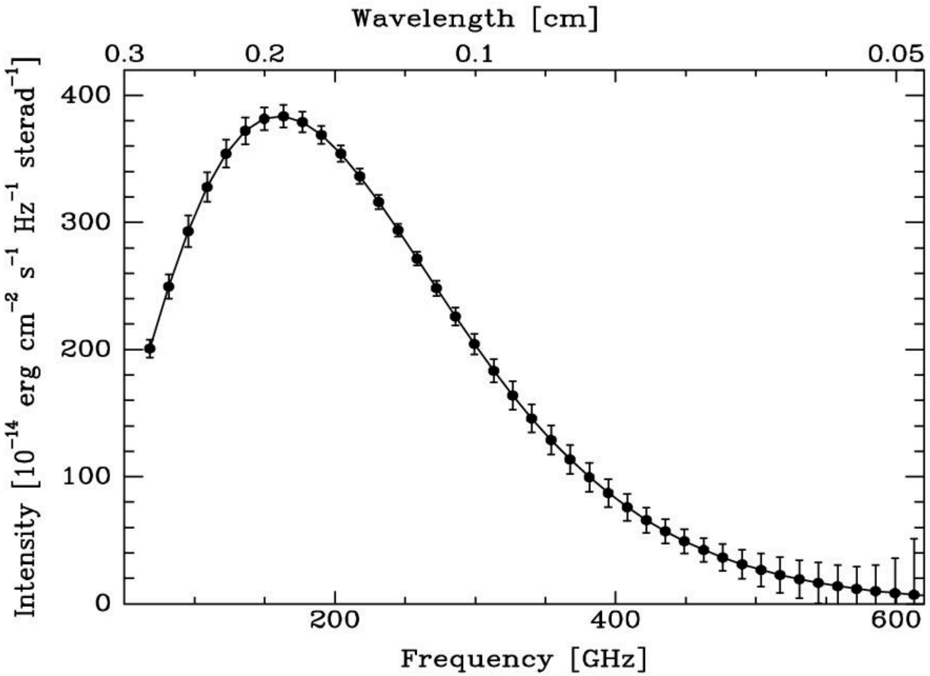


Discuss in detail

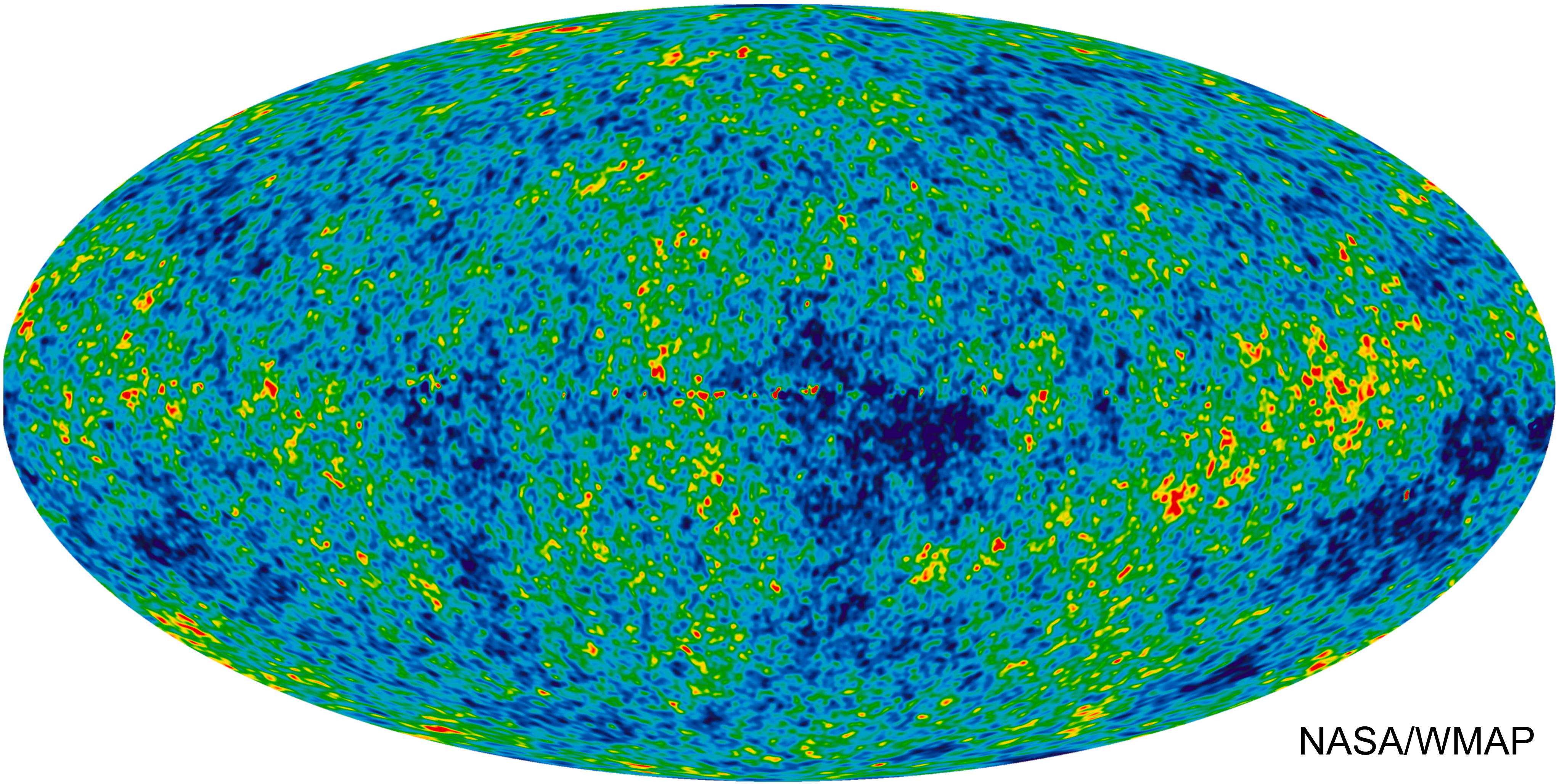
Australian National University

Cosmic Microwave Background

- Going back in time towards the "Big Bang", eventually the temperature of the Universe is high enough to ionize Hydrogen.
- Thompson scattering then prevents us from seeing any further back.
- This radiation, redshifted to ~3mm wavelengths, looks like a near-perfect Planck function:



Fluctuations in the CMB at the level of 1:100,000 were the seeds of galaxies and all structure in the Universe.



NASA/WMAP

Week 11 Summary

Textbook: Sections 8.1, 8.3, 8.4, 8.5, equation 9.28, 9.55 and 9.96 (briefly only!), 10.1, 10.2 and 10.3. This is half of a 3rd year course – a super-brief intro only this week.

- Olbers Paradox – the Universe is not infinite.
- Hubble's law, age and isotropy of the Universe.
- Solution to the Friedman equations: what the scale parameter *R* means, and what the *critical density* is.
- Cosmological redshift from an observational perspective.
- The Cosmic Microwave Background.

ASTR2013: Foundations of Astrophysics
Week 12
Turbulence and Magnetic Fields in Galaxies

Dr. Amit Seta
amit.seta@anu.edu.au

21st Oct 2019

Introduction

Turbulence is ubiquitous in nature, from the coffee in a cup to astrophysical scales. Turbulence in the interstellar medium (ISM) of spiral galaxies is usually driven by supernova explosions (Fig. 1) via shocks waves. Multiple supernovae going off in different locations drives the turbulence in galaxies and this turbulence in turn amplifies magnetic fields. Below we calculate, the driving scale of turbulence (l_0) and the turbulent velocity (v_0). Then we look how this ISM turbulence can generate magnetic fields.

Supernova driven turbulence in spiral galaxies

The radius of the shock wave R (Fig. 2) as it explodes is given by (*Sedov-Taylor solution, derivation to be done in the tutorial*)

$$R \approx 2 \left(\frac{Et^2}{\rho_0} \right)^{1/5},$$

where E is the energy of the explosion, ρ_0 is the density of the medium and t is the time. We assume that the shock injects negligible energy when the velocity of the shock is equal to the local sound speed (the shock becomes part of the medium then). So, the maximum radius of the shock waves before it dissipates roughly equal to the driving scale of the turbulence. Thus, $l_0 \approx R$ when the velocity of shock front is equal to c_s, the sound speed of the medium. The sound speed of the medium

$$c_s = \sqrt{k_B T / m_p},$$

where $k_B = 1.38 \times 10^{-16}\,\mathrm{erg\,K^{-1}}$ in the Boltzmann constant, $m_p = 1.67 \times 10^{-24}\,\mathrm{g}$ and T is the temperature of the medium. For hot gas in the ISM ($T = 10^6\,\mathrm{K}$ and number density of particles $n_0 = 0.1\,\mathrm{cm^{-3}}$),

$$c_s = \sqrt{\frac{1.38 \times 10^{-16} \times 10^6}{1.67 \times 10^{-24}}} \approx 10^{0.5(-16+6+24)}\,\mathrm{cm/\,s} = 10^7\,\mathrm{cm/\,s} = 100\,\mathrm{km/\,s}.$$

To find the driving scale of turbulence l_0, $R = l_0$ at $t \approx R/c_s$,

$$l_0 \approx R \simeq 2 \left(\frac{E}{c_s^2 \rho_0} \right)^{1/3}.$$

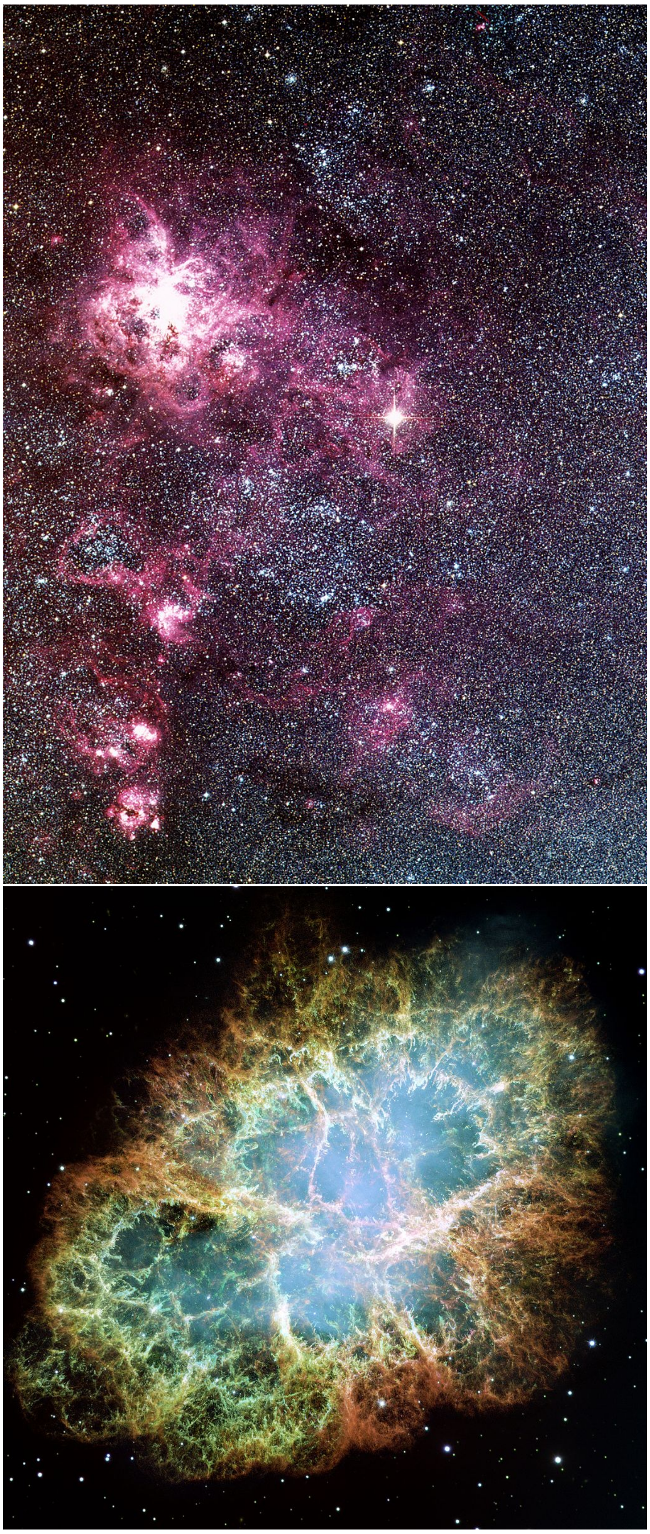


Figure 1: Top: SN1987A, exploding star. Bottom: Crab nebula, supernova remnant.

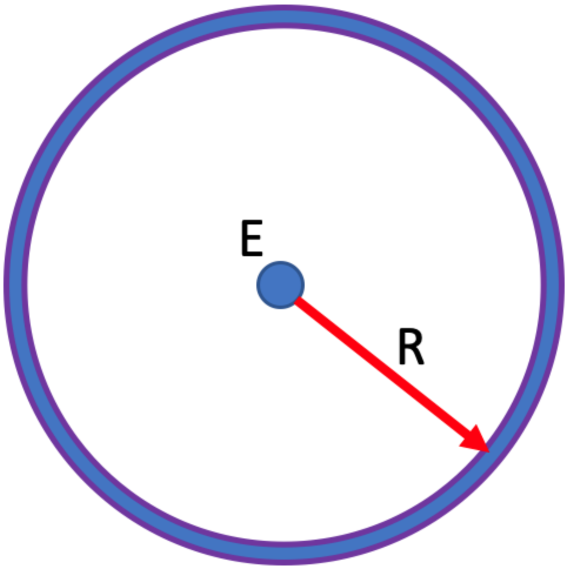


Figure 2: Spherically symmetric shockwave with radius R carrying energy E deposited at the centre by explosion.

For $E = 10^{51}$ erg, $\rho_0 = n_0 m_p$ and $c_s = 100\,\mathrm{km/s}$,

$$l_0 \simeq 2\left(\frac{10^{51}}{10^{14}(0.1\times 1.67\times 10^{-24})}\right)^{1/3}\,\mathrm{cm} \simeq 10^{\frac{51-14+25}{3}}\,\mathrm{cm} \simeq 10^{62/3}\,\mathrm{cm}$$

$$\simeq 10^{2/3}10^{20}\,\mathrm{cm} \simeq 4\times 10^{20}\,\mathrm{cm} \approx 100\,\mathrm{pc},$$

as $1\,\mathrm{pc} \approx 3\times 10^{18}\,\mathrm{cm}$.

To calculate the turbulent velocity, we assume that a fraction of the total supernova energy is converted to the turbulent kinetic energy of the medium. The energy per unit mass of the turbulent medium is v_0^2, the rate of gain of total energy per unit mass is expressed as

$$v_0^2/(l_0/v_0).$$

This must come from supernova explosion, the rate of injection of kinetic energy per unit mass by supernova explosions is expressed as

$$\nu E/M_{\mathrm{gas}},$$

where ν is the frequency of supernova explosion and M_{gas} is the galaxy mass. Considering that the 10% of supernova explosion energy is deposited into the medium, we can balance both energy terms,

$$\frac{v_0^2}{l_0/v_0} = 0.1\frac{\nu E}{M_{\mathrm{gas}}}$$

$$v_0 = \left(\frac{0.1\nu E l_0}{M_{\mathrm{gas}}}\right)^{1/3}$$

For $E = 10^{51}$ erg, $\nu = (30\,\mathrm{yr})^{-1}$ and $M_{\mathrm{gas}} = 4\times 10^9\,\mathrm{M}_\odot$, we get

$$v_0 = \left(\frac{0.1\nu E l_0}{M_{\mathrm{gas}}}\right)^{1/3} = \left(\frac{0.1\times 10^{-9}\times 10^{51}\times 10^{20}}{4\times 10^9\times 2\times 10^{33}}\right)^{1/3}\,\mathrm{cm/s} = \left(\frac{10^{-1-9+51+20}}{10^{1+9+33}}\right)^{1/3}\,\mathrm{cm/s}$$

$$= 10^{18/3}\,\mathrm{cm/s} = 10^6\,\mathrm{cm/s} = 10\,\mathrm{km/s}.$$

Thus, the turbulent velocity of the ISM of a typical spiral galaxy is on an average equal to $v_0 = 10\,\mathrm{km/s}$.

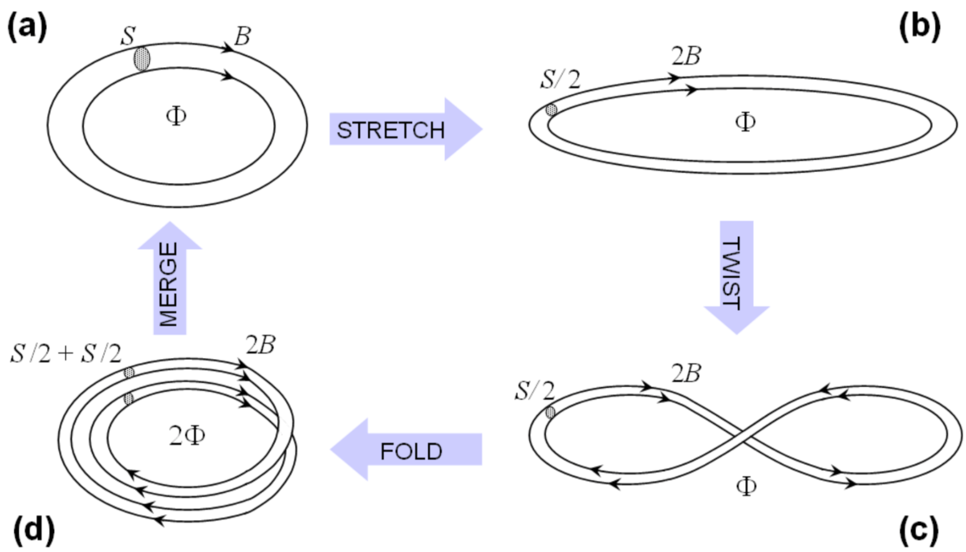


Figure 3: The stretch-twist-fold-merge mechanism (image credits: Anvar Shukurov). S and B are the cross section and magnetic field strength of the initial flux tube. $\phi = SB$ is the magnetic flux associated with the tube.

Magnetic field amplification

The ISM turbulence amplifies magnetic fields. The turbulent kinetic energy of the medium is converted to the magnetic field energy. This process is known as the dynamo theory.

The magnetic field amplification can be explained physically by a stretch-twist-fold-merge (STFM) mechanism (Fig. 3). This was first introduced by Yakov Borisovich Zeldovich. [1] Assuming flux freezing (product of magnetic field and area is conserved, $SB =$ constant, where B is the magnetic field strength and S is the area) and incompressible motions, the algorithm to amplify magnetic field via the STFM mechanism is as follows. First, the magnetic flux tube is stretched to double its length while preserving its volume, (a) $\rightarrow$ (b) in Fig. 3. This increases the magnetic field strength by a factor of two since the cross section is halved. Then the flux tube is twisted to form a figure eight, (b) $\rightarrow$ (c) in Fig. 3, and folded on itself, (c) $\rightarrow$ (d) in Fig. 3. Now, both loops of the tube have the magnetic field along the same direction and together occupy the same volume as the original flux tube. Both loops of the tube are now merged together into one, (d) $\rightarrow$ (a) in Fig. 3. The last step requires magnetic diffusion for process to become irreversible. The magnetic field is doubled for each cycle and increases by a factor of 2^n after n such steps. The growth rate is $\ln 2/T$, where T denotes the period of the STFM cycle. STFM cycles are due to the turbulent fluid motions in the ISM. The growth in magnetic energy is at the expense of the turbulent kinetic energy of the fluid motions. Once the magnetic field becomes strong enough, the Lorentz forces reacts back on the flow. In the STFM mechanism, this would imply difficulty in either the stretching and twisting due to magnetic tension or the merging of loops becoming slower (see reference (1) in the further reading section for details). Thus, then the magnetic field stops growing and saturates. The maximum magnetic field strength that can be achieved is the equipartition value, obtained by balancing the magnetic field energy with the turbulent kinetic energy.

$$\frac{B_{\mathrm{eq}}^2}{8\pi} = \frac{1}{2}\rho v_0^2$$

$$B_{\mathrm{eq}} = \sqrt{4\pi\rho v_0^2}.$$

[1] https://en.wikipedia.org/wiki/Yakov_Zeldovich

For, $v_0 = 10\,\mathrm{km/\,s}$ and $\rho = 0.1 \times 1.67 \times 10^{-24}\,\mathrm{g\,cm^{-3}}$,

$$B_{\mathrm{eq}} = \sqrt{4\pi \times 0.1 \times 1.67 \times 10^{-24} \times 10^{12}}\,\mathrm{G} \simeq 2 \times 10^{-6}\,\mathrm{G} = 2\,\mu\mathrm{G}.$$

This is very close to the observed value. The generated magnetic fields are at the scale of the driving scale ($l_0 \approx 100\,\mathrm{pc}$) due to the isotropic fluid turbulence. The large-scale magnetic fields in the galaxy also requires large scale properties of the galaxies (such as rotation, velocity shear and density stratification). See reference (2) in the further reading section for details.

Further reading

1. Saturation of Zeldovich Stretch-Twist-Fold Map Dynamos, Amit Seta, Pallavi Bhatt and Kandaswamy Subramanian, JPP (Zeldovich Special Issue), 2015, 81, 5
   Link: https://arxiv.org/pdf/1410.8455.pdf

2. Introduction to galactic dynamos, A. Shukurov, In:Mathematical Aspects of Natural Dynamos,eds E. Dormy & A. M. Soward,The Fluid Mechanics of Astrophysics and Geophysics,Vol. 13,Chapman & Hall/CRC, London, 2007, pp. 313–359
   Link: https://arxiv.org/pdf/astro-ph/0411739.pdf