Flux to Specific Intensity Conversion

Dr Mike Ireland

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We want to determine the flux F through a unit area from a uniform, semi-infinite radiation field. This means that the specific intensity is:

$$I(\theta, \phi) = \begin{cases} I_0 \text{ if } \theta > 0 \\ 0 \text{ otherwise} \end{cases} \tag{1}$$

The flux is given by the integral of $I\cos(\theta)$, i.e. a positive number for flux moving upwards through our unit area, and a negative number for flux moving downwards through our unit area. For rays coming from an angle, the amount of flux passing through the area is reduced by $\cos(\theta)$. The two dimensional solid angle differential is given by $d\Omega = \sin(\theta)d\theta d\phi$ in spherical coordinates. This means that our flux integral is:

$$\begin{aligned}
F &= \int_{-\pi/2}^{\pi/2} d\theta \int_0^{2\pi} d\phi \, I(\theta, \phi)\cos(\theta)\sin(\theta) && (2)\\
&= \int_0^{\pi/2} d\theta \int_0^{2\pi} d\phi \, I_0 \cos(\theta)\sin(\theta) && (3)\\
&= 2\pi \int_0^{\pi/2} \frac{1}{2}\sin(2\theta) d\theta && (4)\\
&= \pi I_0 \left[\frac{1}{2}\cos(2\theta)\right]_0^{\pi/2} && (5)\\
&= \pi I_0 && (6)
\end{aligned}$$

Note that to determine a wavelength-dependent flux F_λ, we replace the total intensity I with the specific intensity I_λ.