Australian National University

ASTR2013 – *Foundations of Astrophysics*

Week 3: Stellar Evolution

Following Dan Maoz – Astrophysics in a Nutshell

Mike Ireland

Australian National University

Equations of Stellar Structure

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}, \quad (3.56)$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r), \quad (3.57)$$

$$\frac{dT(r)}{dr} = -\frac{3L(r)\kappa(r)\rho(r)}{4\pi r^2 4acT(r)^3}, \quad (3.58)$$

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r). \quad (3.59)$$

Non-relativistic equations here only !

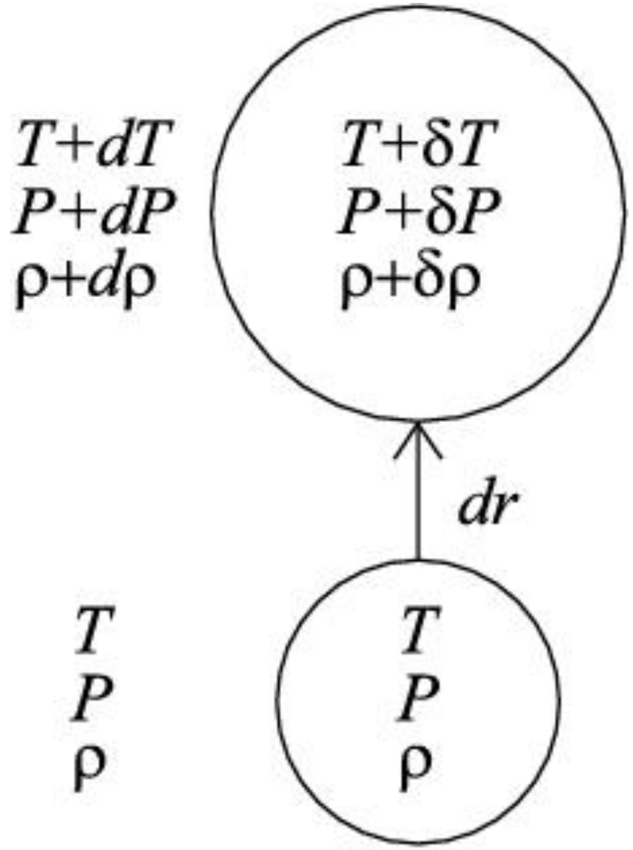
The opacity, nuclear generation rate and pressure are really a complex function of density and temperature.

$$P_{\text{gas}} = nk_B T = \frac{\rho k_B}{\mu u} T$$

EOS (gas only)

Convective Instability

- A parcel of gas that moves upwards without exchanging energy with its surroundings is *expanding adiabatically*.



$$PV^{\gamma} = \text{const.}$$

$$P \propto \rho^{\gamma}$$

$$\frac{d\log(P)}{d\log(\rho)} = \gamma$$

$$\nabla_{\text{ad}} = \frac{d\log(T)}{d\log(P)} = \frac{\gamma - 1}{\gamma}$$

Monatomic Gas

Week 4 Summary

Textbook: Sections 3.8, 4.1, 4.2, 4.3.

1. Theory of Scaling Relations.
2. Key observed stages of stellar evolution.
3. Evolutionary tracks and key theoretical stages of stellar evolution.
4. White dwarfs.

Scaling Relations

Work through on board

- Take 3 first equations of Stellar Structure, and turn them into equations of proportionality with average quantities:

$$M \sim R^3 \rho, \quad P \sim \frac{M\rho}{R}, \quad L \sim \frac{T^4 R}{\kappa\rho}$$

- Assuming gas pressure dominates ($P \sim \rho T$), we re-derive the Virial theorem: $T \sim M/R$. If there is a strong nuclear thermostat on the main sequence, this means $R \sim M$.
- If Thompson scattering dominates (κ=const) then the luminosity relationship gives $L \sim M^3$.
- If radiation pressure dominates, $P \sim T^4$, and we put the 3 equations together to get $L \sim M$.
- See textbook for $L \sim M^4$ for low mass stars.

Observations of Stellar Evolution

Given scaling relations, we know high mass stars have shorter lifetimes, and use observations of stellar clusters to observationally determine what happens to stars as they age.

See Wikipedia for image credits

M67

NGC4755

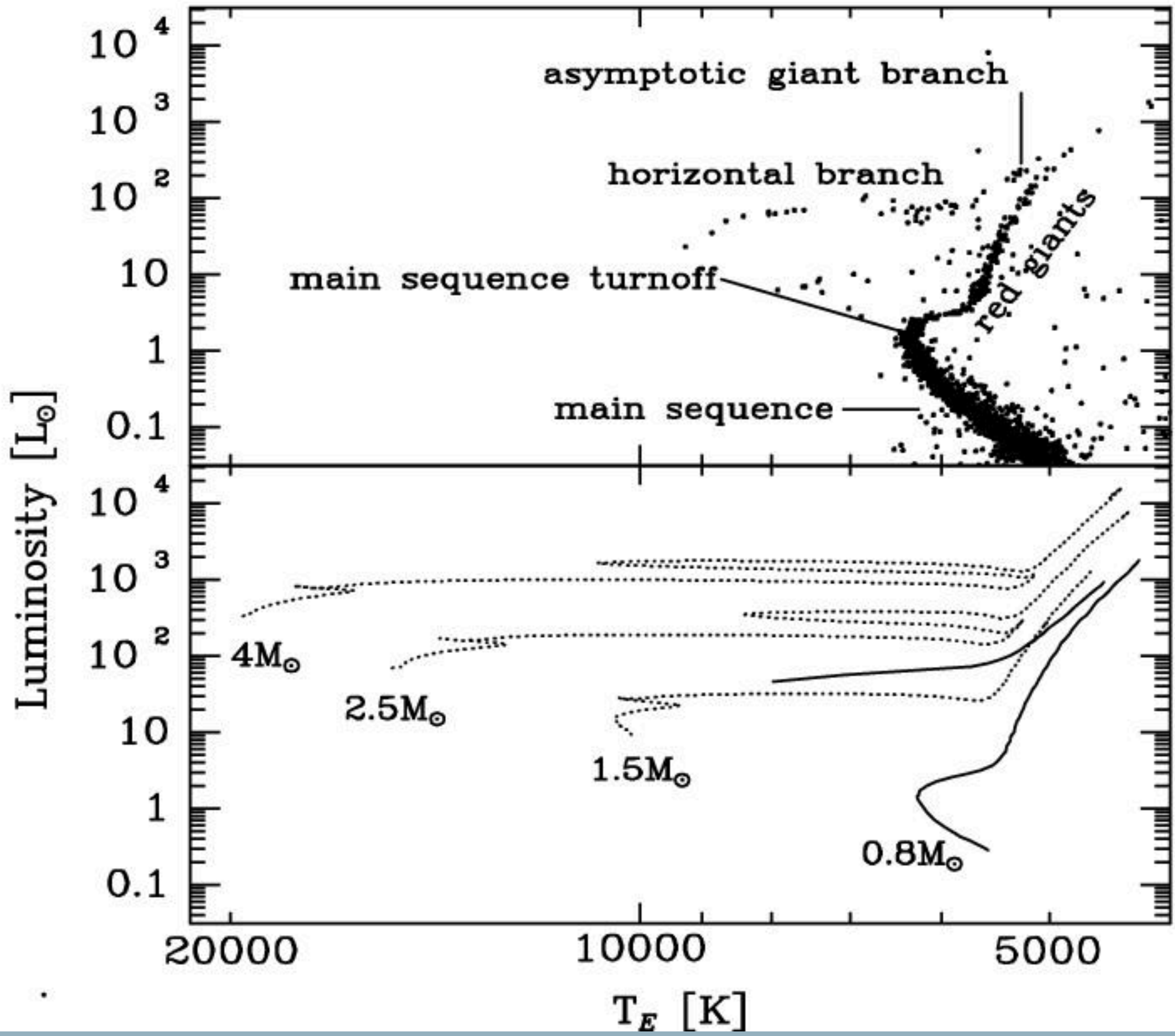
NGC 6819

M15

Isochrone and Evolutionary Tracks

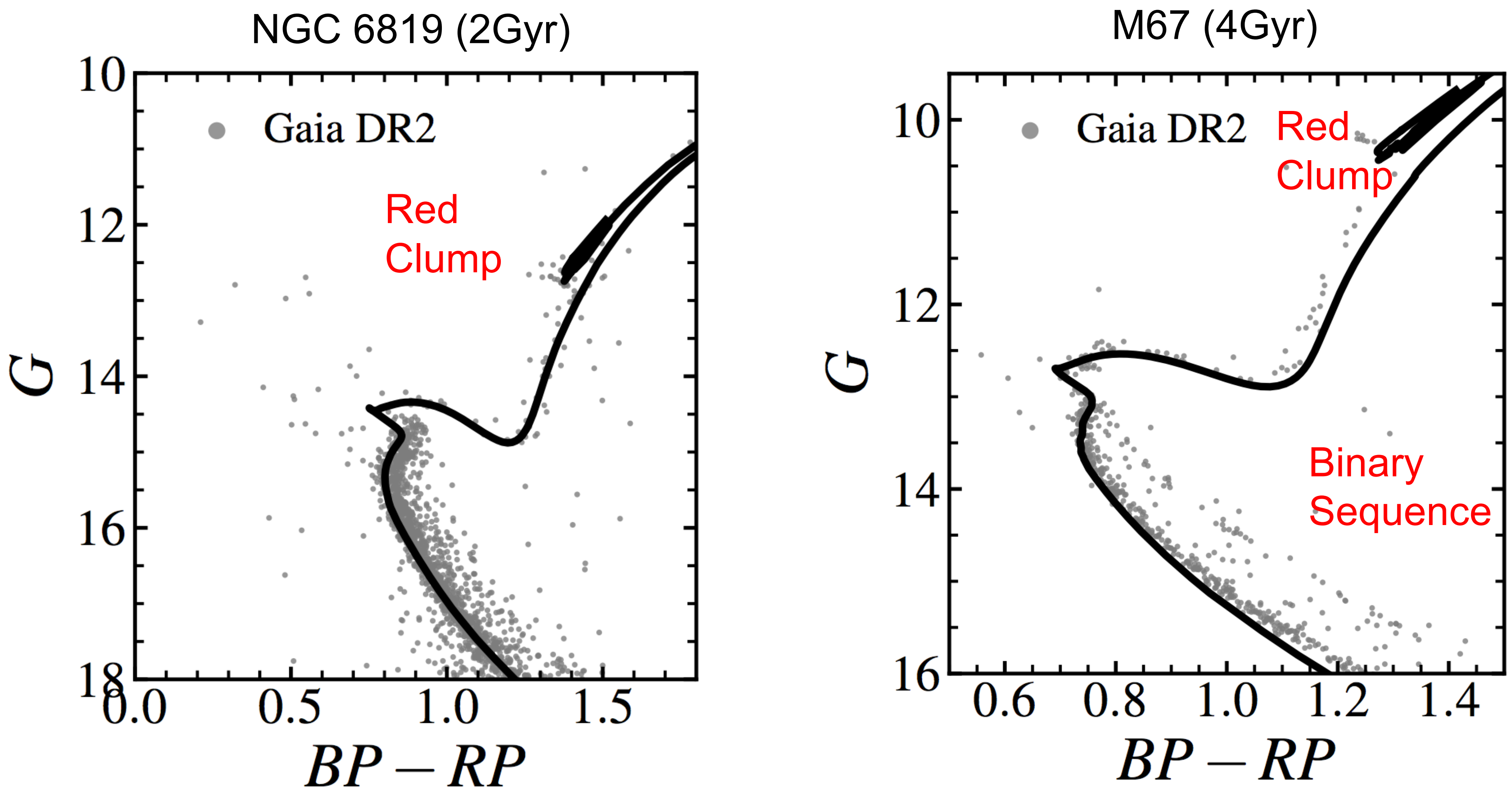
Cluster (~10 Gyr, metal-poor)

Isochrones (also metal-poor)



Open Clusters – Gaia Data

Even the best data affected by fits for *distance* and *dust reddening*.
G measures luminosity (2.5 = factor of 10) and BP-RP measures temperature.



Choi et al. (2018, ApJ, 863, 65)

General Stages of Stellar Evolution (near solar mass)

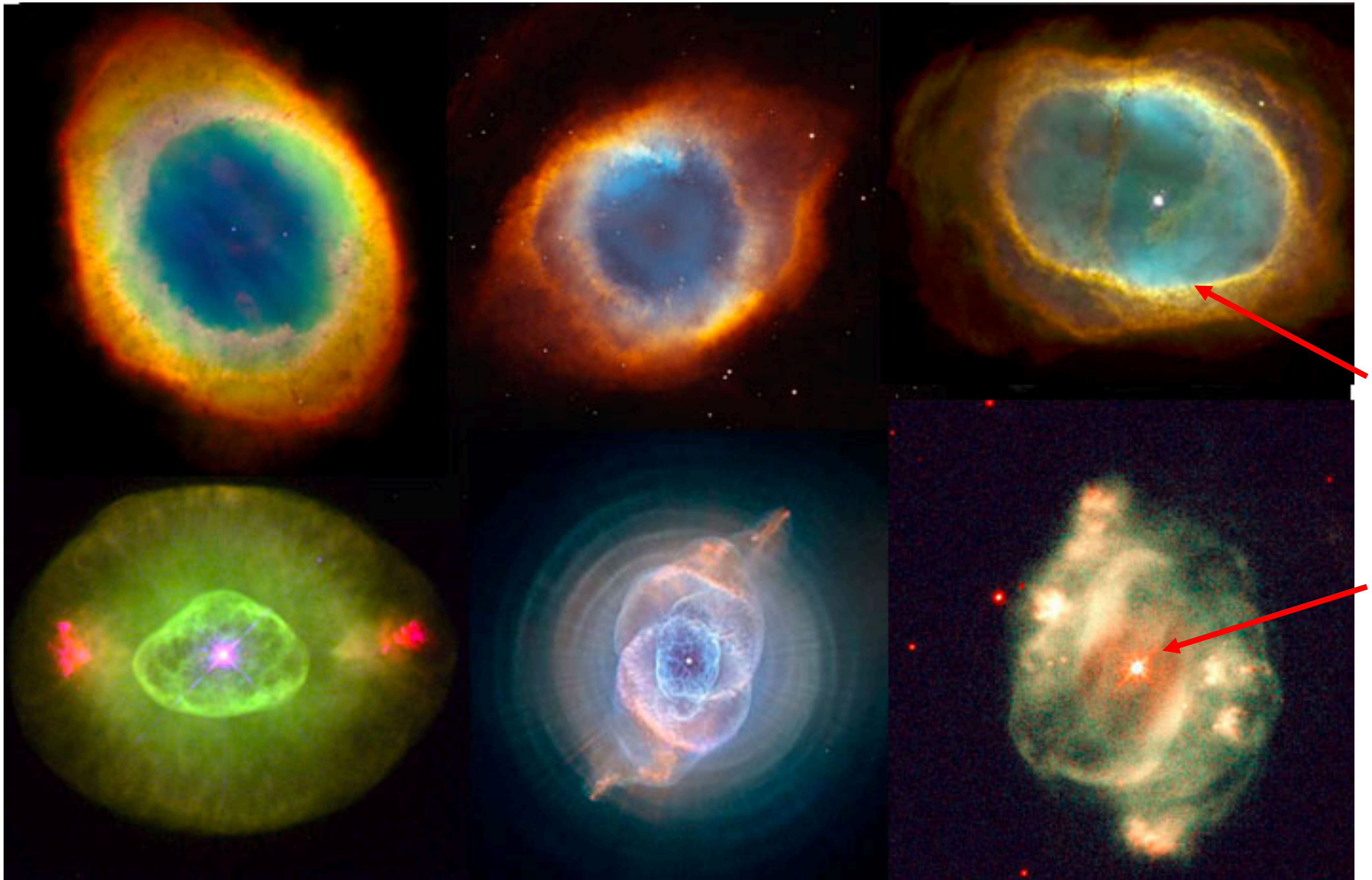
Add Whiteboard Pictures Here…

- A pre-main sequence star is fully convective, doesn't obey the 4th equation of stellar structure and contracts on the Kelvin-Helmholtz timescale.
- The main sequence ends when the core is supported by electron degeneracy pressure and not gas pressure.
- This star becomes a *red giant*, on the K-H timescale if more massive than ~1.3 M_{sun}, or more gradually via the *subgiant stage* for solar or lower masses.
- The giant star has *hydrogen shell burning.*

General Stages of Stellar Evolution (near solar mass)

- When the core temperature and density is high enough, *helium burning* begins in the core.
- [As this is a degenerate core, the reaction runs away as a helium flash]
- The helium-burning star is either a *horizontal-branch* or *red clump* star, depending on metallicity.
- Once He burning to (to C, or even O) is complete, the core contracts until supported by degeneracy pressure.
- Hydrogen burning resumes, and the star and the star resumes the giant branch where it left off (roughly), now an *asymptotic giant branch* star. Occasional substantive He burning is called a *thermal pulse.*
- The resulting high luminosity drives the surface layers off, leaving behind an inert C,N,O core called a *white dwarf*. $1M_{sun}$ star : 0.6 M_{sun} white dwarf
- [the intermediate, rapid phases are called post-AGB, pre-planetary nebula and planetary nebula phases]

Planetary Nebula Phase



Ejected Stellar Atmosphere

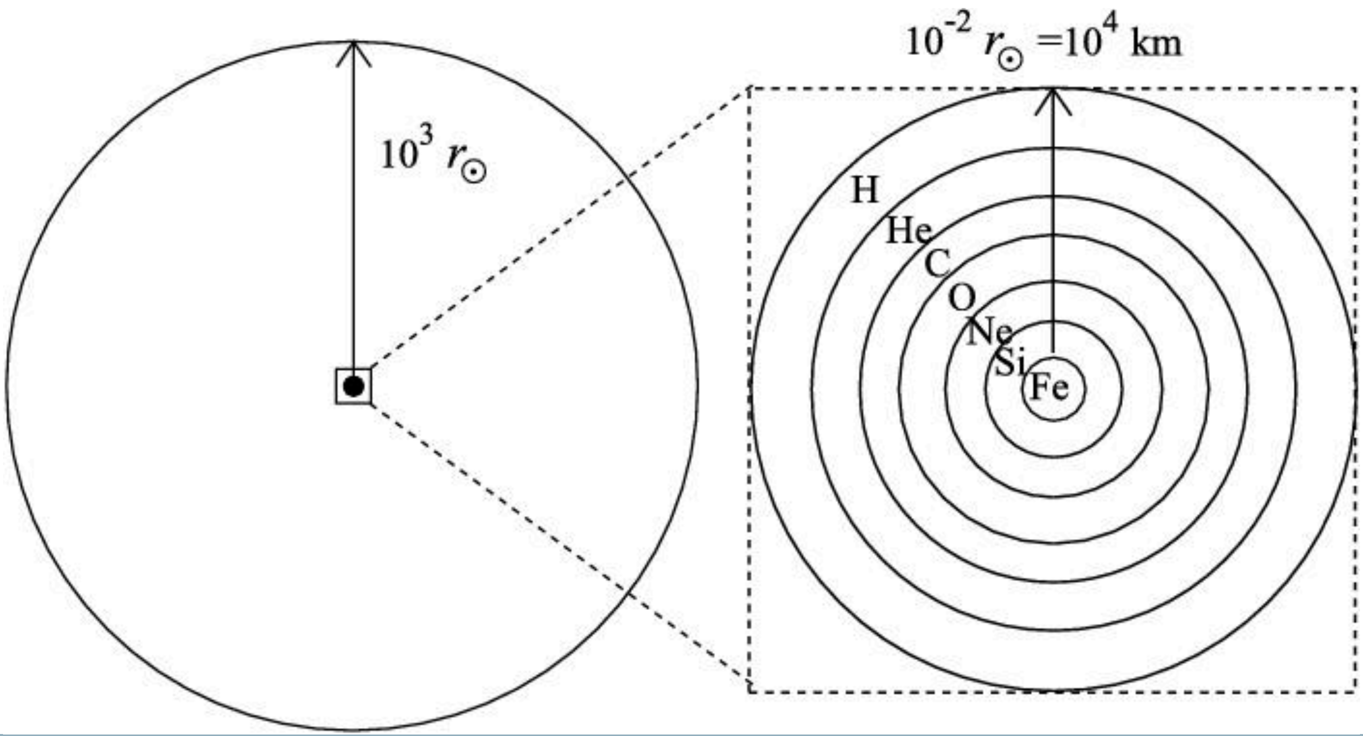
Cooling (still very hot) White Dwarf

Nuclear Generation – Energy Available

- Nuclear reactions produce most their energy in photons (γ rays) and kinetic energy of daughter products including annihilating positrons.
- A smaller but still significant (several percent) energy fraction comes in the form of Neutrinos which typically leave stars.
- Energy available is computed by $E=mc^2$.
  - Proton: 1.00728 amu.
  - Helium-4: 4.0026 amu (1.00065 amu per nucleon)
  - Carbon-12: 12.0000 (1.0000 amu per nucleon)
  - Iron-56: 55.9349 (1 - 0.00116 per nucleon)
  - Gold-197: 196.9666 (1 - 0.00017 per nucleon)
- 0.65% of mass energy to H to He, 0.065% He to C and 0.001% to Fe.

Massive Star Evolution

- For a star more massive than about 8 M_{sun}, the core is so hot that electron degeneracy pressure is never important enough compared with kinetic gas and photon pressure.
- An inert core never forms, and the center of the star has an “onion” structure.
- If it actually looks like the structure below, then it is within a day away from becoming a supernova! (more on this next week)



Australian National University

White Dwarfs

- Electron degeneracy pressure can be derived from simple quantum particle-in-a-box considerations. In the non-relativistic case, this is:

$$P_e = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20 m_e m_p^{5/3}} \left(\frac{Z}{A}\right)^{5/3} \rho^{5/3}.$$

- Note that it has *exactly the same form* as a convective star consisting of a monatomic ideal gas: a white dwarf is the low entropy limit of this functional form.
- If there were no nuclear reactions, this relationship between pressure and density would be all you have to know [called a polytrope with index 1.5].
- If P approaches, rc2, we have to use the relativistic degenerate EOS.....

White Dwarfs

- Electron degeneracy pressure can be derived from simple quantum particle-in-a-box considerations. In the non-relativistic case, this is:

$$P_e = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20 m_e m_p^{5/3}} \left(\frac{Z}{A}\right)^{5/3} \rho^{5/3}.$$

- Note that it has *exactly the same form* as a convective star consisting of a monatomic ideal gas: a white dwarf is the low entropy limit of this functional form.
- If there were no nuclear reactions, this relationship between pressure and density would be all you have to know [called a polytrope with index 1.5].
- If P approaches, ρc^2, we have to use the relativistic degenerate EOS.....

White Dwarfs

- For the ultra-relativistic equation of state where electron degeneracy pressure dominates, the rest mass of the electron no longer matters in the equation of state:

$$P_e = \left(\frac{3}{8\pi}\right)^{1/3} \frac{hc}{4m_p^{4/3}} \left(\frac{Z}{A}\right)^{4/3} \rho^{4/3}.$$

- The scaling relations no longer work with a 4/3 exponent (try it – radius cancels out).
- The electron equation of state becomes relativistic as the mass approaches the Chandrasekhar mass (below) and the radius of the white dwarf approaches 0.

More on this in lab!

$$M_{ch} = 0.21 \left(\frac{Z}{A}\right)^2 \left(\frac{hc}{Gm_p^2}\right)^{3/2} m_p.$$

Week 4 Summary

Textbook: Sections 3.8, 4.1, 4.2, 4.3.

1. Theory of Scaling Relations.
2. Key observed stages of stellar evolution.
3. Evolutionary tracks and key theoretical stages of stellar evolution.
4. White dwarfs.