ASTR2013 Week 11 Tutorial

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In addition to the questions below, I don't know of anyone who actually got through 5.1 (mass function of a Globular cluster), 7.1 (star counts), 7.2 (deriving the gravitational focusing expression).

1. Compare the current critical density of the Universe to the typical density of the solar wind in the vicinity of the earth's radius of 10 protons/cm^3. Is it more important to consider cosmological effects or the mass of the solar wind in precision calculations of planetary orbits and gravitational effects? [NB they are both more than a factor of more than a million below the current precision in measuring GM for the sun]

2. (Textbook 8.2) Measurements of the radial recession velocities of five galaxies in a cluster give velocities fo 9700, 8600, 8500 and 10,000$\dot{\text{k}}\text{m}\,\text{s}^{-1}$.

   (a) What is the distance to the cluster if the Hubble parameter H_0 is $70\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$?

   (b) Estimate, to an order of magnitude, the mass of the cluster if every galaxy is projected roughly half a degree from the cluster center.

3. A hypothetical matter-dominated universe with no dark energy expanding at the critical density has a scale parameter given by:

$$R(t) = R_0 \left(\frac{t}{t_0}\right)^{2/3} \tag{1}$$

   (a) Find an expression for the Hubble parameter $H = \dot{R}/R$.

   (b) Given that $H(t_0) = H_0$, find the age of the Universe in terms of H_0.

   (c) Given that the oldest stars were modelled to be $\sim$10 Gyr old in the 1990s and the Hubble constant is $\sim 70\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$, was such a Universe compatible with observations?

4. An object is observed at a redshift $z = 7$ corresponding to a proper distance d. A *proper distance* is the distance you'd get by having a tape measure stretched at the current time between us and this object. Assume flat space-time (the only kind we're talking about in this course when discussing the Universe).

   (a) If a total of N photons were emitted from the object, how many photons pass through the shell of surface area $4\pi d^2$?

   (b) If a photon luminosity of dN/dt photons/s were emitted from the object, what is the observed flux of photons? [Hint: recall the time dilation formula for cosmological redshift]

(c) If a total luminosity of L was emitted form the object, what is the observed energy flux? (i.e. power per unit area) [Hint: the photons in the previous question are redshifted]

(d) If we define a luminosity distance of $d_L = \sqrt{L/4\pi F}$, how does this relate to the proper distance?

5. Assume a galaxy like the Milky Way has an infinitesimally thin disk with surface density Σ (i.e. in g/cm^2).

(a) For an object of height h above a point P in the disk, draw a diagram showing the mass in an annulus of cylindrical coordinate (ϖ, θ, z) radius between ϖ and $\varpi + d\varpi$ that makes a contribution to the gravitational force on the object.

(b) Write down the expression for the radius r of the annulus from the object.

(c) Integrate the acceleration due to gravity contribution of all annuli to derive the total acceleration on the object due to the disk.