Combined Reference Notes: Exoplanets Course

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1 Pulsar Planets

In each section of the course, we will provide reference notes. These summarise the main points from the video lessons in that section. They are intended to be a reference source - allowing you to quickly pick out key facts and equations without having to re-watch all the videos.

1.1 INTRODUCTION

Exoplanets are planets that orbit other stars. They are hard to see because other stars are far away - much too far to visit with space-probes, given our current technology.

We can therefore only study them with telescopes. The problem here is that the light from any exoplanets is overwhelmed by the light from their stars, which are a billion times brighter.

Four techniques have been used to study them:

- Reflex motion - look not at the planet but at the wiggle it induces in the star.
- Transits - look for the drop in brightness when the planet goes in front of the star
- Gravitational micro-lensing - look for the increase in brightness of a distant star when a planet passes in front of it and focusses its light on us.
- Direct imaging - use clever techniques to spot the planet against the glare of its star.

1.2 PULSARS

Pulsars are neutron stars left over from supernova explosions. Millisecond pulsars spin hundreds of times per second and are thought to be pulsars originally in binary systems, which were spun up by gas from their companion star.

Because of the regular pulses these objects put out, changes in the distance to them can be measured with exquisite precision. This allows us to measure the reflex motion caused by any planets very accurately - and in one pulsar, the reflex motion due to three planets was found.

1.3 MATHEMATICS OF REFLEX MOTION

If you measure the period P of the reflex motion, you can work out how far away from the star the planet must be r using the equation

$$r = \sqrt[3]{\frac{GM_* P^2}{4\pi^2}}$$

where M_* is the mass of the star.

The velocity of the star v_* due to its reflex motion is given by

$$v_* = \frac{2\pi r}{P} \frac{M_p}{M_*}$$

where M_p is the mass of the planet. Note that v_* is the true velocity of the star: what we actually observe v_o is $v_0 = v_* \sin i$ where i is the inclination angle of the orbit.

If you want to measure the radius of the orbit of the star (due to its reflex motion), it is given by

$$\frac{M_p}{M_*} = \frac{r_*}{r}$$

(assuming the mass of the planet is much less than that of the star). Once again, what we measure is not r_* but $r_* \sin i$.)

1.4 EXPLANATION?

How can we explain the existance of pulsar planets? One possibility is that they somehow survived the supernova explosion that formed the pulsar, but that is unlikely for many reasons. A second possibility is that some of the gas fired out by the explosion falls back in, forms an accretion disk and then planets form in this disk, in a similar way to protoplanetary disks. Lack of angular momentum may be a problem for this theory.

We believe that there must have been a second star in the system originally (to spin the pulsar up to millisecond periods) which somehow has gone missing. A third possibility is that this second star was disrupted and the planets formed from its debris. Or finally, the neutron star might have sunk into the core of this second star forming a giant Thorne-Zytkow object, which might have helped keep pre-existing planets and brought them in closer.

1.5 NEW PULSAR PLANETS

Two more have been found recently. One is in a globular cluster, which was probably formed by some sort of capture process as stars in this dense environment come close to each other.

The second is so close to the pulsar and so massive that it could not survive unless it were made of something incredibly dense. Perhaps it is the core of a carbon White Dwarf star which has been stripped of its outer layers, leaving a core of diamond?

2 Finding Planets using Reflex Motion

2.1 OBSERVING THE SIDEWAYS WIGGLE?

If a planet of mass m_p moves in an orbit of radius r around a star of mass m_*, the star will show reflex motion of radius

$$r_* = \frac{r}{1 + \frac{m_*}{m_p}} \sim r\frac{m_p}{m_*}$$

This motion is very small - the change in angle θ as observed from the Earth, a distance D away is

$$\theta = \frac{r_*}{D}$$

where θ is measured in radians.

This equation is explained in much more detail in the first section of our preceding course ”Greatest unsolved mysteries of the universe” - check that out for more details if you wish.

In practice this reflex motion is unobservably small, due to telescope imperfections, atmospheric seeing, diffraction and the random arrival of photons (Poisson noise).

2.2 OBSERVING THE DOPPLER WIGGLE?

A second alternative is to look for the reflex motion back and forth along the line of sight. This would cause a Doppler shift in the wavelengths of any features in the spectrum of the star. The Doppler effect, and the whole concept of spectra were also introduced in the first section of our preceeding course ”Greatest unsolved mysteries of the universe” - check that out for more details if you wish.

The star will be moving at a velocity

$$v_* = \frac{2\pi r_*}{P}$$

which will cause a shift $\Delta\lambda$ in the wavelength λ of any spectral line given by the equation

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

where c is the speed of light.

2.3 OBSERVING TECHNIQUES

Once again, this shift is tiny. It is hard to measure because of many possible distortions in the spectrograph, and in particular the uncertainty and rapidly moving position of the light on the slit of the spectrograph.

Two approaches have been used to overcome this. One puts the incoming light through a fibre-optic cable to scramble its position, and then compares it to reference light from an arc lamp. The second technique puts an iodine cell in front of the spectrograph slit, to imprint reference absorption lines all over the spectrum.

2.4 RESULTS

Planets were found - but very strange ones. Hot Jupiters - planets with roughly the mass of Jupiter, but incredibly close to their stars - in orbits of only a few days, and 70 Virginis type systems - giant planets in highly eccentric orbits.

Neither are anything like our solar system, and both were thought to be impossible because of the snow-line theory, which suggests that giant planets can only form far enough out for ice to stay frozen.

3 More Radial Velocity Planets and Transits

3.1 BETTER RADIAL VELOCITY DATA

As time has gone on, radial velocity searches have become more sensitive, allowing them to detect smaller planets with their smaller wobbles. As the surveys continue for longer, they also become sensitive to larger planets further out, in orbits of many years in length.

3.2 ORIGIN OF HOT JUPITERS

Where do the "hot Jupiters" come from? If they form inside a protoplanetary disk, they may set up ripples in the disk which will cause them to migrate. This could bring them from out beyond the snow-line into the regions close to the star.

If anything, this mechanism is too effective, and should move all planets in - there should be nothing except hot Jupiters.

An alternative is planetary billiards: the planets warp each other's orbits, collide, get flung out, and in general end up in confusing elliptical orbits.

3.3 TRANSIT SEARCHES

If a planet is in a nearly edge-on orbit, it may pass in front of the star once per orbit and cause a slight dip in brightness. If the normal brightness of the start is B and the change in brightness is ΔB, then the radius of the planet r_p is given by:

$$r_p = r_* \sqrt{\frac{\Delta B}{B}}$$

where r_* is the radius of the star.

We can also determine how far out the planet is from the period of the orbit (i.e. the interval between dips in brightness).

Transit searches are hard - images need to be deliberately put out of focus to spread the light over more pixels, and the sensitivity calibrated by other stars in the field of view. Global networks of small, wide-field telescopes like HAT south are the current state-of-the-art. But only 1-10

3.4 PLANET RADII

It turns out that transit searches mostly find hot Jupiters, but most of them are larger than predicted by standard models. The discrepancy is worse for the hottest planets.

The equilibrium temperature T of the surface of a planet can be calculated (this was done in our previous course "Greatest unsolved mysteries of the universe") and comes out as

$$T = \sqrt[4]{\frac{L}{16\pi\sigma D^2}}$$

where L is the luminosity of the star, σ is the Stefan-Boltzmann constant, and D is the distance between star and planet.

It is not known what causes these inflated sizes.

3.5 ROSSITER-MCLAUGHLIN EFFECT

As a planet moves across the disk of a star, it blocks the light from different parts. If the star is spinning, this will cause the apparent velocity of the star to shift. This is called the Rossiter-McLauchlin effect and allows us to determine the relative direction of the star's spin and the planet's orbit.

The two are often mis-aligned, which is troublesome for disk migration theories.

4 More Transit Results

4.1 TRANSITS FROM SPACE

It is hard to measure the brightness of stars with much better than 0.1

Kepler spent three years staring at one part of the sky. Due to gyroscope failures it now has to divide its attention between several regions of the sky for a few months each.

4.2 SUPER EARTHS

In our own solar system, planets fall into two quite distinct categories, the rocky planets and the giant planets. There is nothing between the largest rocky planet (Earth) and the smallest giant planet (Neptune).

Kepler shows that in other solar systems, there are large numbers of planets intermediate between the Earth and Neptune in both radius and mass. These are called 'Super Earths', though 'mini-Neptunes' would be a good name too.

It is not clear what they are like. Some may be giant rocky planets, some are probably gas planets and some may even be water worlds.

4.3 UNBIASSING THE POPULATION

To find out the true population of exoplanets, we need to remove the various biasses which affect surveys. For example, planets in close-in orbits are more likely to transit, and their transits will be larger and hence easier to detect. Thus you would expect close-in planets to be over-represented in any exoplanet surveys.

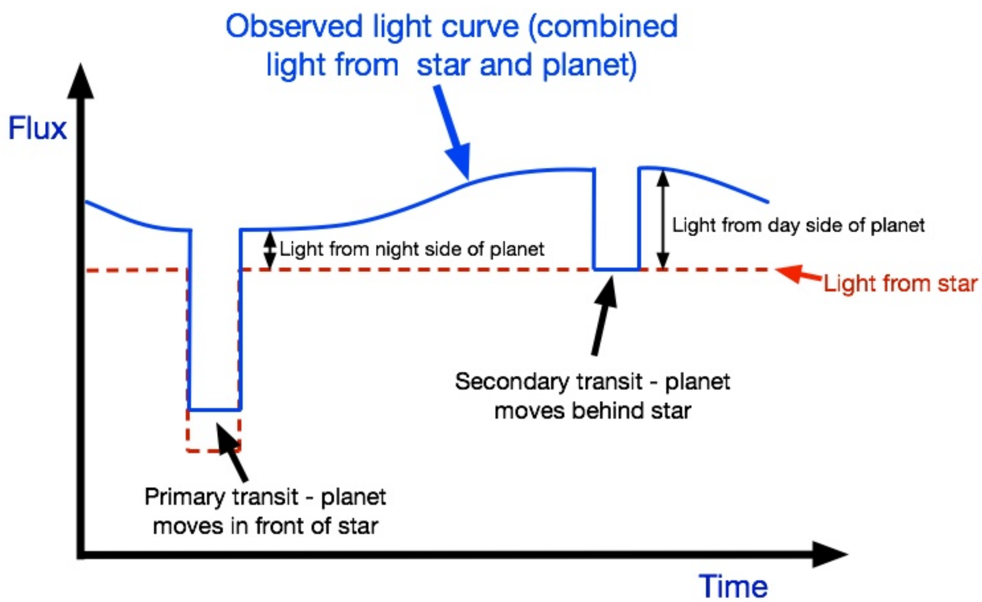
One way to correct for this is "Monte-Carlo" simulations. You guess what the population of exoplanets might be like, simulate this population, then put the simulated results through the discovery process and see which simulated exoplanets would actually have been discovered. Compare this to the planets actually discovered and if the agreement is good, your guessed population is consistent with the data.

A second approach is to divide planets up into categories (e.g those with orbital periods of 2-4 days and masses 0.5 - 0. 8 Jupiter masses). For each category, you can calculate the probability of a planet with those properties being seen. Use this to correct the observed numbers and recover an estimate of the true population.

These methods show that for planets with periods of less than 50 days, the numbers steadily increase towards smaller masses and larger periods.

4.4 SECONDARY TRANSITS

The great precision of space-based light curves allows for more to be learned about planets from the details of the brightness changes.



Secondary transits occur when the planet moves behind the star, and their depth tells us how bright the day side of the planet is. A slope in the brightness between primary and secondary transits tells us the difference between the day- and night-side brightness of the planet. At optical wavelengths the planet just reflects the sunlight, so there is no light from the night-side of the planet. At infra-red wavelengths, the planet will glow depending on its temperature, so the day-to-night brightness difference tells us the temperature difference from the hot to the dark side of the planet.

Most exoplanets measured using this technique are quite dark at optical wavelengths, presumably due to a lack of clouds. Close-in exoplanets have large temperature differences between day and night, while further out ones seem to have less of a difference.

4.5 SPECTRA AND COLOURS

By subtracting a spectrum taken during the secondary transit from one taken just out of the secondary transit, the spectrum of the day side of the planet can be obtained. This is a very difficult measurement, only really possible from space telescopes like Spitzer or the Hubble Space Telescope.

It may also be possible to use spectra taken during a primary transit to detect absorption in the atmosphere of an exoplanet, or in a wind being blown off an exoplanet. In one case this allowed the detection of very strong winds, and in another, a huge tail of hydrogen being blown off the exoplanet.

5 Gravitational Lensing

5.1 GRAVITATIONAL LENSING

Einstein showed that if a light-ray passes a distance r from an object of mass M, then the light-ray will be deflected by an angle θ given by the equation

$$\theta = \frac{4GM}{rc^2}$$

where G is the gravitational constant and c is the speed of light (note - this is an approximation to the full equation, only valid for small angles).

5.2 SEARCHING FOR PLANETS

Survey telescopes monitor the brightness of millions of stars in the bulge of our galaxy. They are waiting for the rare times when a closer star passes very close to the line-of-sight of the background star, and bends the light of the background star.

This lensing will produce multiple images separated from the lensing star by an angle of roughly α (the Einstein Radius) given by:

$$\alpha = \sqrt{\frac{4GM}{Dc^2}}$$

where D is the distance from the Earth to the background star. Note that this is actually just a special case of the true Einstein Radius equation valid when the lens is half way to the background star.

5.3 MICROLENSING

Unfortunately, these angles are much too small to see when stars or planets lens other stars in our own galaxy. But while we can't see the multiple images, we can detect the amplification of the background star - its apparent increase in brightness. This case (gravitational lensing with angles too small to see but causing increases in brightness) is called gravitational microlensing.

It is most commonly calculated using ray-tracing. Simulated light rays are sent out from the background star and bent as defined by the equations above, and the position they land at is recorded in the "image plane", an imaginary plane passing through the Earth.

A single mass causes single spikes in brightness, but when you have multiple masses within a few Einstein Radii of each other you get complex patterns, of short spikes in brightness superimposed on the overall trend.

5.4 RESULTS

Survey telescopes trigger alerts when a star shows a probable microlensing event, whereupon intense follow-up from multiple telescopes around the world begins. This is necessary to spot and measure the very narrow spikes caused by caustic crossings.

So far, quite a wide range of planets have been found using this technique - including a super-Earth (much further out than can be seen by Kepler), and some Jupiter-like planets in Jupiter-like orbits.

It seems, from combining microlensing data with radial velocity results, that solar system analogues (i.e. solar systems with roughly Jupiter-mass planets in roughly Jupiter-like orbits) are seen around roughly 30

5.5 FREE FLOATING PLANETS?

Recently, a number of short brightness spikes have been found which seem to come from planets which are not in orbit around stars (or at least are a long way out). These free-floating planets seem to outnumber stars in our galaxy. Perhaps they were flung out in the game of planetary billiards which may have taken place in young solar systems?

6 Debris Disks

6.1 INFRARED EXCESS

The IRAS astronomy satellite discovered that many stars, most famously Vega, are brighter at far-infrared wavelengths than you'd expect from their stellar emission.

For Vega, this excess emission peaked at around 60 μm wavelength. We can use this to estimate the temperature of the disk, via Wien's displacement law:

$$\lambda_{\text{peak}} T = b = 2.9 \times 10^{-3} \text{m K}$$

where λ_{peak} is the wavelength at which the emission peaks, and T is the temperature. Using this, we find that the excess emission is coming from something very cold, and hence presumably far out.

We can estimate how far out the emission is coming from by using the equation for the equilibrium temperature of a planet derived in the first course:

$$T = \sqrt[4]{\frac{L}{16\pi\sigma D^2}}$$

where L is the luminosity of the star, D the distance between star and planet, and σ is the Stefan-Boltzmann constant.

To produce enough radiation to explain the infra-red excess, given this low temperature, a planet would have to be unfeasibly large. The fraction of the star's light which is intercepted by a planet of radius r and distance from the star D, and hence available to re-radiate is

$$\sim \frac{r^2}{4D^2}$$

But luckily, if the same mass is divided up into smaller chunks of radius r, the fraction of starlight intercepted and re-radiated is proportional to $1/r$.

6.2 DEBRIS DISKS

So it seems that the best way to explain the infra-red excesses is by clouds of small particles far out from the star. Observations indicate that these clouds are typically in disk-like geometries.

These dust disks cannot be the remains of the proto-planetary disk, as either radiation pressure or the Poynting-Robertson effect would very rapidly clear such dust grains out of a solar system.

One possibility is that the dust is deposited by comets. In the case of Beta-Pictoris, we see transient redshifted absorption lines in the star spectrum, indicating that some big gassy things fall into the star hundreds of times a year. These are probably comets - but even with 1000 large comets per year it is not enough to give the amount of dust required.

Most likely, the dust comes from an asteroid belt, composed mostly of large asteroids (100 km or larger). These occasionally collide and are smashed up, producing smaller rocks, which in turn collide and collide until they are ground down to the size of the dust we observe. This dust is then swept out or swallowed by the star, but is constantly replenished by more asteroid collisions.

Something is needed to keep the asteroids stirred up - most likely a planet somewhere near the asteroid belt.

Note that the debris disks seen to date are mostly much cooler, and hence further out than the asteroid belt in our own solar system. They are probably more analogous to the Kuiper Belt in our solar system (the swarm of icy bodies including Pluto and Eris which orbit beyond Neptune). At present, however, we can only detect debris disks with ten times more dust than in our own Kuiper Belt. Roughly 20

7 Adaptive Optics

7.1 LIGHT GOING THE WRONG WAY

A small fraction of the light in any optical system does not go the right way. This is a problem when you are trying to image a faint object (like a planet) close to a bright object (like a star).

The ratio of the brightness of a planet (of radius r) to its star is

$$\frac{1}{4}\left(\frac{r}{D}\right)^2$$

where D is the distance between star and planet.

One cause of this is imperfections in the telescope optics. But this is usually not the dominant problem in professional astronomical telescopes.

A second problem is atmospheric seeing. The air you are looking through is not of uniform temperature. Hot and cold air has a slightly different refractive index, so the hot and cold bubbles of air distort your image.

The best observatory sites on Earth are at high altitude and have very smooth air flow, but even there, light is typically blurred over an angle of around 0.6 arcseconds.

7.2 DIFFRACTION LIMIT

Even from space, however, the light entering your telescope is bent by diffraction. Any wave (like light) usually travels in a straight line as that is the only direction in which all the waves add up in phase (Huygens' principle). But when light has to pass through a finite aperture, light can add up in phase over a wider range of angles, blurring the image.

This diffraction blurring places a fundamental limit on how sharp an image you can get with any telescope, even in space. The blurring angle θ is roughly

$$\theta = \frac{\lambda}{D}$$

where λ is the wavelength of the light and D is the diameter of the telescope.

7.3 ADAPTIVE OPTICS

It is now becoming possible to undo the effect of atmospheric seeing on ground-based telescopes. If you can measure how the incoming wavefronts are being distorted (with a wave-front sensor), you can bounce the light off a deformable mirror which has an equal and opposite distortion, thus in principle restoring diffraction-limited image quality. And as ground-based telescopes are much larger than space ones, this means better image quality than can be achieved from space.

This works fine when you have a bright star near your target (as is the case for planet imaging), as you can use the light from the bright star to measure the wavefront distortion. If you are not looking near a bright star you may have to create your own artificial star, by using a laser to ionise a part of the sodium layer high in our atmosphere.

8 Direct Imaging

8.1 OBSERVE IN THE INFRA-RED

Early attempts at direct imaging could only achieve contrast ratios (the ratio of the brightness of the star to a detectable planet) of a few hundred to one. So to have any hope of seeing anything, they needed to make observations in the infra-red. If planets are hot, they will emit their own light at infra-red wavelengths which might be quite bright.

As mass is assembled to form a giant planet in the first place, gravitational potential energy (when the fragments are far apart) will be converted into kinetic energy (as they fall towards each other) and then into thermal energy (once they collide).

A rough estimate of the gravitational energy U available in forming a planet of mass M and radius r is

$$U = \frac{GM^2}{4r}$$

To work out how much this might heat up a planet, you can compare it to the specific heat capacity times mass. And even a very rough estimate will show you that newborn giant planets may well be very very hot.

A hot planet will radiate energy. This can be estimated using the Stefan-Boltzmann equation: $P = A\sigma T^4$, and it seems that planets will remain hot for periods on the order of ten million years.

The first successful exoplanet image was of a very massive and hot object near a very small and cool star. Almost more like a binary star system than a solar system.

8.2 BROWN DWARFS

There is no clear distinction between a small star and a very large planet. A star with a mass of less than about 10

8.3 MORE DISCOVERIES

A new technique, Angular Differential Imaging, involves using the rotation of the sky relative to the telescope to take out many of the aberrations. This allowed a slew of new discoveries, mostly around stars with debris disks. The planets found are huge, hot, and far out.

One "planet" was found at optical wavelengths by the Hubble Space Telescope, orbiting Fomalhaut. But it reflects too much light to be a planet - it is probably a cloud of dust orbiting a smaller planet.

8.4 MODELLING

Crude spectra of these exoplanets can be obtained, and by comparing these to theoretical models, we can hope to learn something about the composition, temperatures and atmospheres of these planets.

The first step in modelling is to guess the chemical composition. You then need to balance gravity against pressure, and model the energy flow through the different layers of the planet. This will determine the temperature and pressure at a given altitude, which in turn will determine which molecules are present. The molecules provide opacity - i.e. they block heat flow, in turn affecting the temperature and pressure. By looping through all these calculations you can hope to reach a self-consistent solution.

8.5 SURVEYS

More recently, systematic surveys for exoplanets have failed to find many. It appears that very hot, massive and bright planets a long way out are rare. The microlensing results must therefore come from free-floating planets and not planets in the outer parts of solar systems.

The technology is improving fast and we can expect more results soon!

9 Earth-like Planets

An introduction to life in space and the habitability of planets can be found in the last section of our first course.

9.1 FINDING EARTH-LIKE PLANETS

We cannot yet find earth-mass planets in earth-like orbits around Sun-like stars. But it is possible to find planets in the habitable zone (i.e. where the equilibrium temperature is suitable for liquid water) around red dwarf stars.

This is possible because such planets are much closer in, and because the low mass and radius of red-dwarf stars makes reflex motion and transit signals bigger.

Kepler has found a number of potentially habitable planets, but because the Kepler stars are so faint and far away it is currently impossible to work out their mass and density, or to image them.

Some potentially habitable planets have been found about nearby red dwarf stars. In some ways, red dwarfs look like the ideal place for habitable life - they are common, long lived, and seem to have suitable planets. Although it is possible that their lack of UV and their flares might make evolution difficult.

9.2 PLANET COMPOSITION

There are unlikely to be planets with new elements, or even with very different ratios of elements, as the same processes of nuclear fusion are responsible for elements everywhere.

One possible exception: there may be stars with more carbon than oxygen (though this is controversial). If true, all the oxygen would combine with carbon to form carbon monoxide, which would blow away. The remaining carbon would form planets of graphite, diamond and silicon carbide (rather than the oxygen dominated rocks in our own solar system).

A second possibility is that some planets might have much higher levels of radioactive elements than our own, due to enrichment from a supernova just before they formed. This might change the internal heat, volcanic activity and plate tectonics.

9.3 KEEPING AN ATMOSPHERE

Volatile elements (like our atmosphere and ocean) are probably delivered to the Earth by either comets or icy asteroids. But small planets cannot hold on to volatiles like these.

The typical speed v of an atom or molecule in an atmosphere is given by:

$$v = \sqrt{\frac{3kT}{m}}$$

where k is the Boltzmann constant, T the temperature and m the mass of the molecule.

In order to escape from the gravity of a planet, a molecule will need to reach escape velocity, which is:

$$v = \sqrt{\frac{2GM}{r}}$$

where M is the mass of the planet, and r the radius of the planet.

Lighter molecules will move faster and hence have a greater chance of escape. The typical speed of a hydrogen atom in Earth's atmosphere is roughly three times less than the escape velocity. But a small fraction of the atoms will move much faster than the typical (mean) speed and so slowly over time, hydrogen will escape.

9.4 MOUNTAINS

The height of a mountain cannot be too large, or its weight would cause the rocks at its base to flow like a liquid. On Earth, the practical limit is roughly 10 km (depending somewhat on the type of rock), so Mt Everest is pretty close to the limit.

The limiting height h should be proportional to:

$$h \propto \frac{r^2}{M}$$

where r is the radius of a planet and M its mass. If we assume that all planets have the same density (i.e. are all made of rock), then $M \propto r^3$, and:

$$h \propto \frac{1}{r}$$

so smaller planets (like Mars) can have bigger mountains, which seems to fit.

9.5 FUTURE TELESCOPES

To calculate what it would take to actually image details on exoplanets, we need to consider the diffraction limit of telescopes, and how many photons they would detect.

The diffraction limit θ in radians of a telescope of diameter d operating at a wavelength λ is given by $\theta = \lambda/d$.

The angle needed to resolve an object of size r and distance D is $\theta = r/D$.

To work out how many photons you can detect, you need to work out the flux, and then divide by the energy of a photon:

$$e = h\nu = h\frac{c}{\lambda}$$

where h is Planck's constant, ν the frequency, and c the speed of light.