ASTR2013 ass6

Yuan Yuxuan u6772166

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Problem 1

The density of the flat universe at present is given by

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G}$$

where $H_0 \approx 70 km s^{-1} Mpc^{-1}$ is Hubble constant.

The density of baryonic mass make up 4% of total mass, indicating that the density of baryonic matter at present time is: $\rho_{b,0} = 0.04 \cdot \rho_{c,0} = 3.68 \times 10^{-28} kg m^{-3}$

The electron density at this time(including bounded and ionized) is equal to $n_{e,0} = \frac{\rho_{b,0}}{m_H} \approx 0.22 m^{-3}$ where $m_H = 1.00784u$ is the mass of single Hydrogen atom.

a

We know that the relation between scale factor and redshift $R = \frac{1}{1+z}$

The redshift of cosmic microwave background is z = 1000. The scale factor of CMB is hence $R_{CMB} = \frac{1}{1+z} \frac{1}{1001}$

The scaling relation for the number density is that $n \propto R^{-3}$, where R is the scale factor of the universe. The scale factor of the present universe is 1. So the ratio of density of baryonic matter at CMB and present is:

$$\frac{n_{e,CMB}}{n_{e,0}} = \frac{R_{CMB}^{-3}}{R_0^{-3}} = R_{CMB}^{-3} \approx 1 \times 10^9$$

Assuming full ionization, the free electron density at CMB is hence

$$n_{e,CMB} = 2.2 \times 10^8 m^{-3}$$

b

The Thompson cross section is $\sigma_T = 6.7 \times 10^{-25} cm^2$.

The mean free path of the Thompson scatter event is that

$$l = \frac{1}{n_{e,CMB}\sigma_T} = 6.83 \times 10^{19} m$$

mean time between $\tau = \frac{l}{c} = \frac{1}{n_{e,CMB}\sigma_T c} = 2.28 \times 10^{11} s$

Next we calculate the age of universe at CMB, From tutorial problem 3, we know that the time evolution of scale parameter is that:

$$R(t) = R_0 \left(\frac{t}{t_0}\right)^{2/3}$$

where R_0=1 is the scale parameter at the present time, $t_0 = 2/3H_0^{-1}$ is the age of the present universe.

We modified above equation to

$$t = \frac{1}{t_0} R^{\frac{3}{2}} = \frac{2}{3H_0}(\frac{1}{1+z})^{\frac{3}{2}}$$

At CMB(z = 1000), the age of universe is that

$$t_{CMB} = \frac{2}{3H_0}(\frac{1}{1001})^{\frac{3}{2}} = 9.28 \times 10^{12} s$$

The mean time between Thompson scattering events as a fraction of the age of the Universe at that time is $\frac{\tau}{t_{CMB}} = 0.025$

Problem 2

The angular diameter distance in flat universe is given by

$$D_A = 3ct_0 \left[(1+z)^{-1} - (1+z)^{-3/2}\right]$$

a

For a Universe where the age $t_0 = \frac{2}{3} H_0^{-1}$ and small redshift $z << 1$, we use the taylor expansion $(1+z)^l = 1 + lz$

$$D_A = 3ct_0[(1-z) - (1-\frac{3}{2}z)] = 3c\frac{3}{2}H_0^{-1}\frac{z}{2} = \frac{cz}{H_0} = \frac{v}{H_0}$$

where we use the formula for redshift: z = v/c. The Hubble distance is defined as $v = HD$, which tells us that $D = D_A$. Therefore for small redshifts, the angular diameter distance equals the Hubble law distance.

b

To find the maximum angular diameter distance, we take the derivative of D_A with z,

$$\frac{dD_A}{dz} = 3ct_0[-(1+z)^{-2} - (-\frac{3}{2})(1+z)^{-\frac{5}{2}}] = 3ct_0 \frac{3 - 2(1+z)^{\frac{1}{2}}}{2(1+z)^{\frac{5}{2}}} = 0$$

Solving above equation:

$$3 - 2(1+z)^{\frac{1}{2}} = 0$$

which tells us the maximum angular diameter distance occurs at $z = \frac{5}{4}$

The maximum angular diameter distance is

$$D_A = 3ct_0 \left[(1+z)^{-1} - (1+z)^{-3/2}\right] = \frac{2c}{H_0}[(\frac{9}{4}^{-1} - \frac{9}{4}^{-\frac{3}{2}})] = \frac{8c}{27H_0} = 1.27Gpc$$

Problem 3

Conservation of magnetic flux and mass indicate that

$$\Phi = BA = BHD = \text{const}$$

$$M = \rho V = \rho LHD = \text{const}$$

i) When the gas is compressed in the direction perpendicular to the magnetic field, L=const. In that case, $\rho HD = \text{const}$, together with $BHD = \text{const}$ we get,

$$\frac{\rho HD}{BHD} = \frac{\rho}{B} = \text{const}$$

so the dependence of magnetic field strength B on the gas density ρ is

$$B = C\rho$$

ii) When the gas is compressed in the direction parallel to the magnetic field, L is not constant, H=const, D=const. From $BHD = \text{const}$, we get that

$$B = C$$

where C is a constant. There is no dependence of magnetic field strength B on the gas density ρ.