ASTR2013: Foundations of Astrophysics
Week 12
Turbulence and Magnetic Fields in Galaxies

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Introduction

Turbulence is ubiquitous in nature, from the coffee in a cup to astrophysical scales. Turbulence in the interstellar medium (ISM) of spiral galaxies is usually driven by supernova explosions (Fig. 1) via shocks waves. Multiple supernovae going off in different locations drives the turbulence in galaxies and this turbulence in turn amplifies magnetic fields. Below we calculate, the driving scale of turbulence (l_0) and the turbulent velocity (v_0). Then we look how this ISM turbulence can generate magnetic fields.

Supernova driven turbulence in spiral galaxies

The radius of the shock wave R (Fig. 2) as it explodes is given by (*Sedov-Taylor solution, derivation to be done in the tutorial*)

$$R \approx 2 \left(\frac{Et^2}{\rho_0} \right)^{1/5},$$

where E is the energy of the explosion, ρ_0 is the density of the medium and t is the time. We assume that the shock injects negligible energy when the velocity of the shock is equal to the local sound speed (the shock becomes part of the medium then). So, the maximum radius of the shock waves before it dissipates roughly equal to the driving scale of the turbulence. Thus, $l_0 \approx R$ when the velocity of shock front is equal to c_s, the sound speed of the medium. The sound speed of the medium

$$c_s = \sqrt{k_B T / m_p},$$

where $k_B = 1.38 \times 10^{-16}\,\mathrm{erg\,K^{-1}}$ in the Boltzmann constant, $m_p = 1.67 \times 10^{-24}\,\mathrm{g}$ and T is the temperature of the medium. For hot gas in the ISM ($T = 10^6\,\mathrm{K}$ and number density of particles $n_0 = 0.1\,\mathrm{cm^{-3}}$),

$$c_s = \sqrt{\frac{1.38 \times 10^{-16} \times 10^6}{1.67 \times 10^{-24}}} \approx 10^{0.5(-16+6+24)}\,\mathrm{cm/\,s} = 10^7\,\mathrm{cm/\,s} = 100\,\mathrm{km/\,s}.$$

To find the driving scale of turbulence l_0, $R = l_0$ at $t \approx R/c_s$,

$$l_0 \approx R \simeq 2 \left(\frac{E}{c_s^2 \rho_0} \right)^{1/3}.$$

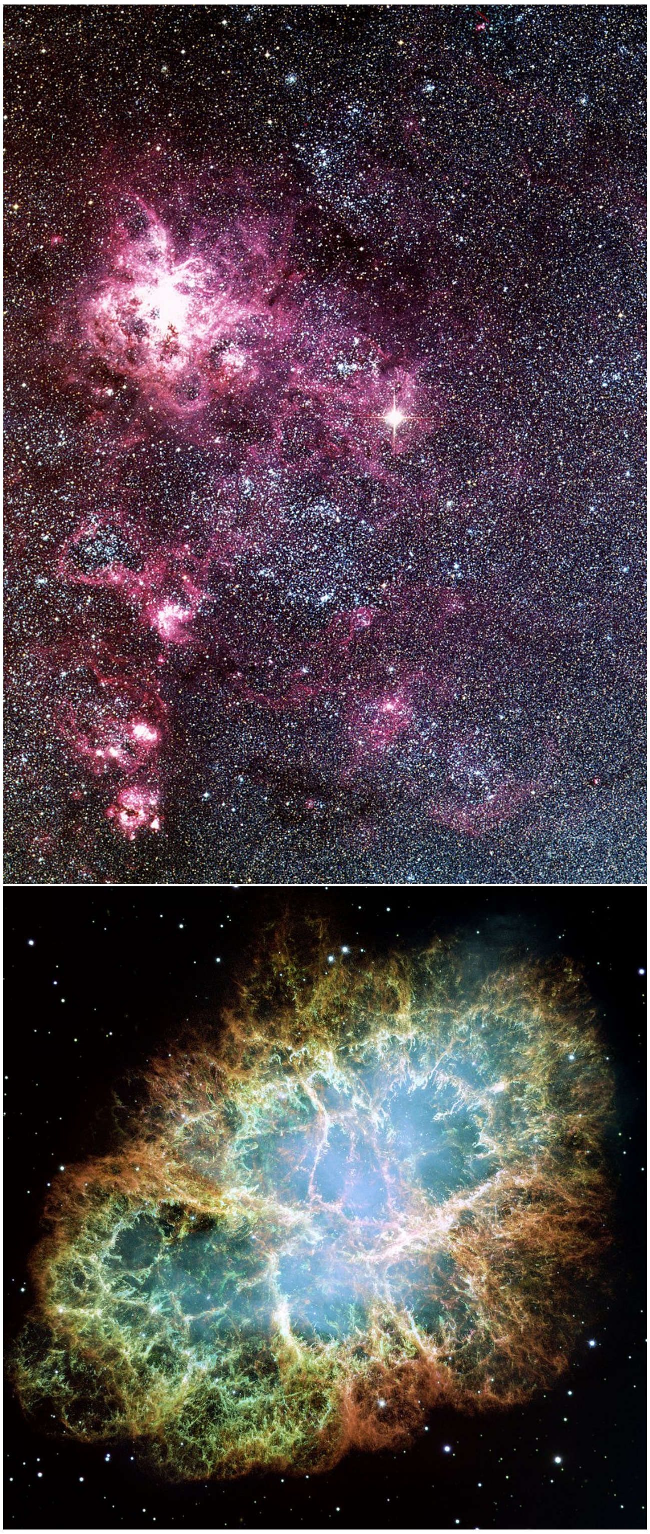

Figure 1: Top: SN1987A, exploding star. Bottom: Crab nebula, supernova remnant.

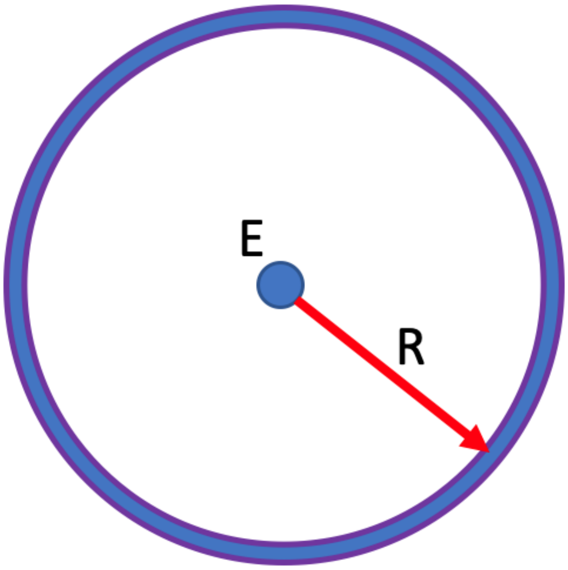

Figure 2: Spherically symmetric shockwave with radius R carrying energy E deposited at the centre by explosion.

For $E = 10^{51}$ erg, $\rho_0 = n_0 m_p$ and $c_s = 100\,\mathrm{km/s}$,

$$l_0 \simeq 2\left(\frac{10^{51}}{10^{14}(0.1 \times 1.67 \times 10^{-24})}\right)^{1/3} \mathrm{cm} \simeq 10^{\frac{51-14+25}{3}}\,\mathrm{cm} \simeq 10^{62/3}\,\mathrm{cm}$$
$$\simeq 10^{2/3} 10^{20}\,\mathrm{cm} \simeq 4 \times 10^{20}\,\mathrm{cm} \approx 100\,\mathrm{pc},$$

as $1\,\mathrm{pc} \approx 3 \times 10^{18}\,\mathrm{cm}$.

To calculate the turbulent velocity, we assume that a fraction of the total supernova energy is converted to the turbulent kinetic energy of the medium. The energy per unit mass of the turbulent medium is v_0^2, the rate of gain of total energy per unit mass is expressed as

$$v_0^2/(l_0/v_0).$$

This must come from supernova explosion, the rate of injection of kinetic energy per unit mass by supernova explosions is expressed as

$$\nu E/M_{\mathrm{gas}},$$

where ν is the frequency of supernova explosion and M_{gas} is the galaxy mass. Considering that the 10% of supernova explosion energy is deposited into the medium, we can balance both energy terms,

$$\frac{v_0^2}{l_0/v_0} = 0.1\frac{\nu E}{M_{\mathrm{gas}}}$$
$$v_0 = \left(\frac{0.1\nu E l_0}{M_{\mathrm{gas}}}\right)^{1/3}$$

For $E = 10^{51}$ erg, $\nu = (30\,\mathrm{yr})^{-1}$ and $M_{\mathrm{gas}} = 4 \times 10^9\,\mathrm{M}_\odot$, we get

$$v_0 = \left(\frac{0.1\nu E l_0}{M_{\mathrm{gas}}}\right)^{1/3} = \left(\frac{0.1 \times 10^{-9} \times 10^{51} \times 10^{20}}{4 \times 10^9 \times 2 \times 10^{33}}\right)^{1/3} \mathrm{cm/s} = \left(\frac{10^{-1-9+51+20}}{10^{1+9+33}}\right)^{1/3} \mathrm{cm/s}$$
$$= 10^{18/3}\,\mathrm{cm/s} = 10^6\,\mathrm{cm/s} = 10\,\mathrm{km/s}.$$

Thus, the turbulent velocity of the ISM of a typical spiral galaxy is on an average equal to $v_0 = 10\,\mathrm{km/s}$.

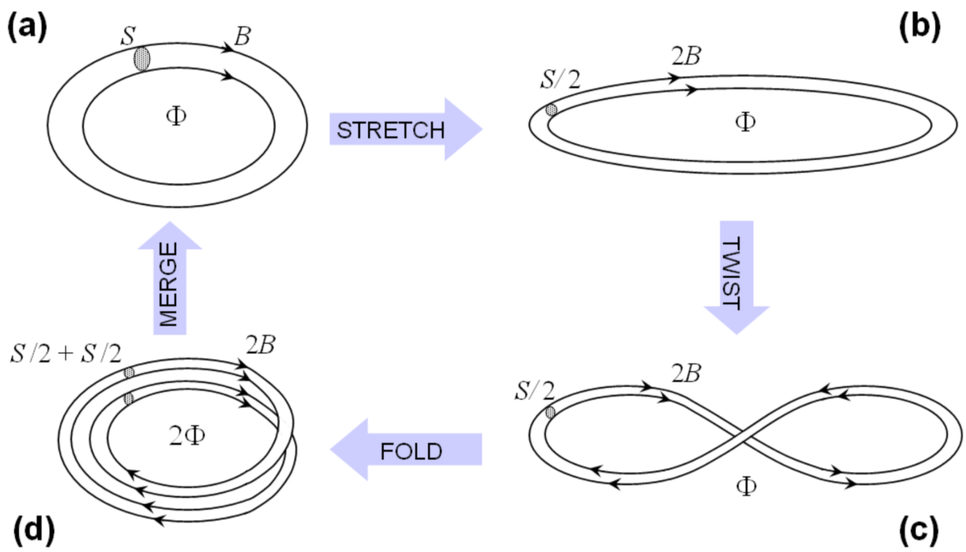

Figure 3: The stretch-twist-fold-merge mechanism (image credits: Anvar Shukurov). S and B are the cross section and magnetic field strength of the initial flux tube. $\phi = SB$ is the magnetic flux associated with the tube.

Magnetic field amplification

The ISM turbulence amplifies magnetic fields. The turbulent kinetic energy of the medium is converted to the magnetic field energy. This process is known as the dynamo theory.

The magnetic field amplification can be explained physically by a stretch-twist-fold-merge (STFM) mechanism (Fig. 3). This was first introduced by Yakov Borisovich Zeldovich. [1] Assuming flux freezing (product of magnetic field and area is conserved, $SB =$ constant, where B is the magnetic field strength and S is the area) and incompressible motions, the algorithm to amplify magnetic field via the STFM mechanism is as follows. First, the magnetic flux tube is stretched to double its length while preserving its volume, (a) $\rightarrow$ (b) in Fig. 3. This increases the magnetic field strength by a factor of two since the cross section is halved. Then the flux tube is twisted to form a figure eight, (b) $\rightarrow$ (c) in Fig. 3, and folded on itself, (c) $\rightarrow$ (d) in Fig. 3. Now, both loops of the tube have the magnetic field along the same direction and together occupy the same volume as the original flux tube. Both loops of the tube are now merged together into one, (d) $\rightarrow$ (a) in Fig. 3. The last step requires magnetic diffusion for process to become irreversible. The magnetic field is doubled for each cycle and increases by a factor of 2^n after n such steps. The growth rate is $\ln 2/T$, where T denotes the period of the STFM cycle. STFM cycles are due to the turbulent fluid motions in the ISM. The growth in magnetic energy is at the expense of the turbulent kinetic energy of the fluid motions. Once the magnetic field becomes strong enough, the Lorentz forces reacts back on the flow. In the STFM mechanism, this would imply difficulty in either the stretching and twisting due to magnetic tension or the merging of loops becoming slower (see reference (1) in the further reading section for details). Thus, then the magnetic field stops growing and saturates. The maximum magnetic field strength that can be achieved is the equipartition value, obtained by balancing the magnetic field energy with the turbulent kinetic energy.

$$\frac{B_{\rm eq}^2}{8\pi} = \frac{1}{2}\rho v_0^2$$

$$B_{\rm eq} = \sqrt{4\pi\rho v_0^2}.$$

[1] https://en.wikipedia.org/wiki/Yakov_Zeldovich

For, $v_0 = 10\,\mathrm{km/\,s}$ and $\rho = 0.1 \times 1.67 \times 10^{-24}\,\mathrm{g\,cm^{-3}}$,

$$B_{\mathrm{eq}} = \sqrt{4\pi \times 0.1 \times 1.67 \times 10^{-24} \times 10^{12}}\,\mathrm{G} \simeq 2 \times 10^{-6}\,\mathrm{G} = 2\,\mu\mathrm{G}.$$

This is very close to the observed value. The generated magnetic fields are at the scale of the driving scale ($l_0 \approx 100\,\mathrm{pc}$) due to the isotropic fluid turbulence. The large-scale magnetic fields in the galaxy also requires large scale properties of the galaxies (such as rotation, velocity shear and density stratification). See reference (2) in the further reading section for details.

Further reading

1. Saturation of Zeldovich Stretch-Twist-Fold Map Dynamos, Amit Seta, Pallavi Bhatt and Kandaswamy Subramanian, JPP (Zeldovich Special Issue), 2015, 81, 5
   Link: https://arxiv.org/pdf/1410.8455.pdf

2. Introduction to galactic dynamos, A. Shukurov, In:Mathematical Aspects of Natural Dynamos,eds E. Dormy & A. M. Soward,The Fluid Mechanics of Astrophysics and Geophysics,Vol. 13,Chapman & Hall/CRC, London, 2007, pp. 313–359
   Link: https://arxiv.org/pdf/astro-ph/0411739.pdf