Combined Reference Notes

Lesson 1: Expanding Space

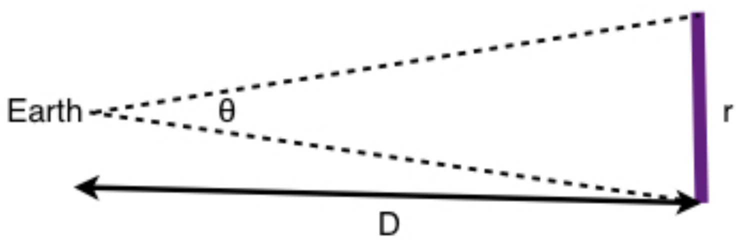
BACKGROUND

In each section of the course we will provide summary notes. These are not a replacement for the lesson, but should allow you to quickly go back and pick up key facts and equations from the lesson, without watching all the videos again.

NOTES

Distances and Sizes

One way to measure distances is if you know the size of an object. If you can then measure its apparent angular size, you can work out the distance:



Given the geometry above, and if $D \gg r$, (almost always the case in astronomy), then

$$r = \theta D$$

as long as θ is measured in radians (a radian is $\frac{180}{\pi}$ degrees).

Angles in astrophysics are often so small that even a degree is too large a unit to be convenient. We typically use arcminutes (one arcminute is 1/60 of a degree) and arcseconds (one arcsecond = 1/60 of an arcminute = 1/3600 of a degree).

Fluxes and luminosities

Luminosity L is the total amount of power put out by some object, and is measured in Watts. Flux f is the power we receive at our telescope, per unit collecting area, and is measured in Watts per square metre. They are related by the equation:

$$f = \frac{L}{4\pi D^2}$$

where D is the distance to the emitting object.

Spectra and the Doppler Effect

A spectrum is a graph of flux per unit wavelength plotted against wavelength. It will often show emission or absorption lines due to particular elements.

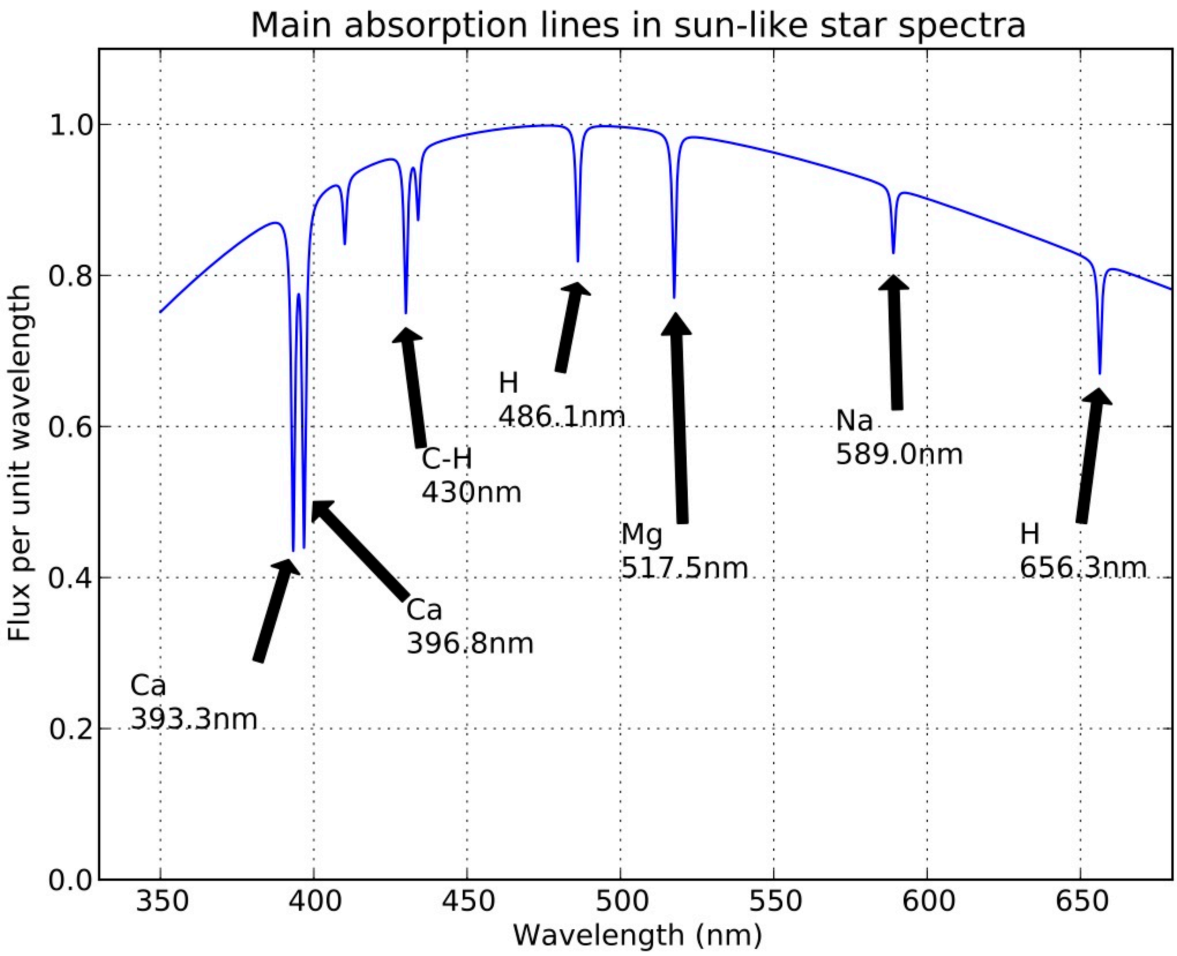
If an object is moving towards or away from you, these spectral lines will be moved in wavelength away from their normal wavelength λ_o. If you observe a line at wavelength λ, you can define a redshift z as:

$$z = \frac{\lambda - \lambda_0}{\lambda_0}$$

If this shift is due to the doppler effect, and the velocity $v \ll c$ (velocity much less than the speed of light - nearly always true), then:

$$z = \frac{v}{c}$$

To measure a redshift, you will need to know what lines to expect, and what their wavelengths are in the laboratory. The following graph shows you some of the typical lines you would see in a star or galaxy. Note that not all stars will show all these lines, and there are a variety of other lines that in certain stars can be strong. The C-H line is due to vibrations in the chemical bond linking carbon to hydrogen in molecules.



Hubble Law

Assuming that the brightest star in every galaxy had about the same luminosity (not a good assumption), Edwin Hubble calculated their relative distances. He found that the distances correlated with redshift. Everything was moving away from us and the speed correlated with how far things were from us.

The standard explanation is that space itself is expanding. Objects are not moving - they are just being carried apart by the expansion of space.

This means that unless more matter is created, the density of the universe must continuously go down (same amount of matter spread over more space). The alternative is that more matter is appearing out of nowhere - this is called the "steady state theory".

The "Steady State Theory" predicts that the universe should always look the same. We actually observe, however, that the universe was different in the past (we can see the past by looking a distant objects). Quasars were more common and the microwave background emission comes from a time when space was opaque.

Peculiar Motions

If galaxies did not have any mass, and space was expanding fast, then every galaxy would move away from us, and their redshift would be proportional to their distance. But in reality, galaxies have mass, and their gravity pulls them towards each other. These motions due to gravity attracting neighbouring galaxies are called peculiar motions (peculiar because they do not fit in with the Hubble Law). The observed motion of galaxies is the combination of the Hubble Law (due to the expansion of the universe) and the peculiar motions.

When galaxies are close together, their mutual gravity is strong, and the amount of space to expand between them is weak. In this case, gravity can overcome the expansion and suck the galaxies together. Our own Milky Way galaxy and the nearby Andromeda Galaxy (M31) are being sucked together by their mutual gravity and will ultimately collide. But when galaxies are further apart, their gravitational attraction is weaker and there is more space between them to expand: in this case the expansion will win.

Lesson 2: Origin of the Universe

SCALE FACTORS

The scale factor a(t) tells you the size of any length in the universe (of things not bound together) relative to its size today. So if for example, a(t) at some time t was 0.5, all lengths are half their present-day value.

Redshift z is given by:

$$z = \frac{1 - a(t)}{a(t)}$$

or alternatively,

$$a(t) = \frac{1}{1 + z}$$

DENSITY AND ENERGY EVOLUTION

The present-day mean energy of photons (the microwave background) is around 10^{-3} eV (an eV is an electron volt = 1.6×10^{-19} J, defined as the energy an electroc acquires when accelerated across a one volt potential difference).

At some point in the past, the energy would be:

$$E(t) = \frac{10^{-3}\mathrm{eV}}{a(t)}$$

Because photons outnumber particles by a factor of around a billion, in the early universe, it is the photons that control the energy - so on average the particles will have this same energy.

The present-day average density of the universe ρ_o is around $5 \times 10^{-28}\mathrm{kg\ m}^{-3}$. In the past (or future) it will be given by:

$$\rho = \rho_o(1 + z)^3$$

MICROWAVE BACKGROUND

At redshifts before around 1000, a fraction of photons had enough energy (13.6 eV) to knock the electrons off hydrogen atoms. Ionised hydrogen (hydrogen with its electrons knocked off) scatters light very strongly - so it would look like a glowing fog. Back then photons would be trapped bouncing between the atoms. Once the universe cooled down enough for the electrons to “recombine” with their nuclei, the universe became transparent and the photons have been flying freely in all directions ever since. This sea of photons is the observed microwave background.

PRIMORDIAL NUCLEOSYNTHESIS

Neutrons have a half-life of 887 seconds when not locked securely away inside a nucleus. So if there had not been nuclear fusion in the first few minutes of the universe, the neutrons would have gone away.

Nuclear fusion didn’t start earlier because most reactions start with deuterium, and in the first 90 seconds, photons have enough energy to blow deuterium apart (the “Deuterium Bottleneck”).

The primordial nuclear reactions produced a universe of 25% Helium and 75% Hydrogen, plus trace amounts of Lithium and Deuterium.

STRUCTURE FORMATION AND ANTIMATTER

The microwave background is extremely uniform - fluctuations of only one part in 100,000. Gravitational instability turned that very smooth universe into the lumpy one we see today. These tiny fluctuations probably came from quantum fluctuations when the universe was VERY young. These were somehow amplified up to the sizes of galaxy super-clusters. We don’t know why the fluctations are the size we observe.

We also don’t know why there is such a large imbalance between matter and antimatter in our universe.

Lesson 3: Dark Energy

BACKGROUND

In the 1990s, it was generally believed that the expansion rate of the universe was slowing down. The big question was how rapidly it was slowing down - rapidly enough to come to a halt and then start shrinking?

AGE OF UNIVERSE

If you assume that space is expanding at a uniform pace, you can estimate the age of the universe.

One way is to look at an object at distance D, with redshift z. Its motion away from us is cz (speed of light times the redshift). So if you extrapolate backwards, it must have been right here on Earth a time $t = \frac{D}{cz}$ ago.

An alternative way to think about it is that when light set out from an object at redshift z, the universe had a scale factor $a(t) = \frac{1}{1+z}$. The time t is just D/c. So if (for example) the scale factor 1 billion years ago was 0.9, then if the expansion was steady, the scale factor would have been 0 at a point in time ten billion years ago.

Some objects were discovered that seemed, according to this theory, to be older than the universe. A universe that started off expanding fast then decelerated would make this worse.

TYPE 1A SUPERNOVAE

This type of supernovae are probably caused when a white dwarf star becomes too massive and causes a run-away thermonuclear explosion.

It turns out that the peak luminosity correlates with how long they stay bright for. So if you measure the duration of staying bright, you can calculate the luminosity. If you measure the flux, you can use the inverse square law to work out the distance to the supernova, much more accurately than by previous methods.

ACCELERATION

By finding supernovae at a range of distances and measuring their redshifts, you can plot the expansion rate of the universe against time. This showed that the universe started off expanding slowly, and has been getting faster and faster.

Some form of long-range repulsive force must be responsible. One possibility is Dark Energy - a form of vacuum energy which has negative pressure.

Lesson 4: Quasars

WHAT IS A QUASAR?

It stands for ’Quasi Stellar Radio Source" - it means a something out in space which is emitting powerful radio waves, but which if you look at it with an optical telescope, looks like a star (i.e. a dot).

When you take a spectrum of one, you see broad emission lines, indicating gas that is swirling around at 10000 km/s, and often very high redshifts, indicating huge distances.

Because of the huge distances, the inferred luminosities of quasars are collossal. For things this far away to have the fluxes we observe, they must have luminosities of around 10^{40} W.

To calculate luminosities for things this far away, you have to allow for the redshifting of the photons as they travel. The inverse-square law relation between flux and luminosity is modified to become:

$$f = \frac{L}{4\pi D^2(1+z)^2}$$

Note that even this isn’t quite correct - you have to worry about what exactly is meant by distance D in an expanding and curved universe. But it will do for purposes of this course.

A quasar 40 Astronomical units away would vaporise the Earth's oceans in a fraction of a second.

To vaporise water (or anything else), you first need to bring it up to its boiling point. Energy needed to raise the temperature of something by ΔT K is $mc\Delta T$, where m is its mass and c its specific heat capacity ($4200 \mathrm{J\ kg^{-1} K^{-1}}$ for water). You then need to convert it from liquid to gas by supplying the latent heat of vaporisation ($2260 \mathrm{kJ\ kg^{-1}}$ for water).

Quasars can vary on timescales of hours to days. In general, if an object varies on a timescale t, it cannot be larger than ct (where c is the speed of light). So quasars are the size of a solar system or less. They also squirt out jets than can be millions of light-years long, which means they must be capable of staying active for a millions of years.

ENERGY SOURCE

Two possible energy sources were considered - nuclear fusion and gravity.

To get the most possible energy from nuclear fusion you would convert 56 Hydrogen atoms into 1 Iron atom. Each hydrogen atom has a mass of 1.00794 u (atomic mass units) while an iron atom has a mass of 55.845 u ($u = 1.6605 \times 10^{-27} \mathrm{kg}$). You will see that the 56 hydrogen nucei weigh about 1% more than the Iron nucleus. This 1% is converted into energy using the equation $E = mc^2$. About a solar mass of Hydrogen would need to be converted into Iron per day - which would rapidly produce a black hole.

To get the maximum possible energy from gravity, you would drop something from infinity down to the event horizon (the radius of no return at which you need to travel at the speed of light to escape). The event horizon radius for a non-rotating black hole is called the Schwarschild Radius and is

$$r_s = \frac{2GM}{c^2}$$

Gravitational potential energy is given by the equation

$$E = \frac{-GMm}{r}$$

Combine these and you find that the kinetic energy of a particle dropped from infinity as it reaches the event horizon is $\frac{1}{2}mc^2$, i.e. 50% of the rest mass can be converted to energy. This is much better than nuclear fusion can produce.

Most likely, infalling gas forms a disk, and friction within this disk causes it to glow, liberating energy.

THE PROBLEM

You can orbit a black hole just like anything else. So how can you make black holes so massive when the universe is so young, when most of the matter you need to feed it with is in perfectly safe orbits a long way away?

Lesson 5: First Light in the Universe

FORMATION OF THE ELEMENTS

The Big Bang produced 75% Hydrogen and 25% Helium. Other elements are made in stars. There is a complicated network of nuclear reactions, with a bottle-neck producing carbon from helium (the triple-alpha reaction which required very high densities to proceed).

Only massive stars produce the heaviest elements (like Iron) which are expelled in supernovae. Core collapse supernovae (when a star collapses due to running out of fuel) produce around 0.1 solar masses of Iron, while thermonuclear supernovae (the rapid nuclear fusion of a white dwarf star) produce around 0.6 solar masses of iron.

N-BODY CODES

These codes allow you to follow the motion of large numbers of particles moving under their mutual gravity (it is not in general possible to analytically solve for the motion of more than 3 bodies, so a numerical method like this is the only possibility). They work time-step by time-step. If your time-step is Δt, then if the starting x coordinate of a particle is x_0, and its starting x coordiate of velocity is v_{x0}, then its position and velocity at the end of the time-step are:

$$x_1 = x_0 + v_{x0}\Delta t$$

and

$$v_{x1} = v_{x0} + a_x \Delta t$$

,

where a_x is the x-component of the acceleration, which you calculate by summing the gravitational forces from all the other particles, using Newton's law of gravity

$$a = \frac{GM}{r^2}$$

Repeat these steps over and over again for each particle. Example code is available in the computer program part of this section.

QUASAR ABSORPTION

Hydrogen gas will absorb photons with an energy of 10.2 eV (enough energy to excite an electron from its ground state to the first excited energy level). Because of the expansion of space, photons emitted at higher energies (shorter wavelengths) are also absorbed, if there is neutral hydrogen around by the time they have been redshifted down to 10.2 eV.

This should produce a sudden cut-off in the spectra of all distant objects. Instead, a series of narrow absorption lines are seen (the Lyman-alpha forest) because most of the hydrogen has been ionised - only a few clouds are left neutral. This ionisation must have been done by hard ultra-violet photons, presumably from the first stars.

WHEN DID REIONISATION HAPPEN?

There are two conflicting clues. The most distant quasars known, those at redshifts above 6, show the absorption-cut-off expected if the universe still had neutral hydrogen everywhere.

On the other hand, the haze in the microwave background suggests that reionisation took place earlier - before redshift 10.

A new generation of telescopes are looking for the 21cm radio emission from this neutral gas.

WHAT CAUSED REIONISATION?

There don't seem to be enough galaxies or quasars at this redshift to do it. But maybe smaller galaxies or individual stars did it. The James Webb Space Telescope may be able to see these first stars.

An alternative approach is to look for low mass stars from this first generation that might be around. They would show a total lack of heavy element lines in their spectra. None have been seen, though some with very little heavy elements are being found.

Lesson 6: Gamma-Ray Bursts

INTRODUCTION

Gamma ray bursts are pulses of gamma-rays coming from out in space. They come from all directions, and are divided into two classes - the short-hard ones and the long-soft ones.

FLUX AND FLUENCE

Flux is energy per unit area per unit time. Fluence is just energy per unit area.

	Total	Per unit time
Physics Terminology	Energy	Power
What an astronomical object emits	Energy	Luminosity
What we detect on Earth	Fluence	Flux

You can describe the radiation put out by some astronomical object either by its energy or its luminosity (which is energy per unit time). For an object which puts out energy at a constant rate, its total energy output is just its luminosity multiplied by how long it lasts. More generally, the energy is the integral with respect to time of the luminosity, and the fluence is the integral with respect to time of the flux.

Luminosity (and flux) are typically used for objects which put out power for long periods, like stars and quasars. Energy (and fluence) are typically used for objects which put out a lot of power but only for a small time, like supernovae and gamma-ray bursts.

The inverse square law works for both.

POSSIBLE SOURCES

To begin with, nobody knew how far away these things were. Three models were investigated:

1) Colliding comets in the Oort cloud. If the kinetic energy of two comets was converted into gamma-rays (unlikely) you could just about produce the required fluences if the comets were large and relatively close in.

2) Neutron star rearrangement. If 1 part in 10000 of a neutron star moved in radius by 1 part in 10000, and all the change in potential energy was converted into gamma rays, then you could get the necessary fluence from anywhere in our galaxy.

3) If a star died, and half the mass crashed down on the centre, the released potential energy would be enough to give the required fluences right across the observable universe.

NUMBER COUNTS

If you distribute identical objects uniformly through space, the closer ones will show larger fluxes or fluences because of the inverse square law from brightness. $f \propto 1/r^2$, so the distance out to which you can see these things $r \propto f^{-1/2}$.

The number of these objects within range of your telescope will be proportional to the volume you survey, which is proportional to r^3. Thus the number seen brighter than f is $\propto f^{-3/2}$. The number of Gamma ray bursts rises slower than this, meaning that they cannot be uniformly spread at all distances.

COUNTERPARTS

New satellites gave accurate enough positions so that optical counterparts could be tracked down. This showed that the gamma-ray bursts were coming from cosmological distances, and hence had stupendous luminosities: up to $10^{48} J$.

These luminosities were too great to be explained in any sensible model, unless we believe that they are beaming the flux in our direction. If an object is moving at speed v, then its radiation is focussed into a cone of angle θ along the direction of motion, where

$$\theta = \sqrt{1 - \left(\frac{v}{c}\right)^2}$$

Because all the radiation is beamed into this small angle, the observed flux or fluence is enhanced by a factor of $\sim \frac{4}{\theta^2}$, where θ is measured in radians.

Long soft bursts seem to come from Type–2 supernova - when dying massive stars run out of fuel and collapse. Short-hard bursts are more mysterious but seem to come from merging compact objects.

Lesson 7: Dark Matter

MEASURING THE MASS OF A GALAXY

One way is by adding up the starlight. Use the inverse square law to work out the luminosity from the observed flux. Multiply this by a mass-to-light ratio (typically around 4 solar masses per solar luminosity) to estimate the mass in stars. The mass-to-light ratio differs for different types of star - a value of around 4 is an average of the sort of stars in our own galaxy (massive stars put out a lot of light per unit mass, while smaller stars have lower masses but their luminosities are lower still).

A second way is to find something orbiting around a galaxy and work out how strong the centripetal force needs to be to hold it in that orbit. Gravity must supply this centripetal force so you can work out how much mass needs to be present.

Centripetal force is given by

$$f = \frac{mv^2}{r}$$

Set this equal to the gravity…

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Thus if you know how far out the orbiting thing is, and how fast it is moving, you can work out the mass.

Note that this technique tells you (approximately) only the mass at radii less than r. By combining measurements at different radii you can estimate how the mass is distributed.

The gravitational technique finds much higher masses than counting starlight - so there must be something extra - dark matter. The discrepancy is greatest on larger scales - most of the dark matter is on the outskirts of galaxies.

OTHER EVIDENCE

Gravitational lensing shows similar masses. So does the x-ray emission from clusters of galaxies.

One rival theory (MOND - Modified Newtonian Dynamics) suggests that dark matter can be replaced by adding a small constant to Newton's gravitational equation. This would be too small a change to pick up on lab or solar-system scales but would explain the data on much larger scales. The bullet cluster is evidence against this.

WHAT IS DARK MATTER?

Mostly it cannot be made of normal matter (as that would be inconsistent with the formation of elements in the early universe).

One possibility is MACHOS (massive compact halo objects). This was mostly eliminated by searches for gravitational microlensing. The current lead contender is some strange, massive but hard-to-detect sub-atomic particle - a so-called WIMP (Weakly Interacting Massive Particle)

Lesson 8: Solar System Formation

SHRINKAGE

Solar systems come from the gravitational collapse of giant molecular clouds. The clouds have to shrink by a factor of around 10000, which is hard due to the random swirly motions of the gas in them.

ANGULAR MOMENTUM

Angular momentum l of an object moving around some centre is defined as $l = mvr$, where m is the mass of the object, r its distance from the centre and v the perpendicular component of its motion (i.e. the component of its velocity which is not towards or away from the centre but at right angles). Note that this equation is only an approximation valid for objects whose size is small compared to r.

Angular momentum is conserved (in the absence of an external torque), so if r decreases, v must increase, with

$$v \propto \frac{1}{r}$$

As we saw in the dark matter section, when an object moves in an orbit fast enough, gravity supplies only the necessary centripetal force, and the object will continue to move in a circular orbit. This occurs at a velocity

$$v = \sqrt{\frac{GM}{r}}$$

Set these two velocities equal and solve for the radius r at which the velocity will be enough to hold a circular orbit and you get

$$r = \frac{r_0^2 v_0^2}{GM}$$

where r_0 is the starting radius and v_0 is the starting speed.

This angular momentum means that giant molecular clouds can't collapse very much at all - they will produce spinning disks that are hundreds of times bigger than our solar system.

PROTOPLANETARY DISK

More realistic n-body simulations partially solve the angular momentum problem, as much of it goes into binary star orbits. You can get an appropriate sized disk - but nearly all the mass will be in the disk and not in the centre (the star).

Inner parts of the disk move faster than outer ones to balance gravity which causes viscosity (fluid friction). This will cause most of the matter to move inwards while angular momentum moves outwards. Unfortunately normal fluid viscosity is pitifully inadequate.

Most likely magnetic fields provide the excess viscosity by linking together different parts of the disk. But this can only work if the disk is ionised, so that particles will spiral around the magnetic field lines.

ASSEMBLY

Dust grains assemble by chemical bonding. Once they are sand or gravel sized, how they continue to stick is a mystery.

Metre-sized rocks should spiral into the star rapidly due to disk drag (the gas orbits a little slower than the rocks as a pressure gradient partially supports it).

Once rocks somehow get past these barriers, the collide with each other in in a chaotic and random way assembling the planets.

Lesson 9: Life in Space

DRAKE EQUATION

The number of intelligent species is given by

$$N_{\text{life}} = N_{\text{stars}} \times P \times H \times L \times I \times T$$

where:

P is the number of planets per star,
H is the fraction of planets that are habitable,
L is the fraction of habitable planets upon which life gets started
I is the probability of life evolving intelligence
T is the time over which an intelligent species hangs around.
We know that there are around 10^{23} stars in the observable universe. But life is really complicated - of all the possible molecular arrangements you could choose for DNA, only around 1 in 10^{400} results in viable life.

PLANET TEMPERATURE

For anything in space, its temperature is determined by the balance of heat coming in and heat going out.

Perfectly emitting objects (so called black bodies) put out radiation given by the Stefan Boltzman equation

$$L = A\sigma T^4$$

where L is the luminosity, A the surface area and T the temperature in K. The Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}\mathrm{W\ m^{-2}K^{-4}}$.

The energy absorbed by a planet is equal to the incident flux times the cross-sectional area (πr^2) for a sphere.

Set them equal to each other and you find an equilibrium temperature

$$T = \sqrt[4]{\frac{L}{16\pi\sigma D^2}}$$

where L is the luminosity of the star and D the distance between planet and star.

This is only a rough approximation - it predicts, for example that Venus should be habitable, which it isn’t.

THREE STRANGE ARGUMENTS

Three interesting, speculative and controversial arguments.

1) It is odd that the lifespan of our Sun (10 billion years) and the time taken to evolve intelligence (4 billion years) are so similar. The first depends on nuclear physics, the second on chemistry. You might expect them to be tens or orders of magnitude different. This coincidence could be explained if actually the mean time to evolve intelligence is much longer than the lifespan of most stars. In this case most planets would not have intelligent life, but those that did, would take a time comparable to the star lifespan to evolve.

2) If humanity expands across the galaxy, the vast majority of the people who will ever live will be on distant planets in the far future. We, stuck on Earth in the dawn of spaceflight, would thus be highly unusual. If, on the other hand, humanity destroyed itself in the near future, most of the humans that ever lived will be alive now, so we are very typical. If you believe we are more likely to be typical than unusual, then humanity will not last for long.

3) The likely time to spread through the galaxy is much shorter than the likely time to evolve intelligence. Thus whatever species evolved intelligence first would colonise the whole galaxy before the second species had its chance. So as aliens didn't arrive here millions of years ago, we must be first.