Australian National University

ASTR2013 – *Foundations of Astrophysics*

Week 3: Physics of Stars

Following Dan Maoz – Astrophysics in a Nutshell

Mike Ireland

Recap – The Virial theorem and Stellar Timescales

$$E_{\rm th}^{\rm tot} = -\frac{E_{\rm gr}}{2}$$

The Virial Theorem for Stars (written in 3 forms)

$$E_{\rm gr} = -2E_{\rm th}^{\rm tot}$$

$$E^{\rm tot} = -\frac{1}{2}E_{\rm gr}$$

- The *dynamical timescale* for any body of mass M and radius R is:

~1 hour

$$\tau_{\rm ff} \approx \tau_{\rm dyn} = \sqrt{\frac{R^3}{GM}}$$

$$\propto \bar{\rho}^{-1/2}$$

- This is approximately the *free-fall* timescale, 1/2π times the orbital period of a body almost scraping the surface, or the sound-crossing time.
- We can take the ratio of the *total internal energy* to the *stellar luminosity* to get the Kelvin-Helmholtz timescale:

~10^7 years

$$\tau_{\rm KH} \approx \frac{1}{2}\frac{GM^2}{R}\frac{1}{L}$$

Week 3 Summary

Textbook: Sections Finishing Chapter 3 (other than quantum mechanics in 3.9 and 3.10).

1. Main Sequence Scaling Relations
2. Equations of Stellar Structure.
3. Equation of State
4. Convective instability and convective energy transport.
5. Nuclear generation by the proton-proton chain and the CNO cycle.

Solar Parameters

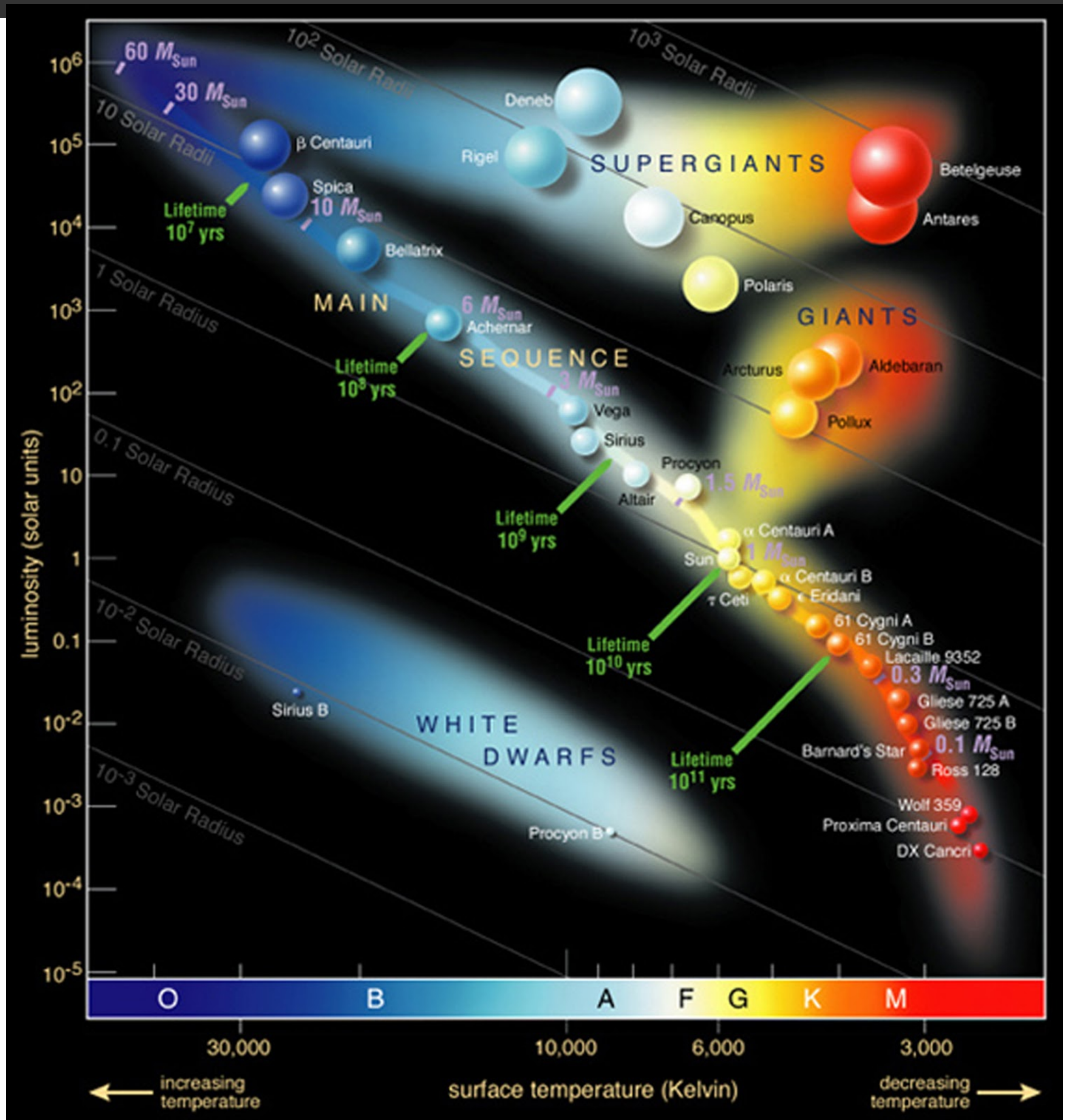
- The sun has an effective temperature of 5772K, and a radius of 6.957 x 10^8W, resulting in a Luminosity of 3.828 x 10^{26}W. Why not a different radius?

$$L = 4\pi R^2 \sigma_{\rm SB} T_{\rm eff}^4$$

- Radioactive dating of rocks in the solar system given an age of 4.6 Gyr, and the sun appears middle-aged compared to other 1 M_{sun} stars. This means its lifetime is 10Gyr. Why not a different age?

Scaling Relations

- Luminosity is roughly ~M^4 for intermediate mass stars, ~M^5 for low mass stars and ~M for high mass stars. Why?
- Life time goes as M3 for intermediate mass stars – assuming nuclear burning you can figure this out for yourself based on the above…
- Mass appears to relate to radius as roughly M~R. Again… why?



Stellar Structure Equations

- Conservation of mass, momentum and energy are the underlying principles, resulting in 4 differential equations as a function of radius coordinate *r* or in other text and codes, mass coordinate M_r.
- Mass Continuity:

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

- Hydrostatic Equilibrium:

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

Pressure in a little more detail

- Pressure actually as 3 components:
  - Ideal gas pressure: $$P_{\mathrm{gas}} = nk_BT = \frac{\rho k_B}{\mu u}T$$
  - Radiation pressure: $$P_{\mathrm{rad}} = \frac{u}{3} = \frac{4\sigma_{\mathrm{SB}}}{3c}T^4$$
  - Degeneracy pressure… important as the electron density approaches 1 electron per cube of the de Broglie wavelength: $$\lambda_e = \frac{h}{mv_e}$$

Energy Transport

- If radiation transports energy, then the derivative of the radiative energy density is proportional to luminosity.

$$\frac{du(r)}{dr} \propto \frac{dT(r)^4}{dr} \propto F(r) = \frac{L(r)}{4\pi R^2}$$

- If internal kinetic energy of gas transports energy perfectly, then the temperature gradient with pressure is the adiabatic gradient:

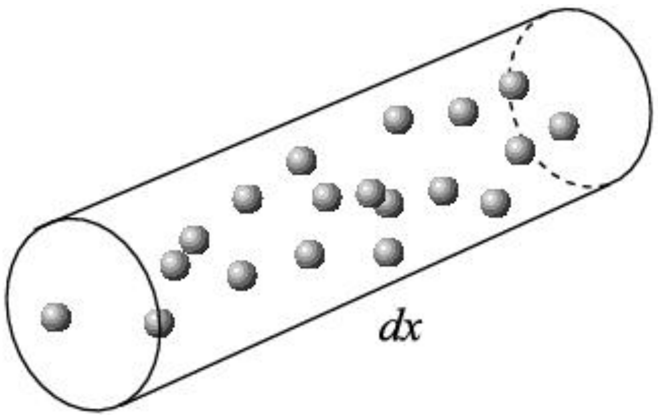
$$\frac{dT}{dr} = \frac{\gamma - 1}{\gamma} \frac{T}{P} \frac{dP}{dr} \qquad \text{from } PV^{\gamma} = \text{const}$$

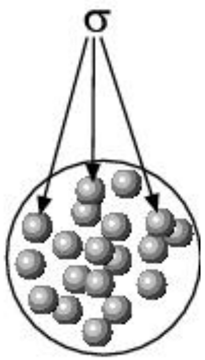
more on this later…

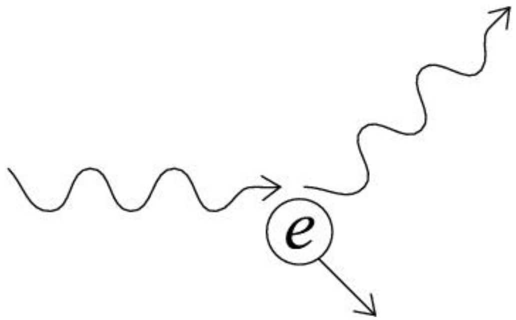
Opacity and Radiative Diffusion

- Electrons, atoms and ions absorb or scatter photons inside a star.
- Each particle has a cross-section $\sigma(\lambda)$ to absorbing radiation of wavelength λ.
- Given a particle number density *n*, we can define a mean-free path:

$$l = \frac{1}{n\sigma}$$







Opacity and Radiative Diffusion

- A mean free path is inversely proportional to density, so we instead typically talk about *opacity:*

$$\kappa = \frac{\Sigma n_i \sigma_i}{\rho}$$

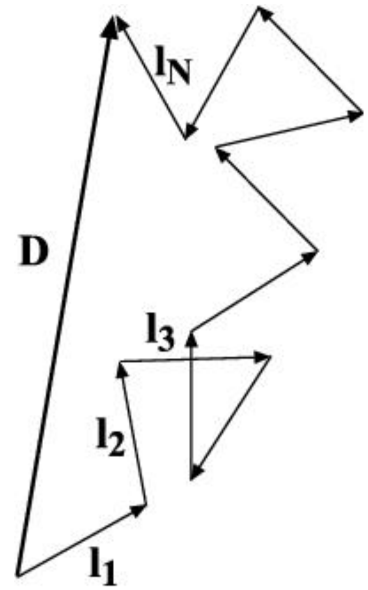
- For mostly ionized gas, Thompson scattering provides a lower limit to opacity:

$$\sigma_\mathrm{T} = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = 6.7 \times 10^{-25}\ \mathrm{cm}^2.$$

- *Free-Free*, and *bound-free* opacities are proportional to density. *Bound-bound* opacities matter much less in stellar interiors.

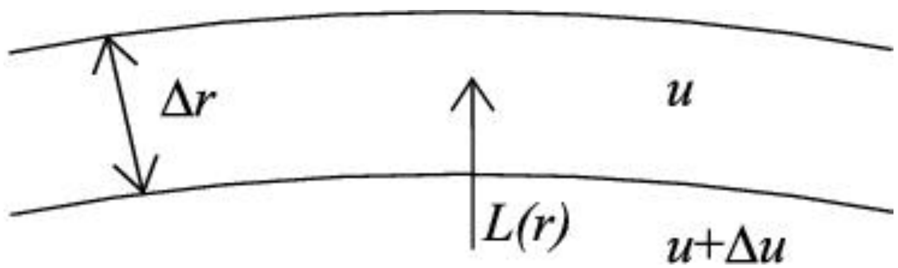
Opacity and Radiative Diffusion

- Photons diffuse through the star, with an average mean free path of 2cm.
- Considering neighboring shells with radiative energy density *u*, changing with *r*, we can write:



$$\frac{L(r)}{4\pi r^2} = -\frac{cl}{3}\frac{du}{dr}$$

- In a more rigorous derivation, the factor of 3 is called the *Eddington Approximation*, and is accurate deep in the star.



Australian National University

Opacity and Radiative Diffusion

- The energy density derivative is:

$$\frac{du}{dr} = \frac{du}{dT}\frac{dT}{dr} = \frac{4\sigma_{\mathrm{SB}}}{c}T^3\frac{dT}{dr}$$

- We can write the mean free path in terms of the *Rosseland Mean Opacity* as:

$$l = (\kappa_R \rho)^{-1}$$

- This gives the radiative energy flow equation:

$$\frac{dT(r)}{dr} = -\frac{3L(r)\kappa_R(\rho, T)\rho(r)}{64\pi r^2 \sigma_{\mathrm{SB}} T^3(r)}$$

(textbook uses *a* instead of σ_{SB})

Australian National University

Energy Generation

- On the *nuclear timescale*, we can approximate a star as being in steady state, with constant energy generation from nuclear reactions.
- Energy generation is conventionally parameterized by ε, which is the power generated per unit mass.
- Energy conservation gives:

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho(r) \epsilon(\rho, T)$$

Equations of Stellar Structure

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}, \tag{3.56}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r), \tag{3.57}$$

$$\frac{dT(r)}{dr} = -\frac{3L(r)\kappa(r)\rho(r)}{4\pi r^2 4acT(r)^3}, \tag{3.58}$$

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r). \tag{3.59}$$

Non-relativistic equations here only !

The opacity, nuclear generation rate and pressure are really a complex function of density and temperature.

Equations of Stellar Structure

- Pressure as a function of density and temperature is and *equation of state*, e.g:

$$P_{\rm gas} = nk_BT = \frac{\rho k_B}{\mu u}T$$

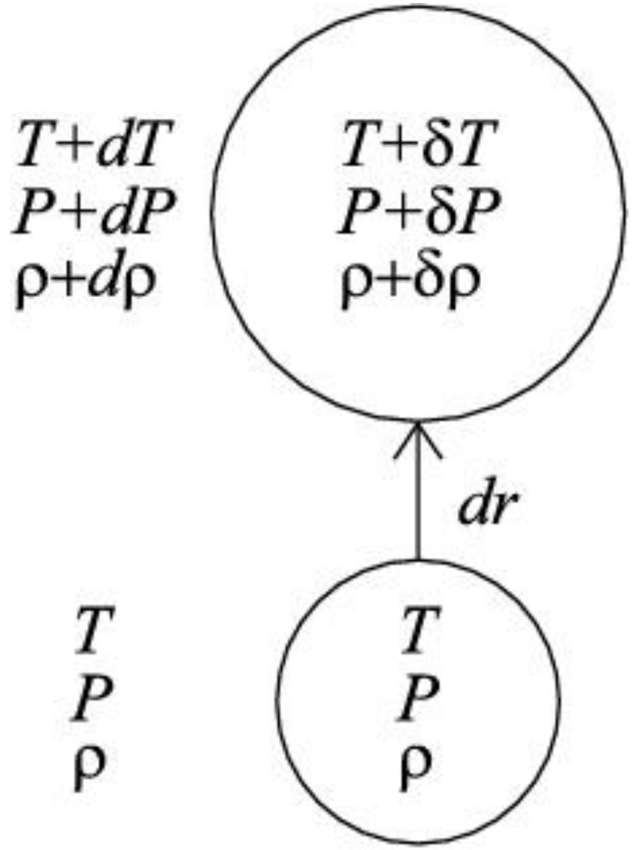
- [noting that $\mu = \mu(\rho, T)$ in general]
- We also need boundary conditions including the obvious ones and something like:

$$L(r_*) = 4\pi R^2 \sigma_{\rm SB} T^4 (r \text{ near surface})$$

- Here "near surface" is often 2/3 of a photon mean-free path from r_*, but needs a multi-wavelength photospheric model to be accurate.

Convective Instability

- A parcel of gas that moves upwards without exchanging energy with its surroundings is *expanding adiabatically*.
- The gradient dP/dT, or the gradient of logarithms for this parcel, is called the *adiabatic gradient*.
- If the parcel expands and cools more than the surrounding gas, it becomes buoyant and accelerates upwards, i.e. it is *unstable*.



Convective Instability

- The adiabatic gradient is therefore the steepest possible temperature gradient in a star. If γ is too small, or κ too large (or both) the star has a convective region.

γ=5/3 for a monatomic gas
γ~1.1 when hydrogen is 50% ionized

Maximum luminosity carried by convection for a gas with energy density u_{gas} = is of order:

$$L_{\mathrm{conv.}} < 4\pi r^2 u_{\mathrm{gas}} v_s$$

$$PV^\gamma = \mathrm{const.}$$

$$P \propto \rho^\gamma$$

$$\frac{d\log(P)}{d\log(\rho)} = \gamma$$

$$\nabla_{\mathrm{ad}} = \frac{d\log(T)}{d\log(P)} = \frac{\gamma - 1}{\gamma}$$

Nuclear Generation

- Nuclear energy generation in the main sequence comes from two key reaction chains. The proton-proton chain:

$$p + p \rightarrow d \ (1.4 \text{ MeV})$$

$$p + d \rightarrow \ {}^3_2\text{He} \ (5.5 \text{ MeV})$$

$${}^3_2\text{He} + \ {}^3_2\text{He} \rightarrow \ {}^4_2\text{He} + 2p \ (12.9\text{MeV})$$

- … and the CNO cycle:

$$p + {}^{12}\text{C} \rightarrow {}^{13}\text{N} + \gamma$$

$${}^{13}\text{N} \rightarrow {}^{13}\text{C} + e^{+} + \nu_e,$$

$$p + {}^{13}\text{C} \rightarrow {}^{14}\text{N} + \gamma,$$

$$p + {}^{14}\text{N} \rightarrow {}^{15}\text{O} + \gamma,$$

$${}^{15}\text{O} \rightarrow {}^{15}\text{N} + e^{+} + \nu_e,$$

$$p + {}^{15}\text{N} \rightarrow {}^{12}\text{C} + {}^{4}\text{He}.$$

More on this with numbers in the labs!

Week 3 Summary

Textbook: Sections Finishing Chapter 3 (other than quantum mechanics in 3.9 and 3.10).

1. Main Sequence Scaling Relations
2. Equations of Stellar Structure.
3. Equation of State
4. Convective instability and convective energy transport.
5. Nuclear generation by the proton-proton chain and the CNO cycle.