ASTR2013 Problem Set 1 - due **2 Aug 2019**

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1. Flux density can be given in terms of frequency: $f_\nu = df/d\nu$, or wavelength $f_\lambda = df/d\lambda$. Derive a formula to convert between these two quantities.

2. The flux density of a star f_λ is measured at two different wavelengths, λ_1 and λ_2. This can be used to measure the temperature of the star. In this question, assume the radiation from the star is described by blackbody emission.

   (a) For $\lambda_1 = 440\,\text{nm}$ and $\lambda_2 = 550\,\text{nm}$ (approximately corresponding to B and V filters in the Johnson system), create in python a labelled log-log plot of this temperature with respect to the measured flux ratio, with the temperature varying between 3,000 and 30,000 K.

   (b) In the Wien limit, show that the temperature is given by:

   $$T = \frac{hc}{k_B}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)\frac{1}{\ln(f_2/f_1) + 5\ln(\lambda_2/\lambda_1)} \tag{1}$$

   Plot this approximate relationship on your plot from (a).

   (c) Assume that the measurements of flux density in (a) are made on a single pixel of a CMOS or CCD camera with a well depth of 11,000 electrons. This means that you can only measure up to 11,000 photo-electrons before the detector saturates. Assume that exposure times are selected so that $\sim$10,000 photo-electrons are collected at each wavelength. To what precision are the two flux densities measured? To what precision is their ratio measured? How accurate could a temperature be based on this flux ratio at a temperature of 4,000 K or 20,000 K?

   [Hint - when multiplying or dividing independent quantities with percentage errors p_1 and p_2, the uncertainties are added in quadrature to get the total uncertainty $\sqrt{p_1^2 + p_2^2}$]

3. A binary star system of two identical stars orbiting their common centre of mass observed with the Gaia satellite has an parallax of 19 mas (milli-arcsecond), a flux density in Gaia's B_p filter at $\lambda = 532\,\text{nm}$ of $5.9\times10^{-13}\,\text{erg/s/cm}^2/\text{Å}$ and a flux density in Gaia's R_p filter at $\lambda = 797\,\text{nm}$ of $4.8\times10^{-13}\,\text{erg/s/cm}^2/\text{Å}$. Adaptive optics imaging observations at a large telescope resolve the two stars at many epochs, giving an orbital semi-major axis of 76 mas and a period of 6.0 years.

   (a) What is the temperature of the each star, assuming blackbody emission? [Hint: use the result from 1a, or the formula from 1b if you didn't complete 1a]

   (b) What is the distance to the stars? Express your answer in pc and km.

   (c) What is the radius of the each star in solar radii? [Hint: you can use the inverse square law and the blackbody flux relation at the surface of the star]

   (d) What is the luminosity of each star in units of solar luminosity? See where the stars land on a HR diagram. What kind of stars are they?