ASTR2013 ass5

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Problem 1

a

The flux of observation should be bigger than telescope flux limit F.

$$F_{obs} = \frac{L}{4\pi r^2} \geq F$$

Therefore we could get

$$r \leq \sqrt{\frac{L}{4\pi F}}$$

So the maximum distance at which the galaxy could be detected.

$$r_{max} = \sqrt{\frac{L}{4\pi F}}$$

b

The Schechter function tells us the number of galaxies per unit L and volume.

$$\phi(L)dL \approx \phi\left(L_*\right)\left(\frac{L}{L_*}\right)^{-1} \exp\left(-\frac{L}{L_*}\right) dL$$

With a telescope flux limit F and luminosity of galaxy L, the maximum volume that we could detect is

$$V = \frac{4}{3}\pi r_{max}^3 = \frac{4}{3}\pi(\frac{L}{4\pi F})^{\frac{3}{2}} = \frac{1}{3\sqrt{4\pi}}(\frac{L}{F})^{\frac{3}{2}}$$

So the number of galaxies between L and L+dL that can be detected is

$$g(L)dL = \Phi(L)dL \cdot V = \frac{L_*}{3\sqrt{4\pi}F^{\frac{3}{2}}}\phi\left(L_*\right) L^{\frac{1}{2}} \exp\left(-\frac{L}{L_*}\right) dL$$

c

The relative(non-normalised) probability distribution function is given in 1.b. The most likely galaxy luminosity that is detected is the luminosity at which the probability distribution function get its maximum.

$$\frac{dg}{dL} = \frac{L_*}{3\sqrt{4\pi}F^{\frac{3}{2}}}\phi\left(L_*\right)\left(\frac{1}{2L^{\frac{1}{2}}} - \frac{L^{\frac{1}{2}}}{L_*}\right)\exp\left(-\frac{L}{L_*}\right) = 0$$

So we get

$$\frac{1}{2L^{\frac{1}{2}}} - \frac{L^{\frac{1}{2}}}{L_*} = 0$$

and $L = \frac{L_*}{2}$

Problem 2

For a single luminosity L and flux limit f_0, the maximum volume that we could detected is given in problem 1

$$V(L) = \frac{1}{3\sqrt{4\pi}}\left(\frac{L}{f_0}\right)^{\frac{3}{2}}$$

If spatial constant luminosity function is $\Phi(L)$, then the number of galaxy of luminosity from L to L+dL we could detect is $V(L)\Phi(L)dL$. The number of objects observed to have a flux greater than f_0 is the integral over all possible luminosity:

$$N(f > f_0) = \int_0^\infty V(L)\Phi(L)dL$$

a

If all objects have luminosity L, spatially constant luminosity function takes the form:

$$\Phi(L') = n_0 \cdot \delta(L' - L)$$

The number of object observed with flux greater than f_0 is that

$$N(f > f_0) = \int_0^\infty V(L')\Phi(L')dL' = \frac{n_0}{3\sqrt{4\pi}}\left(\frac{L}{f_0}\right)^{\frac{3}{2}}$$

So, $N(f > f_0) \propto f_0^{-\frac{3}{2}}$

b

Given general spatially constant luminosity function

$$\Phi(L) = n_0 \cdot f(L)$$

, where f(L) is an arbitrary function of L.

For object with the luminosity from L to L+dL, the number of object observed with flux greater than f_0 is that

$$N(f > f_0) = \int_0^{\infty} V(L)\Phi(L)dL = \frac{n_0}{3\sqrt{4\pi}} \int_0^{+\infty} L^{\frac{3}{2}} f(L)dL \cdot f_0^{-\frac{3}{2}}$$

So, $N(> f_0) \propto f_0^{-\frac{3}{2}}$

c

The relation between flux and magnitude is: $f_0 \propto e^{-\frac{2}{5}m_0}$

From relation of number and flux above we could get:

$$N(f > f_0) = N(m < m_0) \propto e^{\frac{3m_0}{5}}$$

So the number of objects between magnitude m_1 and m_2 is:

$$N(m_1 \sim m_2) \propto (e^{\frac{3m_2}{5}} - e^{\frac{3m_1}{5}})$$

Therefore we could get the ratio of the number of objects between 15 and 16 and between 16 and 17:

$$\frac{N_{15\sim16}}{N_{16\sim17}} = \frac{e^{\frac{48}{5}} - e^{9}}{e^{\frac{51}{5}} - e^{\frac{48}{5}}} \approx 0.55$$

Since the number of galaxies observed per square degree is N between magnitude of 15 and 16, we know that the number between magnitude of 16 to 17 is: So $N_{16\sim17} = 1.82$ N.

d

This means at large distance r, the volume element is less than $4\pi r^2 dr$, indicating a different geometry of the universe other than Euclidean geometry of flat space time.

Problem 3

a

The luminosity induced by accretion is given in lecture note:

$$L \approx \frac{1}{2}\frac{GM\dot{M}}{r_{\text{in}}} = \frac{\dot{M}c^2}{12}$$

Eddington luminosity could be derived by equating gravitational force to force caused by radiation pressure

$$\frac{GMm_p}{r^2} = \frac{L_E \sigma_T}{4\pi r^2 c}$$

$$L_E = \frac{4\pi c G M m_p}{\sigma_T}$$

where σ_T is the cross section of Thompson scattering.

Eddington-limited accretion states that the luminosity due to accretion reach the Eddington luminosity

$$L = L_E$$

From the formula above we could derive the differential equation, relating black hole mass to its accretion rate.

$$\frac{\dot{M}c^2}{12} = \frac{4\pi cGMm_p}{\sigma_T}$$

b

We solve the differential equation in the following step

$$\frac{dM}{M} = \frac{48\pi Gm_p}{c\sigma_T}dt$$

$$lnM = \frac{48\pi Gm_p}{c\sigma_T}t + lnM_0$$

Finally we get

$$M = M_0 exp(\frac{48\pi Gm_p}{c\sigma_T}t)$$

Initial condition $M(t = 0) = 10M_{sun}$ tells us that $M_0 = 10M_{sun}$

After 100 million years, the mass of black hole will be

$$M = 143.49M_{sun}$$