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First Semester 2019
Mathematical Sciences Institute
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Assignment 3

This assignment must be submitted by **3rd May 5pm**. Late Submissions will incur a 5% penalty per working day. Assignment submissions will close on the **10th May 5pm**. Submissions after this time will be invalid.

Question 1 (Fourier transform) [20P]

1. Let

$$A = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad S = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}, \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

a)[5 pts] Show that A is a circulant matrix by showing $AS = SA$. Then find a vector a such that $Ax = a^*x$ for all $x \in \mathbb{C}^4$, i.e., such that the matrix vector product Ax is equal to the convolution a^*x.

b) [5pts] Note that F_4 is the discrete Fourier transform matrix in $\mathbb{C}^{4,4}$. Show that

$$SF_4 = F_4 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & i \end{bmatrix},$$

What are the eigenvalues of S?

c)[5pts] Use the results from above (and the convolution theorem) to compute the Fourier transform of the solution x of $Ax = b$. If there is more than one solution give the Fourier transform of the set of all solutions.

d) [5pts] Let

$$B_4 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & i \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -i \end{bmatrix}, \quad I_2 \otimes F_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix},$$

Compute the matrix-matrix product $M = B_4(I_2 \otimes F_2)$ and find the permutation matrix P_4 for which $F_4 = MP_4$.

Remarks

- Do all computations by hand. (You can use Python to check, but you need to know how todo the computations by hand.)
- It is ok to use any formulas from the the lecture notes, wikipedia etc but in this case you have to indicate in your solution why they hold.

Tutorial for Question 1

- Go through all assignment questions and point out what is required
- Revise definitions and theorems for circulant matrices and convolutions
- Consider in particular examples for $n = 2$ and/or $n = 3$
- Review the formula
$$F_n = B_n \begin{bmatrix} F_2 & \\ & F_2 \end{bmatrix},$$
the application to FFTs and an example where $n = 6$ and $m = 3$.