ASTK 2013 Pro Set 2.

1. From $\frac{d\log P}{d\log \rho} = \gamma$

$$d\log P = \gamma\, d\log\rho$$

By integral we get $\log P = \gamma \log\rho + \log K$, where K is a constant

Hence: $\log P = \log(K\rho^{\gamma})$

$$P = K\rho^{\gamma}$$

2. For a fully convective star, $\frac{dT}{dr}$ is determined by adiabatic index γ and not radiation:

$$\frac{dT}{dr} = \frac{\gamma-1}{\gamma}\frac{T}{P}\frac{dP}{dr} \qquad (2.1)$$

We also have equations for stellar structures:

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2} \qquad (2.2)$$

$$\frac{dM}{dr} = 4\pi r^2\rho \qquad (2.3)$$

We define the dimensionless ~~unit~~ quantity $M' = M/M_{\odot}$ $r' = r/R_{\odot}$ (2.4)

Applying (2.4) to (2.3) we get

$$\frac{dM'}{dr'} = 4\pi r'^2 \rho \frac{R_{\odot}^3}{M_{\odot}} \qquad (2.5)$$

We can rewrite ~~eq.~~ (2.1) as:

$$\frac{1}{T}\frac{dT}{dr} = \frac{\gamma-1}{\gamma}\frac{1}{P}\frac{dP}{dr} \Rightarrow \frac{d\ln T}{dr} = (1-\frac{1}{\gamma})\frac{d\ln P}{dr}$$

$$\frac{d\ln T}{dr'} = (1-\frac{1}{\gamma})\frac{d\ln P}{dr'} = (1-\frac{1}{\gamma})\frac{\gamma\, d\ln\rho}{dr'} = (\gamma-1)\frac{d\ln\rho}{dr'}$$

So $\frac{d\ln T}{dr'} = (\gamma-1)\frac{d\ln\rho}{dr'}$ (2.6)

Finally $\frac{d\ln\rho}{dr'} = \frac{1}{\gamma}\frac{d\ln P}{dr'} = \frac{1}{\gamma P}\frac{dP}{dr'} = -\frac{1}{\gamma P}\frac{GM'\rho}{r'^2}\frac{M_{\odot}}{R_{\odot}}$

$$\frac{d\ln\rho}{dr'} = -\frac{1}{\gamma}\frac{GM'\rho}{P\cdot r'^2}\frac{M_{\odot}}{R_{\odot}} \qquad (2.7)$$

(2.5) (2.6) (2.7) together forms alternative equations for stellar structure based on the three derivatives: $\frac{dM'}{dr'}$, $\frac{d\ln\rho}{dr'}$, $\frac{d\ln T}{dr'}$.

3. Plots are in the Appendix

The two models are significantly different in the near-surface layers. The greatest cause of the difference is that H and He are not fully ionised in the near-surface layers since its temperature is much lower than the center. Hence the simplified EOS is not valid in that region.

4. Convection stops when:

$$\left|\left(\frac{dT}{dr}\right)_{rad}\right| < \frac{\gamma-1}{\gamma}\frac{T}{P}\left|\frac{dP}{dr}\right|$$

Eq of stellar structure tells us:

$$\left(\frac{dT}{dr}\right)_{rad} = -\frac{3L\kappa\rho}{64\pi r^2 \sigma_{SB} T^3}$$

We define $\left|\left(\frac{dT}{dr}\right)_{convective}\right| = \frac{\gamma-1}{\gamma}\frac{T}{P}\left|\frac{dP}{dr}\right|$ and compare it with $\left|\left(\frac{dT}{dr}\right)_{rad}\right|$ in the same plot.

We find that convection stops at $\frac{r}{r_\odot} \approx 1.827$. At this radius, the temperature is $T \approx 4223K$. The detailed code of Q4 is in the Appendix also.

5. By calculation, the temperature and luminosity of ~~sta~~ star are $T=4223K$ and $L=0.96L_\odot$. In HR diagram, this is a dwarf star in the main-sequence. The mass of the star is roughly $0.75M_\odot$. Its subsequent evolution over the Gyr are as follows:

After the main sequence star has burnt out of its hydrogen in the core. The core ~~contrast~~ contracts and inner temperature rises.

causing hydrogen begins to burn surrounding the core. This process increase the luminosity and lower the temperature, which is called the red giant phase. As the red-giant phase progresses, Helium Ignition begins and the star moves quickly to the horizontal branch's left side and evolve slowly to the right. Once He burning is complete, the core contracts until supported by degeneracy pressure, the star ascends the asymptotic giant branch. In this phase ~~Hydro~~ hydrogen burning resumes with occasional ~~substanc~~ substantive He burning called thermal pulse. The resulting high luminosity drives the surface layer off, leaving behind an inert C,N,O core called a white dwarf.

Appendix starts in next page!

~~Append~~