ASTR2013_ass1

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1 Q1

From chain rule we get:

$$f_\nu = \frac{df}{d\nu} = \frac{df}{d\lambda}\frac{d\lambda}{d\nu} = -\frac{d\lambda}{d\nu} f_\lambda$$

where we use the minus sign for correcting for making f_λ positive.
As

$$\lambda = \frac{c}{\nu} \quad \frac{d\lambda}{d\nu} = -\frac{c}{\nu^2}$$

We get the conversion formula:

$$f_\nu = \frac{c}{\nu^2} f_\lambda = \frac{\lambda^2}{c} f_\lambda$$

2 Q2

(a)

```
In [1]: import astropy.constants as c, astropy.units as u
        import numpy as np, matplotlib.pyplot as plt
```

```
In [2]: lambd_1 = 440*u.nm
        lambd_2 = 550*u.nm
        T = np.logspace(np.log10(3000),np.log10(30000),100)*u.K
```

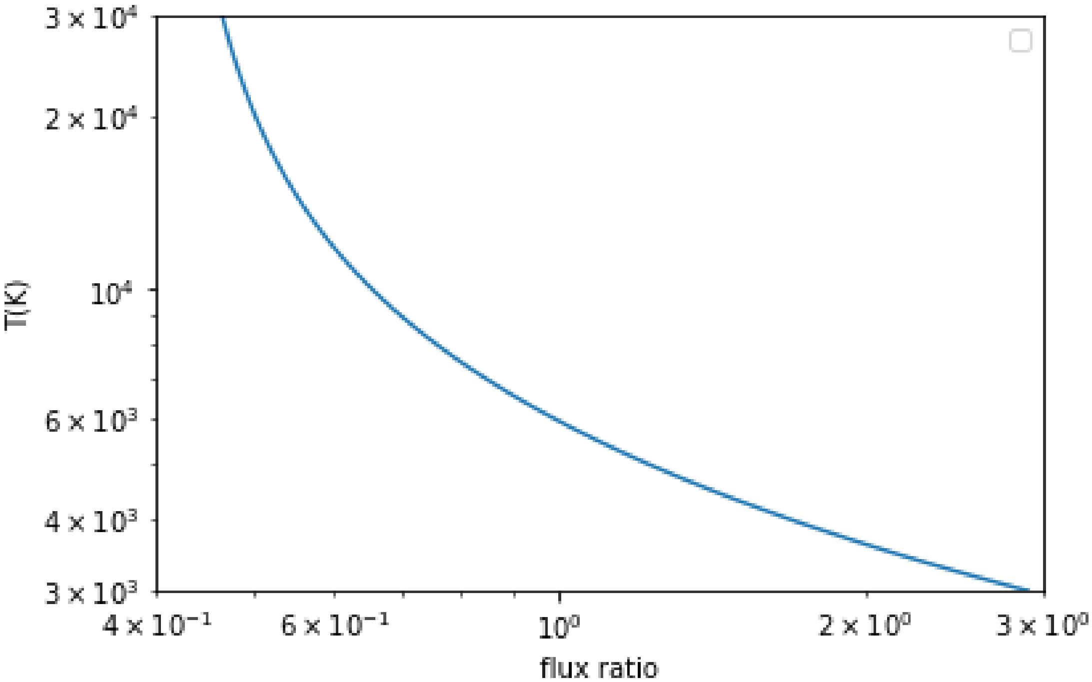
```
In [3]: F = lambda lambd, T: 2*c.h*c.c**2/lambd**5/(np.exp(c.h*c.c/lambd/c.k_B/T)-1)
        F_1 = F(lambd_1, T)
        F_2 = F(lambd_2, T)
```

Log-log plot of temperature vs measured flux ratio with temperature varying between 3000 and 30000K.

```
In [4]: plt.loglog(F_2/F_1, T)
        plt.xlabel('flux ratio');plt.ylabel('T(K)');plt.legend()
        plt.axis([4e-1, 3e0,3000,30000])
```

```
No handles with labels found to put in legend.
```

```
Out[4]: [0.4, 3.0, 3000, 30000]
```



(b)

In Wien's limit, flux ratio writes:

$$f_\lambda = \frac{2hc^2}{\lambda^5} e^{-\frac{hc}{\lambda kT}}$$

For two wavelength, we get:

$$f_1 = \frac{2hc^2}{\lambda_1^5} e^{-\frac{hc}{\lambda_1 kT}}$$

$$f_2 = \frac{2hc^2}{\lambda_2^5} e^{-\frac{hc}{\lambda_2 kT}}$$

Hence the flux ratio writes:

$$\frac{f_2}{f_1} = \frac{\lambda_1^5}{\lambda_2^5} e^{\frac{hc}{kT}(\frac{1}{\lambda_1} - \frac{1}{\lambda_2})}$$

After a few algebra, we can get the temperature is given by:

$$T = \frac{hc}{k_B} \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right) \frac{1}{\ln (f_2/f_1) + 5 \ln (\lambda_2/\lambda_1)}$$

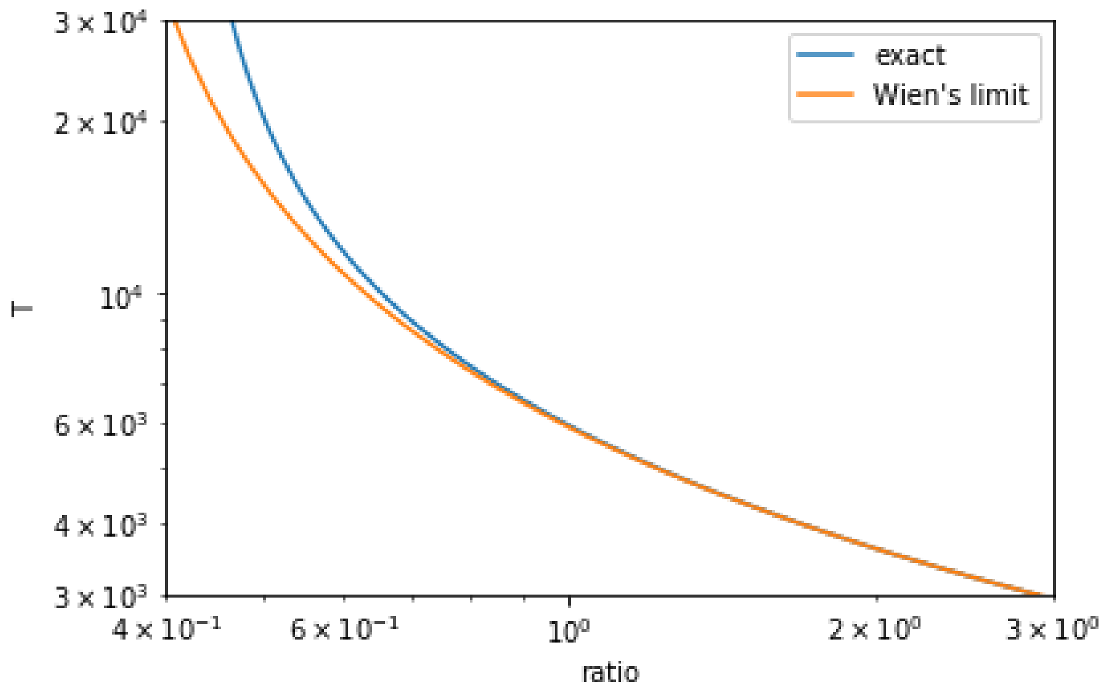
Plot of approximation formula and exact formula

```
In [5]: T_wien = lambda lambd_1, lambd_2, f_r: (c.h*c.c/c.k_B*(1/lambd_1-1/lambd_2)
                                        /(np.log(f_r)+5*np.log(lambd_2/lambd_1))).to(u.K
```

```
In [6]: f_r = np.logspace(-1,1,100)
        T_w = T_wien(lambd_1,lambd_2, f_r)
        #R(lambd_1, lambd_2, T)
```

```
In [7]: plt.loglog(F_2/F_1, T,label="exact")
        plt.loglog(f_r, T_w ,label="Wien's limit")
        plt.xlabel('ratio');plt.ylabel('T');plt.legend()
        plt.axis([4e-1, 3e0, 3000,30000])
```

```
Out[7]: [0.4, 3.0, 3000, 30000]
```



we can see that when temperature is high or the flux ratio is low, wien's limit is not very good.

(c)

From Poisson statistics, the uncertainty of measurement is
$\sqrt{N} \approx \sqrt{10000} = 100$

Then precision of measurement $\approx 100/10000 = 0.01$

precision of flux ratio is approximately $\sqrt{0.01^2 + 0.01^2} \approx 0.01414$

we use the following code to illustate the accuracy of temperature measurement using flux ratio.

At low temperature, we use the wien approimation as it gives a good approximation to the exact case.

At high temperature, we use intepolation module to calculate the temperature.

```
In [8]: from scipy import interpolate
        err=np.sqrt(0.01**2+0.01**2)

        T_ex = interpolate.interp1d(F_2/F_1,T)

        T_1 = 4000*u.K
```

```
T_2 = 20000*u.K
r_1 = F(lambd_2, T_1)/F(lambd_1, T_1)
r_2 = F(lambd_2, T_2)/F(lambd_1, T_2)

T_4000_lower = T_wien(lambd_1, lambd_2, r_1*(1+err))
T_4000_higher = T_wien(lambd_1, lambd_2, r_1*(1-err))

T_20000_lower = T_ex(r_2*(1+err))*u.K
T_20000_higher = T_ex(r_2*(1-err))*u.K

print('Temperature measurement of 4000K fall in range between {:.2f} to {:.2f}:'
      .format(T_4000_lower,T_4000_higher))
print('Temperature measurement of 20000K fall in range between {:.2f} to {:.2f}:'
      .format(T_20000_lower,T_20000_higher))
```

```
Temperature measurement of 4000K fall in range between 3963.14 K to 4032.26 K:
Temperature measurement of 20000K fall in range between 18896.47 K to 21285.47 K:
```

We can see that the precision of measurement of 4000K is about 30/4000=0.75%, whereas for 20000K, the precision is about 1200/20000=6%. The measurement of 4000K is more precise.

3 Q3

(a)

We use interpolation to calculate the temperature of star

```
In [9]: lambd_1 = 532*u.nm
        lambd_2 = 797*u.nm
        T = np.logspace(np.log10(3000),np.log10(10000),100)*u.K
        F_1 = F(lambd_1, T)
        F_2 = F(lambd_2, T)
        T_ex = interpolate.interp1d(F_2/F_1,T)

        f_1 = 5.9e-13*u.erg/u.s/u.cm**2/u.Angstrom
        f_2 = 4.8e-13*u.erg/u.s/u.cm**2/u.Angstrom
        T_star = T_ex(f_2/f_1)*u.K
        print('The temperature of star is:{:.2f}'.format(T_star))
```

```
The temperature of star is:5019.73 K
```

(b)

```
In [10]: D = 1/19e-3*u.pc
         print('The distance to the star is:{:.2f}'.format(D))
         print('The distance to the star is:{:.2f}'.format(D.to(u.km)))
```

```
The distance to the star is:52.63 pc
The distance to the star is:1624040832351153.50 km
```

(c)
Using inverse square law:

$$f_{earth} = \frac{f_\lambda R_{sun}^2}{D^2}$$

we get:

$$R_{sun} = D\sqrt{\frac{f_{earth}}{f_\lambda}}$$

We perform this calculation for two filters.

```
In [11]: #(c)
         lambd = 532*u.nm
         F_earth = 5.9e-13*u.erg/u.s/u.cm**2/u.Angstrom
         ratio = F_earth/F(lambd, T_star)
         R_star = D*np.sqrt(ratio)
         print('The radius of the star is: {:.2f}'.format(R_star.to(u.R_sun)))
```

```
The radius of the star is: 1.58 solRad
```

```
In [12]: lambd = 797*u.nm
         F_earth = 4.8e-13*u.erg/u.s/u.cm**2/u.Angstrom
         ratio = F_earth/F(lambd, T_star)
         R_star = D*np.sqrt(ratio)
         print('The radius of the star is: {:.2f}'.format(R_star.to(u.R_sun)))
```

```
The radius of the star is: 1.58 solRad
```

We can see that they give approximately the same answer.
(d)
We calculate Luminosity using Stefan-Boltzmann law:

```
In [13]: L = c.sigma_sb*T_star**4*4*np.pi*R_star**2
         print('The Luminosity of the star is: {:.2f}'.format(L.to(u.L_sun)))
```

```
The Luminosity of the star is: 1.43 solLum
```

From the temperature and luminosity of the star, we see that it lies in the main sequence in the HR-diagram. They are main sequence star.