1: 知. 线特征残差. $d_\varepsilon = \frac{|(\tilde{P}_i - P_b) \times (\tilde{P}_i - P_a)|}{|P_a - P_b|}$

求 Jacobi.

设 $Y = \frac{(\tilde{P}_i - P_b) \times (\tilde{P}_i - P_a)}{|P_a - P_b|}$,

那么: $\frac{\partial d_\varepsilon}{\partial \tilde{P}_i} = \frac{\partial |Y|}{\partial \tilde{P}_i} = \frac{\partial |Y|}{\partial Y} \cdot \frac{\partial Y}{\partial \tilde{P}_i} = \frac{Y}{|Y|} \cdot \frac{\partial Y}{\partial \tilde{P}_i}$ (参考面特征推导)

$= \frac{Y}{|Y|} \cdot \frac{\partial Y}{\partial \tilde{P}_i} = \frac{Y}{|Y|} \cdot \frac{(P_a - P_b)^\wedge}{|P_a - P_b|}$ ... ①

[参考线特征推导. slides 中的 $d_\varepsilon = Y$, 而作业中 $d_\varepsilon = |Y|$]

$\frac{\partial \tilde{P}_i}{\partial T} = \frac{\partial (T \cdot P_i)}{\partial T}$. 设 ΔT 为 T 的左扰动, $\Delta T = \exp(\delta \xi^\wedge)$,

[视觉 SLAM 14 讲 4.3.5 节]

$= \frac{\partial (T \cdot P_i)}{\partial \delta \xi}$ ..... $= \begin{bmatrix} I & -(RP_i + t)^\wedge \\ 0^T & 0^T \end{bmatrix}$ ... ②

$J_\varepsilon = \frac{\partial d_\varepsilon}{\partial T} = \frac{\partial d_\varepsilon}{\partial \tilde{P}_i} \cdot \frac{\partial \tilde{P}_i}{\partial T}$, 代入公式①, ②得

$J_\varepsilon = \frac{Y}{|Y|} \cdot \frac{(P_a - P_b)^\wedge}{|P_a - P_b|} \cdot \begin{bmatrix} I & -(RP_i + t)^\wedge \\ 0^T & 0^T \end{bmatrix}$

$= \frac{Y}{|Y|} \cdot \frac{(P_a - P_b)^\wedge}{|(P_a - P_b)|} \cdot \begin{bmatrix} I & -(RP_i + t)^\wedge \end{bmatrix}$.

- 为什么去掉 0^T. ⇒ 为了维度一致
- 1: $J_\varepsilon = \frac{Y}{|Y|} \cdot \frac{(P_a - P_b)^\wedge}{|(P_a - P_b)|} \cdot \begin{bmatrix} I & -(RP_i + t)^\wedge \end{bmatrix}$

$= [3 \times 1] \cdot [3 \times 3] \cdot [3 \times 6] = [3 \times 6]$

如果加上 $[0^T, 0^T]$ 就不对了

2..

2. 知面特征 残差 $d_{ke} = \left| (\tilde{P}_i - P_j) \cdot \frac{(P_e - P_j) \times (P_m - P_j)}{|(P_e - P_j) \times (P_m - P_j)|} \right|$.

求 Jacobi

设 $X = (\tilde{P}_i - P_j) \cdot \frac{(P_e - P_j) \times (P_m - P_j)}{|(P_e - P_j) \times (P_m - P_j)|}$

那么 $\frac{\partial d_{ke}}{\partial \tilde{P}_i} = \frac{X}{|X|} \cdot \frac{\partial X}{\partial \tilde{P}_i} = \frac{X}{|X|} \cdot \frac{(P_e - P_j) \times (P_m - P_j)}{|(P_e - P_j) \times (P_m - P_j)|}$ ... ① [保件]

与线特征类似，参考《视觉SLAM 14讲》4.3.5. 可知

$$\frac{\partial \tilde{P}_i}{\partial T} = \begin{bmatrix} I & -(RP_i + t)^\wedge \\ 0^T & 0^T \end{bmatrix} \quad \cdots\cdots ②$$

$$J_{ke} = \frac{\partial d_{ke}}{\partial T} = \frac{\partial d_{ke}}{\partial \tilde{P}_i} \cdot \frac{\partial \tilde{P}_i}{\partial T}$$

$$= \frac{X}{|X|} \cdot \frac{(P_e - P_j) \times (P_m - P_j)}{|(P_e - P_j) \times (P_m - P_j)|} \cdot \begin{bmatrix} I & -(RP_i + t)^\wedge \end{bmatrix}$$