1 Stress, Strain

$\sigma = \frac{F}{A}$

$\epsilon = \frac{\Delta l}{L}$

$\sigma = E\epsilon$

$\frac{F}{A} = \frac{E \cdot \Delta l}{L}, F = \frac{AE}{L} \cdot \Delta l = k\Delta l$

$k = \frac{AE}{L}$

$W = \frac{1}{2}k\Delta x^2 = \frac{1}{2}F\Delta x = \frac{1}{2}\sigma A\epsilon L = \frac{\sigma^2 V_0}{2E} = \frac{\sigma\epsilon V_0}{2}$

$U = \int \sigma d\epsilon$

$W = U \cdot V_0 \rightarrow U = \frac{W}{V_0}$

$\epsilon_{th} = \alpha\Delta T$

2 Equilibrium

$\Sigma F = 0,\ \Sigma M = 0,\ if\ a = 0,\ where\ F = ma,\ and\ M = Fd$

3 Bridges

$T_{supY} = \frac{WL}{2}$

$T_{supX} = \frac{WL^2}{8h}$

$T_{supMax} = \sqrt{(\frac{WL}{2})^2 + (\frac{WL^2}{8h})^2},$

Given loads placed equidistantly over the span

4 Rotation

4.1 Motion

$I_m = my^2,\ for\ a\ single\ point\ mass$

$I_m = \int_M y^2 dm = \rho \int_A y^2 dA$

$I = \int_A y^2 dA$

$I = \frac{bh^3}{12},\ for\ a\ rectangle\ (comes\ from\ integrating)$

$F = ma = my$

$M = F_x y = my^2 \cdot \alpha = I_m \alpha$

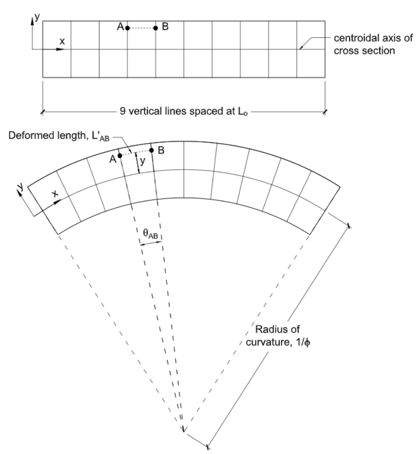
$\omega = \frac{d^2\theta}{dt^2}$

$\theta = \omega_i t + \frac{1}{2}\alpha t^2$

$a = \alpha y$

$M = Fy = m\alpha y^2 = I_m \alpha$

4.2 Bending



$\phi = \frac{d\theta}{dx}$

$L'_{AB} = \phi L_0 \cdot (y + \frac{1}{\phi}) = \phi y L_0 + L_0$

$\epsilon(y) = \frac{\Delta l}{L_0} = \frac{L'_{AB} - L_0}{L_0} = \phi y,\ \epsilon\ at\ the\ centroidal\ axis\ is\ 0$

$\sigma = E\epsilon \rightarrow \sigma(y) = E\phi y$

$\sigma = F/A \rightarrow \Delta F = \sigma(y)\Delta A$

$M = Fd \rightarrow \Delta M = \Delta F y = \phi E y^2 \Delta A$

$M = \int_A \phi E y^2 dA$

$= \phi EI$

5 Harmonic Motion

$Pendulum: \ T = 2\pi\sqrt{\frac{l}{g}}$

$Mass\ Spring: \ 2\pi\sqrt{\frac{m}{k}}$

$f = \frac{1}{T}$

$F = -kx$

$\omega = \frac{2\pi}{T}$

$a = -\omega^2 x$

$x(t) = A \cdot \sin(\omega t + \phi)$

$v = \omega A \cdot \cos(\omega t + \phi)$

$v = \pm\omega\sqrt{x_0^2 - x^2}$

$\frac{d^2x(t)}{dt^2} = -A\omega^2 \sin(\omega_n t + \phi)$

$E_k = \frac{1}{2}m\omega^2(x_0^2 - x^2)$

$E_t = \frac{1}{2}m\omega^2 x^2$

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \rightarrow v = \sqrt{x\frac{k}{m}}$

Gravity

$m\frac{d^2x(t)}{dt^2} + kx(t) = mg$

$x(t) = A\sin(\omega_n t + \phi) + \Delta_0$

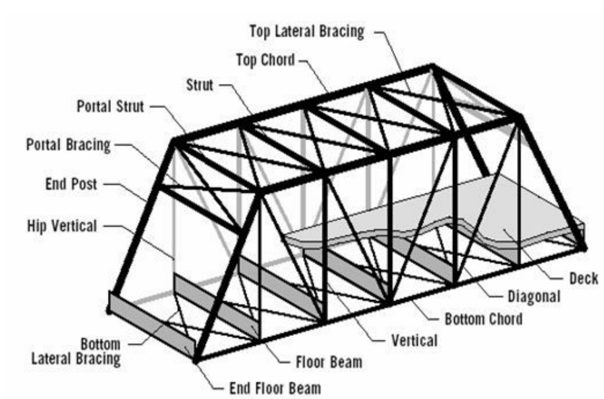
$k = \frac{mg}{\Delta_0}$

$f_n = \frac{1}{2\pi}\sqrt{\frac{mg}{\Delta_0} \cdot \frac{1}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta_0}}$

6 Staticaly Determined Structures

Name	**Permited Degrees of Freedom**	**Restrained Degrees of Freedom**	**Reactions**
Roller	$\Delta(x \oplus y),\ \theta$	$\Delta y = 0$	F_y
Pin	θ	$\Delta x = \Delta y = 0$	$F_x,\ F_y$
Fixed End	N/A	$\Delta x = \Delta y = \Delta\theta = 0$	$F_x,\ F_y,\ M_{xy}$

7 Truss Brigde



1. Select Geometry
2. Determine Loads
   - Gravity
   - Wind
3. Analyze forces in members

4. Design components
5. Determine stiffeness
6. Check dynamic forces
7. Iterate

$P_{central\ joint} = W \cdot \frac{s \cdot W_D}{2}$

$P_{end\ joint} = W \cdot \frac{s \cdot W_D}{4}$

8 Buckling

$\phi = \frac{d\theta}{dx}$

$P \cdot y = P \cdot \Delta_{lat} = M$

$M = EI\phi$

$y(x) = A\sin(\omega x + B)$

$\frac{dy}{dx} = A\omega\cos(\omega x + B)$

$\frac{d^2y}{dx^2} = -A\omega^2\sin(\omega x + B)$

$n\pi = L \cdot \sqrt{\frac{P}{EI}},\ n \epsilon \mathbb{N}$

$P = \frac{n^2\pi^2 EI}{L^2}$

$P_{crit} = \frac{\pi^2 EI}{L^2} = P_E$