

1 Stress, Strain

$$\sigma = \frac{F}{A}$$

$$\epsilon = \frac{\Delta l}{L}$$

$$\sigma = E\epsilon$$

$$\frac{F}{A} = \frac{E \cdot \Delta l}{L}, F = \frac{AE}{L} \cdot \Delta l = k\Delta l$$

$$k = \frac{AE}{L}$$

$$W = \frac{1}{2}k\Delta x^2 = \frac{1}{2}F\Delta x = \frac{1}{2}\sigma A\epsilon L = \frac{\sigma^2 V_0}{2E} = \frac{\sigma\epsilon V_0}{2}$$

$$U = \int \sigma d\epsilon$$

$$W = U \cdot V_0 \rightarrow U = \frac{W}{V_0}$$

$$\epsilon_{th} = \alpha\Delta T$$

$$F_{failure} = A\sigma_y$$

$$F_{allowable} = \frac{A\sigma_y}{FOS} \geq F_{demand}$$

$$A \geq \frac{FOS \cdot F_{demand}}{\sigma_y}$$

2 Equilibrium

$$\Sigma F = 0, \Sigma M = 0, \text{ if } a = 0, \text{ where } F = ma, \text{ and } M = Fd$$

3 Bridges

$$T_{supY} = \frac{WL}{2}$$

$$T_{supX} = \frac{WL^2}{8h}$$

$$T_{supMax} = \sqrt{\left(\frac{WL}{2}\right)^2 + \left(\frac{WL^2}{8h}\right)^2},$$

Given loads placed equidistantly over the span

4 Rotation

4.1 Motion

$I_m = my^2$, for a single point mass

$$I_m = \int_M y^2 dm = \rho \int_A y^2 dA$$

$$I = \int_A y^2 dA$$

$I = \frac{bh^3}{12}$, for a rectangle (comes from integrating)

$$F = ma = m\ddot{y}$$

$$M = F_x y = my^2 \cdot \alpha = I_m \alpha$$

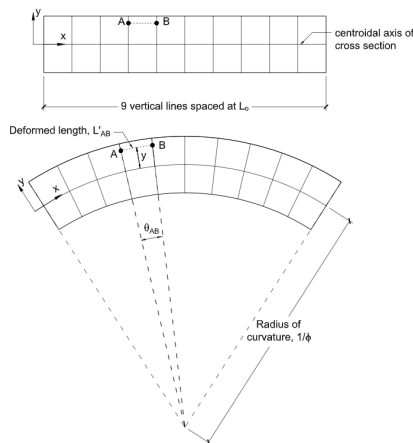
$$\omega = \frac{d^2 \theta}{dt^2}$$

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2$$

$$a = \alpha y$$

$$M = Fy = m\alpha y^2 = I_m \alpha$$

4.2 Bending



$$\phi = \frac{d\theta}{dx}$$

$$L'_{AB} = \phi L_0 \cdot \left(y + \frac{1}{\phi}\right) = \phi y L_0 + L_0$$

$$\epsilon(y) = \frac{\Delta l}{L_0} = \frac{L'_{AB} - L_0}{L_0} = \phi y, \epsilon \text{ at the centroidal axis is } 0$$

$$\sigma = E\epsilon \rightarrow \sigma(y) = E\phi y$$

$$\sigma = F/A \rightarrow \Delta F = \sigma(y)\Delta A$$

$$M = Fd \rightarrow \Delta M = \Delta F y = \phi E y^2 \Delta A$$

$$M = \int_A \phi E y^2 dA$$

$$= \phi EI$$

5 Harmonic Motion

$$\text{Pendulum : } T = 2\pi\sqrt{\frac{l}{g}}$$

$$\text{Mass Spring : } 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{T}$$

$$F = -kx$$

$$\omega = \frac{2\pi}{T}$$

$$a = -\omega^2 x$$

$$x(t) = A \cdot \sin(\omega t + \phi)$$

$$v = \omega A \cdot \cos(\omega t + \phi)$$

$$v = \pm \omega \sqrt{x_0^2 - x^2}$$

$$\frac{d^2 x(t)}{dt^2} = -A\omega^2 \sin(\omega_n t + \phi)$$

$$E_k = \frac{1}{2}m\omega^2(x_0^2 - x^2)$$

$$E_t = \frac{1}{2}m\omega^2 x^2$$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \rightarrow v = \sqrt{x \frac{k}{m}}$$

Gravity

$$m \frac{d^2 x(t)}{dt^2} + kx(t) = mg$$

$$x(t) = A \sin(\omega_n t + \phi) + \Delta_0$$

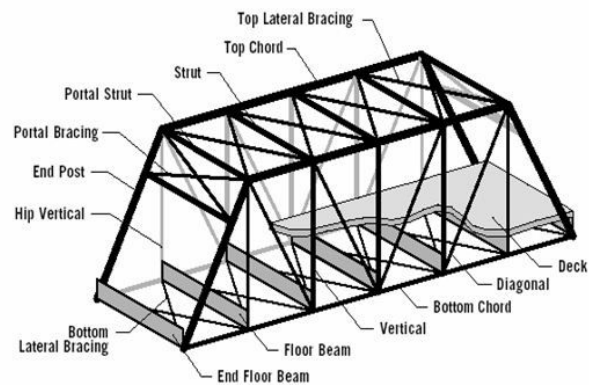
$$k = \frac{mg}{\Delta_0}$$

$$f_n = \frac{1}{2\pi} \sqrt{\frac{mg}{\Delta_0} \cdot \frac{1}{m}} = \frac{1}{2\pi} \sqrt{\frac{g}{\Delta_0}}$$

6 Statically Determined Structures

Name	Permitted Degrees of Freedom	Restrained Degrees of Freedom	Reactions
Roller	$\Delta(x \oplus y), \theta$	$\Delta y = 0$	F_y
Pin	θ	$\Delta x = \Delta y = 0$	F_x, F_y
Fixed End	N/A	$\Delta x = \Delta y = \Delta \theta = 0$	F_x, F_y, M_{xy}

7 Truss Bridge



1. Select Geometry
2. Determine Loads
 - Gravity
 - Wind
3. Analyze forces in members

4. Design components
5. Determine stiffness
6. Check dynamic forces
7. Iterate

$$P_{central\ joint} = W \cdot \frac{s \cdot W_D}{2}$$

$$P_{end\ joint} = W \cdot \frac{s \cdot W_D}{4}$$

Deflection:

$$W_{ext} = \sum_{i=1}^m \int F_i d\Delta_i \text{ } m \text{ Forces}$$

$$W_{int} = \sum_{i=1}^n \int P_i d\Delta_i \text{ } n \text{ members}$$

$$W_{ext} = W_{int}$$

$$F^* = \text{virtual force}$$

$$\Delta_l = \frac{PL}{AE}$$

$$F^* \Delta = \sum P^* \Delta_l = \sum P^* \frac{PL}{AE}$$

8 Buckling

$$\phi = \frac{d\theta}{dx}$$

$$P \cdot y = P \cdot \Delta_{lat} = M$$

$$M = EI\phi$$

$$y(x) = A \sin(\omega x + B)$$

$$\frac{dy}{dx} = A\omega \cos(\omega x + B)$$

$$\frac{d^2y}{dx^2} = -A\omega^2 \sin(\omega x + B)$$

$$n\pi = L \cdot \sqrt{\frac{P}{EI}}, \quad n \in \mathbb{N}$$

$$P = \frac{n^2 \pi^2 EI}{L^2}$$

$$P_{crit} = \frac{\pi^2 EI}{L^2} = P_E$$

$$r = \sqrt{\frac{I}{A}}, \quad r = \text{radius of gyration}$$

$$\sigma_E = \frac{P_E}{A} = \frac{\pi^2 EI}{AL^2} = \frac{\pi^2 E}{(L/r)^2}, \quad L/r = \text{slenderness ratio}$$

$$L/r < 200$$

9 Compression Member

$$\text{failure envelope} = \min\{\sigma_y, \sigma_E\}$$

$$\sigma_{allowable} = \min\left\{\frac{1}{FOS_{crush}} \cdot \sigma_y, \frac{1}{FOS_{buckling}} \cdot \sigma_E\right\}$$

$$\text{Yield : } F_{allowable} = \frac{1}{FOS_{crush}} A \sigma_Y \geq F_{demand}$$

$$A \geq FOS_{crush} \cdot \frac{F_{demand}}{\sigma_Y}$$

$$\text{Buckling : } F_{allowable} = \frac{1}{FOS_{buckling}} A \sigma_E \geq F_{demand}$$

$$I \geq FOS_{buckling} \cdot \frac{F_{demand} L^2}{\pi^2 E}$$

10 Wind

$$W_{wind} = \frac{F_{wind}}{A} = \frac{1}{2} \rho v^2 C_D$$

$$\text{race car : } C_D = 0.2$$

$$\text{sphere : } C_D = 0.75$$

$$\text{boxy object : } C_D = 1.5$$

$$\rho_{air} = 1.2 \text{ kg/m}^3$$

$$C_D = 1.5$$

$$V \geq 170 \text{ km/h} \approx 47.2 \text{ m/s}$$

$$W_{wind \text{ ave}} = 2.0 \text{ kPa}$$

$$A_{tributary \text{ bottom}} = \frac{h \cdot s}{2}, \text{ due to handrail}$$

$$A_{tributary \text{ top}} = \Sigma s h$$