1 Stress, Strain

$\sigma = \frac{F}{A}$

$\epsilon = \frac{\Delta l}{L}$

$\sigma = E\epsilon$

$\frac{F}{A} = \frac{E \cdot \Delta l}{L}, F = \frac{AE}{L} \cdot \Delta l = k\Delta l$

$k = \frac{AE}{L}$

$W = \frac{1}{2}k\Delta x^2 = \frac{1}{2}F\Delta x = \frac{1}{2}\sigma A\epsilon L = \frac{\sigma^2 V_0}{2E} = \frac{\sigma\epsilon V_0}{2}$

$U = \int \sigma d\epsilon$

$W = U \cdot V_0 \rightarrow U = \frac{W}{V_0}$

$\epsilon_{th} = \alpha\Delta T$

$F_{failure} = A\sigma_y$

$F_{allowable} = \frac{A\sigma_y}{FOS} \geq F_{demand}$

$A \geq \frac{FOS \cdot F_{demand}}{\sigma_y}$

2 Equilibrium

$\Sigma F = 0,\ \Sigma M = 0,\ if\ a = 0,\ where\ F = ma,\ and\ M = Fd$

3 Bridges

$T_{supY} = \frac{WL}{2}$

$T_{supX} = \frac{WL^2}{8h}$

$T_{supMax} = \sqrt{(\frac{WL}{2})^2 + (\frac{WL^2}{8h})^2},$

Given loads placed equidistantly over the span

4 Rotation

4.1 Motion

$I_m = my^2,\ for\ a\ single\ point\ mass$

$I_m = \int_M y^2 dm = \rho \int_A y^2 dA$

$I = \int_A y^2 dA$

$I = \frac{bh^3}{12},\ for\ a\ rectangle\ (comes\ from\ integrating)$

$F = ma = my$

$M = F_x y = my^2 \cdot \alpha = I_m \alpha$

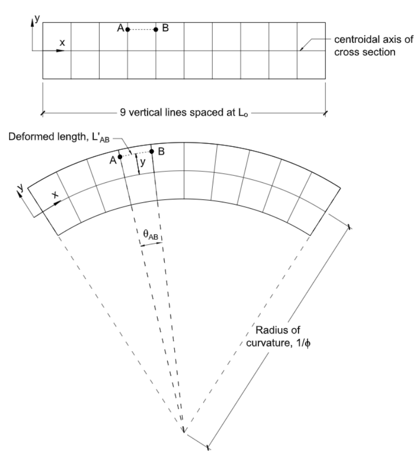
$\omega = \frac{d^2\theta}{dt^2}$

$\theta = \omega_i t + \frac{1}{2}\alpha t^2$

$a = \alpha y$

$M = Fy = m\alpha y^2 = I_m \alpha$

4.2 Bending



$\phi = \frac{d\theta}{dx}$

$L'_{AB} = \phi L_0 \cdot (y + \frac{1}{\phi}) = \phi y L_0 + L_0$

$\epsilon(y) = \frac{\Delta l}{L_0} = \frac{L'_{AB} - L_0}{L_0} = \phi y,\ \epsilon \ at\ the\ centroidal\ axis\ is\ 0$

$\sigma = E\epsilon \rightarrow \sigma(y) = E\phi y$

$\sigma = F/A \rightarrow \Delta F = \sigma(y)\Delta A$

$M = Fd \rightarrow \Delta M = \Delta F y = \phi E y^2 \Delta A$

$M = \int_A \phi E y^2 dA$

$= \phi EI$

5 Harmonic Motion

$Pendulum:\ T = 2\pi\sqrt{\frac{l}{g}}$

$Mass\ Spring:\ 2\pi\sqrt{\frac{m}{k}}$

$f = \frac{1}{T}$

$F = -kx$

$\omega = \frac{2\pi}{T}$

$a = -\omega^2 x$

$x(t) = A \cdot \sin(\omega t + \phi)$

$v = \omega A \cdot \cos(\omega t + \phi)$

$v = \pm\omega\sqrt{x_0^2 - x^2}$

$\frac{d^2x(t)}{dt^2} = -A\omega^2 \sin(\omega_n t + \phi)$

$E_k = \frac{1}{2}m\omega^2(x_0^2 - x^2)$

$E_t = \frac{1}{2}m\omega^2 x^2$

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \rightarrow v = \sqrt{x\frac{k}{m}}$

Gravity

$m\frac{d^2x(t)}{dt^2} + kx(t) = mg$

$x(t) = A\sin(\omega_n t + \phi) + \Delta_0$

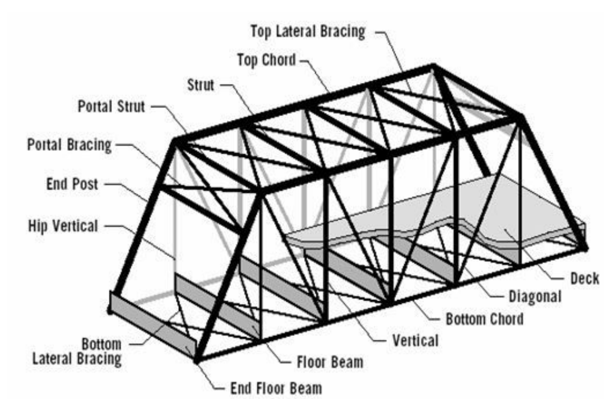
$k = \frac{mg}{\Delta_0}$

$f_n = \frac{1}{2\pi}\sqrt{\frac{mg}{\Delta_0} \cdot \frac{1}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta_0}}$

6 Staticaly Determined Structures

Name	Permited Degrees of Freedom	Restrained Degrees of Freedom	Reactions
Roller	$\Delta(x \oplus y),\ \theta$	$\Delta y = 0$	F_y
Pin	θ	$\Delta x = \Delta y = 0$	$F_x,\ F_y$
Fixed End	N/A	$\Delta x = \Delta y = \Delta\theta = 0$	$F_x,\ F_y,\ M_{xy}$

7 Truss Brigde



1. Select Geometry
2. Determine Loads
   - Gravity
   - Wind
3. Analyze forces in members

4. Design components
5. Determine stiffeness
6. Check dynamic forces
7. Iterate

$P_{central\ joint} = W \cdot \frac{s \cdot W_D}{2}$

$P_{end\ joint} = W \cdot \frac{s \cdot W_D}{4}$

Deflection:

$W_{ext} = \sum_{i=1}^{m} \int F_i d\Delta_i \ m\ Forces$

$W_{int} = \sum_{i=1}^{n} \int P_i d\Delta_i \ n\ members$

$W_{ext} = W_{int}$

$F^\star = virtual\ force$

$\Delta_l = \frac{PL}{AE}$

$F^\star \Delta = \sum P^\star \Delta_l = \sum P^\star \frac{PL}{AE}$

8 Buckling

$\phi = \frac{d\theta}{dx}$

$P \cdot y = P \cdot \Delta_{lat} = M$

$M = EI\phi$

$y(x) = A\sin(\omega x + B)$

$\frac{dy}{dx} = A\omega\cos(\omega x + B)$

$\frac{d^2y}{dx^2} = -A\omega^2\sin(\omega x + B)$

$n\pi = L \cdot \sqrt{\frac{P}{EI}},\ n \epsilon \mathbb{N}$

$P = \frac{n^2\pi^2 EI}{L^2}$

$P_{crit} = \frac{\pi^2 EI}{L^2} = P_E$

$r = \sqrt{\frac{I}{A}},\ r =\ radius\ of\ gyration$

$\sigma_E = \frac{P_E}{A} = \frac{\pi^2 EI}{AL^2} = \frac{\pi^2 E}{(L/r)^2},\ L/r = \ slenderness\ ratio$

$L/r < 200$

9 Compression Member

$failure\ envelope = \min\{\sigma_y, \sigma_E\}$

$\sigma_{allowable} = \min\{\frac{1}{FOS_{crush}} \cdot \sigma_y, \frac{1}{FOS_{buckling}} \cdot \sigma_E\}$

$Yield:\ F_{allowable} = \frac{1}{FOS_{crush}} A\sigma_Y \geq F_{demand}$

$A \geq FOS_{crush} \cdot \frac{F_{demand}}{\sigma_Y}$

$Buckling:\ F_{allowable} = \frac{1}{FOS_{buckling}} A\sigma_E \geq F_{demand}$

$I \geq FOS_{buckling} \cdot \frac{F_{demand} L^2}{\pi^2 E}$

10 Wind

$W_{wind} = \frac{F_{wind}}{A} = \frac{1}{2}\rho v^2 C_D$

$race\ car:\ C_D = 0.2$

$sphere:\ C_D = 0.75$

$boxy\ object:\ C_D = 1.5$

$\rho_{air} = 1.2 kg/m^3$

$C_D = 1.5$

$V \geq 170 km/h \approx 47.2 m/s$

$W_{wind\ ave} = 2.0 kPa$

$A_{tributary\ bottom} = \frac{h \cdot s}{2},\ due\ to\ handrail$

$A_{tributary\ top} = \Sigma sh$

11 Free vibrations in truss

$f_n = \frac{15.76}{\sqrt{\Delta_0}}\ for\ single\ load$

$f_n = \frac{17.76}{\sqrt{\Delta_0}}\ for\ distributed\ load$

Damping : Tendency of a system to lose energy as it vibrates

$\beta =$ Damping ratio

$\frac{md^2x}{dt^2} + 2\beta\sqrt{mk}\frac{dx}{dt} + kx = mg$

$x(t) = Ae^{-\beta\omega_n t}sin(\omega_n t\sqrt{1-\beta^2} + \phi) + \Delta_0$

12 Forced vibrations

$F(t) = F_0 sin(\omega t) + mg$

$\frac{md^2x}{dt^2} + 2\beta\sqrt{mk}\frac{dx}{dt} + kx = F_0 sin(\omega t)mg$

$x(t) = DAF \cdot \frac{F_0}{k}\sin(\omega t + \phi) + \Delta_0,$ At steady state

Dynamic Amplification Factor : $DAF = \frac{1}{\sqrt{(1-(\frac{f}{f_n})^2)^2 + (2\frac{\beta f}{f_n})^2}}$

$\Delta_{max} = \frac{F_{max}}{k}$

$F_{max} = DAF \cdot F_0 + mg$