

## 1 Stress, Strain

$$\sigma = \frac{F}{A} \quad (1)$$

$$\epsilon = \frac{\Delta l}{L} \quad (2)$$

$$\sigma = E\epsilon \quad (3)$$

$$\frac{F}{A} = \frac{E \cdot \Delta l}{L}, F = \frac{AE}{L} \cdot \Delta l = k\Delta l \quad (4)$$

$$k = \frac{AE}{L} \quad (5)$$

$$W = \frac{1}{2}k\Delta x^2 = \frac{1}{2}F\Delta x = \frac{1}{2}\sigma A\epsilon L = \frac{\sigma^2 V_0}{2E} = \frac{\sigma\epsilon V_0}{2} \quad (6)$$

$$U = \int \sigma d\epsilon \quad (7)$$

$$W = U \cdot V_0 \rightarrow U = \frac{W}{V_0} \quad (8)$$

$$\epsilon_{th} = \alpha\Delta T \quad (9)$$

$$F_{failure} = A\sigma_y \quad (10)$$

$$F_{allowable} = \frac{A\sigma_y}{FOS} \geq F_{demand} \quad (11)$$

$$A \geq \frac{FOS \cdot F_{demand}}{\sigma_y} \quad (12)$$

## 2 Equilibrium

$$\Sigma F = 0, \Sigma M = 0, \text{ if } a = 0, \quad (13)$$

$$\text{where } F = ma, \text{ and } M = Fd \quad (14)$$

### 3 Bridges

$$T_{supY} = \frac{WL}{2} \quad (15)$$

$$T_{supX} = \frac{WL^2}{8h} \quad (16)$$

$$T_{supMax} = \sqrt{\left(\frac{WL}{2}\right)^2 + \left(\frac{WL^2}{8h}\right)^2}, \quad (17)$$

Given loads placed equidistantly over the span

### 4 Rotation

#### 4.1 Motion

$$I_m = my^2, \text{ for a single point mass} \quad (18)$$

$$I_m = \int_M y^2 dm = \rho \int_A y^2 dA \quad (19)$$

$$I = \int_A y^2 dA \quad (20)$$

$$I = \frac{bh^3}{12}, \text{ for a rectangle (comes from integrating)} \quad (21)$$

$$F = ma = my \quad (22)$$

$$M = F_x y = my^2 \cdot \alpha = I_m \alpha \quad (23)$$

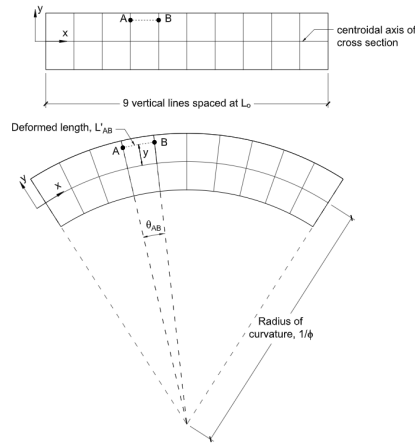
$$\omega = \frac{d^2 \theta}{dt^2} \quad (24)$$

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2 \quad (25)$$

$$a = \alpha y \quad (26)$$

$$M = Fy = m\alpha y^2 = I_m \alpha \quad (27)$$

## 4.2 Bending



$$\phi = \frac{d\theta}{dx} \quad (28)$$

$$L'_{AB} = \phi L_0 \cdot \left(y + \frac{1}{\phi}\right) = \phi y L_0 + L_0 \quad (29)$$

$$\epsilon(y) = \frac{\Delta l}{L_0} = \frac{L'_{AB} - L_0}{L_0} = \phi y, \quad \epsilon \text{ at the centroidal axis is } 0 \quad (30)$$

$$\sigma = E\epsilon \rightarrow \sigma(y) = E\phi y \quad (31)$$

$$\sigma = F/A \rightarrow \Delta F = \sigma(y)\Delta A \quad (32)$$

$$M = Fd \rightarrow \Delta M = \Delta F y = \phi E y^2 \Delta A \quad (33)$$

$$M = \int_A \phi E y^2 dA = \phi EI \quad (34)$$

## 5 Harmonic Motion

$$\text{Pendulum : } T = 2\pi\sqrt{\frac{l}{g}} \quad (35)$$

$$\text{Mass Spring : } 2\pi\sqrt{\frac{m}{k}} \quad (36)$$

$$f = \frac{1}{T} \quad (37)$$

$$F = -kx \quad (38)$$

$$\omega = \frac{2\pi}{T} \quad (39)$$

$$a = -\omega^2 x \quad (40)$$

$$x(t) = A \cdot \sin(\omega t + \phi) \quad (41)$$

$$v = \omega A \cdot \cos(\omega t + \phi) \quad (42)$$

$$v = \pm\omega\sqrt{x_0^2 - x^2} \quad (43)$$

$$\frac{d^2x(t)}{dt^2} = -A\omega^2 \sin(\omega_n t + \phi) \quad (44)$$

$$E_k = \frac{1}{2}m\omega^2(x_0^2 - x^2) \quad (45)$$

$$E_t = \frac{1}{2}m\omega^2 x^2 \quad (46)$$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \rightarrow v = \sqrt{x \frac{k}{m}} \quad (47)$$

### Gravity

$$m \frac{d^2x(t)}{dt^2} + kx(t) = mg \quad (48)$$

$$x(t) = A \sin(\omega_n t + \phi) + \Delta_0 \quad (49)$$

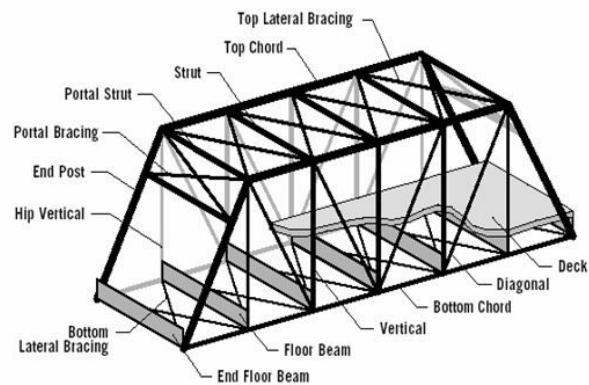
$$k = \frac{mg}{\Delta_0} \quad (50)$$

$$f_n = \frac{1}{2\pi} \sqrt{\frac{mg}{\Delta_0} \cdot \frac{1}{m}} = \frac{1}{2\pi} \sqrt{\frac{g}{\Delta_0}} \quad (51)$$

## 6 Statically Determined Structures

Name	Permitted Degrees of Freedom	Restrained Degrees of Freedom	Reactions
Roller	$\Delta(x \oplus y), \theta$	$\Delta y = 0$	$F_y$
Pin	$\theta$	$\Delta x = \Delta y = 0$	$F_x, F_y$
Fixed End	N/A	$\Delta x = \Delta y = \Delta \theta = 0$	$F_x, F_y, M_{xy}$

## 7 Truss Bridge



1. Select Geometry
2. Determine Loads
  - Gravity
  - Wind
3. Analyze forces in members

4. Design components
5. Determine stiffness
6. Check dynamic forces
7. Iterate

$$P_{central\ joint} = W \cdot \frac{s \cdot W_D}{2} \quad (52)$$

$$P_{end\ joint} = W \cdot \frac{s \cdot W_D}{4} \quad (53)$$

$$\mathbf{Deflection:} \quad (54)$$

$$W_{ext} = \sum_{i=1}^m \int F_i d\Delta_i \quad m \text{ Forces} \quad (55)$$

$$W_{int} = \sum_{i=1}^n \int P_i d\Delta_i \quad n \text{ members} \quad (56)$$

$$W_{ext} = W_{int} \quad (57)$$

$$F^* = \text{virtual force} \quad (58)$$

$$\Delta_l = \frac{PL}{AE} \quad (59)$$

$$F^* \Delta = \sum P^* \Delta_l = \sum P^* \frac{PL}{AE} \quad (60)$$

## 8 Buckling

$$\phi = \frac{d\theta}{dx} \quad (61)$$

$$P \cdot y = P \cdot \Delta_{lat} = M \quad (62)$$

$$M = EI\phi \quad (63)$$

$$y(x) = A \sin(\omega x + B) \quad (64)$$

$$\frac{dy}{dx} = A\omega \cos(\omega x + B) \quad (65)$$

$$\frac{d^2y}{dx^2} = -A\omega^2 \sin(\omega x + B) \quad (66)$$

$$n\pi = L \cdot \sqrt{\frac{P}{EI}}, \quad n \in \mathbb{N} \quad (67)$$

$$P = \frac{n^2\pi^2 EI}{L^2} \quad (68)$$

$$P_{crit} = \frac{\pi^2 EI}{L^2} = P_E \quad (69)$$

$$r = \sqrt{\frac{I}{A}}, \quad r = \text{radius of gyration} \quad (70)$$

$$\sigma_E = \frac{P_E}{A} = \frac{\pi^2 EI}{AL^2} = \frac{\pi^2 E}{(L/r)^2}, \quad L/r = \text{slenderness ratio} \quad (71)$$

$$L/r < 200 \quad (72)$$

## 9 Compression Member

$$failure\ envelope = \min\{\sigma_y, \sigma_E\} \quad (73)$$

$$\sigma_{allowable} = \min\left\{\frac{1}{FOS_{crush}} \cdot \sigma_y, \frac{1}{FOS_{buckling}} \cdot \sigma_E\right\} \quad (74)$$

$$Yield : F_{allowable} = \frac{1}{FOS_{crush}} A \sigma_Y \geq F_{demand} \quad (75)$$

$$A \geq FOS_{crush} \cdot \frac{F_{demand}}{\sigma_Y} \quad (76)$$

$$Buckling : F_{allowable} = \frac{1}{FOS_{buckling}} A \sigma_E \geq F_{demand} \quad (77)$$

$$I \geq FOS_{buckling} \cdot \frac{F_{demand} L^2}{\pi^2 E} \quad (78)$$

## 10 Wind

$$W_{wind} = \frac{F_{wind}}{A} = \frac{1}{2} \rho v^2 C_D \quad (79)$$

$$race\ car : C_D = 0.2 \quad (80)$$

$$sphere : C_D = 0.75 \quad (81)$$

$$boxy\ object : C_D = 1.5 \quad (82)$$

$$\rho_{air} = 1.2 kg/m^3 \quad (83)$$

$$C_D = 1.5 \quad (84)$$

$$V \geq 170 km/h \approx 47.2 m/s \quad (85)$$

$$W_{wind\ ave} = 2.0 kPa \quad (86)$$

$$A_{tributary\ bottom} = \frac{h \cdot s}{2}, \text{ due to handrail} \quad (87)$$

$$A_{tributary\ top} = \Sigma sh \quad (88)$$



## 11 Free vibrations in truss

$$f_n = \frac{15.76}{\sqrt{\Delta_0}} \text{ for single load} \quad (89)$$

$$f_n = \frac{17.76}{\sqrt{\Delta_0}} \text{ for distributed load} \quad (90)$$

$$\textbf{Damping} : \text{Tendency of a system to lose energy as it vibrates} \quad (91)$$

$$\beta = \text{Damping ratio} \quad (92)$$

$$\frac{md^2x}{dt^2} + 2\beta\sqrt{mk}\frac{dx}{dt} + kx = mg \quad (93)$$

$$x(t) = Ae^{-\beta\omega_n t} \sin(\omega_n t \sqrt{1 - \beta^2} + \phi) + \Delta_0 \quad (94)$$

## 12 Forced vibrations

$$F(t) = F_0 \sin(\omega t) + mg \quad (95)$$

$$\frac{md^2x}{dt^2} + 2\beta\sqrt{mk}\frac{dx}{dt} + kx = F_0 \sin(\omega t) + mg \quad (96)$$

$$x(t) = DAF \cdot \frac{F_0}{k} \sin(\omega t + \phi) + \Delta_0, \text{ At steady state} \quad (97)$$

$$\textbf{Dynamic Amplification Factor} : DAF = \frac{1}{\sqrt{(1 - (\frac{f}{f_n})^2)^2 + (2\frac{\beta f}{f_n})^2}} \quad (98)$$

$$\Delta_{max} = \frac{F_{max}}{k} \quad (99)$$

$$F_{max} = DAF \cdot F_0 + mg \quad (100)$$

### 13 Shear forces

$$w(x) = \frac{d}{dx}v(x) \quad (101)$$

$$\Delta V_{12} = \int_{x_1}^{x_2} w(x) dx \quad (102)$$

$$v(x) = \frac{d}{dx}M(x) \quad (103)$$

$$\Delta M_{12} = M_2 - M_1 = \int_{x_1}^{x_2} v(x) dx \quad (104)$$

### 14 Shear Stress

$$\tau = \text{Shear Stress} \quad (105)$$

$$\Delta C = \tau \cdot b \Delta x = \int \sigma dA \quad (106)$$

$$\tau \cdot b \Delta x = \int_{y_0}^{y_{top}} \frac{\Delta M y}{I} dA = \frac{\Delta M}{I} \cdot \int_{y_0}^{y_{top}} y dA \quad (107)$$

$$\tau = \frac{\Delta M}{\Delta x} \cdot \frac{1}{Ib} \cdot \int_{y_0}^{y_{top}} y dA = \frac{\Delta M}{\Delta x} \cdot \frac{Q}{Ib} \quad (108)$$

$$V = \frac{dM}{dx} \quad (109)$$

$$\tau = \frac{VQ}{Ib} \quad (110)$$

$$Q = \text{First Moment of Area taken about the centroid} \quad (111)$$

$$Q = \int_{y_0}^{y_{top}} y dA = \int_{y_{bot}}^{y_0} y dA \approx \sum_{i=0}^n A_i d_i \quad (112)$$

$$A_i = \text{areas of cross section from } y_0 \text{ to top/bottom of cross section} \quad (113)$$

$$d_i = \text{distance from centroid of } A_i \text{ to centroidal axis of cross section} \quad (114)$$

$$Q = \frac{1}{2}by(h-y) \text{ for rectangle} \quad (115)$$

$$Q(0) = Q(h) = 0 \quad (116)$$

$$\frac{dQ}{dy}\left(\frac{h}{2}\right) = 0 \quad (117)$$

## 15 Flexural Stresses

$$\Delta l = 0 \text{ at centroidal axis} \quad (118)$$

$$\phi = \frac{d\theta}{dx} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}, \quad \phi = \text{curvature} \quad (119)$$

$$\epsilon(y) = \phi y \quad (120)$$

$$I = \int y^2 dA \quad (121)$$

$$\sigma = E\epsilon \quad (122)$$

$$\sigma(y) = E\phi y \quad (123)$$

$$\Delta F_i = \sigma_i \Delta A_i = E\phi y_i \cdot \Delta A_i \quad (124)$$

$$N = \sum_{i=0}^n \Delta F_i = 0 \quad (125)$$

$$0 = \sum_{i=0}^n E\phi y_i \Delta A_i \quad (126)$$

$$\lim_{\Delta A \rightarrow 0} \sum_{i=0}^n E\phi y_i \Delta A_i = \int E\phi y dA \quad (127)$$

$$\Delta M_i = \Delta F_i y_i \quad (128)$$

$$M = \lim_{\Delta A \rightarrow 0} \sum_{i=0}^n \Delta F_i y_i = \lim_{\Delta A \rightarrow 0} \sum_{i=0}^n E\phi y_i^2 \cdot \Delta A_i = E\phi \int y_i^2 dA \quad (129)$$

$$M = EI\phi = EI \frac{\sigma}{Ey} = EI \left( \frac{d^2y}{dx^2} \right) \quad (130)$$

$$\sigma = \frac{My}{I}, \text{ Navier's equation,} \quad (131)$$

$$M = \text{bending Moment, } I = \text{Second Moment of area} \quad (132)$$

$$\sigma = \text{Flexural stress, } y = \text{Vertical Distance from centroid} \quad (133)$$

$$\Delta\theta_{12} = \theta_2 - \theta_1 = \int_{x_1}^{x_2} \phi dx \quad (134)$$

$$\delta_{DT} = \lim_{\Delta x \rightarrow 0} \sum_{i=1}^n \phi(x_i)(x_{DT} - x_i)\Delta x = \int \phi(x)(x_{DT} - x)dx \quad (135)$$