1 Stress, Strain

$$\sigma = \frac{F}{A} \tag{1}$$

$$\epsilon = \frac{\Delta l}{L} \tag{2}$$

$$\sigma = E\epsilon \tag{3}$$

$$\frac{F}{A} = \frac{E \cdot \Delta l}{L}, F = \frac{AE}{L} \cdot \Delta l = k\Delta l \tag{4}$$

$$k = \frac{AE}{L} \tag{5}$$

$$W = \frac{1}{2}k\Delta x^2 = \frac{1}{2}F\Delta x = \frac{1}{2}\sigma A\epsilon L = \frac{\sigma^2 V_0}{2E} = \frac{\sigma\epsilon V_0}{2} \tag{6}$$

$$U = \int \sigma d\epsilon \tag{7}$$

$$W = U \cdot V_0 \to U = \frac{W}{V_0} \tag{8}$$

$$\epsilon_{th} = \alpha\Delta T \tag{9}$$

$$F_{failure} = A\sigma_y \tag{10}$$

$$F_{allowable} = \frac{A\sigma_y}{FOS} \geq F_{demand} \tag{11}$$

$$A \geq \frac{FOS \cdot F_{demand}}{\sigma_y} \tag{12}$$

2 Equilibrium

$$\Sigma F = 0,\ \Sigma M = 0,\ if\ a = 0, \tag{13}$$

$$where\ F = ma,\ and\ M = Fd \tag{14}$$

3 Bridges

$$T_{supY} = \frac{WL}{2} \tag{15}$$

$$T_{supX} = \frac{WL^2}{8h} \tag{16}$$

$$T_{supMax} = \sqrt{(\frac{WL}{2})^2 + (\frac{WL^2}{8h})^2}, \tag{17}$$

Given loads placed equidistantly over the span

4 Rotation

4.1 Motion

$$I_m = my^2, \; for \; a \; single \; point \; mass \tag{18}$$

$$I_m = \int_M y^2 dm = \rho \int_A y^2 dA \tag{19}$$

$$I = \int_A y^2 dA \tag{20}$$

$$I = \frac{bh^3}{12}, \; for \; a \; rectangle \; (comes \; from \; integrating) \tag{21}$$

$$F = ma = my \tag{22}$$

$$M = F_x y = my^2 \cdot \alpha = I_m \alpha \tag{23}$$

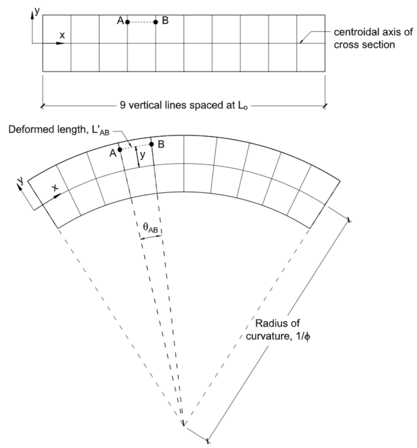
$$\omega = \frac{d^2\theta}{dt^2} \tag{24}$$

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2 \tag{25}$$

$$a = \alpha y \tag{26}$$

$$M = Fy = m\alpha y^2 = I_m \alpha \tag{27}$$

4.2 Bending



$$\phi = \frac{d\theta}{dx} \tag{28}$$

$$L'_{AB} = \phi L_0 \cdot (y + \frac{1}{\phi}) = \phi y L_0 + L_0 \tag{29}$$

$$\epsilon(y) = \frac{\Delta l}{L_0} = \frac{L'_{AB} - L_0}{L_0} = \phi y,\ \epsilon \ at \ the \ centroidal \ axis \ is \ 0 \tag{30}$$

$$\sigma = E\epsilon \rightarrow \sigma(y) = E\phi y \tag{31}$$

$$\sigma = F/A \rightarrow \Delta F = \sigma(y)\Delta A \tag{32}$$

$$M = Fd \rightarrow \Delta M = \Delta F y = \phi E y^2 \Delta A \tag{33}$$

$$M = \int_A \phi E y^2 dA = \phi E I \tag{34}$$

5 Harmonic Motion

$$Pendulum:\ T = 2\pi\sqrt{\frac{l}{g}} \tag{35}$$

$$Mass\ Spring:\ 2\pi\sqrt{\frac{m}{k}} \tag{36}$$

$$f = \frac{1}{T} \tag{37}$$

$$F = -kx \tag{38}$$

$$\omega = \frac{2\pi}{T} \tag{39}$$

$$a = -\omega^2 x \tag{40}$$

$$x(t) = A \cdot \sin(\omega t + \phi) \tag{41}$$

$$v = \omega A \cdot \cos(\omega t + \phi) \tag{42}$$

$$v = \pm\omega\sqrt{x_0^2 - x^2} \tag{43}$$

$$\frac{d^2x(t)}{dt^2} = -A\omega^2 \sin(\omega_n t + \phi) \tag{44}$$

$$E_k = \frac{1}{2}m\omega^2(x_0^2 - x^2) \tag{45}$$

$$E_t = \frac{1}{2}m\omega^2 x^2 \tag{46}$$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \rightarrow v = \sqrt{x\frac{k}{m}} \tag{47}$$

Gravity

$$m\frac{d^2x(t)}{dt^2} + kx(t) = mg \tag{48}$$

$$x(t) = A\sin(\omega_n t + \phi) + \Delta_0 \tag{49}$$

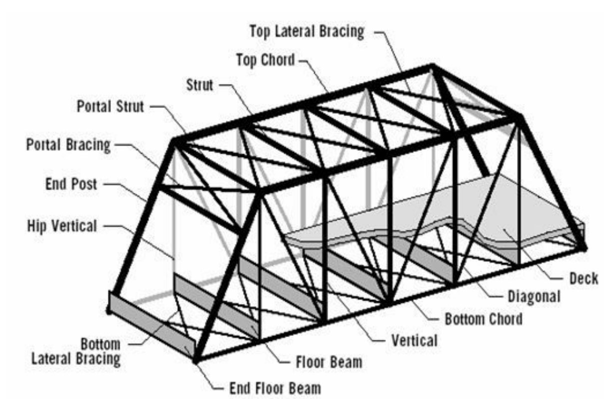
$$k = \frac{mg}{\Delta_0} \tag{50}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{mg}{\Delta_0} \cdot \frac{1}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta_0}} \tag{51}$$

6 Staticaly Determined Structures

Name	**Permited Degrees of Freedom**	**Restrained Degrees of Freedom**	**Reactions**
Roller	$\Delta(x \oplus y),\ \theta$	$\Delta y = 0$	F_y
Pin	θ	$\Delta x = \Delta y = 0$	$F_x,\ F_y$
Fixed End	N/A	$\Delta x = \Delta y = \Delta\theta = 0$	$F_x,\ F_y,\ M_{xy}$

7 Truss Brigde



1. Select Geometry
2. Determine Loads
   - Gravity
   - Wind
3. Analyze forces in members

4. Design components

5. Determine stiffeness

6. Check dynamic forces

7. Iterate

$$P_{central\ joint} = W \cdot \frac{s \cdot W_D}{2} \tag{52}$$

$$P_{end\ joint} = W \cdot \frac{s \cdot W_D}{4} \tag{53}$$

Deflection: (54)

$$W_{ext} = \sum_{i=1}^{m} \int F_i d\Delta_i \; m\ Forces \tag{55}$$

$$W_{int} = \sum_{i=1}^{n} \int P_i d\Delta_i \; n\ members \tag{56}$$

$$W_{ext} = W_{int} \tag{57}$$

$$F^\star = virtual\ force \tag{58}$$

$$\Delta_l = \frac{PL}{AE} \tag{59}$$

$$F^\star \Delta = \sum P^\star \Delta_l = \sum P^\star \frac{PL}{AE} \tag{60}$$

8 Buckling

$$\phi = \frac{d\theta}{dx} \tag{61}$$

$$P \cdot y = P \cdot \Delta_{lat} = M \tag{62}$$

$$M = EI\phi \tag{63}$$

$$y(x) = A\sin(\omega x + B) \tag{64}$$

$$\frac{dy}{dx} = A\omega\cos(\omega x + B) \tag{65}$$

$$\frac{d^2y}{dx^2} = -A\omega^2\sin(\omega x + B) \tag{66}$$

$$n\pi = L \cdot \sqrt{\frac{P}{EI}},\ n \epsilon \mathbb{N} \tag{67}$$

$$P = \frac{n^2\pi^2EI}{L^2} \tag{68}$$

$$P_{crit} = \frac{\pi^2EI}{L^2} = P_E \tag{69}$$

$$r = \sqrt{\frac{I}{A}},\ r = \ radius\ of\ gyration \tag{70}$$

$$\sigma_E = \frac{P_E}{A} = \frac{\pi^2EI}{AL^2} = \frac{\pi^2E}{(L/r)^2},\ L/r = \ slenderness\ ratio \tag{71}$$

$$L/r < 200 \tag{72}$$

9 Compression Member

$$failure\ envelope = \min\{\sigma_y, \sigma_E\} \tag{73}$$

$$\sigma_{allowable} = \min\{\frac{1}{FOS_{crush}} \cdot \sigma_y, \frac{1}{FOS_{buckling}} \cdot \sigma_E\} \tag{74}$$

$$Yield:\ F_{allowable} = \frac{1}{FOS_{crush}} A\sigma_Y \geq F_{demand} \tag{75}$$

$$A \geq FOS_{crush} \cdot \frac{F_{demand}}{\sigma_Y} \tag{76}$$

$$Buckling:\ F_{allowable} = \frac{1}{FOS_{buckling}} A\sigma_E \geq F_{demand} \tag{77}$$

$$I \geq FOS_{buckling} \cdot \frac{F_{demand} L^2}{\pi^2 E} \tag{78}$$

10 Wind

$$W_{wind} = \frac{F_{wind}}{A} = \frac{1}{2}\rho v^2 C_D \tag{79}$$

$$race\ car:\ C_D = 0.2 \tag{80}$$

$$sphere:\ C_D = 0.75 \tag{81}$$

$$boxy\ object:\ C_D = 1.5 \tag{82}$$

$$\rho_{air} = 1.2kg/m^3 \tag{83}$$

$$C_D = 1.5 \tag{84}$$

$$V \geq 170km/h \approx 47.2m/s \tag{85}$$

$$W_{wind\ ave} = 2.0kPa \tag{86}$$

$$A_{tributary\ bottom} = \frac{h \cdot s}{2},\ due\ to\ handrail \tag{87}$$

$$A_{tributary\ top} = \Sigma sh \tag{88}$$

11 Free vibrations in truss

$$f_n = \frac{15.76}{\sqrt{\Delta_0}} \; for\ single\ load \tag{89}$$

$$f_n = \frac{17.76}{\sqrt{\Delta_0}} \; for\ distributed\ load \tag{90}$$

$$\textbf{Damping} : \text{Tendency of a system to lose energy as it vibrates} \tag{91}$$

$$\beta = \text{Damping ratio} \tag{92}$$

$$\frac{md^2x}{dt^2} + 2\beta\sqrt{mk}\frac{dx}{dt} + kx = mg \tag{93}$$

$$x(t) = Ae^{-\beta\omega_n t} sin(\omega_n t\sqrt{1-\beta^2} + \phi) + \Delta_0 \tag{94}$$

12 Forced vibrations

$$F(t) = F_0 sin(\omega t) + mg \tag{95}$$

$$\frac{md^2x}{dt^2} + 2\beta\sqrt{mk}\frac{dx}{dt} + kx = F_0 sin(\omega t) mg \tag{96}$$

$$x(t) = DAF \cdot \frac{F_0}{k} \sin(\omega t + \phi) + \Delta_0, \text{ At steady state} \tag{97}$$

$$\textbf{Dynamic Amplification Factor}: \; DAF = \frac{1}{\sqrt{(1-(\frac{f}{f_n})^2)^2 + (2\frac{\beta f}{f_n})^2}} \tag{98}$$

$$\Delta_{max} = \frac{F_{max}}{k} \tag{99}$$

$$F_{max} = DAF \cdot F_0 + mg \tag{100}$$

13 Shear forces

$$w(x) = \frac{d}{dx} v(x) \tag{101}$$

$$\Delta V_{12} = \int_{x1}^{x2} w(x)\, dx \tag{102}$$

$$v(x) = \frac{d}{dx} M(x) \tag{103}$$

$$\Delta M_{12} = M_2 - M_1 = \int_{x1}^{x2} v(x)\, dx \tag{104}$$

14 Flexural Stresses

$$\Delta l = 0 \text{ at centroidal axis} \tag{105}$$

$$\phi = \frac{d\theta}{dx} = \frac{d}{dx}(\frac{dy}{dx}) = \frac{d^2y}{dx^2},\ \phi = \text{curvature} \tag{106}$$

$$\epsilon(y) = \phi y \tag{107}$$

$$I = \int y^2 dA \tag{108}$$

$$\sigma = E\epsilon \tag{109}$$

$$\sigma(y) = E\phi y \tag{110}$$

$$\Delta F_i = \sigma_i \Delta A_i = E\phi y \cdot \Delta A_i \tag{111}$$

$$N = \sum_{i=0}^{n} \Delta F_i = 0 \tag{112}$$

$$0 = \sum_{i=0}^{n} E\phi y_i \Delta A_i \tag{113}$$

$$\lim_{\Delta A \to 0} \sum_{i=0}^{n} E\phi y_i \Delta A_i = \int E\phi y \, dA \tag{114}$$

$$\Delta M_i = \Delta F_i y_i \tag{115}$$

$$M = \lim_{\Delta A \to 0} \sum_{i=0}^{n} \Delta F_i y_i = \lim_{\Delta A \to 0} \sum_{i=0}^{n} E\phi y_i^2 \cdot \Delta A_i = E\phi \int y_i^2 dA \tag{116}$$

$$M = EI\phi = EI\frac{\sigma}{Ey} = EI(\frac{d^2y}{dx^2}) \tag{117}$$

$$\sigma = \frac{My}{I},\ \textbf{Navier's equation}, \tag{118}$$

$$M = \text{bending Moment},\ I = \text{Second Moment of area} \tag{119}$$

$$\sigma = \text{Flexural stress},\ y = \text{Vertical Distance from centroid} \tag{120}$$

$$\Delta\theta_{12} = \theta_2 - \theta_1 = \int_{x_1}^{x_2} \phi \, dx \tag{121}$$

$$\delta_{DT} = \lim_{\Delta x \to \infty} \sum_{i=1}^{n} \phi(x_i)(x_{DT} - x_i)\Delta x = \int \phi(x)(x_{DT} - x)dx \tag{122}$$

$$\tag{123}$$