1 Wave particle duality

Energy Quanta: (1)

$$u_v dv = \frac{8\pi v^2}{c^3} \cdot \frac{hv}{e^{\frac{hv}{K_b T}} - 1} \tag{2}$$

$$E = hv \tag{3}$$

Photoelectric Effect: (4)

$$\frac{1}{2} m_e v_k^2 = E_k = hv - W \tag{5}$$

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \tag{6}$$

$$R_H = 1.097373 \times 10^7 m^{-1} \qquad n_1 = 1 \Rightarrow Lyman \tag{7}$$

$$n_1 = 2 \Rightarrow Balmer \tag{8}$$

$$n_1 = 3 \Rightarrow Paschen \tag{9}$$

$$n_1 = 4 \Rightarrow Brackett \tag{10}$$

$$n_1 = 15 \Rightarrow Pfund \tag{11}$$

$$n_2 > n_1 \tag{12}$$

De Broglie Relationship: $E^2 = p^2c^2 + m^2c^4$ (13)

for zero rest mass: $E^2 = p^2c^2 \rightarrow E = pc$ (14)

$$hv = h\frac{c}{\lambda} = pc \tag{15}$$

$$\therefore p = \frac{h}{\lambda} \tag{16}$$

Electron diffraction (17)

$$E_K = eV - \frac{p^2}{2m} \tag{18}$$

$$p = \sqrt{2meV} \tag{19}$$

$$\lambda = \frac{h}{\sqrt{2meV}} \tag{20}$$

$$\lambda = 2d\sin(\theta) \tag{21}$$

$$\sin^2(\theta) = \frac{C}{V}, \text{ Where } C = \frac{h^2}{8med^2} \tag{22}$$

Bohr Model (23)

$$F_{centripedal} = F_{electric} \tag{24}$$

$$m\frac{v^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2} \tag{25}$$

$$\therefore v^2 = \frac{e^2}{4\pi\epsilon_0 mr} \tag{26}$$

$$I_{orbit} = n\lambda = 2\pi r \text{ Where n is the quantisation condition} \tag{27}$$

$$\therefore v = \frac{hn}{2\pi mr} \tag{28}$$

$$L = mvr = n\frac{h}{2\pi} = n\hbar \tag{29}$$

As such, allowed radii are described by the expression: (30)

$$r_n = \frac{h^2\epsilon_0 n^2}{\pi me^2} = a_{Bohr}n^2 \tag{31}$$

$$r_1 = a_{Bohr} = \frac{h^2\epsilon_0}{\pi me^2} = 5.3 \times 10^{-11}m \tag{32}$$

$$E_k = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\epsilon_0 r} \tag{33}$$

$$E_T = E_k + E_c = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r^2} = -\frac{e^2}{8\pi\epsilon_0 r} \tag{34}$$

Substiting in r_n gives: (35)

$$E_T = -\left(\frac{me^4}{8\epsilon_0^2 h^2}\right)\cdot\frac{1}{n^2} = \frac{-13.6eV}{n^2},\ 1\ eV \approx 1.602 \times 10^{-19}J \tag{36}$$

$$\Delta E = E_i - E_f = \frac{me^4}{8\epsilon_0^2 h^2}(\frac{1}{n_f^2} - \frac{1}{n_i^2}) \tag{37}$$

$$textsince : E = hv = h\frac{c}{\lambda} \tag{38}$$

$$\frac{1}{\lambda} = \frac{me^4}{8\epsilon_0^2 h^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \tag{39}$$

Waves (40)

$$\Psi = Ae^{i(kx-\omega t)} \quad (41)$$

$$k = 2\pi/\lambda \quad (42)$$

$$\omega = 2\pi v \quad (43)$$

$$c = v\lambda = \frac{\omega}{2\pi} \cdot \lambda = \frac{\omega}{\lambda} \quad (44)$$

$$P(x) \propto |\Psi|^2 dx \quad (45)$$

$$\frac{\partial \Psi}{\partial x} = ikAe^{i(kx-\omega t)} = ik\Psi \quad (46)$$

$$\frac{\partial \Psi}{\partial t} = -i\omega Ae^{i(kx-\omega t)} = i\omega\Psi \quad (47)$$

De Broglie: $p = \frac{h}{\lambda} = \frac{h}{2\pi} \cdot \frac{2\pi}{\lambda} = \hbar k$ (48)

Enstein: $E = hv = h\frac{\omega}{2\pi} = \hbar\omega$ (49)

$$\frac{\partial}{\partial x}\Psi = i\frac{p}{\hbar}\Psi \Rightarrow p\Psi = \{-i\hbar\frac{\partial}{\partial x}\}\Psi \quad (50)$$

$$\frac{\partial}{\partial t}\Psi = -i\frac{E}{\hbar}\Psi \Rightarrow E\Psi = \{i\hbar\frac{\partial}{\partial t}\}\Psi \quad (51)$$

$$\Delta t = \frac{1}{\Delta f} = \frac{h}{\Delta E} \Rightarrow \Delta E \cdot \Delta t \geq h \quad (52)$$

$$\Delta x = \Delta\lambda_{dB} = \frac{h}{\Delta p} \Rightarrow \Delta x \cdot \Delta p \geq h \quad (53)$$

Heisenber's Uncertainty Principle:

It is impossible to specify simultaneously, with precision, both the momentum and the position of a particle.

Shrodinger's Equation Derivation (54)

$$E = E_k + E_p = \frac{1}{2}mv^2 + V = \frac{p^2}{2m} + V \quad (55)$$

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V\Psi(x,t) \quad (56)$$

Time independent Shrodinger's Equation (57)

$$\Psi(x,t) = \phi(t)\psi(t) \tag{58}$$

$$i\hbar \frac{1}{\phi(t)} \frac{\partial \phi(t)}{\partial t} = -\frac{\hbar^2}{2m\psi(x)} \frac{\partial^2 \psi(x)}{\partial x^2} + V \tag{59}$$

$$\therefore\ i\frac{A}{\hbar}\phi = \frac{\partial \phi(t)}{\partial t} \tag{60}$$

$$\phi(t) = Ce^{-i(\frac{A}{\hbar})t} \Rightarrow \frac{d\phi(t)}{dt} = -i(\frac{A}{\hbar})Ce^{-i(\frac{A}{\hbar})t} \tag{61}$$

By unit analysis: $A = E$ (62)

$$\therefore\ E\psi = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi \tag{63}$$

Electron in a box (64)

$$V(0) = V(L) = \inf \tag{65}$$

$$V(x) = 0\ \forall\, x \,|\, 0 < x < L \tag{66}$$

$$\therefore\ -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \tag{67}$$

General Solution: $\psi = A\sin(kx) + B\cos(kx)$ (68)

$$\frac{d^2\psi}{dx^2} = -k^2(A\sin(kx)) + B\cos(kx)) = -k^2\psi \tag{69}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = \left(-\frac{\hbar^2}{2m}\right)\cdot(-k^2\psi) = E\psi \tag{70}$$

$$E = \frac{\hbar^2 k^2}{2m} \tag{71}$$