# Decomposition of Graphs: Strongly Connected Components

Daniel Kane

Department of Computer Science and Engineering
University of California, San Diego

## Graph Algorithms
## Data Structures and Algorithms

# Learning Objectives

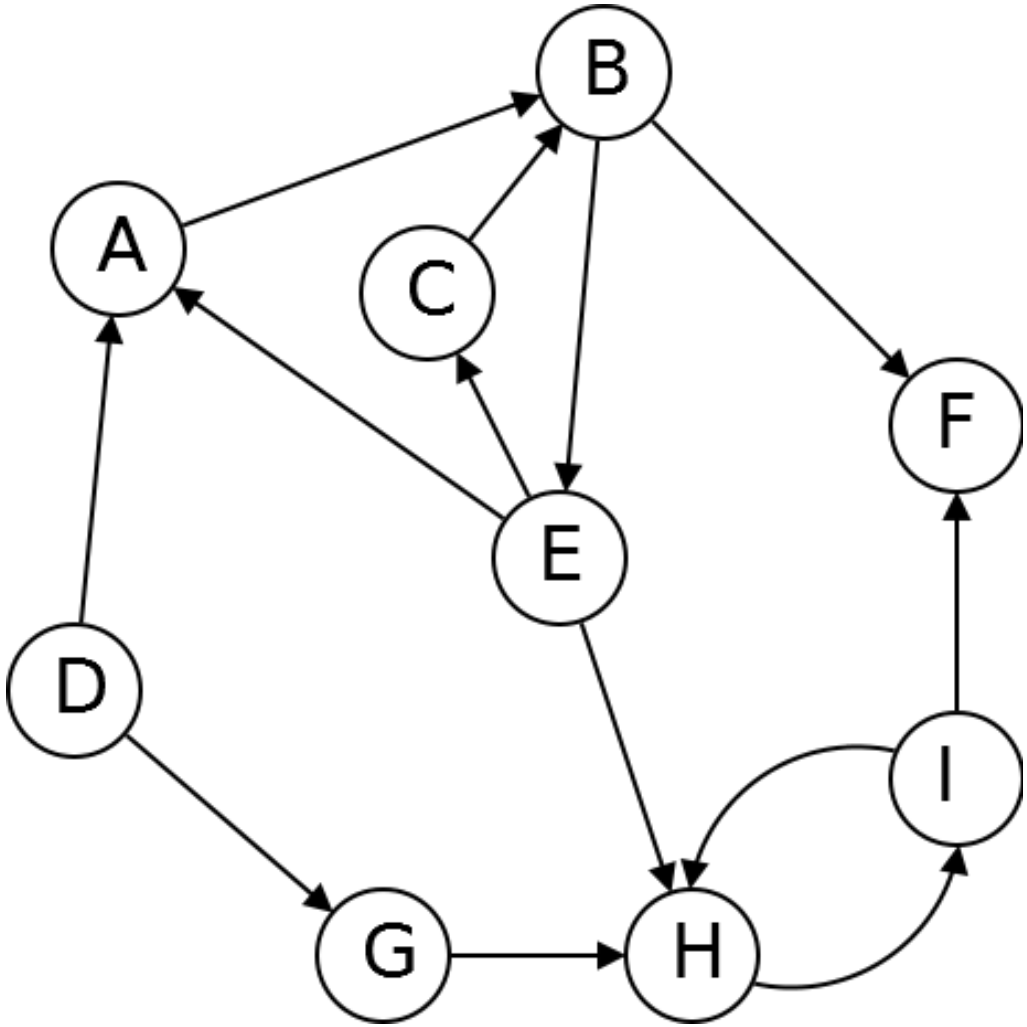
- Understand the definition of a strongly connected component of a directed graph.
- Give some other notions of connectivity within a directed graph.

# Outline

# Connectivity in Digraphs

In undirected graphs, have connected components.
Directed graphs are more complicated.

# Example



# Possible Notions

- Connected by edges in any direction.
- One vertex reachable from another.
- Two vertices both reachable from the other.

# Outline

# Strongly Connected Components

## Definition

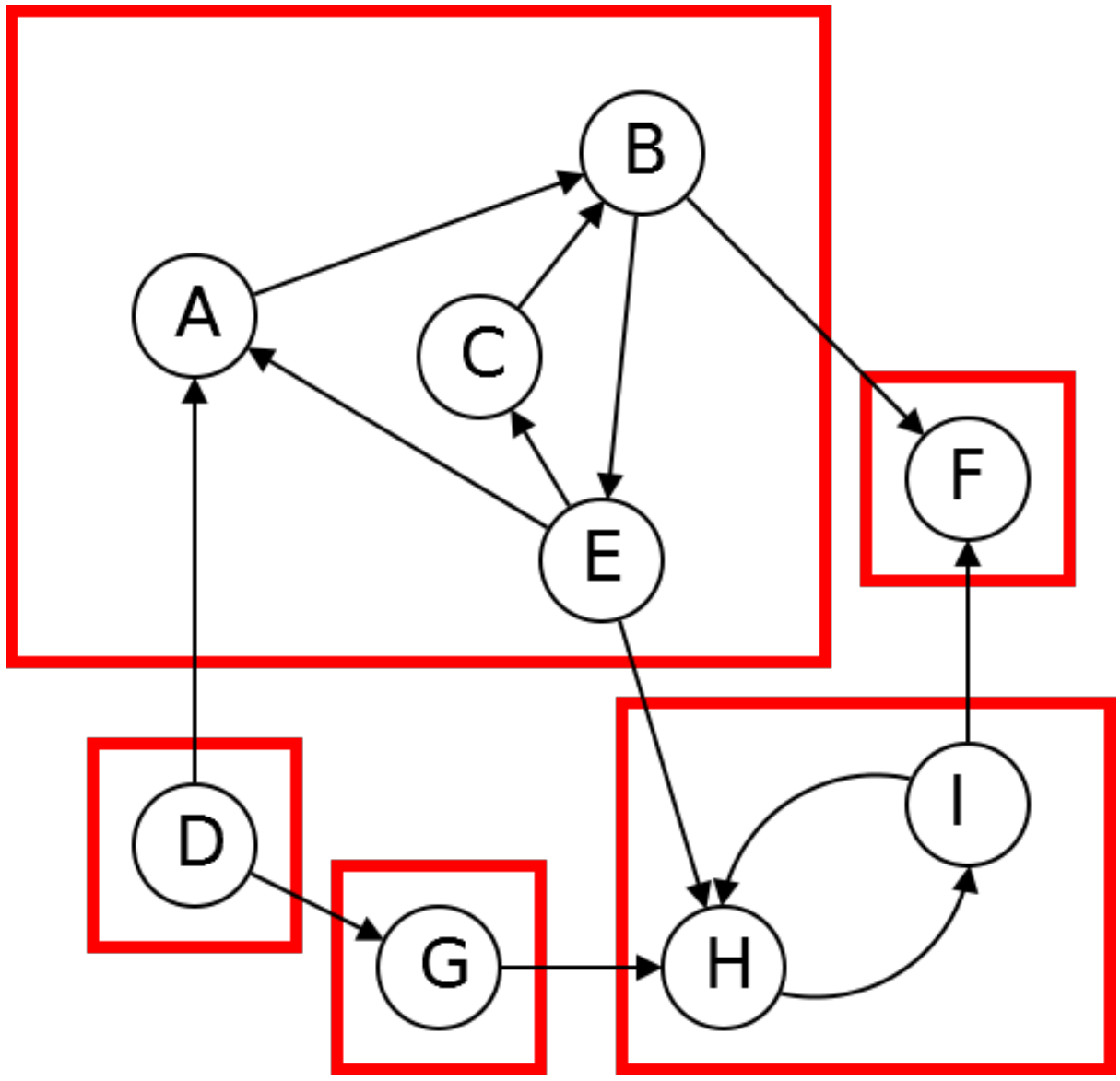
Two vertices $v, w$ in a directed graph are connected if you can reach $v$ from $w$ and can reach $w$ from $v$.

# Result

## Theorem

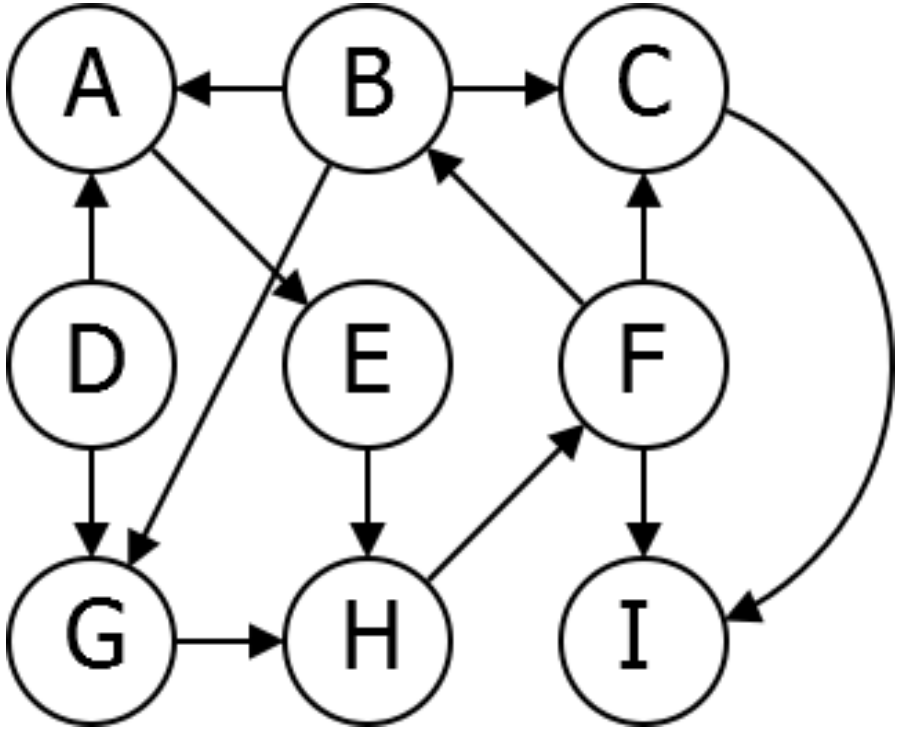
A directed graph can be partitioned into strongly connected components where two vertices are connected if and only if they are in the same component.

# Example



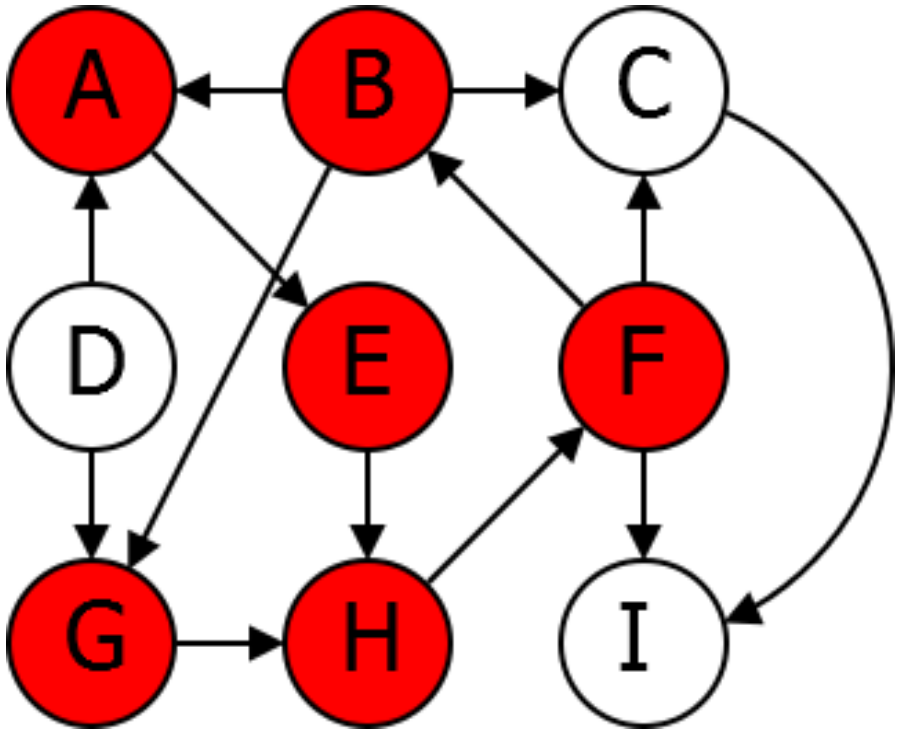
# Problem

What is the SCC of *A*?



# Solution

$A, B, E, F, G, H.$



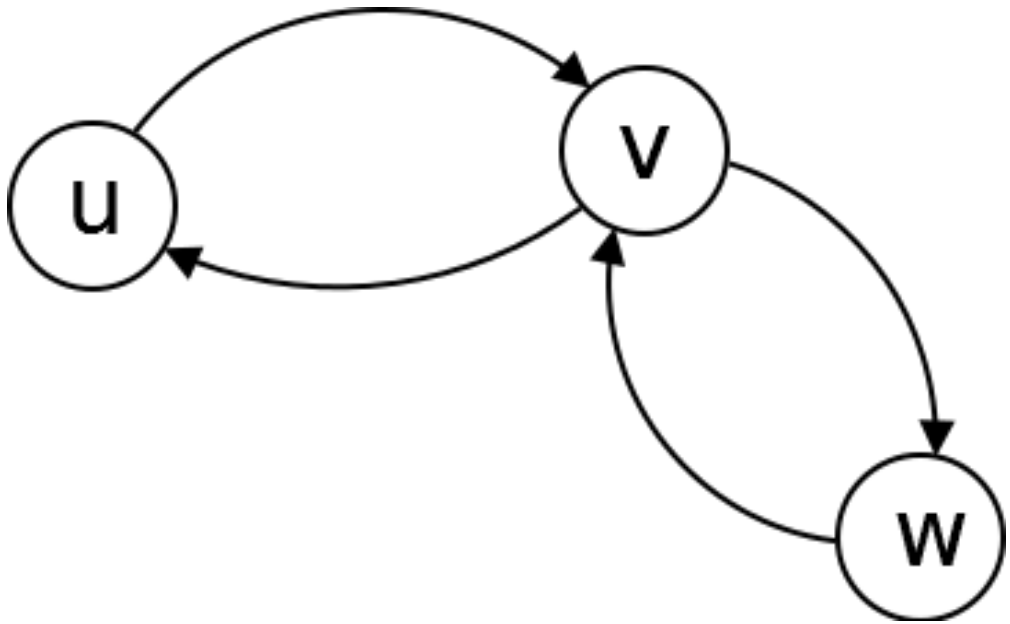
# Result

## Theorem

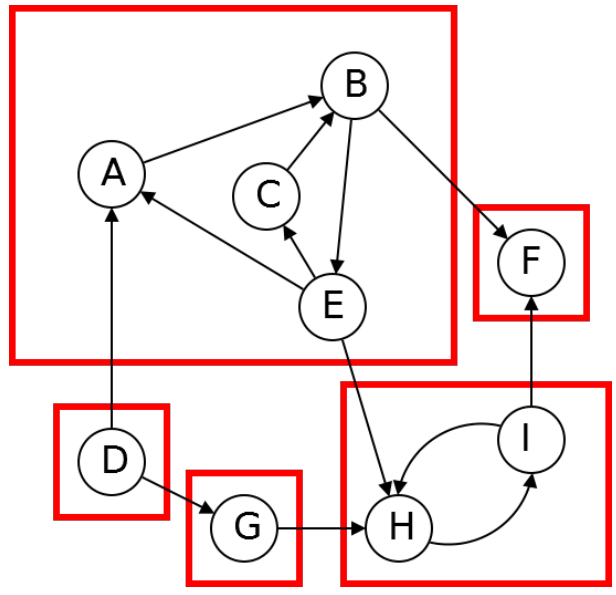
A directed graph can be partitioned into strongly connected components where two vertices are connected if and only if they are in the same component.

# Proof

Need to show an equivalence relation.

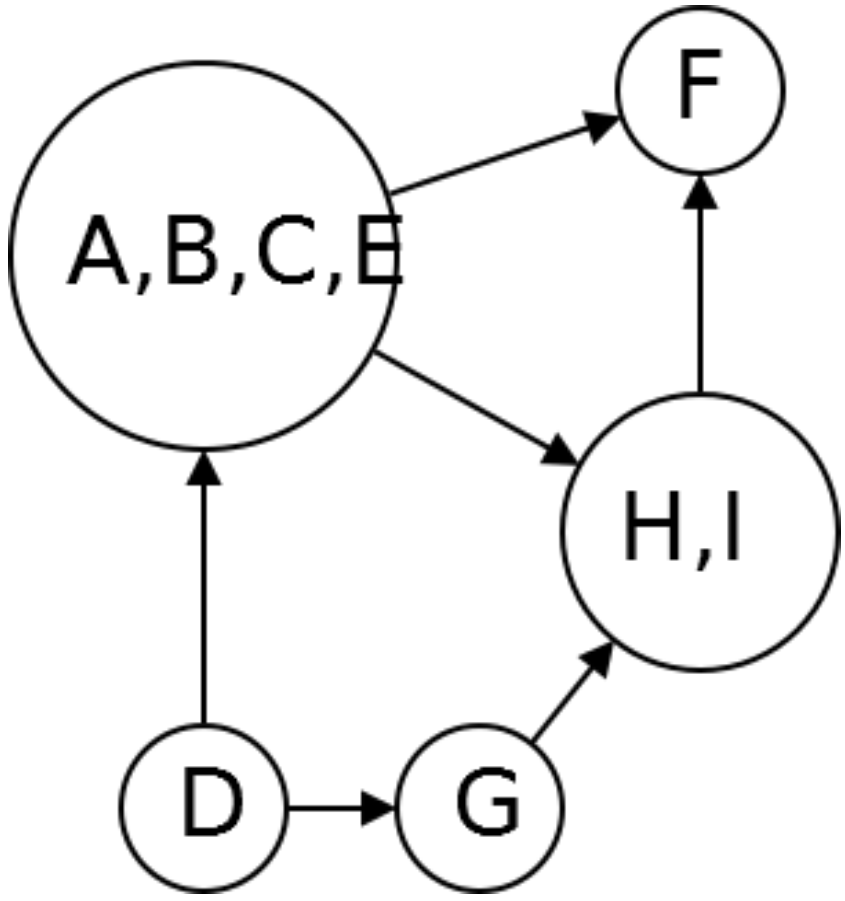


# Metagraph




We can also draw a metagraph showing how the strongly connected components connect to one another.

# Example




Note: It's a DAG.

# DAG

## Theorem

The metagraph of a graph $G$ is always a DAG.

# Proof

**Proof.**

Suppose not. Must be a cycle $\mathcal{C}$. Any nodes in cycle can reach any others. Should all be in same SCC. Contradiction. $\square$

# Summary

- Can partition vertices into strongly connected components.
- Metagraph describes how strongly connected components connect to each other.
- Metagraph always a DAG.

# Next Time

How to compute the strongly connected components of a graph.