

# Stream ciphers (XOR ciphers)

$$X = \{0, \dots, 255\}, \text{ OTP}$$

$$m \in X^n$$

$$E_k(m) = c := m \oplus k$$

$$k \in_R X^n$$

$$D_k(c) = c \oplus k = E_k(c)$$

$\oplus$ XOR	0	1
0	0	1
1	1	0

$$\begin{aligned} c \oplus k &= (m \oplus k) \oplus k \\ &= m \oplus (k \oplus k) \\ &= m \oplus 0 = m \end{aligned}$$

$$\begin{aligned} a \oplus b &= a + b \pmod{2} \\ (a \otimes b) &= a \cdot b \pmod{2} \end{aligned}$$

$$1^\circ a \otimes a = 0$$

$$2^\circ 0 \otimes 0 = a$$

$$3^\circ a \oplus b = b \oplus a$$

$$4^\circ (a \oplus b) \oplus c = a \oplus (b \oplus c)$$

Issue 1<sup>o</sup>: key reuse

$$\begin{aligned} m_1, m_2, \quad c_1 &= m_1 \oplus k \\ c_2 &= m_2 \oplus k \end{aligned}$$

$$\begin{aligned} c_1 \oplus c_2 &= (m_1 \oplus k) \oplus (m_2 \oplus k) \\ &= m_1 \oplus (k \oplus k) \oplus m_2 \\ &= m_1 \oplus 0 \oplus m_2 \\ &= m_1 \oplus m_2 \end{aligned}$$

Key retrieval:

$$m \oplus (c_1 \oplus c_2) = (m_1 \oplus m_2) \oplus m_2 = m_1$$

$$c = m \oplus k$$

$$m \oplus c = (m \oplus m) \oplus k = 0 \oplus k = k$$

$$m \oplus c = (m \oplus m) \oplus k$$

Bit flip attack

Attack in the middle

$$m \oplus k = c \longrightarrow m', (m, c) \longrightarrow c'$$

$$c' = c \oplus m \oplus m'$$

				1	1	0	
m	1	0	1	1	0	0	1
k	1	0	1	0	1	1	1
							⊕
c	0	0	0	1	1	1	0
							⊕
c'	0	0	0	0	0	0	1
							⊕
c''	0	0	0	1	1	1	1
							⊕
k	1	0	1	0	1	1	1
							⊕
m'	1	0	1	1	1	1	0

XOR ciphers are inspired by OTP  
but use prng with seed as key

Solution for key reuse

IV: Initialization vector (public)

K: private key for encrypt

For  $E(c)$   $k = f(IV, K)$

$(= E_k(m))$   
(r, t, ...)

$$(\mathcal{C}, \perp) \xrightarrow{(\mathcal{C}, \perp) \mapsto \mathcal{C}} (\mathcal{C}, \perp)$$