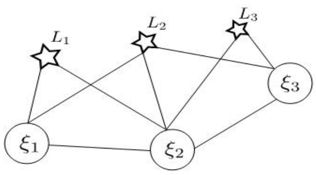
Course4_homework_Information_Matrix

0. Summary

1. Information matrix 绘制

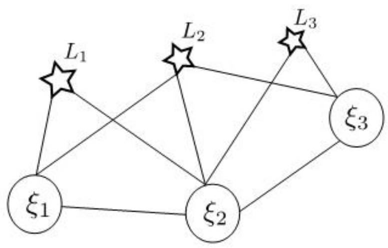
❶ 某时刻，SLAM 系统中相机和路标点的观测关系如下图所示，其中 ξ 表示相机姿态，L 表示观测到的路标点。当路标点 L 表示在世界坐标系下时，第 k 个路标被第 i 时刻的相机观测到，重投影误差为 $\mathbf{r}(\xi_i, L_k)$。另外，相邻相机之间存在运动约束，如 IMU 或者轮速计等约束。



1 请绘制上述系统的信息矩阵 $\mathbf{\Lambda}$.
2 请绘制相机 ξ_1 被 marg 以后的信息矩阵 $\mathbf{\Lambda}'$.

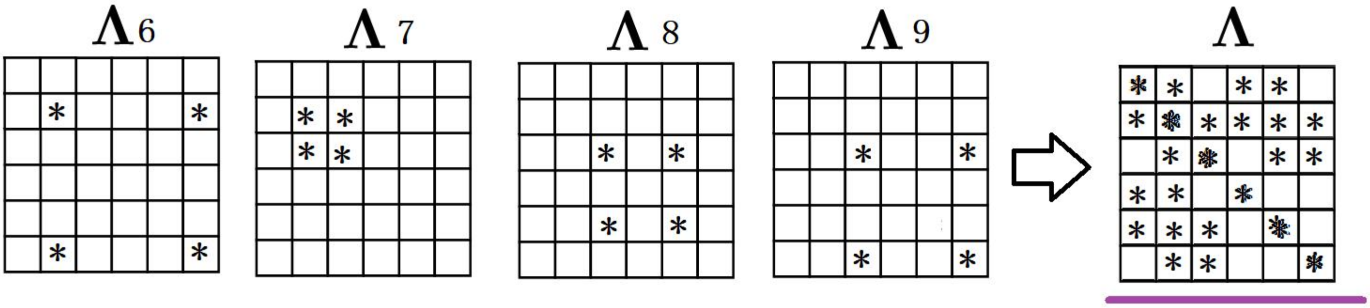
1.1

$$\boldsymbol{\xi} = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ L_1 \\ L_2 \\ L_3 \end{bmatrix}, \quad \mathbf{r} = \begin{bmatrix} \mathbf{r}_{\xi_1 L_1} \\ \mathbf{r}_{\xi_1 L_2} \\ \mathbf{r}_{\xi_1 \xi_2} \\ \mathbf{r}_{\xi_2 L_1} \\ \mathbf{r}_{\xi_2 L_2} \\ \mathbf{r}_{\xi_2 L_3} \\ \mathbf{r}_{\xi_2 \xi_3} \\ \mathbf{r}_{\xi_3 L_2} \\ \mathbf{r}_{\xi_3 L_3} \end{bmatrix} \begin{array}{l} \mathrm{J1} \\ \mathrm{J2} \\ \mathrm{J3} \\ \mathrm{J4} \\ \mathrm{J5} \\ \mathrm{J6} \\ \mathrm{J7} \\ \mathrm{J8} \\ \mathrm{J9} \end{array}$$

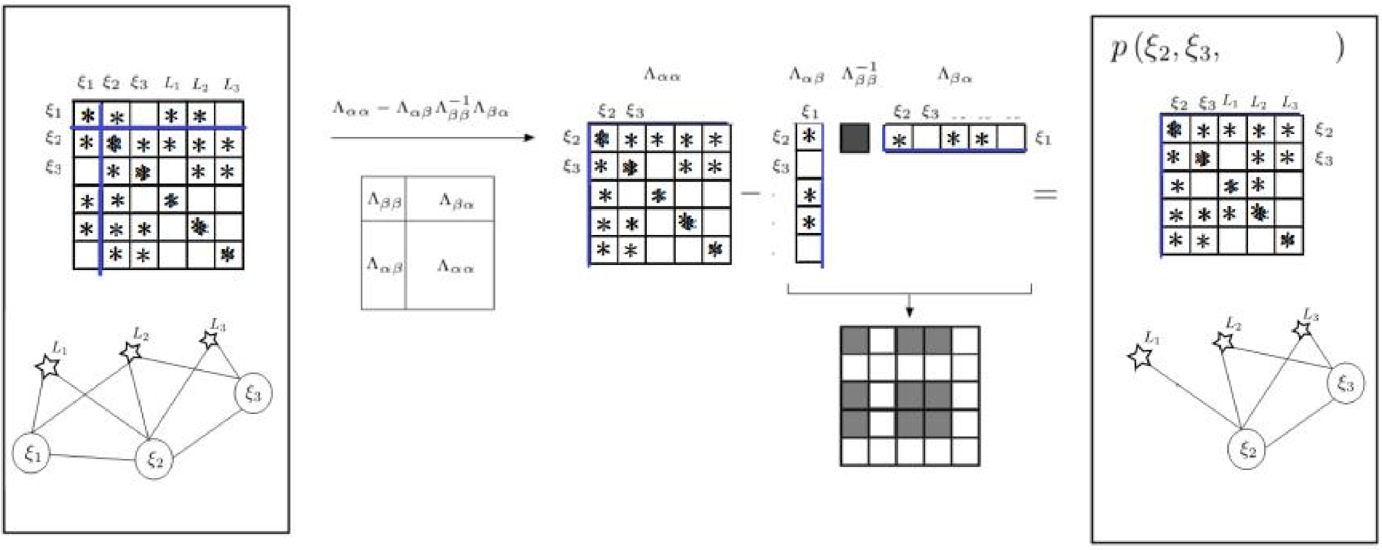


$$\mathbf{\Lambda}1 = \begin{array}{|c|c|c|c|c|c|}\hline * & & & * & & \\ \hline & & & & & \\ \hline & & & & & \\ \hline * & & & * & & \\ \hline & & & & & \\ \hline & & & & & \\ \hline \end{array} \quad \mathbf{\Lambda}2 = \begin{array}{|c|c|c|c|c|c|}\hline * & & & & * & \\ \hline & & & & & \\ \hline & & & & & \\ \hline & & & & & \\ \hline * & & & & * & \\ \hline & & & & & \\ \hline \end{array} \quad \mathbf{\Lambda}3 = \begin{array}{|c|c|c|c|c|c|}\hline * & * & & & & \\ \hline * & * & & & & \\ \hline & & & & & \\ \hline & & & & & \\ \hline & & & & & \\ \hline & & & & & \\ \hline \end{array}$$

$$\mathbf{\Lambda}4 = \begin{array}{|c|c|c|c|c|c|}\hline & & & & & \\ \hline & * & & * & & \\ \hline & & & & & \\ \hline & * & & * & & \\ \hline & & & & & \\ \hline & & & & & \\ \hline \end{array} \quad \mathbf{\Lambda}5 = \begin{array}{|c|c|c|c|c|c|}\hline & & & & & \\ \hline & * & & & * & \\ \hline & & & & & \\ \hline & & & & & \\ \hline & * & & & * & \\ \hline & & & & & \\ \hline \end{array}$$



1.2



2. Information matrix 推导

❷ 阅读《Relationship between the Hessian and Covariance Matrix for Gaussian Random Variables》. 证明信息矩阵和协方差的逆之间的关系。

论文中考虑 gaussian random，并将 pdf 作为优化目标，则得出 Hessian 矩阵=信息矩阵(i.e.协方差矩阵的逆)；
论文中指出，得到信息矩阵的原因是计算二阶导数矩阵时将其他变量视为固定值；
对于 gaussian 分布，等式严格成立；对于 Gamma Random Variable，等式近似成立；

Consider a Gaussian random vector $\boldsymbol{\theta}$ with mean $\boldsymbol{\theta}^\star$ and covariance matrix $\boldsymbol{\Sigma_\theta}$ so its joint probability density function (PDF) is given by:

$$p(\boldsymbol{\theta}) = (2\pi)^{-\frac{N_\theta}{2}} |\boldsymbol{\Sigma_\theta}|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(\boldsymbol{\theta} - \boldsymbol{\theta}^\star)^T \boldsymbol{\Sigma_\theta}^{-1}(\boldsymbol{\theta} - \boldsymbol{\theta}^\star)\right] \tag{A.1}$$

The objective function can be defined as its negative logarithm:

$$J(\boldsymbol{\theta}) \equiv -\ln p(\boldsymbol{\theta}) = \frac{N_\theta}{2} \ln 2\pi + \frac{1}{2} \ln |\boldsymbol{\Sigma_\theta}| + \frac{1}{2}(\boldsymbol{\theta} - \boldsymbol{\theta}^\star)^T \boldsymbol{\Sigma_\theta}^{-1}(\boldsymbol{\theta} - \boldsymbol{\theta}^\star) \tag{A.2}$$

which is a quadratic function of the components in $\boldsymbol{\theta}$. By taking partial differentiations with respect to θ_l and $\theta_{l'}$, the (l, l') component of the Hessian matrix can be obtained:

$$\mathcal{H}^{(l,l')}(\boldsymbol{\theta}^\star) = \left.\frac{\partial^2 J(\boldsymbol{\theta})}{\partial\theta_l \partial\theta_{l'}}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}^\star} = (\boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{-1})^{(l,l')} \tag{A.3}$$

so the Hessian matrix is equal to the inverse of the covariance matrix:

$$\mathcal{H}(\boldsymbol{\theta}^\star) = \boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{-1} \tag{A.4}$$

For Gaussian random variables, the second derivatives of the objective function are constant for all $\boldsymbol{\theta}$ because the objective function is a quadratic function of $\boldsymbol{\theta}$. Therefore, the Hessian matrix can be computed without obtaining the mean vector $\boldsymbol{\theta}^\star$.

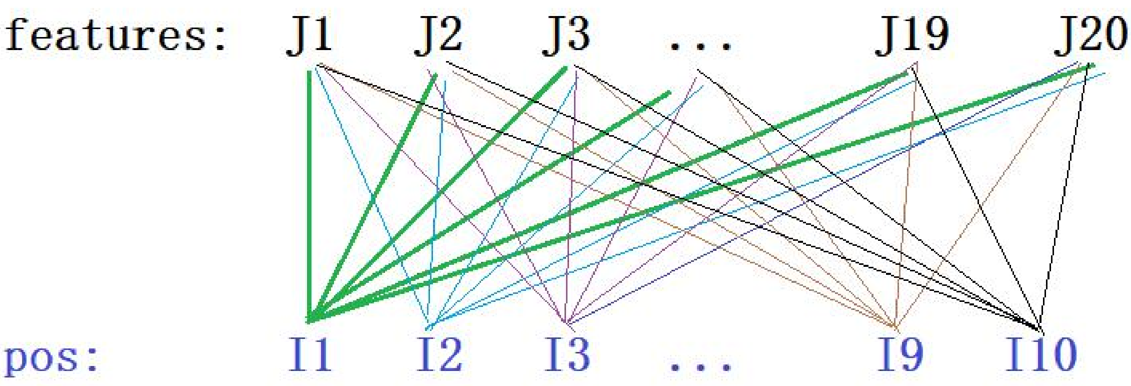
The elements in the Hessian matrix carry the conditional information of the random vector because they are obtained by fixing all other parameters. The diagonal elements are the curvature of the objective function in the corresponding direction. The reciprocals of these diagonal elements are the conditional variances of the uncertain parameters in $\boldsymbol{\theta}$. However, the diagonal elements in the covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$ are the marginal variances of the parameters.

3.code，奇异值；

❸ 请补充作业代码中单目 Bundle Adjustment 信息矩阵的计算，并输出正确的结果。正确的结果为：奇异值最后 7 维接近于 0，表明零空间的维度为 7.

course4-homework

```
int main()
{
    int featureNums = 20;
    int poseNums = 10;
    int diem = poseNums * 6 + featureNums * 3;
    double fx = 1.;
    double fy = 1.;
    Eigen::MatrixXd H(diem,diem);
    H.setZero();
```



Code 如下：

```
hessian_nullspace_test.cpp
                0, fy/z, -y * fy/z_2;
            Eigen::Matrix<double,2,3> jacobian_Pj = jacobian_uv_Pc * Rcw;
            Eigen::Matrix<double,2,6> jacobian_Ti;
            jacobian_Ti << -x* y * fx/z_2, (1+ x*x/z_2)*fx, -y/z*fx, fx/z, 0 , -x * fx/z_2,
                            -(1+y*y/z_2)*fy, x*y/z_2 * fy, x/z * fy, 0,fy/z, -y * fy/z_2;

            H.block( startRow: i*6, startCol: i*6, blockRows: 6, blockCols: 6) += jacobian_Ti.transpose() * jacobian_Ti;
            /// 请补充完整作业信息矩阵块的计算
            H.block( startRow: j*3 + 6*poseNums, startCol: j*3 + 6*poseNums, blockRows: 3, blockCols: 3) += jacobian_Pj.transpose()*jacobian_Pj;
            H.block( startRow: i*6, startCol: j*3 + 6*poseNums,  blockRows: 6, blockCols: 3) += jacobian_Ti.transpose() *jacobian_Pj;
            H.block( startRow: j*3 + 6*poseNums, startCol: i*6 ,  blockRows: 3, blockCols: 6) += jacobian_Pj.transpose() * jacobian_Ti;
        }
    }

//    std::cout << H << std::endl;
//    Eigen::SelfAdjointEigenSolver<Eigen::MatrixXd> saes(H);
//    std::cout << saes.eigenvalues() <<std::endl;
main
```

```
2019.3.4/bin/cmake/linux/bin/cmake --build /media/sf_course4/nullspace_test/cmake-build-debug --target NullSpaceTest -- -j 1
cies of target NullSpaceTest
XX object CMakeFiles/NullSpaceTest.dir/hessian_nullspace_test.cpp.o
X executable NullSpaceTest
et NullSpaceTest
```

运行结果如下：

```
ep@ep-VirtualBox: ~
  0.00634341
  0.00608493
  0.00547299
   0.0053236
  0.00520788
  0.00502341
   0.0048434
  0.00451083
   0.0042627
  0.00386223
  0.00351651
  0.00302963
  0.00253459
  0.00230246
  0.00172459
0.000422374
3.21708e-17
2.06732e-17
1.43188e-17
7.66992e-18
6.08423e-18
6.05715e-18
3.94363e-18
(base) ep@ep-VirtualBox:/media/sf_course4/nullspace_test$
```

说明：

实际计算还需要残差的信息矩阵，此处暂时设置为单位矩阵

以下来自课程 ppt：

$$\sum_{i=1}^{5} \mathbf{J}_i^\top \mathbf{\Sigma}_i^{-1} \mathbf{J}_i \delta \boldsymbol{\xi} = -\sum_{i=1}^{5} \mathbf{J}_i^\top \mathbf{\Sigma}_i^{-1} \mathbf{r}_i \tag{45}$$